05 Apr 1995, 1:30 pm - 3:30 pm

Nonlinear Dynamic Impedance of Pile Group Foundation

K. Miura
Kajima Corporation, Japan

K. Masuda
Kajima Corporation, Japan

T. Maeda
Kajima Corporation, Japan

T. Kobori
Kajima Corporation, Japan

Proceedings: Third International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, April 2-7, 1995, Volume I, St. Louis, Missouri

Nonlinear Dynamic Impedance of Pile Group Foundation

Paper No. 5.46

K. Miura, K. Masuda, T. Maeda, T. Kobori
Kajima Corporation, Akasaka, Minato-ku, Tokyo, Japan

SYNOPSIS The dynamic impedance of a pile group must be determined to perform a dynamic response analysis of the structure it supports. This problem has attracted continued research interest, but most research to date has been on linear soil media. When the excitation level becomes large during an earthquake or a machine type excitation, stress concentration occurs in the soil surrounding the pile, causing the soil to behave nonlinearly. Calculation of pile group impedance necessitates analysis of pile-soil-pile interaction. Any soil nonlinearity around the piles must be taken into account in this analysis. However, the effect of nonlinearity on dynamic impedance has not been examined adequately. This paper describes an analysis method, and presents numerical results for the nonlinear dynamic impedance of a pile group and for the distribution of forces on the pile caps.

Introduction

Pile group impedance is required to perform a dynamic response analysis of a structure supported on a pile foundation. The calculation method should take account of the interaction between the piles through the soil medium. This pile-soil-pile interaction plays a significant role in the overall behavior of the pile group. Consequently, the impedance depends strongly on the excitation frequency. Many impedance analysis methods have been published. One, a Green's-function-based formulation [1-3], may be accurately applicable, but in this method the soil is assumed to be a viscoelastic layered medium. When an excitation becomes large, stress concentration in the soil surrounding the pile causes the soil to behave nonlinearly. The pile-soil-pile interaction through this nonlinear soil is expected to greatly affect the impedance of the pile group, and this should be incorporated into the analysis. No research yet performed has found an adequate answer to this problem.
Nogami et al.[4] and Novak et al.[5] studied the effect of soil nonlinearity using the Winkler idealization. Tanaka et al.[6] applied FEM to the analysis. However, they did not describe distinctly the nonlinear soil's effect on the dynamic impedance of pile group.
This paper describes a Green's-function-based formulation for the nonlinear impedance of pile groups and presents a set of numerical results for the dynamic impedance and for the distribution of forces on the pile caps.

Methodology

The pile group foundation shown in Fig.1 is the object of analysis in the present paper.
The analysis method is based on the Green's-function-based formulation and the following assumptions are made. (1)The pile cap is not in contact with the soil. (2)The soil is a linear medium(L) and the nonlinear soil zone(N) occurs only in the vicinity of the pile. (3)The N zone is concentric with a radius R_1 around the pile shaft and under the pile tip, and the circular interface at R_1 between the N and L zones translates and rotates while keeping its shape. (4)Displacement compatibility is maintained at the interface between the circumference of the pile and the N zone, and also between the N and L zones.

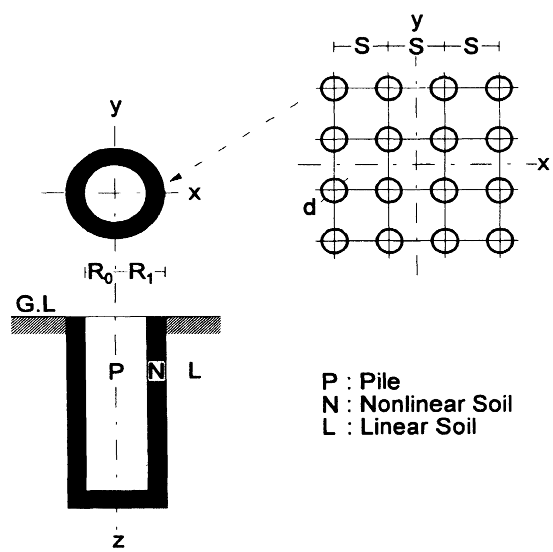

Fig.1 Pile Group Foundation

The analysis is in the frequency domain and the time term $e^{i\omega t}$ is omitted in the expressions below. The piles are modeled as prismatic members and the N zones are also discretized at the same depth as the pile discretization nodes. For an excitation in the x direction, the displacement vector $\{u_p\}$ of the pile nodes can be expressed as:

$$\{u_p\} = [u_x, \phi_y, u_z]^T \qquad (1)$$

where u_x and u_z are displacements in the x and z directions and ϕ_y is the rotational angle about the y axis. Other displacement terms are trivial and are not included. As the circular interface between the N and L zones is assumed to vibrate while keeping its shape, the displacement vector $\{u_N\}$ for nodes of this interface can also be expressed by the same vector entries as Eq.(1). If the piles are subjected to the external force vector f_P, the equation of motion can be written as:

$$\begin{bmatrix} S_{PP} + S_{PN} & -S_{PN} \\ -S_{PN} & S_{NN} + S_{PN} - S_{SC} + S_{LL} \end{bmatrix} \begin{Bmatrix} u_P \\ u_N \end{Bmatrix} = \begin{Bmatrix} f_P \\ 0 \end{Bmatrix} \quad (2)$$

where S_{PP}, S_{PN}, S_{NN}, S_{SC} and S_{LL} are the assembled dynamic stiffness matrices; S_{PP} is for the piles and S_{SC} for the soil columns replaced by both the piles and the N zone; S_{LL} is the inverse matrix of the Green's function matrix G_{LL} and the entries of G_{LL} are calculated numerically by the solution of the thin-layered elements method; and S_{PN} and S_{NN} are the stiffness matrices for the N zone. In detail, S_{PN} represents the lateral stiffness between the pile and the L zone, and S_{NN} is the stiffness between adjoining nodes. The effect of the matrix S_{NN} on the dynamic behavior of the pile group is negligible. When $\{f_P\}=[f_{P1},0]^T$ is an interaction force vector with non-zero entries corresponding only to pile cap degrees of freedom, partitioning the matrices and the vectors in Eq.(2) according to the degrees of freedom '1' corresponding to the pile caps and '2' corresponding to the remaining degrees of freedom, Eq.(2) can be rewritten as:

$$\begin{bmatrix} S^*_{11} & S^*_{12} \\ S^*_{21} & S^*_{22} \end{bmatrix} \begin{Bmatrix} u_{P1} \\ u_2 \end{Bmatrix} = \begin{Bmatrix} f_{P1} \\ 0 \end{Bmatrix} \quad (3)$$

Solution of Eq.(3) leads to the following expression for the forces at the pile caps:

$$\{f_{P1}\} = ([S^*_{11}] - [S^*_{12}][S^*_{22}]^{-1}[S^*_{21}])\{u_{P1}\} \quad (4)$$

If the piles are connected to a common rigid footing at the pile caps, the compatibility constraint matrix [R] can be used to relate the displacements and forces at the pile caps to the displacements and forces of the pile cap at a reference point. If the vectors of the footing displacements and forces are denoted by $\{u_F\}$ and $\{f_F\}$, then:

$$\{u_{P1}\}=[R]\{u_F\} \quad (5)$$

$$\{f_F\}=[R]^T\{f_{P1}\} \quad (6)$$

Using Eq.(4)-(6), the following expression is derived:

$$\{f_F\}=[K_P]\{u_F\} \quad (7)$$

where $[K_P]$ is the impedance matrix of the pile group and the entries of $[K_P]$ can be expressed as:

$$[K_P] = \begin{bmatrix} K_{HH} & K_{HR} & K_{HV} \\ K_{RH} & K_{RR} & K_{RV} \\ K_{VH} & K_{VR} & K_{VV} \end{bmatrix} \quad (8)$$

where, subscripts H, R and V refer to horizontal, rocking and vertical motions, respectively. The matrix $[K_P]$ is symmetric and the cross entries except $K_{HR}(=K_{RH})$ in $[K_P]$ are negligible.

This paper describes the dynamic behavior of single piles and square pile groups in homogeneous(SOIL-1) and layered(SOIL-2) media as shown in Fig.2. The piles are steel pipe of diameter d=101.6cm, thickness t=1.6cm and length L=21m, spaced at 3.0m center to center. The homogeneous medium(SOIL-1) is a uniform half-space with Vs=100m/s, γ=1.60t/m³, ν=0.48 and h=0.02, where Vs, γ, ν and h denote the shear wave velocity, unit weight, Poisson's ratio and damping factor of the linear-hysteretic type, respectively. The layered medium(SOIL-2) comprises two strata. The upper stratum is 20m thick and has the same physical constants as SOIL-1, and the underlying stratum is a uniform half-space with Vs=300m/s, γ=1.70t/m³, ν=0.46 and h=0.02.

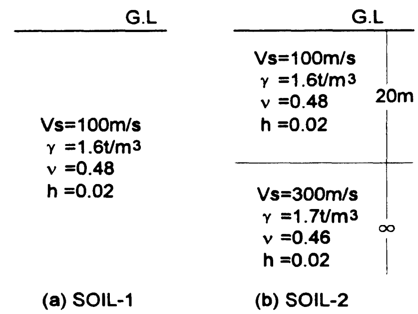

Fig.2 Soil Conditions

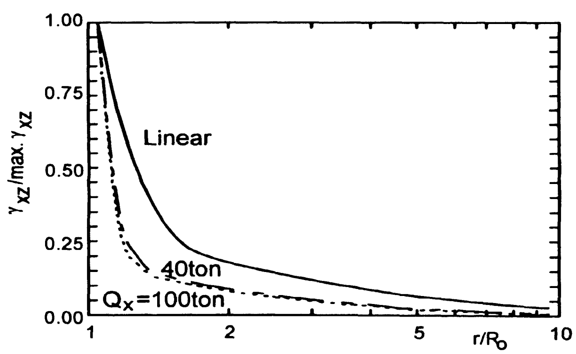

Fig.3 Shear strain distribution

The analysis method described in this paper is formulated on the assumption that soil nonlinearity spreads in the vicinity of the pile. Nonlinear 3-D FEM analyses are conducted to determine whether or not this assumption is valid. A single pile is driven into SOIL-1 and subjected to a static horizontal load Q_x at its cap. Both rotation and vertical displacement at the pile cap are restrained. Analyses are performed for the linear soil and the nonlinear soil where Q_x is increased in steps up to 100tons.

Fig.3 shows the shear strain distribution in the soil on a line lying 5cm below the ground surface in the load application direction. The strains are normalized by the maximum strain at each loading level. Regardless of the load level, the maximum strains appear near the pile and decrease gradually with distance from it. In the nonlinear soil, where Q_x=40 and 100tons, the strains decrease more rapidly than in linear soil case. This shows that excessive strain in soil far from the pile is restrained by the nonlinearity of soil in the vicinity of the pile, and the soil far from the pile remains linear after the soil in the vicinity of the pile becomes nonlinear. Thus, the assumption, that the zone of soil nonlinearity is limited to the vicinity of the pile, is shown to be valid.

The stiffness S_{PN} of the nonlinear soil zone N in Eq.(2) is evaluated as the stiffness connecting laterally the pile to the linear soil zone L. Although Novak el al.[5] evaluated this stiffness matrix using a 2-D wave propagation formulation, the following procedure is adopted to simplify the treatment of nonlinearity. The displacement vector $\delta=[u_x, u_z]^T$ at distance r from the center of the pile in the N zone is assumed to be given by a linear equation as:

$$\delta=\delta_O+\{(r-R_O)/(R_1-R_O)\}(\delta_1-\delta_O) \qquad (9)$$

where $\delta_O=[u_{px}, u_{pz}]^T$ denotes a displacement vector at the circumference of the pile, and $\delta_1=[u_{x1}, u_{z1}]^T$ expresses the displacement vector at the interface between the N and L zones. Using the representative displacement vector $u_N=[u_{xN}, \phi_{yN}, u_{zN}]^T$ of the N zone, the entries of the vector δ_1 can be written as:

$$u_{x1}=u_{xN}, \quad u_{z1}=u_{zN}-\phi_{yN}R_1 \qquad (10)$$

Expanding the displacements given by Eq.(9) into a first order Fourier Series and applying the Finite Element formulation, the stiffness matrix S_{PN} for the N zone can be obtained. The stress-strain relation, such as a plane stress(STS) or a plane strain(STN) condition, is required for this calculation. The formulation in this paper sets the N zone between the pile and the L zone to account for soil nonlinearity. Thus, if the N zone remains linear and has the same physical constants as the L zone, the impedances obtained from the above formulations should coincide with those where the pile is directly connected to the L zone.

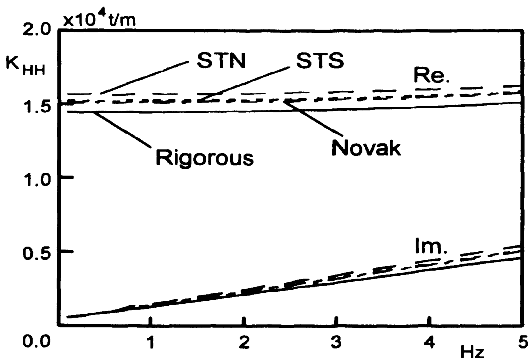

Fig.4 Dynamic Impedances KHH of a Single pile in SOIL-1

Fig.4 and Fig.5 show comparisons of the impedances for a single pile in SOIL-1 and a 2x2 pile group in SOIL-2. The radius R_1 of the N zone is given as R_1/R_o=1.25. In these figures, 'Rigorous' indicates the results under direct connection of the pile to the L zone where there is no N zone, and STN and STS express the results under a plane strain and a plane stress condition, respectively. Furthermore, 'Novak' indicates the results by Novak's stiffness evaluation method. Three evaluation methods, STS, STN and 'Novak', lead to rocking and vertical impedances coinciding with 'Rigorous' ones. Slight discrepancies are recognized in horizontal impedance K_{HH}, but they can be negligible. The results presented hereafter are those for the STS condition.

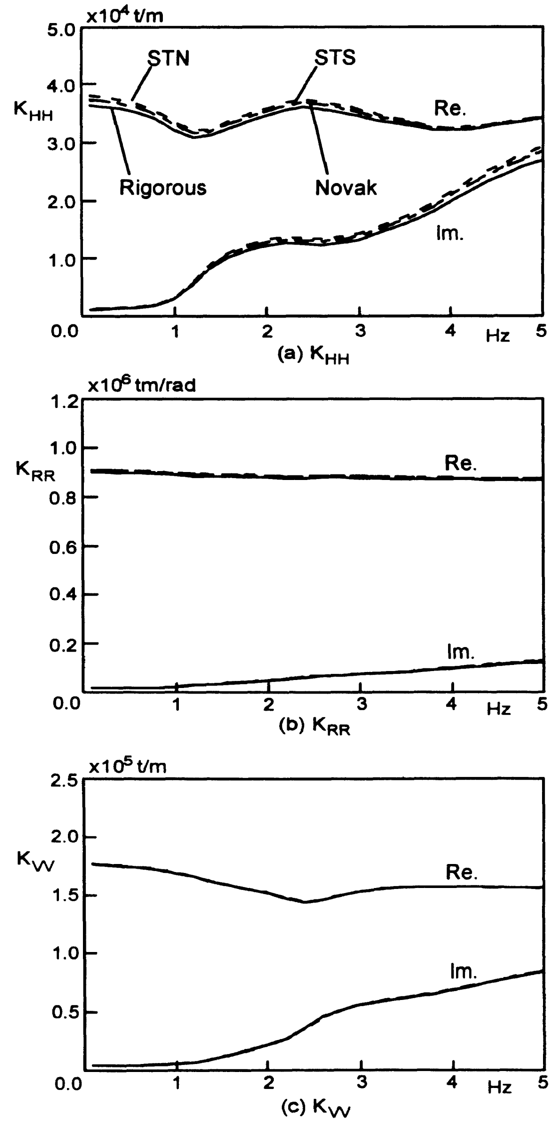

Fig.5 Dynamic Impedances KHH of 2x2 Pile Group in SOIL-2

The progress of nonlinearity is not uniform in the N zone, but depends largely on the position. That is, subdiving the N zone into four segments A to D as illustrated in Fig.6, the strains in segments A and B located in the excitation direction are different from those of C and D in the orthogonal direction. Thus, the progress of nonlinearity in each segment is independently evaluated. The shear rigidity G and the damping factor h of each segment are estimated as those corresponding to the maximum shear strain on G,h-γ(shear strain) curves. The stiffness matrix S_{PN} is given by the resultant of the stiffness matrices of each segment. The calculations of Eq.(2)-(7) are continued iteratively until stable G and h is attained.

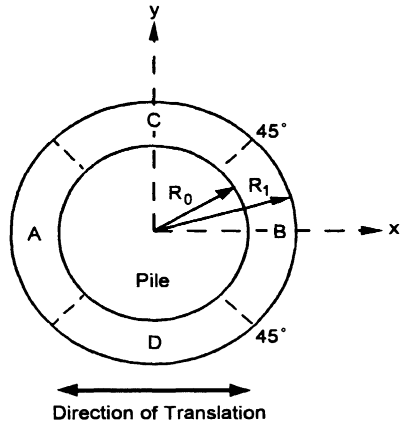

Fig.6 Subdivision of Nonlinear Soil Zone

Results

The horizontal impedances K_{HH} for single, 2x2, 4x4, 6x6 and 8x8 square pile groups in SOIL-2 are examined. The width of the N zone is presumed to be R_1/R_o=1.25 and the G,h-γ relation shown in Fig.7 is employed, which is the relation for a sand.

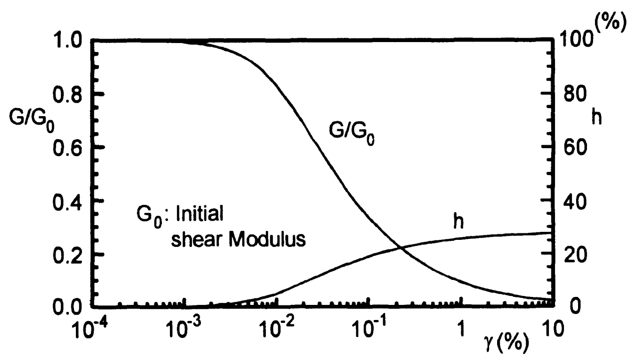

Fig.7 G,h-γ Relation

In a practical application, the R_1/R_o and the G,h-γ relation can be accurately determined from the results of a horizontal static loading test of a pile in situ and of an indoor test on the soil. Each pile is discretized into 11 elements. The excitation is given as a harmonic horizontal displacement δ_x of the rigid footing on the pile caps under the condition that its vertical displacement and rocking are restrained.
Fig.8 shows both linear and nonlinear impedances. Nonlinear impedances occur when δ_x=0.5, 1.0 and 2.0cm. As usual, the real part of the impedance represents the dynamic stiffness, while the imaginary part relates to damping comprising both radiation damping and hysteretic damping of the soil media. The fundamental natural frequency of SOIL-2 is 1.25Hz. The maximum shear strain occurs in segments C and D for a single pile and in segment C of the corner pile of the pile group. The maximum strains in the N zone at 1.0m depth when δ_x=1.0cm at 0.1Hz amount to 2.67% for a single pile, 1.28% for 2x2, 0.68% for 4x4,

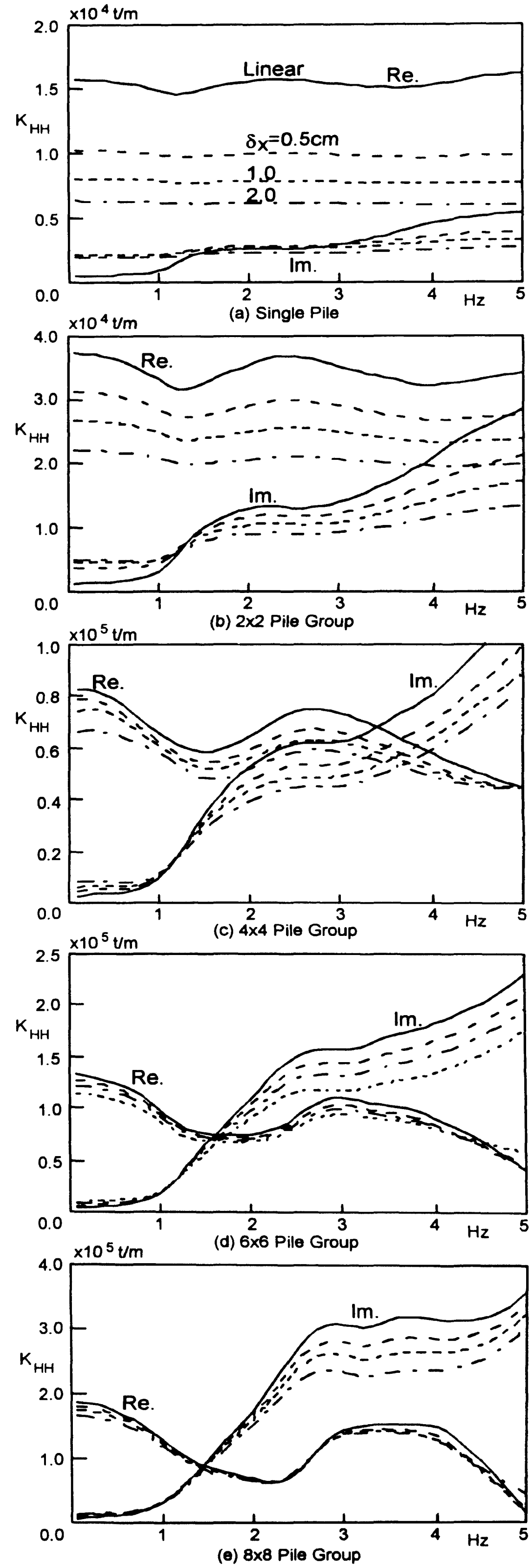

Fig.8 Nonlinear Dynamic Impedance of a Single and Square Pile Group in SOIL-2

0.56% for 6x6 and 0.50% for 8x8 pile group. According to the G,h-γ relation shown in Fig.7, the reduction ratios G/G_0 of the shear modulus at these shear strain levels range from 0.05 (single pile) to 0.13 (8x8 pile group), and the damping factors h range from 25% (single pile) to 24% (8x8 pile group).
The cut-off frequency phenomenon appears clearly in the linear impedance of a single pile. That is, the real part of the impedance conforms a trough at frequencies around the natural frequency, and the imaginary part remains constant at lower frequencies and becomes larger at frequencies above the natural frequency. In the pile groups with fewer piles, such as 2x2 and 4x4, the cut-off phenomenon can also be recognized in the linear impedance. In contrast, as the applied horizontal displacement δ_x increases and nonlinearity in the N zone progresses gradually, the cut-off frequency phenomenon becomes less clear. With progress of soil nonlinearity, the stiffness of the pile group reduces, while the damping related to the imaginary part of the impedance becomes larger at lower frequencies and smaller at higher frequencies. This is because, at lower frequencies where radiation damping is not predominant, the impedance damping is dominated by increase in soil hysteretic damping with progress of nonlinearity. Damping at higher frequencies, where radiation damping is predominant, is prevented from becoming larger because the wave radiated from the pile shafts into the soil is reflected at the interface between the N and L zones and propagates back to the pile.

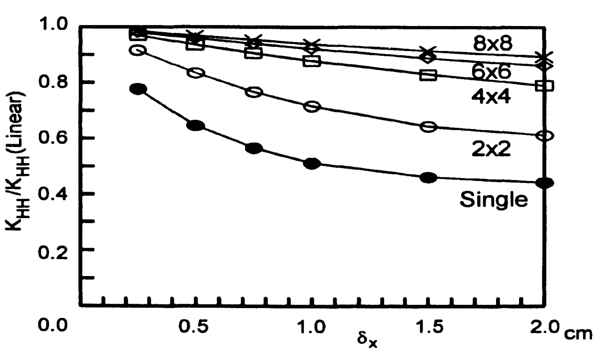

Fig.9 Static Stiffness of Pile group

Fig.9 shows the static stiffness estimated from the real part of the impedance at 0.05Hz. The stiffness of each pile group is normalized by the linear stiffness. The static stiffness naturally decreases with increase in displacement δ_x. More specifically, with increase in δ_x the stiffness of a single pile dwindles more rapidly than that of a pile group, and the stiffness of a pile group with many piles, such as 8x8, is practically unchanged. This feature can be understood as follow: Because the pile vibrates in phase motion with adjacent piles at much lower frequencies than the pile starts to vibrate in anti-phase motion, the stiffness of the pile group is usually less than n(number of piles) times that of a single pile. However, after the nonlinearity is induced near the pile, the interaction of piles through the soil is weakened and the piles start to vibrate nearly independently of each other.

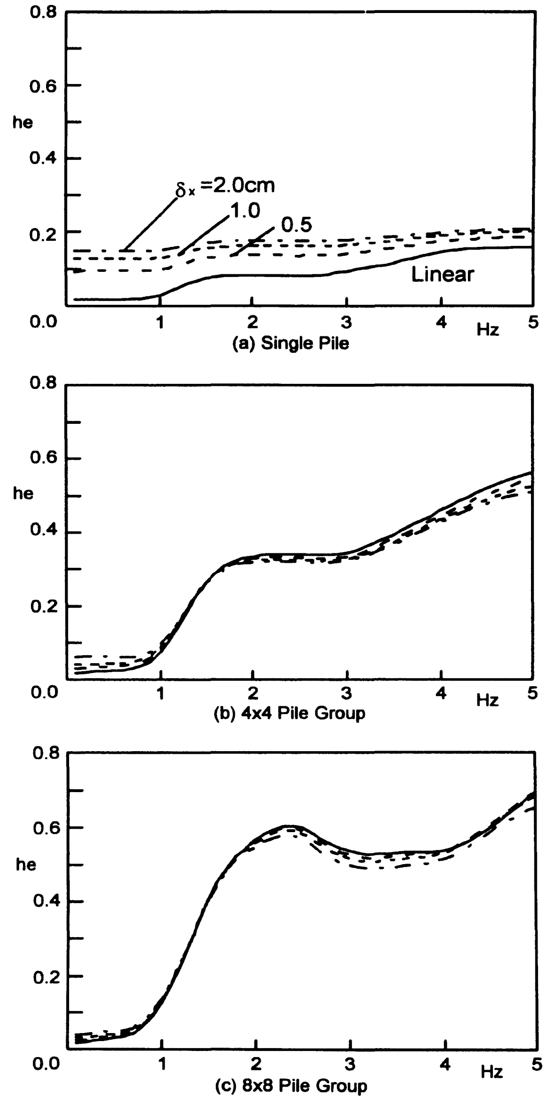

Fig.10 Damping Factor of Impedance

The damping factor of the impedance can be calculated from Eq.(13).

$$h_e=\sin\{0.5\tan^{-1}(K_I/K_R)\} \quad (13)$$

in which K_R and K_I denote the real and the imaginary parts of the impedance.
The feature of the impedance damping factor shown in Fig.10 is similar to that of the imaginary part of the impedance aforementioned. That is, the damping factor becomes larger at lower frequencies and larger at higher frequencies with progress of soil nonlinearity. This tendency reduces as the number of piles is increased. Pile foundations for heavy structures such as high-rise buildings often include a large number of piles. Several hundreds is not rare. In these cases, the pile-soil-pile interaction effect on the impedance due to soil nonlinearity around the pile is greatly weakened and the linear impedance may be regarded as being the same as the nonlinear impedance for practical purposes.

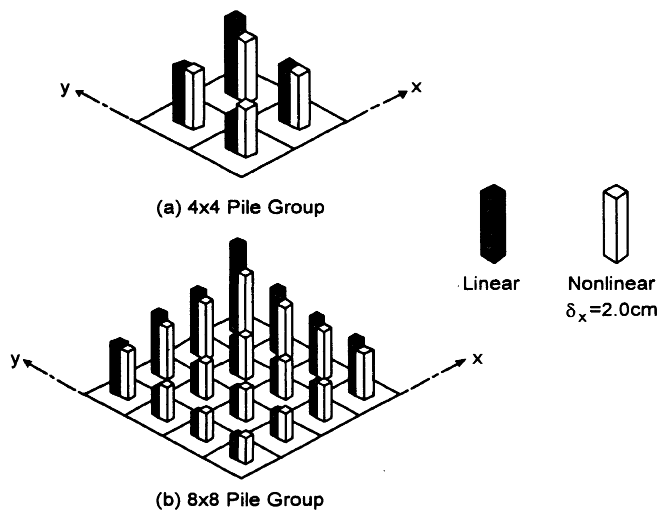

Fig.11 Distribution of Forces on Pile Caps

The distribution of forces on the pile caps at 0.1Hz over a quarter of the foundation is portrayed in Fig.11 for the 4x4 and 8x8 pile groups in SOIL-2. A footing displacement of 2.0cm is caused for the nonlinear analysis. The forces are normalized to adjust the resultant of the pile cap forces in a quarter part to unity.

When the soil is linear, piles far from the center of the footing pick up large shares of the applied load, whereas piles closer to the center pick up less. As nonlinearity progresses, the load shares of the piles approach a uniform distribution.

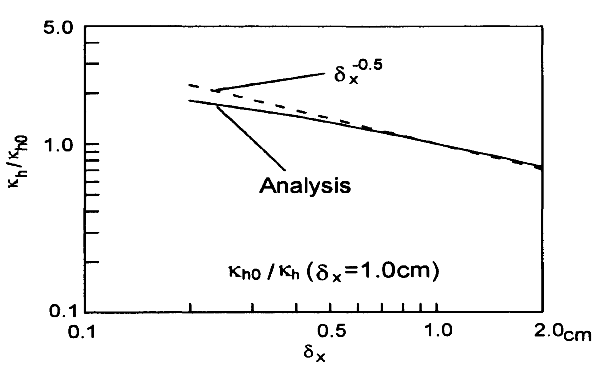

Fig.12 Equivalent kh

Assuming a pile as an infinite beam in a homogeneous soil and applying a horizontal force at a pile cap, while restraining its vertical displacement and rotation, the pile cap displacement δ_x can be expressed by Eq.(14).

$$\delta_x = Q_x/(4EI\beta^3), \quad \beta=\{(\kappa_h d)/(4EI)\}^{1/4} \quad (14)$$

where, d and EI denote the diameter and flexural rigidity of a pile, respectively; and κ_h represents a coefficient of horizontal subgrade reaction of the soil. In situ pile test results were contained and arranged in reference 7, and values of κ_h were extracted which gave Q_x-δ_x relations equivalent to the test ones. Normalizing κ_h by the value at δ_x=1.0cm, the relation between κ_h and δ_x was expressed by Eq.(15) through a regression analysis.

$$\kappa_h/\kappa_{ho} = \delta_x^{-0.5} \quad (15)$$

Fig.12 depicts the κ_h-δ_x relations. 'Analysis' in this figure indicates the relation which is evaluated from the analytical results for a single pile in SOIL-1 using the method described in this paper. The analytical and test results conform well, showing that, the proposed nonlinear impedance analysis method can be applied.

Conclusions

Formulations were presented to calculate the impedance of a pile group taking into account soil nonlinearity around the piles, and numerical results were shown for the impedance and the distribution of forces on the pile caps. As soil nonlinearity around the piles progressed, the following effects were noted.

(1)The real part of the impedance indicating the stiffness of the pile group decreases and the imaginary part representing the damping increases at low frequencies and decreased at high frequencies.

(2)The influence of soil nonlinearity on the impedance of a single pile is remarkable, while it becomes less remarkable for a pile group as the number of piles is increased.

(3)The hysteretic damping of the soil increases and the imaginary part at lower frequencies increases. But at higher frequencies, radiation damping from the pile into the soil medium decreases and the imaginary part decreases slightly.

(4)When the soil was linear, piles farther from the center pick up larger shares of the applied load, whereas piles closer to the center of the foundation pick up less. As nonlinearity progresses, the shares of the applied load approach a uniform distribution.

(5)The comparative study shows that the analytical and test results exhibits a good agreement. Thus, the applicability of the proposed method is confirmed.

References

1)A.M. Kaynia and E. Kausel, 'Dynamic behavior of pile groups', 2nd. int. conf. num. methods offshore piling, Austin,Tx,1982

2)T.G. Davies, R. Sen and P.K. Benerjee, 'Dynamic behavior of pile groups in inhomogeneous soil', J. geotech. eng. ASCE 111,1985

3)K. Miura, et al.,'Dynamic behaviour of pile foundation in homogeneous and non-homogeneous media', Earthquake eng. & Struct. Dyn., Vol.23, 1994

4)T. Nogami, et al., 'Nonlinear pile foundation model for time-domain dynamic response analysis', Proc. 9th world conf. earthquake eng., 1988

5)M. Novak and M. Sheta,'Approximate approach to effects of piles', Proc. ASCE, Vol.114,1988

6)Y. Tanaka and K. Kusakabe, 'Seismic response analysis of nonlinear soil-pile systems with equivalent linearization', Summaries of tech. papers annual meeting AIJ, 1993,(in Japanese)

7)AIJ ed.,'Ultimate strength and deformation capacity of building in seismic design(1990)', AIJ,1990, (in Japanese)