29 Mar 2001, 7:30 pm - 9:30 pm

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ABSTRACT

Since the 1964 Niigata, Japan, earthquake, damages attributed to earthquake induced liquefaction phenomena have cost society hundreds of millions U.S. dollars. Most procedures developed so far predict the potential for earthquake induced liquefaction at the "point" or over the small area, where the soil strength is evaluated. This paper describes a technique to estimate the probability of earthquake induced liquefaction over arbitrary large areas. The proposed technique may be of special interest to both large corporation and insurance company risk management departments, which are looking at estimating earthquake damages over a large area. The area of interest is meshed forming a grid of individual cells, for which the probability of liquefaction is estimated. The probability of liquefaction for a given percentage of the total area is then computed as a system reliability problem.

INTRODUCTION

Earthquake induced liquefaction is defined as a loss of strength in loose saturated soils that exhibit contractive response to loading. The 1964 Niigata, Japan, the 1964 Alaska, and most recently the 1999 Kocaeli, Turkey, and Chi Chi, Taiwan, earthquakes highlighted the devastating effects of liquefaction. Damages attributed to the earthquake induced liquefaction phenomenon have cost society hundreds of millions of U.S. dollars (Seed and Idriss 1982). The most widely used procedure for prediction of liquefaction triggering was developed by Seed et al. (1975). In this empirical procedure, a judgmental curve is plotted in a space of cyclic stress ratio normalized for a magnitude 7.5 earthquake, (CSRN), a measure of earthquake loading, and overburden and energy corrected standard penetration resistance $(N_1)_{60}$, a measure of the soil strength, to divide case histories where surface manifestation of liquefaction was or was not observed. Liao et al. (1988) and Cetin et al. (1999) expanded the work by Seed et al. (1975) into a probabilistic framework to account for the variability and uncertainty inherent to the problem. Given the load and resistance terms, these models predict the probability of liquefaction at the point of interest. There is, however, a need for a rigorous methodology that will integrate the risk at a point to a risk over an area of interest incorporating the uncertainties inherent to the problem. In recent years, especially among insurance companies and corporations, there has been a growing interest in estimating the liquefaction initiation risk of a given percentage of the total surface area. This paper presents a structural reliability approach to estimate the probability of earthquake-induced liquefaction over a finite surface area.

PROPOSED APPROACH

The first step consists of meshing the area considered to form a grid of individual cells. The size and consequently the number of cells depend on the micro-scale adopted, which is problem and application specific. The next step consists on choosing a "point" probabilistic liquefaction model to compute the probability of liquefaction at any given cell. For each cell, the parameters used in the "point" probabilistic liquefaction model are defined as random variables. The collection of these random variables over the entire mesh is represented as a random field. Finally, simulation or system reliability approaches are used to compute the probability of liquefaction of k cells out of n, which represent the spatial extent of liquefaction.

"Point "probabilistic liquefaction model

Herein, the methodology developed by Cetin et al. (2000) is adopted to predict the probability of liquefaction at any given point. The limit state function is given by

$$g(N_{1.60}, CSR, FC, M_w) = N_{1.60}\left(1 + 5.3e^{-5}FC\right) - 13.22\ln(CSR) - 28.47\ln(M_w) + .145FC + 20.23 + \gamma \quad (1)$$

where $(N_1)_{60}$ is the overburden and energy corrected standard penetration blowcount, CSR is the cyclic stress ratio induced by the earthquake, FC is the fines content of the soil, M_w is the earthquake moment magnitude, and □, the random model correction term, is normally distributed with zero mean and standard deviation equal to 3.951. For a given set of random variables $(N_1)_{60}$, CSR, FC, and M_w, the probability of liquefaction is given by

$$p_i = \Phi\left(-\frac{g\left((N_i)_{60i}, CSR_i, M_i, FC_i, \gamma\right)}{\sigma_\gamma}\right) \qquad (2)$$

where $\Phi(\cdot)$ is the cumulative normal distribution function, and σ_γ is the standard deviation of the limit state function $\gamma(\cdot)$.

Liquefaction of k cells out of n

The estimation of the spatial extent of liquefaction is equivalent to the problem of estimating the probability of liquefaction of k cells out of a total of N cells. Let Y be a discrete random variable that takes the value of the number of cells that liquefy on a given earthquake. The spatial extent of liquefaction is thus represented by the probability mass function of the random variable Y. The probability that the random variable Y takes an exact value k is the sum of all possible combinations of exactly k cells liquefying while the other N-k cells do not liquefy. Assume that the probability of liquefaction of each cell is p_f. The two extreme cases are: (a) liquefaction at each cell is independent of liquefaction in all other cells and (b) liquefaction of all cells is fully correlated. For the first case, Y has a binomial distribution, while for latter, Y takes the value 0 with probability $(1-p_f)$, or N with probability p_f. These two cases are illustrated in Figure 1. In reality, however, the liquefaction potentials of two cells are neither independent nor fully correlated. As a result, the calculation of the probability distribution of Y is more cumbersome and can not be represented by a single formula. Symbolically, the probability mass function of Y can be written as:

$$f_y(k) = \sum P\left[\begin{matrix}(\text{k cells liquefy}) \cap \\ (\text{all other N - k cells do not liquefy})\end{matrix}\right] \qquad (3)$$

where the sum is over all possible combinations of k cells chosen out of N. In this paper, system reliability approach is presented to obtain this probability and simulations are used to verify the procedure.

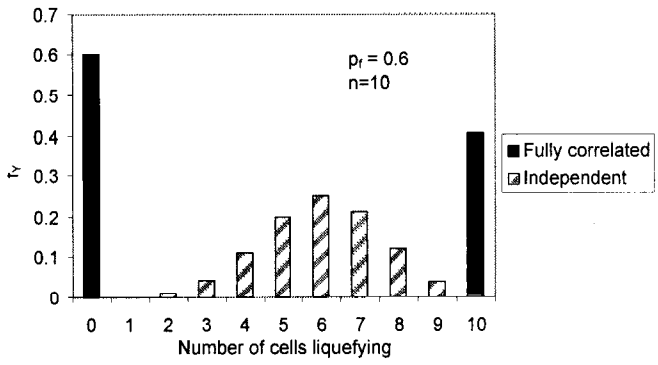

Fig. 1. Probability Mass Function of Y.

System reliability approach

The limit state function representing the probability of liquefaction of a cell *i* is given by Equation 1. Each limit state function g_i can be viewed as a component of a system defined by the collection of limit state functions g_i, i=1, ⋯,N. These limit state functions are mutually correlated because of the underlying correlation of the parameters used in defining g_i. Observe that the parameter representing model uncertainty, γ, is assumed to be independent for each limit state surface. This assumption does not hold if the contribution to model uncertainty arises mainly from inadequate representation of a parameter that is common to all cells, such as moment magnitude (M_w). This issue is relevant and merits further research. Figure 2 shows a sample system where N = 2 and where all the variables are deterministic with the exception of the Standard Penetration Resistance $(N_1)_{60}$. By convention, negative values of the limit state function define failure. Figure 2 also indicates the regions in which liquefaction occurs in both cells, in either cell, or there is not any liquefaction at all.

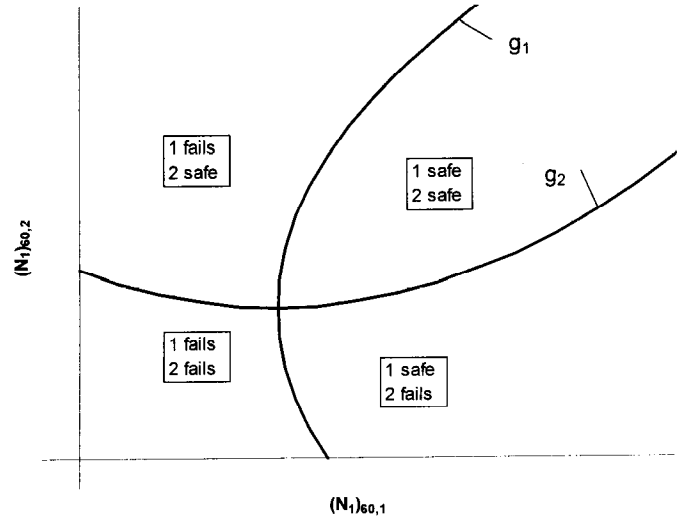

Fig. 2. Illustration of a System with Two Components

The probability inside the summation sign in Equation 3 can be defined as the probability of failure associated with the following

failure domain:

$$\begin{Bmatrix}(g_{l=1}<0)\cap(g_{l=2}<0)\cap....\cap(g_{l=k}<0)\cap\\(g_{j=1}>0)\cap(g_{j=2}>0)\cap...\cap(g_{j=n-k}>0)\end{Bmatrix} \quad (4)$$

where the index *l* corresponds to the indices of the k cells that liquefy, and the index *j* corresponds to the indices of the n-k remaining cells. The solution to the system reliability problem consists in finding the probability that the random variables defining the problem are in the failure domain defined by Equation 4. This probability can be found either through a simulation approach or by the linearization of the failure surfaces (Der Kiureghian 1999). The structural reliability program CALREL (Liu et al. 1989) was used to solve the systems problem. The failure domain in Equation 4 can be rewritten as

$$\begin{Bmatrix}(g_{l=1}<0)\cap(g_{l=2}<0)\cap....\cap(g_{l=k}<0)\cap\\(-g_{j=1}<0)\cap(-g_{j=2}<0)\cap...\cap(-g_{j=n-k}<0)\end{Bmatrix} \quad (5)$$

The probability of failure associated with this failure domain can be identified as the probability of failure of a parallel system defined by the limit state functions $g_{l=1}, g_{l=1}, \ldots, g_{l=k}, -g_{j=1}, -g_{j=2}, \ldots, -g_{j=n-k}$. The probability of failure and the associated generalized reliability index is found by CALREL first by transforming all the variables into the standard normal space. Directional simulation or Monte-Carlo simulation approaches are then used to find the corresponding probabilities of failure in the transformed domain (Liu et al. 1989). The determination of the probability mass function f_Y (Equation 3) requires the re-definition of the failure domain for each combination of k cells chosen out of n. Thus, the program CALREL must be executed each time for each combination of k cells chosen out of n. To reduce the computational effort, a first order approximation (FORM) may be used.

A first order approximation is constructed by first transforming all the random variables into the standard normal space. Let the set of random variables defining the liquefaction problem (i.e. $(N_1)_{60i}$, CSR_i, FC_i, M_i, γ_i, for i=1,⋯,n) be denoted by the random vector $\underline{x}$, and the transformed variables be denoted by the vector $\underline{u}$. The component limit state surfaces are thus denoted by $g_i(\underline{x})$, i=1, ⋯ n. Let the transformed limit state surfaces be denoted by the functions $G_i(\underline{u})$, i=1, ⋯ n. The component limit state surfaces $G_i(\underline{u})$ are linearized at the points of maximum likelihood (design points) within the failure domain defined by each $G_i(\underline{u}) < 0$ (Der Kiureghian 1999). Ideally, the linearization should be done at the points of maximum likelihood defined by the system failure domain (Equation 4). This approach, however, will require a different linearization for each combination of k cells chosen out of n. However, when the surfaces are not strongly nonlinear, linearization at the design point may be sufficient. The effect of nonlinearity has been investigated but is not reported in this paper because of the limitation on space. After linearization, each limit state surface $G_i(\underline{u})$ is replaced by the hyper-plane

$$\beta_i - \underline{\alpha}_i^T \underline{u} = 0 \quad (6)$$

where β_i is the distance form the origin and $\underline{\alpha}_i = \Delta G_i / \left\| \Delta G_i \right\|$ is the unit normal vector (Der Kiureghian 1999). Define $v_i = \underline{\alpha}_i^T \underline{u}$ and vectors $\underline{v} = [v_1 \cdots v_n]$ and $\underline{B} = [\beta_1 \cdots \beta_n]$. The vector $\underline{v}$ is a vector of normal variables with zero mean, unit variance, and correlation matrix given by $\underline{R} = [\rho_{kl}]$, where $\rho_{kl} = \underline{\alpha}_k^T \underline{\alpha}_l$. The first order approximation of the probability associated with the failure domain in Equation 4 is then given by:

$$\int_{\beta_{l=1}}^{\infty}\int_{\beta_{l=2}}^{\infty}\cdots\int_{\beta_{l=k}}^{\infty}\int_{-\infty}^{\beta_{j=1}}\int_{-\infty}^{\beta_{j=2}}\cdots \int_{-\infty}^{\beta_{j=n-k}} \Phi\left(v_{l=1},\cdots,v_{l=k},v_{j=1},\cdots,v_{j=n-k};\underline{\underline{R}}\right) dv_{l=1}\cdots dv_{l=k}\,dv_{j=1}\cdots dv_{j=n-k} \quad (7)$$

The integrand in Equation 7, $\Phi(.)$, is the n-dimensional standard multinormal probability density function. The subscript *l* corresponds to the k cells chosen out of n, and the subscript *j* to the remaining n-k cells. General closed form solutions for this integral exist only for n=2. For larger dimensions, the integral in Equation 7 is solved using simulations with a sequential conditioned importance sampling algorithm (Ambartzumian et al. 1998). The advantage of the linearization approach is that the vector $\underline{B}$ and the correlation matrix $\underline{R}$ are constant for all the combinations of k chosen out of n cells. The proposed approach is implemented using the algorithm shown in Figure 3.

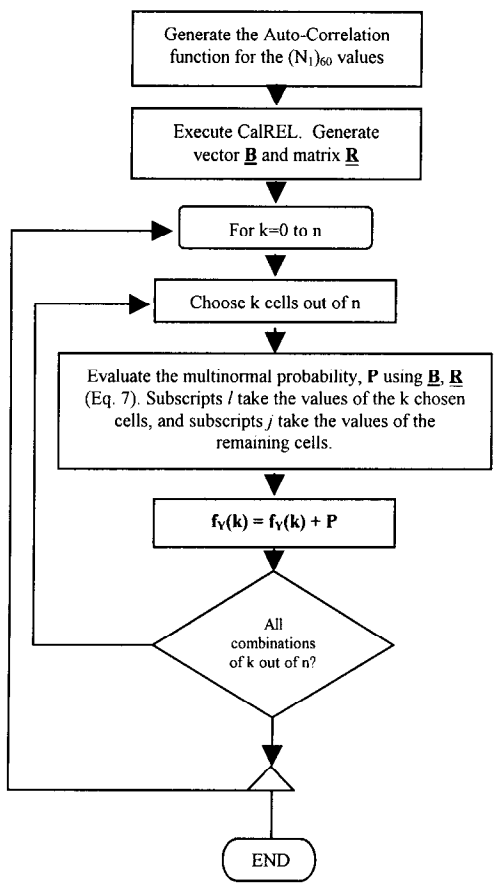

Fig. 3. Algorithm to Determine f_Y

APPLICATION

A sample problem was solved for illustration purposes. The problem consists of 10 cells arranged in a line. The distances between cells are normalized to one. The correlation functions for the random fields $(N_1)_{60}$ and CSR is defined as following:

$$\begin{cases} \rho(h) = 1 - (c_0 + c_1) + c_1 \exp\left(-\left(\frac{h}{a} \right)^2 \right) & \text{for h} > 0 \\ \rho(0) = 1 \end{cases} \qquad (8)$$

where h represents the distance between two points in the random field and parameter, a, describes the correlation scale. The parameters c_1 and c_0 can be evaluated using the limit case as h approaches zero and infinity. By imposing that the correlation vanishes for very large distances, the condition $c_0 + c_1 = 1$ must be satisfied. On the other hand, c_0 controls the maximum correlation that is permitted. LogNormal marginal probability density functions are used for both random fields. The parameter, a, in Equation 8 is varied to illustrate the effect of variations in the auto-correlation function. Larger values of parameter *a* correspond to a larger correlation of equally spaced points. Results are shown in Figure 4. Observe that the simulation and the structural reliability approaches render equal results. The length of the correlation structure has a marked effect on the results. In the limiting case where parameter, a, is equal to zero and infinity the results coincide with those of Figure 1. This illustrates that spatial extent of liquefaction is a function of the spatial auto-correlation of variables that determine liquefaction. Hence, "point" probabilistic liquefaction models are not sufficient to render good estimates of spatial extent of liquefaction.

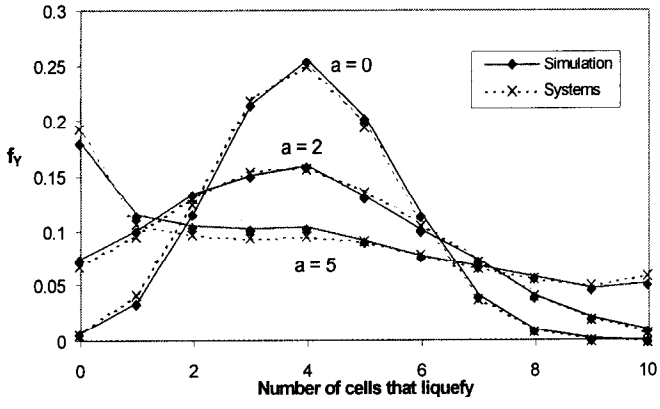

Figure 4. Simulation and Systems Reliability Approaches.

CONCLUSIONS

The approach presented herein constitutes a methodologically simple and sound approach to predict the spatial extent of liquefaction that accounts for the complete stochastic representations of the parameters affecting liquefaction. A simple example illustrated that "point" probabilistic liquefaction models alone alone are inapt to compute the spatial extent of liquefaction.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the help of Prof. A. Der Kiureghian. Without his teaching and his insight, this work would not have been possible.

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