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# Automated Slope Stability Analysis of Zoned Dams

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**SUMMARY:** The study pertains to the pseudo-static stability analysis of zoned dams with geologic discontinuities in the foundation. Sequential unconstrained minimization technique in conjunction with Janbu's generalized procedure of slices has been used for finding the critical slip surface and the corresponding minimum factor of safety. The method has been found to be quite efficient in solving such problems.

## INTRODUCTION

In natural and man-made slopes the slip surfaces that may develop during failure are generally non-circular regardless of whether the failure is caused by static or earthquake forces.

Due to its simplicity pseudo-static stability analysis based on limit equilibrium approach is one of the most widely used technique in computing the factor of safety of slopes under seismic conditions.

Various limit equilibrium methods of slope stability analysis have been excellently reviewed by Mostyn and Small (1987), Nash (1987) and, as such, these are not presented. Successful application of mathematical programming techniques to slope stability analysis using a general slip surface have been made (Martins, 1982; Fredlund, 1984; Mostyn and Small, 1987). Application of calculus of variation technique to slope stability problem is quite controversial (Fredlund, 1984). Even though dynamic programming (Baker, 1980) and linear programming (Munro, 1982) have successfully been applied in analysing stability of slopes they have not been widely used perhaps due to the inherent limitations of the techniques in tackling large nonlinear problems.

Intuitively or otherwise nonlinear programming has been widely used by the geotechnical engineering community in dealing with such problems.

As the efficiency of these techniques is problem oriented an attempt has been made in this paper to demonstrate the successful application of sequential unconstrained minimization technique for automated stability analysis of zoned dams. In addition the effect of earthquake forces (pseudo-static) on the stability of slopes has also been investigated.

## STATEMENT OF THE PROBLEM

Fig. 1 shows the geometry the D/S slope of a zoned dam with a general potential slip surface and with the sliding mass divided into N member of slices.

For the given geometry of the dam section and soil properties, the factor of safety is a function of the shape and location of the potential slip surface. The problem is to determine the shape and location of the slip surface and the associated minimum factor of safety.

## ANALYSIS

### General

The pseudo-static stability analysis of the D/S slope of the dam under earthquake loading is carried out by using the generalized procedure of slices (Janbu, 1973) in conjunction with sequential unconstrained minimization technique for autosearching the critical shear surface and the corresponding minimum factor of safety without a priori restriction on the nature of the slip surface. This is achieved by minimizing the factor of safety with respect to the co-ordinates of the slip surface.

### Earthquake Considerations

Stability of slopes are seriously affected by earthquakes. Earthquake accelerations caused by the ground movement induces an inertial force into the slope material providing an extra overturning moment. The vibrations due to earthquakes may result in the development of pore pressure build up in the slope and thus may cause reduction in the frictional resistance or even liquefaction. Thus the increase in the inertial force as well as the decrease in the shearing strength of the soil may cause failure if the slopes are subjected to ground movement of sufficient magnitude and duration.

In the pseudo-static stability analysis seismic effects are taken into account including a static horizontal force expressed as the product of a seismic coefficient and the weight of individual slice and acting at its centre of gravity. Seed (1973) has presented in detail the selection procedure of seismic coefficient.

As the quake actually imposes displacements rather than forces, the forces resulting from the displacements are dependent in a complicated way on

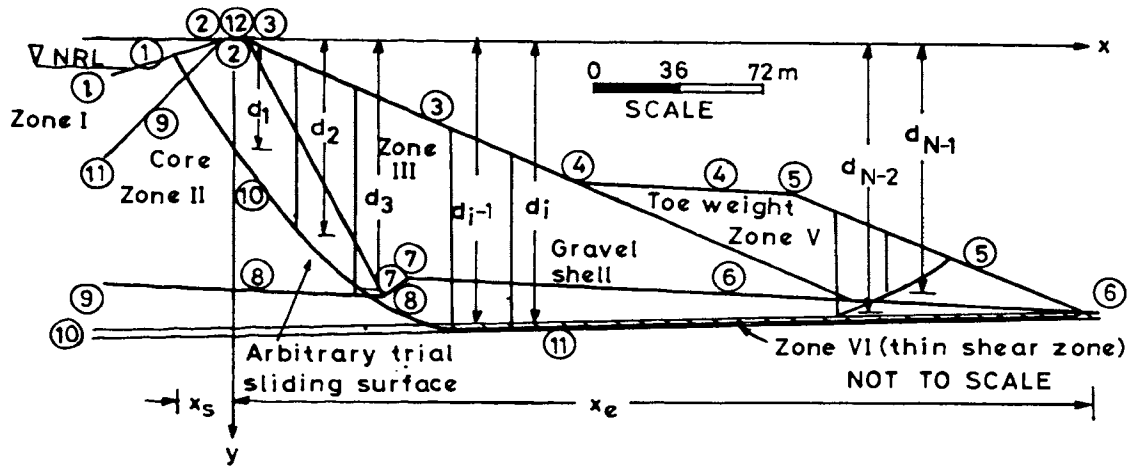


Fig. 1 Idealized Section of a Zoned Dam with the Potential Sliding Mass Divided into Slices.

dynamic stress-strain relationship of the embankment material. Hence the application of the pseudo-static method to analyse the seismic stability of slopes is quite controversial. There is a belief that the method should not be used under any circumstances as it cannot take into account the cyclic nature of forces applied to the slope. However, review of literature to (Mostyn and Small, 1987) suggest that the seismic coefficient method may be suitable for stability analysis of slopes in soils which show no significant loss of strength due to earthquake shaking (usually clayey soils, dry sands and some very dense cohesionless soils).

#### Design Variables and Objective Function

Referring to the Fig. 1 for a fixed number of slices the elements of the design vector are chosen as follows:

$$\bar{D}^T = (d_1, d_2, d_3, \dots, d_{N-1}, x_s, x_e) \quad (1a)$$

writing  $d_N = x_s$  and  $d_{N+1} = x_e$  one obtains

$$D^T = (d_1, d_2, d_3, \dots, d_{N+1}) \quad (1b)$$

So the total number of design variables are  $N+1$ .

The objective function is the factor of safety and for any given slip surface it is computed by using the Janbu's generalized procedure of slices (Janbu, 1973). The factor of safety can be written in terms of the design vectors as

$$F = f(\bar{D}) \quad (2)$$

#### Constraints

To ensure the acceptability of the potential slip surface the following constraints are imposed.

1. The curvature of the slip surface should be concave upward. This requires

$$d_{i+1} - 2d_i + d_{i-1} \leq 0 \quad (3)$$

2. The slip surface should be within the cross-section of the dam. This requires

$$h_i - d_i \leq 0 \quad (4)$$

where  $h_i$  is the y co-ordinate of the intersection point of the top boundary line of the dam section with the vertical line drawn through the point whose y co-ordinate is  $d_i$ ;  $i$  varies from 1 to  $N-1$ .

So the problem is one of  $(N+1)$  design variables and  $(2N-2)$  side constraints.

#### Slices, Width of Slices and Zoning

The encircled key points defining the idealized dam section as shown in Fig. 1 are numbered in order. The lines joining the different key points are also numbered; such numbers are marked by semicircles.

Once the dam geometry is defined by the co-ordinates of the key points, the coefficients of each straight line defining the dam section can be generated on the computer.

The optimal number of slices are to be obtained by a trade off study of computational efficiency and the cost involved. In the present study 24 m slice width has been found to be very satisfactory.

Correctness of the weight calculations of the slices by the developed computer program (Basudhar et al., 1988) has been ensured for different potential slip surface, position of slip surface and comparing the results with manually computed values. The details are reported by Babu (1986) and are not reported here.

#### Existence of Thin Shear Plane in the Dam Foundation

If very thin shear zone is present in the foundation, while searching for the critical slip surface there is a possibility that the surface may lift off from this weak zone ultimately converging to a solution which is much higher than the critical one. This possibility is safeguarded by choosing the initial trial shear surface to lie mostly along the shear zone and also by arbitrarily increasing the shear zone thickness. It has been observed that the critical shear surface does not move much from the actual shear

and, as such, the results are not affected significantly and the technique can be adopted with confidence.

### Optimization Formulation

The problem of finding the critical slip surface and the corresponding minimum factor of safety is stated as a mathematical programming problem as follows.

Find the design vector  $\bar{D}_m$  such that  $F = f(\bar{D}_m)$  is the minimum of  $f(\bar{D})$  subject to

$$g_j(\bar{D}_m) \leq 0; \quad j = 1, 2, \dots, M$$

where  $M$  is the total number of constraints.

### Minimization Procedure

The sequential unconstrained minimization technique using the interior penalty function formulation in combination with Powell's multidimensional search and quadratic fit for finding the minimizing steps, has been used. The basic object of the penalty function method is to convert the original constrained problem into one of unconstrained minimization by blending the constraints into a composite function ( $\Psi$ ). The detailed background of these methods are available in standard textbooks on optimization (Rao, 1984).

For problems with inequality constraints only, the  $\Psi$ -function is defined as:

$$\Psi(\bar{D}, r_k) = F(\bar{D}) - r_k \sum_{j=1}^M \frac{1}{g_j(\bar{D})}$$

where  $F$  is to be minimized over all  $\bar{D}$ , satisfying

$$g_j(\bar{D}) \leq 0; \quad j = 1, 2, \dots, M$$

The penalty parameter  $r_k$  is made successively smaller in order to obtain the constrained minimum of  $F$ .

### RESULTS AND DISCUSSIONS

The following design parameters are used in the analysis:

Unit weight:	Core	20.4 KN/m <sup>3</sup>
	Shell	24 KN/m <sup>3</sup>
	Toe-weight	24 KN/m <sup>3</sup>

Angle of shearing resistance ( $\phi'$ ):

Core	26.5°
Shell and toe-weight	35°
Shear zone	18°

Effective cohesion ( $C'$ ):

Core	0, 20, 40KPa
Shell and toe-weight	0
Shear zone	0

Pore pressure ratio ( $r_u$ ): 0.3, 0.4, 0.5

Seismic coefficient ( $\alpha_g$ ): 0, 0.05, 0.1, 0.12

Numerical results have been obtained by using

DEC 1090 system and the SUMSTAB package (Basudhar et.al., 1988).

Typical results with three different initial trial surface is presented in Table 1. The table

TABLE 1. Factors of Safety for  $c' = 20$  KPa

$\alpha_g$	$r_u$	Set 1			Set 2		Set 3	
		$F_{avg}$	$F_{st}$	$F_{min}$	$F_{st}$	$F_{min}$	$F_{st}$	$F_{min}$
0.0		1.522	1.829	1.512	1.622	1.497	1.741	1.559
5.0	0.3	1.193	1.299	1.204	1.262	1.164	1.407	1.212
10.0		0.982	1.043	0.996	1.026	0.952	1.143	0.999
12.0		0.919	0.968	0.929	0.931	0.899	1.038	0.930
0.0		1.268	1.412	1.267	1.347	1.247	1.501	1.290
5.0	0.4	0.996	1.068	1.001	1.047	0.973	1.165	1.014
10.0		0.823	0.863	0.830	0.835	0.804	0.929	0.835
12.0		0.761	0.783	0.753	0.778	0.753	0.866	0.777
0.0		1.012	1.105	1.009	1.079	0.995	1.199	1.034
5.0	0.5	0.806	0.852	0.813	0.839	0.787	0.931	0.819
10.0		0.663	0.679	0.666	0.675	0.648	0.749	0.675
12.0		0.615	0.632	0.609	0.629	0.604	0.698	0.631

$F$  = Starting factor of safety

$F_{st}$  = Critical factor of safety

$F_{avg}$  = Average of the critical factors of safety values.

shows that the initial starting point design vector has marginal influence on the numerical scheme. Similar results have been obtained for other values of  $C'$ . The average of the critical factor of safety for three input surfaces in each case is calculated and is shown in these tables.

The influence of the number of function evaluation and the penalty parameter ( $r_k$ ) on the solution has been studied. It has been found that as the number of function evaluation increases the objective function ( $F$ ) and the composite function ( $\Psi$ ) converge to the same value. This signifies that the solution scheme is quite efficient in locating the minimum factor of safety under the imposed design constraints. Similar observations have also been made for the decreasing sequence of the penalty parameter (see Basudhar et.al., 1988).

Fig. 2 shows the plot of the minimum factor of safety with pore pressure parameter ( $r_u$ ) for different seismic coefficient and cohesion value. It will be seen from the figure that the factor of safety is linearly related to the pore pressure ratio for various values of the effective cohesion and horizontal acceleration. Bishop and Morgenstern (1960), without considering earthquake forces, have used a linear relationship of the following form:

$$F = m - n r_u$$

where  $m$  is the value of  $F$  for  $r_u = 0$  and  $n$  is the slope of the  $F$  vs.  $r_u$  straight line. The validity of the above relation for slopes subjected to earthquake forces has been checked over a wide range of values for horizontal acceleration.

It is also observed that for a given  $r_u$ , the factor of safety decreases with increasing horizontal acceleration. The rate of decrease for a given  $r_u$  depends on the range of values of horizontal acceleration under consideration. Also for all values

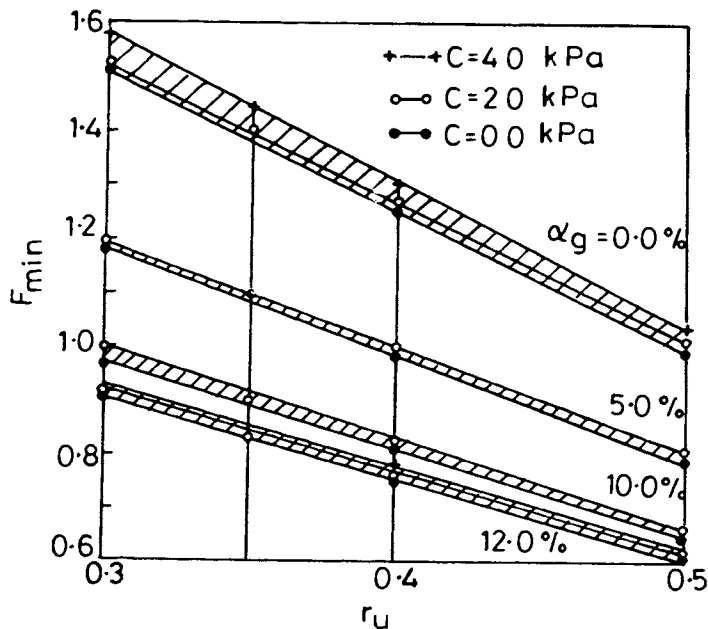


Fig. 2  $F_{min}$  Versus  $r_u$  Relationship.

of horizontal acceleration greater than zero, the magnitude of the effective cohesion does not seem to have any significant influence on the factor of safety. This observation is to be viewed in the light of the fact that for the dam section as shown in Fig. 1 the major portion of the slip surface is controlled by the shear zone with  $C'=0$ .

The linear relationship enables to interpolate/extrapolate for values of  $F$  under different condition of pore water pressure and earthquake loading. Fig. 3 shows the variation of coefficients  $m$  and  $n$  with horizontal acceleration which allows computation of factor of safety for any particular value of  $\alpha_g$ .

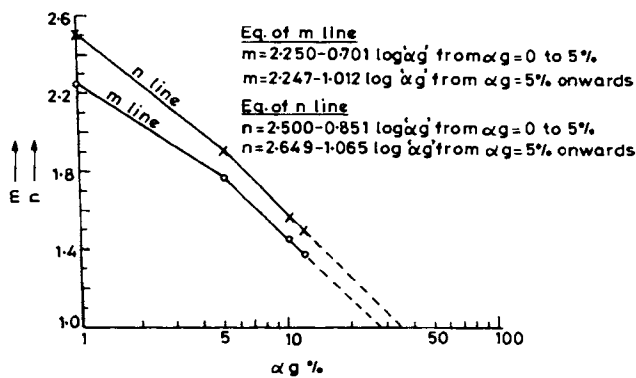


Fig. 3 ' $m$ ' and ' $n$ ' Versus  $\alpha_g$  Relationship.

For gravity forces in excess of 5%, the magnitude of pore water pressures governs the factor of safety; the effect of  $C'$  value for the core material being only marginal. The result is in accordance with the general practice to control pore water pressures through efficient drainage measures in dams subjected to earthquake forces. As such the results shown in Figs. 2 and 3 can be useful aid to decision making in design and construction of dams.

## CONCLUSIONS

Sequential unconstrained minimization technique is quite efficient in locating the generalized critical non-circular slip surface without a priori assumption regarding its shape. For different  $C'$ ,  $r_u$  and  $\alpha_g$  values the relationships between  $F_{min}$  and  $r_u$  is linear. For higher gravity values the magnitude of  $C'$  does not significantly change the factor of safety for the dam under investigation. It has also been seen that for gravity values greater than 5%, for every increase in the gravity value by a factor of 2 the factor of safety is reduced by 20%. Such a chart is quite useful for design purposes. The developed method of analysis is quite effective with respect to speed, accuracy and economy.

## REFERENCES

- Babu, N.S. (1986), Optimization Techniques in Stability Analysis of Zoned Dams, M.Tech. Thesis, I.I.T. Kanpur, India.
- Baker, R. (1980), Determination of the Critical Slip Surfaces in Slope Stability Computations Int. Journal for Numerical and Analytical Methods in Geomechanics, 4, pp. 333-359.
- Basudhar, P.K., Yudhbir and Babu, N.S. (1988), SUMSTAB - A Computer Software for Generalized Stability Analysis of Zoned Dams, Numerical Methods in Geomechanics (Innsbruck 1988).
- Fredlund, D.G. (1984), Analytical Methods for Slope Stability Analysis, Proc. of Fourth Int. Symp. on Landslides, Toronto, 1, pp. 229-250.
- Janbu, N. (1973), Slope Stability Computation, Embankment Dam Engineering, Casagrande Volume edited by R.C. Hirschfeld and S.J. Poulos, John Wiley and Sons, New York, pp. 47-86.
- Martin, J.B. (1982), Embankments and Slopes by Mathematical Programming, J.B. Martins (ed.), Numerical Methods in Geomechanics, D. Reidel Publishing Company, pp. 305-334.
- Mostyn, G.R. and Small, J. (1987), Method of Stabilisation Analysis, Slope Instability and Stabilisation, Walker and Fell (eds.), Balkema, Rotterdam.
- Munro, J. (1982), Plastic Analysis in Geomechanics by Mathematical Programming, J.B. Martin (Ed.), Numerical Methods in Geomechanics, D. Reidel Publishing Company, pp. 247-272.
- Rao, S.S. (1984), Optimization Theory and Application, Wiley Eastern Limited.
- Seed, H.B. (1973), Stability of Earth and Rock-fill Dams During Earthquake, Embankment Dam Engineering, Hirschfeld and Poulos (Eds.), pp. 239-269.