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SYNOPSIS: It has been proved that the shape of foundation and its base side ratio affect the dynamic rigidity of sliding-rotation vibration of bases. Based on plenty of laboratory tests in a small sides the computing formulas of rigidity and damping of base considering the effect of base side ratio are put forward.

INTRODUCTION

Up to now the research of the effect of foundation shape on dynamic parameter is far from enough. In order to correctly design dynamic machinery foundations and structures this problem has been researched by the author in recent years. All together 18 foundations with same base areas of 0.5 m² and different height and base side ratio of foundations were tested under exposed and embedded conditions. All tests were performed under the action of harmonic exciting force. The 18 foundations tested were divided into four groups according to their base shape as follows:

1. The shape of foundations is rectangular, the base side ratios of which are 1:1, 1:2 and 1:3 corresponding to the first group, second group and third group.
2. The shape of the fourth group is circle with radius 0.28 m.

All tests were conducted under the condition of laboratory in which a sand pit was set. In the following a several main phenomena and conclusions are introduced.

I. Vertical Vibration

First all, it must be pointed out that the mechenics model of Mass-Damping-Spring within the range of linearity is adopted as shown in Fig.1. Meanwhile the following assumptions are used:

1. The foundations only possess inertia effect.
2. The base soils only possess the effect of elastisity
3. The damping of base coincides with the viscous damping theory.

According to the assumptions above, the equation of vertical motion of foundation may be expressed as follows:

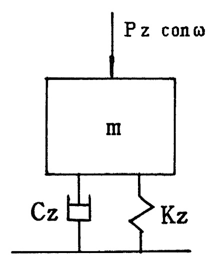

Fig.1 Mechanics Model of Foundation Vibration

$$m\ddot{Z} + C_z\dot{Z} + K_z Z = P_z \cos\omega t \qquad (1)$$

Where: m = the total mass of foundation and machine
P_z = maximum value of exciting force
ω = circle frequency of exciting force
C_z = vertical damping coefficient
K_z = vertical dynamic rigidity of base
$\dot{Z},\ddot{Z}$ = the first and second derivalive of displacement Z

It is well known that the amplitude A of vertical vibration corresponding to equation 1 is as follows:

$$A_z = \frac{P_z}{K_z\sqrt{\left(1-\frac{\omega^2}{\lambda_z^2}\right)^2 + 4D_z^2\frac{\omega^2}{\lambda_z^2}}} \qquad (2)$$

Where:

$$\lambda_z^2 = \frac{Kz}{m} \qquad (3)$$

$$Dz = \frac{Cz}{2\sqrt{m\,Kz}} \qquad (4)$$

The Dz is reffered as to vertical damping ratio When taking derivation of equation 2 to circle frequency and making it to be equal to zero, then the relation between resonance frequency and natural frequency is obtained:

$$\bar{\lambda}_z = \lambda_z\sqrt{1-2Dz} \qquad (5)$$

The vertical damping ratio and rigidity of base may be expressed by the resonance frequency and amplitude obtained:

$$Dz = \sqrt{\frac{1-\sqrt{1-\xi_z}}{2}} \qquad (6)$$

$$Kz = \frac{m\,\bar{\lambda}_z^2}{1-2D_z^2} \qquad (7)$$

Where:

$$\xi z = \frac{1}{1+B_z^2} \qquad (8)$$

$$Bz = \frac{Az,max}{Pz}\, m\,\bar{\lambda}_z^2 \qquad (9)$$

By analysing the test data of vertical vibration of four group foundations, using the formulas above, the variation of damping and rigidity is obtained as listed in Table I.

In order to more clearly show the amplitude response in the whole range of exciting frequencies, herein the Amplitude-Frequency-Curse corresponding to the first group foundations is cited as shown in Fig. 2. For embedded foundations the curve corresponding to the first group foundations with height 1.08 m is described in Fig. 3. In these figures the parameter $b=W/\gamma F^{1.5}$, and $\delta = hm/\sqrt{F}$ (in which W= the total weight of foundation and machine,γ =unit weight of base soil, hm= embedded depth of foundation and F= base area of foundation)

TABLE I. the Effect of Foundation Shape and Its Embedded Depth on Rigidity and Damping Ratio

Num.of Group	Size of Foundation (m)	hm (m)	Dz	Kz (t/m)
1	0.5 0.5 0.135	0.00	0.55	880.5
2	0.5 0.5 0.27	0.00	0.342	864.3
3	0.5 0.5 0.54	0.00	0.185	1432.3
4	0.5 0.5 0.81	0.00	0.098	1926.8
5	0.5 0.5 1.08	0.00	0.059	2329.9
5	0.5 0.5 1.08	0.27	0.130	2626.9
5	0.5 0.5 1.08	0.54	0.218	3098.7
5	0.5 0.5 1.08	0.81	0.229	3738.0
5	0.5 0.5 1.08	1.08	0.473	5454.2
6	0.354 0.707 0.135	0.00	0.531	842.3
7	0.354 0.707 0.27	0.00	0.367	970.5
8	0.354 0.707 0.54	0.00	0.192	1441.5
9	0.354 0.707 0.81	0.00	0.090	1873.6
10	0.354 0.707 1.08	0.00	0.070	2277.7
10	0.354 0.707 1.08	0.27	0.122	2582.8
10	0.354 0.707 1.08	0.54	0.195	2982.1
10	0.354 0.707 1.08	0.81	0.323	3633.3
10	0.354 0.707 1.08	1.08	0.410	4187.1
11	0.298 0.868 0.135	0.00	0.513	749.5
12	0.289 0.868 0.27	0.00	0.339	910.6
13	0.289 0.868 0.54	0.00	0.190	1848.0
14	0.289 0.868 0.81	0.00	0.104	1996.6
15	0.289 0.868 1.08	0.00	0.067	2477.9
15	0.289 0.868 1.08	0.27	0.144	2963.6
15	0.289 0.868 1.08	0.54	0.231	3275.3
15	0.289 0.868 1.08	0.81	0.382	4011.2
15	0.289 0.868 1.08	0.81	0.382	4011.2
15	0.289 0.868 1.08	1.08	0.455	4912.7
16	R=0.28	0.00	0.506	720.1
17	R=0.28	0.00	0.344	895.8
18	R=0.28	0.00	0.170	1372.7

It is clearly seen from Fig. 2 that the resonance frequencies are descreased with the increasing of parameter b. While the figure 3 shows that the resonance frequencies are increased with the increasing of embedded ratio of foundations. It should be shown that the resonance amplitudes are decreased with decreasing of b and increasing of δ.

The variation law of damping and rigidity with parameter b may be seen from Fig.4 and Fig.5 for exposed foundations. Furthermore, the variation of damping ratio may be well expressed as follows:

$$Dz = 0.24/b \qquad (10)$$

The expression 10 is represented in Fig. 4 by solid line.

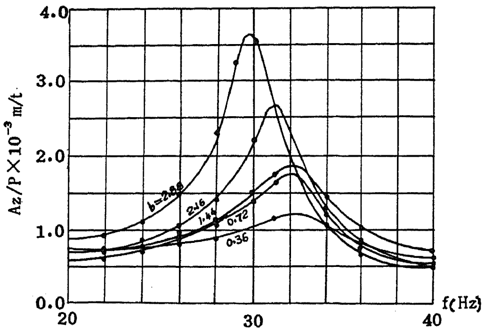

Fig.2 Amplitude-Frequency-Curve under Exposed Foundations and Vertical Vibration

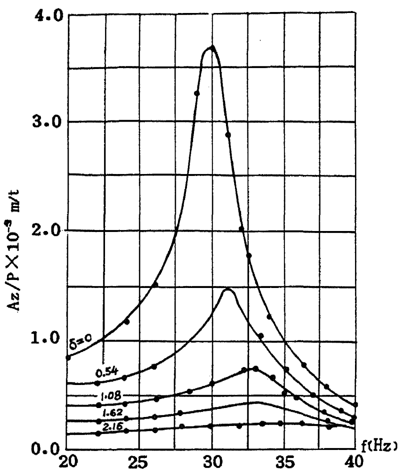

Fig.3 Amplitude-Frequency-Curve under Conditions of Embedded Foundation

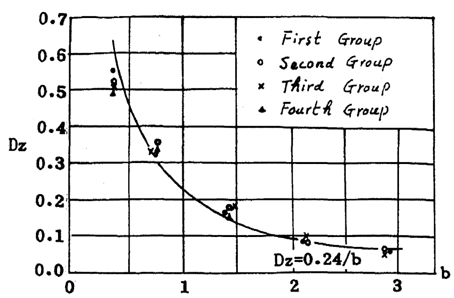

Fig.4 Relation between Vertical Damping Ratio and b under Exposed Foundation

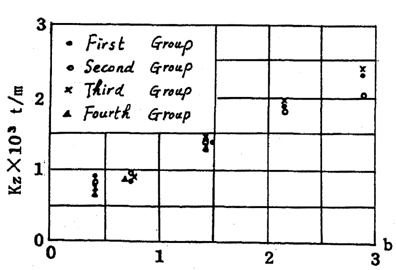

Fig.5 Relation between Vertical Rigidity and b under Exposed Foundation

In order to research the variation law of damping and rigidity of embedded foundation the following two parameters are adopted:

$$\alpha z = \frac{\lambda z}{\lambda zo} \qquad (11)$$

$$\beta z = \frac{Dz}{Dzo} \qquad (12)$$

Where: $\lambda zo, Dzo$ = natural frequency, damping ratio corresponding to exposed foundations

$\lambda z, Dz$ = natural frequency, damping ratio corresponding to embedded foundations

It follows from 18 test foundations that the increasing coefficients of frequency α_z and damping ratio β_z are apparently increased with increasing of embedded ratio of foundations. These test results are well described in Fig.6 and Fig.7 and can be expressed by following two formulas:

$$\alpha z = 0.14\sigma^{1.4} + 1 \qquad (13)$$

$$\beta z = 1.9\delta^{1.5} + 1 \qquad (14)$$

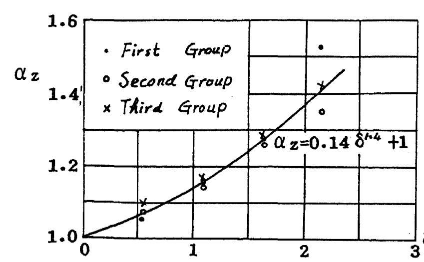

Fig.6 Relationship between Increasing Coefficient of Natural Frequency and Embedded Ratio of Foundation

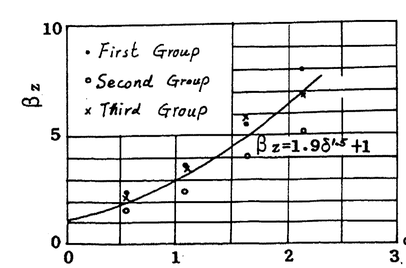

Fig.7 Varying Law of Increasing Coefficient of Vertical Damping Ratio with Parameter δ

By the analysing above mentioned, two main conclusions may be drawn:

1. The effect of mass ratio b on vertical rigidity and damping ratio of base should be considered.

2. When establishing the vertical rigidity, damping ratio, increasing coefficient of frequency and damping ratio, the base area of influence of base side ratio between length and width of foundation on these parameters may be ignored.

II. Vibrations Accompanied by Simultaneous Rotation and Sliding

If vibrations are caused by an exciting moment and horizontal exciting force, the equation of forced vibration of foundation will be written as follows:

$$M\ddot{X} + \phi S \dot{X} + S X = Px \sin\omega t$$

where:

$$M = \begin{bmatrix} m & 0 \\ 0 & J_m \end{bmatrix} \qquad S = \begin{bmatrix} Kx & -h\,Kx \\ -h\,Kx & K + h\,Kx \end{bmatrix}$$

$$X = \begin{Bmatrix} X \\ \varphi \end{Bmatrix} \qquad Px = \begin{Bmatrix} Px \\ M + Px\,h \end{Bmatrix}$$

inwhich: J_m = moment of inertia of the foundation mass with respect to rotation axis

K_x = horizontal rigidity of base, $k_x = C_x F$

K_φ = rotation rigidity of base, $K_\varphi = C_\varphi I$

C_x = Coefficient of elastic uniform shear of base

C_φ = coefficient of elastic nonuniform compression of base

I = moment of inertia of contact base area of foundation with respect to rotation axis

h_2 = distance between the center of gravity of the mass of vibration system and foundation base

h_3 = distance from horizontal force to the center of mass

$\dot{x}$, $\ddot{x}$ = the first and second derivitive of displacement x

$\dot{\varphi}$, $\ddot{\varphi}$ = the first and second derivitive of rotation angle φ

Φ = proportion coefficient of damping

It is well Known that the solution of forced vibration of equation 15 is summarised by the responses of the first and second vibration model. But in order to establish the dynamic parameters the effect of second vibration model on vibration response may be ignored because under the condition of smaller exciting frequency which does not exceed resonance frequency the influence of this model on amplitude of vibration is 5 % smaller than that considering the model. Thus, the problem of two freedom degree may be simplified into one [1] and the equation of motion of vibration system may be written as follows:

$$J_1\ddot{\phi} + C_1\dot{\phi} + K_1\phi = M_1 \sin\omega t \qquad (16)$$

Where:

$$J_1 = J_m + m\,\rho_1^2$$

$$C_1 = \Phi\,[K_\varphi + K_x(\rho_1 - h_2)^2]$$

$$K_1 = K_\varphi + K_x(\rho_1 - h_2)^2$$

$$M_1 = M + P_x(h_3 + \rho_1)$$

$$\rho_1 = \frac{h_2}{1 - \dfrac{\lambda_1^2}{\lambda_x^2}}$$

in which λ_1 is the first natural frequency of vibration system and ρ_1 is the rotation radius corresponding to the first vibration model. Thus, the solution of the equation 16 is entirely similar to the solution of equation 1 which is omitted herein. From equation 16 we may establish damping ratio $D_{x\varphi}$ of sliding-rotation vibration and rigidities K_x and K_φ. It should be pointed out that the following results of analysing are obtained under the assumption of $C_\varphi/C_x=2.86$. From this the rotation radius ρ_1 will be easily established.

In order to research the effect of shape of foundation on sliding-rotation vibration the parameter K is adopted. The K is equal to b/a, in which a is the side parallel to horizontal force and b is perpendicular to horizontal force.

To embedded foundation the variation of rigidity and damping is expressed, in the same way, by the increasing coefficient of frequency and damping ratio, that is:

$$\alpha_{x\varphi} = \frac{\lambda_1}{\lambda_{1,0}} \qquad (17)$$

$$\beta_{x\varphi} = \frac{D_{x\varphi}}{D_{x\varphi,0}} \qquad (18)$$

Where: $\lambda_{1,0}$, $\lambda_{x\varphi,0}$ = first natural frequency, damping ratio corresponding to $h_m = 0$

$\lambda_{1,1}$, $\lambda_{x\varphi}$ = first natural frequency, damping ratio corresponding to $h_m \neq 0$

In order to clarify the variation of amplitude of vibration with varying exciting frequency f (

in H), the Fig. 8 and Fig.9 are cited. The Fig.8 describes the relation of amplitude to mass ratio b and exciting frequency under the condition of $h_m = 0$. It follows from Fig.8 that the greater the mass ratio b, the larger the response of amplitude. The Fig.9 describes the relation of amplitude to embedded ratio δ of foundations and exciting frequencies f. It is seen from this figure that the greater the embedded ratio δ, the smaller the response of amplitude of vibration.

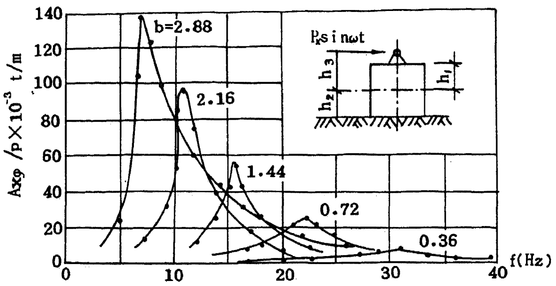

Fig.8 Horizontal Amplitude-Frequency-Curve under Different Value of b

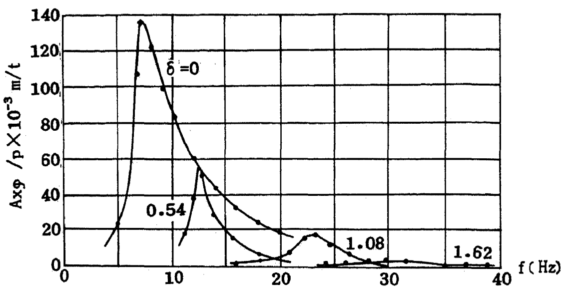

Fig.9 Horizontal Amplitude-Frequency-Curve under Different Value of δ

As for the variation of rigidity and damping with varying b under the condition of exposed foundations the Fig.10 and Fig.11 clearly describe it in detail.

Furthermore, the relationship between damping ratio $D_{x\varphi}$ and mass ratio b may be well expressed as follows:

$$Dx = 0.06/b \tag{19}$$

To embedded foundations the increasing coefficients of natural frequency and damping ratio are described in Fig.12 and Fig.13. Furthermore, these coefficients may be, with saficient accuracy, expressed as follows:

$$\alpha x_{\varphi}=\sqrt{K}\ \delta^{3.8}+1 \tag{20}$$

$$\beta x = 0.5\beta \quad +1 \tag{21}$$

These expressions are described in Fig.12 and Fig.13 in solid line.

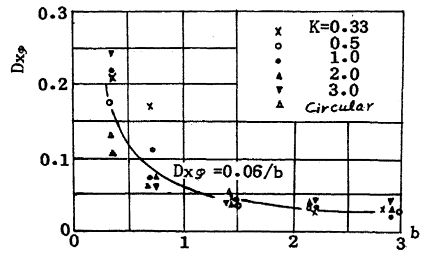

Fig.10 Relationship between Damping Ratio of Sliding-Rotation Vibration and Parameter b under Exposed Foundation

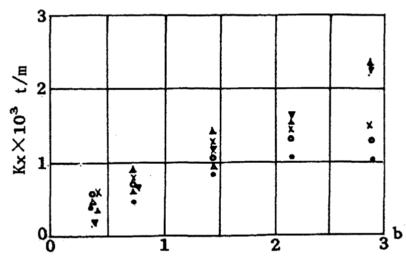
Fig.11 Relationship Between Horizontal Rigidity and Parameter b

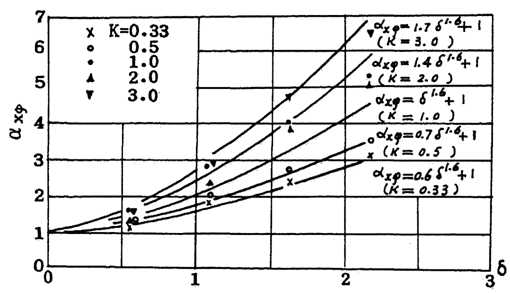

Fig.12 Variation of Increasing Coefficient of Natural Frequency with δ

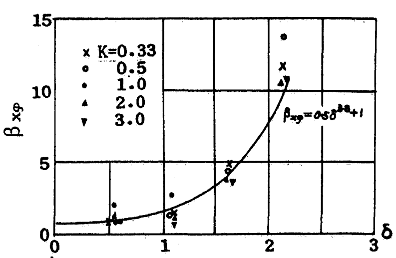

Fig.13 Variation of Increasing Coefficient of Damping Ratio with δ

Conclusions

According to the discussion above the following conclusions may be drawn:

1. Under the condition of exposed foundations the horizontal rigidity K_x, rotation rigidity K_φ and damping ratio $D_{x\varphi,o}$ are not related to the shape of foundation and its base side ratio basically.

2. Under the condition of embedded foundations the increasing coefficeent of damping ratio is not related to the shape and its base side ratio of foundations in principle.

3. To the embedded foundations the increasing coefficient of natural frequency is apparently related to the foundation shape and its base side ratio The greater the value K, the larger the increasing coefficient of natural frequency.

4. The horizontal and rotation rigidity of base are markedly related to mass ratio b. The greater the mass ratio b, the larger the rigidities.

Reference

[1] Wang Xikang, Wu Nanpeng, Jen Ton, and Wang Zongyin, on Analysis of Vibration of Embedded Footings, Research Report 7925, Central Research Inst. of Building and Construction of Metallurgical Ind., 1979