International Conference on Case Histories in Geotechnical Engineering

(2013) - Seventh International Conference on Case Histories in Geotechnical Engineering

02 May 2013, 7:00 pm - 8:30 pm

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Commemorate the Legacy of Ralph B. Peck
Seventh International Conference on Case Histories in Geotechnical Engineering
and Symposium in Honor of Clyde Baker
April 29 - May 4, 2013 CHICAGO

PROBABILISTIC THREE-DIMENSIONAL MODEL OF AN OFFSHORE MONOPILE FOUNDATION: RELIABILITY BASED APPROACH

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ABSTRACT

When wind turbines are to be installed offshore, expensive geotechnical in-situ tests are carried out at the location of each turbine and only a quantile value (typically the 5% quantile) of the measured strength parameters is used as design parameter, *e.g*., the 5% quantile value of the undrained shear strength of the soil. Typically, measurement, statistical and model uncertainties are not taken into account in code-based, deterministic design. Hence, current methodology based design may be expensive, but the reliability of the foundation is unknown. Instead, a reliability-based design process based on stochastic analysis of the soil parameters is proposed to obtain an efficient design with known reliability and smaller costs for tests and construction. In this study a monopile foundation in undrained, over-consolidated clay is considered as an example. A three-dimensional (3D) finite-element model is established and a stochastic model for the undrained shear strength of the soil is proposed using random field theory. The Mohr–Coulomb constitutive model is used to model the soil behavior. Reliability indices of the monopile are obtained through an advanced reliability method and a probabilistic procedure is proposed regarding the 3D design of monopile foundations.

INTRODUCTION

Designing offshore wind turbine foundations concerns several uncertainties due to material properties, measurement techniques and/or modeling procedures. These uncertainties are usually not accounted for, or they are neglected by introducing either partial safety factors on material properties or total safety factor on the resistance and/or on the loads. This is the strategy which is typically utilized in the deterministic design methodologies in the current design codes. In this regard, expensive geotechnical in-situ and laboratory tests are conducted to estimate soil properties, but only deterministic values (*e.g.*, a 5 percent quantile value) of them are used for design. Furthermore, the reliability of the structure remains unknown in this procedure. Instead, by a reliability-based design procedure, a design is obtained where uncertainties are accounted for in a rational way. Furthermore, this can be cost effective using stochastic parameters of uncertain properties which are already estimated through an optimized field investigation for the whole region (*e.g.*, a wind farm). This investigation can be cheaper than individual testing for each wind turbine in a wind farm. It can also be noted that applying a stochastic design approach, partial or total safety factors in the deterministic design can be calibrated or modified and used in future designs.

Several studies were conducted for developing stochastic models of foundations. The bearing capacity of a footing placed on the soil surface was predicted analytically and verified via Monte Carlo simulation (MCS) when considering spatially random fields for the cohesion and the friction angle of the soil (Fenton and Griffiths, 2003). Fenton and Griffiths (2007) also studied the effect of soil spatial variability on the settlement and ultimate load statistics of a pile. Andersen et al. (2011) proposed a reliability-based design procedure for estimating the first natural frequency of an offshore wind turbine founded on a monopile. They applied a random field model for the undrained shear strength of clayey soil. In a similar study by Andersen et al. (2012), an advanced reliability method was proposed to estimate rare events of the first natural frequency of an offshore monopile foundation. Vahdatirad et al. (2011) studied the application of a stochastic dynamic stiffness model for a surface footing for an offshore wind turbine. They used a semi-analytical model in combination with Monte Carlo Simulation (MCS) for estimating the distribution of the footing stiffness. In another study, Vahdatirad et al. (2012) estimated the stochastic stiffness of a laterally loaded offshore monopile modeled by a one-dimensional Finite Element Method (FEM) model. They considered a nonlinear *p-y* curve for the modeling of the soil stiffness and applied an Asymptotic Sampling (AS) method to estimate rare events of the monopile stiffness.

In the present study, a 3D finite-element model for a monopile foundation in undrained, over-consolidated clay is developed and utilized as computational model. The geometrical and material properties of the monopile are close to the real site conditions for monopile foundations for large offshore wind turbines in the North Sea. The rotation at the pile cap is considered as a representative failure mode according to the offshore standard (DNV, 2007). Three failure modes are considered: a serviceability limit state, an ultimate limit state and a fully established failure in soil material (see the section "Model for limit state and design equations"). A reliability analysis is performed for these failure states by means of the AS method.

COMPUTATIONAL MODEL

A 3D finite element model has been constructed in the Abaqus numerical package by scripting in Python. Scripting in Python has the advantage that parametric analysis can be performed and used in the reliability assessment.

Continuum 8-node solid elements (C3D8) were used for soil as proposed by Kellezi and Hansen (2003), as well as Abdel-Rahman and Achmus (2006). Incompatible-mode 8-node solid elements (C3D8I) were used for the monopile in order to model the bending along the pile. A master-slave concept was used for interaction between the monopile and the surrounding soil (Abdel-Rahman and Achmus, 2006). A tie constraint was used between the monopile and the soil elements inside the monopile. The tangential behavior with a friction coefficient of 0.67 was applied for modeling the frictional behavior between the monopile and the surrounding soil. Furthermore, the linear pressure-overclosure relationship with a contact stiffness of $10^{10}\,\mathrm{N/m^2}$ was introduced in order to model the normal behavior at the interaction.

An elastic–perfectly plastic Mohr-Coulomb constitutive model is used for the soil behavior. This is implemented by a user-defined material subroutine (UMAT) which has been written in Fortran. Using this subroutine, material random properties can be defined as solution-dependent state variables (SDV) in each integration point of the soil element.

A stepwise execution is conducted for the finite element analysis. In this regard, the geostatic step is first performed for generation of the initial stress state using soil elements having a submerged unit weight only. Afterwards, the gravity loads of monopile elements with a submerged unit weight are applied in a consolidation step. It is noted that a water density of 1000 kg/m^3 and gravitational acceleration of 9.81 m/s^2 are used for calculating the submerged unit weights. Finally, a combination of shear and bending moment is considered as external loads at the pile cap. It is assumed that the wind force is dominating and applied at a height of 61.5 m above the monopile cap with a horizontal direction. The amount of this load must be considered large enough such that the lateral deformations plastify the soil completely and a full failure mechanism is achieved. This is ensured by the value of 52×10^6 N.

Table 1 shows the geometrical and material properties of the monopile. As shown in this table, a free length above the soil layer is considered for the monopile. This prevents the soil to go over the pile during failure, which is not corresponding to the real situation.

Table 1. Geometrical and material properties of the monopile

Geometrical properties	Outer radius (m)	Thickness (m)	Length (m)	
			Embedded	Free
	3.00	0.06	35.00	4.50
Material properties	Elastic modulus (N/m^2)	Density (kg/m^3)	Poisson's ratio	
	2.05×10^{11}	7872	0.29	

The undrained shear strength (C_u) of clayey soil is modeled by a LogNormal random field. This random field is used as soil cohesion in the Mohr-Coulomb constitutive model. An increasing trend over the depth is considered for the mean value of C_u (see Table 2). Furthermore, a linear correlation between C_u and soil initial elastic modulus E_0 is assumed as $E_0 = k_s C_u$ where $k_s = 200$ is the coefficient for over-consolidated clay. Ideally, a cross-correlation should be applied between C_u and E_0 (Fenton and Griffiths, 2003), but the linear relationship is applied as an approximation.

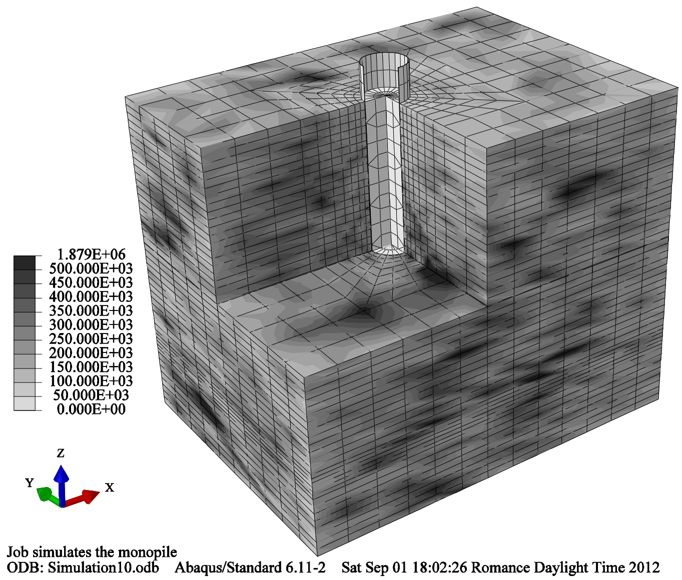

Fig. 1. Mapping of the three-dimensional random field for C_u in the applied finite-element model.

Table 2. Deterministic and stochastic properties of the soil

Deterministic properties		Stochastic properties			
Density (kg/m^3)	2200	Lognormally distributed undrained shear strength, C_u (N/m^2)	COV		0.40
Poisson's ratio	0.499		Mean value, μ (N/m2)		$1.5\times10^5+2000z$ (z is layer depth in meter)
Friction angle (degree)	0.01		Correlation length (m)	Horizontal, δ_x	8.00
Dilation angle (degree)	0.01			Horizontal, δ_y	8.00
Lateral earth pressure coefficient	1.00			Depth, δ_z	2.50
		Elastic modulus, E_0 (N/m^2)			$E_0 = 200C_u$

For generating the random field, the turning bands method (TBM) is utilized. This method was originally proposed by Matheron (1973) and can be used for generation of realizations of a random field in a three-dimensional space by using a sequence of one-dimensional processes along lines crossing the domain. An exponential 3D correlation function (ρ) is used as proposed in (JCSS, 2006):

$$\rho = \exp\left(\pi \times \left(-\frac{|\Delta x|}{\delta_x} - \frac{|\Delta y|}{\delta_y} - \frac{|\Delta z|}{\delta_z}\right)\right) \tag{1}$$

where Δx, Δy and Δz are spatial distances in the x, y and z directions, respectively. Further, $\delta_x = \delta_y$ is the correlation length in the horizontal directions and δ_z is the correlation length in the depth direction (see Table 2). The deterministic and stochastic soil properties are shown in Table 2.

A Matlab script has been developed for generating the random field by TBM. The variables are saved as SDV and mapped on each integration point of a soil element by the UMAT subroutine during the analysis. Figure 1 presents a realization of the random field for C_u. The black regions in this figure show the stronger parts with higher value of C_u, whereas the white regions represent the weaker parts.

Figure 2 illustrates plastic strains around the monopile at the failure state for the same realization as in Fig. 1. This example shows that a fully developed failure mechanism is obtained due to the large lateral deformations. The boundaries of the computational domain are placed far enough away from the pile inasmuch as there are no plastic strains near the boundaries. As shown in Fig. 2, some parts close to the monopile and inside the failure region are not plastified, which are representing the stronger area having higher values of C_u.

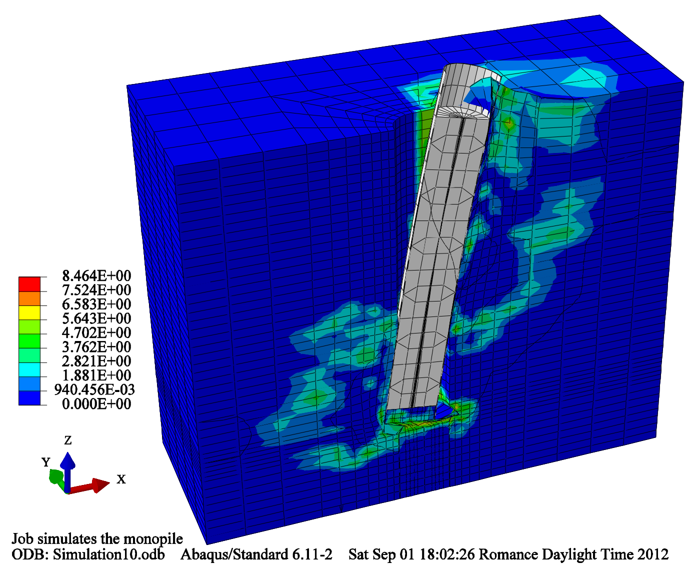

Fig. 2. Plastic strains at fully developed failure mechanism.

MODEL FOR LIMIT STATE AND DESIGN EQUATIONS

A generic form of a limit state function g is defined by two basic variables, namely the load P and the resistance or load bearing capacity Y, given as:

$$g = Y - P \quad (2)$$

This function is defined such that positive values of g correspond to safe states and negative values correspond to failure states. The load P and the resistance Y are supposed to be functions of relevant uncertainties, see below. In this study, Y is assumed to be assessed by the following model:

$$Y = \theta\, R\left(\mathbf{X}, \mathbf{W}\right) \quad (3)$$

where $\mathbf{X}$ is the vector of random variables modeling soil strength parameters (here the undrained shear strength of clayey soil), $\mathbf{W}$ is a set of deterministic parameters such as monopile properties or deterministic soil properties, $R()$ represents the model for the load resistance which in this paper is represented by the FEM model described above. Finally, θ accounts for the model uncertainty, see Table 3.

A representative, simple load model is assumed to consist of several uncertainties (see, *e.g.*, Sørensen & Toft, 2010):

$$P = X_{dyn} X_{\exp} X_{aero} X_{str} L \quad (4)$$

where X_{dyn} accounts for uncertainty related to modeling of the dynamic response, including uncertainty in damping ratios and natural frequencies, $X_{\exp}$ models the uncertainty related to the modeling of the exposure such as the terrain roughness and the land space topography, X_{aero} accounts for uncertainty in assessment of lift and drag coefficients, X_{str} is uncertainty related to the computation of the load-effects-given external load, and L is uncertainty related to the extreme load-effect due to wind loads. The uncertainties in this study are assumed to be representative for normal operation of wind turbines (IEC 61400-1, 2005). The proposed statistical parameters for the uncertainties in Eq. (4) are shown in Table 3.

To obtain the distribution of the annual maximum load effect L with considered coefficient of variation (see Table 3), its characteristic value L_c is determined such that the following design equation is fulfilled:

$$Y_d - \gamma_f L_c = 0 \quad (5)$$

Table 3. Stochastic models for physical, model and statistical uncertainties

Variable	Distribution	Mean	COV	Quantile
R	Lognormal	-	0.50	5%
θ	Lognormal	-	0.50	5%
L	Weibull	-	0.15	98%
X_{dyn}	Lognormal	1.00	0.05	Mean
X_{exp}	Lognormal	1.00	0.20	Mean
X_{aero}	Gumbel	1.00	0.10	Mean
X_{str}	Lognormal	1.00	0.03	Mean

where Y_d is the design value of the load resistance which can be obtained from the FEM response by applying characteristic values of material parameters and γ_f is the partial safety factor for the load effect, see Table 4. Three possibilities are considered to obtain the design value of the load bearing capacity:

1. Model one: Y_d is determined using the characteristic value of soil strength parameter applying partial safety factors for material properties:

$$Y_d = \eta \theta_c R\left(\frac{C_{uc}}{\gamma_m}, \mathbf{W}\right) \quad (6)$$

   where C_{uc} is the characteristic value of undrained shear strength, see Table 2, γ_m is the partial safety factor for the material parameter, see table 4, θ_c is the characteristic value of the model uncertainty θ in table 3, and η is a conversion factor, accounting for bias in the model $R()$.

Table 4. Partial safety factors for design equations (partly based on IEC 61400-1, 2005)

Variable	Value
Partial safety factor for load effect, γ_f	1.35
Partial safety factors for material properties, γ_m	1.3
Conversion factor, η	1.00*
Partial safety factor for load resistance, γ_R	1.3

* Corresponding to no conversion (hidden) in the models.

2. Model two: Y_d is determined from the characteristic value of load bearing capacity applying a partial safety factor for resistance:

$$Y_d = \eta \frac{Y_c}{\gamma_R} \quad (7)$$

where Y_c is the characteristic value of the resistance Y obtained from Eq. (3) and quantile values of $R()$ and θ, and γ_R is the partial safety factor for the resistance, cf. Table 4.

3. Model three: Y_d is determined from the characteristic value of the random variable applying a partial safety factor for the resistance:

$$Y_d = \eta \frac{\theta_c R\left(C_{uc}, \mathbf{W}\right)}{\gamma_R} \tag{8}$$

Based on the above models, a representative limit state function *g* can be written:

$$g = \theta\ R\left(\mathbf{X}, \mathbf{W}\right) - X_{dyn} X_{\exp} X_{aero} X_{str} L \tag{9}$$

In the present study, three limit states for failure are considered based on the rotation of pile cap (DNV, 2007). These levels are expressed as:

1. Serviceability limit state (SLS) where the rotation of the monopile cap is limited to 0.25 degrees.
2. Ultimate limit state (ULS) where the rotation of the monopile cap is limited to 3 degrees.
3. Fully developed failure limit state (FLS) where the lateral deformations of the pile are sufficiently large to plastify the soil completely (total collapse of the soil is achieved).

These failure criterions are considered through the reliability analysis, and the probability of failure for each level of failure is estimated.

RELIABILITY ANALYSIS

Considering the limit state function given in Eq. (9), the annual probability of failure can be written:

$$P_f = P(g \leq 0) \tag{10}$$

Typically, a maximum annual probability of failure of the order 10^{-3} to 10^{-4} is required for critical wind turbine structural components. If crude Monte Carlo simulation (CMCS) is applied, 10^4 to 10^5 realizations are needed to obtain a coefficient of variation of 0.3 for the probability estimate. Simulation of this amount of realizations implies high computational cost inasmuch as one realization takes around 15 minutes. Hence, application of advanced reliability methods is required such that fewer realizations are needed. In this study, asymptotic sampling (AS) is applied to estimate the probability of failure and the corresponding reliability index. AS is an advanced Monte Carlo simulation method originally proposed for high-dimensional reliability analysis by Bucher (2009). Sichani et al. (2011a) developed this method for high-dimensional dynamics problems such as wind turbines. Asymptotic sampling was utilized as an efficient method for estimating low first passage probabilities of high-dimensional nonlinear systems (Sichani et.al, 2011b). Andersen et al. (2012) applied this method to estimation of rare events of the first natural frequency of an offshore wind turbine founded on a monopile. In another study, Vahdatirad et al. (2012) proposed an improved AS method to estimate the stochastic stiffness of a monopile foundation by the FEM.

The basic idea of AS is to generate more simulations in the target region (the failure domain) by increasing the excitation power (Bucher, 2009). For this reason, the standard deviations of the random variables are increased artificially by the factor of $1/f$ to scale the results into the failure region. Then, the scaled reliability index $\beta(f)$ corresponding to scaled results is estimated. $\beta(1)$ represents the un-scaled reliability index at failure (Bucher, 2009). Therefore, this relationship enables an estimation of $\beta(1)$ by extrapolation techniques and curve fitting. The implemented procedure and more details can be found in (Bucher, 2009; Andersen et al., 2012). Herein, the fitting equation proposed by Bucher (2009) is used:

$$\frac{\beta(f)}{f} = A + \frac{B}{f^2} \tag{11}$$

where *A* and *B* are coefficients which are determined through a regression analysis. Then, the reliability index at failure can be estimated as:

$$\beta(1) = A + B \tag{12}$$

Hence, the probability of failure can be expressed as:

$$P_f = \Phi(-\beta(1)) \tag{13}$$

where Φ is the standardized Gaussian distribution function.
Several values of the f factor were considered, including: 1.0, 0.8, 0.6, 0.5, 0.4 and 0.3. Choosing the f factor is related to the desired probability level and having enough points for the curve fitting by Eq. (11). 400 realizations are made at each value of the f factor, leading to 2400 realizations in total. For each realization of the limit state function, one realization of the resistance Y in Eq. (3) is obtained by means of the FEM and the f factor used for increasing the standard deviation of random variables (here C_u). A corresponding realization of the load P in Eq. (4) is obtained be simulation using the same f factor. Having realizations of the resistance Y and the load P, a realization of the limit state value g can be determined from Eq. (2) or Eq. (9).

Figure 3 shows ascending sorted values of the limit state function versus number of realizations. As shown in this figure, the number of samples in the failure domain ($g \le 0$) increases by decreasing the f factor. The related probability of failure to each f can be estimated as:

$$P_f^{\,f_i} = \frac{1}{N}\sum_{m=1}^{N} I_m ; \begin{cases} I_m = 1 \text{ if } g_{f_i}^m \le 0 \\ I_m = 0; \text{otherwise} \end{cases} \tag{14}$$

where $N = 400$ is the number of realizations, $f_i, i = 1, 2, ..., n$, is the considered f factor and $g_{f_i}^m$ is the mth sample of realizations for an f factor of f_i. The corresponding reliability index $\beta(f_i)$ is determined as:

$$\beta(f_i) = \Phi^{-1}(1 - P_f^{\,f_i}) \tag{15}$$

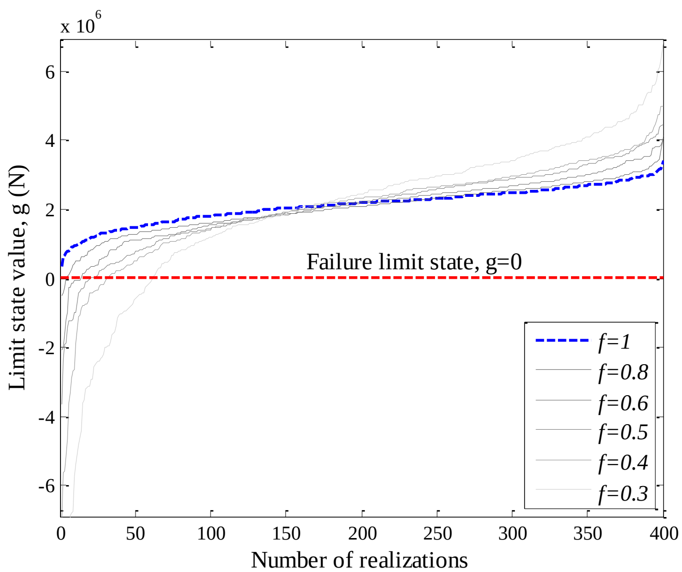

Fig. 3. Ascending sorted limit state values (g) versus number of realizations.

Applying the reliability indices obtained from Eq. (15) into Eq. (11), a system of equations is constructed as:

$$\begin{bmatrix} 1, \dfrac{1}{f_1^{\,2}} \\ 1, \dfrac{1}{f_2^{\,2}} \\ .,. \\ 1, \dfrac{1}{f_n^{\,2}} \end{bmatrix} \times \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \dfrac{\beta(f_1)}{f_1} \\ \dfrac{\beta(f_2)}{f_2} \\ . \\ \dfrac{\beta(f_n)}{f_n} \end{bmatrix} \tag{16}$$

Solving Eq. (16), the coefficients A and B are determined. Next, the un-scaled reliability index $\beta(1)$ and the probability of failure P_f are estimated through Eq. (12) and Eq. (13), respectively.

Three models of the design equation presented in the previous section were used in the reliability analysis. Figures 4 to 6 illustrate the reliability indices of the monopile using the three design equations and three levels of failure defined by limit states SLS, ULS and FLS. The AS fitted curves for finding $\beta(1)$ as well as reliability indices corresponding to different f factors are illustrated in these figures. As shown in these figures, the reliability index $\beta(1)$ using the FLS definition (as expected) for all models is the largest, and the smallest one is obtained by the SLS. This is in agreement with the design concepts inasmuch as the probability of failure in the FLS must be less than those using the ULS or SLS.

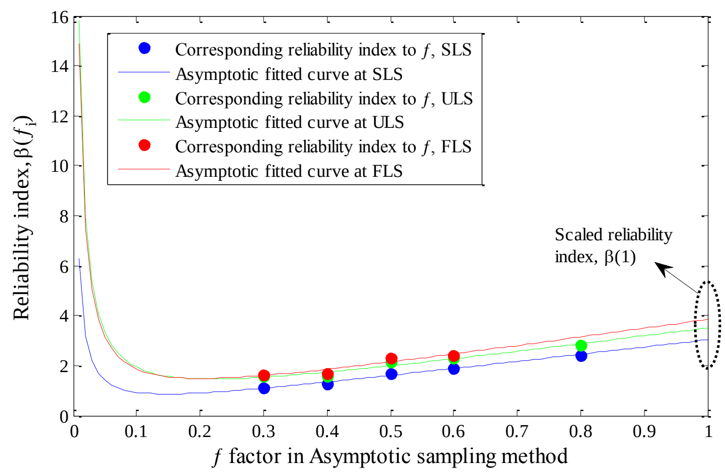

Fig. 4. Reliability indices by AS method for three levels of failure—design equation based on model 1.

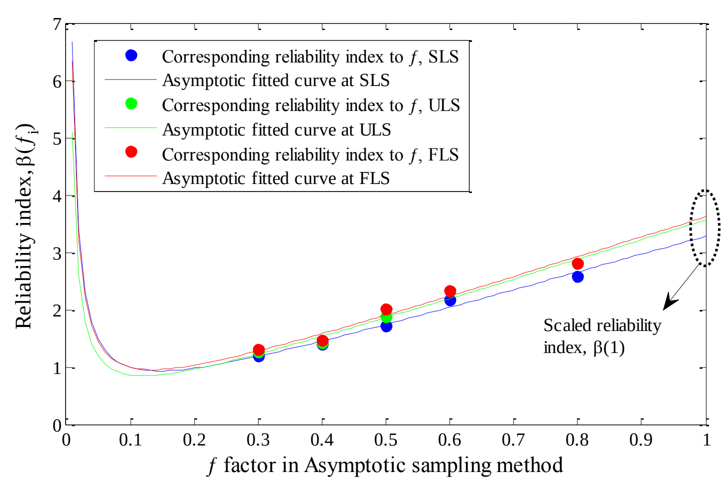

Fig. 5. Reliability indices by AS method for three levels of failure—design equation based on model 2.

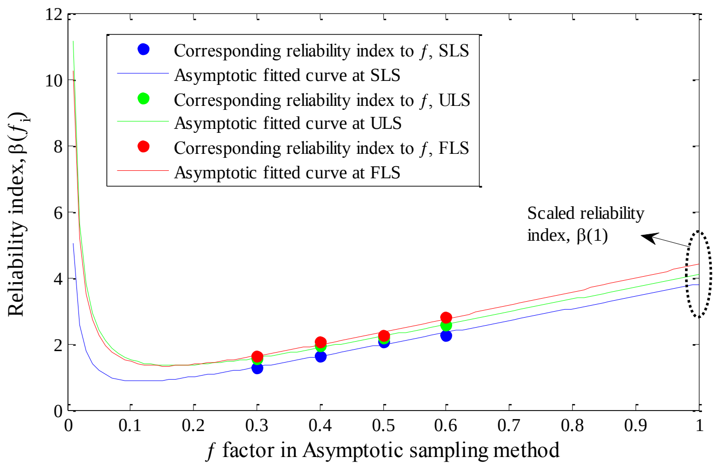

Fig. 6. Reliability indices by AS method for three levels of failure—design equation based on model 3.

Values of the reliability indices and probabilities of failure for the different design models are shown in Table 5. As shown in this table, design model 3 results in smaller probabilities of failure compared to the other models. This illustrates the importance of choosing the model to obtain design values in a deterministic approach, *i.e.* how to apply the partial safety factors in the design equation. Equivalently, the reliability-based procedure can be used to calibrate/modify the partial safety factor for the soil properties such that a given target reliability is obtained.

Table 5. Reliability indices and probability of failures for the three failure modes

Failure modes		Design models		
		1	2	3
SLS	Reliability index (β)	3.0	3.3	3.8
	Probability of failure (P_f)	1.2×10^{-3}	5.2×10^{-4}	7.1×10^{-5}
ULS	Reliability index (β)	3.5	3.6	4.1
	Probability of failure (P_f)	2.2×10^{-4}	1.8×10^{-4}	1.9×10^{-5}
FLS	Reliability index (β)	3.9	3.6	4.4
	Probability of failure (P_f)	5.8×10^{-5}	1.5×10^{-4}	5.3×10^{-6}

CONCLUSIONS

A reliability analysis was performed for an offshore monopile foundation. A stochastic 3D finite element model was developed for undrained, over-consolidated clay. The undrained shear strength of the soil was considered as uncertain having a lognormal distribution based on the concept of random field theory and spatial variation. The turning-bands method was utilized to generate realizations of the random variables in the 3D random field. These variables were mapped on each integration points of the soil elements by a user defined subroutine in Fortran. Three design equations were proposed for reliability analysis at three levels of failure for SLS, ULS and FLS. The asymptotic sampling method was used for performing the reliability analysis. Furthermore, the design equation for model 3 results in the most conservative results. The reliability-based procedure can be used to calibrate/modify the partial safety factor for the soil properties such that a given target reliability is obtained, thus resulting in more optimized designs.

ACKNOWLEDGEMENTS

The authors kindly acknowledge financial support from the Danish Council for Strategic Research within the program "Reliability-based analysis applied for reduction of cost of energy for offshore wind turbines".

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