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# A Variational Formulation for Classical Models of the Slope Stability With Earthquake Effect

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# \ Variational Formulation for Classical Models of the Slope Stability Vith Earthquake Effect

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YNOPSIS: The paper deals with an extension of application of the Method of Apriori Integration ɔ a model of a general slope, which is partly homogeneous and which is loaded also by the earthquake ffect. The most dangerous direction of the power effect of the vibration is shown and for this irection the change of safety factor is discussed. The effect of reduced shear strength is ncluded.

NTRODUCTION

he Method of Apriori Integration was established o the stability analysis according to the clas- :ical models - see KOUDELKA - PROCHÁZKA (1980), 1987), (1990) and PROCHÁZKA (1990). This paper xtends the application of the method to the ana- ysis of earthquake effect for the Sweden mo- el.

'he classical model of Sweden model is suitable lso to the non-cohesive soils. In this case the ircular slip surfaces changes to the plane slip :urface on the slope terrain. The cohesive soils :an lose the cohesion througt the vibration ef- 'ect and, in an extremal case can change to the ıon-cohesive ones. The analysis of soil slopes vhich lose their cohesion can be carried out by :he weaker means then the Method of Apriori Inte- ;ration - see next text.

'he necessary assumption for analysis of the cri- :ical value of measure of stability - the safety 'actor - is the state of the direction of vibra- ,ion effect which gives the maximal power influ- ɘnces to the stability.

STABILITY EFFECT

Two quantities are essential for stability ana- lysis of slopes under earthquake effect loading: the amplitude y and the acceleration a . The- se quantities are also the fundamental ones for the international scale MCS (Mercalli - Cancani- Sieberg). In this scale the values according to MYSLIVEC - KYSELA (1975) are introduced in Fig.1. Dashed line denotes an extrapolation of these quantities for 10 - 12 degrees MCS to obtain also the extremal influences. The vibration can posses an arbitrary direction.

Different from the construction from artificial materials the dynamics of earthquake has two prin- cipal effects to the soil. These are the power (force) effect due to the pulsation of the acce- leration and the reduction of the shearing stif- fness in relation to the maximum amplitude - see Fig. 1b, frequency and the time.

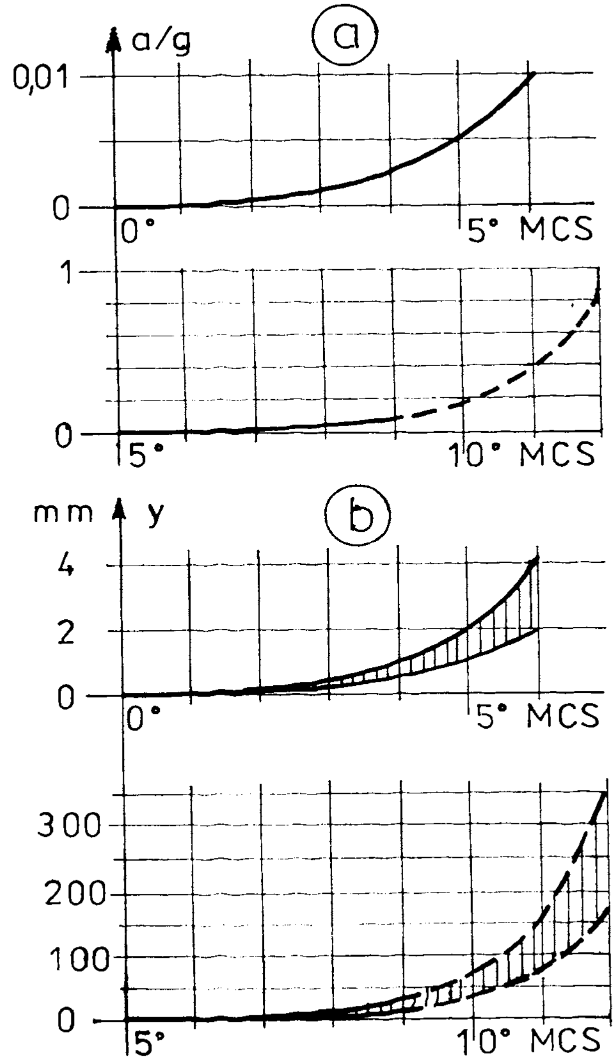


Fig. 1 Values a/g, max. y

The basic effect to the amplitude y and the acceleration a of the earthquake has the direction of the force influence. The direction can be of an arbitrary one. It is an argument in a variational principle as far as it cannot be stated in advance.

The tectonical activities of the earth surface appear by the vibrations of a various power in the vicinity of epicenter. The radius of its influence depends on many circumstances. The main quantity is the intensity of vibrations. Some another important quantity, there is the depth of the epicenter. The direction of vibrations depends on the above mentioned factors. The vertical components of vibrations dominate in the vicinity of the epicenter, horizontal components dominate in a more distant regions. Both of these factors, as well as another less important ones, can be stated in advance and this statement can be done with a succesful precision. Therefore, it is necessary to assume that the direction of vibrations is the argument of the variational tasks including the earthquake effect. Despite of the another geotechnical tasks the influence of this factor is the most principal one in the slope stability problems.

In the general case of the cohesive soil with the internal friction ( $\varphi \neq 0$) the basic change occurs by a transfer from the static state to the dynamic one, where the total shearing strength applies. The maximal shearing strength reduces its value from the top one to the residual one. The distribution of the shearing strength parameters $\varphi_{um}$ and $c_{um}$ according to Fiřt and Kysela - see Myslivec, Kysela (1975) - is shown in Figs. 2a, 2b (dry and moistured soils). These graphs are added by the relation of the similarity coefficient - see Koudelka, Procházka (1990) - on the intensity of vibrations in MCS in Fig. 2c. The ratio $\lambda_{um}/\lambda_{uf}$ characterizes the change of $\lambda$ . The distribution of the ratio states also the ratio of the total residual friction $\mathrm{tg}\,\varphi_{ur}$ and the top coefficient of friction $\varphi_{uf}$.

The distribution of Hamilton´s similarity coefficient $\Pi$ - see Koudelka, Procházka (1990) can be expressed by the relation $\Pi_{ur}/\Pi_{uf}$. Its course is identical with $c_{ur}/c_{uf}$ in Fig. 2b.

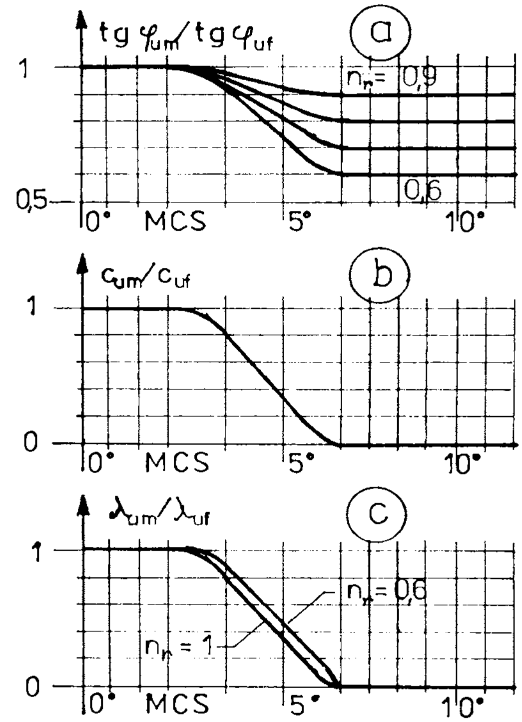


Fig. 2 Reductions of the shearing strength and the coefficient $\lambda$ due to earthquake for $n_r = \mathrm{tg}\,\varphi_{ur}/\mathrm{tg}\,\varphi_{uf}$.

## APRIORI INTEGRATION METHOD

In this chapter the possibility of use of the Apriori Integration Method will be shown on the problem of the slope stability with earthquake effect. The conception of the method originates from the connection of the classical methods (conventional methods) of the slope stability analysis and the variational formulation. We assume that the degree in MCS is given and the related geotechnical parameters are also given (the angle of internal friction $\varphi$ and the cohesion c ). Moreover, the acceleration a is given according to Myslivec, Kysela (1975).

Let two coordinate systems Oxy and Ox´y´ be given. $y = t(x)$ or $y' = t'(x')$, respectively, is the surface of the terrain, $y = f(x)$ or $y' = f'(x')$, respectively, describes the slip surface which admissible shape the part of a circle will be - see Fig. 3.

The safety factor F for the classical Sweden model on a fixed slip surface has the form:

$$F = \frac{N\,\mathrm{tg}\varphi + C}{T} \qquad (1)$$

where N and T are normal and tangential component (with respect to the slip surface) of the gravity. The most probable positon of slip surface and the value of the safety factor are defined by the following condition:

$$F_0 = \min F \qquad (2)$$

where the minimum is considered on all admissibl slip surfaces and on a set of the angles $\alpha_e$ . The expression of the members is denoted in Koudelka, Procházka (1980). In this publication the problem is solved with respect to our denotation in system Ox´y´.

Considering the earthquake effect expressed in the form of geotechnicla parameters and $\alpha_e$, the formulas will hold:

$$T(f, \alpha_e) = \iint_{f(x)}^{t(x)} \gamma(x,y)\; p(x)\; dy\, dx$$

$$N\,\mathrm{tg}\varphi(f, \alpha_e) = \iint_{f(x)}^{t(x)} \gamma(x,y)\; \mathrm{tg}\,\varphi(x)\; q(x)\; dy\, dx$$

$$C(f, \alpha_e) = \int \frac{c(x)}{q(x)} dx \qquad --- \qquad (3)$$

where   is the volume weigth,  c  is the cohesion on a unit of length and

$$p = \frac{x - x_c}{R} \quad , \quad q = \sqrt{1 - p^2}$$

where $(x_c\ y_c)$ are coordinates of the center of the slip surface and R is its radius. Because the relations (3) are formally the same (in 0xy) as the relations of model without earthquake effect (in 0x´y´), the means of the Apriori Integration Method can be used to the explicite expression of the functionals in terms of six functions $F_1, \ldots, F_6$ - see Koudelka, Procházka (1980). Thus, the very accurate and reliable minimization (2) can be proceeded.

## VARIATIONAL SOLUTION

The complexity of the variational solution of the slope stability with the earthquake effect follows from the highten number of variational arguments, as it was described above. In the plane task the variational formulation without earthquake includes eight variables (height h, angle of slope $\beta$, $\gamma$ , c , tg$\varphi$ , $x_c$ , $y_c$ , R ), which number can be reduced to five using the similarity coefficient $\lambda = c/(\gamma\ h\ tg\varphi\ )$ - Koudelka, Procházka (1984 a,b), (1987 a,b), (1990). It was proved that the similar relation hold also in case of homogeneous general slopes - see Koudelka, Procházka (1990). In Koudelka, Procházka (1990) the heterogeneous general slopes are discussed.

The earthquake effect brings some other variational arguments: a - acceleration of earthquake vibrations, $\alpha_a$ - direction of vibrations, (in 3D tasks $\alpha_{a1}$ , $\alpha_{a2}$) and the changes of material properties of soils ( $\varphi_{um}$, $c_{um}$ - shearing strengths).
The simplest task of slope stability has at least eleven variables and after reduction eight arguments.

With respect to the material properties of soils which are projected to the mathematical relations it is suitable to part the solution split to two parts: for noncohesive soils (c = 0) and for the cohesive soils (c ≠ 0).

## SOLUTION FOR NONCOHESIVE SOILS

This solution holds for all sorts of soils under stronger earthquake because all solids under vibrations about 6° MCS behave as noncohesive. Fig. 4a shows the relations of the strength influence of an element in a vicinity of slope surface its safety factor under earthquake effect $F_{eo}$ can be expressed as follows

$$F_{eo} = \frac{N_g - N_a}{T_g - T_a}\ tg\ \varphi_{um} = \frac{N_e}{T_e}\ tg\ \varphi_{um} \qquad (4)$$

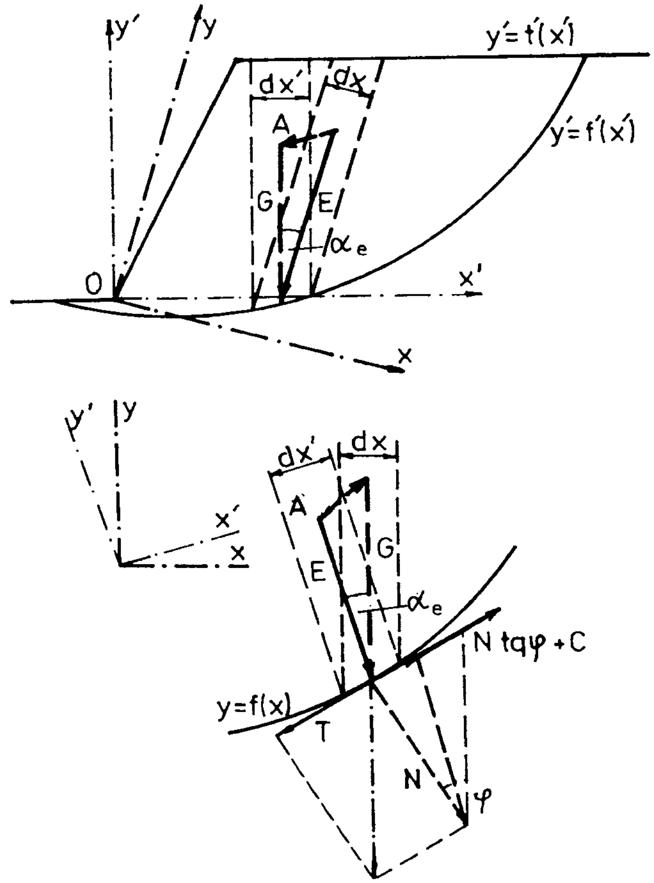


Fig. 3 Model of slope and the acting forces by earthquake

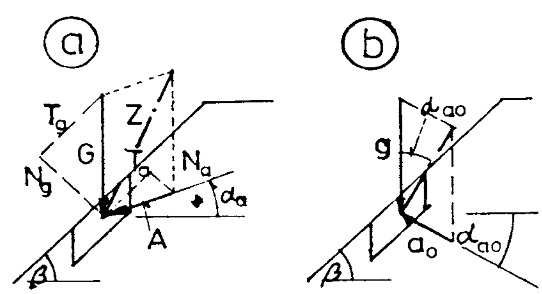


Fig. 4 Strength influence of earthquake

After setting according to Fig. 4 and from the condition of extrem of $F_e$ in relation to $\alpha_a$ we obtain

$$F_{eo} = \frac{\cos\beta\ - \frac{a}{g}\sin\ (\beta - \alpha_a)}{\sin\beta\ + \frac{a}{g}\cos\ (\beta - \alpha_a)}\ tg\ \varphi_{um} \qquad (4´)$$

From equation (4´) it follows that the variational solution depends on $\alpha_a$ , a , $\beta$ ,and tg $\varphi_{um}$ only (g being constant). For the degree of earthquake (which implies a ) and a sort of soil (which implies tg $\varphi_{uf}$ ) the change of properties according to Fig. 2 can be defined (this can be obtained also by means of experiment). The change of properties can deliver tg $\varphi_{um}$ . The variational task gives the most dangerous direction of vibrations ( $\alpha_a$), thus $\alpha_{ao}$ .

In virtue of derivation of $F_{eo}$ with respect to $\alpha_a$ one obtaines

$$\frac{dF_e}{d\alpha_a} = \left[ \frac{a}{g} \cdot \frac{\frac{a}{g} + \sin \alpha_a}{(\sin \beta + \frac{a}{g} \cos(\beta - \alpha_a))^2} \right] \mathrm{tg}\, \varphi_{um} \tag{5}$$

$\alpha_{ao}$ is the solution of $\frac{dF_a}{d\alpha_a} = 0$ .

If we do not consider the trivial solutions for $a/g = 0$ and $\mathrm{tg}\, \varphi_{um} = 0$ the solution is as follows:

$$\sin \alpha_{ao} = - \frac{a}{g} \tag{6}$$

under the condition that

$$\sin \alpha_{ao} \neq \pm \sqrt{1 - \frac{A^2}{1 - A^2 + A^4}} , \tag{7}$$

where $A = \frac{a}{g} \mathrm{cotg}\, \beta$ .

The formula (6) expresses that the lowest value of the safety factor $F_{eo}$ is attained when the deviation of the resultant of forces from the earthquake (gravity and earthquake) from the vertical direction is maximum. This result confirms the assumptions of the basic formulation. The horizontal effect of vibrations is not the most dangerous as sometimes is uncorrect considered.

## SOLUTION FOR COHESIVE SOILS

On basis of the recent papers Koudelka, Procházka (1987). (1988), (1989) and (1990) the safety factor can be expressed similarly as in the case without the earthquake effect. The transformation of the task leads to the relation:

$$F_{eo} = \Pi_{um} F_{ole} \tag{8}$$

instead of $F_{go} = \Pi_{uf} F_{olg}$ , where $\Pi_{um} = c_{um}/\gamma h$ and $\Pi_{uf} = c_{uf}/\gamma h$, $F_{olg}$ and $F_{ole}$ are the unit stabilties (stability numbers) of the model without earthquake effect and the model with the earthquake effect, respectively. For the simple slopes these quantities can be tabulated.

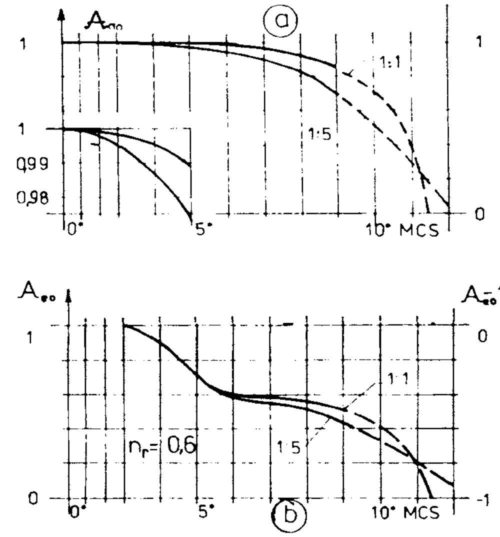


Fig. 5 Coefficients of power effect $A_{ao}$ and total coefficients of earthquake effect $A_{eo}$ of slopes 1:1 and 1:5.

## EXAMPLE

Fig. 5 displays the relation between coefficients $A_{ao}$ and $A_{eo}$ and the intensity of vibrations in degree MCS for noncohesive soils. The relations are:

$$F_{eo} = A_{ao} A_{\varphi} \frac{\mathrm{tg}\, \varphi_f'}{\mathrm{tg}\, \beta} = A_{eo} F_o , \quad A_{\varphi} = \frac{\mathrm{tg}\, \varphi_m'}{\mathrm{tg}\, \varphi_f'} . \tag{9}$$

The coefficient $A_{eo}$ also expresses the relation between safety factors with and without earthquak effect, respectively.

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