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Electrical Characterization of Soil for In-Situ Measurement of Liquefaction Potential

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SYNOPSIS A new method for characterizing the fundamental sand properties with electrical parameters is described. Correlations are established between the electrical parameters and relative density, D_r , cyclic stress ratio, τ/σ'_o , and the parameter $K_{2 \max}$.

An electrical probe, used to measure the electrical parameters in situ, is described. Field measurements, taken with the probe at one of the 1906 San Francisco earthquake sites, indicate that this is a viable alternative for the in situ evaluation of liquefaction potential.

INTRODUCTION

Over the past ten years, the phenomenon of soil liquefaction during earthquakes has become one of the major topics in geotechnical engineering research. Laboratory cyclic tests have been used to develop procedures for the assessment of liquefaction potential of sites underlain by sand deposits (Seed, 1979). These procedures have been based upon the assumption that the resistance to liquefaction of a given sand is a function of its relative density. Recently, however, extensive laboratory studies have shown that a number of factors other than relative density affect liquefaction potential. These include the soil fabric or structure (Ladd, 1974 and 1977, and Mulilis, et al, 1975), degree of overconsolidation (Seed and Peacock, 1971), soil stress history (Seed, 1976) and cementation of particles (Seed, 1979). In order to correctly assess the liquefaction potential of a given soil, these factors must be considered in addition to the relative density. At present, the most widely accepted procedure for the assessment of liquefaction potential is that suggested by Seed (1979). The method is based upon a relationship between the stress ratio, τ/σ'_o , required to cause

liquefaction and standard penetration resistance, N , obtained from the Standard Penetration Test (SPT). This relationship has been established through an extensive collection of SPT data from sites all over the world where liquefaction has occurred. The relationship is shown in Fig. 1. This chart can be used to predict liquefaction potential from SPT measurements. The main advantages of this method are that it is rapid, it is based upon a large amount of field data, and it is widely used and accepted. The main disadvantages are that SPT measurements are not always reliable, the penetration resistance, N , is a scalar quantity and so its relationship with structural properties is questionable, and during the penetration test, soil structure is destroyed and, therefore, cannot be accurately measured.

Attempts have recently been made to correlate the stress ratio required to cause liquefaction with penetration resistance, q_c , measured by the Cone Penetration Test (CPT).

An empirical correlation of this type, based on data from Zhou (1980) is shown in Fig. 2. The advantages of this method are that continuous monitoring of q_c is possible, thin seams of liquefiable soil can be detected, reproducibility of q_c values is good and the CPT is a widely accepted test. The disadvantages are that q_c values are difficult to obtain in dense sands and, as with the SPT method, soil structure is destroyed during penetration.

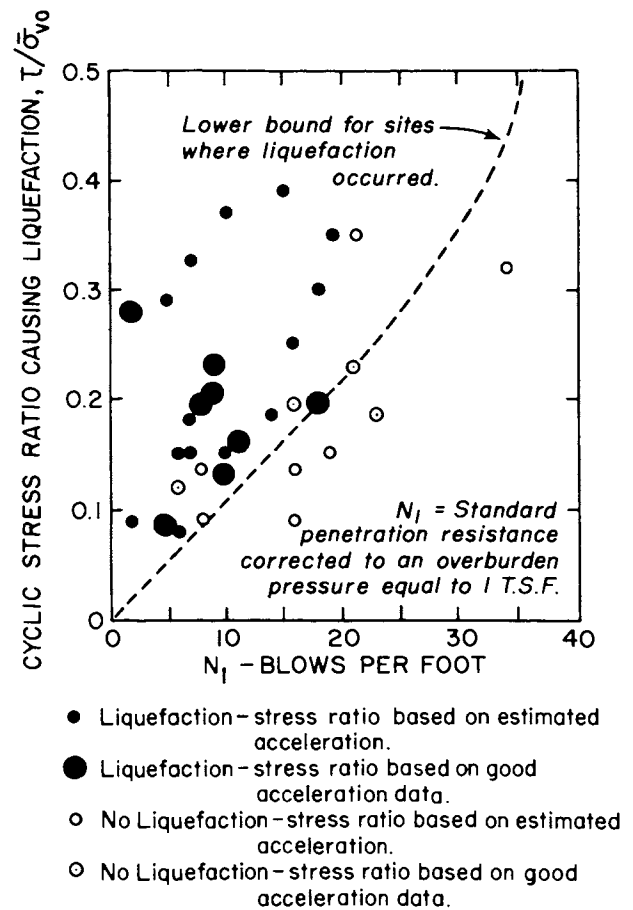


Fig. 1 Correlation between Stress Ratio causing Liquefaction in Field and Penetration Resistance of Sand (after Seed, et al, 1979)

A third method for the assessment of liquefaction potential is based upon a correlation between stress ratio and dilation angle, v , measured by the pressuremeter (Vaid et al, 1981). The relationship is shown in Fig. 3. This correlation was established from a relationship between relative density and dilation angle.

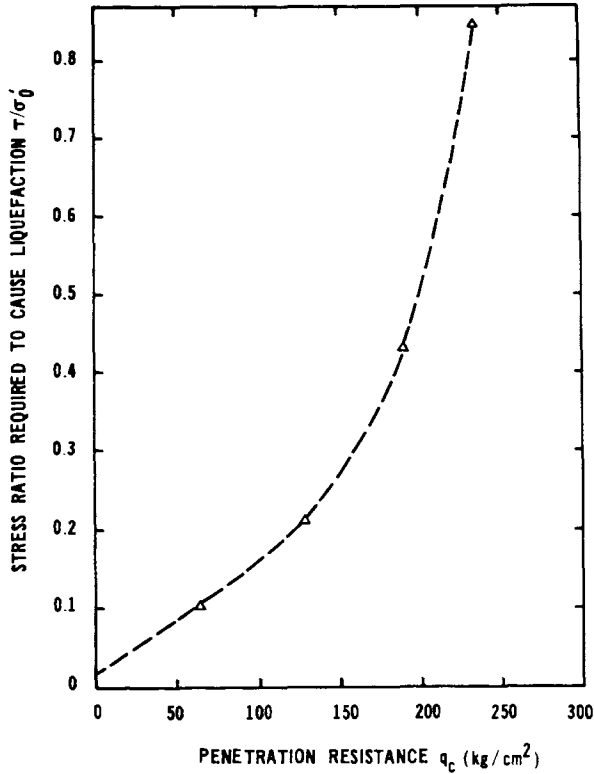


Fig. 2 Correlation between Stress Ratio causing Liquefaction and Cone Penetration Resistance (after Zhou, 1980)

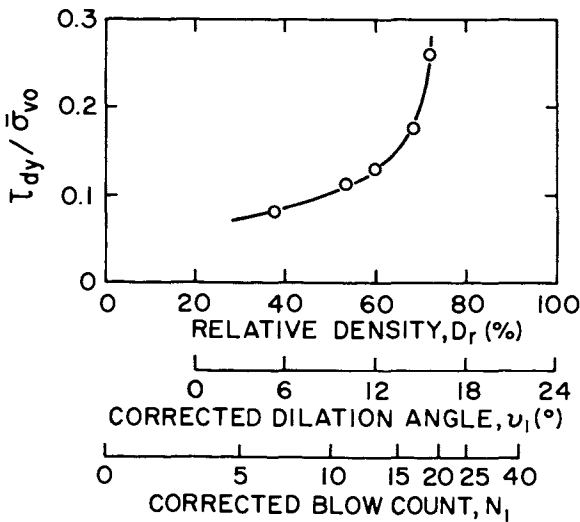


Fig. 3 Liquefaction Resistance of Sand as a Function of Relative Density, Dilation Angle and Penetration Resistance (after Vaid, et al, 1981)

Although the correlation appears to be good, the use of relative density alone as a measure of liquefaction potential has been questioned. Peck (1979) pointed out that, unless the additional factors influencing liquefaction are allowed for, the resistance to liquefaction in the field may be appreciably greater than that predicted, making estimates unnecessarily conservative and expensive. Apart from this criticism, the main disadvantage of the method is that there is a lack of field

data to support it. However, the pressuremeter does have one advantage over other in-situ testing methods, that the parameter which it measures in the field, dilation angle, can also be measured in the laboratory. This enables direct comparison of field and laboratory data.

Recently, Dobry et al (1980) have proposed a method for the evaluation of liquefaction potential based upon the cyclic shear strain developed during an earthquake. This method requires the measurement of shear wave velocity, V_s , to determine maximum shear modulus, G_{max} , from the expression

$$G_{max} = \rho V_s^2 \tag{1}$$

where ρ = mass density of the soil layer under investigation. The shear wave velocity is usually measured by the cross-hole technique. The main disadvantage with this is that values of V_s cannot be obtained in the upper 5-10 feet of soil.

A new method for the in-situ measurement of liquefaction potential is proposed. This method is based on the electrical properties of soil, which are dependent upon not only relative density, but also other factors influencing liquefaction, such as soil structure, stress history and cementation. The method is extremely powerful, since it can be applied using both the stress and the strain approach.

Electrical Characterization of Sand

Extensive studies carried out by Archie (1942), Arulanandan (1975), Arulanandan and Kutter (1978), Arulmoli (1980), Kutter (1978), and Wyllie and Gregory (1953) have shown that a non-dimensional electrical parameter, the Formation Factor, is dependent upon particle shape, size distribution, long axis orientation and contact orientation and also void ratio, degree of saturation and cementation. The formation factor, F_i , is defined as the ratio of the conductivity, σ_s , of solution saturating a sand sample, to the conductivity, σ_{mi} , of the mixture along some direction, i . The average formation factor, \bar{F} , analogous to the mean normal stress is given by

$$\bar{F} = \frac{F_V + 2F_H}{3} \tag{2}$$

where F_V and F_H are the vertical and horizontal formation factors respectively.

An integration technique proposed by Bruggeman (1935) for spherical particles was extended to randomly oriented ellipsoids by Meredith (1959), who showed that the formation factor, F , is related to porosity, n , and a shape factor, f , by

$$F = n^{-f} \tag{3}$$

Similarly, the relationship between average formation factor and porosity can be written as

$$\bar{F} = n^{-\bar{f}} \tag{4}$$

where \bar{f} is the average shape factor. A mean value of the average shape factor, \bar{f}_{mean} , can also be defined as

$$\bar{f}_{mean} = \frac{1}{2} (\bar{f}_{max} + \bar{f}_{min}) \tag{5}$$

where \bar{f}_{max} and \bar{f}_{min} are the extreme values of \bar{f} at extreme porosities.

The Anisotropy Index, A , was introduced by Arulanandan and Kutter (1978) and defined as

$$A = \left(\frac{F_V}{F_H} \right)^{1/2} \quad (6)$$

Lastly, an index called Relative Packing, P_r , was introduced by Arulmoli (1981) and defined as

$$P_r (\%) = 100 \frac{\bar{F} - \bar{F}_{\min}}{\bar{F}_{\max} - \bar{F}_{\min}} \quad (7)$$

where \bar{F}_{\max} and \bar{F}_{\min} are the extreme values of \bar{F} at extreme porosities.

Factors Influencing the Electrical Parameters

To study the effect of the various sand properties on the electrical parameters, formation factor measurements were made in the laboratory using two 6 in. cubical cells. One of these had two vertical electrodes fixed to opposite inside walls of the cell for measurement of F_H and the other had electrodes fixed to the upper and lower inside faces for measurement of F_V .

The single most important sand property which affects the formation factor is porosity. The vertical and horizontal formation factor-porosity relationships for Monterey sand prepared by three different methods are shown in Fig. 4. It

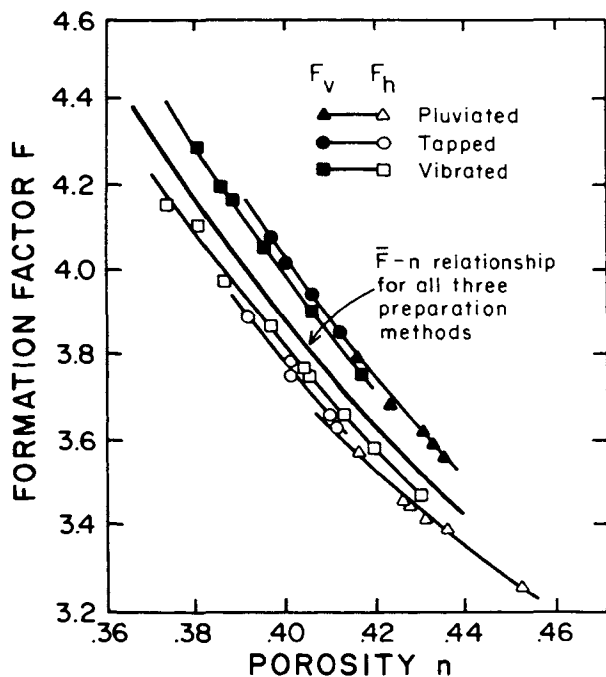


Fig. 4 Vertical, Horizontal and Average Formation Factor vs. Porosity Curves for Monterey '0/30' Sand Prepared by Three Different Methods

can be seen that vertical and horizontal formation factors both decrease as porosity increases for all three preparation methods. It can also be seen that tapping and vibration of samples, methods which tend to align particles vertically, cause a decrease in F_V and an increase in F_H . Thus, a change to a more vertical orientation of particles decreases the

anisotropy index, A , since $A = (F_V/F_H)^{1/2}$. The anisotropy index can, therefore, be used to characterize particle orientation. Arulmoli (1980) has also shown the dependence of A upon contact orientation or 'packing'.

The average formation factor, \bar{F} , has been shown, theoretically, to be independent of particle orientation (Dafalias and Arulanandan, 1979) and thus is a useful parameter to characterize porosity. To illustrate this, the average formation factors were calculated from the curves in Fig. 4 and plotted on the same diagram as shown. It was found that the \bar{F} - n relationship was unique for all three preparation methods.

Another sand property which influences the formation factor is particle shape. It has been shown, experimentally, that the more angular the particle, the higher will be the formation factor (Arulmoli, 1980 and Wyllie and Gregory, 1953). It has also been shown theoretically that the slope of the Log \bar{F} versus Log n line increases with angularity (Dafalias and Arulanandan, 1979). This slope is characterized by the average shape factor, \bar{f} . Experimental evidence tends to support the relationship between angularity and \bar{f} although this is not conclusive.

The effect of cementation on formation factor was shown by Wyllie and Gregory (1953). They showed that cementation had the same effect as decreasing porosity, that is the formation factor was increased.

The effect of degree of saturation on formation factor was shown by Kutter (1978). He showed that a decrease in the percent saturation caused an increase in the formation factor.

Lastly, it should be pointed out that since particle orientation is affected by changes in confining stress, then the formation factor should also be dependent upon confining stress. This has not, however, been confirmed experimentally.

In summary, it has been shown, theoretically and experimentally, that porosity, particle orientation, shape, cementation and degree of saturation can be quantified in terms of the electrical parameters, \bar{F} , \bar{f} and A . It has also been shown by Arulmoli (1981) that the Relative Packing, P_r , is uniquely related to relative density, D_r , for all uniform sands. This relationship is shown in Fig. 5. The electrical

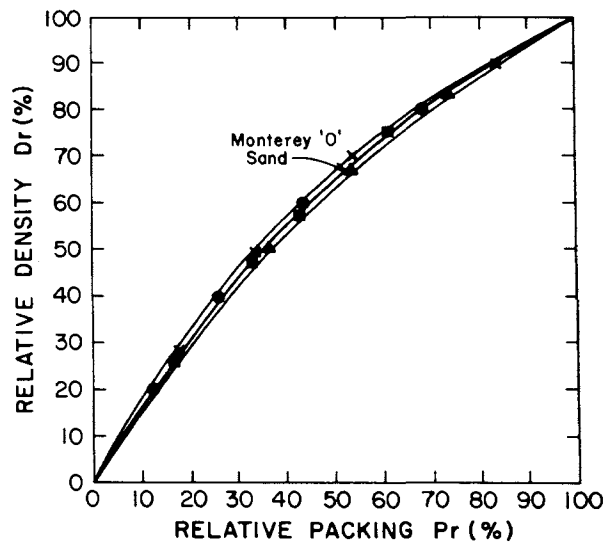


Fig. 5 Relationship between Relative Density and Relative Packing for Uniform Sands (after Arulmoli, et al, 1981)

parameters are, therefore, a very useful means of characterizing the fundamental grain and aggregate properties of sand which influence its mechanical behavior.

USE OF ELECTRICAL INDICES IN THE ASSESSMENT OF LIQUEFACTION POTENTIAL

So far, it has been shown that the electrical parameters \bar{F} , \bar{f}_{mean} and A are dependent upon the important grain and aggregate properties of sand which influence its mechanical behavior. It is, therefore, reasonable to assume that there should be some correlation between the electrical parameters and certain mechanical properties of sand such as liquefaction potential, friction angle, permeability and compressibility. The assessment of liquefaction potential by use of the electrical parameters will now be investigated.

Assessment of Liquefaction Potential Using the Stress Approach

To investigate the relationship between the stress ratio required to cause liquefaction, τ/σ'_0 , and the electrical parameters, a site which liquefied during the 1906 San Francisco earthquake was chosen for study. The site chosen was Lawson's Landing near Bodega Bay. Samples were taken from the site and reconstituted in the laboratory, where electrical measurements were made and cyclic triaxial tests carried out. A relationship was established between τ/σ'_0 and the combined electrical parameter $(\frac{A}{\bar{F}})^2 \frac{1}{\bar{f}_{\text{mean}}}$. This

relationship varied, depending on the number of cycles to liquefaction (representing a certain magnitude of earthquake). The relationships representing a 7.5 and 8.25 magnitude event are shown in Fig. 6. Thus, by measuring the electrical

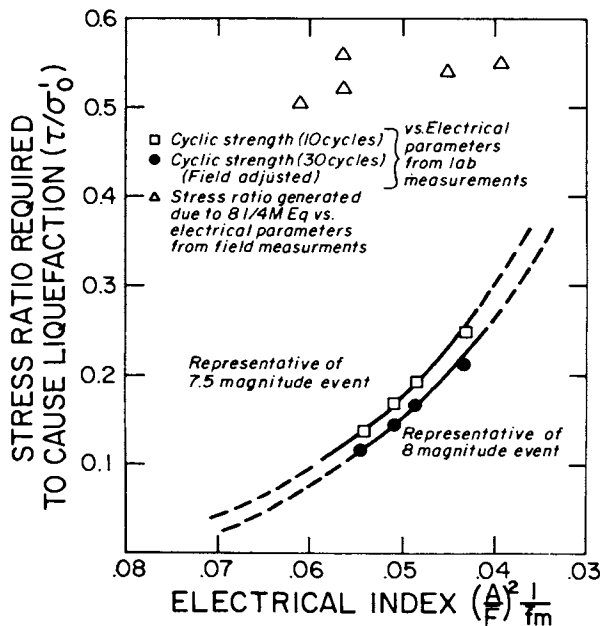


Fig. 6 Correlation between Stress Ratio and the Electrical Index for Two Different Earthquake Magnitudes (after Kleinfelder, 1981)

properties of a certain sand, the stress ratio required to cause liquefaction can be determined from Fig. 6. A correlation of this type has also been established by Arulmoli et al (1981) with various sands for which the liquefaction characteristics have been extensively studied and reported.

Assessment of Liquefaction Potential Using the Strain Approach

The method proposed by Dobry et al (1980) for the evaluation of liquefaction potential is based upon an estimation of the threshold peak ground surface acceleration, $(A_p)_t$, given by:

$$(A_p)_t = 1.154 \times 10^{-4} \frac{G_{\text{max}}}{\sigma_v r_d} \quad (8)$$

where G_{max} = the maximum value of shear modulus for the soil, σ_v is the total vertical stress, and $r_d \leq 1$ is the soil flexibility coefficient defined by Seed and Idriss (1971). The design earthquake acceleration for the site, A_p , is then compared with $(A_p)_t$ to determine whether or not liquefaction will occur. This method requires measurement of G_{max} which is usually carried out using the cross-hole technique. However, Arulmoli et al (1981) have shown that G_{max} can also be obtained by electrical measurement. G_{max} is given by:

$$G_{\text{max}} = 1000 K_{2\text{max}} (\sigma'_m)^{1/2} \quad (9)$$

where $K_{2\text{max}}$ is a parameter largely dependent on relative density and σ'_m is the mean effective confining pressure. Since the unique relationship between P_r and D_r has already been shown in Fig. 5, a relationship between $K_{2\text{max}}$ and P_r can be

developed. This correlation has been verified in the laboratory and in the field by Kleinfelder and Associates (1981). Fig. 7 shows a schematic representation of the laboratory cross-hole measurement of shear wave velocity. Field measurements of shear wave velocity were obtained by the down-hole technique. These are shown in Fig. 8. Electrical measurements were also made to determine the relative packing, P_r . These laboratory and field measurements, together with data obtained from other sources, were used to establish the relationship between $K_{2\text{max}}$ and P_r as shown in Fig. 9. Thus, electrical measurements can be used to evaluate G_{max} and predict liquefaction potential using the strain approach.

It has, so far, been shown that certain correlations exist between the electrical parameters and certain important soil properties such as cyclic stress ratio required to cause liquefaction and the parameter $K_{2\text{max}}$. To make use of the

correlations for prediction of liquefaction potential in the field, the electrical measurements should be made in situ. An electrical probe may be used for this purpose.

AN ELECTRICAL PROBE

An electrical probe, Geoelectronics Model GE-100, can be used to make electrical measurements on soils in situ (Arulanandan, 1977). Details of the probe and its operation are given by Arulmoli et al (1981). To justify the use of this instrument for prediction of liquefaction potential, it was first checked in the laboratory and then used to take measurements in the field.

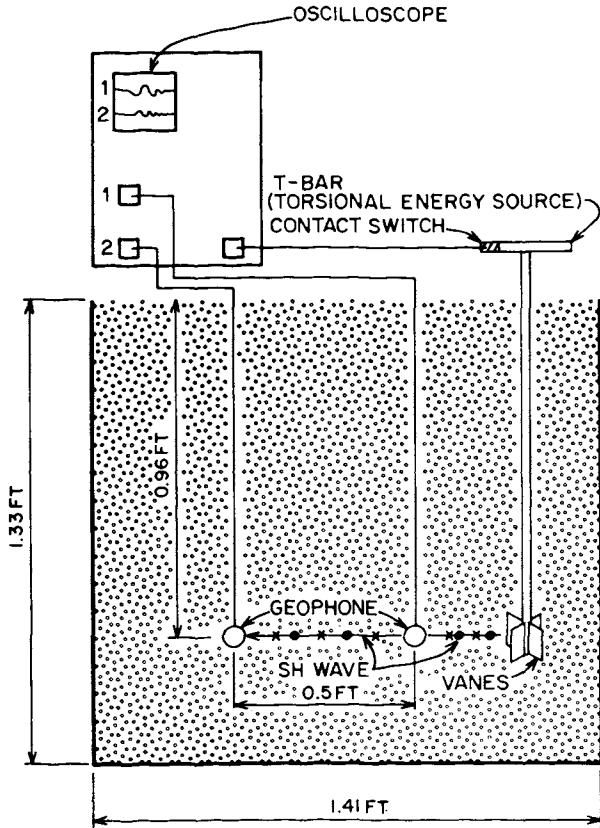


Fig. 7 Schematic Representation of the Cross-Hole Laboratory Test (after Kleinfelder, 1981)

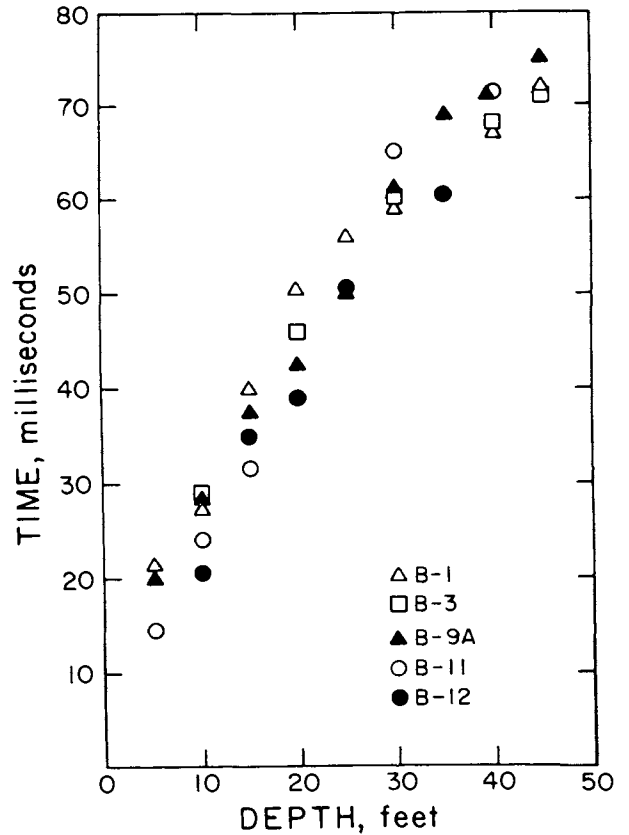


Fig. 8 Shear Wave Velocity Measurements Obtained in the Field by the Down-Hole Technique (after Kleinfelder 1981)

Laboratory Verification of Probe Measurements

The probe was used in the laboratory to measure values of average formation factor for samples of Monterey sand. The $F-n$ relationship shown in Fig. 4 was used to predict values of porosity which were then compared with the prepared porosities. The agreement was found to be within acceptable accuracy. Details are given by Arulmoli et al (1981), who also showed that the probe could predict relative density and stress ratio required to cause liquefaction with acceptable accuracy.

Applicability of the Probe to Field Measurement

Formation factor measurements were made at various depths with the probe at the Lawson's Landing site. The variation of average formation factor, \bar{F} , with depth and the associated values of relative density, D_r , obtained from the $D_r - P_r$ relationship of Fig. 5 are shown in Fig. 10. The stress ratio generated by a proposed 8.25 magnitude earthquake was then determined at each depth using the simplified procedure given by Seed and Idriss (1971). These values of stress ratio were then plotted against the combined electrical parameters as shown in Fig. 6. It can be seen that the probe would indicate a definite susceptibility to liquefaction of the site.

As a comparison, SPT measurements were made at the site and plotted on Seed's chart as shown in Fig. 10. This method indicates a moderate susceptibility to liquefaction. Kleinfelder and Associates (1981) also carried out analyses using the strain approach (Dobry et al, 1980) and the analytical procedure proposed by Seed and Idriss (1967). Both of these methods indicated that the site was susceptible to liquefaction.

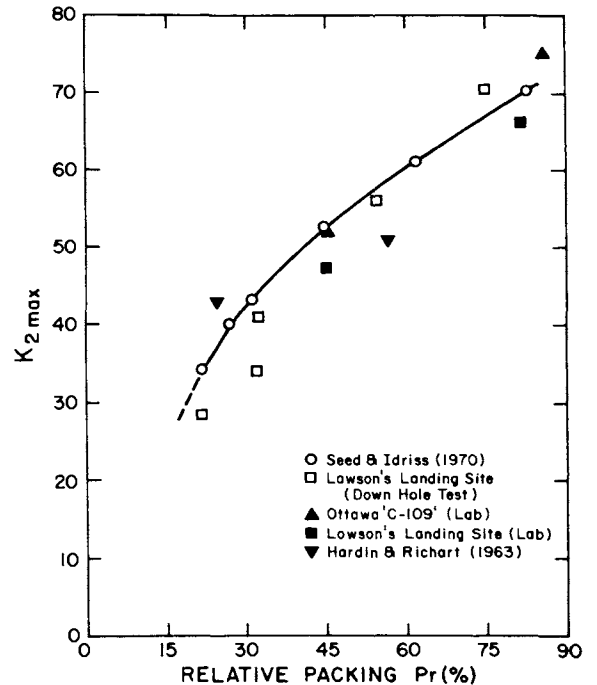


Fig. 9. Correlation Established between K_{2max} and Relative Packing, P_r

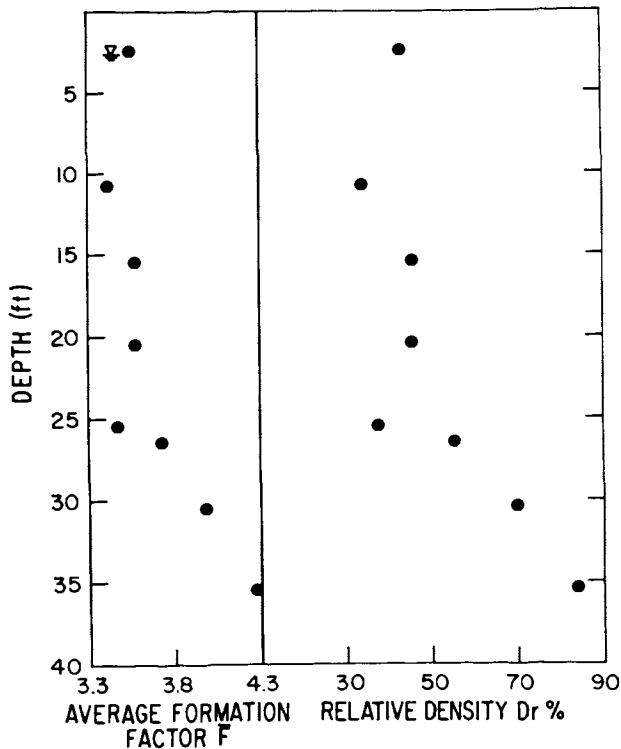


Fig. 10 Variation of Average Formation Factor and Relative Density with Depth

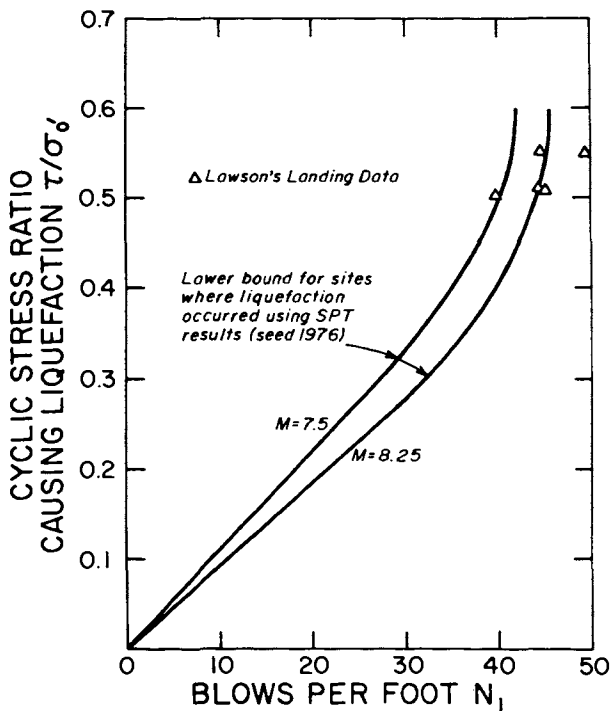


Fig. 11 Use of SPT Values Obtained in the Field to Predict Liquefaction Potential

SUMMARY AND CONCLUSIONS

A non-destructive electrical method for indexing the grain and aggregate properties of sand has been presented.

Three electrical parameters, \bar{F} , A and \bar{f}_{mean} have been introduced. The average formation factor, \bar{F} , has been shown to uniquely define the porosity of uniform sands; the anisotropy index, A , enables quantification of particle orientation; and the mean value of average shape factor, \bar{f}_{mean} , characterizes particle shape. Correlations between these parameters and D_r , τ/σ'_0 and $K_{2\text{max}}$ were presented.

An electrical probe, capable of measuring the electrical parameters in situ, was introduced. Based upon the previously established correlations, the probe was used to predict the liquefaction susceptibility of the Lawson's Landing site. The probe compared favourably with other methods for predicting liquefaction potential and thus offers a viable alternative for in situ site evaluation.

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