Mar 9th, 3:00 PM - 3:45 PM

Preparing Middle Grades Teachers to Use Drawn Models for Developing Arithmetic with Rational Numbers

Andrew Izsak
University of Georgia

Erik Jacobson
University of Georgia

Follow this and additional works at: https://digitalcommons.georgiasouthern.edu/stem

Part of the Curriculum and Instruction Commons, Educational Methods Commons, Junior High, Intermediate, Middle School Education and Teaching Commons, and the Science and Mathematics Education Commons

Preparing Middle Grades Teachers to Use Drawn Models for Developing Arithmetic with Rational Numbers

Andrew Izsák & Erik Jacobson

Department of Mathematics and
Science Education

University of Georgia

1st Annual Georgia Scholarship of STEM
Teaching and Learning Conference

March 9, 2012

Research Based Course for Preservice Middle Grades Teachers

- Focus: Developing knowledge for using drawn models of fraction arithmetic

- NSF Funded Grants
  - Coordinating Students' and Teachers' Algebraic Reasoning
  - Does it Work?
  - Diagnosing Teachers' Multiplicative Reasoning

Why Focus on Teachers' Knowledge for Using Drawn Models?

- Theoretical perspectives on teachers' knowledge
    - Shulman's (1986) Pedagogical Content Knowledge
    - Ball et al's. (2008) Mathematical Knowledge for Teaching
- Standards documents
    - National Council of Teachers of Mathematics (2000)
    - Common Core State Standards Initiative (2010)

Research on Teachers' Knowledge of Fraction Arithmetic

- **Difficulties modeling situations**
  - Give a situation that illustrates 1 3/4 ÷ 1/2

  - 1 kg of detergent makes 15 kg of soap. How much soap does .75 kg of detergent make?

- **Difficulties reasoning with drawn models**
  - Responding to students' questions during instruction

Pedagogical Purposes for Drawn Models: Divergent Perspectives

- Which students should use drawn models?
  - Visual learners
  - Students who are struggling with numeric methods
  - All students

Pedagogical Purposes for Drawn Models: Divergent Perspectives

- What are appropriate goals?
  - Show an answer to a given problem
  - Infer a general numeric method from a pattern
  - Deduce a general numeric method from operations on quantities

Fraction Division

- For the following division statement:
    - Generate a word problem
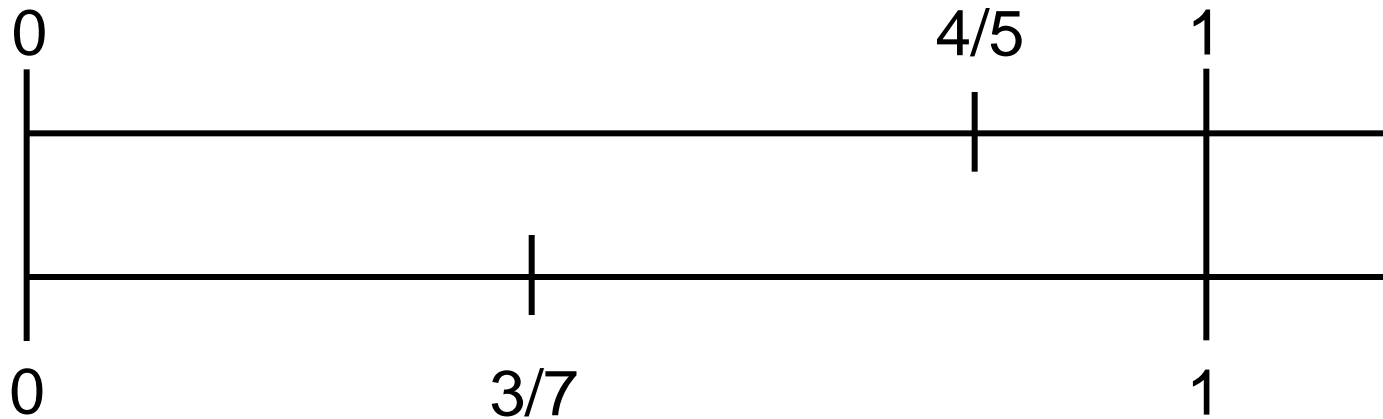   - Use a drawn model to explain the quotient
        - 4/5 ÷ 3/7 = 28/15

Two Division Models for 4/5 ÷ 3/7

- Measurement division
  - How many 3/7ths are in 4/5ths?

- Partitive division
  - If 4/5ths is 3/7ths of a group, how much is a whole group?

Model One:
How Many 3/7ths are in 4/5ths?

- Same wholes

```
0                                    4/5      1
|————————————————————————————————|————————|————
|
|————————————————————|—————————————————————|————
0                   3/7                     1
```

Model One:
How Many 3/7ths are in 4/5ths?

- Partition the whole into 1/5ths and 1/7ths
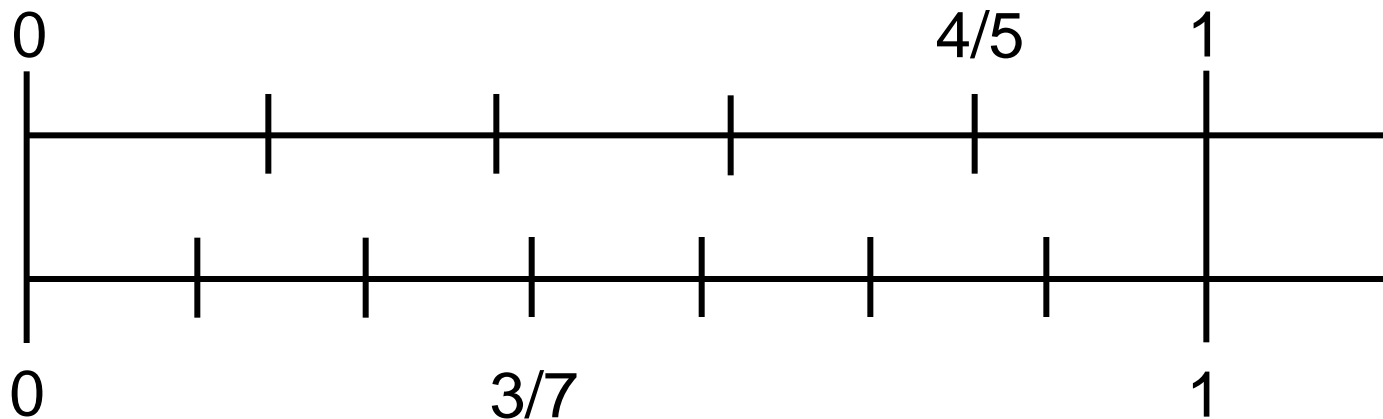
Model One:
How Many 3/7ths are in 4/5ths?

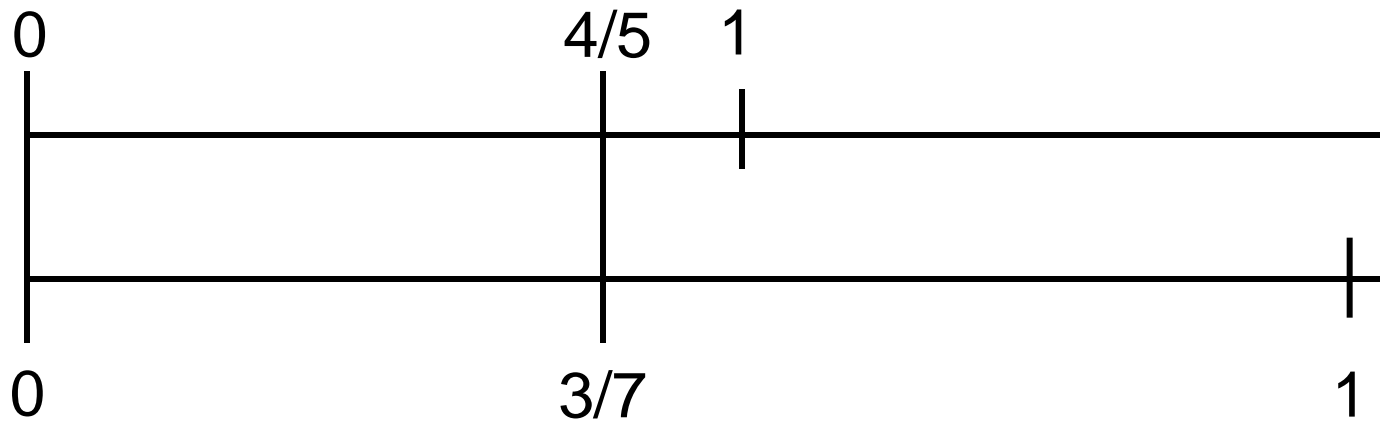- Use denominators to create a common partition of the whole
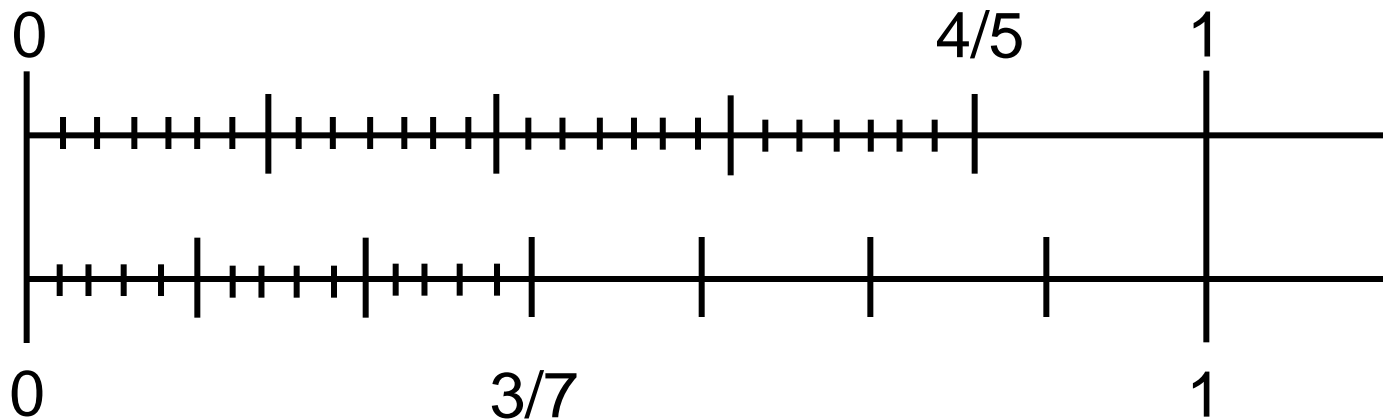
Reasons for Using the Partitive Model

- Not in wide use in U.S. schools

- Builds on students' experiences with sharing

- The invert-and-multiply algorithm expresses operations on quantities directly

- Prepare students for proportional reasoning
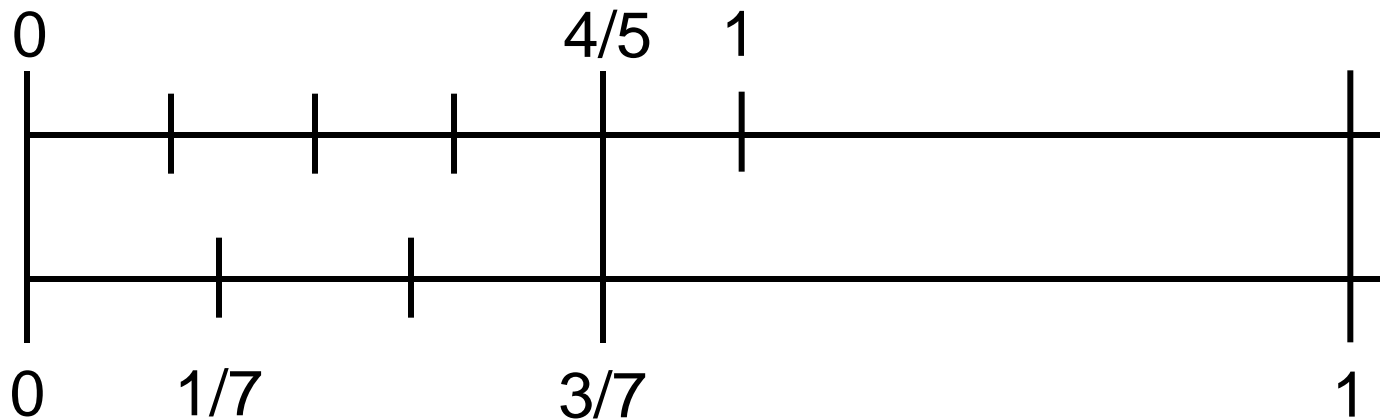
Model Two:
How Many in One Group?

- Different wholes for different quantities

```
0                4/5   1
|————————————————|——|————————————————|

|————————————————|————————————————————|—
0                3/7                   1
```
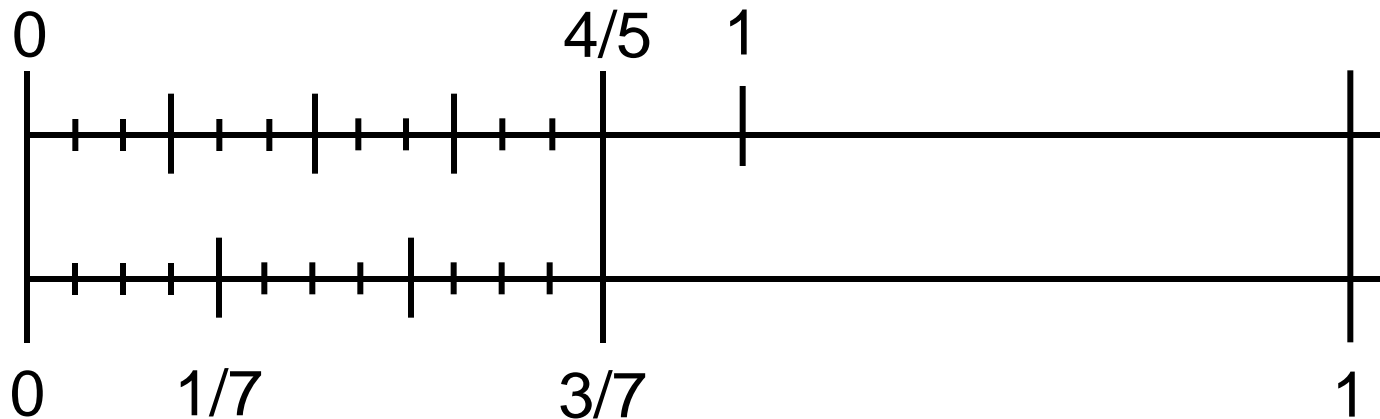
Model Two:
How Many in One Group?

- Partition the wholes into 1/5ths and 1/7ths

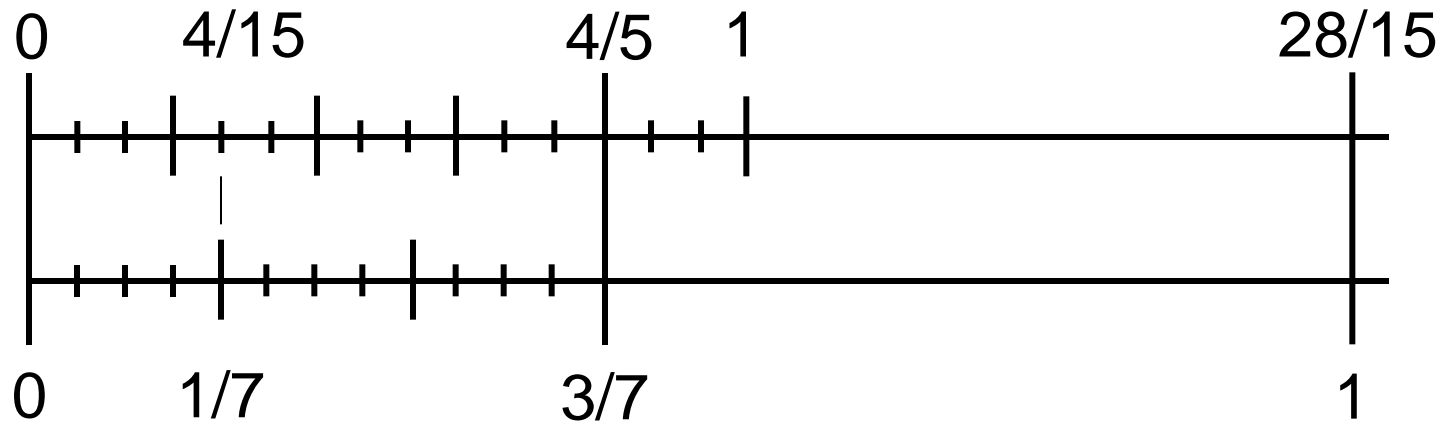# Model Two:
How Many in One Group?

- Use numerators to create a common partition of 4/5ths and 3/7ths

Model Two:
How Many in One Group?

- Attend to the appropriate wholes

| 0 | 4/15 | | 4/5 | 1 | | 28/15 |

| 0 | 1/7 | | 3/7 | | | 1 |

Main Themes for EMAT 5280 at UGA

- Referent Units
  - Understand units to which numbers refer
- Partitioning
  - Using whole-number multiplication to guide partitioning
- Iterating
  - Interpret $\dfrac{A}{B}$ to mean A copies of $\dfrac{1}{B}$
- Appropriateness
  - Identifying multiplication and division situations

Students

- 28 preservice middle grades teachers
- Juniors
- Little to no experience using drawn models for fraction arithmetic
- College mathematics:
  - MATH 1101 (Math Modeling)
  - MATH 1113 (Pre calculus)
  - MATH 2200 (Analytic Geometry & Calculus)

Data

- Pretest and posttest
- Written homework and exams
- Video recorded instruction
- Video recorded interviews: 4 focal pairs
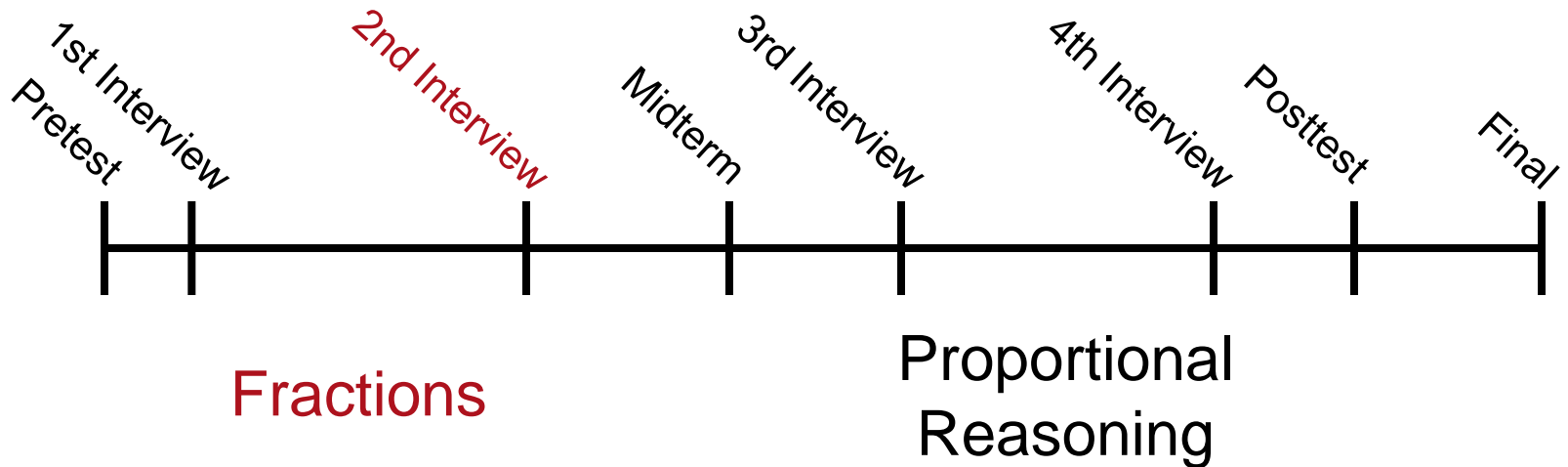
Initial Partitioning Tasks

- Paper Folding: Connecting Multiplication to Partitioning

  – Predict how many equal parts you would get if you fold a strip of paper in half, in thirds, and in thirds again.

  – Devin folded a strip of paper into 4 equal parts. If he wants to create 18 equal parts how many parts could he fold his paper into next?

Initial Partitive Division Tasks
How Many in One Group?

- 12 cookies are shared among 3 friends. How much does one friend get?

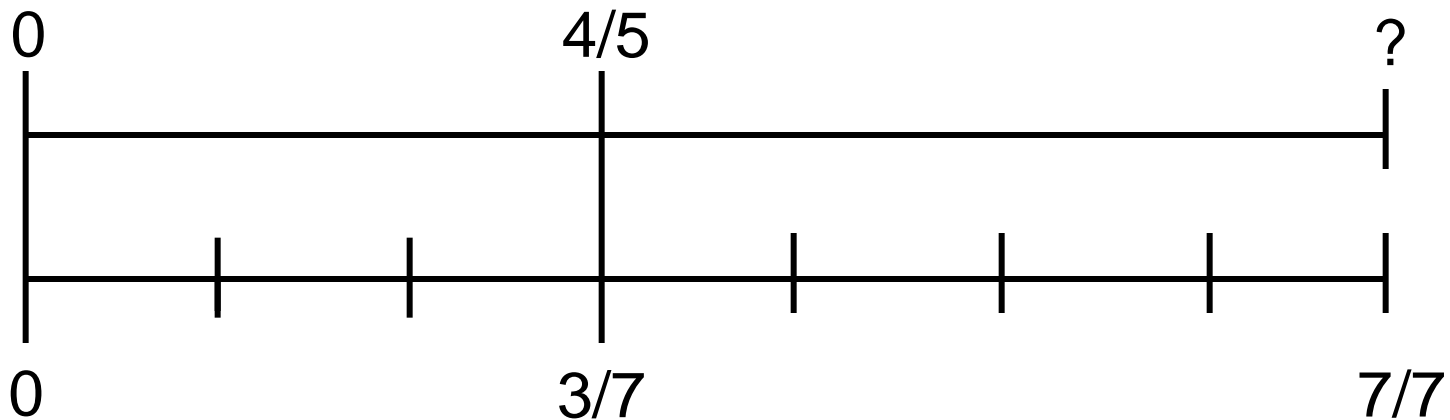- 9 dollars is 3/7 of the cost for dinner. How much does dinner cost?

Later Partitive Division Task

- Use a double number line to solve:

    4/5 tablespoons of oil are needed for 3/7 of a salad dressing recipe. How many tablespoons of oil are needed for one recipe?

Lisa and Tanya

- Set up a double number line:

0   4/5   ?

0   3/7   7/7

- Use denominators 7 and 5 to partition into 35
- Focus on lining up equivalent fractions:

$$\frac{28}{35} \neq \frac{15}{35}$$

Lisa and Tanya (Continued)

- Tanya proposes dividing 4/5 by three to find how much oil for 1/7 of the recipe

- Tanya does not see how to partition 4/5 into three parts

- Tanya proposes dividing 1 Tbsp into 15 parts

- What should be the same?

- Referent unit error

Discussion

- Standards demand expertise of teachers
- Difficulties not in finding finer partitions
- Rather in coordinating partitioning and referent units
- Collecting similar data on preservice secondary students
- This content is learnable