International Conference on Case Histories in Geotechnical Engineering
(2004) - Fifth International Conference on Case Histories in Geotechnical Engineering

16 Apr 2004, 8:00am - 9:30am

Retaining Walls Under Seismic Loading

Vijay K. Puri
Southern Illinois University, Carbondale, Illinois

Shamsher Prakash
Missouri University of Science and Technology, prakash@mst.edu

Retno Widanarti
Southern Illinois University, Carbondale, Illinois

Proceedings: Fifth International Conference on Case Histories in Geotechnical Engineering
New York, NY, April 13-17, 2004

RETAINING WALLS UNDER SEISMIC LOADING

Vijay K. Puri
Professor
Civil Engineering
Southern Illinois University
Carbondale, IL 62901, USA

Shamsher Prakash
Professor Emeritus
Civil Engineering
University of Missouri
Rolla, MO 65401, USA

Retno Widanarti
Graduate Student
Civil Engineering
Southern Illinois University
Carbondale, IL, 62901, USA.

ABSTRACT

The design of retaining walls in seismic areas poses a complex problem. The traditional design approach usually consists of calculation of a factor of safety against sliding, overturning and bearing capacity failure. This is generally enough for static loads. During seismic loading, the retaining walls tend to get displaced from their original position. The performance of quay walls during the past earthquakes emphasizes this fact. For safe design of retaining walls in seismic areas, the calculation of static and dynamic earth pressure behind the retaining walls is the first requirement. Realistic calculation of displacement of the retaining wall is an equally important aspect. The paper presents a simple method for calculation of static and dynamic active force on the rigid retaining wall. The method follows the pseudo-static approach of analysis and includes the effects of cohesion of the backfill and the friction between the backfill and the wall face. The resultant earth pressure obtained by this method has been compared with the experimentally observed values during small scale tests on retaining walls by other investigators. The displacement must not exceed specified allowable values. A brief discussion of calculation of displacement of rigid retaining walls is also included.

KEYWORDS: Wall, Rigid, Retaining. Seismic, Displacement, design

INTRODUCTION: Many types of structures are used to retain soil. Rigid masonry walls, flexible retaining walls, cantilever sheet piles and anchored bulkheads are some examples. The stability analysis of these structures requires estimation of static and dynamic lateral pressures. The observation of failures of retaining structures during past earthquake (Amano, Azuma and Ishii, Hyashi, 1956; Duke and Leeds, 1967; Kubo and Nakase, 1966; and Steedman, 1998) clearly bring out the importance of displacements that the retaining structures may possibly undergo due to seismic loading. (Table1). The displacements shown in Table 1 may be due to a variety of reason associated behavior of soil under dynamic loading. There are two key aspects in the design of retaining walls for earthquake loading:

(a). Calculation of static and dynamic lateral pressures.

(b). Calculation of likely displacement of the retaining wall.

The lateral earth pressure for static case is generally determined using either Rankine's (1857) or Coulomb's (1773) method. These methods are available in all textbooks on soil mechanics (Das, 2001; Prakash, Rajan and Saran (1979), and Taylor (1948). The earthquake induced forces on the retaining wall are generally computed using the modified Coulomb's approach in which the earthquake force on the backfill is replaced by an equivalent static force. This is known as Mononobe-Okabe method (Mononobe, 1929; Okabe, 1926; and Prakash, 1981). Mononobe-Okabe's method is suitable for cohesionless backfills. A solution for determination of static and dynamic active earth pressure for c-ϕ soil was developed by Prakash and Saran (1966) and Saran and Prakash (1968) and Prakash (1981). The approach by Prakash and Saran (1966) provides a convenient method for determination of static and dynamic lateral pressures for a typical soil. However it has the following limitation:

1. The effect of the vertical component of acceleration been neglected.
2. The backfill surface is assumed to be horizontal, which may actually be inclined in many cases.
3. The unit adhesion between the back face of the retaining wall and soil was assumed to be equal to the unit cohesion of the soil

A method for calculation of dynamic active pressure for a c-ϕ soil accounting for the effect of the following factors is presented here (Fig. 1).

The effect of cohesion, c, and adhesion, c_a.

The inclination of the backfill, i

Horizontal and vertical seismic coefficients, α_h and α_v respectively.

Surcharge, q

Inclination of the wall face, α

Depth of tension cracks, H_c

Table 1. Damage to retaining structures and quay walls

Earthquake and Date	Magnitude	Harbor Location	Damage	Displacement (meter)	Reference
Kitaizu (11/25/1930)	7.1	Shimizu	Failure of gravity	7.93	Amano, Azuma and Ishii (1956)
Shizuoka (7/11/1935)		Shimizu	Retaining wall collapse	4.88	
Tonankai (12/7/1944)	8.2	Shimizu	Sliding of retaining wall	3.05 - 3.96	
		Nagoya	Outward movement of bulkhead with relieving platform		
		Yokkaichi	Outward movement of pile supported deck		
Nankai (12/21/1946)	8.1	Nagoya	Outward movement of bulkhead with relieving platform	3.96	
		Osaka	Failure of retaining wall above relieving platform	4.27	
		Yokkaichi	Outward movement of pile supported deck	3.66	
		Uno	Outward movement of gravity wall		
Tokachioki (3/4/1952)	7.8	Kushiro	Outward movement of gravity wall	5.49	Duke and Leeds (1963)
Chile (5/22/1960)	8.4	Puerto Montt	Complete overturning of gravity walls.	4.57	
			Outward movement of anchored bulkheads	0.61 – 0.915	
Niigata (6/16/1964)	7.5	Niigata	Tilting of gravity wall	3.05	Hyashi, Kubo and Nakase (1966)
			Outward movement of anchored bulkheads	0.305 – 2.13	

DERIVATION OF EQUATIONS: A schematic diagram of the earth pressure problem is shown in Fig.1. ABEC is an assumed failure wedge. Considering the unit length of the wall, $(P_A)_{dyn}$ = total active force, R = soil reaction, I_F = horizontal inertia force, W = weight of assumed failure wedge, W_t= resultant of weight W and I_F, C = cohesion force, C_a =adhesive force, q = surcharge, α = inclination of wall face with vertical ($\alpha \geq 0$) and i = inclination of the backfill ($0 \leq i < \phi$).

Weight of the wedge W,

$$W = \frac{1}{2}\gamma H^2 \{\tan\alpha + \tan\theta + \frac{n}{\cos i \cos(\theta + i)}[(2 + n) \times \tan\alpha \cos\theta + 2\sin\theta] + \frac{\sin^2(\alpha + \theta)\sin i}{\cos^2\alpha \cos\theta \cos(\theta + i)}\} \quad \text{..........(1)}$$

where

$$n = \frac{H_c}{H} \quad \text{...(2)}$$

$$H_C = \text{depth of tensile cracks} = \frac{2c}{\gamma}\sqrt{K_A} \quad \text{...................(3)}$$

In which K_A = Rankine's active earth pressure coefficient, and γ = unit weight of soil.

$$H = H_1 - H_c \quad \text{..(4)}$$

$$\text{Surcharge, } Q = \frac{qH}{\cos(\theta+i)}[\frac{\sin(\alpha+\theta)}{\cos\alpha} + n\tan\alpha \cos\theta] \quad \text{............(5)}$$

Cohesive force, $C = c\frac{H}{\cos\alpha}\frac{\cos(\alpha - i)}{\cos(\theta + i)}$ …………………..(6)

Adhesive force $C_a = ec\frac{H}{\cos\alpha}$ …………………………….(7)

In which

$$e = \frac{c_a}{c}$$

Where c_a = unit adhesion between the back face of the wall and the backfill, and c = unit soil cohesion.

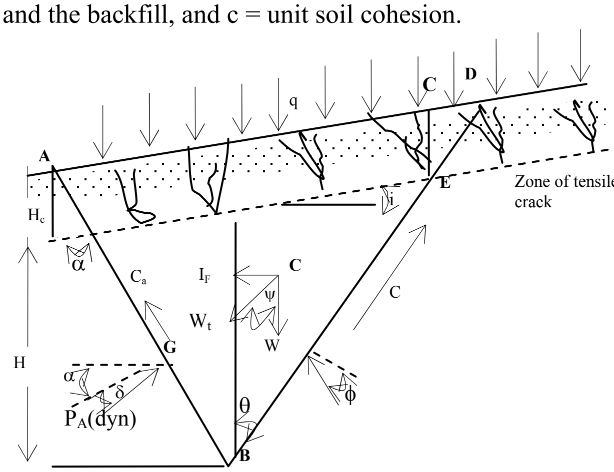

Fig. 1 Force Acting on the Assumed Failure Wedge

Horizontal Inertia force, $I_F = (W + Q)\,\alpha_{h}$……………….....(8)

Vertical Inertia force, $I_{Fv} = (W + Q)\,(1\pm\alpha_v)$.......................(9)

Applying the conditions for static equilibrium, namely $\sum F_x = 0$ and $\sum F_y = 0$, one obtains

$$(P_A)_{dyn}\cos(\alpha + \delta) - R\cos(\phi + \theta) - (W + q)\alpha_h$$

$$+ C\sin\theta - C_a\sin\alpha = 0 \text{ ……………………………....(10)}$$

$$(P_A)_{dyn}\sin(\alpha+\delta) + R\sin(\phi+\delta) + C\cos\theta$$
$$+ C_a\cos\alpha - (W+Q)(1\pm\alpha_v) = 0 \text{ …..........................(11)}$$

Multiplying Eq. (10) by $\sin(\phi + \theta)$ and Eq. (11) by $\cos(\phi + \theta)$ and simplifying, a relationship for $(P_A)_{dyn}$ can be obtained, or

$$(P_A)_{dyn} = \frac{1}{2}\gamma H^2 (N_{a\gamma})_{dyn}$$
$$+ qH(N_{aq})_{dyn} - cH(N_{ac})_{dyn} \text{ ……………(12)}$$

in which

$$(N_{ac})_{dyn} = \left[\frac{\cos(\alpha - i)\cos\phi}{\cos\alpha\cos(\theta + i)} + \frac{e\cos(\alpha + \phi + \theta)}{\cos\alpha}\right]$$
$$\times\frac{1}{\sin(\alpha + \delta + \phi + \theta)} \text{ ………………………..(13)}$$

$$(N_{aq})_{dyn} = \left[\frac{\sin(\alpha + \theta)}{\cos\alpha} + n\tan\alpha\cos\theta\right]\times$$
$$\left[\frac{\alpha_h\sin(\phi + \theta) + (1 \pm \alpha_v)\cos(\phi + \theta)}{\sin(\alpha + \delta + \phi + \theta)\cos(\theta + i)}\right. \text{ ……..……...……(14)}$$

and

$$(N_{a\gamma})_{dyn} = \frac{1}{\sin(\alpha + \delta + \varphi + \theta)}\left\{\tan\alpha + \tan\theta\right.$$
$$+ \frac{n}{\cos i\cos(\theta + i)}[(2 + n)\tan\alpha\cos\theta + 2\sin\theta]$$
$$\left.+ \frac{\sin^2(\alpha + \theta)\sin i}{\cos^2\alpha\cos\theta\cos(\theta + i)}\right\}\times$$

$$[\alpha_h\sin(\phi + \theta) + (1 \pm \alpha_v)\cos(\phi + \theta)] \text{ ………………...(15)}$$

It may be noted that the right-hand side of Eq. (13) does not contain α_h and α_v and, therefore, the value of N_{ac} will be the same for static and dynamic cases.

The static active earth pressure $(P_A)_{stat}$ may be obtained as follows

$$(P_A)_{stat} = \frac{1}{2}\gamma H^2 (N_{a\gamma})_{stat}$$
$$+ qH(N_{aq})_{stat} - cH(N_{ac})_{stat} \text{ ………………..(16)}$$

where

$$(N_{ac})_{stat} = (N_{ac})_{dyn} \text{ ……………………………….…..(17)}$$

Relationship for $(N_{aq})_{stat}$ and $(N_{a\gamma})_{stat}$ may be obtained from Eq. (14) and (15), respectively, by substituting $\alpha_h = 0$ and $\alpha_v = 0$. Thus

$$(N_{aq})_{stat} = \left[\frac{\sin(\alpha + \theta)}{\cos\alpha} + n\tan\alpha\cos\theta\right]\times$$
$$\left[\frac{\cos(\phi + \theta)}{\sin(\alpha + \delta + \phi + \theta)\cos(\theta + i)}\right] \text{ …………………(18)}$$

$$(N_{a\gamma})_{stat} = \frac{1}{\sin(\alpha + \delta + \phi + \theta)}\left\{\tan\alpha + \tan\theta\right.$$
$$+ \frac{n}{\cos i\cos(\theta + i)}[(2 + n)\tan\alpha\cos\theta + 2\sin\theta]$$
$$\left.+ \frac{\sin^2(\alpha + \theta)\sin i}{\cos^2\alpha\cos\theta\cos(\theta + i)}\right\}[\cos(\phi + \theta)] \text{ …....…..(19)}$$

The value of $(P_A)_{dyn}$ and $(P_A)_{stat}$ obtained from Eq. (12) and (16), respectively, are for a given assumed failure wedge. In order to obtained the maximum values of the total dynamic earth force, $(P_A)_{dyn}$ the earth pressure coefficient $(N_{aq})_{dyn}$, $(N_{a\gamma})_{dyn}$, and $(N_{ac})_{dyn}$ were optimized. A computer code was developed for this purpose. It must be mentioned here that these earth pressure coefficients were individually optimized and then $(P_A)_{dyn}$ was obtained by superimposing their effect, i.e., using Eq. (17). The same procedure was followed for the maximum value of static earth force, $(P_A)_{stat}$. From known values of $(P_A)_{dyn}$ and $(P_A)_{stat}$, the dynamic increment $(\Delta P_A)_{dyn}$ can be obtained as

$$(\Delta P_A)_{dyn} = (P_A)_{dyn} - (P_A)_{stat} \qquad (20)$$

EFFECT OF VARIOUS PARAMETERS ON DYNAMIC EARTH PRESSURE

Using the procedure developed in the preceding section, calculation can be made for specific cases to show the effect of parameter such as $e = c_a'/c$, i, and α_v on the dynamic active earth force on retaining walls. These were the factors which were not considered in the published studies presently available for a typical c- ϕ type soil.

Effect of e

Figure 2 shows plot of $(\Delta P_A)_{dyn-e}/(\Delta P_A)_{dyn-e=0}$ for retaining wall with H = 5 m, $\alpha = 0^o$, i = 0, and q = 0. The constant properties of backfill are;

$\phi = 30^o$ $\quad \delta = 2\phi/3$ $\quad \alpha_h = 0.2$

$\gamma = 17.5$ kN/m^3 $\quad \alpha_v = 0$

The cohesion of the back fill was varied. From 5 kN/m^2 to 10 kN/m^2 and e was varied from 0 to 1. It is seen from this figure that for the values cohesion used in this calculation, the magnitude of dynamic active earth pressure increment increases with increase in ‘e’ value. Similar trend is seen from the data in Figure 3 which shows the variation of $(\Delta P_A)_{dyn-e}/(\Delta P_A)_{dyn-e=0}$ for 10 m high wall for values of c varying from 10 to 30 kN/m^2. It may be concluded that assumption of e =1, leads to somewhat conservative values of $(\Delta P_A)_{dyn}$.

Effect of the Inclination of backfill, i

The effects of the inclination of the backfill on the dynamic active force are shown in Fig.4 and Fig. 5. In obtaining these plots, the following constants parameters were assumed:

H = 10m $\quad \delta = 2\phi/3$ $\quad \alpha = 10^o$

$\gamma = 18$ kN/m^3 $\quad \phi = 30^o$ $\quad \alpha_h = 0.2$

$\alpha_v = 0$

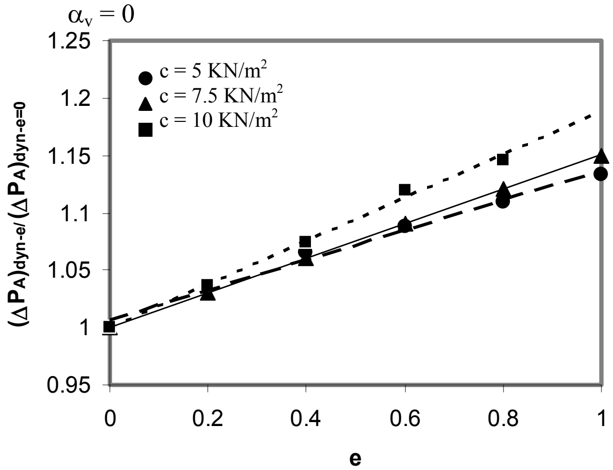

Fig. 2 $(\Delta P_A)_{dyn-e}/(\Delta P_A)_{dyn-e=0}$ versus e

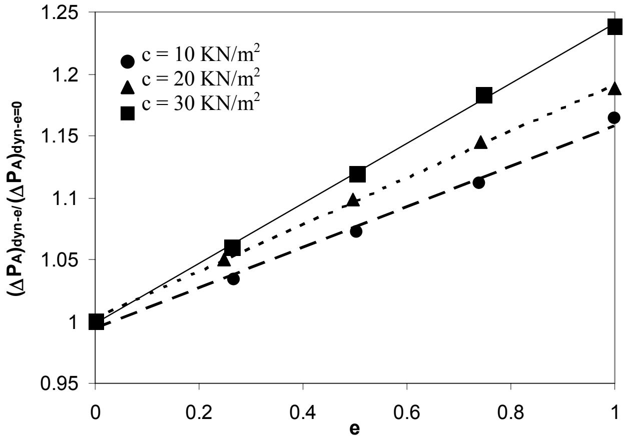

Fig. 3 $(\Delta P_A)_{dyn-e}/(\Delta P_A)_{dyn-e=0}$ versus e

In Fig. 4, the magnitude of q = 50 kN/m^2, c = 0, and the angle i was varied from zero to 15^o. In a similar manner in Fig. 5, the magnitude of q = 0, c = 20kN/m^2, and α were varied from zero to 15^o. These plots show that the value of $(\Delta P_A)_{dyn-i}/(\Delta P_A)_{dyn-i=0}$ increases with the increase in magnitude of i. This is primarily due to the fact that, for a given retaining wall, an increase in the positive value of i increases the weight of the failure wedge, and it generates higher dynamic pressure increments

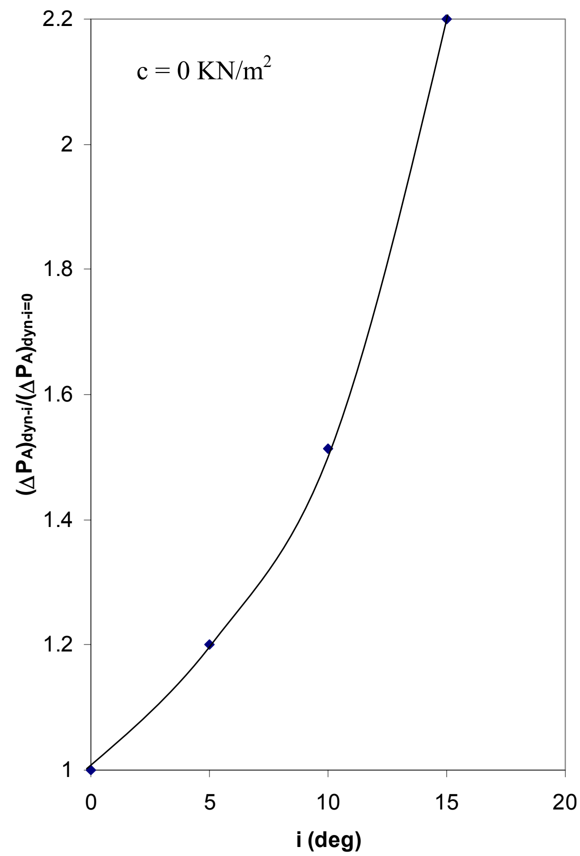

Fig. 4 $(\Delta P_A)_{dyn-i}/(\Delta P_A)_{dyn-i=0}$ versus i

Effect of vertical seismic coefficients, α_v

Fig. 8 shows plots of $(\Delta P_A)_{dyn-\alpha v}/(\Delta P_A)_{dyn-\alpha v=0}$ against α_v/α_h. In developing these plots, it was assumed that

$H = 0$ m	$c = 0$
$\alpha = 10^o$	$q = 0$
$\phi = 30^o$	$\gamma = 18$ kN/m^3
$\delta = 2\phi/3$	$i = 0$

From these plots it can be seen that the dynamic force increment depends on the magnitude of α_v/α_h for $\alpha_h < 0.5$. When α_h is small, the dynamic force increment increases with the increase of α_v. However, for $\alpha_h \geq 0.5$, the magnitude of α_v has an insignificant effect

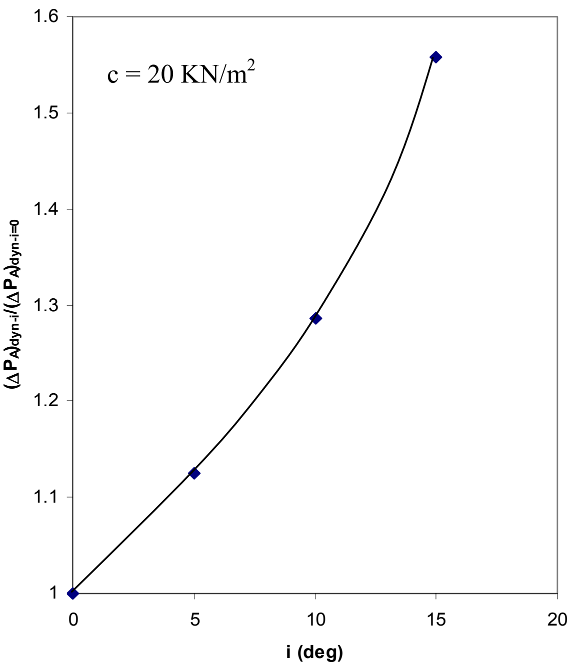

Fig. 5 $(\Delta P_A)_{dyn-i}/(\Delta P_A)_{dyn-i=0}$ versus i c = 20 kN/m^2, $\alpha_h = 0.2$ and $\alpha_v = 0$)

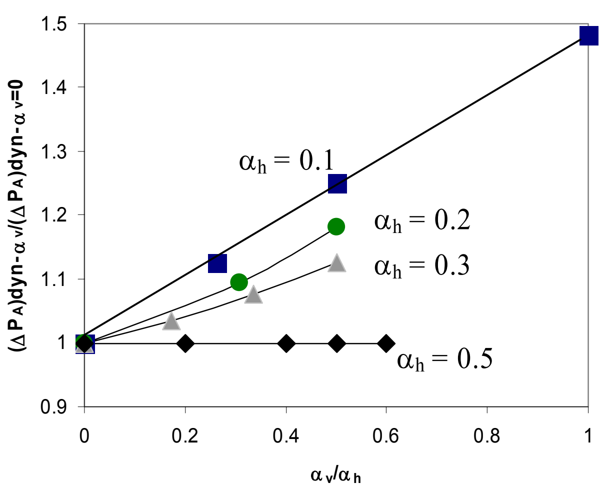

Fig. 6 $(\Delta P_A)_{dyn-\alpha v}/(\Delta P_A)_{dyn-\alpha v=0}$ versus α_v/α_h

COMPARISON WITH EXPERIMENTAL DATA

Sherif, Ishibashi and Lee (1982) reported result of measurements of dynamic active earth pressure on a 1 m high rigid retaining wall. The backfill properties are as follows

Unit weight, $\gamma = 16.28$ kN/m^3

Angle of internal friction, $\phi = 40.9^o$

Angle of wall friction $\delta = 23.9^o$

Slope of backfill = 0^o

The wall was subjected to sinusoidal acceleration of up to 0.5 g. The results are shown in Fig. 7. Note that

$$K_{AE} = \frac{(P_A)_{dyn}}{\frac{1}{2}\gamma H^2} \text{ ....(21)}$$

The result obtained from Mononobe-Okabe theory and from the present study for the case of the model test are also shown in Fig. 7

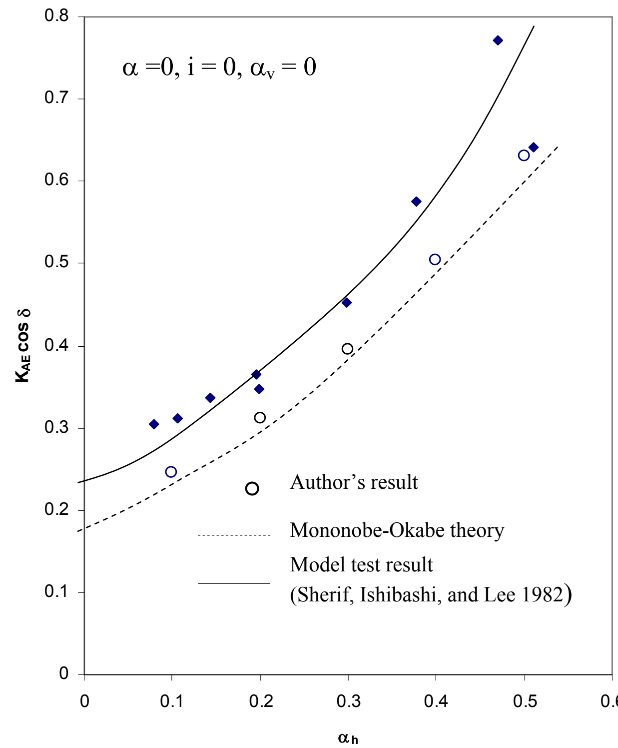

Fig.7. Comparison of theory with model test results

The results from the author's calculations are close to Mononobe- Okabes theory which is to be expected.

POINT OF APPLICATION OF RESULTANT ACTIVE THRUST

The original Mononobe-Okabe solution had assumed that the resultant active thrust acts at a distance of H/3 from the bottom of the wall, similar to the static case ($\alpha_h = \alpha_v = 0$). The laboratory observations indicate that the resulting active thrust acts somewhat higher than H/3 measured from the bottom of the wall. Seed and Whitman (1970) have suggested that for the case of rotation about the bottom of the wall, the static pressure may be assumed to act at H/3 and the dynamic increment at 0.6 H from the base of the wall. For a wall undergoing rotation about the top, the resulting active thrust may be assumed to act at 0.55H from the bottom of the wall (sheriff and Fang, 1984)

Sherif, Ishibasi and Lee (1982) have suggested that for wall undergoing translation, the line of the static active thrust may be assumed to act at 0.42 H and dynamic increment at 0.48 H above the bottom of the wall. For all these case the active thrust and dynamic increment are assumed to act at angle'δ' with the normal to the wall face.

DISPLACEMENT ASPECT OF RETAINING WALLS

The usual design procedure for a retaining wall does not ensure that its displacement will be within tolerable limits during an earthquake. Richard and Elms (1979) developed a design procedure for gravity retaining walls based on limiting displacement using the concept of sliding block analysis (Newmark, 1965) and analysis of earthquake records (Franklin and Chang, 1977). Nadim and Whitman (1983) proposed a slight modification to Richard and Elms procedure primarily to account for the effect of ground amplification. Whitman and Liao (1985) observed that while Richards and Elms procedure is relatively simple, uncertainties may arise due to limitations in determination of actual soil properties, assumptions in modeling and from nature of expected ground motion. Wu (1999) reviewed the available models for computing the retaining wall displacement and concluded that these are not sufficient to predict displacement in a realistic manner.

It may be mentioned that the displacement of a rigid retaining wall may be entirely due to sliding, due to rocking (rotation) or due to combined sliding and rocking. Prakash, Wu and Rafnsson (1995) developed comprehensive solutions for seismic displacement of rigid retaining walls accounting for the effect of ground motion, soil properties and non - linearity of soil behavior. They considered cases of retaining wall undergoing sliding displacement only, rocking displacement only and wall displacement occasioned by coupled sliding and rocking. They also provided charts for estimating of wall displacement for the benefit of the design engineers.

CONSLUSION

A procedure has been presented to determine the magnitude of the static and dynamic active thrust for a typical c-ϕ soil accounting for the effect of wall friction, adhesion between the soil and the wall face and the inclination of the backfill surface. The paper also highlights to the need for determination of displacement of retaining wall due to earth loading. There is a need to develop realistic models to determine the displacement of retaining wall subjected to earthquake loads.

REFERENCES:

Amano, R., Azuma, H., and Ishii, Y. (1956). "Aseismic Design of Quay Walls in Japan," *Proceedings,* 1st World Conference on Earthquake Engineering, Berkeley, California.

Coulomb, C.A. (1776). "Essai sur une Application des Regles de Maximis et Minimis a quelques Problemes de Statique, relatifs a I'Architecture,*"Mem. Roy. des Sciences,* Paris, Vol 3, p. 38.

Das, B.M. (2001). "Principles of Geotechnical Engineering," Brooks/Cole, 5th edition.

Das, B.M. (1992). "Principles of Soil Dynamics," PWS-Kent, Boston.

Duke, C.M., and Leeds, D.J. (1963). "Response of soils, Foundations, and Earth Structure," *Bulletin of the Seismological Society of America*, Vol 53, No. 2, pp. 309-357.

Franklin, A.G., and Chang, F. K. (1977). "Earthquake Resistance of Earth and Rockfill Dams," *Report 5, Miscellaneous Paper S71-17*, Soils and Pavement Laboratory, US. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi.

Hayashi, S., Kubo, K., and Nakase, A. (1966). "Damage to Harbor Structure in the Nigata Earthquake," Soils and Foundation, Vol.6, No. 1, pp. 26-32.

Mononobe, N. (1929). "Earthquake-Proof Construction of Masonry Dams," *Proceedings*, World Engineering Conference, Vol.9, pp. 274-280.

Nadim, F., and Whitman, R.V. (1983). "Seismically Induced Movement of Retaining Walls," *Journal of Geotechnical Engineering*, ASCE, Vol. 109, No. GT7, pp. 915-931.

Newmark, N.M. (1965). "Effect of Earthquakes on Dams and Embankments," *Geotechnique*, Vol. 15, No. 2, pp. 139-160.

Okabe, S. (1926). "General Theory of Earth Pressure," *Journal of the Japanese Society of Civil Engineers*, Vol. 12, No.1.

Prakash, S. (1981). "Soil Dynamics," McGrawHill, New York.

Prakash, S., Ranjan, G., and Saran, S.(1979). "Analysis and Design of Foundations and Retaining Structures," Sarita Prakashan, New Dellhi, India.

Prakash, S. and Saran, S., (1966). "Static and Dynamic Earth Pressures Behind Retaining Walls," *Third Symposium on Earthquake Engineering*, Roorkee, India, Vol. 1, pp. 277-288.

Prakash, S., Wu, Y., and Rafnsson, E.A. (1995). "Displacement Based Aseismic Design Charts for Rigid Walls," Shamsher Prakash Foundation, Rolla, Missouri, USA.

Rankine, W.J.M. (1857). "On the Stability of Loose Earth," Philos. Trans. Royal Society, London, Vol. 147, pp. 9-27.

Richards, R., and Elms, D.G. (1979). "Seismic Behavior of Gravity Retaining Walls," *Journal of the Geotechnical Engineering Division*, ASCE, Vol. 105, No. GT4, pp. 449-464.

Saran, S., and Prakash, S., (1968). "Dimensionless Parameters for Static and Dynamic Earth Pressure for Retaining Walls," *Indian Geotechnical Journal*, Vol. 7, pp. 295 – 310.

Seed, H.B., and Whitman, R.V. (1970). " Design of Earth Retaining Structures for Dynamic Loads," *Proceeding* , Specialty Conference on Lateral Stresses in The Ground and Design of Earth Retaining Structures, ASCE, pp. 103-147.

Shakya, D.A.(1987). "Engineering Solutions for the Displacement of Rigid Retaining Walls Subjected to Earthquake Loads," M.S. Thesis, University of Missouri, Rolla.

Sherif, M.A., Ishibashi, I., and Lee, C.D. (1982). "Earth Pressure Against Rigid Retaining Walls," Journal of the Geotechnical Engineering Division, ASCE, Vol. 108, No. GTS, pp. 679 – 696.

Sherif, M.A., and Fang, Y.S. (1984). "Dynamic Earth Pressures on Walls Rotating About the Top," *Soils and Foundations*, Vol. 24, No. 4, pp. 109-117.

Steedman, R.S. (1998). "Seismic Design of Retaining Walls," *Proceeding* Institution of Civil Engineers, Geotechnical Engineering, Vol. 14, pp. 12-22.

Taylor, D.W. (1948). "Fundamentals of Soil Mechanics," John Wiley and Son, NY.

Whitman, R.V., and Liao, S. (1985). "Seismic Design of Gravity Retaining Walls," U.S. Army Waterways Experiment Station, Miscellaneous Paper GL – 85 – 1

Wotring, D., and Andersen, G. (2001). "Displacement Based Design Criteria For Gravity retaining Walls In Light of Recent Earthquake," *Proceeding*, 4th Int. Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics and Symposium in Honor of Professor W.D. Liam Finn, San Diego, March 26-31, pp. 7.33.

Wu, Y., and Prakash, S. (2001). "Seismic Displacements of Rigid Retaining Walls," *Proceeding* 4th International Conference on Recent Advanced in Geotechnical Earthquake Engineering and Soil Dynamics and Symposium to Honor Professor W.D. Liam-Finn, San Diego, March 26-31, pp No. 7.05.

Wu, Y. (1999). "Displacement – Based Analysis and Design of Rigid Retaining Walls during Earthquake," *PhD. Dissertation*, University of Missouri, Rolla.