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EFFECT OF THE SOIL SPATIAL VARIABILITY ON THE STATIC AND DYNAMIC STABILITY ANALYSIS OF A LEBANESE SLOPE

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ABSTRACT

The accidental topography and heterogeneous Lebanese geology in addition to the active seismicity have initiated the static and dynamic stability analysis of Lebanese slopes.

In this paper, the stability of a sandy Lebanese slope situated at Mansourieh near Beirut is investigated using deterministic and probabilistic approaches. The characterization of the variability of the slope soil properties is done based on geological investigation, as well as geophysical (Resistivity and Ambient noise) and geotechnical tests performed on this slope. Three dimensional 3D static deterministic analyses is performed to determine the overall safety factor of the slope and to find the location of the critical failure surface. The deterministic model is based on numerical simulations using the finite difference code FLAC3D. Then, two-dimensional probabilistic analysis is carried out on the critical section obtained from the 3D model. In the probabilistic analysis, the soil properties are modeled using the random field theory. An efficient uncertainty propagation methodology based on the expansion optimal linear estimation EOLE method is used to discretize the random field. Concerning the dynamic analysis, it is implemented in order to determine the amplification at the top of slope, where the looseness of the soil there may amplify the earthquake acceleration. The results have shown a small safety factor as well as high amplification. The importance of using the probabilistic approach versus the deterministic one is also presented and discussed.

INTRODUCTION

The accidental topography and heterogeneous Lebanese geology in addition to the active seismicity have initiated the static and dynamic stability analysis of Lebanese slopes. In Lebanon, this risk is particularly acute, and has been documented in scientific literature since the very first comprehensive geological studies of the country (Dubertret, 1945; 1968). In this small Mediterranean country, landslide risk is compounded by a combination of factors (Khawlie and Hassanain, 1979; 1984a,b). In addition to the country's steep topography, those factors are the Sand or Clay to Marly formations (Searle, 1972; Tavitian, 1974), the karstic conditions. Those factors are further amplified by its Mediterranean meteoric regime of heavy concentrated showers, thus resulting in more than three thousands recorded failures. Those instabilities are further exacerbated by improper land use, largely the result of incomplete of ill-adapted zoning rules and regulations.

Because of this combination of factors, the objective of research on this topic needs to be a comprehensive evaluation of Landslide Hazard Assessment and Risk Evaluation. Such work would use hazard zonation mapping of areas of landslide risks, and will focus on integrating GIS in the evaluation of slope stability. Such critical zones will be characterized based on their geotechnical, geophysical and geological soil properties. Then, an analysis will be carried out to evaluate the Static and dynamic stability analysis of the behavior of these slopes, using both deterministic and probabilistic methods.

The application of this case will provide lessons learned for the general context. The results of those two analyses will provide the data and information necessary to help define main critical issues that need to be addressed in integrating landslide hazards into the development and planning process.

METHODOLOGY OF THE ANALYSIS

The Analysis is carried out in three steps. First, the extent of the specific study site is delimited, and the soil properties carried out. Second, a Static Analysis is carried out to determine the slope's safety factor and identify any zones of concern. Third, a Dynamic Analysis is carried out, focusing on those zones of concern. The result of those analyses will then provide an detailed understanding of the current behavior of the slope, and thus serve as a basis for any forecasts on its future behavior, or inform any future analysis.

The determination of the site's extent is carried out based on geological investigation. A geotechnical investigation is then carried out to determine soil properties. However, since there are practical limitations to carrying out extensive testing across the slope, an analysis is carried out to determine those properties across the entire site while taking into account any uncertainties.

The Static Analysis is carried out through in two stages. First, a three dimensional (3D) static deterministic analysis is carried out. The deterministic model is based on numerical simulations using the finite difference code FLAC3D. Second, the probabilistic analysis is carried out the critical two-dimensional (2-D) section obtained from the 3D-model. In the probabilistic analysis, the soil properties are modeled using Random Field theory. An efficient uncertainty propagation methodology based on the Expansion Optimal Linear Estimation (EOLE) method (Sudret and Der Kiureghian, 2000) is used to discretize the random field.

After the Static Analysis has been completed, the Dynamic Analysis is carried out to determine, at the top of slope, any amplifications of seismic ground acceleration. This type of analysis is particularly critical in actively seismic regions such as Lebanon, where the topography changes on small scales, and thus creates many conditions where seismic accelerations can be amplified. Such "site-effects" therefore magnify, and the local scale, the effect of otherwise manageable earthquakes. In many sites, this happens at the top of the slope, where the looseness of the soil generally amplifies the earthquake acceleration, thus causing a significant reducing in the Safety Factor determined by Static Analysis.

LOCATION AND TOPOGRAPHY OF THE SITE

The site under consideration for the present application is located near Beirut, the Lebanese Capital. The slope is located in the Mansourieh suburb of Beirut, near 33°51'45" North and 35°33'48" East, and rises to an elevation that does not exceed 154 m Mean Sea Level (MSL). The slope is recorded to have already moved in 1994. While apparently stable today, its stability remains critical; it can be easily upset either by any seismic event, or by any ill-adapted human activity.

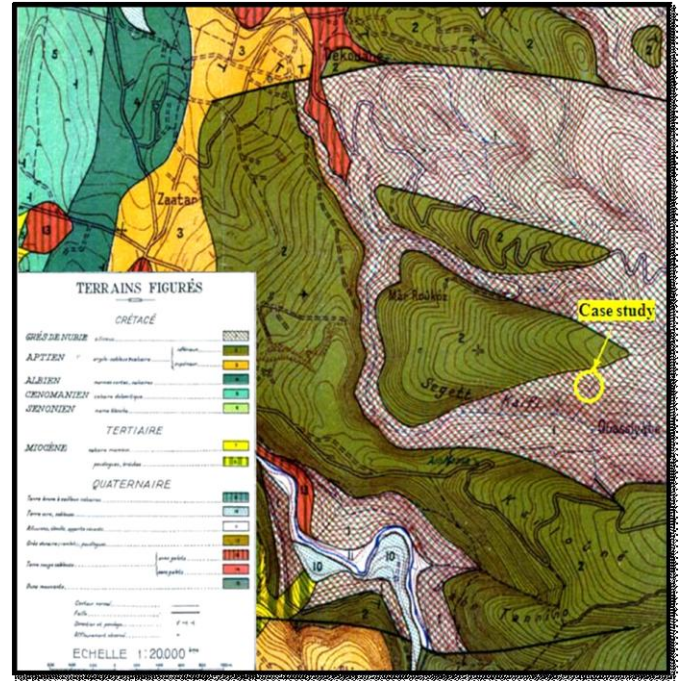


Fig.1 Geologic Map Showing Site Location.

The site under investigation extends over a rectangular area of about 500 m by 200 m. In spite of its documented instability, the site contains many inhabited structures located along the slope. The inhabitants report a comparatively higher perception of otherwise minor earthquakes, which appears to suggest a localized amplification effect.

GEOLOGY AND SOIL CHARACTERIZATION

The site is located in a Cretaceous formation, overlaying a Siliceous Nubian Sandstone formation ("Grès de Nubie Siliceux" – C1) by geologic maps (Dubertret, 1968), as well as the Lebanese "Centre National de Recherche Scientifiques" (CNRS).

Because of the size of the site, it was not practical to carry out extensive geotechnical investigation. It was decided to carry out a detailed investigation on a portion of the site, and to carry out statistical analysis to determine soil properties across the entire site. This approach was designed to address three types of uncertainty created by this specific case; the Epistemological Uncertainty that results from lack of information, the Random Uncertainty caused from the natural variability of the site, and the Computational Uncertainty relate to Modeling errors (Kulhawy, 1992). Our approach (Figure 2) was designed to address all those uncertainties.

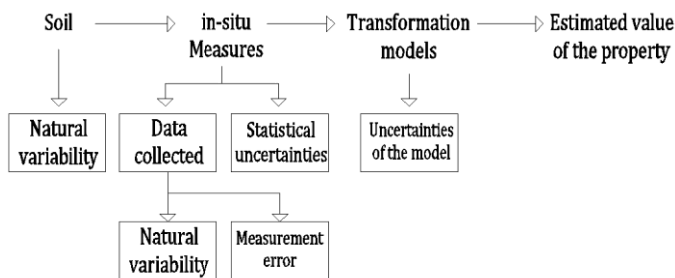


Fig.2. Types of Uncertainties in Geotechnical Engineering.

In determining soil properties across the slope, both Epistemological and Random uncertainty needed to be addressed. The geotechnical investigation included both In-Situ and Laboratory testing. In-Situ tests included Soil Electrical Resistivity Test (SRT) and Standard Penetration Tests (SPT). The Laboratory tests were Sieve/Gradation Analysis and Direct Shear Tests, which were carried out on selected soil samples.

Those tests were selected to investigate two aspects; first, to determine both soil properties in specific locations (SPT, Gradation, Direct Shear), and second, to determine the extent of variability of soil properties (SRT). This geotechnical investigation focused on providing a clear view of a cross-section of the site, as shown in Figure (3).



Fig.3. Site Overview, with location of Tests.

First, the geotechnical tests determined the physical properties of the soil along the slope and sections of the site. Along this cross section, the soil profile of the slope was determined to generally consist of clayey sand. There were layers of gravel inclusions (0m to 3m), and a relatively higher proportion of clays, consisting mostly of an expansive blue-gray clay between 6 m and 7.5 m. This clay is known locally as the “Roumieh Clay”.

Second, the Soil Resistivity Testing (SRT) confirmed the SPT sampling and the lab results of the direct shear tests, indicating that the soil was mostly a Sand / Clayey-Sand. The entire slope was determined to be located with the same geologic

layer, where the basic properties that affect electrical conductivity (porosity, shape of grain size, clay content) do not change significantly. For this reason, it is reasonable to assume that the engineering properties of the soil are similarly correlated as the electrical Resistivity, and their variability is therefore similar (4).

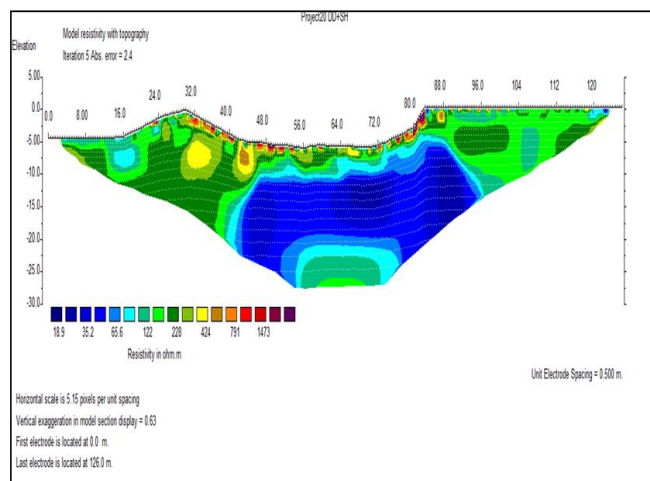


Fig.4. Soil Resistivity Test across the Slope.

In general, properties typically vary with the location of in-situ test and soil samples, because of both the spatial inherent variability in soils us and the uncertainties in the measurement process. Based on the results from the ERT, the variability of the soil physical properties was found to be variable in a manner that could be predicted by statistical distribution functions.

Modeling of Random Fields

In general, complex soil formation processes determines the extent of homogeneity of soil properties. Even in cases where soil properties are variable, when there are no drastic variations in soil types, sampling from adjacent locations in a relative homogeneous soil mass produce similar results with variation. This is the case of the slope under consideration, where the soil profile shows no significant inhomogeneities. In this case, once enough tests are carried out to determine soil properties, the range of variation of those properties can be effectively described by their correlation structure within the framework of random fields (Vanmarcke, 1983).

This correlation structure is defined by the scale of fluctuation, or “Autocorrelation Distance”. This is the main parameter necessary to representing the natural variability of soil properties. It is also a useful indicator of dependencies across both time and space. It can therefore be used to assess the distance required between sample points and that allows ensuring that the values are effectively uncorrelated. It has to be noted that, in order to ensure that the autocorrelation distance (θ) of a random field obtained for a certain soil type is meaningful and adequately reflects the soil variability, it is

necessary that the sampling distance (s) is such as: $s < \theta/2$ (Popescu 1995).

Data Analysis and Determination of Geotechnical Properties

The software toolkit in MATLAB was used to find the theoretical model for autocorrelation function that best fit the data. The statistical analysis of a random field, representing a soil property, is achieved through the determination of the mean, standard deviation, Probability Density Function, and the autocorrelation function of this field. The probability density function (PDF) of a sample data is estimated using least square method. This is done by using the distribution fitting tool in MATLAB to “fit” the sample Probability Density Function (PDF) to an existing PDF type that gives least values of error on its parameters. In general, the probability density function of data related to soil parameters follows a lognormal distribution (Phoon and Kulhawy; 1996, 1999).

In the slope considered, the range of variability of soil properties across and along the slope could therefore be represented by Log-Normal distribution, following standardized approaches for Sands and Clayey Sands (Cherubini et al., 1993; Lacasse and Nadim, 1996). The “fitting” process, using the least square method, gives two important outputs, the coefficient of determination (R^2) which provides a measure of how well new values are likely to be predicted by the fitting autocorrelation function and the parameters of the fitting autocorrelation function used to determine the Autocorrelation Distance of the field. A cosine decaying autocorrelation function was found for the soil properties with a vertical distance of autocorrelation equal to 2.5 m obtained from the SPT test and a horizontal one equal to 5m obtained from the Resistivity test. The resulting ranges of variation of soil physical properties are shown in Table 1.

Table 1 - Values of the Engineering Properties.

	Average	Standard Deviation	COV
C	4 kPa	2 kPa	50%
ϕ	30°	3°	10%

3D STATIC SLOPE STABILITY

The determination of the 3D-static Safety Factor was carried out in three steps. First, a digitized model of the slope’s terrain was obtained. Then, a 3-D Mesh was automatically generated by a custom-built software. Finally, the 3-D Mesh was used in a Finite Difference (FD) software to carry out the analysis.

First, the topographic map for the 10,000 m² site was obtained in a digitized format. The file was obtained from Google Earth as a KML file « Keyhole Markup Language », and was found to have adequate resolution for the scale of the slope considered.

Second, the topography had to be “rendered” in a format suitable for the FD analysis. This could not be done automatically, because both of the complexity of the topography, and of the need to reflect the distribution function of the soil’s physical properties. A purpose-built software was therefore developed to automatically divide the 10,000 m² area into a series of cubic elements, of 5m x 5m horizontal cross-section, and of varying height. The automatically-generated mesh was thus made up of 4,141 points that defined a 3-D cubic grid.

Third, the FD analysis was carried out on the discretized 3-D mesh. The Finite Difference (FD) software used was “FLAC3D” (Fast Lagrangian Analysis of Continua), which is based on a Lagrangian computation of a discretized mesh. One key advantage of FLAC3D is its internal option “FISH” that allows to define a specific failure mechanism.

The FD Analysis was carried out in such a way as to take into account the natural variability of soil properties, and to ascertain their effect on the slope’s Safety Factor. In order to study the effect of the variability of each parameter on the safety factor, different “runs” were made with FLAC3D, each time varying one parameter across the slope, and “seeding” the cells with the values derived from the autocorrelation function (Figure 5).

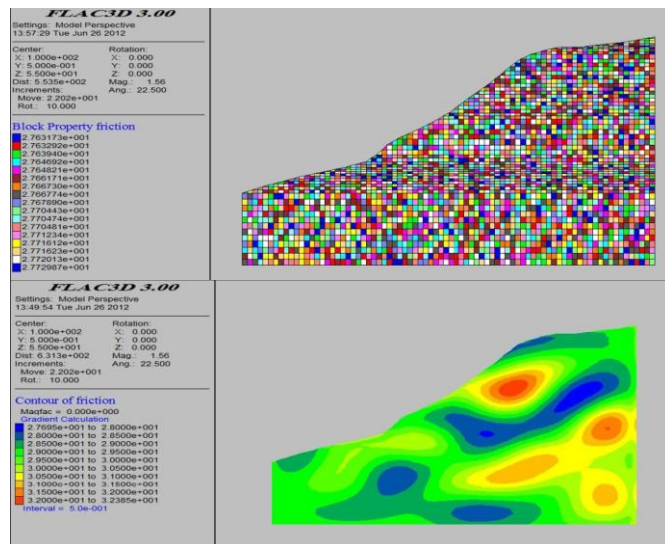


Fig.5. Variation of the Friction Angle ϕ Across a Section of the Slope in the First “Run”.

It was found that the variation of soil parameters (C, ϕ) had an important effect of the Safety Factor (SF). Specifically, it was the friction angle of the soil (ϕ) that proved to be the most determining parameter in this case. In addition, by comparing the results of this approach to a “classical” deterministic analysis (where the safety factor was found equal to 1.1), it was shown that the Safety Factor obtained by this probabilistic approach is lower, thus better explaining the reported recent

behavior of the slope (Figure 6).

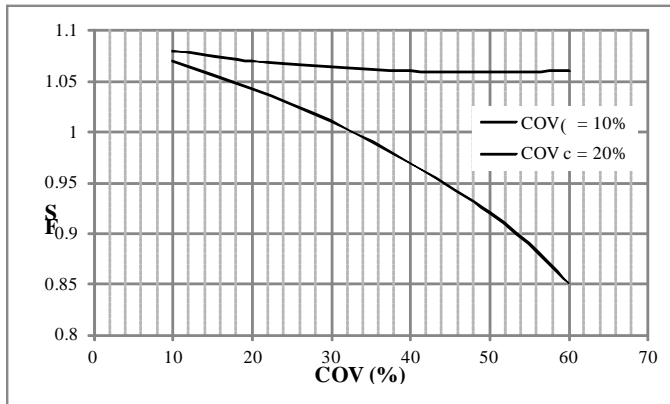


Fig.6. Safety Factor as function of Soil Properties.

The final results thus obtained by FLAC3D showed a Factor of Safety of 1.1 in some critical zones, thus validating the general perception that the slope is unstable (Figure 7).

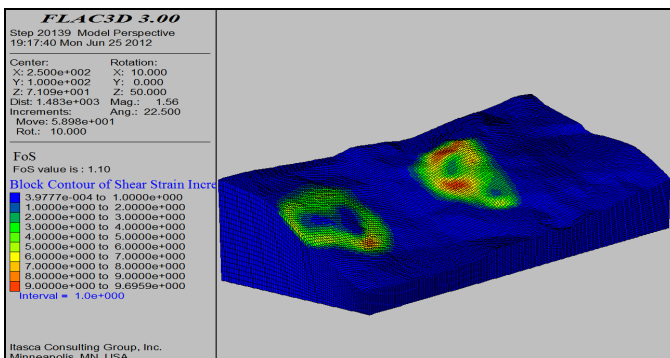


Fig.7. Results of the 3-D Static Analysis.

In addition to computing the Safety Factor, the computational procedure allowed us to identify the areas in the slope that were relatively weaker. Those were the zones where the Shear Stresses were relatively much higher. Relatively weaker zones could be easily revealed by plotting the slope in 3D, and color-coding the different blocks to show the “Shear Strength Increment” across the slope. This showed not only one, but two critical zones, C1 and C2.

The first zone, C1, corresponds to the location where the 1994 slope stability failure took place. At the bottom is now located a newly constructed popular restaurant.

The second zone, C2, has not moved yet. It is located just beneath apartment blocs often rented out to student of the engineering school of Université Saint Joseph.

DYNAMIC ANALYSIS

The low safety factor is particularly critical in seismically

active areas such as Lebanon. This is mainly because of the country’s frequent earthquakes, which have two effects on such slopes. First, when an earthquake occurs, the resulting ground acceleration will lead to temporally higher shear stresses across the slope, thereby increasing instability for a long enough period to cause failure. The second effect is a location-specific magnification of earthquake ground accelerations. Such “Site Effects” are due to the country’s highly variable topography and sub-soil geology, which modifies a seismic signal’s spectral, spatial and temporal properties. In general, the “Site Effect” can either magnify or dampen the seismic ground acceleration.

In the case under consideration, it appears that the “Site Effect” magnifies the seismic signal. Indeed, inhabitants in and near the site generally report a relatively higher perception of seismic events, even of the most minor events that go unnoticed in other nearby areas of Mansourieh. This appears to indicate that, in the case of the Mar-Roukoz slope, the “Site Effect” is a critical factor that needs to be specifically investigated, especially since the static analysis showed such a low safety factor.

In order to do this, the Dynamic Analysis was carried out in four steps; (1) selection of relevant locations, or points, where the Site Effect is likely to be the most critical, (2) numerical modeling and dynamic analysis to determine the “recorded” accelerations at those specific points, (3) development of a site-specific response spectra. Then, (4) the slope’s dynamic behavior can be analyzed.

Selection of Relevant Points

First, we used the same topography as in the static analysis, and focused our analysis around the two critical zones. This was done by specifying points (Table 2) located around the two critical locations, as previously identified by the 3D Static Analysis (Figure 8).

Table 2 - coordinates of points chosen (UTM).

Points	Easting	Northing	Altitude
A	737025	3749920	150.45
B	737155	3749955	171.42
C	737215	3749995	211.75
ZONE	737270	3750020	225.23

The Dynamic Analysis can then focus on sections defined along those points, in such a way as to characterize the adverse Site Effects around the critical zones. The specific amplification is determined at those selected points, and a response spectrum developed.

Numerical Modeling and Dynamic Analysis

For the purposes of the Dynamic Analysis, it was necessary to

define the initial parameters of the problem, and to select the relevant seismic signal.



Fig.8. Points Selected for the Evaluation of Seismic Amplification.

The initial parameters of the problem were both the types of boundary conditions and the damping ratio. On the lateral limits of the site, “Free Field” absorbing boundary conditions were applied. The damping ratio takes into account the partial dissipation of vibration energy due to internal frictions in natural dynamic systems. In this case, it was taken as 5%; this is the upper limit for geological materials, where the range is 2% to 5%, and is a reasonable for the slope’s specific geologic context.

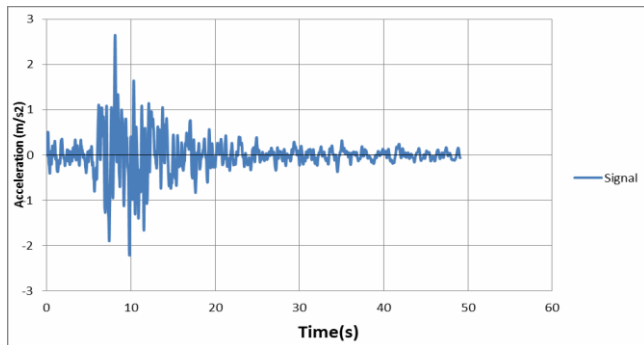


Fig.9. Relevant Seismic Signal as Registered at the Bhannes Seismic Observatory.

Because local inhabitants often report relatively higher perception of earthquake events, it was decided to select a signal that corresponded to the relevant event. This was the case of the Tyre earthquake, which locals inhabitants perceived relatively highly, in spite of the fact that it had occurred far to the South, 16 km from the antique city of the Tyre, on the Lebanese coast. The signal was therefore selected from the records of Lebanon’s Seismic Observatory in Bhannes (33.35 N; 35.36 E), itself located 60 km far from the epicenter of the relevant earthquake (Figure 9).

In our analysis, the seismic signal is applied at the base of the modeled slope as acceleration. The signal was amplified 100

times to correspond to a ground acceleration equivalent to 25% earth gravity, which is the currently required design parameter specified by the Lebanese construction code.

Site-Specific Response Spectra

The Site Effect was determined by measuring a ratio, at specified points, between two maximum horizontal accelerations; the acceleration of the top of the slope (A_{max}), and the acceleration of the underlying Nubian Sandstone rock formation (A_{Rmax}), which is the direct result of the seismic event (Figure 10).

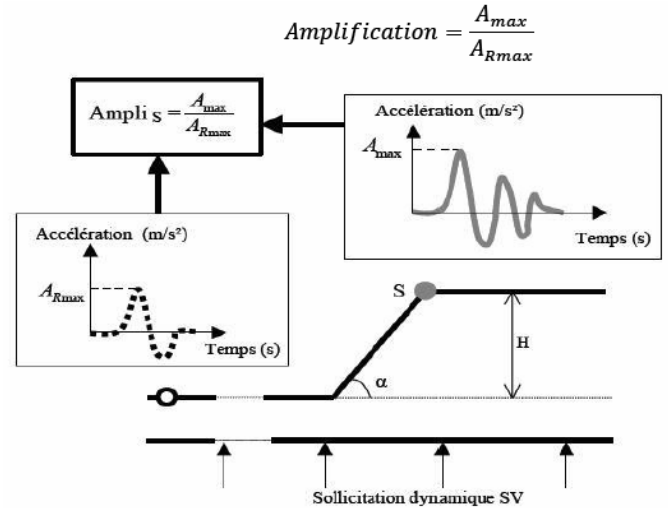


Fig.10. Determination of the Amplification due to Site Effect.

The amplifications were thus determined at each of the selected points. For each one of those points, the seismic signal was derived from the recorded signal. For each one of those seismic signals, the extreme values (minimum and maximum) were measured from the adjusted plot, the Absolute Maximum determined, and the Amplification was thus computed (Table 3).

Table 3 – Computed Amplifications at the Selected Points.

Points	Max	Min	Absolute Max	Amplification
Source	2.6487	-2.21239	2.6487	1.00
A	2.988	-4.08	4.08	1.55
B	1.27	-2.185	2.185	0.83
C	2.092	-2.01	2.01	0.76
ZONE	2.161	-3.015	3.015	1.15

From the response spectra developed for each point, an “envelope” of response spectra could then be determined for the entire site. The greatest amplification was found to correspond to a range of frequency between 0.3 Hertz and 2.2 Hertz. The configuration of the site was found to amplify the seismic acceleration by a factor of 1.7.

To facilitate structural analysis, an “envelope” of response spectra can be developed. This will help analyze zones that are similarly structurally active. In the case of the most critical zones, the currently inhabited “Foyer” and “Zone” sites, where an “envelope” of spectra was developed that also incorporates the “Source” signal (Figure 11).

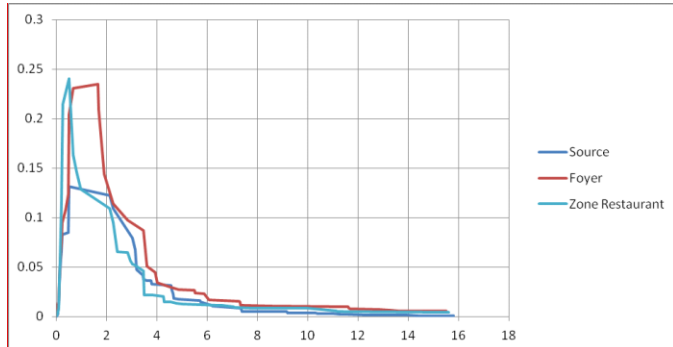


Fig.11. Envelope of Response Spectra.

CONCLUSION

The current case illustrates well the need for an evolution in the approach to slope stability problems, particularly those issues concerning large scale slopes. Such an evolution needs to lead to “micro-zoning”, where land utilization parameters are merely defined by economic concerns, but also includes engineering design at its core.

Such “micro-zoning” will need to incorporate three key approaches; the use of probabilistic methods of investigation, the development of site-specific seismic Response Spectra.

Probabilistic methods of investigation have two core advantages over traditional methods. First, they better take into account the natural variability of soil profiles. This may not be a critical factor on smaller scales, but it is a determining factor on medium to large scales where it extensive geotechnical testing may not a practical option. This was indeed illustrated in the current case, where the factor of safety was shown to significantly change with variations in soil parameters that are similar to those that occur naturally.

The importance of locally adapted response spectra has also been demonstrated. In this case, Site Effects caused ground accelerations to be significantly higher than that caused by the initial earthquake. The acceleration also varied significantly across the large scale of the slope. The present analysis showed that relatively small changes in location will have a determining impact on the structural viability of constructions, something that was only a concern on much large scales.

In today’s context of rapid development and urban expansion, one cannot afford to wait for problems to happen in order to

address them. It is very possible that, at the scale of such slopes, local solutions may have detrimental large scale effects. For this reason, a boarder image is needed. This can be done by ensuring that approaches need to be integrated with Geographic Information Systems to provide a better guideline for development.

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