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Foundation Failure Case Histories Reexamined Using Modern Geomechanics

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ABSTRACT

Case histories have played an important role in guiding development of geotechnical engineering during a time when theory was not sophisticated enough to model even simple problems with an acceptable level of rigor. As the discipline transitions from overwhelming reliance on empiricism to a greater reliance on science, it is useful to reexamine the best known case histories as a general check on modern methods of analysis. In the engineering of foundations in clay, three case histories – the collapses of the Transcona and Fargo grain elevators and the near collapse of the leaning tower of Pisa – stand out. We will see that limit analysis, which is a method of analysis based on two theorems from plasticity theory that allow bounding the collapse load from above and below, produces collapse load estimates that match closely the estimated collapse loads for the two failed grain elevators. It does so without giving the analyst much latitude in selection of input parameters, not requiring the elaborate assumptions needed when attempts are made to use an excessively simplified theory to analyze a real problem. We will also show, using the problem of a leaning tower, how resort to a complete analysis of a boundary-value problem, using a method like the finite element method, is sometimes required in determining the critical ultimate limit state.

INTRODUCTION

Proper design of foundations requires a strong basis on mechanics but should also be corroborated by the satisfactory performance of foundations. Well documented case histories are useful in such corroboration. Experiments also allow us to compare predicted and measured foundation response; however, experiments are often restricted to model tests or to single-element tests (such as the load test of a pile). In contrast, case histories that are sufficiently rich in details allow comparisons with simulations of entire foundation-structure systems, adding a measure of realism to the validation of analyses and design methods.

Not every case history needs to be a complete account of successful or unsuccessful design and construction of a structure. Unfortunately, real structures are rarely instrumented and soil profile characterization is rarely done so completely that a traditional case history can be useful to a complete validation of a theoretical analysis. While a traditional case history informs, may reveal serious missteps or, more rarely, may reveal a limit state that is surprising and not typically considered in design, it rarely serves as validation of a theoretical method of analysis.

In this paper, we focus on the combination of case histories with science-based methods of analysis as a powerful way of advancing methods of design. We will explore two cases of bearing capacity failure (the collapse of grain elevators in Transcona and Fargo), discuss an alternative collapse limit state (leaning stability) that threatened the Tower of Pisa. To all cases, we apply modern methods of analysis.

By selecting case histories of historical relevance to geotechnical engineering, we illustrate how our progress in developing predictive methods based on the mechanics of soils and structures can be tested by analysis of case histories. We conclude the paper by laying out some principles regarding both the planning of detailed field experiments and for using case histories usefully in the testing and validation of predictive methods.

THEORETICAL METHODS

Limit Analysis

It has been roughly sixty years since Drucker, Greenberg and Prager (1951) published their ground-breaking lower and upper bound theorems of plasticity theory, on which limit analysis is based. Limit analysis always had the potential to produce excellent solutions to collapse problems, typified in soil mechanics by the bearing capacity problem. However, the numerical techniques required for finding very close lower and upper bounds on collapse loads, thus closely defining the collapse loads, were not available until very recently.

Limit analysis takes advantage of the lower and upper bound theorems of plasticity theory to bound the rigorous solution to a stability problem from below and above. The lower bound theorem states that collapse does not occur for a statically admissible stress field - a stress field that is stable (i.e., does not violate the yield criterion at any point) and statically admissible (i.e., is in equilibrium with the surface traction and body forces). This can be written in the form of the virtual work equation as:

$$\int_S T_i^L v_i dS + \int_V X_i^L v_i dV = \int_V \sigma_{ij}^L \dot{\varepsilon}_{ij} dV \le \int_V D(\dot{\varepsilon}_{ij}) dV = \int_V \sigma_{ij} \dot{\varepsilon}_{ij} dV \qquad (1)$$

where σ_{ij}^L = statically admissible stress field in equilibrium with the tractions T_i^L and the body forces X_i^L; σ_{ij} = actual stress field; $\dot{\varepsilon}_{ij}$ = actual strain rate field; v_i = actual velocity field; and D($\dot{\varepsilon}_{ij}$) is the plastic dissipation associated with the strain rate field $\dot{\varepsilon}_{ij}$. It should be noted that, in the lower bound theorem, only the equilibrium condition and the stress boundary conditions are satisfied. No kinematics is taken into account.

The upper bound theorem states that collapse is either imminent or already underway for a kinematically admissible velocity (or strain rate) field – a velocity field which is both unstable [i.e., the rate of external work calculated from the velocity (or strain rate) field exceeds or equals the internal power dissipation)] and kinematically admissible (i.e., the velocity field satisfies the velocities specified at the boundary of the soil mass). This can be written as follows:

$$\int_S T_i^U v_i^U dS + \int_V X_i^U v_i^U dV = \int_V \sigma_{ij}^U \dot{\varepsilon}_{ij}^U dV = \int_V D(\dot{\varepsilon}_{ij}^U) dV \ge \int_V \sigma_{ij} \dot{\varepsilon}_{ij}^U dV \qquad (2)$$

where v_i^U = kinematically admissible velocity field compatible with the strain rate field $\dot{\varepsilon}_{ij}^U$; σ_{ij}^U = stress field in equilibrium with the upper bound loading T_i^U and X_i^U; σ_{ij} = actual stress field; and D($\dot{\varepsilon}_{ij}^U$) is the plastic dissipation associated with the strain rate field $\dot{\varepsilon}_{ij}^U$. The upper bound theorem satisfies the flow rule, the compatibility condition and the velocity boundary conditions, but not the equilibrium condition or traction boundary conditions. In (1) and (2), the inequalities are due to the principle of maximum power dissipation. The stress fields in (1) and (2) are in terms of effective stresses since power is dissipated only through the soil skeleton.

Finite element limit analysis combines the limit theorems with finite elements to produce a discrete mathematical programming problem. The numerical formulations used in this investigation originate from those developed by Sloan (1988, 1989), but has evolved significantly over the past two decades to incorporate the major improvements described in Lyamin and Sloan (2002a, 2002b) and Krabbenhoft et al. (2005, 2007). In brief, these formulations use linear stress (lower bound) and linear velocity (upper bound) triangular finite elements to discretize the soil mass. In contrast to conventional displacement finite element analysis, each node in limit analysis mesh is unique to a particular element so that statically admissible stress (in the lower bound case) and kinematically admissible velocity discontinuities (in the upper bound case) are possible along shared edges between two adjacent elements. Both formulations result in convex mathematical programs, which (considering the dual form of the upper bound problem) can be cast in the following form:

$$\begin{aligned} &\text{maximize} \quad \lambda \\ &\text{subject to} \quad \mathbf{A\sigma} = \mathbf{p}_0 + \lambda \mathbf{p} \\ &\qquad\qquad f_i(\boldsymbol{\sigma}) \le 0, \quad i = \{1, \ldots, N\} \end{aligned} \qquad (3)$$

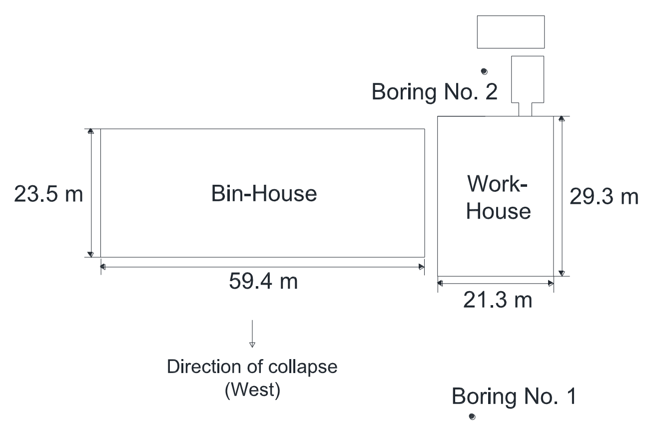
where λ is a load multiplier, $\boldsymbol{\sigma}$ is a vector of stress variables, **A** is a matrix of equality constraint coefficients, $\mathbf{p}_0$ and $\mathbf{p}$ are vectors of prescribed and optimizable forces, f_i is the yield function for stress set *i*, and *N* is the number of stress nodes. The solutions to problem (1) can be found efficiently by using general Interior-Point methods (IPM) or specialised conic optimization solvers (SOCP).

The end product of the optimization problem is the stress field leading to the maximum lower bound to the collapse load and the velocity (displacement) field consistent with the lowest upper bound to the collapse load achievable with the finite element mesh used in the analysis. If these two bounds are close enough, the collapse load is rather precisely known. As we will see, that is always the case with 2D computations. In 3D computations, it is known that the lower bound is closer to the collapse load than the upper bound is.

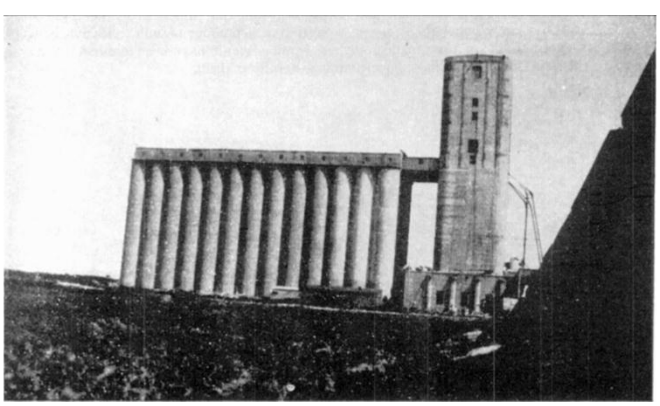
CASE HISTORY Ⅰ: TRANSCONA GRAIN ELEVATOR FAILURE, 1913

The Structure, the Failure and the Soil Profile

The Canadian Pacific Railway Company started construction of the Transcona Grain Elevator in 1911 in North Transcona, 7 miles northeast of Winnipeg, Canada, and roughly 230 miles north of Fargo, North Dakota. Construction ended in September 1913. The elevator was composed of a reinforced concrete work house and a bin house, which were connected by a bridge and conveyor belt. The conveyor belt operated in a low cupola at the top of the bin-house. The work house was 21.3 m wide, 29.3 m long and 54.9 m tall and rested on a raft foundation with base located 3.66 m below the ground surface. The bin house had five rows of 13 bins and was constructed on a reinforced-concrete raft foundation that was 23.5 m wide and 59.4 m long and had the base placed also at a depth of 3.66 m. The thickness of this mat foundation was 0.6 m. The bins were 28.0 m in height and 4.27 m in diameter. Fig. 1 shows a plan view and a photo of the structure.



(a)



(b)

Fig. 1 The Transcona grain elevator: (a) the elevator foundation plan and (b) the structure before collapse (after White, 1953). Used with permission from ICE Publishing.

Before construction of the grain elevator, the only test known to have been performed was a plate load test at the design depth of the foundation. The bearing soil was deemed similar to soil found in the Winnipeg area, on which tall structures had been erected. The notion that the depth of influence of a plate load test is tied to the size of the plate may not have been well understood at the time; as a result, the effect of a weaker clay layer located below the shallower, relatively stiff clay layer on which the plate load test was performed was not contemplated.

After construction, operations started; the amount of grains was approximately evenly distributed between the bins. On October 18, 1913, settlement of the bin house was first observed (Allaire, 1916). There were 875,000 bushels of wheat in the elevator at that moment, corresponding to a load of 231,400 kN. If we combine these 231,400 kN with the dead weight of the structure, a total load of 409,400 kN was applied at the base of the raft. Considering these loads to have been distributed uniformly over the raft foundation, the applied load on the mat foundation could be estimated as 293 kPa.

As soon as settlement started, it increased steadily, but slowly, to about 0.30 m within an hour. After that, the structure tilted toward the west during the next 24 hours until its lean was 26 degrees, 53 minutes from the vertical. A 7.5-9.0 m wide strip of ground on the east side of the bin-house bulged up about 1.2-1.5 m (except on the south side, where the work house was located), while the west side settled as much as 9 m below its original level (Fig. 2). As the photos in Fig. 1 and Fig. 2 attest, all of this happened slowly enough that White (1953) could take photographs during the event and have an accurate recollection of what happened.

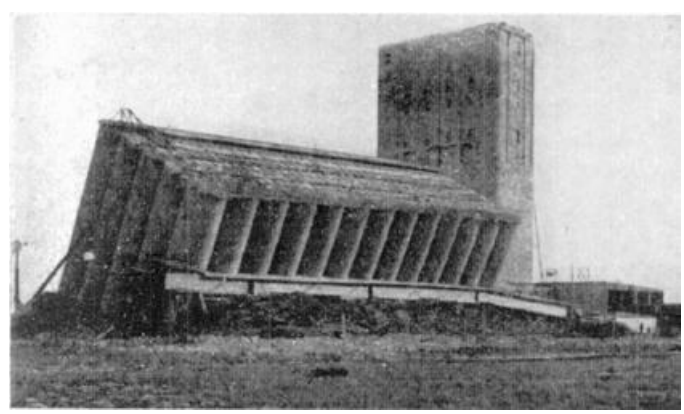


Fig. 2 The collapse of the Transcona Grain Elevator (White, 1953). Used with permission from ICE Publishing.

After the failure, several wash borings were made near the site that showed that rather uniform deposits of clay existed beneath the bin house and elevator. In 1951, there were additional borings performed near the work house. As a result, it was found that the ground was mainly composed of two thick clay layers. The unit weights of these layers were taken as 18.8 kN/m^3 by Peck and Bryant (1953). The ground water level was located at a depth of 2.56m in boring No. 1. By testing undisturbed samples from different depths, the unconfined compressive strength, natural water content and liquid and plastic limits were obtained for different layers and are shown in Table 1. Using half of the unconfined compressive strength values obtained, the undrained shear strength profile with depth could be estimated as shown in Fig. 3 . The locations of borings No. 1 and No. 2 are shown in Fig. 1(a). The lines in Fig. 3 represent the shear strength profile used in the analysis of this failure, discussed next.

The undrained shear strength s_u was assumed constant with depth within each of the layers identified in Fig. 3; the vertical lines in this figure represent the values of s_u assumed in the analysis for each layer, which are given numerically in Table 2. The layer of fractured limestone was treated as a frictional material. A high friction angle, of as much as 50 degrees, would likely be appropriate for the fractured limestone. Analyses show, however, that even a layer with friction angle as low as fifteen degrees would deflect the slip mechanism up into the overlying clay, so characterization of the fractured limestone turns out not to be critical to the results of the analysis (and the friction angle is noted as greater than 15 degrees in Table 2).

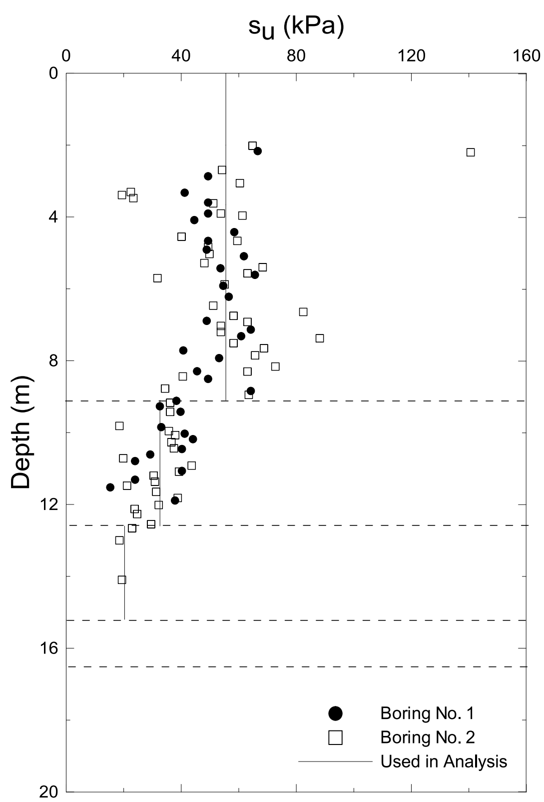


Fig. 3 Undrained shear strength with depth (based on borings No. 1 and No. 2).

Table 1. Soil profile of the Transcona elevator case

Layer No.	Depth (m)	Description	Properties	
1	0-9.10	Clay	Unit weight (kN/m^3)	18.8
			Average water content (%)	45
			LL (%)	105
			PL (%)	35
2	9.10-12.6	Clay	Unit weight (kN/m^3)	18.8
			Average water content (%)	57
			LL (%)	105
			PL (%)	35
3	12.6-15.2	Clay and gravel	Unit weight (kN/m^3)	18.8
4	15.2-16.5	Fractured limestone	Unit weight (kN/m^3)	19
			ϕ (^{o})	>15

Table 2 The value of undrained shear strength for each soil layer below the Transcona elevator foundation.

Layer No.	Depth (m)	Undrained shear strength (kPa)
1	0-2	55.5
2	9.1-12.6	32.6
3	12.6-15.2	20.3
4	15.2-16.5	Treated as frictional

<u>Reassessment of the Case History using Modern Methods</u>

We will start by analyzing the Transcona elevator collapse using 2D limit analysis. 2D analysis would be suitable for a plane-strain problem (one in which the shear strain components in one plane and the strain component normal to that plane are all zero). Assumption of plane strain is a frequent assumption in soil mechanics, even when it does not strictly apply. Every problem is in fact three-dimensional; the cross section of the problem in this case refers to the cross section corresponding to the smaller plan dimension (the width B of the foundation). Fig. 4 shows the finite element mesh used for the 2D limit analyses. Fig. 5 shows the same mesh distorted after collapse (in truth, an image based on the velocities at failure and thus on the last displacement increments), and Fig. 6 shows the plastic energy dissipation as a result of the collapse. Both the distorted mesh and the plot of energy dissipation show where shearing localized at collapse. The distorted mesh does not represent accumulated displacement but rather results from scaling up the incremental displacements at collapse, giving a qualitative view of slip pattern at collapse. The lower bound on the unit limit load was 284 and the upper bound was 296 kPa. Since the estimated unit load at the time of collapse is 293 kPa, there appears to be a good match; however, neither our estimates of shear strength nor the estimate of load at the time of collapse is free of error. Additionally, the foundation is not long enough with respect to its width for the problem to approximate a plane-strain problem. Accordingly, we also performed 3D limit analysis of the same problem.

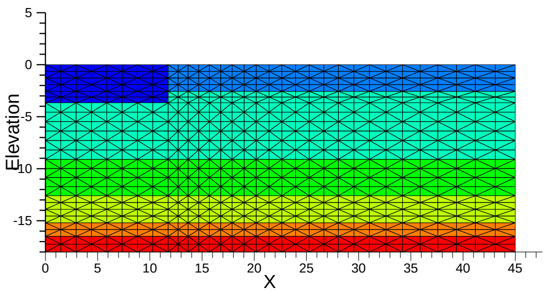


Fig. 4 Two-dimensional (plane-strain) mesh for the Transcona elevator.

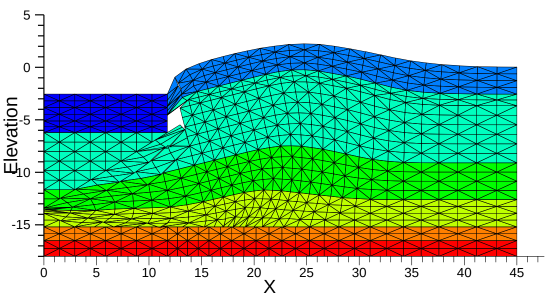


Fig. 5 Distorted two-dimensional (plane-strain) mesh for the Transcona elevator.

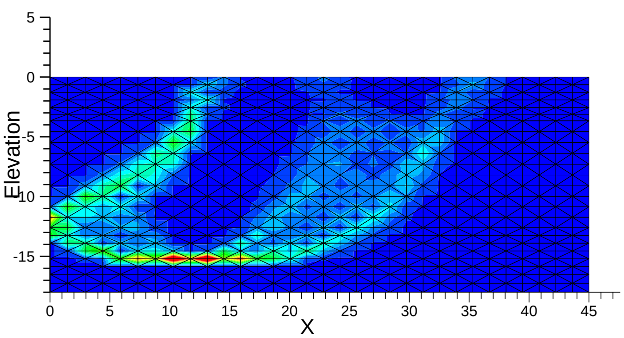


Fig. 6 Contours of plastic power dissipation from the two-dimensional (plane-strain) analysis of the Transcona elevator failure.

The mesh for the 3D analysis is shown in Fig. 7, the distorted mesh in Fig. 8, and the power dissipation plot in Fig. 9. The resulting lower and upper bounds on limit unit bearing capacity are 310 and 376 kPa. The larger difference between the bounds compared with that for the 2D analysis reflects the greater challenges of a 3D analysis. Comparisons of 3D finite element limit analysis with the solutions of problems that can be solved exactly (such as by Salgado et al. 2004 and Lyamin et al. 2007) show that the lower bound is significantly closer to the collapse load, so that we can expect the collapse load, as calculated using 3D FELA to be of the order of 320-330 kPa, less than 10% greater than the estimated load at collapse.

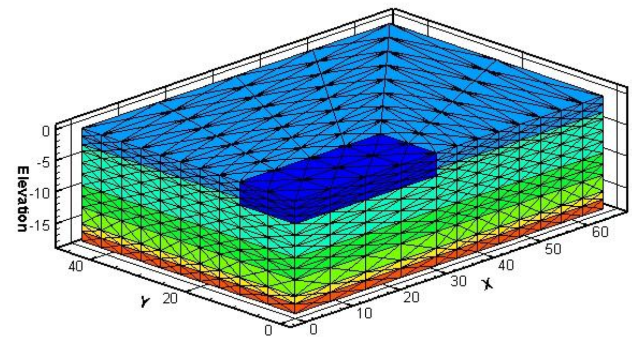


Fig. 7 Three-dimensional mesh for the Transcona elevator.

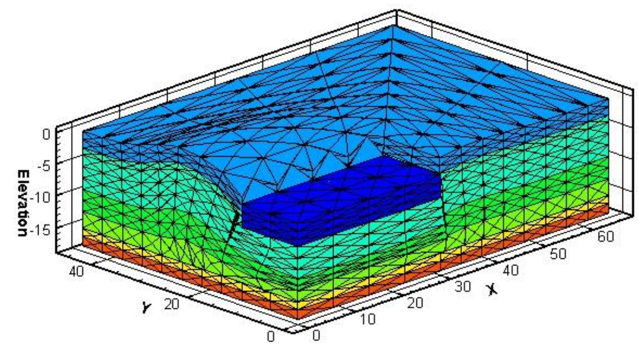


Fig. 8 Distorted three-dimensional (plane-strain) mesh for the Transcona elevator.

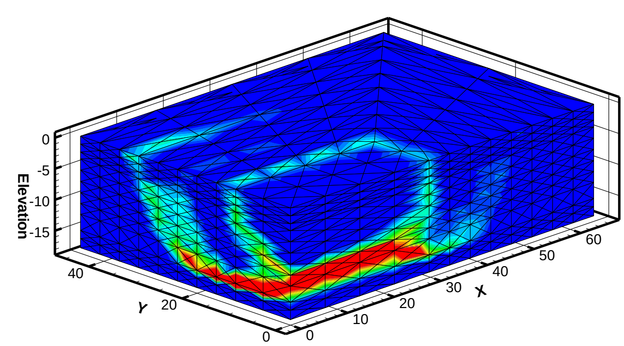


Fig. 9 Contours of plastic power dissipation from the three-dimensional (plane-strain) analysis of the Transcona elevator failure.

The advantage of using an analysis that is rigorous and requires no assumption to make it applicable to the problem at hand is that the only judgment that is required regards the values of shear strength to use in the analysis. The geometry is set and is easily input into the analysis. The estimated collapse load based on the amount of grain believed to be in the elevators at the time of collapse is subject to some uncertainty, so the difference of the order of 10% between calculated and observed bearing capacity is quite satisfactory. To put this comparison in perspective, Peck and Bryant (1953) used the bearing capacity equation proposed by Skempton (1951) which already contained early forms of shape and depth factors to estimate the collapse load. The equation that they used was:

$$q_n = q_{bL}^{net} = N_c c_u = 5\left(1+\frac{B}{5L}\right)\left(1+\frac{D}{5B}\right)c_u \qquad (4)$$

In order to use this equation, they needed to estimate a value of shear strength that would represent the entire soil mass, since it does not allow for layered soil, which of course requires considerable judgment. Peck and Bryant (1953) used two values of shear strength: a weighted average value of the undrained shear strength over the total thickness of the clay layers and the smallest shear strength for all layers. The effect of the strength of fractured limestone was neglected. A bearing capacity of 314 kPa, which is 7.2% greater than the observed unit load at collapse, was calculated using the average shear strength. Use of the smallest undrained shear strength produced a value of 240 kPa, which is 18% lower than the collapse unit load.

Since the variation of shear strength with depth affects the depth and shape of the slip mechanism in a way that cannot be foreseen without a suitable method of analysis (Salgado 2008), this is typically a difficult estimate to make based on judgment and without the aid of a more sophisticated method of analysis. Another element of uncertainty regarding the shear strength is that, based on the tests on samples from a limited number of borings, we established a depth profile of shear strength and assumed no variation of it in the horizontal direction. Lastly, given the composition of the clay at the site, with a nonnegligible percentage of montmorillonite, for example, its residual friction angle should be lower than its critical-state friction angle, which would suggest that some degree of progressive failure might have played a small role in the failure. This would be consistent with our collapse load estimated being slightly greater than the load believed to have been applied to the foundations at the time of collapse.

CASE HISTORY II: FARGO GRAIN ELEVATOR, 1955

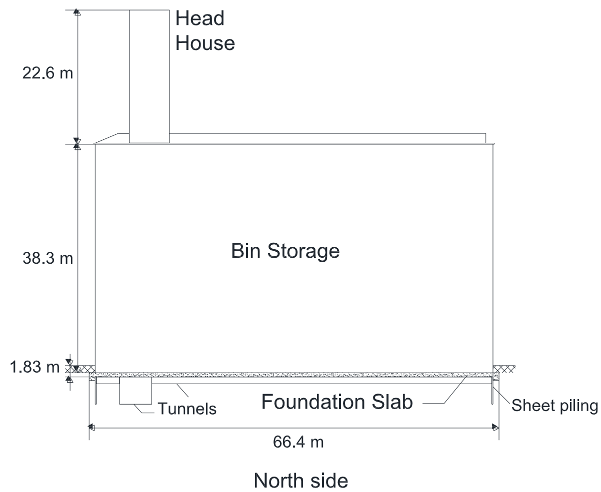
The Structure, the Failure and the Soil Profile

The Fargo grain elevator was a reinforced concrete grain elevator built during the summer and fall of 1954 about two miles west of Fargo, North Dakota (located in roughly the same geologic setting and 230 miles south of Transcona). It consisted of 20 circular bins that were arranged in two rows of 10 bins each and attached structures (Fig. 10(a) and (b)). The height and inside diameter of the bins were 37.2 m and 5.8 m. This structure rested on a reinforced concrete raft foundation that was 15.8 m wide, 66.4 m long and 0.71m thick. The outer 0.91 m edge around the structure was thickened to 1.32m. Except for this thickened edge, the base of the foundation was located 1.83 m below the ground surface. The raft foundation was interrupted locally by tunnels. There were also sheet piles that had been installed around the foundation; they should have had negligible effect on the performance of the raft.

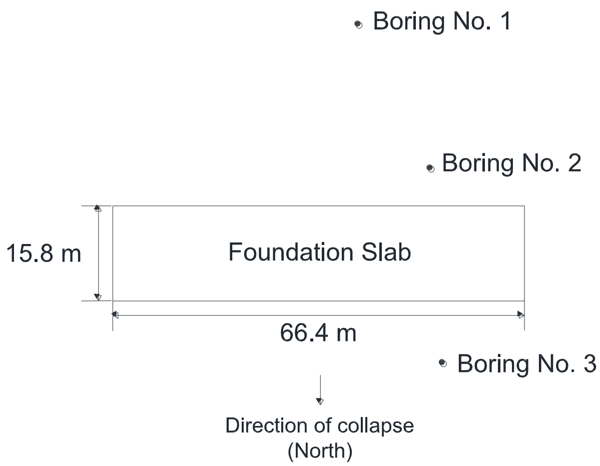
Until fall of 1954, only a small amount of grain had been stored in the elevator. The first time the elevator was filled with a large quantity of grain was April, 1955. Filling started then, and, in the early morning of June 12, 1955, the elevator collapsed and disintegrated. On May 10, 1955, after major filling had started, seven elevation benchmarks were installed, and the settlements of the foundation were recorded together with applied loads once a week after that. Fig. 11 shows the resulting load-settlement curve. The collapse happened when the unit load at failure was estimated at 260 kPa, with an estimated eccentricity of 0.96 m west and 0.03 m south from the centroid of the raft. Subtracting the weight of the 1.8m of soil excavated from the site, the net unit load at the time of failure was approximately 228 kPa. Failure produced a mass of concrete debris and grain on the north side of the original location of the structure. The ground bulged up as much as 1.83 m on the south side of the structure.

After collapse, three borings were performed at the site: boring No. 1 and 3 in zones largeky unaffected by the collapse and boring No. 2 in a zone disturbed by the collapse (Fig. 10 (b) shows the locations of the three borings). They revealed that the soil profile below the structure had three clay layers and one thin sand layer.

The undrained shear strength depth profile in the clay layers was estimated from the results of the field vane tests. In arriving at shear strength values from vane measurements, it is important to take into account rate effects, which increase with plasticity index (PI) (Bjerrum, 1972). Azzouz et al. (1983) suggested a correction factor that includes end effects, which would lead to an overestimate of shear strength if ignored. The undrained shear strength can then be obtained by multiplying the shear strength measured with the field vane by that correction factor.



(a)



(b)

Fig. 10 Fargo Grain Elevator: (a) Section view (b) Plan view.

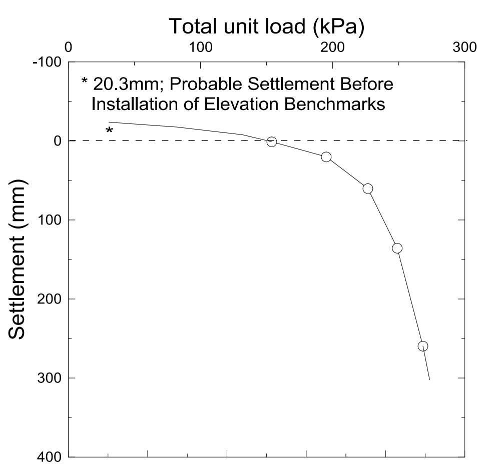


Fig. 11 Unit load versus settlement.

For the bottom clay layer, a credible range for $ds_u/d\sigma'_v$ was established based on the available CU test results, and this value optimized (resulting equal to 0.17) to best fit the vane test results. The natural water content, the density and the liquid and plastic limits were also obtained for various depths within each of the layers. The highest plasticity index observed for each clay layer was used to estimate the correction factor from Fig. 12, resulting in values of 0.75 for layer 1, 0.7 for layer 2 and 0.62 for layer 4. For the sand layer, only SPT results are available (Fig. 13 and Table 3). The ground water level was located at the depth of 1.90m. Although the borings extended down to a depth of approximately 20 meters, we know that, in this area of the country, glacial till is found at depths of approximately 30 meters and sound rock at depths exceeding 60 meters. On that basis, in Fig. 13, we have extended the shear strength observed down to a depth of 30 meters, where glacial till then begins. The shear strength profile used in our analysis of this failure is indicated in Fig. 13 through straight line segments; it is given numerically in Table 4.

Reassessment of the Case History using Modern Methods

Previous attempts to analyze this case history relied on the bearing capacity equation (Nordlund and Deere 1970). Different forms of the bearing capacity equation are applicable to clay with s_u that is either constant or increases linearly with depth (Salgado 2008); the present soil profile cannot be fit into either of these cases. An additional deviation is the presence of the sand layer. Nordlund and Deere (1970), like Peck and Bryant (1953), used the Skempton (1951) bearing capacity equation.

The friction angle for the sand layer was assumed as 25 degrees, a value that is clearly too low based on present knowledge of sand behavior. The slip surface was assumed to reach down to a depth of 2B/3 below the base of the footing. The method of slices was used to evaluate the shear strength of the sand layer. A weighted average of the shear strength between the base of the footing and the depth of 2B/3 below the base was used to calculate the bearing capacity. Nordlund and Deere (1970) explored three methods of estimating shear strength: from unconfined compressive tests on untrimmed samples, from field vane tests and from field vane tests with correction for progressive failure. The calculated unit limit bearing capacities were 229kPa, 344 kPa and 281 kPa; respectively, 12% lower, 32% higher and 8% higher than the collapse load. This means that an attempt to estimate the load at collapse, even with the foreknowledge of the right answer, using the traditional bearing capacity equation would yield an uncertainty of about 44% (minus 12% to plus 32%) of the collapse load. The progressive failure correction relied on taking mobilized shear strengths at the same levels of strain as estimated from laboratory tests. Load eccentricity was ignored.

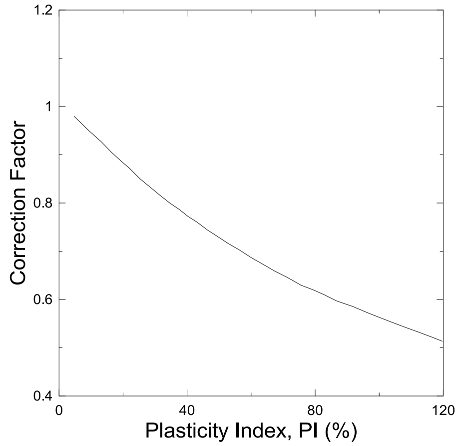


Fig. 12 Field vane correction factor (after Azzouz et al., 1983).

We have again used limit analysis to bound the collapse load from below and above. Starting with plane-strain analysis, Fig. 14 shows the mesh used in the analyses. Fig. 15 shows the soil mass distortion at collapse, and Fig. 16 shows the power dissipation throughout the soil mass. Both of these figures give an indication of the nature of the deformation associated with collapse. The lower bound limit unit bearing capacity ranged from 290 to 295 kPa as we varied the friction angle of the sand layer between 30 and 34 degrees, a credible range for loose sand.

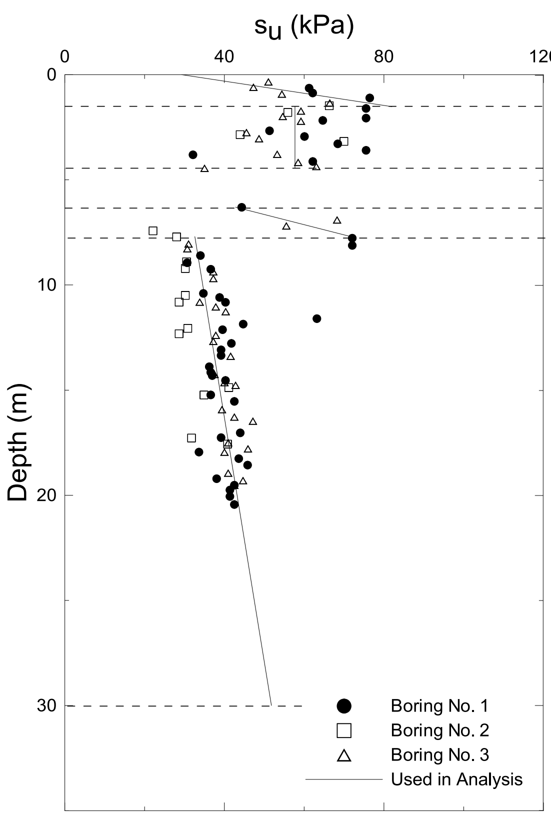


Fig. 13 Shear Strength with depth (Boring No. 1, 2 and 3).

Table 3 Soil profile of the Fargo Grain Elevator case

Layer No.	Depth (m)	Description	Properties	
1	0-1.52	Silty clay	γ (kN/m^3)	17.29
			(%)	30.5
			LL (%)	75
			PL (%)	30
2	1.52-4.45	Stratified clay and silt	γ (kN/m^3)	17.29
			wc (%)	43.2
			LL (%)	40-90
			PL (%)	30
3	4.45-6.30	Loose sand	wc (%)	31.9
			N_{SPT}	5-13
4	6.30-7.70	Desiccated clay crust	γ (kN/m^3)	14.93
			wc (%)	47.2 (6.1-7.7) 65 (7.7-19.51)
			LL (%)	105-115
			PL (%)	37
	7.70-30	Clay	Sensitivity	4

Note: wc = average water content; γ = unit weight

Table 4 Shear strength profile

Layer No.	Depth (m)	s_u or ϕ
1	0-1.52	29.3-82.0 kPa (linear increase)
2	1.52-4.45	57.7 kPa
3	4.45-6.30	ϕ = 30-34^o
4	6.30-7.70	42.8-71.7 kPa (linearly increase)
	7.70-30	32.6-51.8 kPa (linearly increase)

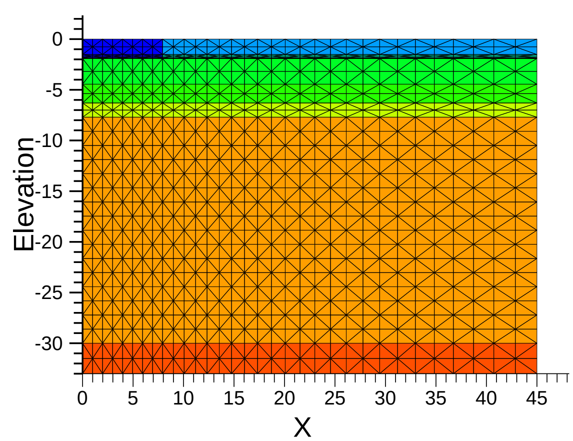


Fig. 14 Two-dimensional (plane-strain) mesh for the Fargo elevator.

Fig. 17 shows the 3D mesh. Fig. 18 and Fig. 19 provide indications of the deformation field within the soil at collapse. The lower bound was calculated as 310 kPa, and the upper bound as 378 kPa. As indicated earlier, the collapse load is expected to be much closer to the lower bound. This leaves our lower bound estimate about 15% higher than the estimated load at the time of collapse.

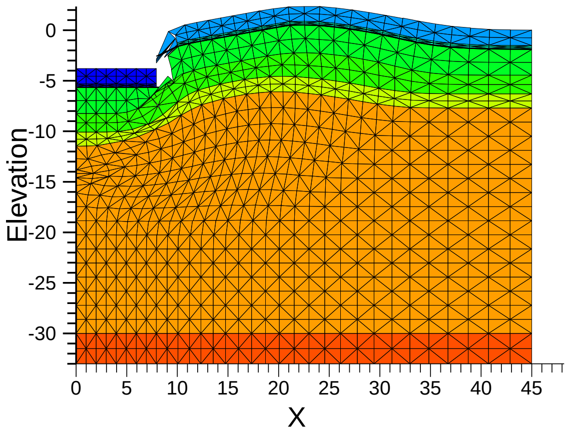


Fig. 15 Distorted two-dimensional (plane-strain) mesh for the Fargo elevator.

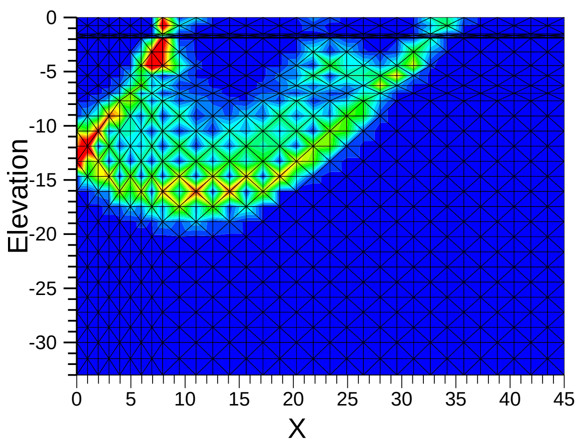


Fig. 16 Contours of plastic power dissipation from the two-dimensional (plane-strain) analysis of the Fargo elevator failure.

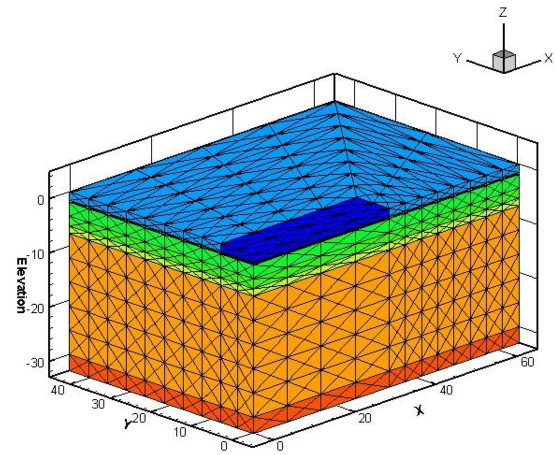


Fig. 17 Three-dimensional mesh for the Fargo elevator.

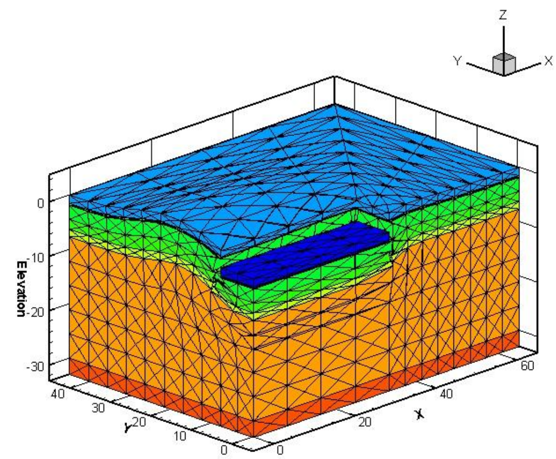


Fig. 18 Distorted three-dimensional (plane-strain) mesh for the Fargo elevator.

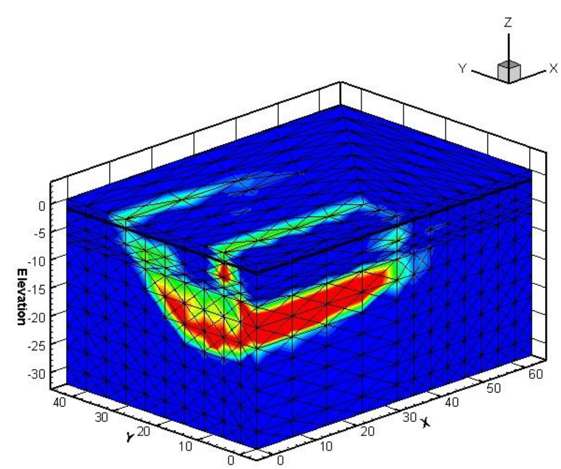


Fig. 19 Contours of plastic power dissipation from the three-dimensional (plane-strain) analysis of the Fargo elevator failure.

The Tower of Pisa Case History

The Tower of Pisa case history has been well covered in the literature. We follow here Salgado (2008) and provide only a summary of the essential facts needed for the focus of our discussion: the use of modern analysis methods to ascertain the controlling limit state for the Tower of Pisa. The discussion is largely based on work by Burland Potts (2003),

The Tower of Pisa is located in the city of Pisa, in Tuscany, Italy. The city is located on the Arno River, northwest of Rome. The Tower stands 54m tall and weighs 142,000 kN. The foundation is a spread foundation in the form of a hollow cylinder with outer diameter equal to 19.58m and inner diameter equal to 4.47m (Mitchell et al 1977). The hollow space appears to have been filled with rubble and mortar at the time of construction (Burland et al. 2003).

Construction of the Pisa Tower to its present height was done in several stages in the course of centuries. The first stage extended from 1173 to 1178. No construction activity took place during the next century. Construction was restarted in 1272, lasting until 1278. By 1272, it was evident that the Tower had started to lean, and masons attempted to correct for the leaning by placing stones on plumb (along a vertical alignment), not according to the Tower alignment. Because of this, the Tower is curved, much like a banana. Another century passed, and, in 1370, construction was completed after another decade of work. It is estimated that the lean of the Tower at that time was 3.5 degrees, corresponding to an angular distortion equal to 0.061 or approximately 1/16.

The tower is located on top of 300m of sediments deposited both by the Arno river and by the sea, at the time when the city was located in a coastal lagoon. Focusing on the layers nearer to the ground surface, the Tower rests on about 9 meters of dense river silts underlain by approximately 30 meters of marine clay. The foundations of the tower are shallow, approximately 20m in diameter and 3m in depth. Because the silt layer was more compressible on the south side of the Tower, the settlement developed faster there than on the north side, resulting in the Tower present inclination.

It is interesting to note the reason why no bearing capacity failure ever occurred. The century-long waiting periods between construction of the three stages of the Tower allowed the silts and clays to compress and strengthen (because denser soils are stronger), such that the soil was able to sustain the loads associated with subsequent construction. By 1838, the Tower had settled in excess of 3 meters and the base of the tower had completely disappeared into the ground. An architect named Gherardesca did not like the fact that people could no longer see the base of the Tower and had a walkway excavated around the Tower. This decision was certainly not a good idea from an engineering standpoint, as the removal of ground support only accelerated the Tower inclination. By 1911, the inclination had reached 5.4 degrees; by 1990, it was 5.5 degrees, with no signs of stabilization.

Altogether, there have been 17 commissions set up to assess the stability of the Tower of Pisa over the nineteenth and twentieth centuries. The process was always quite political and usually resulted in no measures being implemented. The 17th commission was set up in 1990, with Professor Michele Jamiolkowski of the Technical University of Turin as the chair. Creation of the commission happened at a time when memories of the collapse of the Tower of Pavia, a city located just North of Milan, Italy, were fresh. This was certainly helpful in overcoming political resistances that had been a problem for previous commissions.

One of the first moves of the new commission was to reinforce the lowest story of the Tower using prestressed steel wires. This was done as long delays related to the politics surrounding the work of the commission were expected, and there was concern that the masonry composing the southern wall of the Tower was severely overstressed. In order to temporarily stabilize this rotation of the Tower, a post-tensioned concrete ring was built around the base of the Tower and 6000kN of lead ingots stacked on it on the north side. The lead ingots did stabilize and even reverse the lean slightly. The lead ingots were not intended as a permanent solution, as the intent was always to reopen the Tower to visitation by tourists, and the ingots were considered a visually unattractive solution. We will return to this measure in the context of our limit state discussion later. The commission later decided to proceed with soil extraction from under the north side of the Tower as a definitive solution for its stabilization.

In late 1996, pilot tests of the under-excavation technique were done. The technique had been successfully used in the stabilization of the Mexico City Cathedral in the 1980's. The idea is simple (see Figure 2 22): to carefully and gradually remove soil from underneath the north side of the Tower so that it will settle, therefore reducing the lean. However, there were members of the commission that had reservations about the technique. A concern expressed by one member of the commission was that soil under-excavation might actually accelerate the leaning and even lead to collapse of the Tower by removing support (load-carrying capacity) from the Tower foundations and further stressing the already overloaded south side. A more natural expectation (subject to considerations discussed later), however, would be that careful, slow extraction of soil would allow overlying soil to move down to occupy the newly created space, moving the Tower down with it. This is indeed what happened, for the tilt decreased sharply with the start of drilling at the end of 1999, extending throughout the year 2000, and finally stabilizing in 2001. The Tower now has the same inclination it had in 1800.

Bearing Capacity or Leaning Stability

While in 1991 it was still believed that it would take tens of years for the Tower to reach a state of collapse, we now know that the geotechnical experts on the commission in charge of stabilizing the Tower came to believe in subsequent years that there were moments of grave danger of a collapse. In fact, according to Potts (2003), the Tower was likely dangerously close to collapse in the 1990-91 period. However, Mitchell (1977) argued that the factor of safety against bearing capacity failure appeared to be sufficient at all times of the Tower's existence (even if others have questioned by how much it exceeded one at certain times of the Tower's life). So what was the nature of the feared collapse of the Tower?

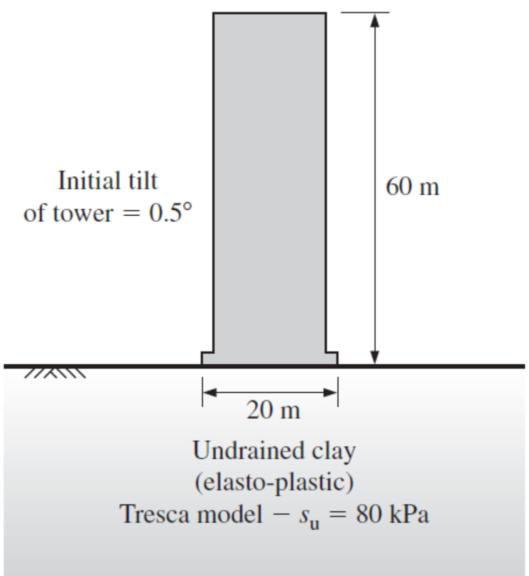


Fig. 20 Tower with an initial lean of 0.5 degrees on top of a Tresca soil (clay) with undrained shear strength of 80 kPa (redrafted after Potts 2003). Used with permission from ICE Publishing.

The beginnings of an answer lie in the Mitchell et al. (1977) observation that the maximum shear stress at points or within zones in the clay layer matched or exceed the estimated soil shear strength, even if the factor of safety was equal to 2. This indicated that a plastic zone had likely formed below the south end of the Tower. Potts (2003) provides a simple example that illustrates the type of collapse that the Tower of Pisa could have experienced. This type of collapse, known as leaning instability, is closely associated not only with the shear strength of the soil, which we use in our bearing capacity calculations, but also with the soil stiffness, as represented by its shear modulus. Fig. 20 shows a tower (with geometry quite similar to that of the Tower of Pisa) on top of a clay with s_u = 80kPa. The tower is built with an initial inclination of 0.5˚. A finite element analysis then simulates the soil-tower response as the weight of the tower is gradually increased for three different values of soil stiffness: G = 10s_u, G = 100s_u, and G = 1000s_u. Note that, since the value of s_u is the same in all three cases, the weight at which collapse would occur would be the same if bearing capacity were the mechanism of collapse. Instead, as shown in 9, collapse takes place for a much lower tower weight in the case of low stiffness (G/s_u = 10). It also happens quite suddenly. Why the difference?

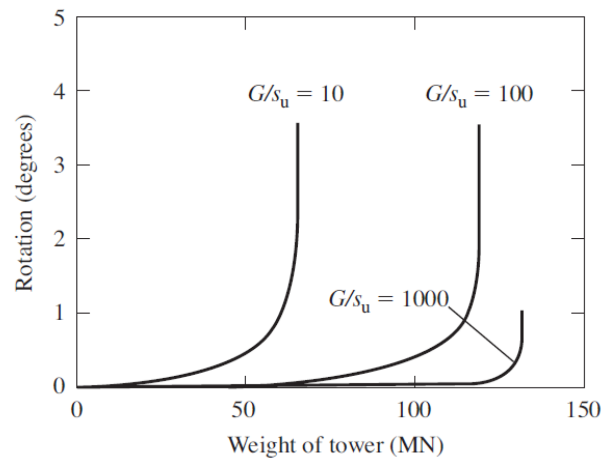


Fig. 21 Weight leading to collapse of the tower versus relative stiffness G/s_u of the foundation soil (redrafted after Potts 2003). Used with permission from ICE Publishing.

Fig. 22 shows a plot of incremental displacements at the last loading increment (just before collapse) for the case of low stiffness. It also shows that the shear stress in the soil becomes equal to the shear strength (forming a plastic zone, represented as a shaded zone) in only a portion of the soil deposit, and a plastic mechanism does not form. So how does collapse occur? In essence, it occurs through an overturning failure. The low stiffness of the soil as it enters the plastic range does not provide enough support beneath the right edge of the tower after a plastic zone forms there to balance the moment of the weight of the tower with respect to the center of the foundation. Contrast that with the full plastic mechanism that forms below the tower in the high-stiffness-ratio case, shown in Fig. 23. Here, clearly, we have a bearing capacity failure, which is a very different mechanism from leaning instability, hence the difference in tower weights for which these two failures are observed.

Understanding the potential mechanism of instability for the Tower of Pisa was not merely an academic exercise. For example, using lead ingots on the north side of the Tower to stabilize it, as described earlier, would only work, as argued by Potts (2003), if the prevailing mechanism was leaning instability, in which case the lean would reduce upon placement of the ingots. If a bearing capacity failure had been in progress, the ingots would actually have precipitated failure. The same can be stated regarding underexcavation.

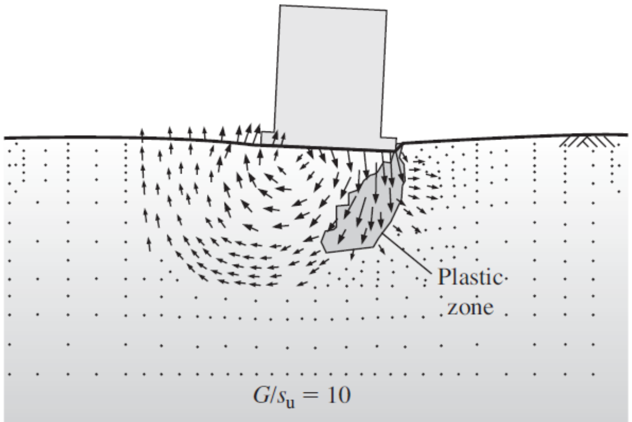


Fig. 22 Displacement field and plastic zone for soft (low-shear modulus) soil (redrafted after Potts 2003). Used with permission from ICE Publishing.

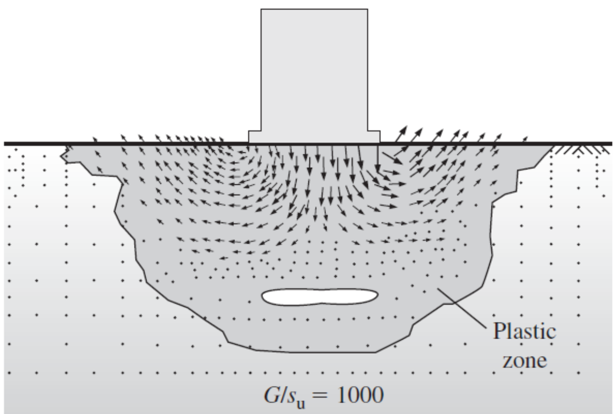


Fig. 23 Displacement field and plastic zone for stiff (high-shear modulus) soil (redrafted after Potts 2003). Used with permission from ICE Publishing.

CONCLUSIONS

The practice of foundation design has relied extensively on empiricism and relatively simple analyses (such as the bearing capacity equation). This was needed because crucial elements in a rigorous analysis of soil mechanics problems were missing until very recently: computation power, rigorous methods for analyzing elasto-plastic boundary-value problems and realistic constitutive models.

Our discipline is in a state of transition. The science of soil mechanics has developed considerably in the last 20-30 years. The progress in the science is gradually finding its way into practice, which still overwhelmingly relies on traditional methods. The evaluation of case histories using modern methods of analysis is a useful way to show the usefulness of these methods. In this paper, we have used finite-element limit analysis (FELA) and the finite element method (FEM) to reveal features of foundation engineering problems that would not otherwise be detectable with simple methods. We have done so without resorting to sophisticated constitutive models, relying instead on the well-known Tresca yield surface for clay in all three case histories and linear elasticity in the last of three case histories examined, but in more complex problems, certainly those involving frictional soils, more realistic soil models would be required.

In the two cases in which a bearing capacity collapse was observed, we showed that use of the bearing capacity equation is awkward, requiring a number of assumption to make it applicable to the problems. For example, the bearing capacity equation does not accept soil layering (and, in fact, forms of it exist only for either uniform strength with depth or linear increasing strength with depth) and cannot mix sand and clay in the same soil deposit. In the last case history, we showed that use of the bearing capacity equation to assess the potential collapse of a tall, leaning structure would be completely incorrect if the ratio of soil strength to soil stiffness is high, which means only a more sophisticated method of analysis could realistically be used.

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