13 Mar 1991, 1:30 pm - 3:30 pm

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Peiji Yu
The Research Institute of Water Conservancy and Hydroelectric Power, Beijing, China

E. B. Wylle
University of Michigan, Ann Arbor, MI

F. E. Richart Jr.
University of Michigan, Ann Arbor, MI

Proceedings: Second International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, March 11-15, 1991, St. Louis, Missouri, Paper No. 11.7

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Peiji Yu
Senior Research Engineer, The Research Institute of Water Conservancy and Hydroelectric Power, Beijing, China

E.B. Wylie
Professor and Chairman, Department of Civil Engineering, University of Michigan, Ann Arbor, MI

F.E. Richart, Jr.
W.J. Emmons Professor Emeritus of Civil Engineering, University of Michigan, Ann Arbor, MI

SYNOPSIS: A computer program, originally prepared to evaluate stresses and displacements below an axisymmetric surface footing subjected to torsional loadings, has been modified to accommodate embedded footings. The material surrounding the footing can be considered as elastic, nonlinear inelastic, or nonlinear inelastic including slip. Layered systems can also be treated.

Good agreement was obtained in comparisons with published solutions for elastic systems, as obtained by the finite element method, and by an approximate method. In particular, comparisons were made for variations at maximum amplitude of rotation and dimensionless frequency at maximum amplitude of rotation as functions of the embedment ratio.

The influence of soil nonlinearity and slip at the footing boundary were computed for conditions similar to those for a circular embedded footing previously tested in the field. Comparisons of computed and field results showed the importance of including soil nonlinearity and slip at the footing periphery when evaluating test data.

INTRODUCTION

The torsional dynamic response of circular rigid footings has been studied for a number of years, but most of the work concerned the response of surface footings. Up to the present time the writers know of no rigorous analytical solution for the response of embedded footings. An approximate solution for vibrations of embedded footings was given by Novak and Sachs (1973), and the finite element solution of this problem was reported by Waas (1972). Both of these studies dealt with the response of footings in an elastic medium. Richart and Whitman (1967) pointed out that nonlinear effects might introduce important differences between the theory and test results for the response of surface footings. Weissmann (1971) noted the importance of considering effects of material damping of soils and of slip between the surface footing and subsoil. Novak and Sachs (1973) reiterated that the inclusion of the slip effect appeared necessary. Henke and Wylie (1982) described a numerical procedure of solving the torsional dynamic response of footings resting on the surface of a linear or nonlinear half space. The procedure offered an attractive opportunity to treat this problem in more generalized conditions. In this paper the procedure is developed for solving response of rigid footings embedded in a linear or nonlinear half space or in a stratum on rock. The results for the homogeneous elastic half space were computed first and were compared with published results. Then the response of an experimental footing was compared with three calculated conditions: linear elastic, nonlinear inelastic, and nonlinear with slip between the footing and the soil.

NUMERICAL PROCEDURE

The numerical procedure used in this paper was fundamentally the same as that developed by Henke and Wylie (1982). However it was modified for studying the embedded rigid footing as shown in Figure 1. Henke's (1980) contribution was the development a characteristic-like method for solving the multi-dimensional axisymmetic torsional wave equation. For the torsional displacement U, the particle velocity V, and the shear stresses τ_r, τ_z fields, illustrated in Figure 2, Henke derived four total

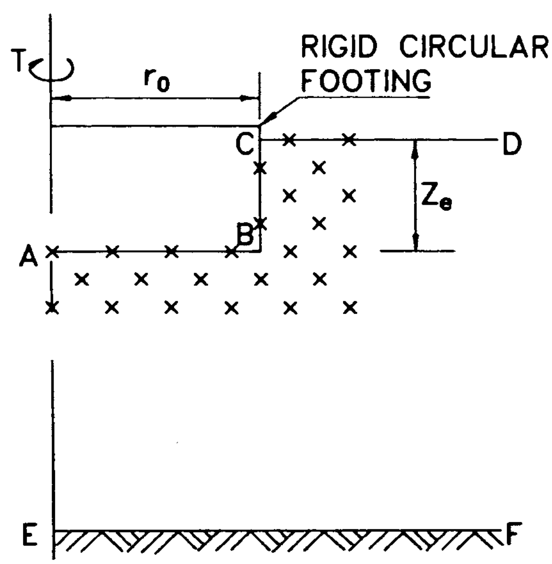

Fig. 1 Embedded Footing and Staggered Grid

derivative equations each of which was valid along a respective integration path as shown in Figure 3. The four total derivative equations, called compatibility equations, may be integrated and solved numerically for the four unknowns, τ_z, τ_r, V, and $\partial\tau_z/\partial r$ of node P at time t + Δt. For the nodes along the boundaries there are only two integration paths, and therefore only two compatibility equations. The other two equations are given by the boundary conditions. The conditions for boundaries AB, CD, and AE in Figure 1 were discussed by Henke, et al (1982). The boundary conditions for the sideface BC of the footings are $V = r_o\omega$ (r_o = radius of footing and ω = angular velocity of footing) and $\tau_z(t + \Delta t) = \tau_z(t)$ if there is no slip between the footing and soil. When slip occurs along the sideface the boundary conditions become $|\tau_r|=|\tau_{rslip}|$ and $\partial\tau_r/\partial Z = 0$. Henke and Wylie (1982) derived the equation relating the shear stresses as follows:

$$\frac{\partial\tau_r}{\partial Z} - \frac{G_{rt}}{G_{zt}}\frac{\partial\tau_z}{\partial r} + \frac{1}{r}\frac{G_{rt}}{G_{zs}}\tau_z = 0 \qquad (1)$$

Then the second condition for the boundary BC is

$$\frac{\partial\tau_z}{\partial r} = \frac{1}{r}\frac{G_{zt}}{G_{zs}}\tau_z \qquad (2)$$

in which G_{zt} and G_{zs} are the tangent and secant shear moduli in the Z-θ directions, respectively. The motion equation of the footing is

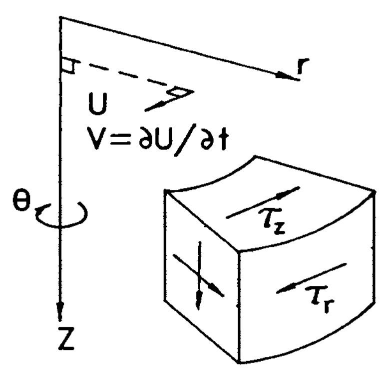

Fig. 2 Axisymmetric Torsional Displacement Velocity, and Stress Fields

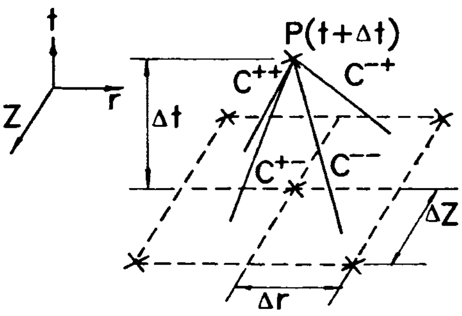

Fig. 3 Four Integration Paths Through Node P

$$M_A(t) + M_R(t) = I\frac{d\omega}{dt} \qquad (3)$$

in which I = mass moment of inertia of the footing; M_A = applied moment; and M_R = reacting moment. M_R consists of the reacting moment, M_{RR}, along the sideface, and the reacting moment, M_{RZ}, beneath the base of the embedded footing. M_{RZ} is equal to M_R in Henke and Wylie (1982). Because the staggered grid is used, the reacting moment M_{RR} along the sideface of the footing is

$$M_{RR} = \sum_{i=1}^{m}(4\pi\, r_o\,\Delta Z)\,\tau_{ri} \qquad (4)$$

in which m = the number of nodes along the sideface; τ_{ri} = the stress τ_r at those nodes; and ΔZ = the grid spacing in Z direction.

When integrating the four derivative equations Henke used a numerical approximation in accordance with the trapezoidal rule as follows:

$$\int_A^P \frac{V}{r}\,dr = \frac{1}{2}\left(\frac{V_p}{r_p} + \frac{V_A}{r_A}\right)(r_p - r_A) \qquad (5)$$

An improved approximate integration of this term is used in this paper:

$$\int_A^P \frac{V}{r}\,dr = \frac{1}{2}(V_p + V_A)\,\ell n\,\frac{r_p}{r_A} \qquad (6)$$

With the integration in Eq. (5) some numerical oscillations appeared in the method during this study, particularly during investigations involving nonlinear materials. These were eliminated with the integration in Eq. (6); however, even then, it was necessary to assign a reasonable value to the radius r(1) for boundary AE in Figure 1 to maintain the accuracy of the integration.

A computer program named CHARFOUND was presented based on Henke's procedure. It is a generalized program for the torsional response of footings. The footing was assumed to be circular and rigid. The medium, in which the footing was embedded or on which it was placed, could be linear elastic or nonlinear inelastic. For the nonlinear inelastic condition it was assumed that the shearing stress-strain curves of soils followed the Ramberg-Osgood equations as noted in Streeter, et al (1974), Richart (1975). The medium could be a half space or a stratum on rock. Slip, along the interface of the footing and the medium, could be considered on either the sideface or the base of the footing or on both. The variations of shear modulus and mass density of soils along the depth could also be considered. In addition to the response of the footing, the particle velocity V, shearing stresses τ_r and τ_z, and displacement U of any node, and the average shearing stress and strain of any subcell in the medium could be obtained. An energy balance computation (Wylie and Henke, 1979) was used to judge the reliability of the numerical results.

RESPONSE OF FOOTINGS IN ELASTIC HALF SPACE

It has been demonstrated that the response of surface footings on an elastic half space might be interpreted as a mass-spring-dashpot system (Lysmer and Richart, 1966, Richart and Whitman, 1967). The same system may also be used to interpret the response of embedded footings in an elastic half space. Then the damped frequency, f_d, and the damping ratio, D, may be evaluated from the free vibration decay curves (Richart, et. al., 1970), obtained from measurements or calculations. The static torque-rotation relationships of the rigid cylinder embedded in the elastic half-space was published by Luco (1976), and is given in Table 1. In Table 1, Z_θ = the embedment depth of the footing, $T_o = 16 G_o r_o^3 \theta_s/3$, G_o = shear modulus of the half space, and θ_s = static rotational displacement produced by the applied torque T. The dimensionless resonant frequency a_m, and the dimensionless rotation, A_m, can be determined by the equations,

$$a_m = 2\pi\ f_m\ r_o\ (\rho/G_o)^{1/2} \tag{7}$$

$$A_m = (3T_o/16\ T) M_m \tag{8}$$

in which ρ = mass density of the half space, and

$$f_m = f_d\ (1-2D^2)^{1/2}\ /\ (1-D^2)^{1/2} \tag{9}$$

$$M_m = 1/(2D(1-D^2)^{1/2}) \tag{10}$$

Table 1. Normalized Static Torque for Various Embedment Ratios (Z_e/r_o) (After Luco, 1976)

Z_e/r_o	0	0.125	0.250	0.500	1.000
T/T_o	1.00	1.45	1.81	2.48	3.73

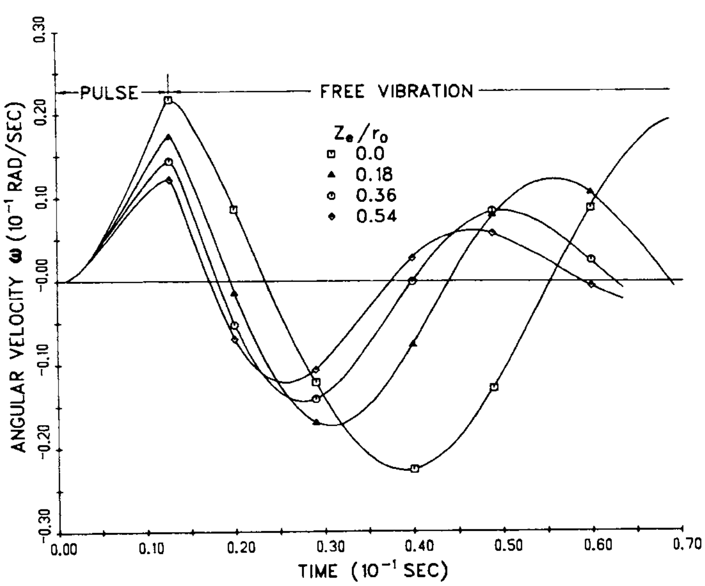

Fig. 4 Free Vibration Curves for Different Embedment Depths (Inertia Ratio B_θ = 4)

The free vibration-decay curves of circular footings embedded in a homogeneous elastic half space and with various embedment ratios were computed. One set of the curves is shown in Figure 4 and the calculation is presented in Table 2. The values of a_m and A_m vs Z_e/r_o for two inertia ratios B_θ [= $I/(\rho r_o^5)$] are plotted in Figure 5 and Figure 6. Also plotted in the two figures are the results by Novak and Sachs (1973) and Waas (1972). It may be observed in Figures 5 and 6 that though these results were obtained from quite different procedures, agreement among them is quite good. It is interesting to note that for the inertia ratio B_θ = 1.83, the dimensionless resonant frequency, frequency, a_m, increases to a peak value then decreases as the embedment ratio, Z_e/r_o, increases.

Table 2. Calculation of a_m and A_m (B_θ = 4.0)

Z_e/r_o	f_d (Hz)	D	f_m (Hz)	a_m	M_m	$3T_o/16T$	A_m
0.0	15.7	0.046	15.7	0.982	10.89	0.188	2.042
0.182	20.2	0.112	20.0	1.256	4.52	0.117	0.529
0.364	22.4	0.169	22.1	1.386	3.05	0.088	0.268
0.545	24.0	0.222	23.4	1.469	2.37	0.071	0.168
0.727	25.0	0.273	24.0	1.504	1.98	0.058	0.115
0.908	25.8	0.323	24.3	1.524	1.73	0.052	0.090

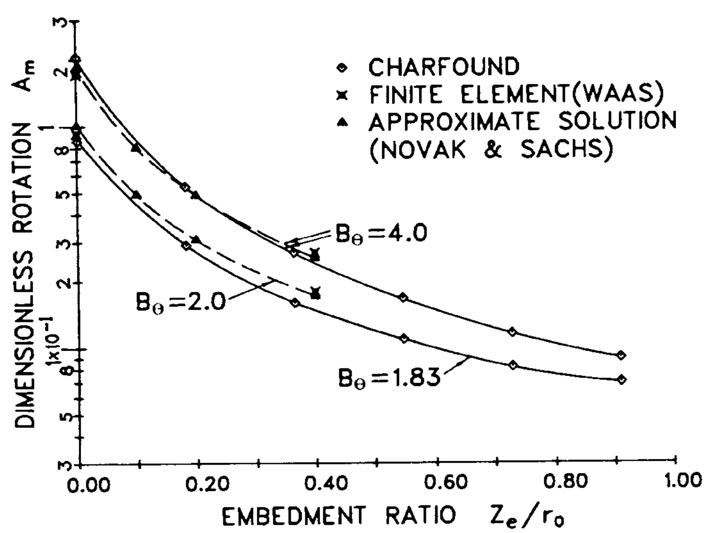

Fig. 5 Resonant Rotational Amplitude vs. Embedment Ratio

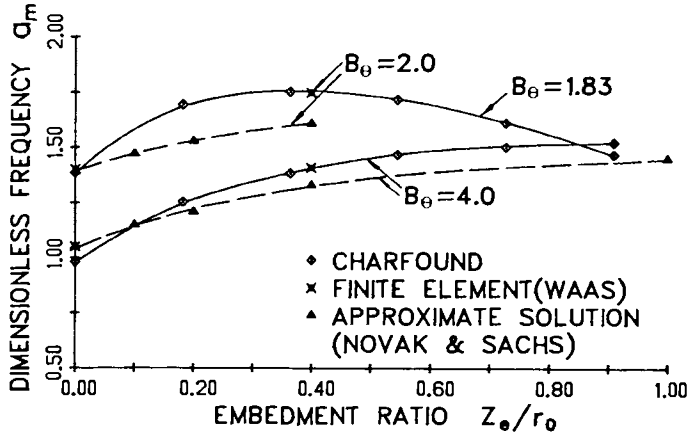

Fig. 6 Dimensionless Resonant Frequency vs. Embedment Ratio

RESPONSE OF FOOTINGS IN NONLINEAR INELASTIC MEDIUM

It is difficult to get generalized results for the response of footings in a nonlinear inelastic medium because there are many variables. Only a special problem was analyzed in this paper. Experimental results for the response of an embedded footing were obtained from the report by Fry (1963). The same footing and soil data were used in the numerical computation for direct comparison with the experimental results. The circular footing was made of reinforced concrete with radius, r_o = 3.65 ft. and embedment depth, z_e = 2.08 ft. The total weight of the footing, including the vibrator, was 18,465 lb and the mass moment of inertia was I = 3823 ft-lb-sec^2. The soil at the site where the experiments were performed was a nonplastic uniform fine sand with average mass density ρ = 3.23 lb-sec^2/ft^4. The shear moduli, determined through shear wave velocities measured at the site on the original soil, varied linearly with depth as shown in Figure 7.

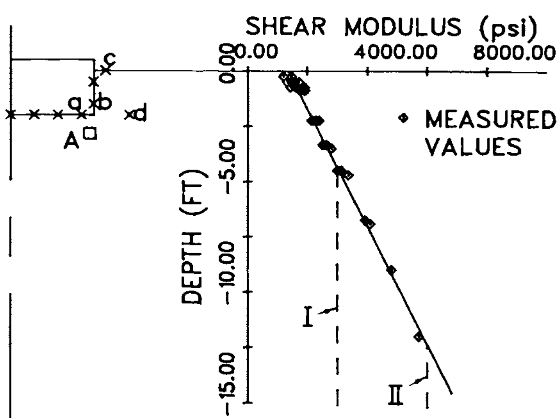

Fig. 7 Shear Modulus vs. Depth and Computation Section

The program CHARFOUND can be used for layered soils with various shear moduli and mass densities. However, the larger the differences of shear wave velocities the larger the error in results caused by interpolation. Alternatively, any interpolation errors may be minimized by taking a finer grid and a smaller time step. Two variations of shear moduli as shown in Figure 7 were compared. The number of nodes beneath the footing, RN, was 4 for the section I and 6 for the Section II. The thickness of the soil was assumed to be 16.7 ft. The footing responses for restraint by the two soil cross sections were similar and showed that the shear moduli of the layers around and directly beneath the footing were much more important to the dynamic response than the shear moduli of underlying layers. Thus section I was chosen for analyzing the problem.

Three conditions of the medium were considered: linear elastic, nonlinear inelastic without slip; and nonlinear inelastic with slip along the sideface of the footing. The parameters in the Ramberg-Osgood equation, α, R, C_1, and τ_m, were taken, respectively, as 0.01, 5, 0.8, and 6250 lb/ft^2 for the elastic condition; and as 1, 3, 0.8, and 318 lb/ft^2 for the nonlinear inelastic condition. Usually the slip stress along the sideface is smaller than the slip stress beneath the base for an embedded footing because of the smaller normal stress, and the disturbance of the soil. Slip was considered only along the sideface since, when the torque applied on the footing is large enough to cause slip, it will occur along the sideface first. For this condition slip stresses were assumed to vary linearly with depth and were taken as 43 and 129 lb/ft^2 for the two nodes on the sideface.

The forced vibration results, computed with the excitation frequency f = 22 Hz and the torque amplitude of T = 39,650 lb-ft, are presented in Figures 8 to 10. Only two and a half cycles of vibrations were plotted in these figures. Figure 8 shows that the rotational displacements of the footing for the nonlinear condition was greater than for the elastic condition, and that slip between the footing and the soil increased the rotational amplitude further.

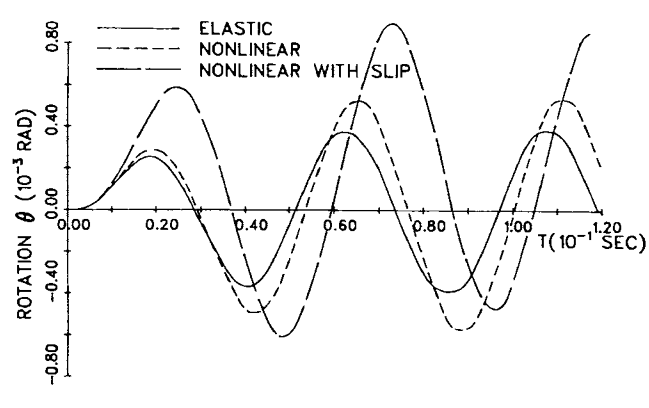

Fig. 8 Rotational Displacement vs. Time

The shear stress τ_z at the node "a" (Figure 9(a)) and τ_r at the node "b" (Figure 9(b)) are shown in Figure 9. When there was no slip along the sideface $(\tau_r)_b$ was greater than $(\tau_z)_a$ and was equal to about 330 lb/ft^2. This would require a normal pressure greater than 450 lb/ft^2 if the internal friction angle of the sand was assumed to be 36 degrees. It was clear that the value of $(\tau_r)_b$ was impossible for the footing and the soil conditions. Slip along the sideface limited $(\tau_r)_b$ and caused $(\tau_z)_a$ to increase and made the results more reasonable.

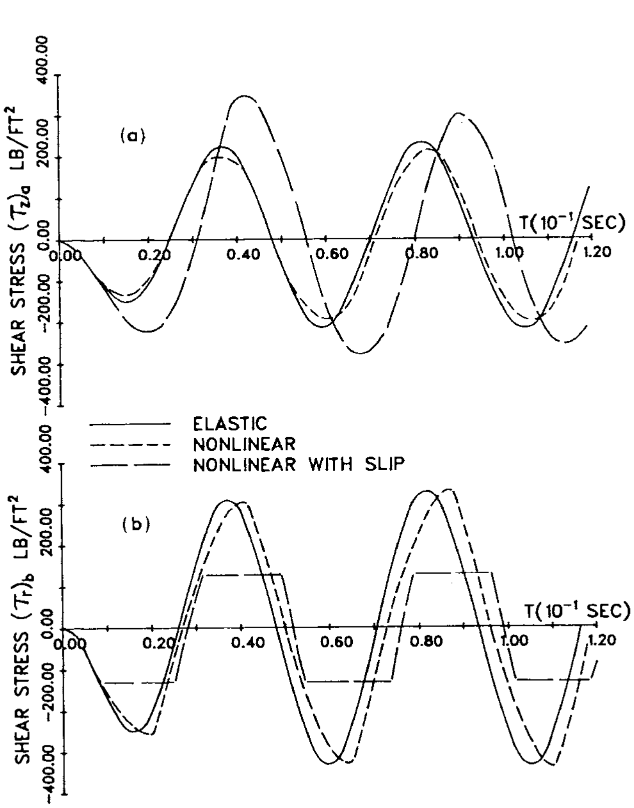

Fig. 9 Shearing Stress at Two Nodes on Interface

Figure 10 presents the displacement at node "c" (Figure 10(a)) and the shear stress at the node "d" (Figure 10(b)) in the soil. Slip along the sideface of the footing decreased the displacements at the soil surface and it also decreased the stresses in the soil around the footing. The average shearing stress-strain curves of the subcell A beneath the footing (Figure 7) are shown in Figure 11. In Figure 11 (a) the stress-strain curves for the elastic condition showed a small hysteretic loop compared with the linear line assumed, and the loops for the nonlinear condition in Figure 11(b) were a little wider and flatter than the assumed Ramberg-Osgood curves. Both of these variations were caused by interpolation and could be minimized be using a finer grid. Figure 11(c) shows that slip between the footing and the soil made the stress-strain curves more hysteretic as expected. Results of the energy bal-

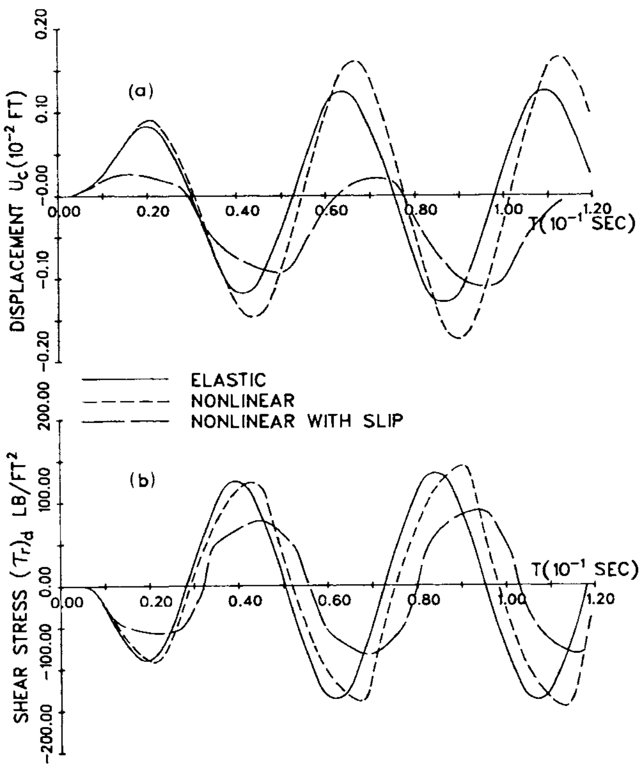

Fig. 10 Displacement and Shearing Stress at Two Nodes in Soil

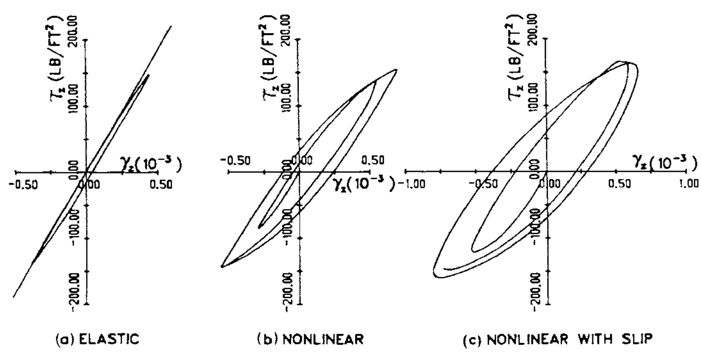

Fig. 11 Average Shearing Stress-Shearing Strain Curves in Subcell "A"

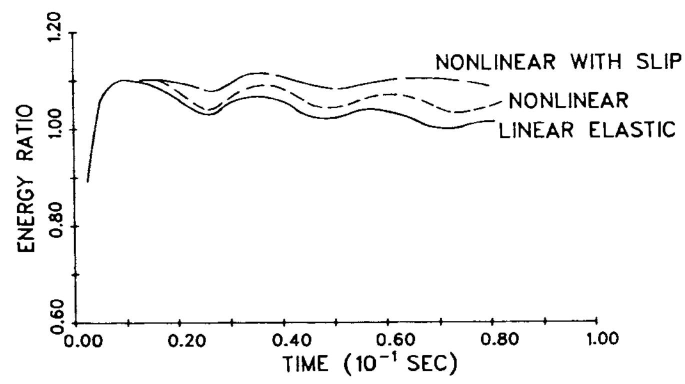

Fig. 12 Energy Ratio vs. Time

ance computation, plotted in Figure 12, show energy ratios varying between 1.0-1.1 for all conditions. This acceptable energy balance adds credibility both to the procedure and to the computational results.

By computing the response of the footing for various excitation frequencies the response curves of the footing for the three conditions were obtained. These were plotted in Figure 13 along with the experimental curve (Fry, 1963, Figure 106). The curves of phase angle vs. frequency are presented in Figure 14. It should be noted that all of these curves were for the case of rotating-mass excitation, which meant torque amplitudes were proportional to the square of the excitation frequencies.

From Figure 13 and 14, it is evident that consideration of the nonlinearities of the footing and the soil system, including nonlinear properties of soils and slip between the footing and soil, made the numerical and experimental results match more closely. It was not deemed important to obtain a precise fit between the two; uncertainties in the soil parametric values and the bonding conditions between the footing and the soil made this impractical. The most important finding is that the nonlinear shearing stress-strain properties of soils and the bonding conditions between the footing and soils must be considered when analyzing the dynamic response of footings.

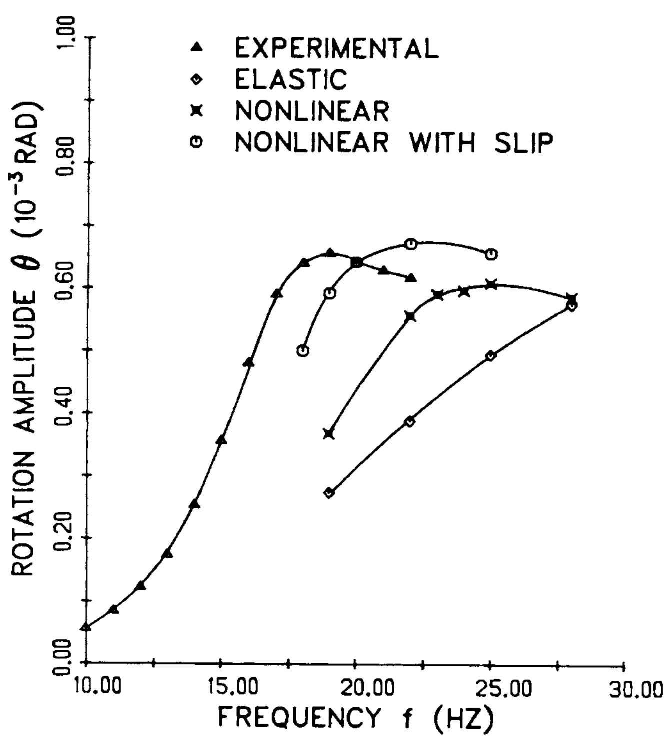

Fig. 13 Rotation Amplitude vs. Frequency

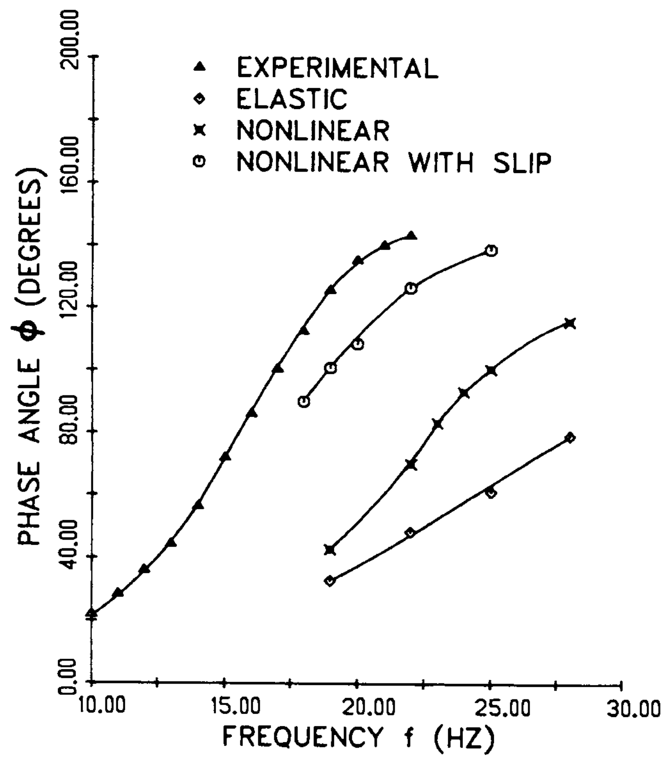

Fig. 14 Phase Angle vs. Frequency

CONCLUSIONS

A program named CHARFOUND had been developed for analyzing the response of circular rigid footings. This program is based on a characteristic-like method for solving the multidimensional axisymmetic torsional wave equations, developed be Henke and Wylie (1982). The program can evaluate the torsional response of footing under various conditions, including surface footings and embedded footing with or without slip at the sideface and on the base. The soil around the footing may be treated as elastic, or nonlinear inelastic, and may be considered as a half-space of a finite layer over rock beneath the footing. The program could also be extended to treat an anisotropic medium.

The torsional response of footings under various conditions were computed. The results for the condition of a homogeneous elastic medium were compared with published results and the agreements were good. The results for the nonlinear inelastic condition showed how the nonlinearities of the footing and soil system affected the response of the footing and the stresses, strains and displacements in the soil. The comparison between experimental and numerical results have proved that it is necessary to consider the nonlinearities of the footing and soil system, and possible slip along the vertical side face, the obtain reasonable results.

ACKNOWLEDGEMENTS

This study was supported by the National Science Foundation Grant NSF-PFR-8017429, The University of Michigan, and the Research Institute of Water Conservancy and Hydroelectric Power of the People's Republic of China. The support be these organizations is gratefully acknowledged.

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