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Hoyoung Seo
Purdue University, West Lafayette, IN

Monica Prezzi
Purdue University, West Lafayette, IN

Rodrigo Salgado
Purdue University, West Lafayette, IN

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SETTLEMENT ANALYSIS OF AXIALLY LOADED PILES

Hoyoung Seo
Purdue University
West Lafayette, IN-USA 47906

Monica Prezzi
Purdue University
West Lafayette, IN-USA 47906

Rodrigo Salgado
Purdue University
West Lafayette, IN-USA 47906

ABSTRACT

In pile design, settlement controls the design in most cases because, by the time a pile has failed in terms of bearing capacity, it is very likely that serviceability will have already been compromised. This notwithstanding, pile foundations are often designed based on the calculations of ultimate resistances reduced by factors of safety. This is in part due to the lack of accessible realistic analyses for estimation of settlement, especially for piles installed in layered soil. This paper presents a new settlement analysis method for axially loaded piles in multilayered soil and analyzes two case histories for which load tests were performed on nondisplacement piles. The analysis follows from the solution of the differential equations governing the displacements of the pile-soil system obtained using variational principles. The input parameters needed for the analysis are only the pile geometry and the elastic constants of the soil and pile. A user-friendly spreadsheet program (ALPAXL) was developed to facilitate the use of the analysis.

INTRODUCTION

Pile design has traditionally relied on calculations of ultimate resistances reduced by factors of safety that would indirectly prevent settlement-based limit states. While pile design is often done without explicit settlement checks, analyses that can accurately calculate settlement for a given load will offer opportunities for more cost-effective design in the future. In this paper, we will examine the analytical basis for calculating the pile head settlement of nondisplacement piles subjected to axial loads.

Available settlement analyses either assume that the soil resistance can be represented by a series of disjointed springs (the spring stiffness is determined through theoretical, experimental or empirical means) or that the soil is a continuum. The approach with springs (Seed and Reese 1957; Coyle and Reese 1966; Murff 1975; Randolph and Wroth 1978; Kraft et al. 1981; Armaleh and Desai 1987; Kodikara and Johnston 1994; Motta 1994; Guo and Randolph 1997; Guo 2000) has the advantage that approximate analytical or simple numerical solutions of pile settlement can be obtained (Randolph and Wroth 1978; Armaleh and Desai 1987; Motta 1994). The continuum approach has traditionally required expensive numerical techniques, such as the boundary integral method, the finite layer method or the finite element method, to obtain solutions (Poulos and Davis 1968, Mattes and Poulos 1969, Butterfield and Banerjee 1971, Poulos 1979, Rajapakse 1990, Lee and Small 1991).

Efforts were made over the last decade to solve the problem of axially loaded piles in multilayered soil with mathematical rigor. Vallabhan and Mustafa (1996), based on the principle of minimum total potential energy, proposed a simple closed-form solution for an axially loaded, nondisplacement pile installed in a two-layer elastic soil medium. Lee and Xiao (1999) expanded the solution of Vallabhan and Mustafa (1996) to multilayered soil and presented semi-analytical solutions. Recently, Seo and Prezzi (2007) obtained explicit analytical elastic solutions for a pile in multilayered soil. The advantage of this continuum-based analysis is that it captures the three-dimensional nature of the pile-soil interaction and produces the pile load-settlement response in seconds; this analysis is detailed next.

ANALYSIS

Problem definition and basic assumptions

The analysis considers a single circular pile embedded vertically into a multilayered elastic soil deposit (Fig. 1). There are altogether N discrete soil layers, and the bottom (base) of the pile rests at the interface of the m^{th} and $(m+1)^{th}$ layer ($m < N$). The pile has a length L_p with a diameter B ($=2r_p$, where r_p is the pile radius) and is subjected to an axial load Q_t at the pile head, which is flush with the ground surface. A cylindrical coordinate system (r-θ-z) is used. The z axis coincides with the pile axis, and the positive z direction points downward.

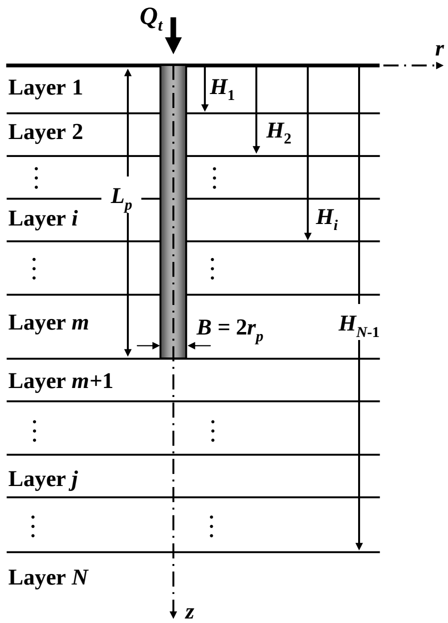


Fig. 1. Geometry of the pile-soil system

H_i denotes the vertical distance from the ground surface to the bottom of any soil layer i (the subscript i denotes the i^{th} layer); thus, the thickness of layer i is given by $H_i - H_{i-1}$ with $H_0 = 0$. All soil layers extend to infinity in the horizontal direction, and the bottom (N^{th}) layer extends to infinity downward in the vertical direction. The soil medium is assumed to be elastic and isotropic, homogeneous within each layer, with elastic properties described by Lame's constants λ_{si} and G_{si}. The pile is assumed to behave as an elastic column (i.e., an elastic axial compression element) with Young's modulus E_p. There is no slippage or separation between the pile and the surrounding soil or between the soil layers. The horizontal soil displacements in the soil mass due to the axial load Q_t are neglected in the analysis because, in general, these are very small compared with the vertical soil displacements.

Soil displacement

The vertical displacement u_z at any point within the soil mass is assumed to be a product of two separable variables and is represented as follows:

$$u_z(r,z) = \phi(r)w(z) \tag{1}$$

where w(z) is the axial pile displacement function, and $\phi(r)$ is a dimensionless soil displacement decay function varying along r; this function describes the decrease in the soil displacement with increasing horizontal distance from the pile axis. The $\phi(r)$ is assumed to be equal to one at the pile-soil interface ($r = r_p$). This ensures proper pile-soil contact. Furthermore, the displacements in the soil must vanish at infinite horizontal distances from the pile; therefore, $\phi(r)$ is assumed to be zero at $r = \infty$.

Principle of minimum potential energy

With the assumed displacement fields of Eq. (1), strains are calculated and subsequently related to stresses using elasticity theory. The soil potential energy density is expressed in terms of the elastic constants and strains. Since the strains can be expressed in terms of the displacement functions w(z) and $\phi(r)$, the expression of the potential energy Π contains these functions (Seo and Prezzi 2007). Applying the principle of minimum potential energy (according to which the first variation $\delta\Pi$ of the potential energy is equal to 0 at equilibrium) yields the governing differential equations.

Soil displacement decay function

The governing differential equation for the soil displacement decay function is obtained by taking the variation of ϕ and then equating the coefficients of these variations to zero:

$$\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} - \left(\frac{\gamma_r}{r}\right)^2 \phi = 0 \tag{2}$$

where

$$\frac{\gamma_r}{r_p} = \sqrt{\frac{n_s}{m_s}} \tag{3}$$

and m_s and n_s are given by:

$$m_s = \sum_{i=1}^{N} G_{si} \int_{H_{i-1}}^{H_i} w_i^2 dz \tag{4}$$

$$n_s = \sum_{i=1}^{N} (\lambda_{si} + 2G_{si}) \int_{H_{i-1}}^{H_i} \left(\frac{dw_i}{dz}\right)^2 dz \tag{5}$$

Solution of the differential equation of ϕ gives (Seo and Prezzi 2007):

$$\phi(r) = \begin{cases} \dfrac{K_0\left(\dfrac{\gamma_r}{r_p} r\right)}{K_0(\gamma_r)} & \text{for } r_p \le r < \infty \\ 1 & \text{for } 0 \le r \le r_p \end{cases} \tag{6}$$

where $K_0(\cdot)$ is the modified Bessel function of the second kind of zero order.

Pile displacement

The form of the differential equation governing pile displacement w(z) is given as follows:

$$-(E_iA_i + 2t_i)\frac{d^2w_i}{dz^2} + k_iw_i = 0 \quad (7)$$

where $A_i = A_p$; $E_i = E_p$ for i = 1… m; $E_i = \lambda_{si} + 2G_{si}$ for i = m+1… N; and

$$k_i = \pi G_{si}\frac{\left[K_1(\gamma_r) + \gamma_r K_0(\gamma_r)\right]^2 - (\gamma_r^2 + 1)\left[K_1(\gamma_r)\right]^2}{\left[K_0(\gamma_r)\right]^2} \quad (8)$$

$$t_i = \frac{1}{2}\pi r_p^2(\lambda_{si} + 2G_{si})\frac{\left[K_1(\gamma_r)\right]^2 - \left[K_0(\gamma_r)\right]^2}{\left[K_0(\gamma_r)\right]^2} \quad (9)$$

where $K_1(\cdot)$ is the modified Bessel function of the second kind of first order. The constants k_i and t_i represent the shear and compressive resistances offered by the soil mass against pile settlement. The general solution of Eq. (7), which is a second-order linear differential equation, is given by:

$$w_i(z) = B_i e^{\zeta_i z} + C_i e^{-\zeta_i z} \quad (10)$$

where $\zeta_i = [k_i/(E_iA_i+2t_i)]^{0.5}$ and B_i and C_i are integration constants. The axial force $Q_i(z)$ in the pile shaft at a depth z in the i^{th} layer is obtained from:

$$Q_i(z) = -(E_iA_i + 2t_i)\frac{dw_i}{dz} = -a_iB_ie^{\zeta_i z} + a_iC_ie^{-\zeta_i z} \quad (11)$$

where $a_i = \zeta_i(E_iA_i+2t_i) = [k_i(E_iA_i+2t_i)]^{0.5}$. The integration constants B_i and C_i can be determined analytically from the boundary conditions (Seo and Prezzi 2007).

Modification of soil moduli

One of the assumptions of the analysis described above is that there is zero horizontal displacement in the soil. Therefore, because of this assumption, the analysis predicts a stiffer pile response than is expected in reality. In Eqs. (5) and (9), the term $(\lambda_{si} + 2G_{si})$ represents the soil constrained modulus, which approaches infinity as the soil Poisson's ratio approaches 0.5. This creates an artificial stiffness that can be eliminated by using a modified value for the shear modulus (a similar procedure was proposed by Randolph (1981) for laterally loaded piles and Basu et al. (2007) for axially loaded piles with rectangular cross section). First, we make $\lambda_{si} = E_{si}\nu_{si}/[(1+\nu_{si})(1-2\nu_{si})]$, where E_{si} is the soil Young's modulus of the i^{th} layer, equal to zero (which is equivalent to making the soil Poisson's ratio $\nu_{si} = 0$). We then replace G_{si} by a modified shear modulus G_{si}^* in the analysis. Finally, we match the results for the pile response obtained from our analysis with those obtained from finite element analysis (FEA) performed for identical pile and soil conditions using ABAQUS. The following expression for G_{si}^*:results:

$$G_{si}^* = 0.75G_{si}\left(1 + 1.25\nu_{si}^2\right) \quad (12)$$

Iterative solution scheme

The γ_r parameter in Eqs. (8) and (9), which depends on the pile settlement w and its derivative dw/dz (Eqs. (4) and (5)), must be determined before we calculate the parameters k_i and t_i, which, in turn, are needed in the solution of Eq. (7) for the pile displacement. Hence, an iterative solution scheme is required. In the first iteration, an initial value is assumed for γ_r, and the pile displacement and its derivative (obtained from the axial force) are calculated. At the end of the iteration, a new γ_r value is obtained using the calculated pile displacement and the values of its derivative; the calculated value of γ_r is compared with the assumed initial value. If the difference is greater than the prescribed tolerance, iterations are continued, with the calculated value of γ_r taken as the new input in the calculations. Successive iterations are continued until the value of γ_r obtained from two consecutive iterations falls below the prescribed limit. This iterative solution scheme is provided in the form of a flow chart in Fig. 2.

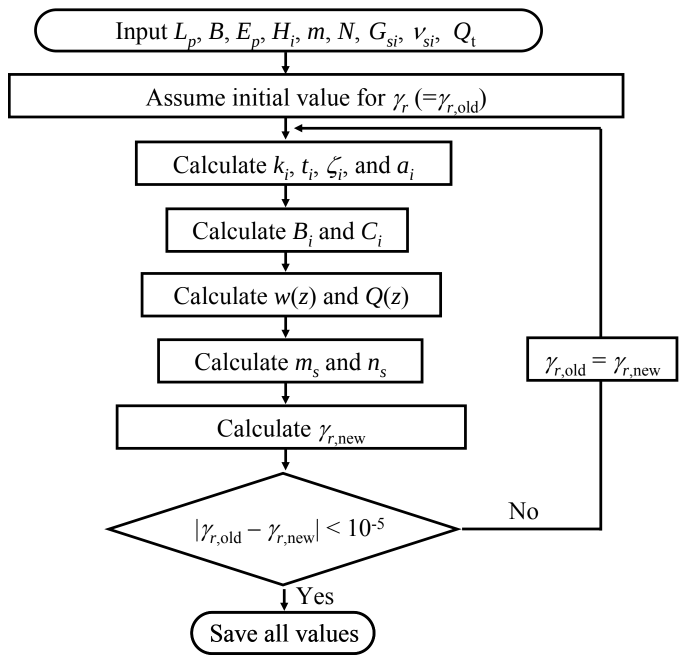
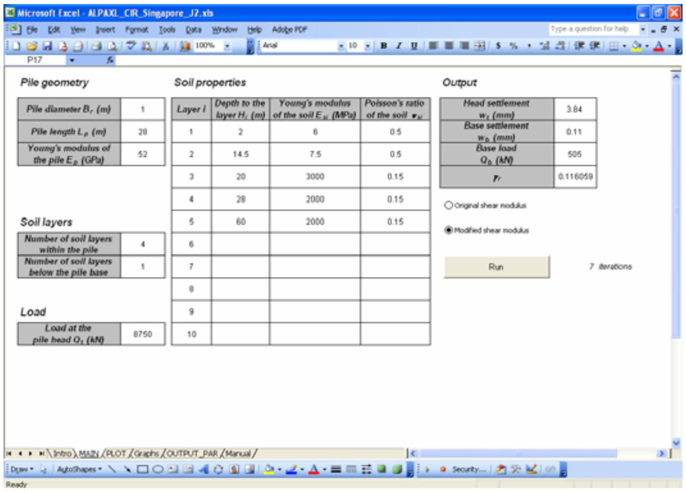


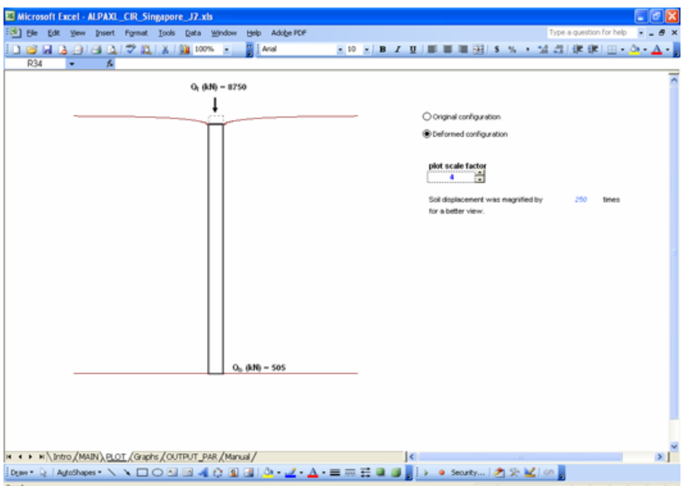
Fig. 2. Flowchart for the iterative procedure

Development of a user-friendly program (ALPAXL)

To facilitate the use of our analysis, a user-friendly spreadsheet program (ALPAXL) was developed. This program is based on the solution scheme presented above and uses built-in functions of EXCEL. ALPAXL provides the results of the analysis, the deformed configuration of the pile-soil system and the load-settlement curve in seconds. Figure 3 shows screenshots of ALPAXL. It can be downloaded at http://cobweb.ecn.purdue.edu/~mprezzi.



(a)



(b)

Fig. 3. Screenshots of the spreadsheet program ALPAXL: (a) input section; (b) output section

RESULTS

<u>Comparison with previous pile settlement studies</u>

We compare the results from our study with numerical or analytical solutions available in the literature (Poulos and Davis 1980; Fleming et al. 1992; Mylonakis 2001). Figure 4 shows normalized pile head stiffness K_N ($K_N = Q_t/(w_t E_p B)$), where w_t = settlement at the pile head) as a function of normalized pile length (L_p/B) for an ideal end-bearing pile. The pile-soil modulus ratio E_p/G_s is equal to 3000. The pile base is assumed to rest on a rigid layer; the soil above the rigid layer is homogeneous with a shear modulus G_s and a Poisson's ratio ν_s = 0.5. The curves shown in Fig. 4 were obtained with the analysis presented in this paper and from previous studies by Poulos and Davis (1980), Fleming et al. (1992) and Mylonakis (2001). Fleming et al. (1992) did not specifically address the case of ideal end-bearing piles. However, by considering the shear modulus below the base to tend to infinity in the equation for the magical radius r_m (Randolph and Wroth 1978), the results in Fig. 4 can be obtained. Figure 4 shows that, for E_p/G_s = 3000, the normalized pile head stiffness decreases with increasing L_p/B and that the results of the analysis presented in this paper are in good agreement with those from the previous studies.

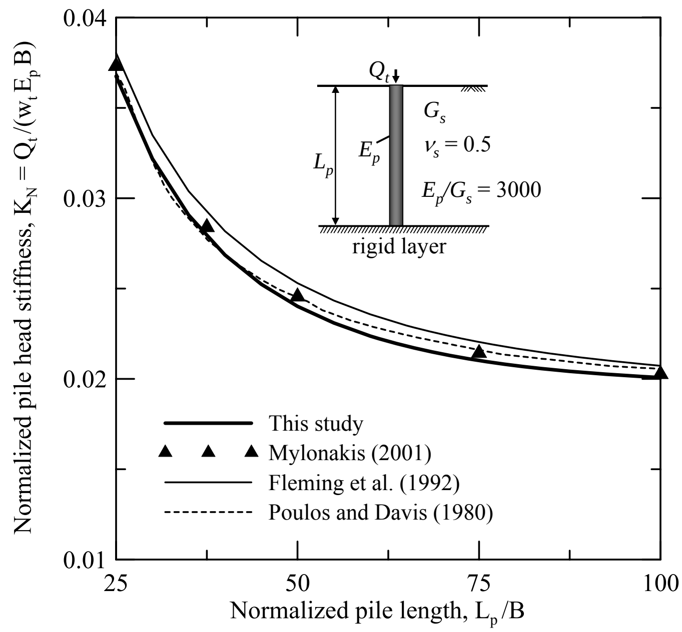


Fig. 4. Comparison of normalized pile head stiffness versus normalized pile length of ideal end-bearing piles (E_p/G_s = 3000)

CASE STUDIES

We compare the results of our analysis with two case studies reported in the literature. The analysis was carried out using ALPAXL.

<u>Micropile (Italy)</u>

Russo (2004) presented a case history on micropiles used for underpinning a historical building in Naples, Italy. The micropiles were installed in a complex soil profile (there are thick layers of man-made materials accumulated over millennia at the site). The soil profile and representative values of cone resistance q_c for each soil layer are shown in Fig. 5.

According to Russo (2004), the micropile installation steps were: 1) drilling of a 200-mm-diameter hole using a continuous-flight auger, 2) inserting a steel pipe equipped with injection valves, 3) filling the annular space between the pipe and the soil with grout, 4) grouting the pile shaft through each valve using a double packer, and 5) filling the steel pipe with grout. A micropile (0.2m in diameter and 19m in length) was load-tested. Two anchor piles were used to provide reaction to the loading frame, and the compressive load was applied on the test pile with a hydraulic jack. The vertical displacement of the pile head was measured by LVDT's, and the axial strain along the shaft was measured by vibrating-wire strain gages.

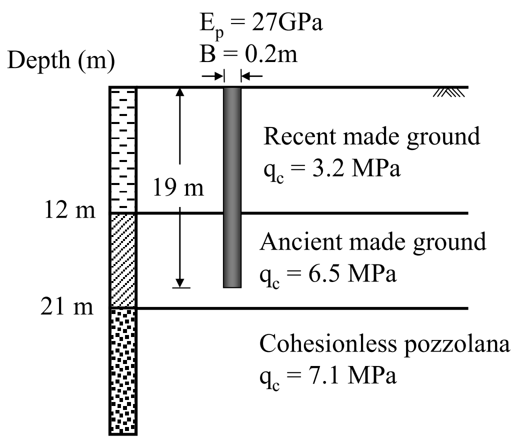


Fig. 5. Soil profile at the micropile test site.

Russo (2004) compared the pile load test results with those obtained from finite element analysis. The Young's moduli of each soil layer were back-calculated from the FEA. Although Russo (2004) did not provide information on the geometry and properties of the steel pipe left inside the micropile, its outer diameter and inner diameter were assumed to be 33.4mm and 25.4mm, respectively. Accordingly, assuming that the Young's moduli of the steel and grout are 200GPa and 25GPa, the equivalent Young's modulus of the composite steel-grout cross section is calculated to be approximately 27GPa. Table 1 shows the input values used in the analysis. We used four soil layers in the analysis with the bottom of the second layer flush with the base of the pile. The Poisson's ratio was assumed to be 0.3 for all the soil layers.

Table 1. Input values for the analysis of the microplile load-tested in Italy (B = 0.2m; L_p = 19m; E_p = 27GPa)

Layer	H_i (m)	E_{si} (MPa)	ν_{si}
1	12	50	0.3
2	19	117	0.3
3	21	117	0.3
4	50	138	0.3

Figure 6 shows both the measured and calculated load versus settlement curves. Figure 7 shows measured and calculated load-transfer curves for applied loads equal to 51, 253, and 542kN. These figures show that there is very good agreement between the calculated and measured values, although the calculated values for the pile head settlement become smaller than the measured values for loads greater than about 400kN.

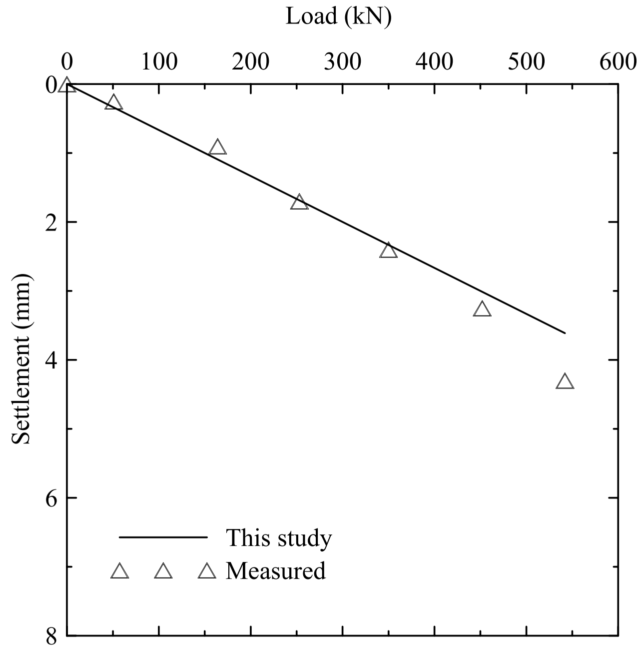


Fig. 6. Load-Displacement curve at the pile head (Italy case)

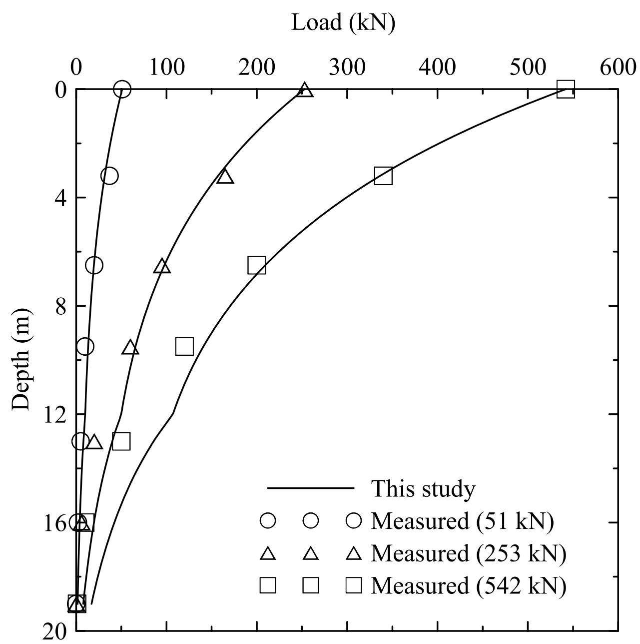


Fig. 7. Load-transfer curves for applied loads equal to 51, 253 and 542 kN (Italy case)

Figure 8 shows the calculated vertical soil displacements at the level of the pile head and base. There is practically no settlement at the level of the pile base for a pile head settlement of 2% of the pile diameter because of the high compressibility of the micropile because of its high-slenderness ratio.

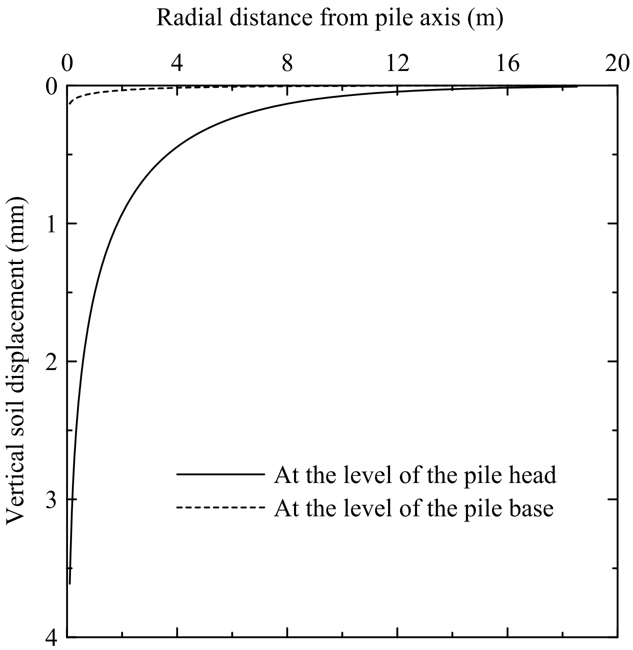


Fig. 8. Calculated vertical soil displacement as a function of radial distance from the pile axis at the level of the pile head and base (Italy case)

Drilled shaft in rock (Singapore)

Chang and Wong (1987) reported the results of instrumented load tests on drilled shafts installed in weathered sedimentary rocks of the Jurong Formation in Singapore. The test pile J2 (diameter B = 1.0m) was located next to a river. The top two meters of the soil consisted of a soft silty clay (N_{avg} = 2, where N_{avg} represents the average of the SPT blow counts for the layer), which was underlain by a soft, peaty clay (N_{avg} = 2.5) layer extending down to a depth of 14.5m. The pile was cast in a cased hole drilled through the soft clay layers and embedded in a weak, highly weathered shale (N_{avg} = 100 ~ 150). The pile extended down to a depth of 28m below the ground surface. Figure 9 shows the pile and subsurface profile. The pile was instrumented with six pairs of vibrating-wire strain gages. The representative Young's modulus of the pile was 52 MPa. The pile was designed to carry an axial load of 3500 kN and tested to 2.5 times the design load two weeks after its installation using the slow maintained-load test method.

The elastic properties of the soil and rock layers were not available in the original paper by Chang and Wong (1987). For the rock layers, input values for the Young's moduli were obtained from Kim et al. (1999) since they reanalyzed the pile load test results reported by Chang and Wong (1987) to develop load-transfer functions for drilled shafts installed in weathered rock. The Young's modulus values of the upper and lower shale used in their analysis were 3000 and 2000 MPa, respectively (Kim et al. 1999). For the clay layers, the Young's modulus was estimated from the undrained shear strength s_u, which, in turn, was estimated from the N_{SPT} values following Stroud (1974); the estimated s_u for the upper and lower clay layers were 12 and 15 kPa, respectively. According to Calanan and Kulhawy (1985), values for the E_s/s_u ratio generally range between 200 and 900, with an average value of 500. Using E_s/s_u=500, E_s values for the upper and lower clay layers were determined to be 6 and 7.5 MPa, respectively. The Poisson's ratio was assumed to be 0.5 for the clay layers and 0.15 for the rock layers. The input values used in the analysis are summarized in Table 2.

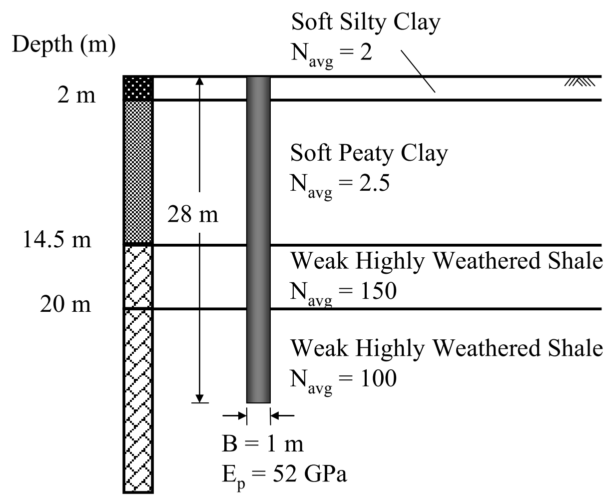


Fig. 9. Soil profile and test pile (Singapore case)

Table 2. Input values for the analysis of the drilled shaft J2 load-tested in Singapore (B = 1.0m; L_p = 28m; E_p = 52GPa)

Layer	H_i (m)	E_{si} (MPa)	ν_{si}
1	2	6	0.5
2	14.5	7.5	0.5
3	20	3000	0.15
4	28	2000	0.15
5	60	2000	0.15

Figure 10 shows the predicted and measured load-settlement curves for the test pile. The results from our analysis are in good agreement with the measured data. In particular, at the design load level (Q_t = 3500kN), the calculated settlement was almost the same as the measured value. Figure 11 shows the predicted and measured load-transfer curves. The results from both the load test and our analysis indicate that most of the applied load was carried by shaft friction along the pile-rock interface. Although the load-transfer curves obtained from the analysis deviates from the measured data as the load increases, overall there is reasonable agreement.

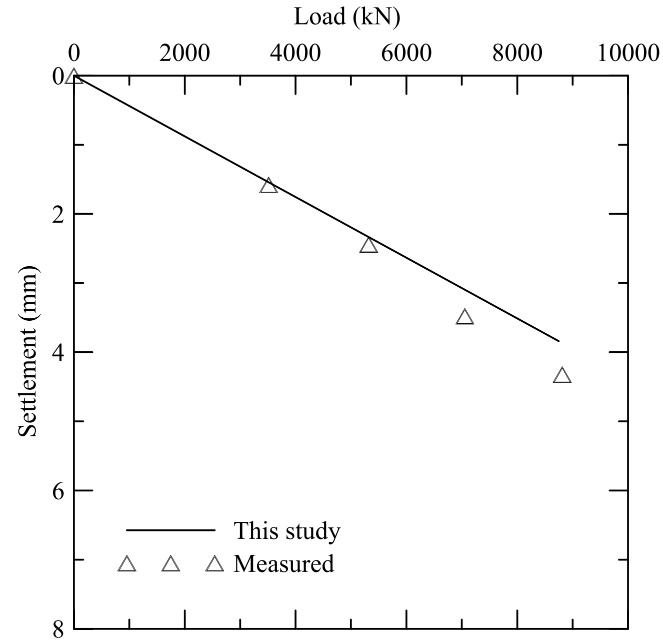


Fig. 10. Load-pile head settlement curve (Singapore case)

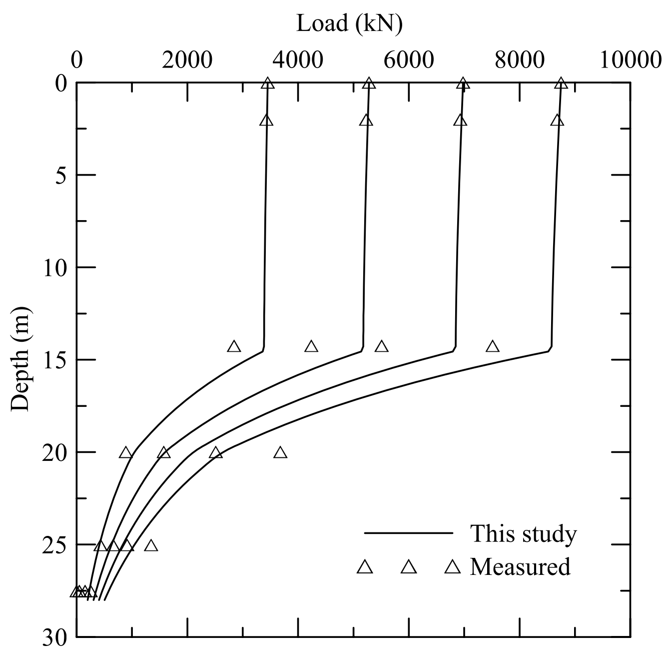


Fig. 11. Predicted and measured load-transfer curves (Singapore case)

SUMMARY AND CONCLUSIONS

Pile design has relied on calculations of ultimate resistances reduced by factors of safety that would indirectly prevent settlement-based limit states. This is in part because of the absence of realistic analysis tools allowing calculation of settlement given an axial load on the pile. Analyses that can accurately calculate settlement for a given load offer an opportunity for more cost-effective design in the future.

In this paper, a new analysis to estimate pile settlement in multilayered soil was presented. This analysis is based on the solution of the governing differential equations for pile and soil displacements obtained using the principle of minimum potential energy and calculus of variations. The analysis produces pile displacement and axial force as functions of depth and vertical soil displacement as a function of the horizontal distance from the center of the pile if the following are known: the pile cross-sectional dimensions and length, thicknesses of the soil layers, Young's modulus of the pile material, the Young's moduli and Poisson's ratios (or any elastic pairs) of the soils in the various layers, and the magnitude of the applied axial force. A user-friendly spreadsheet program (ALPAXL) was developed to facilitate the use of the analysis.

Comparisons were made with the numerical or analytical solutions available in the literature. The results from our analyses for end-bearing piles showed good agreement with those from previous studies. Furthermore, two case histories were analyzed. The predicted pile head settlement and load-transfer behavior compared well with the measured data.

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