IDENTIFYING UNDAMAGED-BEAM STATUS BASED ON SINGULAR SPECTRUM ANALYSIS AND WAVELET NEURAL NETWORKS

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Abstract. In this paper, the identifying undamaged-beam status based on singular spectrum analysis (SSA) and wavelet neural networks (WNN) is presented. First, a database is built from measured sets and SSA which works as a frequency-based filter. A WNN model is then designed which consists of the wavelet frame building, wavelet structure designing and establishing a solution for training the WNN. Surveys via an experimental apparatus for estimating the method are carried out. In this work, a beam-typed iron frame, Micro-Electro-Mechanical (MEM) sensors and a vibration-signal processing and measuring system named LAM_BRIDGE are all used.

Keywords. Singular spectrum analysis, frequency-based filter, wavelet neural networks, identifying structure

1. INTRODUCTION

Automatic monitoring structural condition during its service life which is also called Structural Health Monitoring (SHM) is a necessary work. The core of any health monitoring system is the ability to automatically identify structural damages consisting of localizing, predicting damages occurred in the structure and also estimating severity of them [1]. These are in terms of response and performance under operational and environmental loadings in real time based on feedback information from the structure. In SHM systems, as usual, the current structure's status is compared with its undamaged condition to realize changes in dynamic response of the structure at these two times [2, 3]. Hence, undamaged-structure identifying at the initial time is firstly carried out to build a database of the intact structure for comparing and estimating. To do well this target, there are two related key factors. The first one is building exact datasets while the other relates to identifying the structure via these data sets. Recently, for these factors, in order to improve the ability to face with noise and multi-dimensions of the databases, to face with uncertainty and nonlinear aspects of structure's dynamic response, SSA, wavelet transform and neural networks are all used widely, independently or associatively.

Regarding data sets used for identification, as usual, they are measured data sets with noise. The noise in these relates to many reasons such as the precision of the measurement devices, noise on the measurement devices, environmental conditions of the measurement devices, and unknown nonlinear characteristics of the actuators [4]. Hence filtering noise and using signal-analysis tools are carefully treated. One of the effective approaches to handle this problem is to use SSA which is considered as a new non-parametric tool for data analysis [5, 6]. SSA is a technique for time series analysis based on the principles of multivariate statistics. It decomposes a given time series into a set of independent additive time series and analyzes them via frequency. Based on a procedure of principal component analysis, the method projects an original time series onto a vector basis obtained from the series itself. In this process, the set of these series can be seen as a slowly varying trend representing the signal mean at each instant. As a result, noise which can be filtered from the original data set together with the special features of the structure expressed by vibration signals can be extracted [5].

Related to the identifying, it is a process of modeling an unknown system based on a data set of input–outputs. For mechanical structures, this is done in the form of identifying structural parameters such as stiffness, vibration factors such as displacement, frequencies, mode shapes, and damping ratios, and stress and so on [7–11]. Since most of real physical systems are nonlinear, ill-defined and uncertain, hence it is difficult to directly establish models by conventional mathematical means [12]. In addition, as above mentioned, in order to estimate the status of a structure based on data-driven methods, one has to establish a correlative relation at two times: the undamaged-structure time and check-in time. This is really difficult if this process is based on traditional ways because it is not able to exactly repeat an exciting status at two different times. Hence mathematical models including artificial neural networks (NN) technique are frequently used to overcome the impending challenges with different degrees [13–15]. In [13], an NN was used for identifying and predicting the onset of corrosion in concrete bridge decks taking into account parameter uncertainty. In [14] an NN model was used to infer the location and the extent of structural damages. Another method deals with the structural damage detection using measured frequency response functions (FRFs) as input data for NN presented in [15]. In the data sets, the compressed FRFs were used as the NN input variables instead of the raw FRF data while the output was a prediction for the actual state of the structure. A further advantage of this particular approach was found to be the ability to deal with relatively high measurement noise.

The wavelet transform method analyzes the signal into two dimensions, time-frequency or space-frequency by using mother functions with two parameters a, and b. The first one, a, called the scale can establish a variable width- window which can play a role similar to frequency while the other, b, called the location parameter can change analyzed location. As a result, wavelet transforms can be able to locally analyze signals to find out irregular events in dynamic response signals such as vibration signals of the damaged structure in both time or frequency domain [16]. It is known that NN is a powerful tool for handling problems via data sets of large dimension. Nevertheless, the implementation of NN suffers from the lack of efficient constructive information related to both, determining the parameters of neurons and choosing network structure. The combining of wavelets and neural networks such as in the term of wavelet neural networks (WNN) is a solution for improving partly the above issues. Based on this combining, wavelets and neural networks can remedy the weakness of each other. Recently, WNN has been proposed in various works, including managing health of structures [17–21]. In [21], prediction and identi?cation of nonlinear dynamical systems based on WNN models were shown. In these models, the traditional-fuzzy rules from Takagi–Sugeno–Kang fuzzy system were established by participating of wavelet basis functions that have the ability

to localize both in time and frequency domains.

Consequently, in this paper, a solution for identifying undamaged-beam status based on SSA and a WNN is presented. First, a database is built from measured data sets and SSA which works as a frequency-based filter. A mathematical tool for identifying is then designed via the WNN techniques. For this work, establishing a wavelet frame such that the size of WNN to be reduced, designing a structure of the WNN based on the created wavelet frame and giving an appropriate solution for training the WNN are all carried out. In the WNN, each wavelet works as a neuron for both, processing and storing data. Effectiveness of the proposed method is estimated from surveys via an experimental apparatus consisting of a beam-typed iron frame, MEM sensors and a vibration signal processing and measuring system named LAM_BRIDGE made by the Lab. of Applied Mechanics (LAM), HCM City University of Technology.

2.   BUILDING DATABASE VIA SSA

To extract information correlated with system response, the main steps are performed as follows. First, SSA builds a matrix, called the trajectory matrix from the original time series in a process called embedding. This matrix consists of vectors obtained by means of a sliding window that traverses the series. The trajectory matrix is then subjected to singular value decomposition (SVD) which decomposes the trajectory matrix into a sum of unit-rank matrices known as elementary matrices. Each of these matrices can be transformed into a reconstructed time series by a process known as diagonal averaging. The obtained time series is called principal components in which the sum of all the principal components is equal to the original time series. In next step known as grouping, the selection of the principal components that represent the trend of the signal is carried out. By using this step, the principal components are selected with which to reconstruct the trend of the original series. In this work, to perform automatically the selection of the principal components that represent the trend, the frequency spectra of each principal component is transformed and compared. As a result, the components which concentrate most of their power in the lowest frequency ranges represent the trend of the signal [5].

In order to establish a database via the measured data sets, in this paper, SSA for extracting information correlated with dynamic response state of mechanical systems is used. It can be observed that the role of this work is as a filtering process. As a result, only vibration signals expressing vibration features have contributed to the database. In other words, in principle, the trend of a time series can be described as a function that re?ects slow, stable, and systematic variation over a long period of time. Hence, by selecting the principal components that represent the trend via the frequency spectra of each principal component will rebuild the data set conveying the special features of structure dynamic response. The sign for this selection is the power in the lowest frequency ranges as mentioned in [6]. Besides in case that vibration frequency is known, this frequency is, of course, a special sign for this arm. This content will be detailed in section 4.3.

3.   DESIGNING THE WNN

3.1.   Mathematical formulation

Let T_Σ be the filtered data set by SSA consisting of P input-output data pairs $(\bar{x}_h, y_h)$, $\bar{x}_h \in \Re^n, y_h \in \Re^1$, expressed by an unknown function f at points $\bar{x}_h, y_h = f(\bar{x}_h)$ in which $\bar{x}_h = [x_{h1} x_{h2} ... x_{hn}]$.

The dynamic system function $f(\bar{x}_h)$ is approximated by the wavelet transform functions and wavelet coefficients, in a general form as follows [22–26]:

$$\bar{f}(\bar{x}_h) = \sum_{i=1}^{M} W_i \sum_{a,b}^{\psi_{a,b}} (\bar{x}_h) \tag{1}$$

In (1), M is the number of wavelets which will be discussed in the next subsection, W_i, $i = 1...M$, represents the discrete wavelet transform; $\psi_{a,b}(.)$ is the two-dimensional wavelet expansion functions obtained from the mother wavelet function $\phi(t)$ by simple scaling a and translation b as below:

$$\psi_{a,b}(\bar{x}_h) = K\phi\left(\frac{\bar{x}_h - a}{b}\right), \quad a, b \in \Re^M \tag{2}$$

in which, K is a positive parameter.

To satisfy the orthogonality condition, the mother wavelet function has to satisfy restricting constraints: the integral of the wavelet must be equal to zero and integral of the square of the wavelet function must be equal to one. Such constraints make the orthogonal wavelets non-differentiable. In the proposed WNN model, errors between the approximated and actual outputs are minimized using a mathematical optimization approach which requires derivatives of the wavelet function. As such, a non-orthogonal differentiable wavelet function, the Mexican hat function (the Ricker wavelet) [24], is used in the WNN model.

The value of the Ricker wavelet function corresponding to h-the data sample is calculated by

$$\phi(t_{hk}) = \frac{1}{\sqrt{2\pi}\sigma^3}(D - \frac{t_{hk}^2}{\sigma^2})\exp\left(-\frac{t_{hk}^2}{2\sigma^2}\right). \tag{3}$$

Parameters in (3) are calculated as follows:

$$t_{hk} = \frac{\sum_{j=1}^{n}|w_{kj}x_{hj} - b_k|}{a_k}, \quad h = 1...P; \ k = 1...M; \tag{4}$$

$$\sigma = \frac{\sum_{h=1}^{P}\sum_{k=1}^{M}\bar{t}_{hk}}{PM} \quad ; \tag{5}$$

D is a positive coefficient, which is chosen D=1 in next surveys of this paper. In (5) $\bar{t}_{hk}$ is the value of t_{hk} in (4) corresponding to the initial rough set of (a, b) belonging to the finite wavelet frame whose quantity will be illustrated in subsection 3.2.

By using (3, 4, 5), expression (1) can be rewritten by another form as follows:

$$\bar{f}(\bar{x}_h) = \hat{y}_h = \sum_{k=1}^{M} v_k\phi(t_{hk}) + d_0 \tag{6}$$

in which v_k, w_{kj} $(k = 1...M, j = 1...n)$ and d_0 are parameters; $\hat{y}_i$ is the i-th appropriated output.

By definition an error function E as below:

$$E = \frac{1}{2}\sum_{h=1}^{P}(y_h - \hat{y}_h)^2 = \frac{1}{2}\sum_{h=1}^{P}\left(y_h - \sum_{k=1}^{M}v_k\phi(t_{hk}) - d_0\right)^2, \tag{7}$$

from (4) and (7), the following is given:

$$E = \frac{1}{2} \sum_{h=1}^{P} \left(y_h - \sum_{k=1}^{M} v_k \phi \left(\sum_{j=1}^{n} |w_{kj} x_{hj} - b_k| \Big/ a_k \right) - d_0 \right)^2 \tag{8}$$

The approximating as in (1) is great satisfaction if $\bar{f}(\bar{x}_i) \to y_i$. To perform this, the optimal value of the elements in the parameter vector p defined as below:

$$p = [a_k \ b_k \ v_k \ w_{kj} \ d_0]^T, \ k = 1...M, j = 1...n \tag{9}$$

needs to be estimated such that

$$\hat{y}_i = \sum_{k=1}^{M} v_k \phi(g_{ik}) + d_0 \to y_i. \tag{10}$$

It means

$$E(p) \to 0. \tag{11}$$

Be noted here that the optimal resolution of (11) can be depicted by an adaptive optimal discretization via the well-known NN technique. The adaptive discretization consists in determining the optimal parameters of vector p according to the features of the data set T_Σ. The NN which is established based on a training process and a training data set as shown in Figure 1 is used for this arm. The NN has one hidden layer and one linear neuron in the output layer, in which wavelet ψ works as an activation function of M hidden neurons, so-called a wavelet neural networks, WNN. By this way, the optimal value of $[a_k \ b_k \ v_k \ w_{kj} \ d_0]^T, \ k = 1...M, j = 1...n$, is estimated via the training process of the WNN.
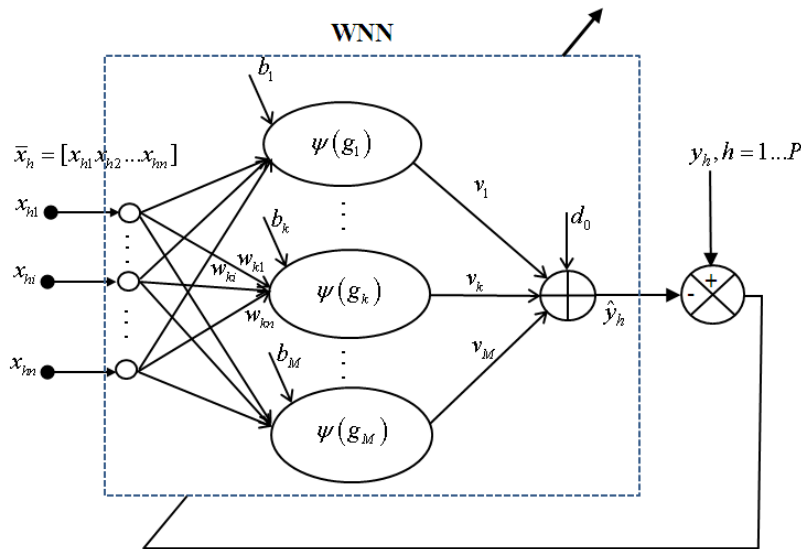

Figure 1: Structure and the solution for calculating the optimalparameter set of the WNN

3.2. Establishing a finite wavelet frame

Regarding the target to be the best approximating the function $y = f(\bar{x})$ via the wavelet transform represented by (1), reality shows that the redundant wavelet bases result in making the size of the system identification WNN model prohibitively large, increasing the computational cost. Questions need to be answered here are how many wavelets are required? (Meaning $M =?$) and what are the wavelets used in this work? For practical implementation, the theoretically in?nite wavelet frame must be truncated into a ?nite frame (signed I) for the training process. Here, this phase can be seen as to build the initial rough set of (a, b). For this focus, in this paper the methods presented in [22, 26] are considered. However, instead of using input-output data space as these methods for creating the fine set of (a, b) only the input data space for the rough set is used and the fine one will be established via the neural-networks training technique.

Be noted here that the non-orthogonal Mexican hat wavelet function (3) is not compactly supported but is rapidly vanishing. Thus, its support, $S_{a,b}$, defined by Daubechies [23] as in (12) is used for establishing I.

$$S_{a,b} = \left\{ \bar{x} \in X : |\psi_i(\bar{x})| > \varepsilon \max_{\bar{x}} |\psi_i(\bar{x})| \right\}, \quad i = 1 \ldots L, \tag{12}$$

where X is the entire input state space of T_{Σ}; ε is a given small positive index and L is the default number of elements (a, b) as follows [22]:

$$a_p = 2^{-p}, \quad b_{p,q} = q2^{-p}, \quad p, q \in N* \tag{13}$$

In order to remove the empty wavelets whose supports do not contain any data, first, find the value set I_i related to the input data $\bar{x}_i$ as follows:

$$I_i = \left\{ (a,b) : \bar{x}_i \in S_{a,b} \right\}, \quad i = 1 ... L; \tag{14}$$

Subsequently the nonempty wavelet frame is obtained via the union operator $\cup$ as below:

$$W = \left\{ \psi_{a,b} : (a,b) \in I = \cup_{i=1}^{L} I_i \right\} \tag{15}$$

3.3. Training the WNN

As above mentioned, the optimal value of parameters belonging to vector p is established via the training process of the WNN. In this paper an update rule for $[a_k \ b_k \ v_k \ w_{kj} \ d_0]^T$, $k = 1...M, j = 1...n$, during the training process is established based on the well-known Gradient Descent Algorithm.

Based on the Taylor expansion of the error function (8) it can infer

$$E(p_r) \approx E(p_{r-1}) + \nabla E^T|_{p=p_{r-1}}(p_r - p_{r-1}) \tag{16}$$

where the subscription r depicts the r-th loop. From (16), it can infer that in order to reduce the error function depicted in (11), the following expression needs to be satisfied:

$$p_r = p_{r-1} - \eta \nabla E|_{p=p_{r-1}} \tag{17}$$

In (17), η is a positive parameter called momentum used to adjust the convergence rate of the training phase. Expression (17) describes the update rule for each parameter in vector $p = [a_k \ b_k \ v_k \ w_{kj} \ d_0]^T$, $k = 1 \ldots M, j = 1 \ldots n$, during the training process.
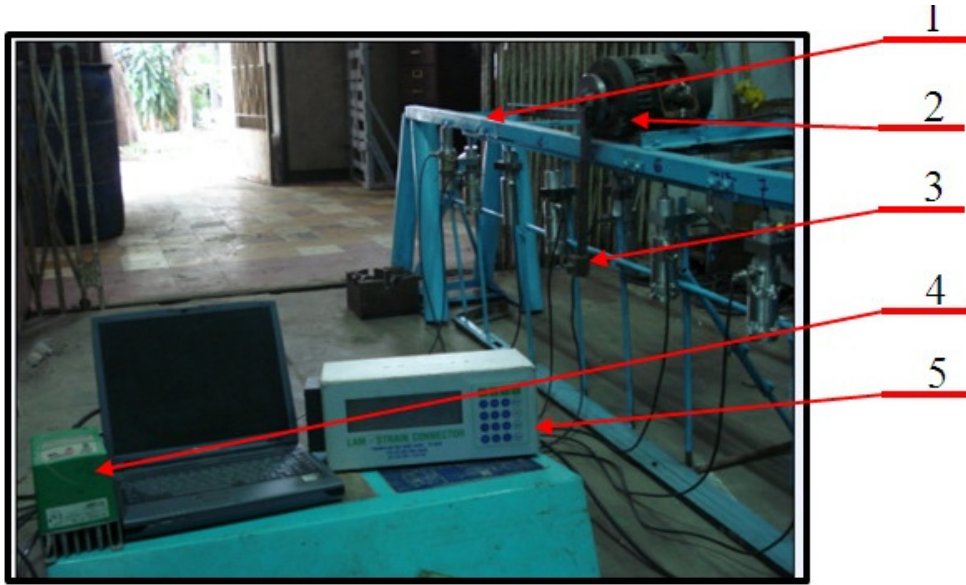
Figure 2: The experimental model

4.   EXPERIMENTS

4.1.   Experimental model

The experimental model is shown in Figure 1 which consists of a beam-typed iron frame with a length of $L = 3$ m (1), a motor D (2) carrying a mass M (3) of 0.3528 kg, a frequency converter V (4) and a measuring system. The measuring system uses 3-axis MEMS acceleration sensors, LIS3DH, linear displacement sensors, HS10, a computer, and the vibration signal processing and measuring system LAM_BRIDGE (5) made by the Lab. of Applied Mechanics (LAM), HCM City University of Technology. In this experimental model, M is fixed on the frame at many different positions located by the co-ordinate (x) along the length L. The center deviation level of M, Md, can be easily varied by varying the distance d from position of fixed M to the rotation axle center of D. V is used to change the angular velocity ω of D. Thus, the factors related to the vibration-exciting statuses (VES) are 3 parameters (x, d, ω).

4.2.   Measuring vibration acceleration

In surveys below, only MEM sensors are used which are fixed at locations $3L/4$, $L/2$, $L/4$, and $L/5$. At the undamaged-frame time, by fixing the motor D at the position $x = 1.5$ m as well as the center deviation value $Md = 0.14112$ kgm, adjusting angular velocity such that $\omega \leq 1.047$ rad/s, using a sample frequency to be 500 Hz, the vibration acceleration signals of the frame at MEM-fixed locations are measured. As a result, the data set as in Figure 3 named TL4 corresponding to the undamaged frame has been built.

By using the same way and the value of the parameters as above mentioned except the changing in the position of motor D with the new location to be $x = 2.85$ m. The MEM fixed at the location of L/5 is utilized to measure the horizontal vibration acceleration of the frame. The noisy signal in this case is the data line in Figure 4 which is called the dataset TL5. The TL5 will be used to verify
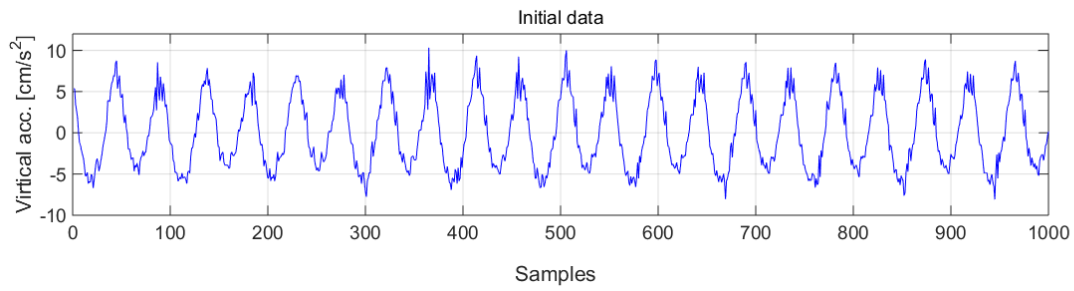
Figure 3: Initial data set in the time domain, TL4

the ability to identify the noisy dataset of the WNN while the TL4 will be filtered noise by SSA to use for estimating the effectiveness of the proposed method in the identifying undamaged-frame status.

4.3.   Identifying the noisy dataset

In order to estimate more objectively the proposed method in this and the next survey, the algorithm for establishing adaptive neuro-fuzzy systems (ANFIS) named ENFS presented in [3] is also utilized. Regarding the ENFS, let be summarized as follows. In order to build ANFIS from datasets, firstly, clusters of the same data samples in the jointed-input-output data space are established via a clustering algorithm. The created cluster data space is then used to build fuzzy sets to which an initial structure of the neuro-fuzzy inference system is created. Finally, this structure is optimized via a training process. ENFS is also a useful mathematical tool for identifying systems via datasets.
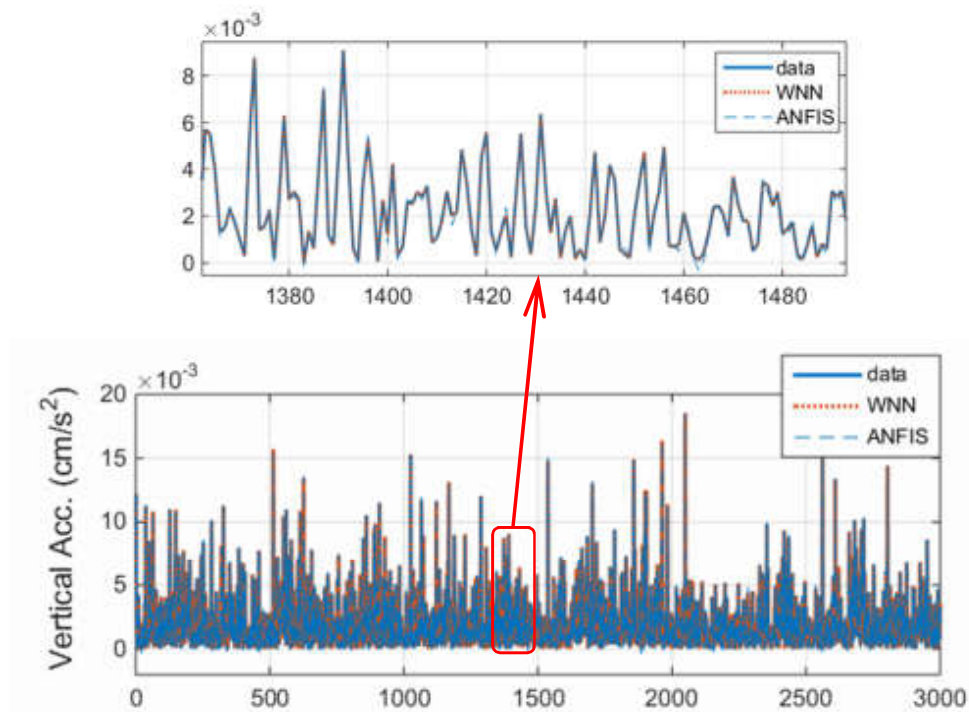

Figure 4: The data output of the TL5 and outputs of the WNN and ENFS

In order to establish an identifying process via the TL5, firstly in the TL5, five pass and present samples are used to predict the next future point, means that input is $[x(k-4),\ x(k-3),\ldots,x(k)]$ and output is $x(k+1)$. The WNN and ENFS are then used to identify the new dataset created from the noisy dataset TL5. The data output and outputs of the WNN and ENFS are shown in Figure 4. The result depicts the two methods can identify well the dataset. Particularly, the WNN is more accuracy than ENFS with the mean absolute error (MAE) (18) of them, respectively, are $1.1700.10^{-5}$ and $1.2138.10^{-4}$ (mm/s^2).

$$\text{MAE} = \frac{1}{P}\sum_{i=1}^{P}|y_i - \hat{y}_i| \tag{18}$$

where P is the number of samples in the data set; y_i and $\hat{y}_i$ are the data output and estimate output, respectively.

4.4. Identifying undamaged-frame status

In this subsection, the filtered 1000-sample data set, TL4, is used for identifying dynamic response of the beam when it is considered to be undamaged via the proposed WNN. There two phases here to be performed as follows.
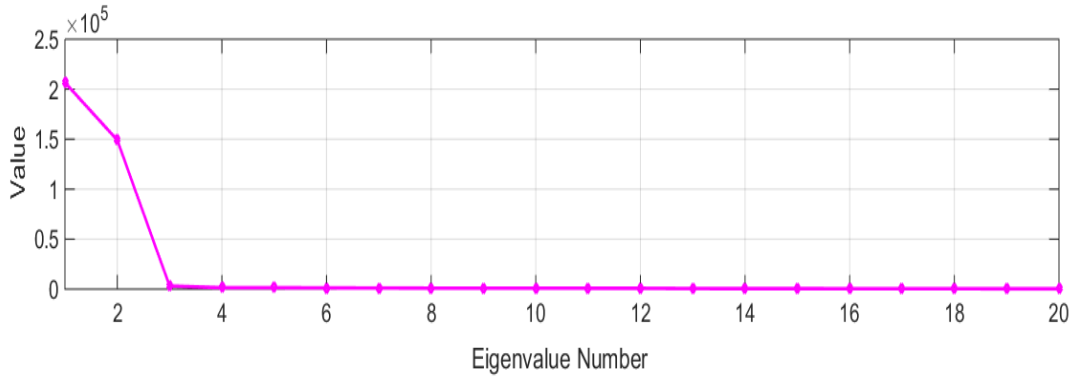

Figure 5: Value of eigen values

First, by using SSA with a window length of $L = 20$, 20 principal components were separated from the initial data set TL4 corresponding to 20 eigen values which is shown in Figure 5. The part extracted from this result related to 5 eigen values, from the first to the fourth together with the last one, is shown in Figures 6 and 7. Namely, the principal components numbered from 1 to 4 and 20 are illustrated in Figure 6. Transforming these five ones in the time domain into the frequency domain resulted in corresponding frequency spectrum (Power Spectral Density, PSD) which are depicted in Figure 7. In the frequency domain it can observe that the vibration frequency depends deeply on the corresponding eigen value. Smaller values of eigen value are closely correlative with the high vibration frequency ranges of the signal, and conversely. This is the key aspect of signal analysis shown in the rest of this section.

Based on the theory of SSA as mentioned in section 2, to which the frequencies coinciding the trend of the signal are the lowest frequency ranges related to the vibration frequency of the frame. Besides, in cases that system's vibration frequency to be estimated then the trend of the signal is, certainly, closely correlated with this value. Let note here that in this survey, the vibration frequency of the frame is known. During this survey, vibration frequency is always smaller or equal to 10Hz. Based on this feature, by considering obtaining results depicted figures 6 and 7 we can handle the trend of the initial signal to be the set depicted in both figures 6a and 6b; all the other ones are noise. It means that the data set is combined between 6a and 6b expresses best the dynamic response of the undamaged frame which will be used for training the WNN in the next subsection. The filtered dataset and the result compared between the initial and the filtered one shown in Figures 8 and 9.
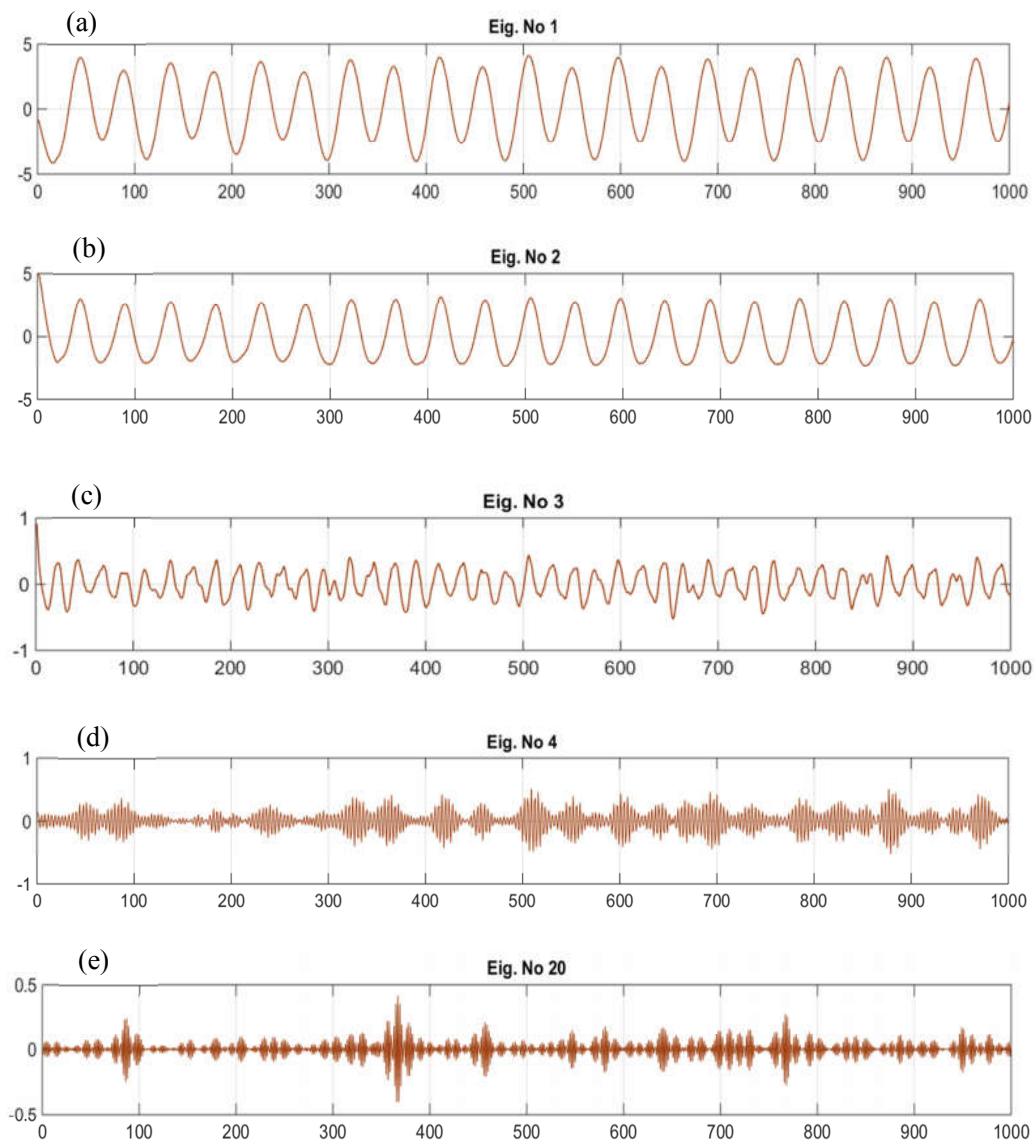
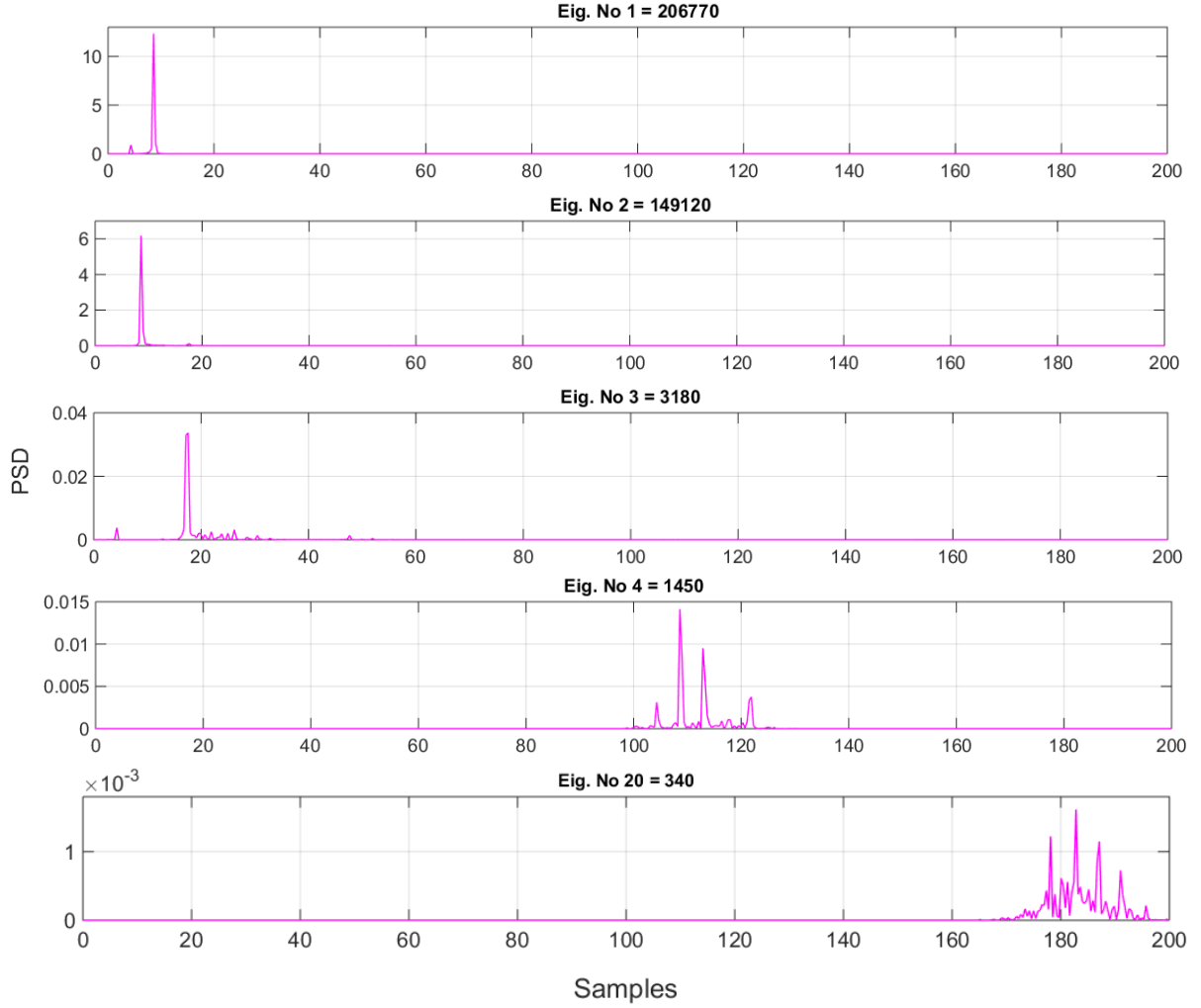Figure 6: Principal components corresponding to Eig. 1-4 and 20 in the time domain

Figure 7: Power Spectral Density, PSD, of each principal component

Instead of using an input-output-typed data set as usual for identifying, in this survey, the proposed method is estimated via a time-series prediction problem as presented in $[4, 27]$. First, in the TL4, five pass samples are used to predict the next point, means that input is $[x(k-4), x(k-3), ..., x(k)]$ and output is $x(k+1)$. By this way, from the TL4 two data subsets are established. The first one having 900 samples is used for building FLSs, and the other one having 95 samples is utilized for testing. Results obtained in the training process shows in Figures 8, 9 and Table 1. Compared results show that prediction errors related to the proposed method are smaller than that of the ENFS. MAE of the WNN and ENFS respectively to be 0.0326 and 0.0401 (mm/s^2) as depicted in Figure 10 and Table 1. The test set with 95 samples is then used for the WNN and ANFIS which have been built based on the 900-sample data set as above mentioned. Results in the testing phase are shown in Figure 11 and Table 1. In this phase, MAE of the WNN to be 0.0756 (mm/s^2) is significantly smaller than that of the ENFS which is 0.1059 (mm/s^2). These results show the comparative effectiveness of the proposed WNN.
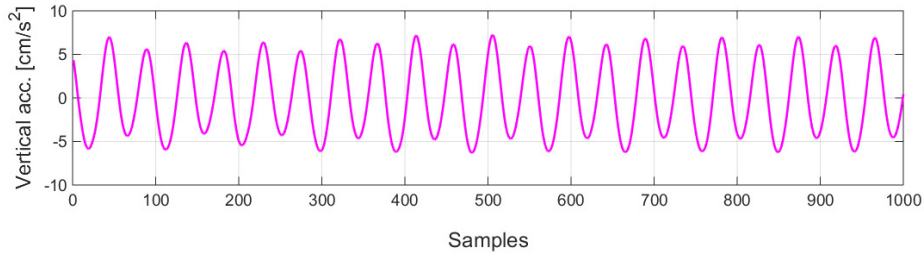
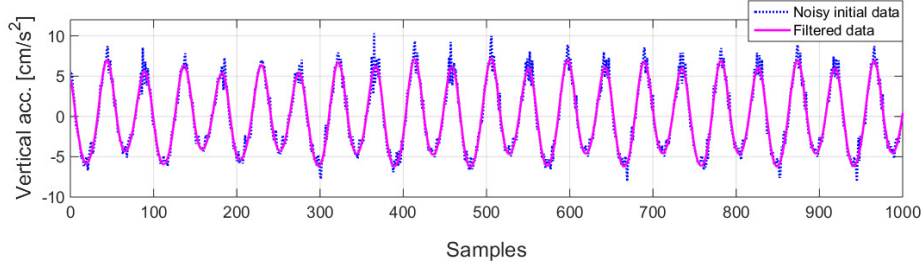Figure 8: The result obtained based on SSA: the filtered data set TL4

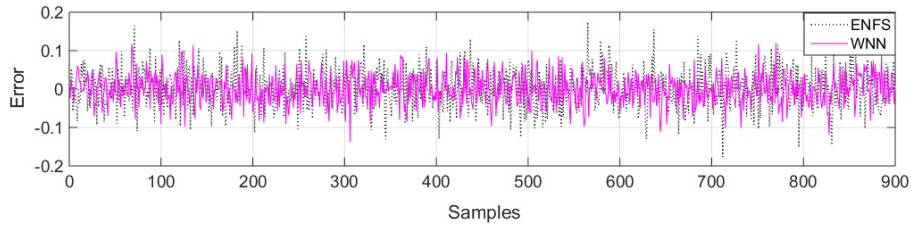Figure 9: The result obtained based on SSA: the comparison result

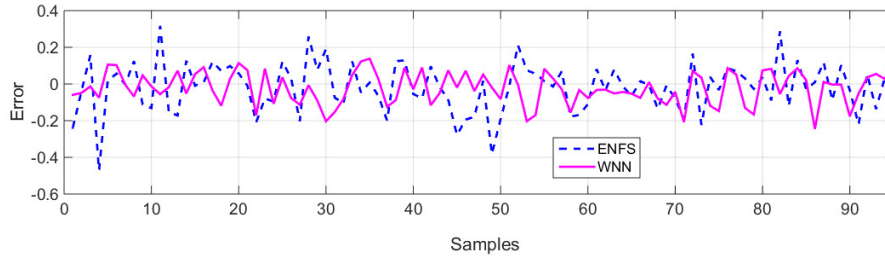Figure 10: Prediction error $(\mathrm{mm/s^2})$ of each method in the training process

Figure 11: Prediction error $(\mathrm{mm/s^2})$ of each method in the testing phase

MAE $[\mathrm{mm/s^2}]$		
Training		Testing
ANFIS	0.0401	0.1059
WNN	0.0326	0.0756

Table 1: Mean absolute error

5. CONCLUSION

The solution for identifying the undamaged-beam status using the SSA and WNN technique has been presented in which the SSA worked as a filter while the proposed WNN took part in identification. By using SSA, the beam's vibration-measured data set is separated into subsets based on their vibration frequency ranges. The beam's vibration frequency is used to find out the subset that best expresses the structure's dynamic response. Subsequently, the WNN model identifies the dynamic response of the structure based on this filtered subset.

In order to estimate the effectiveness of the proposed method, an experimental apparatus has been used for the real surveys. This experimental apparatus consists of a beam-typed iron frame, MEM acceleration sensors and the vibration-signal processing and measuring system LAM_BRIDGE. Results show that SSA is a useful tool for building frequency-based filters, especially in cases structure's vibration frequency to be given. For the WNN, it can be used for identifying system offline. However, appropriate solutions for improving convergent time in the training process need to be continuously considered. This will be presented in the next paper.

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