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PAPER 5.01

MODELING UNCERTAINTY IN SEISMIC STABILITY AND EARTHQUAKE INDUCED DISPLACEMENT OF EARTH SLOPES UNDER SHORT TERM CONDITIONS

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ABSTRACT

Different models were developed for evaluating the probabilistic three-dimensional (3-D) stability analysis of earth slopes and embankments under earthquake loading using both the safety factor and the displacement criteria of slope failure. These models are formulated and incorporated within a computer program (PTDDSSA). The probabilistic models evaluate the probability of failure under seismic loading considering the different sources of uncertainties involved in the problem.

A sensitivity analysis was conducted on the different parameters involved in the developed models by applying those models to a well-known landslides (Congress Street) under different levels of seismic hazard.

The hypocentral distance and earthquake magnitude were found to have major influence on the earthquake induced displacement, probability of failure (i.e. probability of allowable displacement exceedance), and dynamic 2-D and 3-D safety factors. Key Words: Reliability Models, 3-D Stability Analysis, Earth Slopes, Earthquakes.

INTRODUCTION

Different models for design and analysis of slopes and embankments under earthquake loading were developed using both the safety factor and the displacement criteria.

Both deterministic and probabilistic approaches were incorporated in the analysis. The probabilistic approach is more suitable for evaluating failure risk considering the different sources of uncertainties involved in the problem i.e. the soil strength, randomness of earthquake occurrence, etc.

Well verified deterministic models/ procedures available in the literature for evaluating/estimating the permanent displacements activated by earthquakes were used in the study. These models were selected among different models available in the literature by comparing predictions using each model to actual earthquake induced displacements of geotechnical structures for international cases of earthquakes incidents. In the developed probabilistic approach; different failure models were derived to evaluate the probability of failure under seismic loading. These were formulated and incorporated within a computer program (PTDDSSA) capable of obtaining two and three dimensional safety factors and probability of slope failure under static condition, two and three dimensional slope safety factors and probability of slope failure under dynamic condition, earthquake induced acceleration, the limiting

slope/embankment acceleration, the earthquake induced displacement, and the probability of allowable displacement exceedance (i.e. slope failure under seismic loading).

A thorough sensitivity analysis is carried out on the different parameters involved in the developed models by applying those models to a well-known landslide (Congress Street landslide).

The availability of such program is very useful in evaluating the safety and for remediation of earthquake triggered landslides of different areas, including cut slopes and earth fill

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embankments in urban areas and along major highways. Moreover, they will be very useful for land use planning and development.

PROBABILISTIC STABILITY ANALYSIS OF SLOPES

In the case of slope stability analysis, M_R = resisting moment; M_D = Driving moment. When M_R is less than M_D , a shear failure occur.

If M_D and M_R are random variables then in the twodimensional (2-D) model, the 2-D safety factor at a specific location $x = x_0$, SF(x₀), becomes

 $SF(xo) = M_R (xo)/M_D (xo)$

The probability of slope failure P_f is given by: $P_f = P_r(SF(x_0) < 1.0)$

Then reliability of the system is the probability of survival = $1-P_f$.

The reliability index, β , is a convenient measure for evaluating the safety of a slope. In terms of the mean SF and standard deviation SF of the factor of safety, the reliability index β is:

 $\beta = (SF - 1)/SF$

In order to compute the probability of failure we have to assume a probability distribution for SF. The distribution of SF depends on the joint distribution of the shear strength parameters which is generally not available. However, if for convenience we assume SF to have Gaussian distribution, then the probability of failure P_f becomes:

 $P_f = 1 - \phi(\beta)$

Where ϕ (.) is the cumulative distribution function of the standardized Gaussian distribution.

According to Yucemen and Al-Homoud (1990) for a certain soil property U, three parameters are introduced to describe the spatial variability of U; \overline{U} , \widetilde{U} and λ_u : average values, standard deviation and the scale of fluctuation respectively. The scale of fluctuation was firstly introduced by Vanmarcke (1977); it is the distance over which the soil property U shows a relatively strong correlation.

As a result of spatial averaging of soil parameters, a reduction in the standard deviation and variance occur, the reduction factor = r_u for standard deviation and r_u^2 for the variance. Vanmarcke (1977) give the methods used to obtain these values for 1-D and 2-D analysis.

Further technical details on corrective factors, scales of fluctuation and other above reported statistical parameters and their methods of estimation are given by Al-Homoud (1985), Yucemen et al. (1973) and Al-Homoud and Yucemen (1988).

PROBABILISTIC MODEL FOR 3-D SLOPE STABILITY ANALYSIS

Assumptions

- 1. Failure surface is cylindrical (Figure 1).
- 2. Location and width of sliding mass are at their critical value.
- 3. The soil properties are statistically homogenous over the soil volume.
- 4. Cross sections along axis of the slope are the same.
- 5. Uncertainty in unit weight of soil and slope geometry is negligible.

Inherent Variability of Resisting Moment M_R

The randomness of resistance moment M_R is described

by its mean μ_{M_R} , standard deviation and scale

fluctuation, λ_{M_p} .

Those statistical parameters depend on the spatial average of the shear strength properties. Spatial average of shear strength differs from point value of shear strength.

Vanmarcke (1977), suggested that the 3-D formulation will deviate from the 2-D formulation by taking into account the end effects.

DEVELOPED PROBABILISTIC MODELS FOR YIELD AND EARTHQUAKE INDUCED SEISMIC COEFFICIENTS

The probabilistic model proposed by Yucemen and Vanmarcke (1983) to simulate the threshold acceleration which produces a state of equilibrium for a potential sliding mass is based.

There are many functions and empirical formulas available in the literature for the attenuation of peak ground acceleration with distance away from the epicenter of an earthquake with a given magnitude. For simplicity and the sake of presenting the development of the procedure, Esteva (1974) simple attenuation equation is used in this study.

Moreover, according to Housner and Jennings (1982), the earthquake base acceleration record can be approximated by a sinusoidal motion.

DEVELOPED PROBABILISTIC MODELS FOR EARTHQUAKE INDUCED DISPLACEMENT BASED ON NON-EXCEEDANCE OF A LIMITED VALUE CRITERION

No Slip Criteria (Model I)

This model requires that, to be in safe condition, no slip should occur. The problem becomes such that; there will be fewer up-crossings or the occurrence of maxima become a Poisson event and the probability of at least one slip occurs = $P(\Delta_t>0)$ where

 Δ_t : total displacement According to Yucemen and Vanmarcke (1983), the probability of failure P_f equals to P_f = P(Δ_t >0) = 1-Exp[-(L-b)S Ω_m]

S: duration of earthquake.

L: total arc length

B: total slope width

 $\Omega_{\rm m}$: Number of maxima above $A_{s,b}(x_o,t)$



Model Based on Total Displacement Exceeding An Allowable Value Criteria (Model II)

The permanent displacement under earthquake loading can be estimated from methods of random vibration theory or from empirical models calibrated against available data of actual slides (e.g. Newmark 1965, Nadim and Whitman 1983, Wong 1982, etc.).

Al-Homoud and Tahtamoni (1999) compared predictions using different block-on-plane models available in the literature and actual values of earthquake induced displacements (D) for a group of international landslide

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incidents that occurred during major earthquakes worldwide. Statistical analysis of the results showed that Nadim and Whitman (1983) advancement of the Newmark block-on-a plane model is the most reasonable prediction model for estimating the permanent earthquake induced displacement (D).

Therefore the model of Nadim and Whitman (1983) is adopted in this study and is incorporated within model II.

Nadim and Whitman (1983) developed a method to evaluate the displacement of a rigid body (block) on a ground surface considering the uncertainties related to ground motion, base resistance and influence of the basic model (i.e. rigidplastic).

Using FOSM approximation, the mean (\overline{D}) and the variance (\widetilde{D}^2) of the earthquake induced displacement (D) evaluated using the above model were derived. Similarly, the mean and standard deviation for the total displacement (D') are derived.

Based on the above and assuming that $(\Delta a - d_1)$ has a Gaussian distribution, the expression for probability of failure was derived.

Probabilistic Model Based on Newmark Formula and Gaussian Distribution (Model III)

The problem was simplified using Newmark (1965) formulas for estimating (N).

As discussed previously, Nadim and Whitman (1983) model is adopted in this study for evaluating earthquake induced displacement (D).

Using FOSM approximation, the mean value of D, \overline{D} and its variance \widetilde{D}^2 were derived.

Considering a Gaussian distribution for Δ_a , the probability of failure was obtained as follows:

$$P_{f} = P(\overline{D} > \Delta_{a}) = 1 - \Phi(\frac{\Delta_{a} - \overline{D}}{\widetilde{D}})$$

PROBABILISTIC MODELS BASED ON BETA DISTRIBUTION (MODEL IV)

Limited Displacement Based Model

For engineering geomechanics, the Beta distribution have proven to be very useful in acquiring a mathematical description of the frequency of a set of measurements (Harr, 1977). The Beta distribution is extremely versatile and is capable of modeling a wide variety of distribution shapes. In addition, Beta distribution have finite maximum and minimum values which is characteristic of all geotechnical variables.

For our problem, which is evaluating the probability of

allowable displacement exceedance ($Pr(D>D_{all})$), the derivation made by the authors is as follow.

Therefore the probability of having the earthquake induced displacement less than an allowable value (D_{all}) is evaluated. Limited Factor of Safety Based Model

Beta distribution is used to obtain the probability of failure, where failure is defined as the case when the factor of safety is less than unity.

1 The Input and Output Parameters of FIDDSSA Analysis: Case: Adopted from P7D3SA a) Static (Yucemen and Al-Homoud, 1990) ntput Input Geometric Parameters Slope Most Critical Sliding Laver Soil Properties: 1. Means, C.O.V's and Surface: 1. Center of Rotation Scale of Fluctuation of the Soil Properties, 2. Radius of Rotation 3. Critical Width of 2. Average Water Table and C.O.V. of Pore Water Failure - Mean 2-D and 3-D Safety Pressure, 3. Mean Corrective Factors Factors and the Corresponding Squared C.O.V of the c.o.v's. Resisting Moment Number of Slices Probability of Slope Trial Centers Failure (Guassian and Beta Distributions).

THE PTDDSSA PROGRAM

PTDSSA (Probabilistic Three Dimensional Slope Stability Analysis) program developed by Yucemen and Al-Homoud (1990) for static conditions is advanced in this study for the dynamic case incorporating all the derivations and dynamic models developed in this study.

A new computer program named (PTDDSSA)(Probabilistic Three Dimensional Dynamic Slope Stability Analysis) is developed to carry out the computations associated with the dynamic probabilistic models presented previously. The program can analyze slopes and embankments located in multilayered deposits under short-term and long-term conditions. This program is capable of obtaining the static (same as PTDSSA) and dynamic safety factors and probabilities of failure.



The input and output parameters of PTDDSSA for static and dynamic analysis are given in Tables 1 (a) and 1(b), respectively.

The local site effect is calculated internally in models I, II, IV, whereas; in model III, the amplification factor has to be given as an input.

Upon obtaining the results of the dynamic analysis using PTDDSSA the decision maker is free to assess the slope condition/safety based on whatever criteria he/she choose as an acceptable safety level under static or dynamic conditions. This decision depends on the adopted codes of practice in each country and the importance of project under consideration (e.g. a slope of earth dam). The output values that should be considered by the decision maker are : 3-D static and dynamic safety factors, earthquake induced displacement, and probability of slope failure or allowable displacement exceedance.

Once, the decision is made regarding the stability conditions, the next step will be selection of remedial actions to be taken.

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A flow chart of PTDDSSA program is given in Figure 2.





Figure 2 Flow Chart of the Developed Frobabilistic Three Dimensional Dynamic Slope Stability Analysis (PTDDSSA) Program.

APPLICATION TO ACTUAL FAILURE LANDSLIDES

Introductory Remarks

PTDDSSA program is applied to an actual world class failure case: Congress Street open cut in Chicago that took place under short-term conditions. This landslides is selected for the purpose of this study because of its fame and the availability of the statistical data of the geotechnical parameters needed by the different models developed in this study and because it was studied in the past under static conditions (e.g. Yucemen (1973) and Al-Homoud (1985)).

A parametric analysis is carried out to study the sensitivity of the PTDDSSA output to the different input parameters under earthquake loading. This includes the input parameters that control the severity of earthquake hazard, and were incorporated in the equations of the failure models incorporated in PTDDSSA. These are: the hypocentral distance, the earthquake strong motion duration, the scales of fluctuation in distance and time, the earthquake magnitude, and the value of allowable displacement.

Also a parametric analysis is carried out to study the sensitivity of the PTDDSSA results to water table elevation, pore water pressure uncertainty, angle of friction, cohesion, inherent variability in cohesion and friction, horizontal scale of fluctuation in cohesion and friction, corrective factor for modeling error and its coefficient of variation, corrective factor for progressive failure and its coefficient of variation and corrective factors of cohesion and friction and their coefficient of variations.

Moreover, a comparison is made between the PTDDSSA results for the different models incorporating the normal (Gaussian) distribution used in evaluating the probability of slope failure, and those incorporating Beta distribution. Here, the probability of allowable displacement exceedance and the probability of failure when the safety factor is less than unity were obtained.

Congress Street Landslide (A Case of Short Term Stability Condition)

Description of Landslide

This landslide took place in 1952 in Chicago while working in the excavation of open cut for the Congress Street. The failure had a length of 60 m and a width of 250 m, and it took place in saturated glacial clay with no time for water to dissipate. Skempton and Hutchinson (1969) described the most critical failure surface. The Congress Street landslide was located in gritty blue clay, which is divided into three layers: stiff gritty due clay, medium gritty blue clay, and stiff to very stiff gritty blue clay (Figure 3). The different corrective factors to account for different sources of discrepancies between laboratory and in-situ soil properties were evaluated by Yucemen et al. (1973).

Estimation of Failure Probability Under Static Condition

Al-Homoud (1985) carried out a 3-D probabilistic analysis for this landslide. The analysis of Congress street landslide is carried out considering total stress analysis ($\varphi=0.0$) class of stability. This analysis gave the following results: 2-D safety factor is 1.059, critical failure width is 56.0 m, and probability of slope failure is 0.506. The most critical failure surface depicted from the analysis is shown in Figure 4.

Estimation of Dynamic Failure Probability Based on Allowable Displacement Exceedance

Using the developed PTDDSSA program, encoded the derivations of the different

models, a parametric analysis is carried out to study the sensitivity of the results to the different input parameters (e.g. parameters that controls the severity of earthquake hazard). The limiting acceleration is found to be 0.193 g.

DISCUSSION

Comparison between Normal (Gaussian) Distribution and Beta Distribution in Evaluating the Probability of Allowable Displacement Exceedance (P.D.E)

Using PTDDSSA program, a comparison is made between the probability of failure or (P.D.E.) obtained for Gaussian and Beta distributions. The hypocentral distance is set to be variable and the P.D.E. is obtained for a set of values of hypocentral distance as shown in Figure 5. As stated previously the Gaussian distribution gives here also higher values of P.D.E compared to those obtained using the Beta distribution.



Figure 3 Approximate Cross-Section of the Cut and the Approximate Position of the Actual Slip Surface at Congress Street Open Cut (Al-Homoud, 1985).





Moreover, by varying the earthquake strong motion duration, Figure 6 show that the Gaussian distribution gives higher values of P.D.E compared to those obtained using the Beta distribution.









For analysis where the scale of fluctuation in distance λ_x and in time λ_t were varied; Figures 7 and 8 show that the Gaussian distribution gives higher values of P.D.E compared to those obtained using the Beta distribution.

The differences between the Normal or Gaussian distribution and the Beta distribution is due to the nature of the Beta distribution which gains its shape from its parameters (q.r). These are obtained from the mean and the standard deviation of the calculated displacement. However, the Normal distribution can be considered as a special case of the Beta distribution (Mean=0.0 and the standard deviation = 1.0), i.e. it has a definite shape (Bell-Shaped).

Moreover, the Beta distribution has finite limits (D_{min} and D_{max}) while the Normal distribution has infinite limits ($-\infty$, $+\infty$). These facts has influenced the evaluated (P.D.E.) value.



Comparison between the Probability of Safety Factor Less than Unity Using the Normal (Gaussian) and β -Distributions

The values of probability of safety factor less than unity obtained using PTDDSSA for Congress Street landslide was as follows. For Gaussian distribution = 0.506 and for Beta distribution = 0.387.

It can be seen from these results that the Beta distribution gives lower values of probability failure compared to those obtained using the Gaussian distribution.

Comparison between Different Models Developed to Evaluate the Probability of Allowable Displacement Exceedance

A comparison between the four models developed to evaluate probability of allowable displacement exceedance is made for the international case by carrying out analysis using PTDDSSA program for the following allowable displacement values = 0.1 m, 0.3 m, and 0.9 m.

The results show that Model I tends to give higher values of probability failure compared to those obtained using model II and IV. Also model II gives higher values of probability of failure compared to model IV. This observation is consistent with previous results. The reason behind these facts is that model I is a special case of model II which assume a zero value for allowable displacement. Hence model I tends to give higher probability of failure. For model IV, reasons were discussed previously.

A better way to study the trend of model III is to calculate the percent difference in the value of predicted probability of allowable displacement exceedance obtained using model III compared to the value obtained using model II. Model III is the only model among models I, II, and IV which assumes that the displacement takes the normal distribution density function while model IV and I assumes that it has Beta distribution, and Poisson process distribution, respectively.

Based on the above, for $\Delta_{all} = 0.1$ m; the maximum and minimum differences are respectively 26.95% and 19.08%. For $\Delta_{all}=0.3$ m, the maximum and minimum differences are respectively 31.30% and 18.63%. For $\Delta_{all}=0.9$ m, the maximum and minimum differences are respectively 68.69% and 21.85%.

These results indicate that as the allowable displacement increase, the difference between models II and III increase. For small values of allowable displacement; the difference (as defined previously) tends to be negligible. This resulted from the fact that model III is not affected by the strong duration value nor the scales of fluctuation of time and distance. Model III is so simple compared with model II, as it deals only with basic parameters, allowable displacement, peak ground acceleration and velocity and amplification factor of model II. Model III do not consider time to failure nor number of maxima above the critical limiting acceleration. Moreover, model III uses empirical equations to obtain the limiting acceleration, while the other models use back analysis (e.g. 3-D factor of safety equal to 1.0) to obtain the limiting acceleration. Nevertheless, model III can be used for its simplicity and is observed to give approximate results specially when the allowable displacement is small. Hence model III (the simplest model) can be used in analysis and design of earth structures of infrastructures for which the design dictate small allowable displacement during an earthquake.

SUMMARY AND CONCLUSIONS

Based on the developed analyses, interpretations and discussions in this study, the following conclusions are reached:

- 1. For model I incorporated in the developed PTDDSSA program, which assume failure to occur as first slip occur (i.e. Poisson process), the probability of failure is the highest among the other models which assume failure to occur when a certain allowable displacement is exceeded.
- 2. Model II incorporated in PTDDSSA program, for which failure is based on exceeding an allowable value criteria and Gaussian distribution gives higher values of probability of failure or allowable displacement exceedance than Model IV, for which failure is defined

based on total displacement exceeding an allowable value criteria and the Beta distribution.

- 3. Model III is the simplest model, which is based on Nadim and Whitman (1983) method, an advancement to Newmark (1965) block-on-plane model. It gives results that are consistent with the results obtained using model I, II, and IV with maximum difference less than 30% specially at small values of allowable displacement.
 - Models that were based on an allowable displacement limit, such as models II, III, and IV were found to be more reasonable than those which do not have such a limit, e.g. model I.
- 5. The hypocentral distance have major influence on the earthquake induced displacement, probability of failure, and dynamic 2-D and 3-D safety factors. As the hypocentral distance increases; the earthquake induced displacement and probability of allowable displacement exceedance decreases and safety factor increases.
- 6. As the earthquake strong shaking period increases, the probability of allowable displacement exceedance increases.
- 7. As the scale of fluctuation in both distance and time increases; the probability of allowable displacement exceedance decreases.
- 8. The effect of earthquake magnitude is the same as the effect of the hypocentral distance on displacement, safety factors, and the probability of allowable displacement exceedance.
- 9. As the allowable displacement increases, the probability of allowable displacement exceedance decreases.
- 10. As the undrained shear strength increases, the safety factor increases, the critical failure width increases, the earthquake displacement decreases and the probability of failure decreases.
- 11. As the coefficient of variation of undrained shear strength and scale of fluctuation increases, the probability of failure increases, the earthquake induced displacement as well as the critical failure width decreases slightly, and the squared coefficient of resisting moment increases.
- 12. As the corrective factors (for modeling error, progressive failure, etc.) increase, the safety factor increases, the probability of failure decrease, the earthquake induced displacement decreases, and the critical failure width increases.

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LIST OF SYMBOLS

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LIST OF SYMBOLS		S	Strong Shaking Period
		\overline{SF}	Static Safety Factor
A _{r,b}	Limiting Acceleration Coefficient for Models	\overline{SF} 'T	Dynamic Safety Factor Natural Period
A_{sh}	Earthquake Induced Acceleration Coefficient	q,r	Coefficient of Beta Distribution
	for Models (I, II, and IV)	β	Slope Angle
An	Peak Ground Acceleration Coefficient on	λ	Natural Frequency
F	Rock.	Ω(x _i)	Coefficient of Variation of a Property xi
b	Critical Slope Width	$\Delta_{\rm xi}$	Coefficient of Variation of a Property xi Due
D	Earthquake Induced Displacement Using		to the Limited Number of Samples
	Probabilistic Approach	$\Omega_{\rm m}$	Number of Maxima above A _{r,b} by u(x,t)
L	Slope Width	Δ_{all}	Allowable Displacement
$M_{\mathbb{R}}$	Resisting Moment	φ()	Gaussian or Normal Distribution Function
$M_{\rm D}$	Driving Moment	β (q,r)	Beta Distribution Function.
$\mathbf{P_f}$	Probability of Failure		
r _u (x _i)	Reduction Factor Resulting from Sptial Averaging of		
	Soil Parameter xi		