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ABSTRACT: For the smooth functioning of turbomachines, the design of the rotor, the bearings, the supporting structure, and the foundation should be carried out as an integrated system. The frequency domain method in conjunction with FEM is chosen for the analysis. The modified influence matrix boundary condition method is applied to reduce the number of equations to be solved. Results for 4 idealized cases are compared. It is shown that the interaction effects are significant in the lowest frequency domain.

INTRODUCTION

There has been rapid growth in the use of turbomachinery. As these are high speed machines, careful attention must be paid to the dynamic response of rotor-bearing-pedestal system. For the machine to run smoothly the mechanical design of the rotor, the bearings and the supporting structure should be such that they act as a single system and together respond to acceleration, critical speed, synchronous whirl, instability, and so on. Each element of soil-foundation-frame-machinery system interacts with each other and hence, an integrated analysis has been presented.

LITERATURE REVIEW

Reissner (1936) studied the effect of a harmonic vertical force assuming uniform stress distribution under the surface footing. Sung (1953) and Quinlan (1953) presented solutions for displacement with uniform, parabolic and 'rigid base' stress distributions. Arnold et al. (1955) considered all the four modes of vibrations treating one at a time while Hseish (1962) extended these to cover all the six coupled modes of vibration. Kobori et al. (1963) solved the vibration problem of rectangular footing. Novak et al. (1973) computed series solutions for stiffness and damping of piles using Baranov's work. Lysmer and Kuhlemeyer (1967) developed an approximately equivalent viscous boundary to absorb the waves radiated in infinite, homogeneous, isotropic body. Valliappan et al. (1976) extended it to an anisotropic material. Lysmer and Wass (1972) provided accurate boundary conditions for layered soil mass for plain strain condition and Kausel et al. (1975) extended it to axisymmetric case wherein the lower boundary was represented by an infinitely stiff layer. Wilson and Brebbia (1976) worked out the turbomachinery problem assuming the foundation mat to be rigid and neglecting the effect of soil. Aneja and Dimmik (1977) conducted model tests with the frame and found the FE solution to give good results.

NUMERICAL TECHNIQUES

There are three methods to analyse any dynamics problem with FEM: (i) Direct integration, (ii) Mode superposition technique, and (iii) Frequency domain method.

Direct integration of differential equation of wave propagation can be done only in small steps numerically. Otherwise, it often leads to numerical instability. Large CPU time is warranted and hence, this method is not recommended. To use mode superposition technique, the stiffness and damping matrices should satisfy some orthogonality conditions with respect to the modal matrix and should be independent of frequency. Since, in geotechnical problems the semi-infinite space cannot be discretised as it is, imaginary viscous boundaries are assumed to absorb the stress waves, which are definitely a function of frequency. Also to use mode superposition method, one should already know the relative importance of various eigen modes, which is not always possible. Hence, this technique also is not preferred. Herein, the third method is preferred as its demands are relatively less compared to the other two techniques.

FREQUENCY DOMAIN METHOD

Though this method is strictly applicable for steady state vibration problems, transient cases can also be analysed representing the completely arbitrary forcing function by a Fourier series or in the limit, exactly, as a Fourier integral. Fast Fourier transform technique can be used to efficiently synthesize the results. The method is based on the assumption that both the impedance and the response functions are of the same harmonic, with a phase difference. An equivalent static problem is formulated as below.

The variational principle states that, among all possible time histories of displacement configurations which satisfy compatibility and the kinematic boundary conditions, and which also satisfy conditions at time t_1 and t_2, the history which is the actual solution makes the Lagrangian functional a minimum, e.g.,

$$\delta \int L \, dt = 0 \quad (1)$$

where L is the Lagrangian given by

$$L = T - U - W_p \quad (2)$$

with T = kinetic energy, U = strain energy, W_p = potential energy. In matrix notation, the kinetic energy density is defined as

$$dT = \frac{1}{2} \rho \{\dot{u}\}^T \{\dot{u}\} \, dV \quad (3)$$

where $\dot{u}$ = derivative of displacement with respect to time, V = volume, ρ = mass density.

For a linear elastic body,

$$L = \frac{1}{2} \iiint_V (\rho\{\dot{u}\}^T \{\dot{u}\} - \{\varepsilon\}^T [C] \{C\} + 2\{u\}^T \{\bar{x}\}) dV + \iint_{S_1} \{u\}^T \{\bar{T}\} \, dS_1 \quad (4)$$

where $\{\varepsilon\}$ = strain vector, [C] = constitutive matrix, $\{\bar{T}\}$ = surface load vector, over the area S_1, and $\{\bar{x}\}$ = body force.

Also $\{u\} = [N] \{q\}$ (5)

with [N] = shape functions, {q} = nodal displacement vector.

If [B] represents strain-displacement matrix then,

$$\{\varepsilon\} = [B] \{q\} \quad (6)$$

and

$$\{\sigma\} = [C][B] \{q\} \quad (7)$$

Substituting Eqs. (5), (6) and (7) into (4) and applying variational principle,

$$\int_{t_1}^{t_2} (\{\delta q\}^T \iiint_V [B]^T [C][B] dV \{q\} - \{\delta\dot{q}\}^T \iiint_V \rho [N]^T [N] \, dV \{q\} - \{\delta q\}^T \iiint_V [N]^T \{\bar{x}\} \, dV - \{\delta q\}^T \iint_{S_1} [N]^T \bar{T} \, dS_1) \, dt = 0 \quad (8)$$

Since $\{\delta q\}$ is arbitrary one gets

$$[m] \ddot{q} + [k] \{q\} = Q \quad (9)$$

where m, mass matrix $= \iiint_V \rho [N]^T [N] \, dV$,

k, stiffness matrix $= \iiint_V [B]^T[C][B] \, dV$, and

Q, load vector $= \iiint_V [N]^T \{\bar{x}\} \, dV + \iint_{S_1} [N]^T \{\bar{T}\} dS$

If the damping can be assumed to be of the linear viscous nature, then

$$[m] \{\ddot{q}\} + [D] \{\dot{q}\} + [k] \{q\} = \{Q\} \quad (10)$$

where [D] is the damping matrix.

To apply the frequency domain analysis

$$\{Q\} = \{Q_o\} \, e^{i\omega t} \quad (11)$$

$$\{q\} = \{q_o\} \, e^{i(\omega t - \emptyset)} = \{\bar{q}\} \, e^{i\omega t} \quad (12)$$

Substituting Eqs. (11) and (12) into Eq. (10) and simplifying one gets

$$[A] \{\bar{q}\} = \{Q_o\} \quad (13)$$

where [A] is the complex stiffness matrix

$$[A] = -\omega^2 [m] + i\omega [D] + [k] \quad (14)$$

Equation (13) can be solved in frequency domain as

$$Q(t) = \int_{-\infty}^{\infty} \bar{Q}(\omega) \, e^{i\omega t} \, d\omega \quad (15)$$

Therefore,

$$[m] \{\ddot{q}\} + [D] \{\dot{q}\} + [k] \{q\} = \int_{-\infty}^{\infty} \bar{Q}(\omega) e^{i \ t} \, d\omega \quad (16)$$

The solution of this is assumed to be of the form

$$q(t) = \int_{-\infty}^{\infty} Y(\omega) \, e^{i\omega t} \, d\omega \quad (17)$$

The resulting set of equations to be solved is

$$[-\omega^2[m] + i\omega [D] + k]Y(\omega) = \bar{Q}(\omega) \quad (18)$$

q(t) can be retrieved from $Y(\omega)$ by an inverse Fourier analysis. The evaluation of complex loads $Q(\omega)$ and complex displacement can be done very easily by Fast Fourier Transform.

The amplitude of dynamic displacement can be calculated by

$$|\bar{q}| = \{Re^2(\bar{q}) + im^2(\bar{q})\}^{1/2} \quad (19)$$

and the phase difference, relative to a reference vibration is

$$\emptyset = -\arctan\{im(\bar{q})/Re(\bar{q})\} \quad (20)$$

Elements

Soil is represented by ZIB8 (Zienkiewicz-Irons Brick 8-noded) elements. Imaginary viscous boundaries are assumed to absorb the stres waves. The foundation mat is represented by 8-noded plate elements developed by Rock and

Hinton (1974). Frames are as usually represented by frame elements. Lumped mass formulation is preferred since it gives good results with lesser work compared to consistent mass formulation. Instead of solving for a particular set of forces coming from a given turbine, the general procedure as used by Brebbia and Wilson (1976) is preferred. In this method, the system response is solved with unit loads at different nodes.

Idealisation of Semi-infinite Medium

Since it is impossible to discretise the semi-infinite space as it is, an equivalent system as proposed by Lysmer et al. (1967, 1972) is analysed. At the assumed boundaries viscous dashpots are provided and their coefficients calculated.

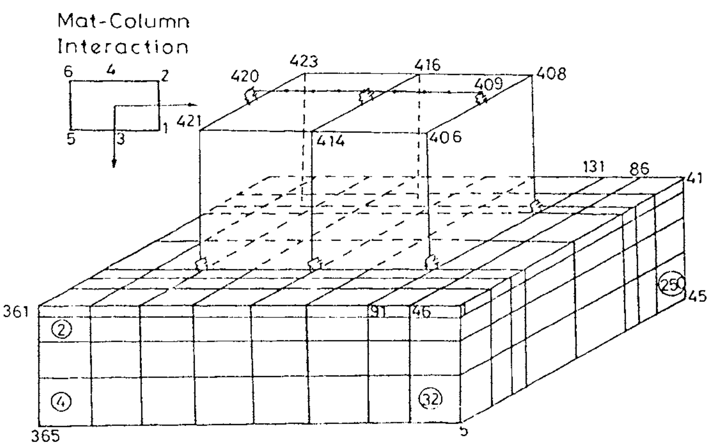

Fig. 1 Finite Element Descretization.

Step-by-step Procedure

The complete structure-foundation-soil-system considered for analysis is shown in Fig. 1. The soil-mat system has 405 nodes and 1215 d.o.f. The frame and rotor have 24 nodes and 144 d.o.f. 18 d.o.f. at nodes 1,2,...,6 are common to both the systems. Thus the total number of complex simultaneous equations to be solved is 1215 + 144 - 18 = 1341 which even with a lesser semi-bandwidth of 250 demands for a memory of 650 - 700 K units. Using the modified influence matrix boundary condition method, the analysis is carried out with a memory of only 40 K. Advantage is taken of the soil-mat system being symmetric about both x and y axes.

(a) The dynamic stiffness matrix of outermost slice (Fig. 2) of one quarter block (say II) is formed by usual FEM. Nodes not connected to the remaining part of the system (31 - 50) are condensed out. Treating this as an element stiffness matrix, it is assembled with stiffness matrix of the next slice, and condensation is again done for unwanted d.o.f. Repeating the same procedure for all the 4 slices, dynamic stiffness matrix for one quarter block is found.

(b) Element numbering for block I is done as in Fig. 3 (by reflecting the nodes of block II). With this numbering the stiffness matrix remains the same for both blocks I and II. Assembling these two again, condensation is performed for d.o.f. not connected to remaining part of the system. This forms the combined dynamic stiffness matrix of blocks I and II.

(c) The dynamic stiffness matrix of complete mat-soil system can again be arrived as from (b). All the d.o.f. except those connected with the superstructure are condensed out. The resulting 18 x 18 matrix is the impedance matrix of soil-mat combined system at nodes 1-6.

(d) This 18 x 18 matrix can be added to superstructure matrix and the resulting global matrix is solved for any type of loading in the superstructure. Once the soil-mat impedance matrix is obtained, the analysis can be done for different properties of superstructure without much computational effort.

RESULTS AND DISCUSSION

Impedance matrices of the soil-mat combined system with data listed in Table 1, are established for the nodes at which the columns are attached to the mat, for various frequencies (ω = 50, 100,...,400). In these matrices, any element in the i-th row, j-th column represents that force that has to act at the node (i) of the mat (intersection of the column axis with mat) in order to produce just one unit vibration amplitude at node (j), with all other amplitudes being zero. Using these matrices, the superstructure is analysed. Vertical amplitudes are computed while dynamic forces of $0.025^2 \sin \omega t$ are acting on the discs in vertical direction and horizontal response is obtained when dynamic forces of $0.025^2 \cos \omega t$ are acting on the discs in horizontal direction (perpendicular to the axis of rotor). This is due to the assumption that the cross stiffness of the bearing fluid is negligible.

The magnification factors have been obtained for various frequencies for the following four cases:

(i) Soil-mat-frame-rotor combined system. Soil is assumed to have weight also.

(ii) Soil-mat-frame-rotor combined system but assuming soil to be weightless elastic medium.

(iii) Frame-rotor system-soil is not considered. Frame is assumed to be fixed at the bottom.

(iv) Rotor alone-ends are assumed to be free.

In general, assuming the frame to be fixed at the bottom overestimates the displacement amplitude (Fig. 4a). Assuming the soil to be massless elastic medium, underestimates actual amplitudes. At higher frequencies, while the free rotor gives higher amplitudes, the other three cases produce

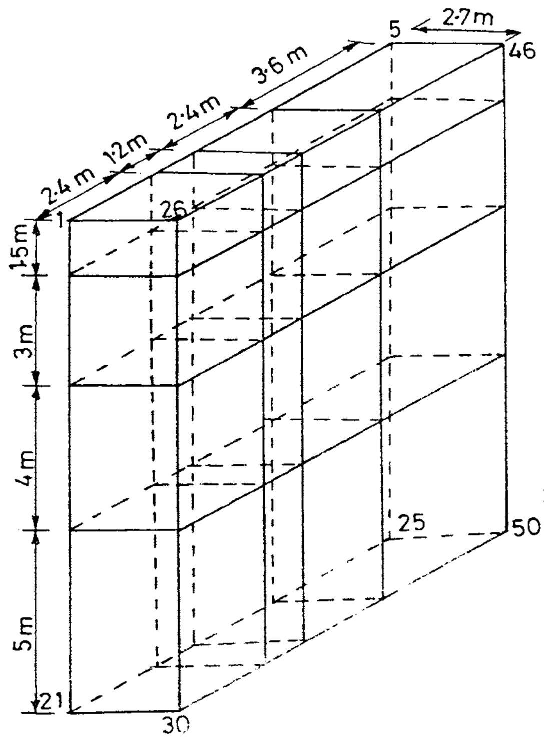

Fig. 2 Condensation for One Block.

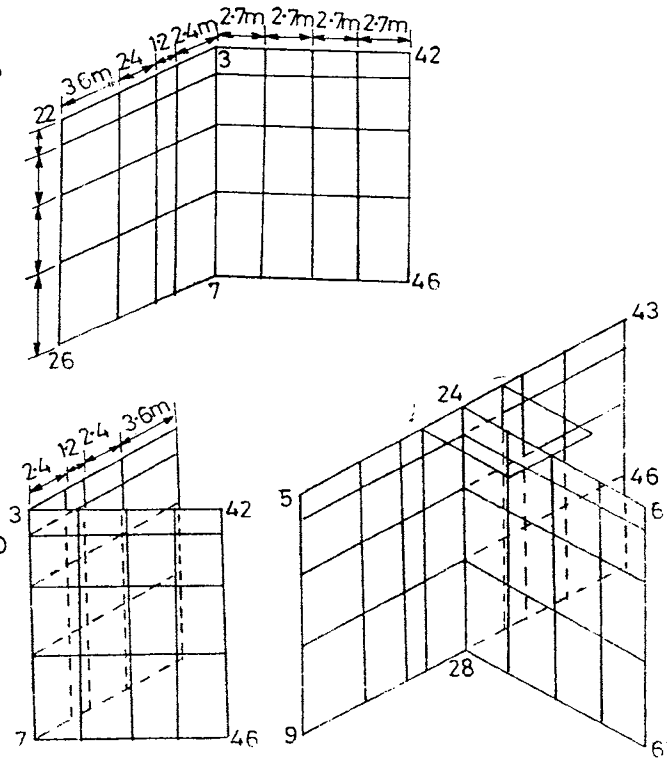

Fig. 3 Condensation of Blocks.

TABLE 1

Beams	EA = 38561 kg EI = 9545 kg-m^2 GJ = 5508 kg-m^2 Weight/m = 1441 kg
Columns	EA = 28889 kg EI = 4023 kg-m^2 GJ = 2321 kg-m^2 Weight/m = 1082 kg
Rotor	E = 2.1 E + 10 kg/m^2 Soft dia. = 0.4572 m ρ = 7900 kg/m^3 Weight of one disk = 2269 kg
Casing	Weight = 2974 kg/m
Fluid bearings	Stiffness varies linearly from 0 at ω = 0 to 1487.26 kg/m at ω = 200 and then remains same Damping varies linearly from 0 at ω = 0 to 10.3 kg-sec/m at ω = 200 and then remains same
Soil	V_p = 480 m/sec V_s = 250 m/sec ρ_s = 1700 kg/m^3 E = 27919892 kg/m^2 ν = 0.3138773

Note: Data for soil (dense sand and gravel) are obtained from Barkan, 1961.

nearly same amplitudes. Upto the frequency order of 200, elastic soil with mass gives lowe magnification factors compared to cases 3 and 4 For low operating frequency machines, soil-structure interaction has marked effect on the response of rotor.

The flexibility of the frame is higher in horizontal direction than the vertical direction an hence, the amplitudes also are higher (Fig. 4b) For low order frequencies case 3 shows more amplitudes but at higher frequencies free rotor gives maximum amplitudes. In general, assuming the soil to be massless elastic medium underestimates the magnification factors. However, in frequency range higher than 250 all the cases show nearly similar effect.

The CPU time elapsed for each step has been shown in Table 2. Apart from the lesser demand of core, the saving in CPU time due to reflection of nodal numbers is expected to be around 65 to 70 percent of one-shot solution. Even though the external forces in the example are symmetric with respect to an axis, the method is general and once the impedance matrix of the part of a system is established, the remaining part can be analysed for any general type of loading.

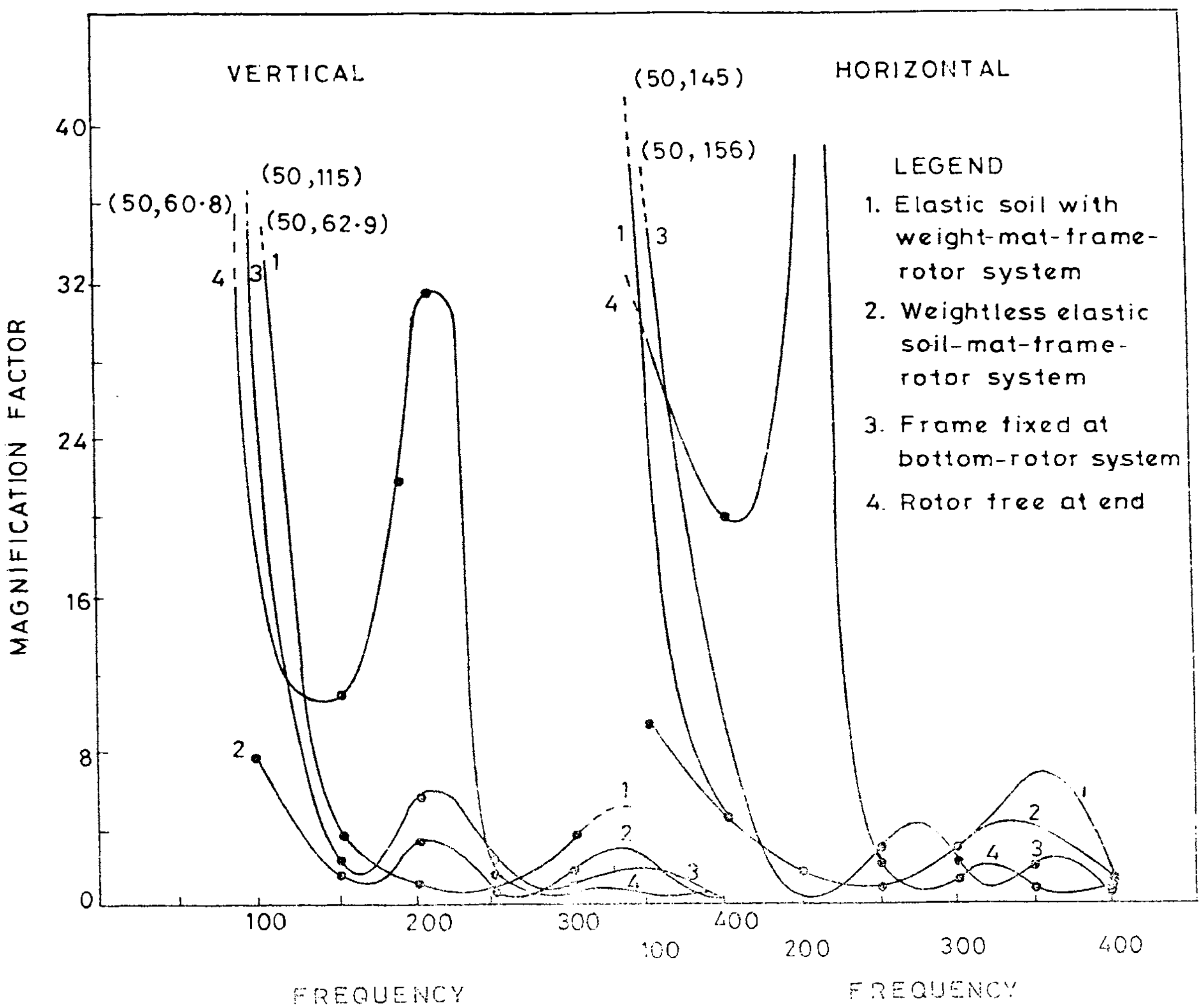

Fig. 4 Magnification Factor Versus Frequency.

CONCLUSIONS

A theoretical approach has been formulated and programmed that makes it possible to analyse the complete soil-mat-frame-rotor-oil film system, considering the interaction of various components. Influence matrix boundary condition is efficiently used to reduce the core and CPU time. The mat impedance matrix is formulated to represent the whole supporting medium beneath the frame. It was found that this interaction markedly affects the response of rotor at the lowest frequencies.

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TABLE 2

Description	Total d.o.f.	Condensed d.o.f.	CPU time (secs)	Maximum core required
1. To form stiffness matrix of outermost slice and to condense unwanted d.o.f.	100	60	29.4	30 K
2. To form stiffness matrix of second outermost slice assembled with (1) and to condense unwanted d.o.f.	165	60	33.5	30 K
3. To form stiffness matrix of third outermost slice assembled with (2) and to condense unwanted d.o.f.	180	57	33.9	30 K
4. To complete one quarter block stiffness matrix and to condense unwanted d.o.f.	198	60	41.7	30 K
5. To form stiffness matrix of two-quarter blocks and to condense unwanted d.o.f.	201	60	76.0	40 K
6. To form final impedance matrix of soil and mat from (3)	147	127	43.4	40 K