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SYNOPSIS This paper presents results of a numerical study on the effectiveness of open trenches in reducing the ground vibration caused by propagating Rayleigh waves. The analysis is performed in the frequency domain and under conditions of plane strain using the Boundary Element Method. The soil is modelled as a homogeneous isotropic linear elastic half space. In order to represent the screening effect due to the installation of the barrier, the index ER is introduced. ER is defined as the ratio of the rate of energy transmission over the area to be protected in the presence of the trench and that without the trench. By means of ER, the influence of several parameters on the screening efficiency of the trench is investigated in some detail.

INTRODUCTION

In the past few decades many investigations, both experimental and numerical, have been conducted to determine the effectiveness of trenches and sheet-wall barriers as vibration isolators of structures or machine foundations and to develop guidelines for the design of these barriers. Barkan (1962) first reported on an application of an open trench and sheet-wall barrier to isolate a building from traffic-induced vibrations. Woods (1968) performed a series of field experiments on *active isolation* (isolation at the source) and *passive isolation* (screening at a distance from the source) achieved by trenches. Model tests were carried out by Woods et al. (1974) and Liao and Sangrey (1978).

Besides these experimental studies, a number of researches have employed numerical methods to analyze vibration screening efficiency of open and solid barriers. A comparison between the results of field tests and theoretical calculations may be found in Haupt (1981).

Fuyuki and Matsumoto (1980), using a finite difference scheme, investigated Rayleigh wave scattering by a rectangular open trench. The Finite Element Method (FEM) in the frequency domain has been widely used. Lysmer and Waas (1972) assessed modifications in seismic surface motion caused by a trench in an elastic layer resting on a rigid base. Haupt (1977) conducted an extensive study on the dynamic behaviour of in-filled trenches of various shapes and sizes in an elastic half plane. Segol et al. (1978) performed a plane strain analysis on the reduction of ground vibration by trenches in a layered soil. A time domain Finite Element Method was used by May and Bolt (1982) to examine the effect of placing barriers in the travel path of P, SV and SH waves.

More recently, the Boundary Element Method (BEM) has been extensively applied to study vibration isolation problems. As known, BEM is very well suited for analysing wave propagation in soils involving infinite domain because the radiation condition at the far field is automatically satisfied. Furthermore, for linear problems only the boundary of the domain needs to be discretized. Emad and Manolis (1985) and Beskos et al. (1986) employed BEM in the frequency domain for studying some vibration screening problems using trenches, under plane strain conditions. Ahmad and Al-Hussaini (1991), and Al-Hussaini and Ahmad (1991) performed a detailed parametric study in order to assess the influence of various parameters on the screening of vertical and horizontal ground vibrations by rectangular wave barriers in homogeneous and layered soils. Dasgupta et al. (1988) and Banerjee at al. (1988) extended the methodology to three-dimensional problems.

In this paper, the Boundary Element Method is used to analyze Rayleigh wave scattering by open trenches under conditions of plane strain. The soil medium is assumed to be a homogeneous isotropic linearly elastic half space. Usually, this assumption is considered suited for solving many geotechnical problems involving overconsolidated cohesive soils. Displacements and stresses are calculated over an area beyond the barrier, where significant attenuation effects take place. In order to represent ground vibration reduction due to the installation of the trench an index (ER) is introduced. This index is defined by the ratio of the rate of energy transmission relative to the screening zone in the presence of the trench and that without the trench. By means of ER, the influence of several parameters, such as the geometrical shape and dimensions of the trench, on the screening effect of Rayleigh waves is investigated in some detail. Although the results obtained in this study are relative to idealized conditions, the conclusions drawn may prove useful in the design of open trenches as vibration isolators.

NUMERICAL METHOD

In steady state dynamic conditions, where the excitation and response are both time-harmonic, the equation of motion of a linear isotropic elastic medium can be expressed in terms of displacement amplitudes u_i, as

$$V_S^2 u_{i,jj} + (V_P^2 - V_S^2) u_{j,ji} + w^2 u_i = 0 \qquad (1)$$

where V_S=velocity of shear wave, V_P=velocity of compression wave, and ω=circular frequency. In Eq.(1) zero body forces are assumed. The variation with time is obtained multiplying u_i by the factor $exp(i\omega t)$, being $i = \sqrt{-1}$.

The basic idea of the Boundary Element Method is to transform the partial differential equation (1) into a boundary integral equation. The frequency domain boundary integral equation for a plane linear elastic body with boundary S, is (KOBAYASHI and NISHIMURA, 1982; BREBBIA and DOMINGUEZ, 1989):

$$c_{ij}(\zeta)\cdot u_i(\zeta,\omega) = \int_S \left[U^*_{ij}(x,\zeta,\omega)\cdot t_i(x,\omega) - T^*_{ij}(x,\zeta,\omega)\cdot u_i(x,\omega)\right] dS \qquad (2)$$

where u_i and t_i are the displacements and the tractions at the boundary S; U^*_{ij} and T^*_{ij} denote the components of the displacement and traction vectors, respectively, at a point x in direction i due to unit harmonic force applied at point ζ in direction j (fundamental solutions); $c_{ij}=\delta_{ij}$ for ζ inside the domain, and $c_{ij}=1/2\delta_{ij}$ for a smooth boundary point; δ_{ij}=Kronecker's symbol.

For a region with arbitrary geometry and boundary conditions it is very difficult to integrate Eq.2 analytically. However, a discretization of S into a finite number of boundary elements makes it possible to integrate the integral equation numerically. Although the use of higher-order boundary elements would be better, simple constant elements are used in this study. This considerably simplifies the calculation of the displacements and stresses at the boundary S. With the aforesaid approximation, Eq.2 can be formulated as the complex matrix equation

$$\mathbf{T}\,\mathbf{u} = \mathbf{U}\,\mathbf{t} \qquad (3)$$

being **u** and **t** displacement and traction vectors at the boundary S, while the matrices **U** and **T** result from the integration of the fundamental solutions over the boundary elements. In Eq.(3) the unknown is represented by the displacement vector **u**.

If an incident wave excitation problem is to be analyzed, some explanations need to be added. In this case, we can write the total displacement field as

$$\mathbf{u}=\mathbf{u}_O+\mathbf{u}_D \qquad (4)$$

where $\mathbf{u}_O$ is the displacement vector of the free field solution (incident and reflected displacements), and $\mathbf{u}_D$ represents the scattered displacement vector which must be considered to account for the presence of the trench. The free field solution can be readily determined from relevant expressions (MIKLOWITZ, 1978), hence the only unknown of the problem is the scattered field. Since this latter satisfies the equation of motion, Eq.3 can be solved in terms of the scattered displacements, imposing zero total tractions at the boundary. The total displacement field is then calculated by composing the free field and scattered solutions. Comparisons with existing wave propagation results obtained using other numerical methods have shown that BEM is reliable and gives accurate results (CONTE, 1990).

Once the boundary solution is obtained, Eq.2 can be also used to find displacements of any point within the domain. The interior stresses can be obtained using the strain displacement and stress strain relations (BREBBIA and DOMINGUEZ, 1989; CONTE et al., 1993). For calculating the stresses on the boundary it is convenient, on the contrary, to employ a different procedure (MANOLIS and BESKOS, 1988) that consists of combining the constitutive law

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) \qquad (5)$$

where λ and μ are the elastic material constants; and the boundary conditions

$$\begin{aligned} t_i &= \sigma_{ij} n_j \\ u_{i,p} &= u_{i,j} p_j \end{aligned} \qquad (6)$$

in matrix form as

$$\mathbf{A}\,\mathbf{B} = \mathbf{C} \qquad \mathbf{(7)}$$

where **A** is a 7x7 matrix contains the material constants and unit vectors **n** and **p**; **C** contains the tractions t_i and the directional derivates of the displacements $u_{i,p}$; and **B** is the unknown vector of σ_{ij} and spatial derivatives of the displacements $u_{i,j}$.

STATEMENT OF PROBLEM

Earthquakes, machine foundations, traffic, pile driving or blasting generate seismic waves, which propagating in soil from the source, transmit energy to other regions. For the case of passive isolation, when the barrier is located in an area remote from the source of vibrations, the major portion of the energy is carried out by Rayleigh waves that propagate in a restricted zone close to the surface of the ground.

Figure 1 shows the horizontal and vertical displacement amplitudes of the Rayleigh wave in a half plane (free field condition) versus the scaled depth y/L_R, being L_R the Rayleigh wavelength. The vibration amplitudes are nondimensionalized by the horizontal displacement amplitude U_{xo} at the ground surface. In figure 2 the variation with depth of the amplitudes of the stresses σ_x σ_y and τ, normalized by the shear modulus μ, is shown for $U_{xo}=L_R$. As can be seen (Fig.1) the horizontal displacement decays rapidly with depth, while the vertical component at first increases somewhat with depth and then decays rapidly. Furthermore, it is interesting to note that the Rayleigh wave leads to a normal stress amplitude σ_x which is not zero at the free surface (Fig.2). The amplitude of σ_x is significant and, as pointed out by Wolf (1985), should be taken into account in the seismic design of earth retaining structures. At a depth greater than the wavelength, both displacement and stress amplitudes are small. Therefore, ground vibrations caused by Rayleigh waves can be effectively reduced by placing a suitable wave barrier in front of the area to be protected (Fig.3). The barrier will reduce vibration effects by reflection, scattering and diffraction of wave energy. Wave barriers may consist, in general, of a solid, fluid or void zone of small depth in the ground. The most effective barriers would be, of course, the open trenches which are those that transmit the minimum wave energy.

The amplitude of the surface displacements and stresses induced by incident Rayleigh waves at an open trench of dimensions $D=L_R$ and $W=0.4L_R$ are shown in figures 4 and 5, respectively. As said earlier, surface displacements at each point are normalized by the horizontal displacement amplitude of the "free field" at the same point. The results indicate that significant amplification effects occur in front of the trench, if compared with the case in which the trench is not present (Figs.1 and 2). However, surface displacements exhibit more pronounced modifications than the stress σ_x (σ_y and τ are zero at the horizontal free surface). The screening effect involves

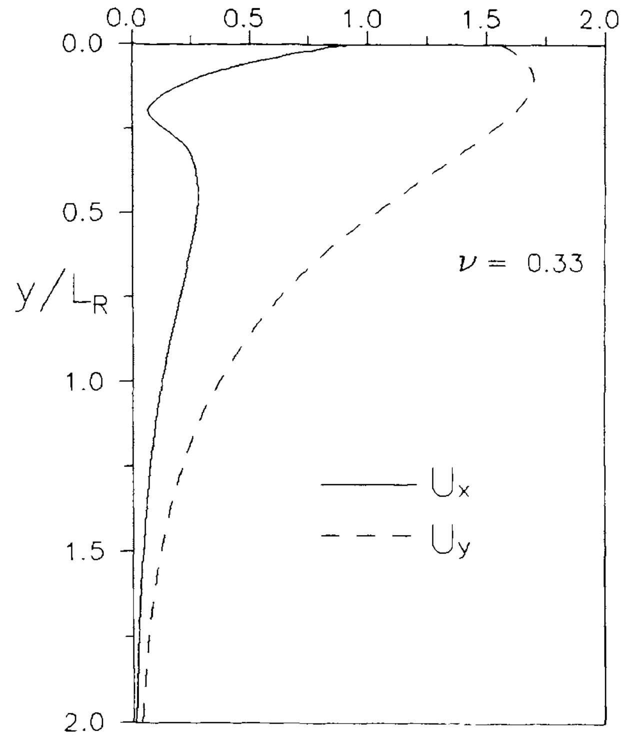

Fig.1 Displacement amplitudes versus depth for Rayleigh wave in a half plane

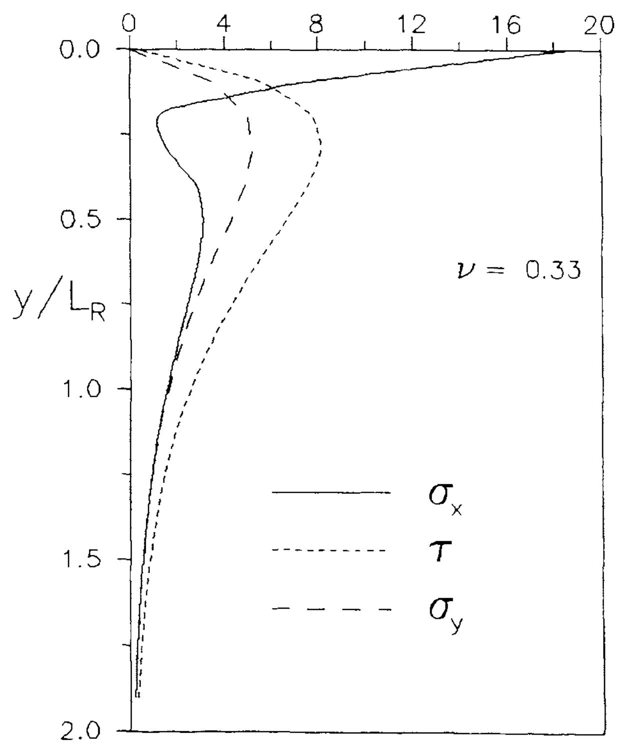

Fig.2 Stress amplitudes versus depth for Rayleigh wave in a half plane

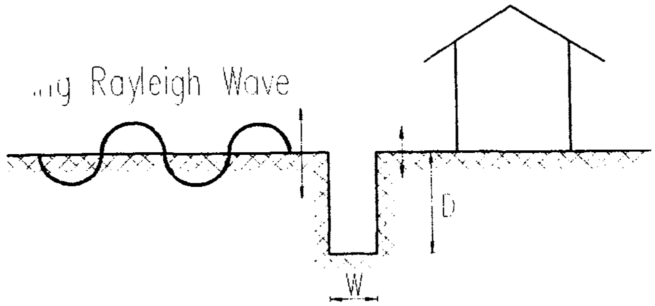

Fig.3 Schematic diagram of screening of Rayleigh wave by an open trench

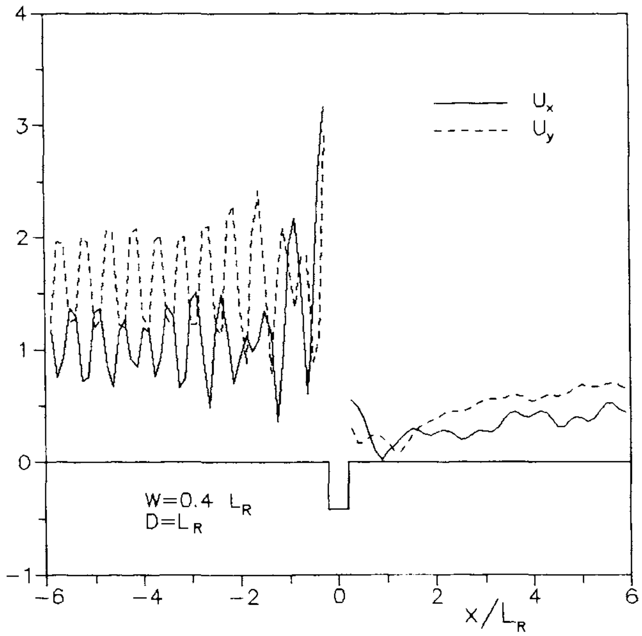

Fig.4 Surface displacement amplitudes induced by incident Rayleigh waves

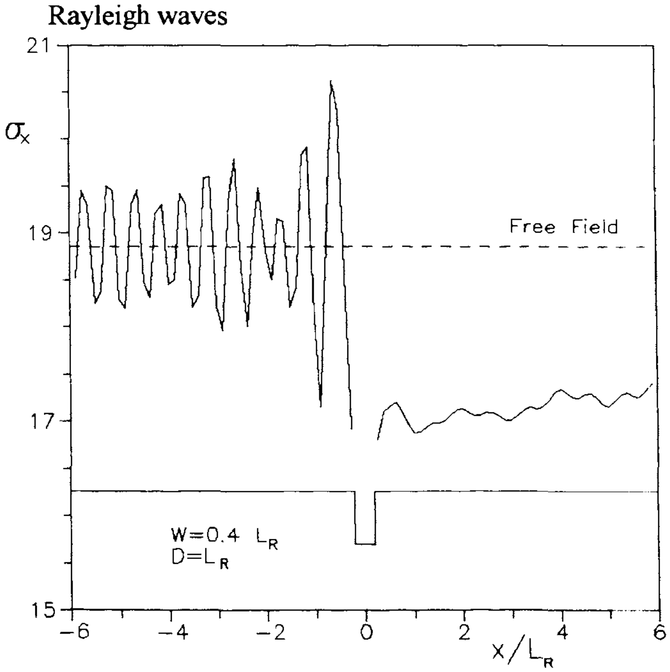

Fig.5 Surface stress amplitude induced by incident Rayleigh waves

an area just beyond the barrier, where displacement and stress amplitudes are significantly reduced with regard to those of the free field condition. Furthermore, as can be noted, σ_x is again not zero at the ground surface (Fig.5).

Consequently, if we consider a Rayleigh wave of finite beam thickness Δ close to the ground surface, the rate of energy transmission at a given x can be calculated by the product of the real part of the force $\sigma_x\Delta\ exp(i\omega t)$ and of velocity $\dot{U}_x\ exp(i\omega t)$, averaged over a period $T=2\pi/\omega$ (the other forces are about zero approaching at the ground surface, and hence their work can be ignored). Integrating over the area to be protected extending to a distance L beyond the trench, results (WOLF,1985)

$$E=-\frac{\Delta}{2}\int_0^L\left(\mathrm{Re}\left[\sigma_X\right]\cdot\mathrm{Re}\left[\dot{U}_X\right]+\mathrm{Im}\left[\sigma_X\right]\cdot\mathrm{Im}\left[\dot{U}_X\right]\right)dx \qquad (8)$$

where $\dot{U}_x=i\omega U_x$. In practice, E represents, the amount of energy propagating in the horizontal direction integrated over an idealized beam close to the free surface, and averaged over a period T. In order to express the screening effect produced by the trench, the parameter ER is then introduced. ER is defined as the ratio of energy E computed in the presence of the trench and that without the trench. As an example, ER=0.40 means 60% reduction in energy transmission due to the installation of the barrier.

APPLICATIONS

The numerical method described in the previous sections is employed here to investigate the influence of same parameters upon the screening effect of an open trench. It premises that all the dimensions of the problem are normalized with respect to the Rayleigh wavelength. This enables the effect of the frequency to be taken directly into account in the analysis.

The first application concerns the extent of the reduction zone beyond the barrier. Figure 6 shows the energy ratio versus the normalized distance L/L_R for two trenches with depth $D=0.5L_R$ and $D=L_R$, respectively. Both trenches have width $W=0.4L_R$. It is apparent from the figure that, in the $D=0.5L_R$ case, ER increases somewhat with L but after a distance of about $5L_R$ reaches a fairly constant value. When $D=L_R$, on the contrary, ER increases almost linearly with L. It was found that the effect of L on ER is different as the dimensions and geometrical shape of the trench change. Therefore, in all our applications it was decided to calculate

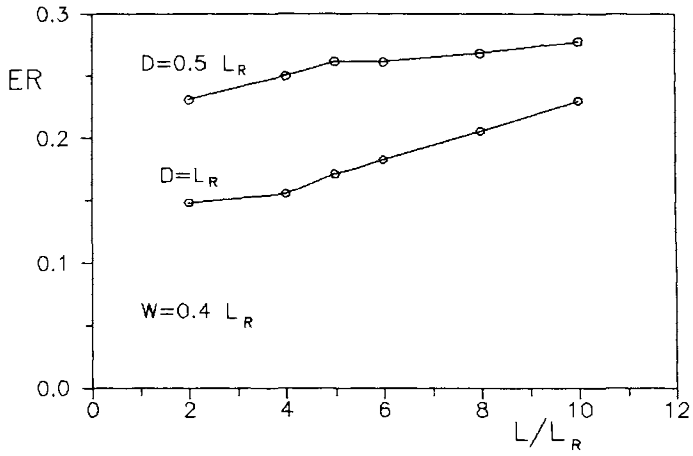

Fig.6 Effect of L on ER

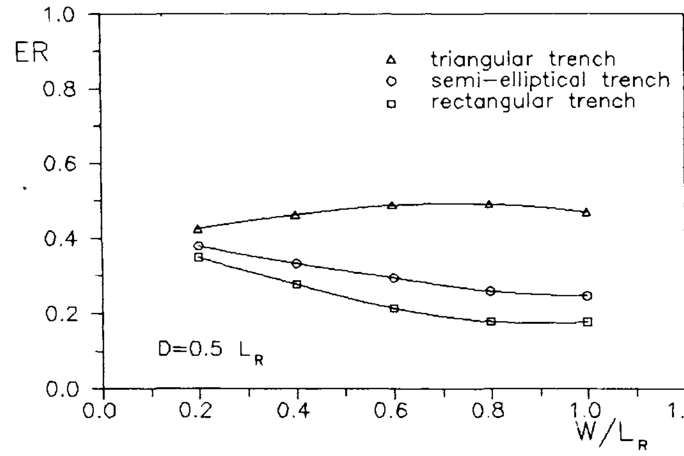

Fig.8 ER versus the normalized trench width

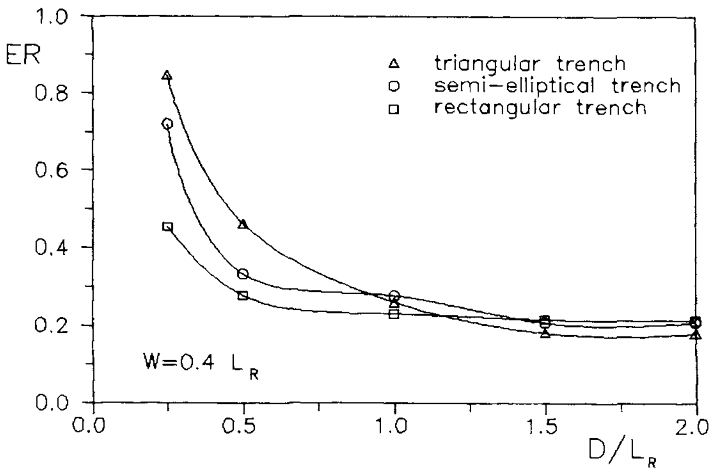

Fig.7 ER versus the normalized trench depth

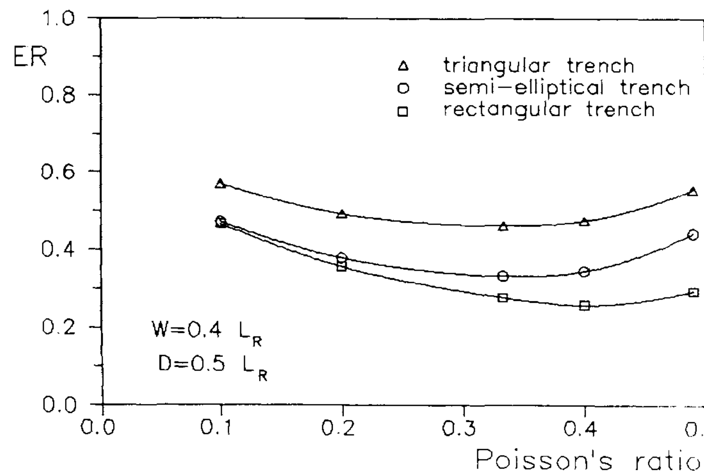

Fig.9 Effect of Poisson's ratio of soil on ER

the energy E (Eq.8) over an area extending to a distance of $10L_R$, beyond the barrier.

Figures 7 to 8 report the results obtained by varying the dimensions and the geometrical shape of the trench. Trenches with rectangular, triangular and semi-elliptical cross-section were considered. The results are given for a Poisson's ratio ν=0.33. The influence of both D and the geometrical shape on the screening effectiveness of the trench can be deduced from figure 7. As expected, increasing depth is in general beneficial. In fact, ER values decrease from about 0.85 to 0.20 as D increases. However, for $D>L_R$ an increase in depth does not produce significant improvements in screening efficiency. This is why the incident Rayleigh wave energy decays with depth, and hence reflection and scattering of seismic waves do not cause further modifications in the response of a deep trench. Furthermore, when $D>L_R$, the rate of energy transmission is also independent of the geometrical shape of the trench. On the contrary, when the trench is shallow ($D<0.6L_R$) the index ER strongly depends on the depth D. In this case, the geometrical shape of the trench markedly affects the screening properties of the trench, and the rectangular trench is the most efficient barrier in reducing seismic effects.

The effect of the trench width is documented in figure 8. The results are relative to a shallow trench, since in this case a significant amount of Rayleigh wave energy is allowed to pass below the barrier. Although a slight dependence of W can be noted in the ER values when the trench is narrow ($W<0.5L_R$), from the figure it appears that the width W is not a very important parameter, if compared with D. In fact, an increase in W has only a small influence upon the screening efficiency of the trench. This means that, for practical purposes, the influence of W can be ignored.

Finally, the index ER for different values of the Poisson's ratio of soil is calculated (Fig.9). As can be seen, ER in general decreases with ν, and hence the effectiveness of the barrier improves. However for ν close to 0.5 (incompressible material) ER somewhat increases. Furthermore, the influence exerted by the trench shape upon its effectiveness is again evident.

CONCLUDING REMARKS

The screening efficiency of open trenches under plane strain conditions is investigated by means of the index ER. This index is defined as the ratio of the rate of energy transmission over the area to be protected in the presence of the trench and that without the trench. The amplitudes of surface motion and stresses are evaluated using the Boundary Element Method. A parametric study on the main factors affecting the effectiveness of trenches is performed. On the basis of the results obtained some remarks may be made:

1. The trench depth D is a very important parameter for judging the effectiveness of a trench. Increasing D is, in general, beneficial. However, for $D>L_R$, an increase in depth does not produce significant reductions of seismic effects, and ER values are also independent of the geometrical shape of the barrier.
2. For a shallow depth ($D<0.6L_R$), a trench of rectangular cross section is more effective in reducing ground vibrations than a trench of triangular or semi-elliptical shape of the same dimensions.
3. The trench width may be regarded as a parameter of minor importance. Poisson's ratio of the soil slightly influences the results.

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