International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics

1991 - Second International Conference on Recent Advances in Geotechnical Earthquake Engineering & Soil Dynamics

13 Mar 1991, 1:30 pm - 3:30 pm

On Predictions and Performance of Machine Foundations

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Proceedings: Second International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, March 11-15, 1991 St. Louis, Missouri, Paper No. 11.36

On Predictions and Performance of Machine Foundations

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SYNOPSIS: A model for predicting the amplitude versus frequency response for surface footings under vibrations is presented. The model considers the effects of soil nonlinearity and confining pressures on soil modulus and has been verified by comparing model predictions with performance field tests.

The elastic half space analog solutions have been used to develop a computer program for this prediction model (Manyando, 1990). Soil nonlinearity has been accounted for by incorporating an iterative procedure in the computer program. The computer program was used to predict the footing response in vertical, torsional and coupled rocking and sliding vibrations. Predicted resonant amplitudes, resonant frequencies and the total sweeps have been compared with the measured values.

It has been found that predictions by this model closely match the measured data. Vertical and coupled rocking and sliding vibrations predictions closely match measured data when material damping is neglected. Predictions for torsional vibrations are observed to be satisfactory when 10 percent material damping is used and the slip at the base of the footing is neglected.

INTRODUCTION

The design of a machine foundation requires the determination of its natural frequency and amplitudes of vibration at the operating frequency. Very little information is currently (1991) available on the comparison of predicted and measured response of surface footings under dynamic loads. The most comprehensive study on the subject was carried out by Richart and Whitman (1967) who used the elastic half space analog approach and analyzed the data reported by Fry (1963). Other previous analyses include those by Novak (1970), Novak and Beredugo (1971, 1972), Moore (1971, 1985), Stokoe and Richart (1974), Vijayvargiya (1981), and Prakash and Puri (1981).

The results obtained by Richart and Whitman (1967) have shown that the elastic half space analog predicts the footing response well enough to justify its use for all practical purposes. Novak (1970) suggested that, when the parameters used in the predictions are derived directly from field experiments, the tests should be evaluated considering soil nonlinearity. In addition, it has been recognized that soil modulus is dependent upon strain amplitude (Ishihara, 1971, and Prakash, 1981) and the mean effective confining pressure (Hardin and Black, 1968). Therefore, a realistic analysis should take into account the effects of soil nonlinearity and confining pressures on soil modulus.

All the studies cited above used the elastic half space analog for their predictions and did not consider soil nonlinearity. Prakash and Puri (1981) also used the same method but accounted for these factors. They concluded that not enough tests had been used in their comparisons to warrant realistic conclusions.

A prediction model that takes into account the effects of soil nonlinearity and confining pressures on soil modulus is presented below.

PROPOSED PREDICTION MODEL

This prediction model is also based on the elastic half space analog approach. Vertical vibrations are predicted using the analog by Lysmer and Richart (1966). Torsional vibrations are predicted using the equations presented by Richart, Hall, and Woods (1970) and coupled rocking and sliding vibrations by Hall's analog (1967). A computer program for the prediction model has been developed by using appropriate analog equations for vertical, torsional, and coupled sliding and rocking vibrations.

SHEAR MODULUS

The quality of any response prediction depends upon the accurate determination of the soil parameters to be used in the equations. The most important parameter for this analysis is the shear modulus of the soil. In the analog solutions, the spring constant and the damping ratio are expressed in terms of the shear modulus, G. Therefore, the determination of the appropriate value of shear modulus is of paramount importance in any realistic analysis. In order to obtain a reasonable estimate for shear modulus, it is neccessary to consider the main factors that affect it. Shear modulus has been studied in detail by many investigators, Hardin and Black (1968), Seed and Idriss (1970), and Vucetic and Dobry (1991). Hardin and Black (1968) listed several factors that affect shear modulus and those that have been considered here are (1) soil type, (2) strain level, and (3) confining pressure.

<u>Soil Type and Confining Pressure.</u> Hardin and Black (1968) proposed a relationship between low strain shear modulus (G_{max}) and simple soil properties that can be reduced to

$$G_{max} = F(\bar{\sigma}_o)^{0.5} \qquad (1)$$

in which F is a factor which depends on the soil type, previous stress history, void ratio, e, and the plasticity index of the soil. Hardin (1978) recommended that this equation be used for an anisotropic state of stress by taking the mean effective confining stress $\bar{\sigma}_o$ as

$$\bar{\sigma}_o = \frac{(\bar{\sigma}_1 + \bar{\sigma}_2 + \bar{\sigma}_3)}{3} \qquad (2)$$

In this analysis the stresses have been determined at a depth equal to one-half the footing diameter and a correction to G_{max} has been applied accordingly.

Strain Level: Seed and Idriss (1970), Ishihara (1971), Prakash and Puri (1981), and Vucetic and Dobry (1991) have presented data that show that shear modulus is dependent upon strain level. As a matter of convenience a plot of normalized shear modulus (defined as the value of shear modulus G at a particular strain, divided by G_{max} at a strain of 10^{-6}) versus shear strain is normaly used. A plot of that type has been shown in Figure 1 .

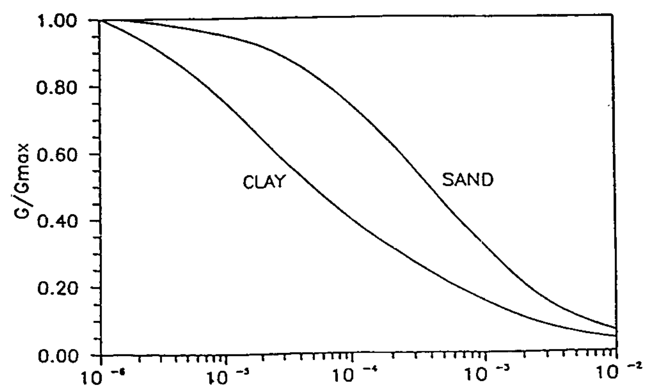

Figure 1. Modulus reduction with strain for sands, low placicity silts, and Clays* (after Seed and Idriss, 1970).

* This curve has been revised in subsequent publications (Vucetic and Dobry, 1991) and therefore has not been used in subsequent publications.

Shear strains that occur in soil depend mainly upon the amplitude of vibration or settlement, which in turn depends upon superimposed loads, foundation geometry, and soil characteristics. These factors are accounted for Prakash and Puri (1981) approximated the shear strain in vertical vibration, γ_z, to be equal to the ratio of the amplitude of vibration or settlement of the footing to the width of the footing. This assumption leads to the approximation of shear strain by normal strain and has been shown to give reasonable results (Manyando and Prakash, 1989). For the current model this definition of shear strain will be used for vertical vibrations. Shear strain for torsional vibrations has been considered to be equal to the rotational displacement at the edge of the base of a surface footing divided by the radius of the footing; this is equivalent to pure shear when the resulting geometrical relationship is considered. The shear strain for coupled rocking and sliding vibrations has been considered to be equal to the rotation about the lateral axis of the combined center of gravity; this is approximated by the ratio of the horizontal displacement at the top of the footing to the height of the footing.

The relationships between shear modulus and shear strain for different types of soil have been obtained by fitting equations of the form reported by Ishihara (1971) to normalized shear modulus versus shear strain curves. The equations are then incorporated into the computer program as callable subroutines. Currently (1991) the relationship can be conveniently obtained by fitting equations to the curves reported by Vucetic and Dobry (1991).

DAMPING IN SOILS

Damping of the system affects the response predictions at near resonance frequencies. Damping is also strain dependent as is the shear modulus. There are two kinds of damping in soils: (1) the loss of energy due to interparticle friction, i.e., material damping and (2) the dissipation of energy associated with the geometry of the foundation-soil system, i.e., geometrical damping.

Geometrical damping: In the elastic half space analog solutions, geometrical damping ratio has been derived for each vibration mode. Figure 2 shows the equivalent damping ratio for rigid circular footings oscillating on the elastic half space. Analog damping values have been used for geometrical damping in these analyses.

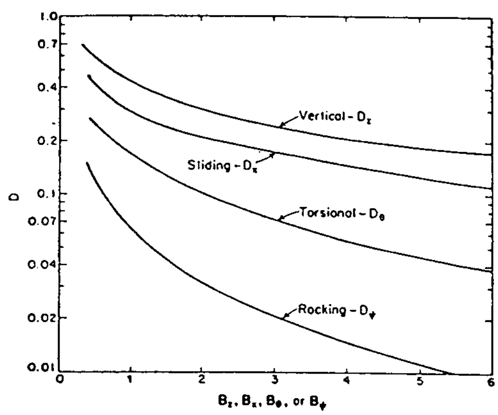

Figure 2. Equivalent damping ratio for rigid circular footings oscillating on the surface of the elastic half space. (Richart, Hall, and Woods)

Material damping: The nature of the material damping of soils has been accepted to be hysteretic rather than viscous, meaning that most of the energy loss is attributed to friction between soil particles (Dobry, 1989). The variation of material damping ratio with shear strain for different soils as reported by Vucetic and Dobry, (1991) is shown in Figures 3. These figures have been used in this study to estimate material damping ratio at a given strain.

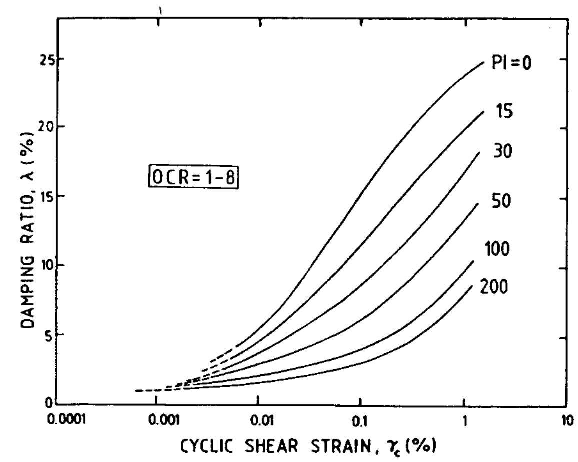

Figure 3. Variation of material damping ratio with shear strain for saturated soil. (after Vucetic and Dobry, 1991)

FORMULATION OF PREDICTION MODEL

The major considerations in this model are:

1. The model is based on elastic half space analog solutions,
2. The model accounts for soil nonlinearity (variation of modulus with shear strain),
3. The model corrects for effects of confining pressures on soil modulus, and
4. The model accounts for nonlinearity of material damping (variation of damping with shear strain).

Based on the above, the prediction model formulation is presented below:

1. Estimate the maximum allowable amplitude of vibration (A_{max}).
2. Compute shear strain γ by using the definition of strain proposed for each case.
3. Obtain the value of shear modulus corresponding to the strain obtained in step 2 using the normalized modulus vs. shear strain relationship (incorporated in the computer program).
4. Correct this value of G for confining pressure by using the simplified equation by Hardin and Black (1968) as follows:

$$\left(\frac{G_1}{G_2}\right) = \left(\frac{\bar{\sigma}_{o1}}{\bar{\sigma}_{o2}}\right)^{0.5} \tag{3}$$

in which G_1 and $\bar{\sigma}_{o1}$ are the shear modulus and the mean effective confining stress at a depth equal to half the width of the footing or the equivalent radius of the footing before the foundation and machine are placed, and G_2 and $\bar{\sigma}_{o2}$ are the corrected shear modulus and the mean effective confining stress at a depth equal to half the width of the footing during the dynamic test.

5. Use the corrected shear modulus to obtain the system spring constant and damping using the analog solutions.
6. Compute the maximum amplitude of vibration using the lumped parameters solutions.
7. Perform an iteration process by comparing the computed amplitude in step 6 to the estimated amplitude in step 1. If the difference between the two amplitudes is within an allowable range, proceed to step 8. If not, take the average of the two and use it as the new estimate for step 1 and repeat steps 2 through 7 until successive computed amplitudes converge to within a desirable range. (A desired convergence between the first estimated (measured) amplitude and the final computed value may not be obtainable in some typical cases).
8. Compute the vibration amplitudes for the required frequency range using the appropriate analog solutions to obtain the total response.

By using this formulation, computer programs have been written for predicting the footing response for vertical, torsional and coupled rocking and sliding vibrations. The formulation for each mode is as follows:

VERTICAL VIBRATIONS

1. Estimate the maximum amplitude of vibration (A_{zmax}). The peak amplitudes from the measured response curves were used for this study. The allowable amplitude specified by the machine manufacturer could be used.
2. Compute shear strain $\gamma = A_{zmax}$/footing diameter (Prakash and Puri, 1981)
3. Obtain the value of normalized shear modulus (modulus ratio) corresponding to the strain from Figure 1 and multiply it by G_{max} to obtain the shear modulus for this strain level.
4. Correct this value of G for confining pressure using Equation 3.

$$\left(\frac{G_1}{G_2}\right) = \left(\frac{\bar{\sigma}_{o1}}{\bar{\sigma}_{o2}}\right)^{0.5} \tag{3}$$

5. Use the corrected modulus to compute the vertical spring constant and damping ratio in the lumped parameters equations, (Equations 4 and 5).

$$k_z = \frac{4Gr_o}{1-\mu}. \tag{4}$$

and

$$c_z = 3.4r_o^{\,2}\frac{\sqrt{G\rho}}{1-\mu}. \tag{5}$$

where ρ is the mass density of the soil, r_o is the equivalent radius of the footing, and μ is Poisson's ratio for the soil.

6. Compute the maximum amplitude of vibration using Equation 6

$$A_z = \frac{P_z}{\sqrt{(k_z - m\omega^2)^2 + (c_z\omega)^2}} \tag{6}$$

in which P_z is the magnitude of the vertical excitation force, m is the total vibrating mass and ω is the excitation frequency.

7. Perform an iteration process by comparing the computed amplitude in step 6 to the estimated amplitude in step 1. If the difference between the two amplitudes is within an allowable range, proceed to step 8. If not, take the average of the two and use it as the new estimate for step 1 and repeat steps 2 through 7 until successive predicted amplitudes converge.
8. Compute the total response over the required frequency range using Equation 6 for the final iteration strain level.

TORSIONAL VIBRATIONS

A similar procedure has been followed for developing the prediction model for torsional vibrations. The solution for this mode is as follows:

1. Estimate the maximum amplitude of vibration (A_ψ max). The peak amplitudes from the measured response curves were used for this study. The allowable amplitude as specified by the machine manufacturer can be used.
2. Compute shear strain using Equation 7 proposed in this study

$$\gamma_\psi = \frac{(A_{\psi x} r_o)}{r_o} = A_\psi \quad (7)$$

   in which A_ψ is in radians.
3. Obtain the value of normalized shear modulus (modulus ratio) corresponding to the strain from Figure 1 and multiply it by G_{max} to obtain the shear modulus for this strain level.
4. Correct this value of G for confining pressure using Equation 3

$$\left(\frac{G_1}{G_2}\right) = \left(\frac{\bar{\sigma}_{o1}}{\bar{\sigma}_{o2}}\right)^{0.5} \quad (3)$$

5. Use the corrected modulus to obtain the torsional static spring constant k_ψ and damping ratio c_ψ using the lumped parameters equations, Equations 8 and 9 .

$$k_\psi = \frac{16Gr_o^3}{3} \quad (8)$$

   The damping term is given by the expression

$$c_\psi = 1.6r_o{}^4 \frac{\sqrt{G\rho}}{1 + B_\psi}. \quad (9)$$

   where B_ψ is the modified inertia ratio.
6. Compute the maximum amplitude of vibration using Equation 10

$$A_\psi = \frac{Mz}{\sqrt{(k_\psi - m\omega^2)^2 + (c_\psi\omega)^2}}. \quad (10)$$

   in which M_z is the magnitude of the twisting moment about the z axis, and m is the total vibrating mass.
7. Perform an iteration process by comparing the computed amplitude in step 6 to the estimated amplitude in step 1. If the difference between the two amplitudes is within an allowable range, proceed to step 8. If not, take the average of the two and use it as the new estimate for step 1 and repeat steps 2 through 7 until successive predicted amplitudes converge.
8. Compute the total response over the required frequency range using Equation 10 for the final iteration strain level.

COUPLED ROCKING AND SLIDING VIBRATIONS

The solution to the equations of motion for coupled rocking and sliding have been obtained by solving the following equations of motion, (Prakash and Puri, 1988). Equation of motion for sliding:

$$m\ddot{x} + c_x\dot{x} + k_x x - Lc_x\dot{\phi} - Lk_x\phi = P_x e^{i\omega t} \quad (11)$$

and the equation of motion for rocking

$$M_m\ddot{\phi} + (c_\phi + L^2c_x)\dot{\phi} + (k_\phi L^2 k_x)\phi - Lc_x\dot{x} - Lk_x x = M_y e^{i\omega t} \quad (12)$$

in which m is the total vibrating mass, k_x is the horizontal static spring constant, c_x is the horizontal damping constant, P_x is the magnitude of the horizontal excitation force, x is the displacement in the horizontal direction, ϕ is the rotational displacement about the combined center of gravity, M_m is the mass moment of inertia of the footing about the center of gravity, k_ϕ is the rocking static spring constant, c_ϕ is the rocking damping constant, M_y is the moment about the y axis, and L is the distance from the base of the footing to the combined center of gravity.

Assumimg the following:

$$x = A_x e^{i(\omega t - \alpha)} \quad (13)$$

$$\phi = A_\phi e^{i(\omega t - \alpha)} \quad (14)$$

$$P_x = P_x e^{i\omega t} \quad (15)$$

$$M_y = M_y e^{i\omega t} \quad (16)$$

and taking the first and second derivatives of x and ϕ and substituting for these terms in Equations 11 and 12 results into 4 equations with 4 unknowns. Hence a solution can be obtained. The resulting 4 simultaneous equations can be expressed in matrix form as follows:

$$\begin{bmatrix} A & -B & -C & D \\ B & A & -D & -C \\ -C & D & E & -F \\ -D & -C & F & E \end{bmatrix} \begin{bmatrix} A_{x1} \\ A_{X2} \\ A_{\phi 1} \\ A_{\phi 2} \end{bmatrix} = \begin{bmatrix} Px \\ 0 \\ My \\ 0 \end{bmatrix} \quad (17)$$

in which
$A = k_x - m\omega^2$
$B = c_x\omega$
$C = Lk_x$
$D = Lc_x\omega$
$E = L^2k_x + k_\phi - M_m\omega^2$
$F = c_\phi\omega + L^2c_x\omega$

The matrix in Equation 17 has been solved to obtain the natural frequencies and amplitudes of vibrations.

Coupled Natural Frequencies: The coupled natural frequencies are obtained by equating the right hand side of the matrix in Equation 17 to zero and solving the Eigen value problem that results. An iteration subprogram has been developed and used to obtain the coupled natural frequencies (Manyando, 1990).

Coupled Amplitudes of Vibrations: The amplitudes of motion have been predicted by using the Gauss-Jordan elimination method (James, Smith, and Wolford, 1985) with partial pivoting. A computer program based on Gauss-Jordan elimination procedure has been written and used to compute the response (Manyando, 1990). The amplitudes of vibrations are obtained from Equations 18 and 19 for the horizontal and rocking components, respectively.

$$A_x = \sqrt{A^2_{x1} + A^2_{x2}} \tag{18}$$

$$A_\phi = \sqrt{A^2_{\phi 1} + A^2_{\phi 2}} \tag{19}$$

The phase angles are obtained as

$$\alpha = \tan^{-1}(A_{x2}/A_{x1}) \tag{20}$$

for sliding and

$$\beta = \tan^{-1}(A_{\phi 2}/A_{\phi 1}) \tag{21}$$

for rocking.

The formulation for coupled rocking and sliding is as follows:

1. Estimate the maximum amplitude of vibration (A_ϕ max). The peak amplitudes from the measured response curves were used for this study. The allowable amplitude as specified by the machine manufacturer can be used.
2. Compute shear strain using Equation 22 proposed in this study

$$\gamma_{\phi =} \frac{(A_h)}{H} = A_\phi \tag{22}$$

   in which A_h is the horizontal displacement at the top surface of the footing and H is the total height of the footing.
3. Obtain the value of normalized shear modulus (modulus ratio) corresponding to the strain from Figure 1 and multiply it by G_{max} to obtain the shear modulus for this strain level.
4. Correct this value of G for confining pressure using Equation 3

$$\left(\frac{G_1}{G_2}\right) = \left(\frac{\bar{\sigma}_{o1}}{\bar{\sigma}_{o2}}\right)^{0.5} \tag{3}$$

5. Use the corrected modulus to obtain the system spring constants and damping constants using the lumped parameters. Equations 23 and 24 are used for sliding and and Equations 25 and 26 for rocking as follows:
   Sliding

$$k_x = 32Gr_o \frac{(1-\mu)}{(7-8\mu)} \tag{23}$$

   and

$$c_x = 18.4r_o{}^2\sqrt{G\rho}\ \frac{(1-\mu)}{(7-8\mu)}\,. \tag{24}$$

   Rocking

$$k_\phi = \frac{8Gr_o^3}{3(1-\mu)}\,. \tag{25}$$

   and

$$c_\phi = \frac{0.8r_o^4\sqrt{G\rho}}{(1-\mu)\sqrt{1+B_\phi}}\,. \tag{26}$$

   where B_ϕ is the modified inertia ratio for rocking.
6. Compute the maximum amplitude of vibration using Gauss-Jordan elimination method with partial pivoting.
7. Perform an iteration process by comparing the computed amplitude in step 6 to the estimated amplitude in step 1. If the difference between the two amplitudes is within an allowable range, proceed to step 8. If not, take the average of the two and use it as the new estimate for step 1 and repeat steps 2 through 7 until successive predicted amplitudes converge.
8. Compute the total response in terms of horizontal amplitudes versus frequency over the required frequency range using Equation 27 .

$$A_h = A_x + \bar{h}A_\phi \tag{27}$$

   in which A_x and A_ϕ are obtained from Equations 18 and 19 , respectively, and $\bar{h}$ is the distance from the combined center of gravity of the system to the top surface of the footing.

Typical predictions have been compared with measured results. The results are shown in Figures 4 and 5 for vertical vibrations, Figures 6 and 7 for torsional vibrations and Figures 8 and 9 for coupled rocking and sliding vibrations.

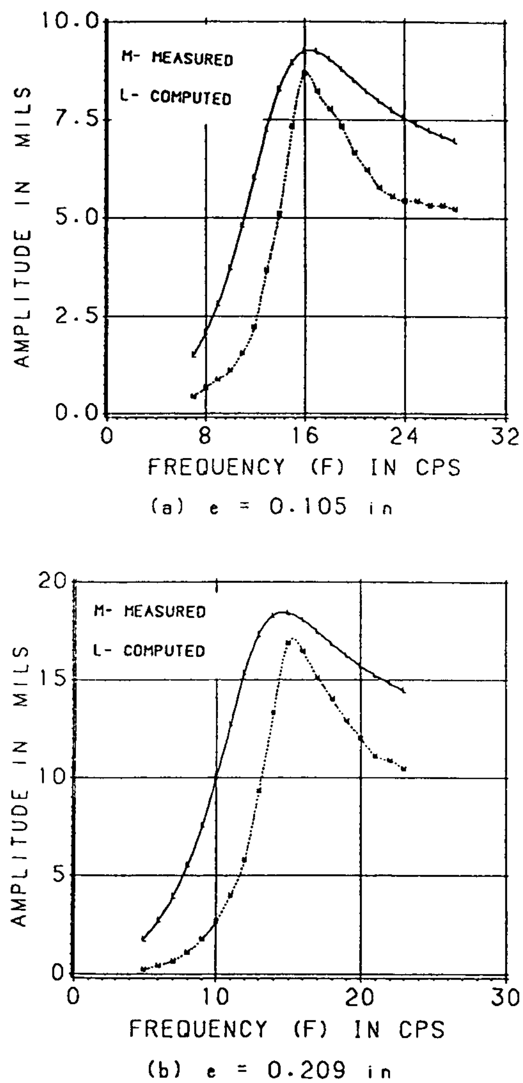

Figure 4. Measured and predicted response of vertical vibrations for different values of eccentricity (a) e = 0.105 and (b) e = 0.209 inches, Eglin, base 1-1.

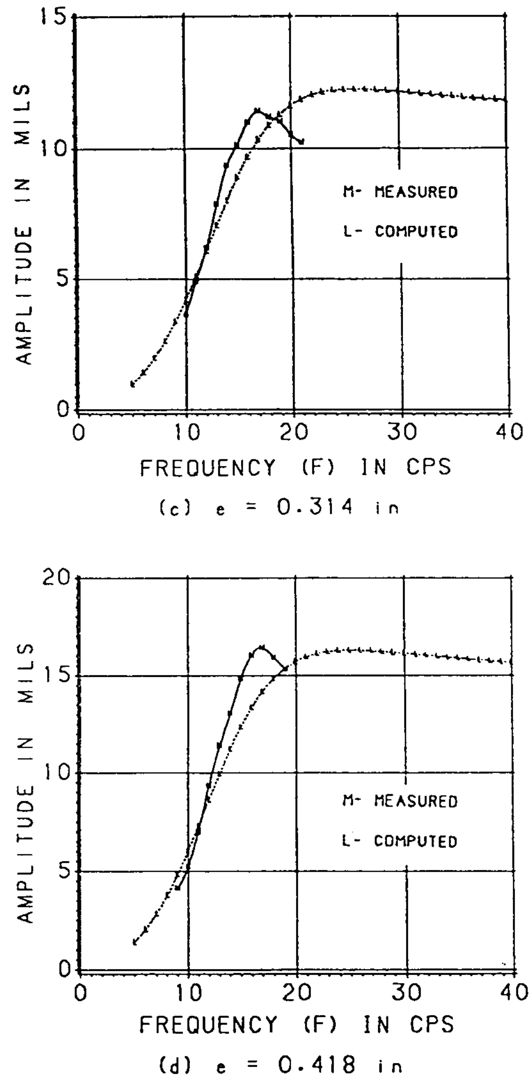

Figure 5. Measured and predicted response of vertical vibrations for different values of eccentricity (c) e = 0.314 and (d) e = 0.418 inches, Vicksburg, base 3.

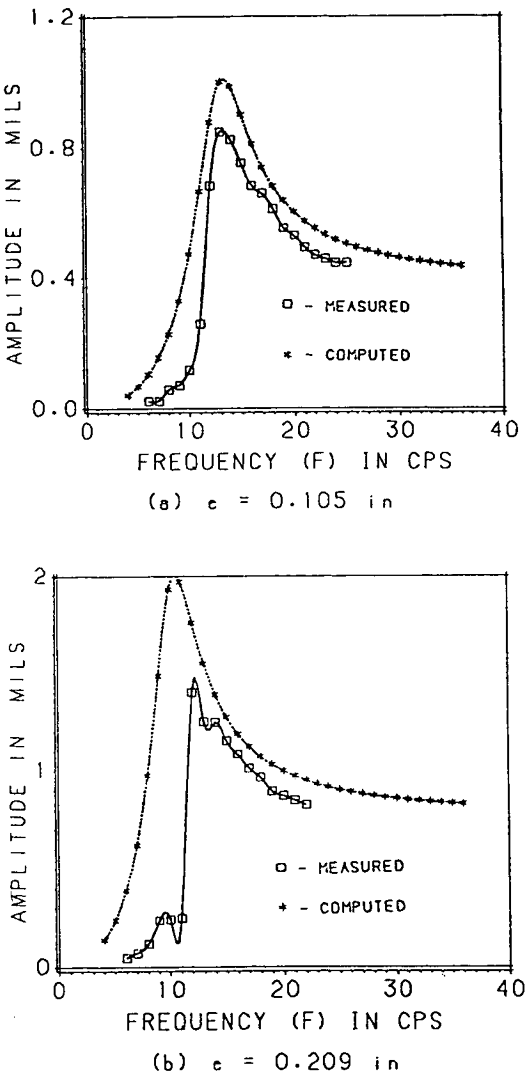

Figure 6. Measured and predicted response of torsional vibrations for different values of eccentricity (a) e = 0.105 and (b) e = 0.209 inches, Eglin, base 1-1.

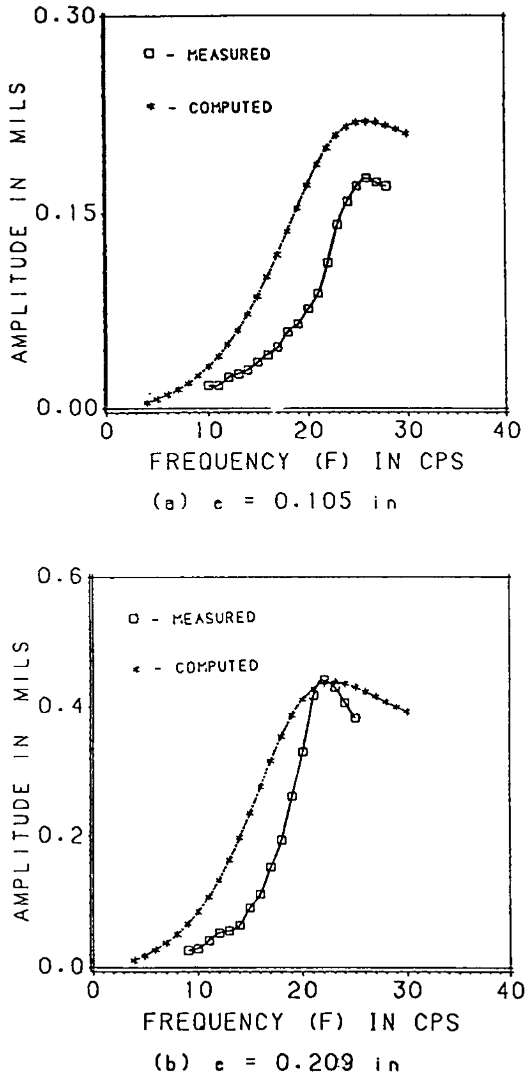

Figure 7. Measured and predicted response of torsional vibrations for different values of eccentricity (a) e = 0.105 and (b) e = 0.209 inches, Vicksburg, base 3.

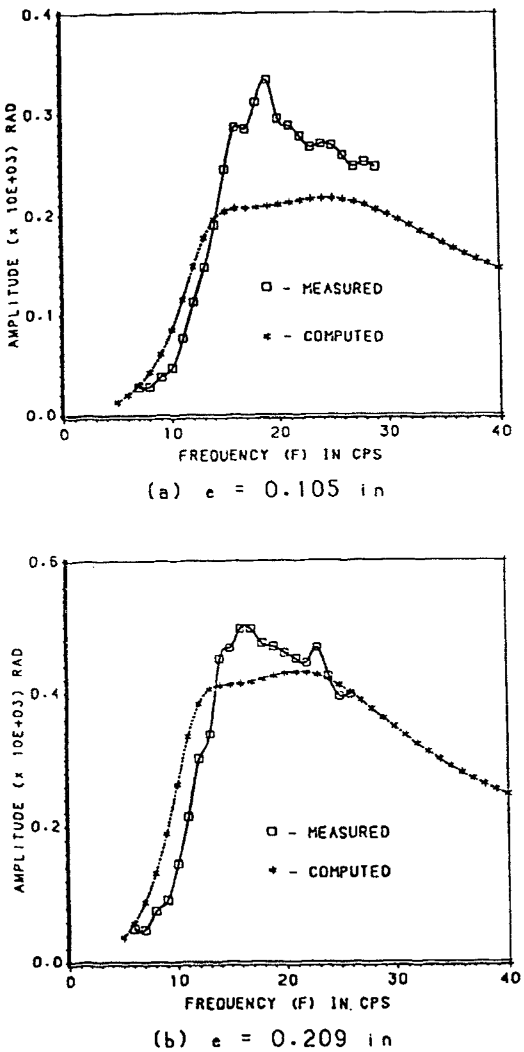

Figure 8. Measured and predicted response of coupled rocking and sliding vibrations for different values of eccentricity (a) = 105 and (b) e = 0.209 inches, Eglin, base 2.

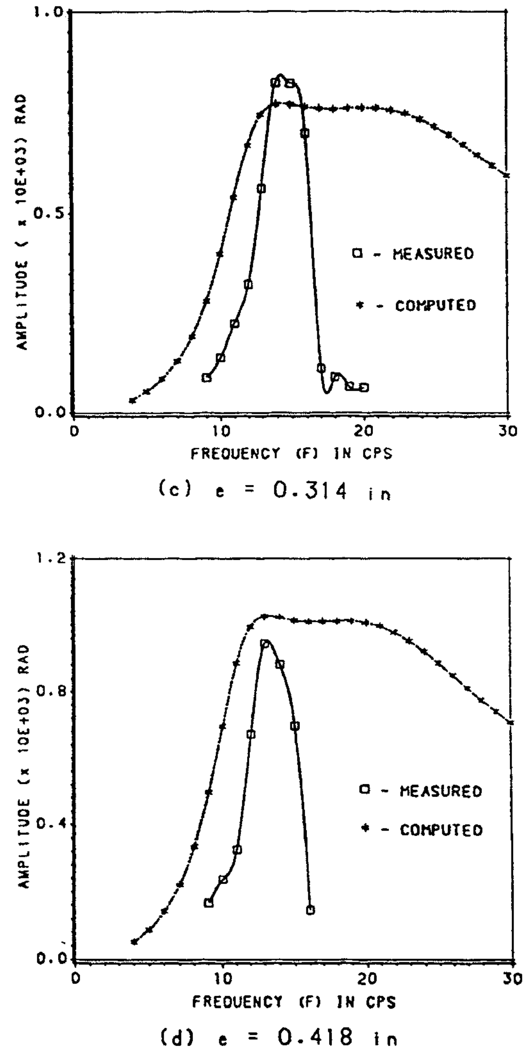

Figure 9. Measured and predicted response of coupled rocking and sliding vibrations for different values of eccentricity (c) e = 314 and (d) e = 0.418 inches, Vicksburg, base 2.

Conclusions

A prediction model that accounts for the effects of soil nonlinearity and confining pressures on soil modulus has been formulated. A computer program based on the new prediction model has been developed. The validity of the prediction model has been verified by comparing predictions with measured data. Predictions by this model are within a factor of 1.5 times the measured values as compared to a factor of about 2.0 reported by Richart and Whitman (1967).

Vertical vibration predictions by this model match the performance quite well for a footing size and loading conditions giving a mass ratio greater than 1.0. That is, predictions closely match the measured results when the effects of strain and confining pressures on soil modulus are included. However, it still appears that there is a need to reevaluate the analog damping expressions to cover a wider range of footing size and loading conditions. Material damping may be neglected in the analysis of vertical vibrations.

In the case of torsional vibrations material damping should be considered in the analysis. The value should be selected with respect to the following: (1) the strain level, (2) inertia ratio, (3) footing geometry, and (4) soil type. For the data analyzed, 10 percent material damping is recommended for use. This value has been shown to provide good predictions by Weissmann (1971) who considered slip at the base of the footing and Manyando (1990) who neglected the slip. The current analysis shows that torsional vibrations may be predicted satifactorily by ignoring the base slip of the footing.

For coupled rocking and sliding vibrations, the best match between predictions and performance is obtained when material damping is neglected. The amplitudes of vibration at the first resonant frequency are much larger than those at the second natural frequency and therefore the former are more critical for design. The strain calculated using the total horizontal displacement rather than the vertical displacement at the edge of the footing has been found to give better predictions.

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