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Fuming Wang
Zhengzhou University of Technology, China

Yigong Ji
Dalian University of Technology, China

Maotian Luan
Dalian University of Technology, Dalian 116024, Liaoning Province, P. R. China

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Fuming Wang
Zhengzhou University of Technology
Zhengzhou 450002, Hena Province, P. R. China

Yigong Ji and **Maotian Luan**
Dalian University of Technology
Dalian 116024, Liaoning Province, P. R. China

ABSTRACT

In this paper, a simplified approach based on spline semi-analytical method is proposed for dynamic response analysis of visco-elastic layered half-space foundation. For varied geometrical and physical properties, the compliance function of layered non-homogeneous foundation is studied. Then an efficient and reliable numerical procedure for back-calculating material properties of pavement through the data measured from in-situ Falling Weight Deflectometer (FWD) tests is presented. The simulation results for in-situ data show that the proposed methodology is workable and applicable.

INTRODUCTION

Dynamic analysis of visco-elastic layered half-space subsoil is of great importance in foundation vibration and dynamic soil-structure interaction. In last decades, various analytical and numerical methods have been developed. In addition, with wide applications of the Falling Weight Deflectometer (FWD) for non-destructive tests (NDT), intensive studies on parameter identification of pavement have been performed recently in order to evaluate its structural capacity. In fact, pavement and subsoil can be simulated as visco-elastic layered half-space (Stubbs *et al.* 1994; Kang 1998). Therefore vibration analysis of layered foundation subjected to dynamic excitation is the fundamental issue of back-calculation of pavement parameters based on the measured data from FWD.
In this paper, a simplified numerical procedure based on spline semi-analytical method is presented for evaluating dynamic response of visco-elastic multi-layered half-space subsoil. Then dynamic impedance or compliance of layered foundation on base is computed for different combined cases. Afterwards, by virtue of the data measured from in-situ Falling Weight Deflectometer (FWD) tests (Boutros *et al.* 1985), the proposed procedure in conjunction with a system identification method is applied for back-calculating material properties of pavement. Verification of the proposed methodology is made through an example analysis.

FUNDAMENTAL AND FORMULATIONS

Modeling of Visco-Elasticity

In visco-elasticity (Molenkamp *et al.* 1980), behavior of materials is generally expressed in terms of complex modulus coupling modulus and viscous damping together which is defined as below

$$E^* = E(1+2\mathrm{i}\beta) \qquad (1)$$

where $\mathrm{i}=\sqrt{-1}$, E is Young's elasticity modulus, and β is hysteretic damping ratio, which is independent of excitation frequency.

Spline Semi-Analytical Procedure for Dynamic Analysis of Layered Foundation

Refer to Fig.1, a non-homogenous visco-elastic subsoil founded on rigid base is subjected to an uniformly-distributed surface load with unity intensity $q=\frac{1}{ab}e^{\mathrm{i}\omega t}$. The rectangular loading area is of dimension a and b in the x- and y-directions

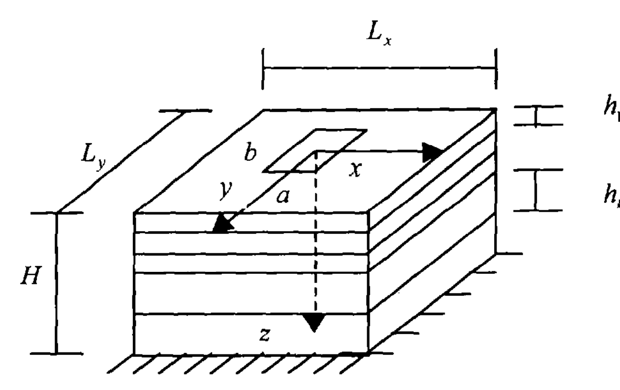

Fig. 1. Elastic multi-layered foundation

respectively. At two vertical sides, the boundaries are placed sufficiently far away from the loading area so that the reflected radiating stress waves from the boundaries are damped out before reaching the reign of interest. The displacement functions u, v, w for layered half-space with any number of nodal surface are represented by the product of simple spline functions and Fourier's series.

$$u = \sum_{m=1}^{R}\sum_{n=1}^{S}[\phi]X_{mn}\{a\}_{mn}e^{i\omega t} = u_0 e^{i\omega t} \tag{2}$$

$$v = \sum_{m=1}^{R}\sum_{n=1}^{S}[\phi]Y_{mn}\{b\}_{mn}e^{i\omega t} = v_0 e^{i\omega t} \tag{3}$$

$$w = \sum_{m=1}^{R}\sum_{n=1}^{S}[\phi]Z_{mn}\{c\}_{mn}e^{i\omega t} = w_0 e^{i\omega t} \tag{4}$$

in which X_{mn}, Y_{mn}, Z_{mn} represent Fourier's series terms expressed for horizontal and vertical loading by Ji *et al.* (1999), $\{a\}_{mn}=[a_0\ a_1\ \cdots\cdots\ a_N]^T$, $\{b\}_{mn}=[b_0\ b_1\ \cdots\cdots\ b_N]^T$ and $\{c\}_{mn}=[c_0\ c_1\ \cdots\cdots\ c_N]^T$ are variables to be determined, $[\varphi]=[\varphi_0\ \varphi_1\ \cdots\cdots\ \varphi_N]$ represent simple spline function matrix with its component as following (Ji *et al.* 1999)

$$\phi_k = \varphi_{1,k}\left(\frac{z}{h_k} - k\right), \quad k = 0, 1, \cdots\cdots, N \tag{5}$$

Where h_k is the thickness of the k-th layer. Simple spline function is expressed as

$$\varphi_1(z) = \begin{cases} z+1 \ , & z \in [-1, 0); \\ 1-z \ , & z \in [0, 1]; \\ 0 \ , & z \notin [-1, 1] \end{cases} \tag{6}$$

Equation (2), (3) and (4) can be written in matrix form

$$\{\delta\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{m=1}^{R}\sum_{n=1}^{S}[N_{mn}]\{d\}_{mn}e^{i\omega t} = [N]\{d\}e^{i\omega t} \tag{7}$$

The stresses of soil can be given

$$\{\sigma\} = [D]\{\varepsilon\} = [D][B]\{d\}e^{i\omega t} \tag{8}$$

where $[D]$ is elastic coefficient matrix, $[B]$ is the strain-displacement matrix. In fact, soil stiffness usually varies with depth. In each layer, the following non-homogeneous distribution of shear modulus is considered

$$G(z) = G_0\left(1 + \lambda\frac{z}{H}\right)^r \tag{9}$$

where G_0 is soil shear modulus at the ground surface, λ, r are parameters used for describing non-uniform distribution mode of shear modulus along depth in order to simulate the nonlinear dependency of shear modulus on confining pressure. The soil elastic matrix $[D]$ can be expressed

$$[D] = \begin{bmatrix} d_1 & d_2 & d_3 & 0 & 0 & 0 \\ d_2 & d_1 & d_3 & 0 & 0 & 0 \\ d_3 & d_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} G(z) \tag{10}$$

where $d_1 = 2(1-\nu)/(1-2\nu)$, $d_2 = d_3 = \nu/(1-\nu)$, ν is soil Poisson's ratio.

According to complex damping principle, horizontal and vertical dynamic responses can be obtained by solving Lagrangian equations of the system. Because of the orthogonality of Fourier's series X_{mn}, Y_{mn}, Z_{mn}, all terms will be uncoupled and each term can be analyzed separately. Therefore dynamic equations can be expressed in the following form while the time factor $e^{i\omega t}$ is removed

$$\left(-\omega^2[M]_{mnmn} + (1+2i\beta)[K]_{mnmn}\right)\{d\}_{mn} = \{f\}_{mn} \tag{11}$$

where

$$[M]_{mnmn} = \sum_{k=1}^{N}\int_{h_k}\rho_k \int_{-\frac{L_y}{2}}^{\frac{L_y}{2}}\int_{-\frac{L_x}{2}}^{\frac{L_x}{2}}[N]_{mn}^T[N]_{mn}\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$[K]_{mnmn} = \sum_{k=1}^{N}\int_{h_k}\int_{-\frac{L_y}{2}}^{\frac{L_y}{2}}\int_{-\frac{L_x}{2}}^{\frac{L_x}{2}}[B]_{mn}^T[D][B]_{mn}\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$\{f\}_{mn} = \sum_{k=1}^{N}\int_{h_k}\int_{-\frac{L_y}{2}}^{\frac{L_y}{2}}\int_{-\frac{L_x}{2}}^{\frac{L_x}{2}}[N]_{mn}^T\frac{1}{ab}\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

in which ρ_k is the density of the k-th layer.

System Identification Method for Pavement Parameter Backcalculation

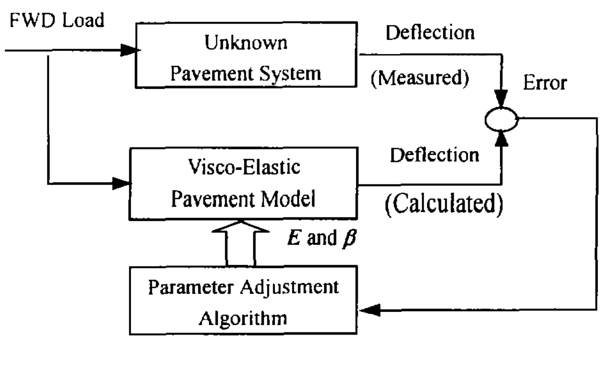

Fig. 2. System identification method

For pavement and subsoil, two types of material parameters, i.e., modulus and damping ratio, are to be identified. The pavement and subsoil are usually composed of base course, surface course and subgrade with a constant damping ratio independent of frequency and can be modeled as a layered subsoil system. The system identification technique, in conjunction with the mechanical model of pavement structure and FWD test data, is employed to estimate material parameters of different layers. It is implemented by minimizing a relative error function that quantifies the difference between the computed compliance function by the proposed procedure for dynamic response analysis of layered system and the experimentally-derived compliance function from FWD testing data. The flowchart of this procedure is given in Fig. 2.

Assuming that the model of pavement be defined by elastic modulus and damping ratio of all layers under consideration with the total number 2*N* of parameters, the deformation vector of the system W^{c} will be determined by

$$W^{c} = f(E_1, \beta_1; E_2, \beta_2; \cdots\cdots; E_N, \beta_N) \qquad (12)$$

Then the data of the *j*-th sensor of the FWD system is expressed as $W_j^{c} = f_j(E_1, \beta_1; E_2, \beta_2; \cdots\cdots; E_N, \beta_N)$. If any function $f_j(E_1, \beta_1; E_2, \beta_2; \cdots\cdots; E_N, \beta_N)$ is expanded using a Taylor's series in the vicinity of $\boldsymbol{E}$ and $\boldsymbol{\beta}$ in a given value of frequency, the error function will be given as following while neglecting higher order terms based on modified Newton's iteration procedure

$$\begin{aligned} e_j &= f_j(\boldsymbol{E} + \Delta\boldsymbol{E}; \boldsymbol{\beta} + \Delta\boldsymbol{\beta}) - f_j(\boldsymbol{E}; \boldsymbol{\beta}) = \nabla f_{Ej}\Delta\boldsymbol{E} + \nabla f_{\beta j}\Delta\boldsymbol{\beta} \\ &= \frac{\partial f_j}{\partial E_1}\Delta E_1 + \frac{\partial f_j}{\partial \beta_1}\Delta\beta_1 + \cdots\cdots \frac{\partial f_j}{\partial E_N}\Delta E_N + \frac{\partial f_j}{\partial \beta_N}\Delta\beta_N \end{aligned} \qquad (13)$$

For the system with *m* sensors, governing equations can be established

$$[F]\{\Delta X\} = \{e\} \qquad (14)$$

where $[F]$ is called the sensitivity matrix, $\{\Delta X\}$ represents parameter vector and $\{e\}$ is the difference vector between measured deformations and computed deformations of the system,

$$\{\Delta E\} = [\Delta E_1 \ \ \Delta\beta_1 \cdots\cdots \Delta E_N \ \ \Delta\beta_N]^{\mathrm{T}}$$

$$\{e\} = [e_1 \ \ e_2 \cdots\cdots e_m]^{\mathrm{T}}$$

$$[F] = \begin{bmatrix} \frac{\partial W_1}{\partial E_1} & \frac{\partial W_1}{\partial \beta_1} & \cdots\cdots & \frac{\partial W_1}{\partial \beta_N} \\ \frac{\partial W_2}{\partial E_1} & \frac{\partial W_2}{\partial \beta_1} & \cdots\cdots & \frac{\partial W_2}{\partial \beta_N} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial W_m}{\partial E_1} & \frac{\partial W_m}{\partial \beta_1} & \cdots\cdots & \frac{\partial W_m}{\partial \beta_N} \end{bmatrix}$$

Equation (14) may be of ill condition from mathematical viewpoint, the singular value decomposition (SVD) technique (Wang and Lytton 1993) is used to solve this equation.

NUMERICAL RESULTS

Dynamic Compliance Functions of Visco-Elastic Foundation

From Equations (2), (3) and (4), horizontal and vertical displacements, u and w, can be determined for a square footing (i.e., $a = b$, $L_x = L_y$). The dimensionless complex compliance functions of base can be represented as following

$$F_m^{xx}(a_0) = aG_1u_0/P_x = f_m^{xx}(a_0) + \mathrm{i}g_m^{xx}(a_0) \qquad (15a)$$

$$F_m^{zz}(a_0) = aG_1w_0/P_z = f_m^{zz}(a_0) + \mathrm{i}g_m^{zz}(a_0) \qquad (15b)$$

where $f_m(a_0)$ and $g_m(a_0)$ are real and imaginary parts of $F_m(a_0)$, u_0 and w_0 are respectively the amplitudes of horizontal and vertical displacements at the base center, P_x, P_z are the amplitudes of the total horizontal and vertical excitations respectively, $a_0 = {\omega a}/{v_s}$ is defined as the dimensionless frequency, $v_s = \sqrt{G_1/\rho_1}$ is shear wave velocity in the surface soil, G_1, ρ_1 is shear modulus and mass density of the surface layer, a is the width of base.

Comparative Studies

Computations and analyses in the conditions of $H/a = 2$, $L_x/a = 60$, $\nu = 0.4$ are made using the proposed spline semi-analytical approach. The numerical results are compared with other available theoretical solutions. Shown in Fig.3 and Fig.4 are vertical compliance functions of visco-elastic homogenous foundation for $\beta = 5\%$ and $\beta = 10\%$ obtained respectively by present method and finite-layer method (Swaddiwudhipong *et al.* 1991). A good agreement between these two methods can be observed. At the same time, the effect of material damping on vibration behavior of base are manifested by comparing the computational results given in Fig.3 and Fig.4.

Furthermore, the effect of layered non-homogeneity of visco-elastic foundation on dynamic compliance functions of base is studied. Firstly, a three-layer foundation is considered. The computed vertical compliance function in the condition of $H/a = 75$, $L_x/a = 75$, $\nu = 0.35$, $\beta = 2.5\%$ is given in Fig.5, in which H_1, H_2 and H_3 are respectively the thickness of the three layers from surface to bottom. For this example, $H_1 : H_2 : H_3 = 1:13:24.8$, $G_1 = G_3$. Five cases are considered, i.e., (1) Case 1: $G_2/G_1 = 0.1$; (2) Case 2: $G_2/G_1 = 0.2$; (3) Case 3: $G_2/G_1 = 0.5$; (4) Case 4: $G_2/G_1 = 0.8$; and (5) Case 5: $G_2/G_1 = 1.0$.

Then non-uniform distribution of shear modulus along depth is taken account. The calculated vertical compliance functions in the case of $H/a = 75$, $L_x/a = 75$, $\nu = 0.35$, $\beta = 2.5\%$ are displayed in Fig.6 and Fig.7. When $r = 1.0$, five cases for different values of the factor λ, i.e., $\lambda = 0.5$, 1.0, 2.0, 5.0, 10, are considered in computations and the results are given in Fig.6. And while $\lambda = 1.0$ and exponent index r varies from 0 to 1.0, e.g., $r = 0$, $1/2$, $2/3$, 1.0, the computed vertical compliance functions are shown in Fig.7.

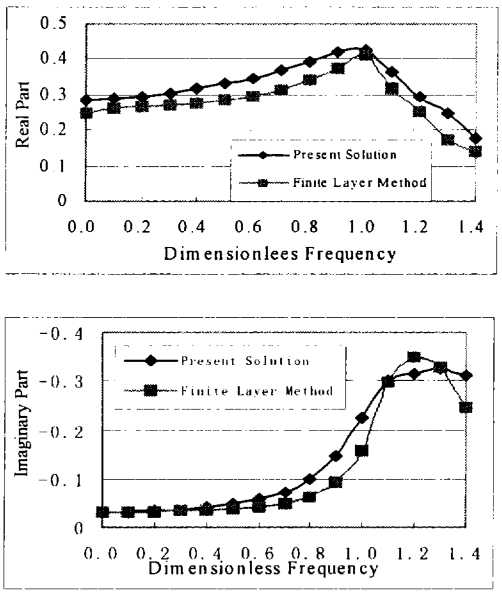

Fig.3. Vertical dynamic compliance function

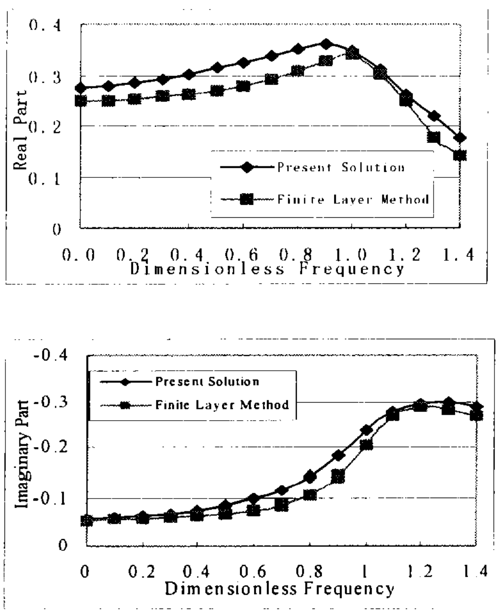

Fig.4. Vertical dynamic compliance function

The following insights can be deduced: (a) Compared with the solutions available, the proposed method can predict surface deflections or compliance functions with a favorably-acceptable accuracy. (b) The non-homogeneity of subsoil has considerable influences on dynamic response of base. The influence of the surface layer soil seems more serious than that of deeper layers. Hence more attentions to the upper layers should be paid in engineering practice. (c) Material damping of soil is indeed an important factor in governing dynamic response of soil foundation. At the same time, a far soil domain should be chosen in order to damp out the reflected radiating stress waves from the boundaries before reaching the region of interest. Therefore the proposed method is applicable to dynamic response analysis of base and non-homogenous soil foundation.

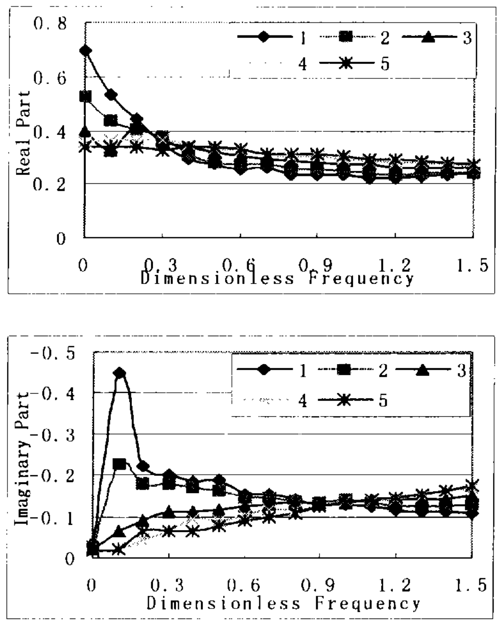

Fig.5. Vertical dynamic compliance function

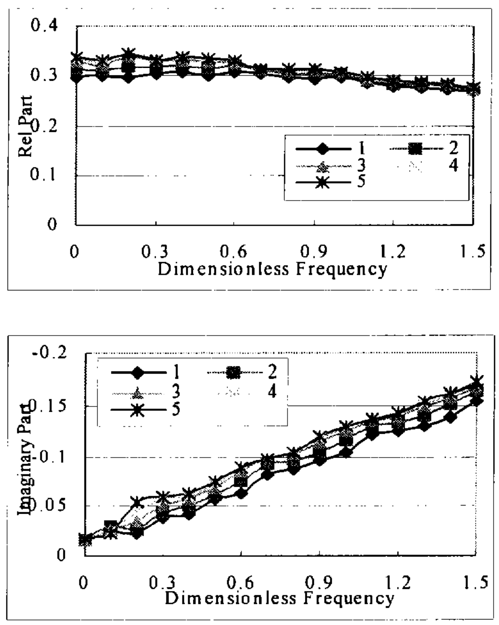

Fig.6. Vertical dynamic compliance function

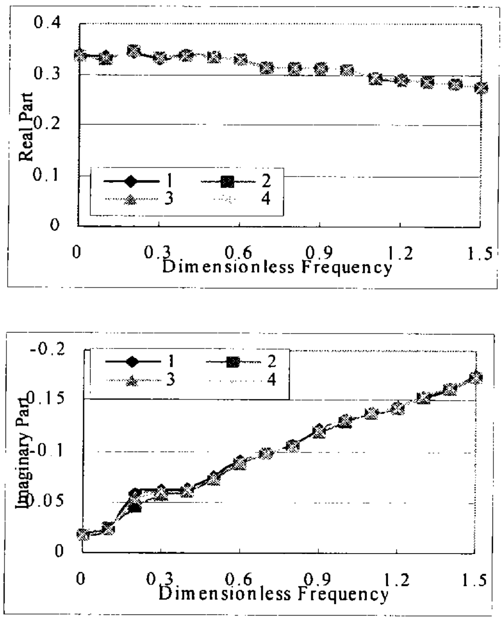

Fig. 7. Vertical Dynamic compliance function

<u>Pavement Parameter Back-Calculation Based on Field Tests</u>

It has been recognized by Stubbs *et al.* (1994) that for pavement and subsoil problem, the damping ratios in base course and surface course, i.e., β_1, β_2, are insensitive to the change of deflection. Therefore it is assumed that $\beta_1 = \beta_2 = 5\%$ as proposed by Boutros *et al.* (1985) in identifying the remaining parameters at the frequencies of 2.4Hz and 4.8Hz proposed by Kang (1998). The basic parameters of pavement are listed in Table 1. The back-calculated parameters are given in Table 2.

Table 1 Material Properties for Pavement Section at DB29-31

Layer	Thickness (m)	Poisson's Ratio	Density (kN/m^3)
CTB	0.150	0.25	18
LTC	0.460	0.35	18
SG	7.390	0.40	18

Note: CTB=Cement Stabilized Base; LTC=Lime-Treated Clay; SG= Subgrade.

Table 2 Backcalculation for Visco-Elastic Foundation

Test Site	CTB	LTC	SG
29	$E_1 = 779.7$	$E_2 = 659.1$	$E_3 = 116.4$ $\beta_3 = 5\%$
30	$E_1 = 413.4$	$E_2 = 381.7$	$E_3 = 90.3$ $\beta_3 = 4\%$
31	$E_1 = 553.2$	$E_2 = 434.1$	$E_3 = 126.7$ $\beta_3 = 3\%$

Note: *E* in MPa.

CONCLUSIONS

A semi-analytical procedure based on spline functions has been proposed to evaluate dynamic compliance functions of generally layered subsoil. The effects of non-homogeneity of soil properties along depth on foundation vibration behavior are comparatively studied on the basis of numerical results. Then the proposed procedure is combined with system identification method to back-calculate pavement parameters. From example analyses, the present methodology seems workable in pavement parameter identification.

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