Reliability of a Wave Equation Analysis in the Estimation of Static Bearing Capacity of a Vertical Pile--a Case Study

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SYNOPSIS: Application of wave equation, in the analysis of pile behavior, under dynamic loading has started with Smith [9]. This paper presents a modified explicit numerical scheme, based on Smith's approach, which retains the simplicity and minimum storage requirement of Smith's scheme but offers a faster and more accurate solution near critical time step. Bearing capacity of pile is estimated through Poulos [8] scheme, after evaluating dynamic properties of soil from set value through this modified scheme. Observed load test results establish a good promise for this analysis.

INTRODUCTION

Estimation of static bearing capacity of a single vertical pile is very essential for foundation engineer. Field pile load testing directly speaks for its bearing capacity but the large amount of cost involved in each testing puts a limitation on its number. Theoretical formulas [4], dynamic driving formulas [10] with their individual simplified idealization, do come handy to the field engineer for the assessment of this highly complicated problem. The use of the theory of one dimensional wave propagation [3,7] has now found increased application for the analysis of the driving behavior of piles and for predicting their static bearing capacity. Since pile driving causes failure of the soil, the idea of correlating the measurement made during pile driving to estimate the pile capacity seems apparently logical. Though the dynamic nature of penetration deviates from the static response of soil, considerable effort has been devoted to find suitable correlation. In this study, the actual field observed bearing capacity of piles has been compared with numericaly predicted value, based on a modified Smith's approach of wave equation analysis.

GEOTECHNICAL CONDITIONS

A detailed soil investigation of the particular site was conducted and it was found that the stratification is very uniform in nature. A typical soil profile is shown in Fig. 1. The sand layer which is main feature of this deposit is poorly graded uniform, mixed with fine sand or silt. Top 2m is covered with greyish brown silty clay (SPT N=5). Below this layer the N value of claye silt layer increases from 12 to 30 upto a depth of 20m from ground surface. Deeper strata offers N value as high as 60 to 80. Water table was found 2m below ground level. Engineering properties of the soil are given below:

Specific gravity of soil	2.52
Mean grain size	0.18mm
Maximum void ratio	0.91
Minimum void ratio	0.46

The deposit in the site is in medium to loose state upto a depth of 10m, below which higher N value indicate dense state.

TEST PROGRAM

Test piles were of 450mm diameter, cast-in-situ concrete piles. Steel casing tubes, Length 24m, Outside diameter 450mm, Youngs Modulus=2.2 x 10^6 kg/cm^2, were driven by 3.5t hammer. The hammer was dropped from a height of 1.3m by the usual method of releasing the clutch of the driving machine. There might be some possibility of slight variation in the height of fall but a close watch was always kept to maintain the uniformity. Number of blows needed for the penetration of each meter of depth was noted throughout. Set value was obtained from the time-transient displacement curve, in last few blows, on the driven casing tube. Depth of embedment, verticality of the casing tube, leakage through bottom shoe etc. were noted. After driving the casing tube, upto the required depth, reinforcement cage was inserted and concrete was poured in three stages. Casing tube was lifted slowly with short fall of hammer. Actual volume of concrete consumed was compared with the calculated volume required, to check against the formation of necking.

The piles were test loaded upto failure, about 45 days after casting. Cyclic load tests (Fig.2) were carried out on tese piles by putting load with Hydraulic jack on finished pile head at cut-off level. Reaction was obtained from loads placed on steel girders. Movement of pile head under the application of each loading and subsequent unloading, were noted by 4 diagonally placed dial gauges on pile head. Load tests were conducted as per IS: 2911, Part 1 [11].

A typical load-settlement behavior for test pile TP 5 is shown in Fig. 3 and the results has been presented in Table 1. It may be observed that the average load carrying capacity of Pile is 160t with 60% skin friction and 40% point bearing, with 6mm elastic settlement (gross-net) at 160t.

NUMERICAL WAVE EQUATION ANALYSIS

Smith's [9] proposed approach to the wave equation was the discrete idealization of pile body as an assemblage of (Fig. 4) lumped, concentrated weights W(1) through W(p), which are connected by weightless spring K(1) through K(p-1), representing pile stiffness. Resistance offered by soil was considered to be visco-elasto-plastic. Time was also discretised into small intervals. In general, the system is considered to be composed of:(i) A ram, (ii)A cap block, as cushioning material, (iii) A pile cap, (iv) A pile and (v) The supporting medium or soil.

The numerical scheme, developed by Smith [9] in its finite difference form is as follows:

D(m,t)=D(m,t-1)+V(m,t-1)
C(m,t)=D(m,t)-D(m+1,t)
F(m,t)=C(m,t). KP(m)
R(m,t)=[D(m,t)-D'(m,t)].KS. [1+J(m). V(m,t-1)]
V(m,t)=V(m,t-1)+[F(m-1,t)-F(m,t)-R(m,t)].g.dt/W(m)

where D,C,F,R,V are displacement, compression of internal spring, force in internal spring, total soil resistance and velocity respectively of m pile element at time t. KP and KS are pile and soil stiffness modulus. J is the damping coefficient and dt is the time increment.

Numerical scheme [1] sweeps over time from the known impact velocity of the first element and with initial at-rest condition of all other pile and soil elements. The iteration of the Smith's scheme terminates when either of the following conditions is satisfied:

1. Pile tip, after penetrating to some maximum downward value starts rebounding upward.
2. Velocities of all the elements are negative or zero.

Set is defined as the maximum tip displacement minus the plastic yielding Quake factor 'q'.

Smith's scheme is a spatially discrete model which is integrated forward in time to generate a transient dynamic response. This scheme solves one dimensional wave equation, $C_p^2 \cdot d_{xx} - d_{tt} = f(x,t)$, which characterizes dynamic axial load in pile body. Here d denotes the time dependent displacement of pile at a distance x and at time t, and the suffix denotes the respective partial differentiation. The constant C_p is propagation velocity of elastic wave through pile material.

The present study is based on Smith's scheme with following modifications:

(i) R(m,t)=[D(m,t)-D'(m,t)], KS +q.KS.J(m).V(m,t-1)

(ii) Displacement and velocities are found out at the end of full time interval (n).dt, (n+1).dt etc., but forces and accelerations are obtained at half time intervals (n+1/2).dt, (n+3/2).dt etc.

(iii) Iteration continues till the oscillations of all elements become negligibly small

(iv) Set is defined as permanent downward movement of the pile.

(v) Total static part of the dynamic resistance offered by soils considered as the maximum of [D(m,t)-D'(m,t)].KS, over all soil elements at each instant of time.

(vi) Tip soil is a tension free element

The finite difference form of the semidiscrete wave equation can be represented as **M** d•• + **KP**. d=**F**, where d is the displacement vector representing nodal degrees of freedom, d•• is the nodal acceleration vector, **M** is the diagonal mass matrix for this lumped parameter model and **KP** is the linear stiffness matrix. The non homogeneous term F, which represents the soil resistance on the pile motion, is the vector of nodal 'loads'. At any instant, n.dt of the discrete time domain, it should always satisfy the dynamic equilibrium equation: **M**.d•• (n)+ **KP**.d(n)=**F**(n), where the term in the bracket indicates the vector at n instant of time. In this modified scheme, first a central difference expressions are introduced for the acceleration d•• in terms of the velocity d• and then this velocity is expressed by central difference expression over displacement.

M.d••(n)=**F**(n)-**KP**.d(n)
[d•(n+1/2)-d• (n-1/2)] / dt = $\mathbf{M}^{-1}$. [**F**(n)-**KP**.d(n)]
d•(n+1/2)=d•(n-1/2) + $\mathbf{M}^{-1}$. [**F**(n)-**KP**.d(n)].dt
d(n+1)=d(n)+dt.d• (n+1/2)

Here the term **F** represent instantaneous soil resistance, which brings non-linearity in the partial differential wave equation. In original Smith's scheme the viscous part of it was expressed as product of the displacement and velocity. The present scheme considers the viscous part as velocity dependent only. Moreover, the modified scheme retains the simplicity of Smith's assumption that the total soil resistance may be considered as time independent constant during the travel of the wave through any pile element from the beginning to the end of discrete time interval dt. The change in the soil resistance is evaluated at the end of each interval. Within the frame of these assumptions the term f(x,t) of the one dimensional wave equation has been considered as a constant **F** in the semidiscrete wave equation **M**.d••+**KP**.d=**F**. To evaluate d(n+1) of any instant this explicit scheme utilizes one just-previous value of d(n) and one just-previous value of d•(n-1/2). The velocities d• (n-1/2) are carried out at half time point. The diagonal nature of the mass matrix **M**, in this lumped parameter model, offers advantage of the speed of computer calculation and explicit nature of the scheme reduces the computer storage requirement.

STABILITY OF THE MODIFIED SCHEME

The homogeneous part of the discrete scheme [6] may be written as d• (n+1/2)=d•(n-1/2)-dt.$\mathbf{M}^{-1}$.**KP**.d(n), which when substituted in the central difference expression of velocity, d(n+1)=d(n)+dt.d•(n+1/2) offers:

d(n+1)=d(n)+dt.[d•(n-1/2)-dt.$\mathbf{M}^{-1}$.**KP**.d(n)]
=d(n)+dt.d•(n-1/2)-$(dt)^2$.$\mathbf{M}^{-1}$.**KP**.d(n)

Since, d(n) =d(n-1)+dt.d•(n-1/2),this offers:

d(n+1) =d(n)+[d(n)-d(n-1)]-$(dt)^2$.$\mathbf{M}^{-1}$.**KP**.d(n)

implies, d(n+1)-2d(n)+d(n-1)+$(dt)^2$.$\mathbf{M}^{-1}$.**KP**.d(n)=0

The solution is sought as d(n)=(del). exp[p.n.dt], where, del=arbitrary displacement vector,p is undetermine d.Considering for any real physical system there exist a separate and distinct non-zero delta for each eigenvalue-eigenvector, this expression offers: exp(p.dt)-2+exp(-p.dt)+$(dt)^2$.$\mathbf{M}^{-1}$.**KP**=0

For any eigenvalue **L** of $\mathbf{M}^{-1}$.**KP** the characteristic equation for 'del' becomes: exp(p.dt)-2+exp(-p.dt)+$(dt)^2$.**L**=0

To check against the growth of original error, with increasing t, in the solution of wave equation, which is bounded for all time t, it is necessary and sufficient that $-1 \leq e^{p\,dt} \leq 1$

This imposes a strict condition that $(dt)^2 \cdot L \leq 4$,ie,$(dt)^2 \leq 4/L_{max}$, where $\mathbf{L}_{max}$ is the maximum of all the eigenvalues of $\mathbf{M}^{-1}$.**KP**, which is $L_{max} = 4.C_p^2/(dx)^2$ for lumped mass idealisation of discretised linear wave equation with uniform mesh discretisation 'dx'. Under this condition critical time step, $t_{cr}=dx/C_p$. So long the time step of integration is kept lower or equal to t_{cr} the numerical solution should converge to true solution. The space-time finite difference nodes of one dimensional wave equation fall exactly on the characteristic curve when the time step of integration dt is $dx/C_p = \sqrt{[W(m)/(KP(m).g)]}$and should offer exact solution.

COMPARISON OF TWO SCHEMES

To compare the efficiency of these two schemes following problem was identified;

The pile consists of 24 elements of same mass and same stiffness and 3 elements for anvil, cap and capblock. Soil stiffness KS is set to zero. Initial displacements are 0 for all elements and for the first 10 pile elements initial velocity is set to unity. Remaining 10 pile element velocity is set to zero. The solution of this uniaxial deformation of a linearly elastic material is the propagation of one-half of the impact velocity into the stationary material and a reduction by same magnitude into the impacting material. Fig 5 compares the velocity wave front obtained from these two schemes. As the error in energy balance of the modified scheme is small it does not contain saw-tooth wave pulses of Smith's original scheme. The approximation brought into the partial differential wave equation by different idealisations, drifts the numerical solution away from its true behaviour. The original frequency content w gets distorted, either upward or downward into an approximated w* depending on the nature of mass and time discretisation. In the present problem, one is interested to know the history of stress wave propagation, through the pile body, because of the impact offered on the pile head by the falling hammer. The entire frequency content, needed to represent the shock wave propagation, is of prime importance. Lumped parameter mass idealisation of spatial discretisation [5,2] depresses the approximate frequency but the central difference operator shifts it upward. Thus the combination of these approximation, adopted in the present modified scheme, tries to compensate each other. This scheme retains the simplicity of Smith's scheme along with a more accurate solution near the critical time step. Fig 6 compares the time displacement behaviour of the pile element just above ground through these two schemes.

Smith's scheme while assessing the stiffness of the soil modulous the total load carrying capacity of the pile Ru was assumed and it was distributed between side skin friction Rf and point bearing Rp (Ru=Rf+Rp) from the knowledge of the sharing, obtained from cyclic load test results. Soil stiffness of the peripherial and tip soil KS_f and KS_p were obtained as KS_f=Rf/q and KS_p=Rp/q. Set values, obtained from this distribution of soil resistance were related with assumed total resistance Ru. It is implicit in this assumption that all the soil elements at some instant, will simultaneously undergo a deformation which is greater or equal to the quake factor, thus generating full capacity of resistance in each element, simultaneously, whose summation is the assumed load carrying capacity. In reality, static part of the total dynamic resistance offered by soil is the summation of the product of the instantaneous displacement of soil and its stiffness, over all the soil elements. Fig 7 shows that the mobilasition of maximum soil resistance is not instantaneous. As the depth increases, phase lag increases and the tip soil offers intermittent beat type resistance on the pile motion.

ANALYSIS OF THE TEST RESULT

Test results have been analysed as follows:

Step1. Modified scheme was used to generate Set-Dynamic shear stiffness curve, with quake factor 1.25mm&J=.0002sec/mm Shear stiffness associated with the free length of the casing tube, above ground, was set to zero. Number of blows for the last meter of penetration for TP-8 was 330, which indicate final set value as 3.0mm. When this set value is entered through Fig. 8 offers dynamic shear resistance of soil as 990t/m which corresponds to soil shear modulous G =1500t/m^2.

Step2. Poulos [8] has offered the estimation of elastic settlement of pile under the application of vertical load. The load required to produce 6mm elastic settlement for TP-5 (Length =17m, Dia=0.45m,E_p=2.1 x 10^6t/m^2,E_s=2.G(1+.25)=3750t/m^2 was found out as 163t. Observed load from cyclic load test results have been presented in Table 1.

DISCUSSION AND CONCLUSION

Table 1 shows that the numerical prediction of the load carrying capacity of pile is in good agreement for TP-8, TP-5 and TP-10. But for the short length pile TP-7 this scheme under estimates by 44%. This may be because of the variation in local geological condition which has helped to increase the pile capacity by soil set-up.

This may be concluded from this study:

i) Modified Smith's approach is numerically more stable

ii) All peripherial soil element do not offer their individual maximum resistance simultaneously.

iii) Defining Set as permanent downward displacement of pile under last blow, the modified scheme correlates well this set with the dynamic shear stiffness property of the peripherial soil.

iv) Evaluating soil static modulous, from this dynamic shear stiffness, Poulos approach may very well predict the load carrying capacity at a given elastic settlement of pile.

v) Proper consideration should be given to the development of soil strength because of compaction of soil through driving of the casing tube and thixotropic increase in soil shear strength.

TABLE 1. LOAD CARRYING CAPACITY OF PILE

No.	Pile Length (m)	Observed Load (t)	Set (mm)	Calculated Load (t)
1. TP-8	21	160	3.0	163
2. TP-5	17	160	3.8	158
3. TP-4	17	180	4.0	153
4. TP-7	13	96	6.7	53
5. TP-10	21	160	3.2	163

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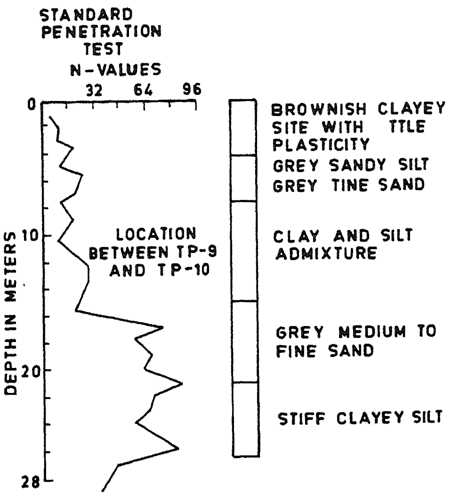


FIG.1 _ GEOTECHNICAL PROFILE

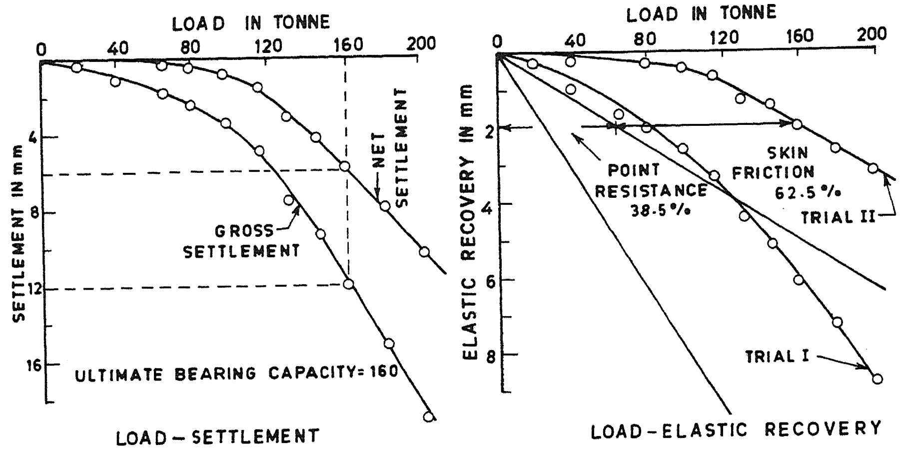


FIG.3 _ CYCLIC LOAD TEST RESULT FOR TP-5

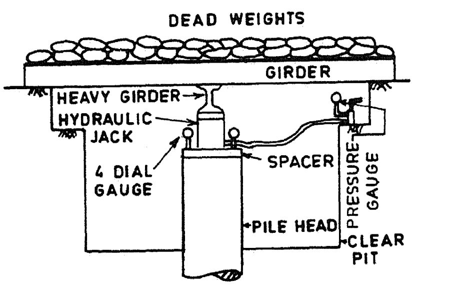
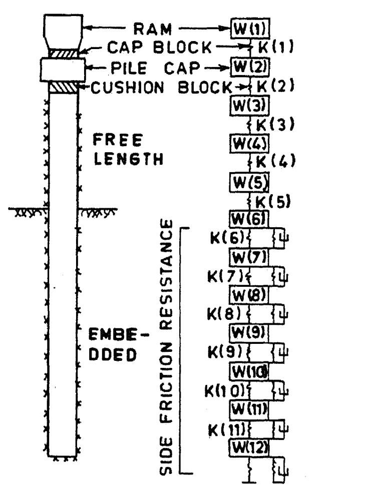


FIG.2 _ PILE LOAD TEST ARRANGEMENT



(a) ACTUAL PILE (b) IDEALIZED PILE

FIG.4 _ LUMPED MASS MODEL

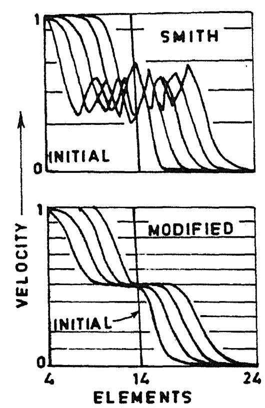


FIG. 5 _ VELOCITY WAVE FRONT

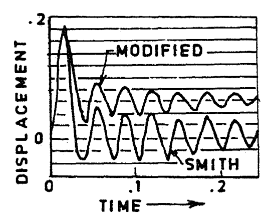


FIG.6 _ TIME DISPLACE-MENT BEHAVIOR

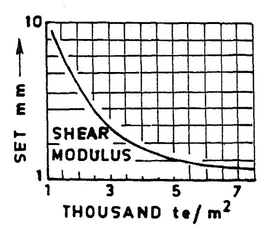


FIG. 8 _ SET VS SOIL RESISTANCE

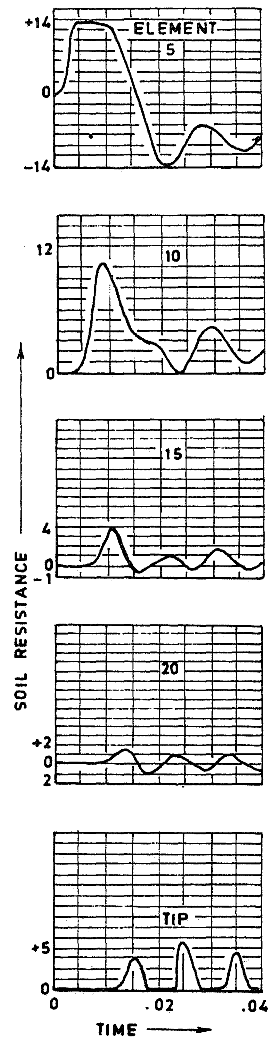


FIG.7 _ MOBILISATION OF SOIL RESISTANCE