Caisson Foundations for Cellular Telephone Monopoles

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SYNOPSIS: Design and construction procedures that were implemented for the installation of caisson foundations to support cellular telephone monopoles at fifteen sites throughout metropolitan New York are presented herein. Subsurface investigation procedures, development of geotechnical design criteria, methods of structural analysis and caisson design, and actual construction installation of the caissons have been provided along with a comparison of soil conditions and their impact on design and construction of the caissons.

INTRODUCTION

As the cellular telephone industry expands its coverage areas and upgrades services to existing users, the construction of new cell sites (transceiver stations) has proliferated across the country. In the Northeast, and particularly within the metropolitan New York area, the demand for cellular telephone services has prompted one major cellular telephone provider to undertake a three year construction program to develop approximately 200 new sites with a completed value in excess of $150 million.

Typically, a cell site is comprised of a shelter to house communications equipment and some type of elevated structure for mounting of antennas. In densely populated areas, existing structures such as multistory buildings or water tanks are commonly used for placement of antennas. In less developed areas where structures of adequate height do not exist, or the locations of such structures do not coincide with the optimum transmission locations, erection of a new antenna-supporting structure is necessary. One type of structure commonly used by the cellular industry for antenna mounting at such cell sites is a single cantilevered pole, known as a monopole.

The weight of the monopole structure including the antenna mounting platform is nominal, resulting in small compressive loads applied at the foundation level. However, these structures are subjected to substantial overturning moments caused by wind and ice loading that must be accommodated by the foundation. To support these loading conditions, accommodate a wide variety of soil types, and provide an expedient and economical installation method, the use of a single large diameter caisson or drilled pier has proven to be the preferred type of monopole foundation.

The metropolitan New York area, encompassing New York City, Long Island, Westchester County, and Northern New Jersey, as shown in Figure 1, exhibits a broad range of subsurface conditions. These include sands of varying density, soft alluvial deposits of clay and silt, glacial till, decomposed rock, and man made fills. Although the superstructure type, loading conditions and performance criteria are essentially standardized, the wide variation of subsurface conditions has precluded a standard approach whereby one type of caisson design and method of construction could be implemented uniformly. Instead, a wide variety of caisson foundation types were necessary to accommodate site specific foundation conditions.

These caissons are constructed with a variety of methods that varied from dry uncased excavations to conventional augured casing installations to bentonite slurry drilling with tremie concrete placement methods. In addition, special design considerations and construction procedures were necessary to minimize the impact of caissons installed directly adjacent to existing building foundations.

Procedures used to evaluate subsurface conditions, develop foundation design criteria, perform structural design of caisson foundations, and the actual construction procedures implemented to install the caissons at 15 sites throughout the metropolitan New York area are presented herein. Additionally, a comparison of soil conditions encountered at various sites and their impact on design and construction of the caissons will also be discussed.

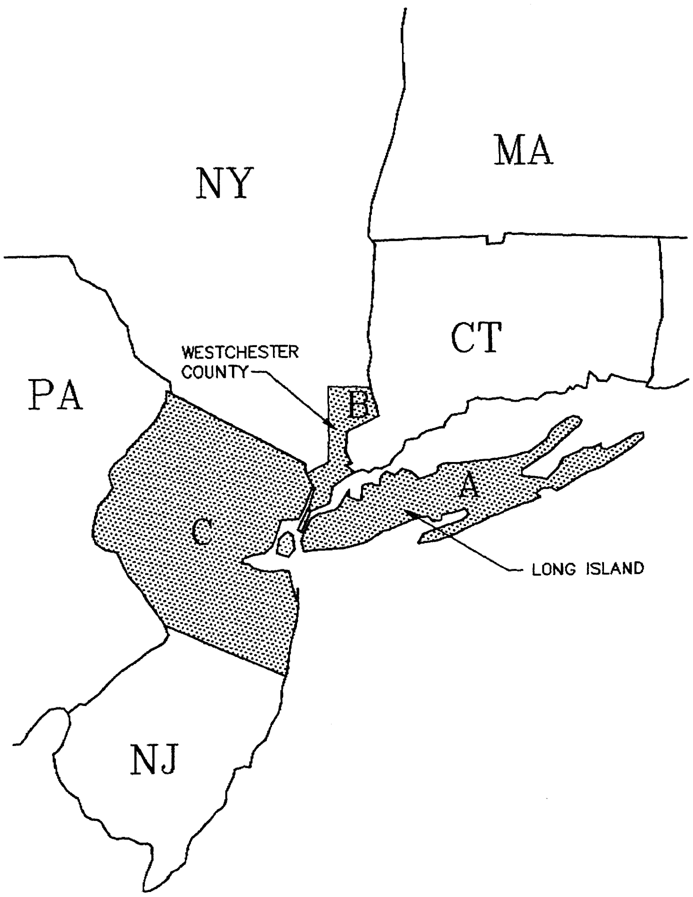

FIGURE 1 - Metropolitan New York Area

MONOPOLE CONFIGURATION AND LOADING CONDITIONS

The monopoles utilized in this program range from 35 to 150 feet in height, with a majority being 100 feet tall. Each was fitted with a triangular-shaped structural steel platform at the top to facilitate mounting of the antennas.

The pole shaft is uniformly tapered, with a diameter of one to two feet at the top, increasing to between three and five feet at the base, depending on the total height. The cross-section is either 12 or 16 sided, and is fabricated from steel. Except for the platform, the structure resembles in many ways the poles used for mounting of lighting fixtures or power lines in various parts of the country. A typical installation is shown in Figure 2.

The most significant loading condition to which the pole is subjected is wind load acting on the shaft, platform, antennas and other appurtenances, and ice which may accumulate on the structure. The magnitudes of wind and ice loads are based on design criteria recommended by EIA(1991), which has been established as an American National Standard. The load definition is consistent with the familiar approach presented in ASCE(1990), which was formerly published as ANSI A58.1 and incorporated into numerous building codes.

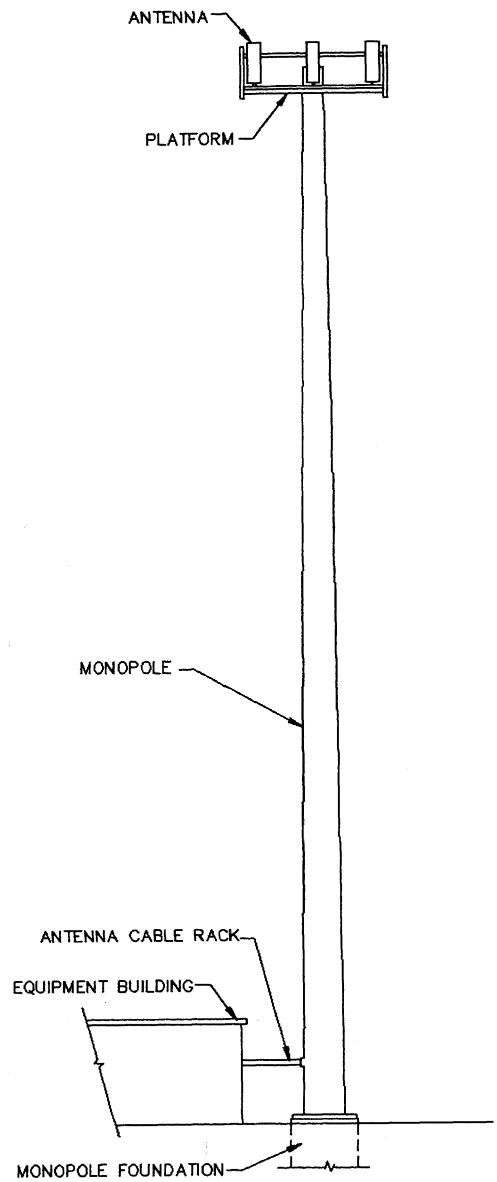

FIGURE 2 - Typical Monopole Installation

Within the metropolitan New York area, the minimum wind speed required for design (based on a 50 year mean recurrence interval) varies from 70 to 90 mph. However, it was decided at the start of the program that a single conservative loading criterion should be used for monopole design. This was to facilitate use of a "stock" pole at any location, thereby reducing lead time and the overall construction duration.

The pole manufacturer normally performs the necessary analysis to determine service load reactions at the foundation level, including shear, overturning moment, and compression. Foundation reactions are presented in Table 1 for a range of monopole heights. It is noted that a major portion of these reactions is due to the presence of the platform atop the pole.

TABLE 1
MONOPOLE BASE REACTIONS

MONOPOLE HEIGHT(ft)	AXIAL (k)	SHEAR (k)	MOMENT (k ft)
75	13	14	760
100	20	19	1320
125	32	31	2530

SUBSURFACE INVESTIGATION

A geotechnical investigation was performed at each cell site within the limits of the proposed monopole structure to identify types and distribution of subsurface materials, delineate engineering characteristics of the subsurface materials and establish caisson foundation design criteria. Generally, each site investigation consisted of one boring drilled to a depth which varied from 25 to 42 feet. Borings were advanced through overburden soils using either a hollow stem power auger or a roller bit with casing and water. Standard penetration testing (SPT) and split spoon sampling were generally performed at five foot intervals in accordance with ASTM D1586. Where rock was encountered, an Nx size diamond core barrel was used to advance the borehole a minimum of 10 feet into sound rock.

An abbreviated laboratory testing program was usually performed for each site to verify field classification of samples and establish index properties of the foundation materials. Gradation analysis, moisture content, and Atterberg limits testing were routinely performed. Unconfined compressive strength testing of soil samples and uniaxial compression testing of rock cores were only occasionally performed to develop additional shear strength characteristics of the foundation materials on marginal sites.

REGIONAL SUBSURFACE CONDITIONS

Subsurface conditions vary considerably throughout the metropolitan New York area. The geologic conditions can be characterized, however, by several distinct regions.

Long Island, including Brooklyn and Queens (identified as Region A in Figure 1), is generally comprised of granular coastal plain sedimentary deposits. These sediments are comprised largely of stratified sand and silt layers with varying amounts of gravel. The distribution and extent of these sediment deposits is fairly consistent, although moderate variation in silt content was found at the various sites investigated. Soil densities generally varied from compact to medium dense as determined during Standard Penetration Testing (SPT). Groundwater levels varied from 10 to 35 feet below grade and were considered to be representative of static conditions based on the pervious nature of the soil deposits.

Embayment deposits of organic clays and silts overlying the granular sediments were encountered along the coastal areas of Queens. These organic deposits are generally 5 to 10 feet in thickness. In many of these tidal areas, fills of between 5 and 10 feet thick have been placed over the embayment deposits to reclaim these areas for development.

Westchester County and the boroughs of Manhattan and the Bronx (Region B in Figure 1) are characterized by predominantly granular deposits overlying glacial till. Directly underlying this glacial till is bedrock. The granular deposits are primarily tan sands and gravel with little to moderate amounts of silt. The glacial till consists of a random matrix of grey and grey-brown sand, gravel, silt and clay that exists in various proportions throughout the region. The till is generally quite dense, with SPT values consistently greater than 50 blows per foot, and is underlain by bedrock. The depth to bedrock is widely varied throughout the region. Bedrock formations encountered during the site investigations included granitic and granodiarite gneiss in the upper and central areas of Westchester and mica schist in the more southerly areas. Depth to groundwater varied substantially, and when encountered, was usually due to perched conditions.

Subsurface conditions in Northern New Jersey (indicated as Region C in Figure 1) are similar to Westchester, although soil color and bedrock type differ markedly. Additionally, the granular deposits overlying the glacial till are generally deeper in extent than those encountered at the Westchester sites. The glacial tills are a heterogeneous mixture of reddish brown sands, gravel, silt and clay, with sand predominating the mix proportion. The underlying bedrock was most commonly found to be part of the Brunswick shale formation, which is generally weathered at the contact surface and becomes more massive with depth. Discontinuities in the shale were generally at low angles to the horizontal. Groundwater conditions were quite variable, with depths ranging from three feet to greater than 35 feet.

FOUNDATION DESIGN CRITERIA

Selection and development of geotechnical design criteria for the caisson foundations is a function of soil type, loading conditions and structural design methodology. The soil type determines which of the general shear strength parameters, friction angle and/or cohesion, are necessary to characterize the strength and behavior of the foundation materials. As mentioned previously, the loading conditions considered most critical are overturning due to substantial lateral loading of the monopole and, to a lesser extent, compressive loads imposed by the weight of the structure.

Since lateral capacity of the caisson to accommodate overturning moments was considered to be the most critical parameter, development of foundation design criteria had to consider the method of analysis for caisson design. Initially, hand solutions developed by Broms were used to size caissons and determine concrete reinforcing requirements. Subsequently, the analysis and design of caissons was performed with commercially available computer programs, LPILE and STIFF1, which are described by Reese and Wang(1989) and by Wang and Reese(1987). These two analysis methods require somewhat different input data which dictated the type of soil parameters developed as discussed herein.

Determination of allowable bearing capacity to support the compressive loads of the monopole and lateral soil resistance to accommodate overturning were based on empirical relationships developed from the boring and laboratory test data obtained at each site. In the determination of allowable bearing capacity, only point bearing was considered. Side resistance (skin friction) was not evaluated because this additional capacity was generally not needed due to the nominal compressive design loads. For point bearing, a minimum design depth was specified to ensure bearing on a suitable foundation subgrade.

Based on procedures developed by Reese and O'Neill(1988), SPT values were input directly into the following relationship to determine ultimate point bearing of caissons founded on granular soils:

$$q_{ult} = 0.6\,N_{spt} \quad (1)$$

where q_{ult} = ultimate bearing capacity (tsf)

N_{spt} = average uncorrected SPT value (blows/ft) obtained directly beneath the caisson base to a depth of twice the base diameter

Notes:

1. q_{ult} obtained from this relationship limits settlement to five percent of the base diameter.
2. For caisson diameters over 50 inches, reduce q_{ult} by 50/D, where D is caisson diameter in inches, to maintain settlement within acceptable limits.

For cohesive soils, point bearing was determined from the relationship:

$$q_{ult} = N_c c_u \quad (2)$$

where c_u = average undrained shear strength (tsf)

N_c = $6.011 + 0.2\,(L/D) \leq 9$ with L = shaft length(ft)

Values of undrained shear strength are computed over a depth of 1 to 2 diameters below the caisson base, and were developed based on established empirical relationships with SPT results and Atterberg limits test data.

Table 2 contains values of allowable bearing capacities determined for the various sites along with soil classifications and unit weights, SPT results, and general shear strength parameters (ϕ and c). Values of allowable bearing capacity are based on a factor of safety between 2.5 and 3.0, depending upon the extent of field and laboratory data obtained for a given site and past experience with similar foundation material types.

TABLE 2
FOUNDATION MATERIAL CHARACTERISTICS

SITE NO.	SOIL TYPE	N_{spt} (blows/ft)	γ_d (pcf)	ϕ	$c^{(2)}$ (pcf)	q_a (tsf)
A-1	SP	26	120	34°	0	4.0
A-2	SP	17	112	31°	0	4.0
A-3	SM	24	118	34°	0	6.0
A-4	SP	20	114	33°	0	4.5
A-5	SP	10	106	28°	0	3.5
A-6	SP	27	120	35°	0	4.0
B-1	SM	>50	130	38°	150	10.0
B-2	MICA SCHIST	28%(3)	145	42°(4)	0	60(5)
B-3	SM	>50	130	38°	0	10.0
C-1	SM	>50	128	38°	0	6.0
C-2	SM	>100	134	38°	250	7.0
C-3	ML	26	125	32°	250	4.0
C-4	SHALE	0%(3)	140	40°(4)	0	11.0
C-5	SM	>50	128	38°	0	8.0

NOTES:

(1) Soil description based on Unified Soil Classification System

(2) Cohesion based on effective strength parameters

(3) RQD value determined at foundation bearing depth

(4) Based on shear strength of discontinuities

(5) Maximum allowable bearing capacity as determined from NYC Building Code

Preparation of foundation design criteria for use with Broms' method of analysis required development of lateral earth pressure resistance diagrams with depth. For granular soils, ultimate lateral soil resistance varies with depth and is determined from the relationship:

$$p = 3D\gamma Z K_p \quad (3)$$

where p = ultimate lateral resistance (pounds per foot of depth)

γ = effective unit weight of soil (pcf)

D = caisson diameter (ft)

Z = depth along caisson (ft)

K_p = Rankine coefficient of passive earth pressure

For cohesive soils, ultimate lateral soil resistance is uniform with depth and determined by:

$$p = 9c_u D \tag{4}$$

The aforementioned lateral soil resistance design criteria were input into Broms' equations to develop maximum resistance of the caissons as discussed further on.

The use of software to assist in designing the caissons necessitated development of alternate foundation design criteria for use as input. Briefly, soil deflection under lateral loading is modeled with p-y (load vs. deflection) curves in the LPILE program. These p-y curves can either be input manually or generated from preprogrammed p-y curves for several different soil conditions from the LPILE data base. These preprogrammed families of p-y curves were developed by the LPILE authors based on extensive full scale field load test results. The basic types of soil conditions are:

- Dry and Submerged Sands
- Saturated Soft Clays
- Saturated Stiff Clays
- Dry Stiff Clays

To facilitate analysis and minimize design costs, computer generated p-y curves were used for design and analysis of the caissons at the various sites.

Geotechnical design parameters that are similar for both Broms' and LPILE methods of analysis include shear strength, (friction angle and/or cohesion) and effective unit weight of the soils. For the LPILE analysis using the computer generated p-y curves, additional geotechnical input included the modulus of horizontal subgrade reaction (k_h). These values were determined based on established empirical relationships with SPT results.

For clays, values of strain (ϵ_{50}) corresponding to one half the maximum principal stress difference determined from unconfined or triaxial compression tests are also needed. Typical values of ϵ_{50} were obtained from established relationships correlating shear strength (c_u) to strain.

STRUCTURAL DESIGN PROCEDURE

Magnitudes of the lateral loads imposed on the monopole and transferred to its foundation are developed in accordance with parameters established by the EIA as discussed previously.

Using the foundation loading data and geotechnical design criteria, the following procedure was used to design the caisson. The procedure described herein differs from that presented in ACI(1985) by incorporating the behavior of layered soils, and focuses on the use of computer software to perform the analysis.

1. Establish minimum diameter of the foundation based on the size of the base plate and clearances for anchor bolts and reinforcing. Minimum vertical reinforcement required by ACI(1989) is calculated as:

$$A_{s,min}=0.005A_c \tag{5}$$

2. Model the soil as a layered medium, based on material types, ground water, and stiffness parameters. These parameters are expressed as a load-deformation relationship (p-y curve) indexed to the soil type and factored by the horizontal subgrade modulus.

3. Depending on the soil stiffness, the maximum moment will occur at a point approximately 1.5 diameters below grade. Estimate this maximum moment and calculate a trial value for the effective stiffness of the caisson and rebars as:

$$K_{eff}=E_c I_{eff} \tag{6}$$

   where I_{eff} is the transformed moment of inertia of the cracked section under the given moment and axial load. In practice, this is calculated by the STIFF1 program, which can also incorporate the stiffness contribution of a permanent steel casing.

4. Run the LPILE analysis to determine deflection of the foundation at grade and maximum bending moment under the given service loads. The deflection is compared to an allowable value of 3/8", and the moment is compared to the estimated value used in the stiffness computation. Determine if additional caisson stiffness or a correction to the estimated stiffness value is needed.

5. Adjust the caisson length, diameter, or vertical reinforcement to obtain the required stiffness. Repeat the analysis until the deflection criterion is satisfied and convergence is obtained.

6. Modify the soil parameters to assess the sensitivity of the design and confirm that predicted behavior will be consistent throughout a reasonable range of variation.

7. Verify that the structural capacity is adequate in accordance with ACI(1989), and that soil pressures are within acceptable limits.

A typical caisson foundation for a monopole is shown in Figure 3.

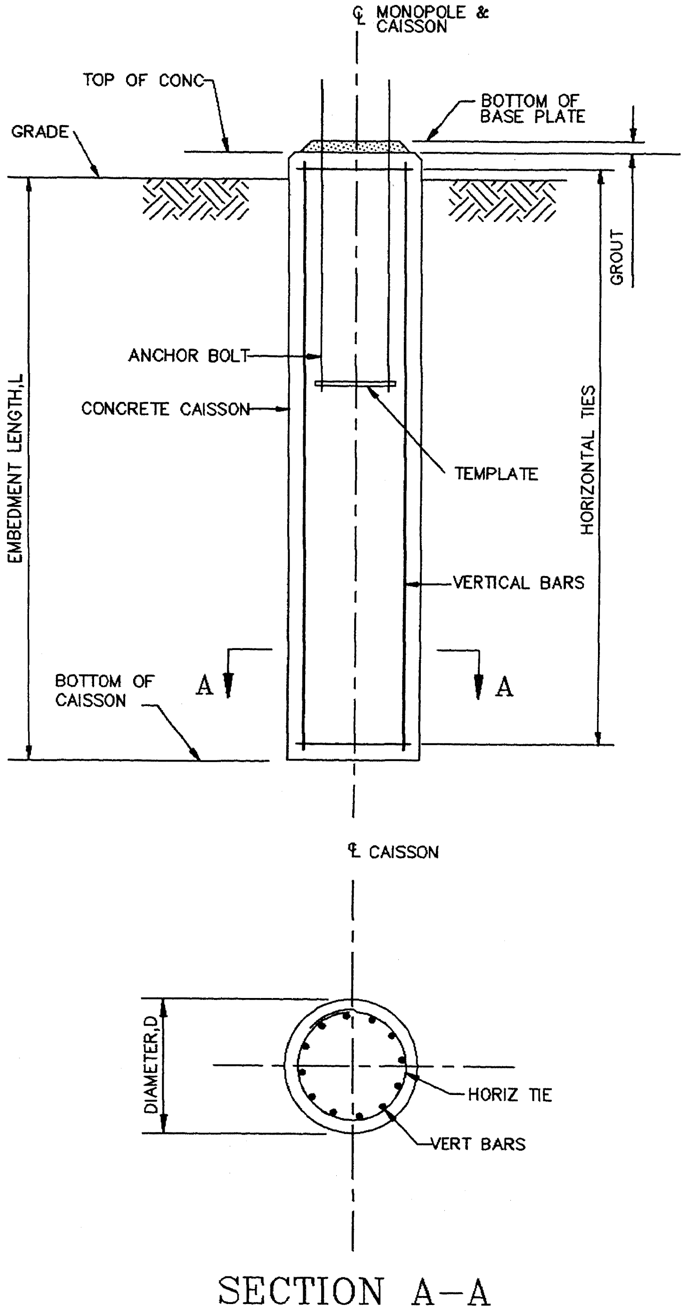

FIGURE 3 - Caisson Foundation

At this point, the economy of the design is reviewed to determine if appreciable savings could be achieved by altering the soil parameters. This can be accomplished in some instances by excavating the natural soil around the top of the caisson to a reasonable depth (three to four feet) and replacing it with compacted backfill, to increase the horizontal subgrade modulus of the surficial soils. The lateral extent of excavation and replacement is shown in Figure 4. This soil replacement method is also effective in reducing the required caisson length by as much as five to ten feet.

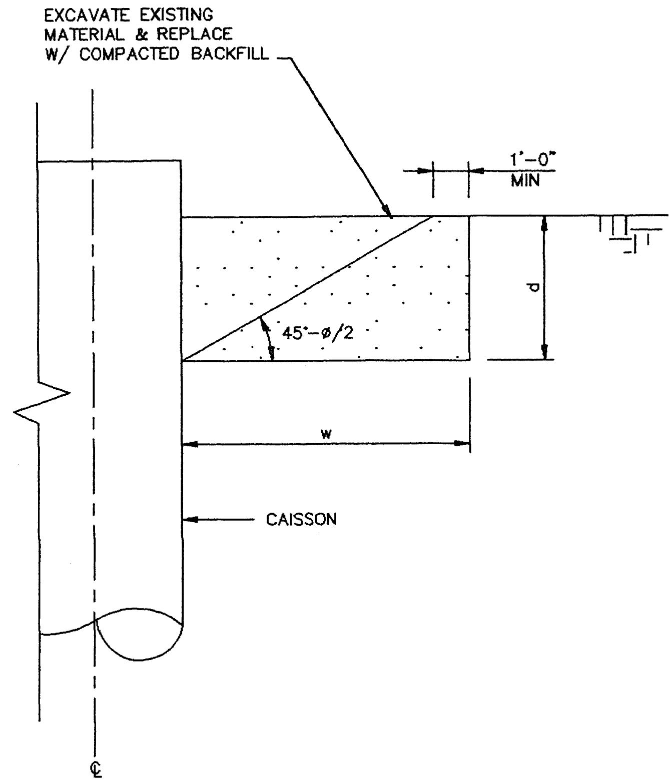

FIGURE 4 - Soil Replacement

Table 3 contains physical characteristics of the monopole structures and caisson foundations for fifteen representative cell sites in the metropolitan New York area, including monopole height and resultant caisson dimensions.

It is not uncommon for these monopoles to be located close to existing buildings and other structures. Site B-2 required a unique design to accommodate its proximity to an existing building. The centerline of this monopole is only 6 feet from the existing basement wall, which extends approximately 7 feet below grade.

Preliminary analysis indicated that the minimum diameter of the caisson would be 5.5 feet, which would place the edge of the caisson within 3 feet of the building. Due to the age and condition of the building foundation wall, it was necessary to avoid transferring the substantial lateral load from the monopole to the adjacent wall. Therefore, a caisson with a double steel casing separated by a layer of compressible material as shown in Figure 5 was designed to absorb lateral loads induced by the monopole. The 5.5 foot diameter inner casing extended full depth to the top of sound rock to enclose the caisson concrete. The larger outer casing extended to just below the building footing. The four inch annular space between the two casings was filled with polystyrene granules which can compress and absorb the lateral deflection and minimize load transfer from the monopole. A neoprene rubber gasket was installed at the top of the annular space to protect the compressible material and minimize intrusion of water.

TABLE 3
CAISSON DESIGN DATA

SITE NO.	MONOPOLE HEIGHT (ft)	DIAMETER (ft)	LENGTH (ft)	A_S/A_C
A-1	100	5.0	25	.009
A-2	100	6.0	30	.009
A-3	125	7.0	35	.009
A-4	132	6.5	35	.009
A-5	100	5.0	30	.006
A-6	100	5.5	25	.007
A-7	60	5.0	30	.009
B-1	150	5.5	16	.007
B-2	150	5.5	20	.007*
B-3	125	**	**	**
C-1	75	5.5	25	.009
C-2	100	6.0	30	.009
C-3	100	6.0	30	.008
C-4	100	6.0	25	.006
C-5	35	4.0	18	.006

*Plus permanent steel casing
**Not yet determined

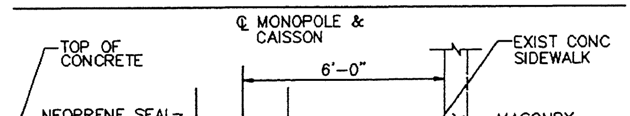

FIGURE 5 - Caisson Installation at Site B-2

CAISSON CONSTRUCTION PROCEDURES

The preferred method of caisson construction for a given site is primarily a function of soil type, although other factors such as groundwater level and proximity of the caisson to existing building foundations will affect the procedures used to install the caisson. The most common methods can be categorized into three groups as follows:

1. Uncased excavations
2. Cased augered holes
3. Slurry displacement installations

Construction of caissons in uncased excavations has been limited to sites with subsurface conditions comprised of stiff cohesive glacial till soils with no groundwater. Only two of the fifteen sites contained in Table 2 (B-1 and C-2) were constructed as uncased excavations in this manner. Advancement of the caisson excavation is accomplished basically by dry augering to the necessary bearing depth. Short term sidewall stability under these circumstances was not a problem as installation of reinforcing and concrete placement was performed soon after completion of the excavation. To ensure the aforementioned sequence of events, the contract documents prohibited any uncased excavations from being left open overnight.

At site B-1, the relatively simple uncased excavation process was complicated when a large boulder was encountered 5 feet above the proposed bearing stratum after the caisson was relocated 15 feet away from its original location. Caisson design modifications to account for the reduced depth included belling of the caisson base and enlarging the diameter of the top portion of the caisson by an additional six feet to a depth of four feet. Implementation of these modifications occurred over several days and as a result, the use of temporary casing became necessary for sidewall stability.

Cased augered holes were the most common construction method used to install the caissons in predominately granular soils where groundwater was not encountered. As the caisson excavation was advanced, temporary casing was lowered to maintain sidewall stability. The casing usually extended to a depth of at least one-half to two-thirds of the caisson length, depending on soil conditions. During placement of concrete, the temporary casing was withdrawn incrementally as the concrete was placed. Since maintenance of anchor bolt alignment was critical to ensure proper orientation of the monopole and antennas with respect to true north, constant checking of the anchor bolts was necessary during the concrete placement and casing withdrawal process.

There were a few sites where casing was left permanently in place due to the proximity of the caisson to existing structures and logistical considerations associated with the installation process. Caissons at sites A-4 and A-6 were each located within 5 feet of existing structures that are supported on shallow footing foundations. To prevent possible undermining of these foundations, the use of temporary surface casing was stipulated in the contract documents. Because of limited equipment access and the concern for protection of the existing structures, the contractor elected to leave the casings in place.

The use of bentonite slurry to advance caisson excavations was used primarily in granular soils with high groundwater levels that were typically encountered on Long Island and in Queens. In most circumstances, sidewall stability above groundwater was maintained with temporary casing. Once groundwater was encountered, a bentonite slurry mix was placed into the excavation. The slurry level was generally maintained several feet above the groundwater level to maintain a positive head and alleviate the possibility of base disturbance due to liquefaction. Once the excavation was advanced to the appropriate depth, the reinforcing cage and anchor bolt assembly was inserted into the excavation and the concrete was placed using tremie methods. The displaced bentonite slurry was usually disposed of onsite.

CONCLUSIONS

The growth of the cellular telephone industry has led to the development of numerous antenna sites in the metropolitan New York area. Single caisson foundations have been found to be the most desirable method of supporting monopoles used as antenna supporting structures.

As part of this construction program, the structural configuration and loading conditions were standardized, enabling the design to focus on site-specific subsurface conditions. Three distinct regions were identified, and foundation design criteria were developed based on geotechnical investigations.

Although simplified field investigation and laboratory testing procedures were used to develop the foundation design criteria for these structures, the quality of the data obtained has turned out to be more than adequate.

A structural design procedure based on computer analysis methods allowed rapid evaluation of the sensitivity of the overall caisson design to the various soil parameters. A major benefit derived from this capability was obtained in several instances where surficial soil excavation and replacement was used to decrease predicted caisson deflections. This was accomplished at a significant reduction in cost compared to deepening or enlarging the caisson.

The various construction procedures employed have also been discussed. By implementing careful planning and supervision at each site prior to and during construction, satisfactory caisson installations resulted at all locations. This was accomplished despite the accelerated construction schedules and the inevitable obstacles that were encountered during construction within the urban/suburban environment.

The performance of each of the completed monopoles has been satisfactory in the two years since the start of the program, which included several severe wind storms, and is expected to continue for the useful life of the structures.

SYMBOLS

A_c	gross concrete area
A_s	area of reinforcing steel
c	cohesion
c_u	undrained shear strength
D	caisson diameter
E_c	modulus of elasticity for concrete
I_{eff}	effective moment of inertia of transformed cracked section
K_{eff}	effective caisson flexural stiffness
k_h	modulus of horizontal subgrade reaction
K_p	Rankine coefficient of passive earth pressure
L	embedment length
N_c	bearing capacity factor
N_{spt}	average uncorrected blow count
p	ultimate lateral resistance
q_a	allowable bearing capacity
q_{ult}	ultimate bearing capacity
z	depth along caisson
γ	effective unit weight of soil
γ_d	dry unit weight of soil
ϵ_{50}	strain
ϕ	angle of internal friction

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