30 Mar 2001, 10:30 am - 12:30 pm

Vertical Earthquake Response Analysis of Xiaolangdi Earth-Rock Dam With 3-D Shear Wedge Model

Zhenzhong Shen
Hohai University, China

Zhiying Xu
Hohai University, China

VERTICAL EARTHQUAKE RESPONSE ANALYSIS OF XIAOLANGDI EARTH-ROCK DAM WITH 3-D SHEAR WEDGE MODEL

Shen Zhenzhong
Hohai University
Nanjing, Jiangsu, CHINA

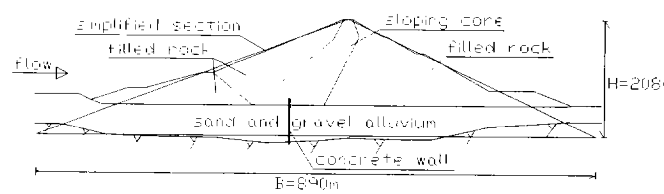
Xu Zhiying
Hohai University
Nanjing, Jiangsu, CHINA

ABSTRACT

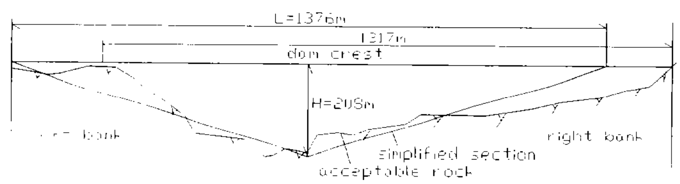
The constructing Xiaolangdi dam is the highest earth-rock dam in China. Its maximum height is about 154 m, and the length of dam axis is about 1500 m. In this paper, by application of the three-dimensional shear wedge theory and the Bubnov-Galerkin approach method, the vertical earthquake response analysis is performed for the earth dam. Its dynamic behaviors such as maximum dynamic displacement, velocity, acceleration and stress *etc* are obtained. Under the action of the designed earthquakes, the fundamental period of the vertical vibration of the Xiaolangdi earth-rock dam is about 0.803 s, the maximum acceleration response is about 0.976 g, and the maximum absolute acceleration is about 1.476 g.

INTRODUCTION

The Xiaolangdi water project, which is being built, locates at the exit of last valley on the Yellow River. It is about 130 km far from the Sanmenxia water project in upstream, and the downstream is the Huanghuaihai Plane. It controls about 95 percents of basin area at Huayuankou on the Yellow River, including about 91.2 percents of total discharge and almost 100 percents of total sediment. The normal water level of the Xiaolangdi reservoir is 275.00 m, and its total storage capacity is 12,650 Mm^3. The dam of the water project is a clay sloping core rockfill dam. Its maximum height is about 154 m, and its length of dam axis is about 1500 m. The average upstream and downstream slope gradients of the typical cross section of the dam are about 1:2.64 and 1:2.10 respectively, shown as in Fig. 1 (a). The valley of dam site is approximate triangle, whose average slope gradients of the two banks are about 1:3.31, shown as in Fig. 1 (b). The overburden layer of dam foundation is sand and gravel, whose average thickness is about 36.0 m.



(a) Typical section



(b) Longitudinal section

Fig. 1 Schematic diagram of the Xiaolangdi earth-rock dam

The research result of vertical earthquake response has not been found by now. The authors proposed a simplified theoretical method of three-dimensional vertical earthquake response analysis for non-homogeneous earth dams in triangular canyons (Shen and Xu, 1997). Here the method is used to analyze the vertical vibration behaviors of the Xiaolangdi earth-rock dam. Because of the similar properties of the dam body and its foundation overburden, the sand and gravel layer is considered as one part of the dam. Thus the height of the dam is about 208.0 m, which is composed of the height of dam body and the depth of the foundation overburden layer.

BASIC PRINCIPLE

Basic assumptions

The maximum longitudinal section and the maximum cross section of the earth dam and the calculation coordinate system are shown as in Fig. 2. The following assumptions are adopted. (1) The bedrock is rigid. (2) The direction of ground motion is vertical. (3) The interaction between dam and water in reservoir is negligible. (4) The dam materials, with the uniform mass densities, are linearly elastic. (5) The shear module of dam materials are non-uniform elastic, which increase as the (l/m)th power of the depth (Abdel-Ghaffar, 1981):

$$G(y) = G_0 (\frac{y}{H})^{\frac{l}{m}} \tag{1}$$

where G_0 is the maximum shear modulus of the dam material at the base; H is the height of the dam and l/m is called non-homogeneous index here, such as 0,1/3,2/5,1/2 and 1 *etc.*

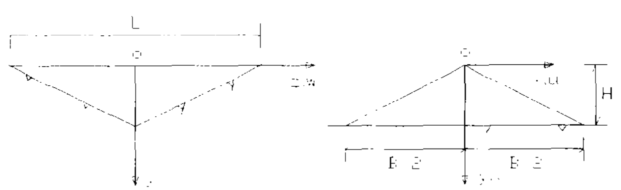


Fig. 2 Schematic diagram of maximum sections and calculation coordinate system

Some calculation formulas

Differential equation of vertical vibration of earth dam For the coordinate system shown as in Fig. 2, the differential equation of vertical vibration of earth dam can be written as following:

$$\frac{\partial^2 v}{\partial t^2} + \frac{c}{\rho}\frac{\partial v}{\partial t} - \frac{G_0}{\rho H^{\frac{l}{m}}}\left[\xi(1+\frac{l}{m}) y^{\frac{l}{m}-1}\frac{\partial v}{\partial y} + \xi y^{\frac{l}{m}}\frac{\partial^2 v}{\partial y^2} + y^{\frac{l}{m}}\frac{\partial^2 v}{\partial z^2}\right] = -\ddot{v}_g(t) \tag{2}$$

where v is the relative displacement to bedrock in y direction; c is the coefficient of damping; ρ is the density of the dam material; $\ddot{v}_g(t)$ is the acceleration of earthquake motion of the bedrock in y direction, t is the time, $\xi = 2(1+\mu)$ and μ is the Poisson's ratio of dam material.

Boundary condition

$$\begin{cases} E\dfrac{\partial v}{\partial y} = 0 & \text{at} \quad y = 0 \\ v = 0 & \text{at} \quad y = H \pm Kz \end{cases} \tag{3}$$

where $K = 2H/L$, L is the length of dam crest, E is the Young's modulus of the dam material and $E = 2(1+\mu)G$.

First mode shape function (Shen and Xu, 1997)

$$\phi(y,z) = \frac{1}{H^4}(y+H+Kz)(y-H-Kz)\times (y+H-Kz)(y-H+Kz) \tag{4}$$

First natural frequency

$$\omega = \frac{30m^2}{H}\sqrt{\frac{G_0}{\rho}}\sqrt{\frac{\alpha}{\beta}} \tag{5}$$

where the parameters α and β are defined as following

$$\alpha = (K^2+3\xi)l^2 + (9K^2+23\xi)lm + (16K^2+36\xi)m^2$$

$$\beta = (l+m)(l+3m)(l+4m)(l+5m)(l+6m)(l+8m)$$

First mode participation coefficient

$$\eta_1 = \frac{\int_0^H \int_{-\frac{1}{K}(H-y)}^{\frac{1}{K}(H-y)} \phi_1 y dy dz}{\int_0^H \int_{-\frac{1}{K}(H-y)}^{\frac{1}{K}(H-y)} \phi_1^2 y dy dz} \quad (6)$$

Displacement response

$$v(y,z;t) \approx \phi_1(y,z)\frac{-\eta_1}{\omega_1'}\int_0^t \ddot{v}_g(\tau)e^{-\lambda_1\omega_1(t-\tau)}\sin[\omega_1'(t-\tau)]d\tau \quad (7)$$

Velocity response

$$\dot{v}(y,z;t) \approx \phi_1(y,z)\times \left[-\eta_1 \int_0^t \ddot{v}_g(\tau)e^{-\lambda_1\omega_1(t-\tau)}\cos[\omega_1'(t-\tau)]d\tau - \lambda_1\omega_1 T\right] \quad (8)$$

Acceleration response

$$\ddot{v}(y,z;t) \approx -\ddot{v}_g(t) + \phi_1(y,z)\eta_1\{\frac{(1-2\lambda_1^2)\omega_1}{\sqrt{1-\lambda_1^2}}\times \int_0^t \ddot{v}_g(\tau)e^{-\lambda_1\omega_1(t-\tau)}\sin[\omega_1'(t-\tau)]d\tau + 2\lambda_1\omega_1\int_0^t \ddot{v}_g(\tau)e^{-\lambda_1\omega_1(t-\tau)}\cos[\omega_1'(t-\tau)]d\tau\} \quad (9)$$

Stress response

$$\sigma_y(y,z;t) \approx E\phi_{1y}'\frac{-\eta_1}{\omega_1'}\int_0^t \ddot{v}_g(\tau)e^{-\lambda_1\omega_1(t-\tau)}\sin[\omega_1'(t-\tau)]d\tau \quad (10)$$

$$\tau_{zy}(y,z;t) \approx G\phi_{1z}'\frac{-\eta_1}{\omega_1'}\int_0^t \ddot{v}_g(\tau)e^{-\lambda_1\omega_1(t-\tau)}\sin[\omega_1'(t-\tau)]d\tau \quad (11)$$

In the above formulae, ϕ_{1y}' and ϕ_{1z}' are the first partial derivatives of $\phi_1(y,z)$ about y and z respectively; λ_1 is the damping ratio of dam material, $\lambda_1 = c/2\rho\omega_1$; $\omega_1' = \omega_1\sqrt{1-\lambda_1^2}$. After substituting the equation (4) into equation (6) and performing the integration, It is obtained that $\eta_1 = 1.856$. T can be got from the following integration:

$$T(t) \approx \frac{-\eta_1}{\omega_1'}\int_0^t \ddot{v}_g(\tau)e^{-\lambda_1\omega_1(t-\tau)}\sin[\omega_1'(t-\tau)]d\tau \quad (12)$$

According to the engineering experiences, the higher modes have little effects on earthquake responses of the dam, and only a few lower modes (1~3 orders) are adopted for practical requirement. Because the first mode shape acts as a crucial role in the earthquake responses, only the first mode shape is adopted. This is the meaning of simplified dynamic analysis here. The equations (7)~(12) can be used to calculate the all kinds of earthquake responses at arbitrary times and the corresponding time-response curves can be obtained. But in engineering, the maximum values of earthquake responses are important. Thus, the technique of earthquake response spectrum can be used to calculate the maximum values of earthquake responses. The corresponding calculation formulas are shown as in following:

$$v_{max} \approx |\eta_1\phi_1|S_d \quad (13)$$

$$\dot{v}_{max} \approx |\eta_1\phi_1|S_v \quad (14)$$

$$\ddot{v}_{max} \approx |\eta_1\phi_1|S_a \quad (15)$$

$$\sigma_{y,max} \approx E|\eta_1\phi_{1y}'|S_d \quad (16)$$

$$\tau_{zy,max} \approx G|\eta_1\phi_{1z}'|S_d \quad (17)$$

where S_d, S_v and S_a are displacement response spectrum, velocity response spectrum and acceleration response spectrum respectively.

EARTHQUAKE RESPONSES OF THE XIAOLANGDI EARTH-ROCK DAM

Calculation parameters

The Xiaolangdi earth-rock dam mainly includes two materials. The one is the filled rock of dam shell and the another is clay of sloping core. In addition, the overburden layer of dam foundation is sand and gravel. For the simplified dynamic analysis, this paper calculates the average dynamic parameters of the whole dam body by use of the weighted average principle.

Density and Poisson's ratio According to the volume percents of all kinds of dam materials, it is easy to obtain the average density and Poisson's ratio, *id.* $\rho = 2192$ kg/m^3, $\mu = 0.43$. Some parameters of dam materials are shown as in Table 1.

Table 1 Dynamic properties of the soils of the Xiaolangdi earth-rock dam

Material	Filled rock	Clay of sloping core	Sand and gravel of foundation
Volume / %	58.3	12.9	28.8
Density / (kg·m^{-3})	2160	2070	2313
Poisson's ratio	0.42	0.48	0.42
K'	2000	470	1400
n	0.5	0.5	0.5
G_{max} / MPa	785	184	549
λ	Variation with shear strain (Shown as in Table 2)		

Shear modulus According to the test data, the maximum shear modulus of dam material can be got from following formula (Lin, 1997):

$$G_{max} = K'P_a\left(\frac{\sigma'_{max}}{P_a}\right)^n \tag{18}$$

where P_a is the atmospheric pressure, $P_a = 100$ kPa; σ'_{max} is the average effective consolidation stress; K' and n are the experiment parameters, shown as in Table 1. The average effective consolidation stress at dam base can be evaluated from following equation:

$$\sigma'_{max} = \frac{1}{3}\gamma' H(1+2\zeta) \tag{19}$$

where γ' is the effective unit weight of dam material; ζ is the consolidation ratio, $\zeta = 0.45$. Substituting the parameters at the dam base into equation (19), the average effective consolidation stress is obtained, *id.* $\sigma'_{max} = 1.54$ MPa. Because the shear modulus decreases with the increment of shear strain, the average shear modulus during the whole earthquake duration may be evaluate by a half of the maximum shear modulus. Thus the average shear modulus at the dam base is

$$G_0 = (785\times0.583+184\times0.129+549\times0.288)\times50\%$$
$$= 320.0 \quad \text{(MPa)}$$

Damping ratio During the earthquake endurance period, the damping ratio is variable with the variation of the shear strain. The relationships between the damping ratios and shear strains of the materials of the Xiaolangdi earth-rock dam are shown as in Table 2, which are given by the indoor tests.

Table 2 The damping ratios of the materials of the Xiaolangdi earth-rock dam

Shear strain	Filled rock	Clay of sloping core	Sand and gravel of foundation
5×10^{-6}	0.018	0.035	0.020
1×10^{-5}	0.020	0.035	0.020
5×10^{-5}	0.035	0.040	0.020
1×10^{-4}	0.040	0.043	0.025
5×10^{-4}	0.065	0.055	0.040
1×10^{-3}	0.080	0.065	0.050
5×10^{-3}	0.080	0.115	0.085
1×10^{-2}	0.200	0.160	0.110

Earthquake data

The distant earthquake, near earthquake and earthquake in dam site area are used to analyze the dynamic behaviors, which are proposed by the experts of the World Bank for the Xiaolangdi water project in 1990 and 1991. The earthquake magnitudes, peak accelerations and earthquake durations of the three earthquakes are shown as in Table 3. The corresponding time-acceleration curves of earthquakes are combined artificially by use of the mathematical model of non-stable stochastic process, whose acceleration response spectra of earthquakes are shown as in Table 4.

Table 3 Parameters of distant earthquake, near earthquake and earthquake in dam site area

Earthquake type	Distant earthquake	Near earthquake	Earthquake in dam site area
Earthquake magnitude	8.0	7.5	6.25
Distance from focus / km	90	29	10
Peak acceleration / g	0.16	0.25	0.50
Earthquake duration / s	30.0	19.0	15.7

Dynamic analysis

First natural frequency Assuming $l/m=1/2$ and substituting the parameters into equation (5), the first natural frequency of the Xiaolangdi earth-rock dam can be got easily, *id.* $\omega_1=7.82$ rad/s. By use of the formula of period $T_1=2\pi/\omega_1$, the fundamental period can be obtained, *id.* $T_1=0.803$ s. According to the average damping ratio $\lambda_1=5\%$, the acceleration response spectra of the three earthquakes for the Xiaolangdi earth-rock dam can be got from Table 3. The results are shown as in Table 5.

Earthquake responses The maximum responses can be calculated from equations (13)~(17), shown as in Table 5, where the displacement response spectrum and velocity response spectrum are obtained from the acceleration response spectrum with the following formulae:

$$S_d=\frac{S_a}{\omega_1^2},\quad S_v=\frac{S_a}{\omega_1} \tag{20}$$

Table 4 Acceleration response spectra of distant earthquake, near earthquake and earthquake in dam site area ($\lambda=5\%$)

Period / s	Acceleration response spectra / g		
	Distant earthquake	Near earthquake	Earthquake in dam site area
0.010	0.160	0.250	0.500
0.030	0.160	0.250	0.500
0.050	0.208	0.355	0.661
0.075	0.251	0.458	0.830
0.100	0.291	0.528	0.960
0.150	0.342	0.610	1.127
0.200	0.376	0.650	1.194
0.250	0.400	0.675	1.209
0.300	0.414	0.688	1.186
0.350	0.424	0.685	1.145
0.400	0.434	0.670	1.058
0.500	0.442	0.619	0.882
0.600	0.448	0.566	0.744
0.650	0.450	0.540	
0.700	0.448	0.515	0.623
0.800	0.439	0.460	0.526
0.900	0.426	0.414	0.452
1.000	0.411	0.375	0.396
1.500	0.320	0.247	0.229
2.000	0.254	0.173	0.154
3.000	0.170	0.106	0.086
4.000	0.128	0.069	0.056
5.000	0.103	0.049	0.040

The maximum absolute acceleration includes the two parts of the earthquake response acceleration and the acceleration of bedrock motion, shown as in following formula:

$$a_{\max}=|\ddot{v}_{\max}|+|\ddot{v}_{g,\max}| \tag{21}$$

It is shown from the analysis results. The maximum displacement response, maximum velocity response and maximum acceleration response occur at the point of the center of the valley at dam crest, but the maximum stress response occurs at the point of 40 percents of dam height and in the vicinity of bank.

Table 5 Earthquake responses of the Xiaolangdi earth-rock dam

Earthquake type	Distant earthquake	Near earthquake	Earthquake in dam site area
Acceleration response spectrum S_a / g	0.439	0.460	0.526
Velocity response spectrum S_v / (m·s^{-1})	0.550	0.576	0.659
Displacement response spectrum S_d / m	0.070	0.074	0.084
Maximum displacement response u_{max} / m	0.130	0.137	0.156
Maximum velocity response $\dot{u}_{max}$ / (m·s^{-1})	1.021	1.069	1.223
Maximum acceleration response $\ddot{u}_{max}$ / g	0.813	0.854	0.976
Maximum absolute acceleration a_{max} / g	0.973	1.104	1.476
Maximum shear stress response $\sigma_{y,max}$ / kPa	849.7	898.3	1019.7
Maximum shear stress response $\tau_{zy,max}$ / kPa	89.8	94.9	107.8

CONCLUSION

By use of the proposed three-dimensional simplified dynamic analysis method, this paper performs the dynamic analysis of vertical vibration of the Xiaolangdi earth-rock dam, and some dynamic behaviors are obtained. The fundamental period of the dam is about 0.803 s, the maximum acceleration response is about 0.976 g, and the maximum absolute acceleration at the dam crest is about 1.476 g.

The method of dynamic analysis of earth-rock dam proposed in this paper is very easy, and the work is less much more than the finite element method. It can give the maximum earthquake responses quickly, which have reference value to both the design and construction and safety running of earth-rock dam.

REFERENCE

Abdel-Ghaffar, A.M. and Aik-Siong Koh. [1981]. Longitudinal vibration of non-homogeneous earth dams. @ Earthquake Engineering and Structure Dynamics. Vol. 9, pp.279-305.

Lin Xiushan. [1997]. Proceeding of Xiaolangdi Water Project on the Yellow River. @ Yellow River Hydraulic Press, Zhenzhou.

Shen Zhenzhong and Xu Zhiying. [1997]. Vertical vibration of inhomogeneous earth dams in triangular canyons. @ Proc. of the 9th Inter. Conf. of the Association for Computer Methods and Advances in Geomechanics. Wuhan, China. A. A. Balkema Press, Potterdam. pp.1783-1787.