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ABSTRACT

A new technique using dimensional analysis (D.A.) is presented here to draw profile of phreatic line through an earthen dam. A universal equation is formulated using D.A. to get numerous points on a steady-state phreatic surface. The prediction of phreatic line using D.A. is then compared with those of conventional methods by Kozeny, A. Casagrande, Stello and also with centrifuge model test results and large scale prototype field results to evaluate applicability of this equation. It has been observed that D.A. can predict the results reasonably well. Thus Dimensional Analysis method can prove to be an easy and sufficiently accurate method to predict solutions to complex and multiparameter problems.

INTRODUCTION

Butterfield (1999) has explained the procedure for application of D.A. to civil engineering problems. Taking a clue from this work authors applied D.A. method to locate phreatic surface through homogeneous as well as zoned earthen dam. Various methods, graphical and analytical, developed by Kozeny, A. Casagrande, L. Casagrande, Van-Iterson, Dupuit, Pavlovsky, Stello etc. are available in literature to estimate the phreatic surface through an earthen dam. A trial and error procedure is also used to draw flow nets. This method is simple and useful in many types of application. However, it is cumbersome for problems involving unconfined flows in inhomogeneous soil (Chang 1988). The D.A. equation to draw phreatic line eliminates such trial and error method and graphical method. The phreatic line as determined by D.A. equation is then compared with the existing methods using various geometrical combinations, laboratory model test results and field observations.

BASIC FORMULATION OF D.A.

The aim of D.A. is to reduce to a minimum the dimension space in which the behavior of a specific system might be studied by combining and arranging systematically the assumed governing variables (v) = (v_1, v_2, v_3,……,vn) encompassing total of m independent primary dimensions (D)= (D_1, D_2, D_3,….., D_N) into N= (n-m) D.G.s, that are (π_1, π_2, π_3,……, π_N), N being less than v. Primary dimensions are expressed in mass (M), length (L), time (T), etc.

In case of homogeneous and non-homogeneous earthen dam, the variables used in the analysis are explained below with respect to figure (1).

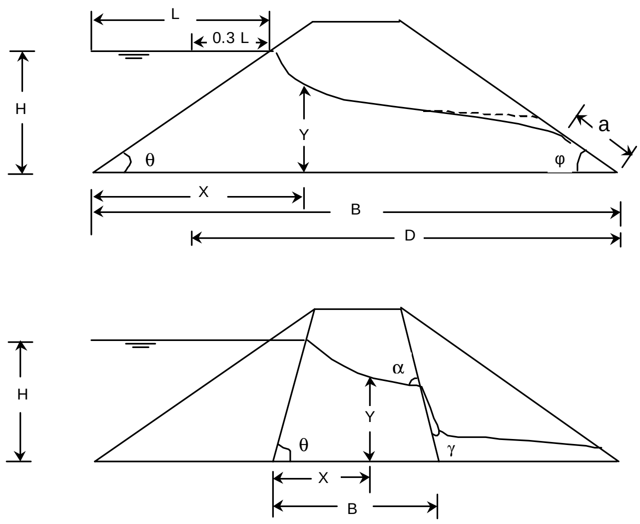


Fig. 1 Dam sections (Homogeneous and Non-Homogeneous)

Here H= height of water level (in meter)
B= bottom width of dam section in case of homogeneous dam and bottom width of core section in case of non-homogeneous dam section (in meter)

θ= Angle of upstream sloping portion in case of homogeneous dam section and angle of upstream face of core in case of non-homogeneous dam section (in degrees)
X = horizontal distance (in meter)
Y = corresponding vertical distance of point on phreatic line.
Here we have v = { H, B, θ, X, Y} hence n = 5
Writing the dimensions of the variables

{v} = $[M^0LT^0, M^0LT^0, 0, M^0LT^0, M^0LT^0]$, m = 1, i.e. no. of repeating variables forming a set Q, which can not be reduced further.
Thus no. of dimensionless Pi – group is
N = (n-m) = (5-1) = 4
now R = set of variables in v which have dimensions totally distinct from each other.
Here in v, the dimensions of all the variables are same hence; any one variable can be selected as R.
Therefore R = {B}
Q is to be selected from R, therefore Q = {B}
Isolated variables are {H, θ, X, Y}
Therefore the dimensionless groups are

$\pi_1 = \{B, H\}$ $\pi_2 = \{B, \theta\}$

$\pi_3 = \{B, X\}$ $\pi_4 = \{B, Y\}$

Therefore, represent

$$\pi_1 = (B^a, H^b) \quad (1)$$

Therefore in dimensional form

$$[M^0L^0T^0] = [MLT]^a \times [MLT]^b$$

Hence comparing the indices of LHS and RHS, for M, L, & T,
Now b = 1, then a = -1
Substituting the values in equation (1)

$$\pi_1 = (B^{-1} \times H) = H/B$$

Similarly $\pi_2 = \theta$

$\pi_3 = X/B$

$\pi_4 = Y/B$

Now, let $\pi_4 = \varphi(\pi_1, \pi_2, \pi_3)$, where φ is an unknown function.

Therefore, we can write

$$\pi_4 = \beta_1 (\pi_1)^{\beta 2} (\pi_2)^{\beta 3} (\pi_3)^{\beta 4} \quad (2)$$

Here a separate π term i.e. π_5 has been introduced and raised to the power of above equation to take into consideration the difference in the permeability of core and shell section in case of non-homogeneous dam section. Thus

$$\pi_5 = K^{\beta 5}$$

where K = Ks/Kc = (permeability of shell material) / (permeability of core material)
Therefore, equation (2) becomes,

$$\pi_4 = [\beta_1 (\pi_1)^{\beta 2} (\pi_2)^{\beta 3} (\pi_3)^{\beta 4}]^{\pi 5} \quad (3)$$

$$Y/B = [\beta_1 (H/B)^{\beta 2} (\theta)^{\beta 3} (X/B)^{\beta 4}]^{\pi 5} \quad (4)$$

To find the values of constants β_1, β_2, β_3, β_4 and β_5 a guiding point has been selected from a fictitious Non-homogeneous dam section having H=90m, B=220m, θ=45^0, K=5, X=330m, and Y=16m. The values of X and Y are selected by taking guidance from the Stello's method.
Substituting these values in equation (4) we get

$$0.0727 = [\beta_1 (0.4090\,\beta_2)(45\,\beta_3)(1.5\,\beta_4)]^{\pi 5} \quad (5)$$

In logarithmic form

$$-2.6214 = \pi_5 [\log\beta_1 - 0.8940\beta_2 + 3.8066\beta_3 + 0.4054\beta_4] \quad (6)$$

where $\pi_5 = K^{\beta 5} = 5^{\beta 5}$

Solving equation (5) and (6) by trial and error procedure we get
$\beta_1 = 6.0538$, $\beta_2 = 1.4$, $\beta_3 = -0.55$, $\beta_4 = -0.655$, and $\beta_5 = 0.2301$

Substituting above values in eq. (4), we get

$$Y = B\,[6.0538\, H^{1.4} \theta^{-0.55} B^{-0.745} X^{-0.655}]^{\pi 5} \quad (7)$$

where $\pi_5 = K^{0.2301}$

Using the equation (7) a reasonably accurate top seepage line can be plotted for homogeneous as well as non-homogeneous dam sections. The steps to be followed while drawing the seepage line using D.A. equation are explained as follows.

PHREATIC LINE FOR HOMOGENEOUS DAM SECTION

In case of homogeneous dam section, following steps shall be followed while drawing the phreatic line using D.A. equation.
Step 1
Start drawing the line using D.A. equation (7) from point of intersection of U/S slope and U/S water level (say A), considering K=1.
Step 2
Draw the line until it meets the D/S slope of the section.
Step 3
Apply the correction at exit point as suggested by Van-Iterson, i.e.

$$a = \frac{D}{\cos\phi} - \sqrt{\frac{D^2}{\cos^2\phi} - \frac{H^2}{\sin^2\phi}}$$

where a, D, H and ϕ are as shown in fig. 1
Step 4
At point A, phreatic line is improved & oriented to meet the physical requirements such that it meets the point of intersection of water level with U/S slope of dam section; i.e. the starting point of the phreatic line.

PHREATIC LINE FOR ZONED SECTION

Figure 2 shows non-homogeneous dam section with H=90m, B=220m, K=20, U/S slope of core (θ)= 1:1 i.e. 45^0, D/S slope of core = 1:1, D/S slope of shell = 2.5:1.
Step 1
In order to draw the phreatic line for this section start from point A (fig. 2). Calculate various values of Y for different

values of X and taking K =1, draw the line until it meets the point B; which is the intersection point of seepage line with D/S slope of core.

Step 2

To draw the further profile in shell portion take K=20 and start drawing seepage line from the D/S part of shell portion from any point say F on the phreatic line, which is found out using D.A. equation for any value of X within the shell portion.

Draw the line for various values of X towards the boundary of shell and core CC' until it meets the boundary CC' at point E.

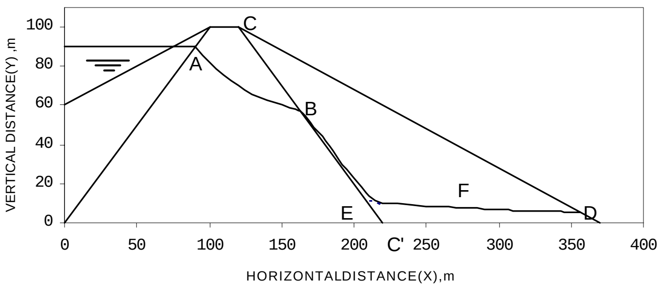


Fig. 2 Phreatic line using D.A. for Non-Homogeneous section

Step 3

There are some rules for entrance and exit condition of seepage line along the boundary CC'. Measure the angle of intersection of line in the core section with boundary CC' at point B which is α (fig. 1). Use equation $\tan \alpha / \tan \gamma = K$. Knowing all the parameters except γ, determine γ which is the angle of intersection of line in the shell portion with the boundary CC' or the exit angle as shown in fig. 1.

Step 4

Draw the straight line from the point B at an angle equal to γ with the boundary CC' and join this line by a smooth curve to the already drawn seepage line EF.

Step 5

Completely draw the seepage line near D/S part of shell, starting from point F until it intersects D/S slope of core at point D.

Step 6

At point A, phreatic line is improved & oriented to meet the physical requirements such that it meets the point of intersection of water level with U/S slope of core; i.e. the starting point of the phreatic line.

COMPARISON

The phreatic line drawn by D.A. for homogeneous and non-homogeneous dam sections is compared with conventional methods (Kozeny, A. Casagrande, & Stello) and also by Centrifuge model test results (Sutherland et al. 1984) and case study. In table 1and 2, the geometrical parameters of various dam sections used for comparison are shown. In fig. 3a-e, the seepage line for homogeneous section drawn using D.A. equation is compared with other conventional methods.

Table 1. Homogeneous dam sections

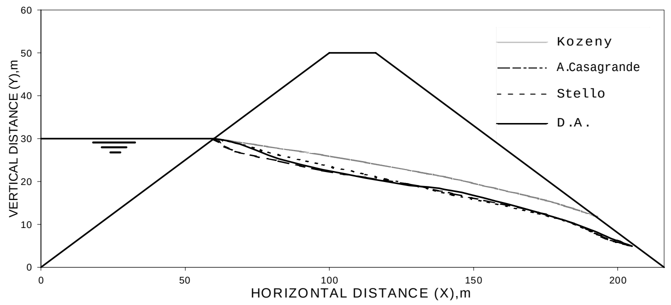
Fig. No.	U/S slope	D/S slope	Dam Ht. (HD)	Water level (H)	Base width	H/HD
3a	2:1	2:1	50	30	216	0.60
3b	2:1	1.5:1	60	25	229	0.41
3c	2.5:1	2.5:1	70	55	372	0.78
3d	3:1	3:1	70	60	442	0.85
3e	3.5:1	3.5:1	80	75	585	0.93
4	2:1	2:1	23	17.25	95	0.75
5	2:1	3.3:1	20.33	11.67	126	0.57
6	3:1	2.5:1	58	50	332	0.86

Table 2. Non-Homogeneous dam sections

Fig. No.	U/S slope (core)	D/S slope (core)	D/S slope (shell)	Dam Ht.	Water level	Base width	K
7a	1:1	1:1	2.5:1	100	90	220	20
7b	1:1	1:1	2.5:1	100	90	220	5
7c	1:1	1:1	2.5:1	100	50	220	5
7d	0.7:1	0.5:1	3.0:1	30	20	41	5
7e	0.7:1	0.5:1	3.0:1	30	20	41	15
8	0.6:1	0.6:1	2.5:1	47.35	39.93	60.62	CD

* All dimensions are in meter

CD = Chimney Drain



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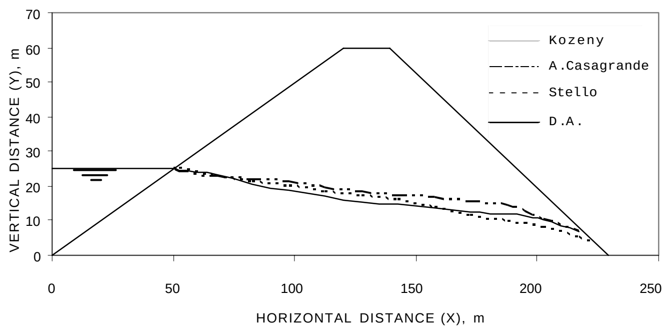

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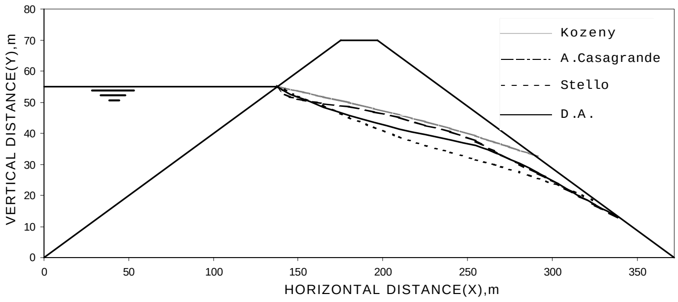

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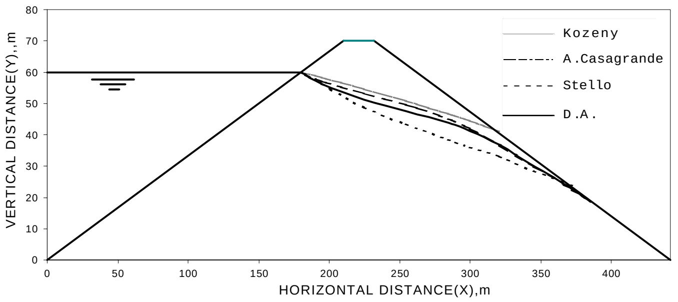

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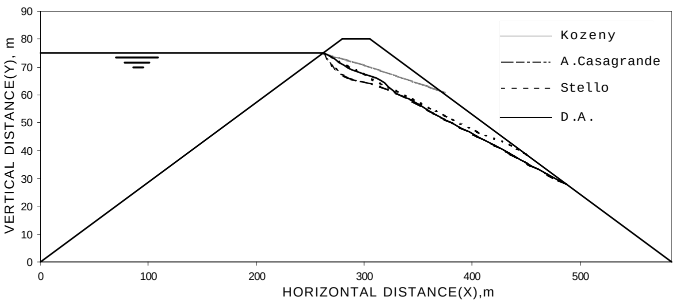


e)

Fig. 3 Homogeneous dam sections

Figure 4, 5 and 6 shows the comparison of D.A. with centrifuge model test results and actual phreatic line for Jumbo dam and Saluda dam respectively. The base of the dam section in fig. 5b and 6b is joined by a straight line for the convenience.

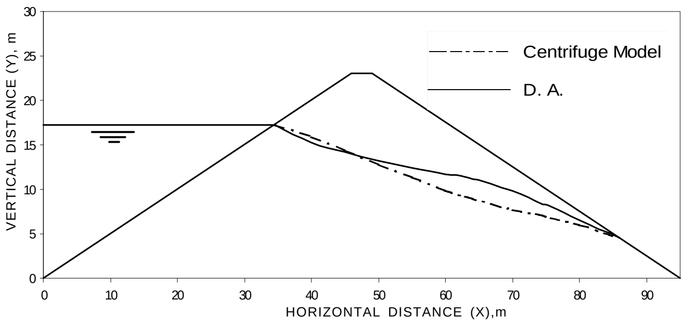
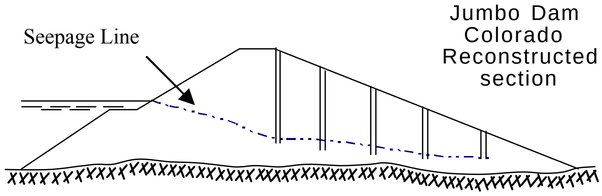


Fig. 4 Comparison of D.A. with Centrifuge model test results



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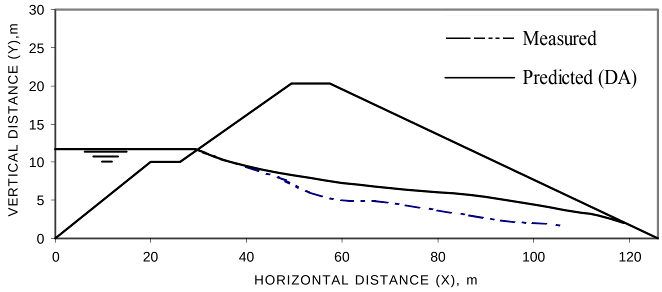
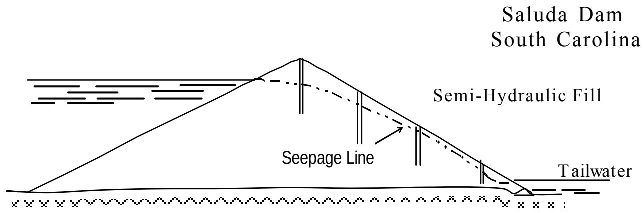


b)

Fig. 5 Comparison with Jumbo dam, Colorado



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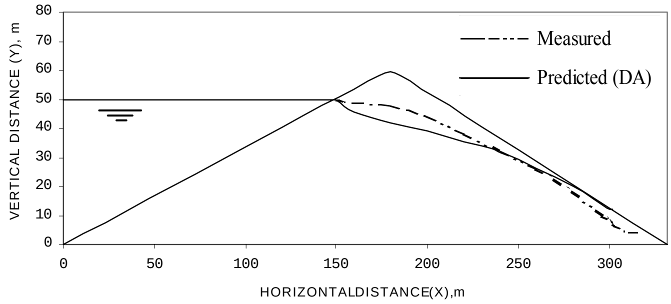
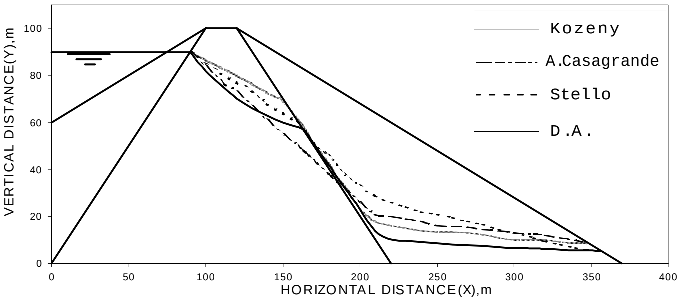


b)

Fig. 6 Comparison with Saluda dam, South Carolina

Figure 7a-e shows the comparison of seepage lines for non-homogeneous dam sections. For zoned section, in order to account for the effect of different soil permeability in core (Kc) & shell portion (Ks), the phreatic line has been drawn for different values of K ranging from 5 to 20. Since for K = 20, the top seepage line by all methods closely follows the base of the dam, the authors feel that there is no need to consider K > 20. Similarly for K < 5 there is not much difference in the profile & the dam section is as good as homogeneous section (Bharatsingh & Sharma,1976)



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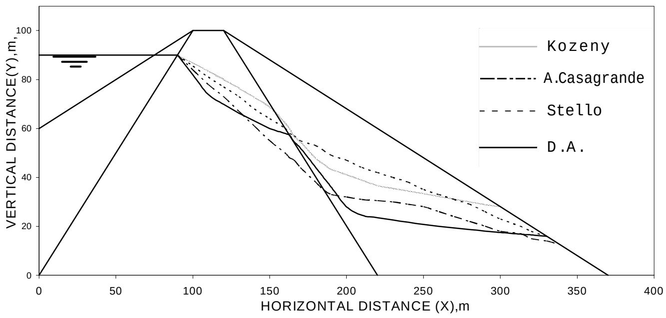

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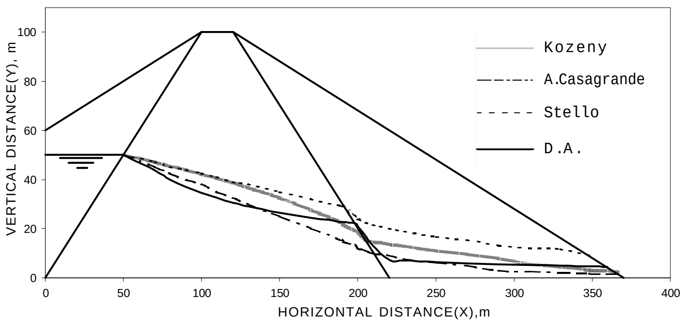
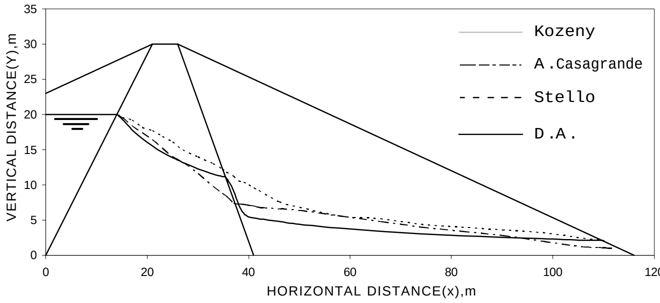


c)

A different conjectured dam section is selected as shown in fig. 7d with H=20m, B=41m, K=5, U/S slope of core (θ) = 0.7:1 i.e. 55^0, D/S slope of core = 0.5:1, D/S slope of shell = 3.5:1. The height of dam section here is only 30m compare to 100m of previous sections. The comparison of profile drawn by D.A. with the other methods indicates applicability of D.A. for geometrically different sections also.

Thus with such variation in H, K, B, & θ, the location of phreatic line for the zoned section using D.A. has been



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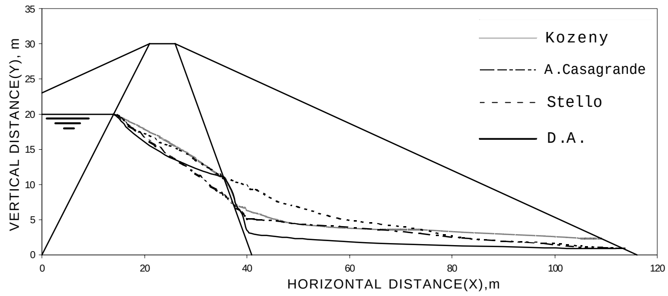

e)

Fig. 7 Non-Homogeneous dam sections

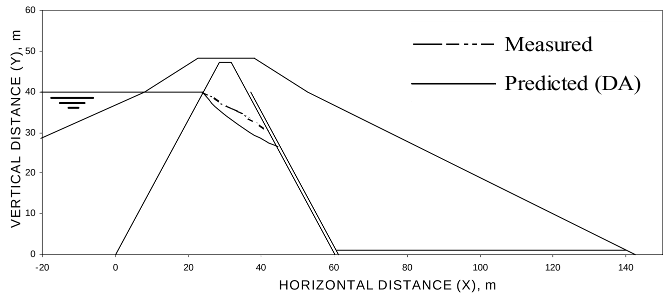

Fig. 8 Comparison with Camanche dam

verified. The dam sections and other parameters used for the comparison of zoned section have been summarized in table 2.

CONCLUSION

The comparison (Fig. 3a-e) reveals that the D.A. equation can be applied effectively to draw phreatic line for geometrically different homogeneous dam section. The phreatic line drawn using empirical correction (Van-Iterson correction) closely follows with the Centrifuge model test results (with configuration A and 90 g scaling) and actual field results of Jumbo dam and Saluda dam (Creager et al. 1944)

The comparison for non-homogeneous sections shows that the same D.A. equation can be effectively used for dam section with zones of different permeabilities. The line thus drawn usind D.A. equation is sensitive to variation in the soil permeability of different sections.

The time required to draw the phreatic line using above D.A. equation is very less and is less tedious compared to other conventional graphical methods.

The incorporation of various influencing parameters in the D.A. analysis can be done without much effort.

It is thus possible to formulate a D.A. equation for other problems with a prerequisite that the various parameters involved in the problem should be known either experimentally or assumed using conventional analysis, along with their inter relationship with each other.

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