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THE UNIVERSITY OF MISSOURI AT ROLLA

THE STATISTICAL MECHANICS  
OF KINK INTERACTIONS

by

Michael Brewer McNeil

A THESIS SUBMITTED  
IN PARTIAL FULFILLMENT OF THE  
REQUIREMENTS FOR THE DEGREE OF  
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## ABSTRACT

Using a potential derived from continuum elasticity, the interactions between kinks on a dislocation line are studied as special cases of the problem of a one-dimensional system of interacting particles. By the use of constant pressure ensembles, the principal kink-interaction problems of physical interest are solved. The application of these solutions to the theoretical analyses of work hardening and internal friction is explored and comparisons made to previous work in which kink interactions were neglected; it is found that, as would be expected, variations of equilibrium kink populations with external stress cannot account for the Bordoni peak, which must therefore be identified with a relaxation time phenomenon. A new type of internal friction is predicted, with a logarithmic decrement independent of frequency but showing a sharp amplitude dependence opposite in slope to that previously predicted from consideration of kinks.

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## PARTIAL LIST OF SYMBOLS USED

A	Helmholtz free energy
a	Nearest-neighbor lattice spacing (sometimes referred to simply as "lattice spacing")
$\beta$	$(kT)^{-1}$
b	Burgers vector (absolute magnitude)
$\bar{b}$	Burgers vector
B	$(kT)^{-1}b^2$
c	Measure of lateral extension of kink
d <sub>l</sub>	Element of length
D <sub>i</sub>	Internal partition function of species i
D'	Unimpeded dislocation length (average)
$\epsilon$	Cutoff radius for potential
E	Internal energy
F <sub>ij</sub>	Transform function for interaction between species i and j
G	Gibbs free energy
H <sub>k</sub>	Transform function for modified attractive Coulomb interaction
h	Planck's constant
k	Boltzmann's constant
K <sub>i</sub>	Modified Bessel function of the third kind
L	Length of system
L <sub>o</sub>	Typical length for kink pair formation



$L'$	Separation of kink groupings
$M$	Number of systems in an ensemble
$\mu$	Shear modulus (averaged over crystal directions)
$\mu_i$	Chemical activity of species $i$
$N$	Number of particles in a system
$N_i$	Number of particles or kinks of species $i$
$N_{ij}$	Number of nearest neighbor pairs between particles or kinks of types $i$ and $j$
$N_0$	Forest dislocation density
$\bar{p}$	Momentum vector $p_1 \dots p_N$
$P$	Pressure
$p_i$	Momentum of particle $i$
$\bar{q}$	Position vector $q_1 \dots q_N$
$q_i$	Position of particle $i$
$Q$	Canonical ensemble partition function
	Phase space probability factor
$S$	Entropy
$s$	$(kT)^{-1}p$
$\sigma$	Resolved shear stress (scalar)
$\underline{\sigma}$	Stress (tensor)
$T$	Absolute temperature
$U_I$	Interaction energy
$W$	Number of possible states of a system
$X$	Linear coordinate
$X_i$	Generalized extensive variable
$Y$	Linear coordinate
$Y$	Constant pressure grand canonical partition function

## CHAPTER 1

### INTRODUCTION

The subject of this dissertation is the application of constant pressure ensemble formulations to the study of kinks in dislocations. Before beginning such a theoretical analysis, it is appropriate to define kinks and to explain why they are important. Consider a dislocation line constrained to have an average direction which does not correspond to a close-packed direction of the crystal lattice. If the Peierls forces<sup>(66)</sup> which constrain it to lie in close-packed directions are stronger than the line-tension effect<sup>(69)</sup> which tends to minimize the total dislocation length, the dislocation line will consist of a number of straight segments lying along close-packed directions and connected by short segments where the dislocation line crosses from one close-packed direction to another. This situation is illustrated in Figure 1. These short connecting segments are called kinks if the orientation of the Burgers vector is such that they can move freely up and down the dislocation; otherwise they are called jogs. In the continuum limit, of course, the distinction disappears<sup>(47,89)</sup>. Kinks will be called positive if the segment on the left hand side has a smaller Y-coordinate than the segment on the right (see Figure 1); otherwise they will be called negative.

That the detailed microscopic structure and behavior

of kinks have measurable effects on the properties of solids was first proposed by Bordoni<sup>(15,16)</sup>, who studied the low-temperature internal friction properties of metals and found a component of the internal friction which depends on the degree of prior strain but not on the impurity content; this he attributed to the creation and annihilation of closely-spaced positive-negative kink pairs (see Figure 2). The first measurements were on copper, but the effect has been found in a number of other metals, notably lead<sup>(59)</sup>, germanium<sup>(44,79)</sup>, and silicon<sup>(79)</sup>.

Although other mechanisms were suggested from time to time<sup>(59)</sup>, it appears that kink behavior of some type is the only adequate explanation for the experimental observations<sup>(78,87)</sup>; thus one may say that kinks are worthy of study if one desires to understand the mechanism of low-temperature internal friction. It has also been suggested that kink motion is of great importance in high-temperature internal friction<sup>(74)</sup>, work hardening<sup>(6)</sup>, creep<sup>(32)</sup>, and polygonization<sup>(82,83)</sup>. In all these cases, however, interpretation of the existing data in terms of microscopic mechanisms is to a greater or lesser extent in doubt; in this work, comparison with experimental data is made only for the cases of internal friction and work hardening.

The attempt to formulate mathematically the interactions of kinks leads to severe difficulties. Kroupa and Brown<sup>(47)</sup> and Seeger and Schiller<sup>(76)</sup> have shown that, in the elastic

approximation, kinks of like sign repel and kinks of unlike sign attract one another according to a Coulombic potential (the analysis of Kroupa and Brown is summarized in Appendix A). This analysis appears to be sound in the case of kinks when the dislocation line connecting the kinks does not contain any intervening kinks; whether it holds for kinks which are not "nearest neighbors" is debatable. Brown<sup>(47)</sup> thinks it does; Granato<sup>(37)</sup> asserts the opposite. Ardell et al.<sup>(3)</sup> assert that, in fact, kinks do not interact at all except when quite close to one another; they do not, however, support this assertion with a calculation of any interaction energies. Eshelby<sup>(33)</sup> has shown that a complete neglect of kink interactions leads one to conclusions which contradict accepted experimental phenomena and feels that, while the potential is not strictly Coulombic, it can be represented as proportional to  $|r + c|^{-1}$ , where  $c$  is a measure of the lateral extension of the kink<sup>(34)</sup>.

In the light of this confusion regarding the pairwise interaction of kinks it is natural that, despite Eshelby's objections, the first computations of kink behavior were based on the assumption that kinks do not interact at all. Seeger et al.<sup>(75,76,77)</sup> have shown that the Bordoni peak can be explained qualitatively by assuming that dislocations continually generate kink pairs (of opposite sign), which in general collapse immediately (see Figure 2); they assume an attractive potential between kinks of unlike sign, but make no assumptions regarding its functional form.

Brailsford<sup>(18,19,20)</sup> shows that a different kink mechanism can be invoked to explain the same data; he specifically excludes kink interactions from his theory, and further assumes that the lateral extension of a kink is small. This latter assumption is hard to attack or defend, largely because so little is known about the dislocation core, despite various recent efforts to examine this area theoretically<sup>(28,41,55,85,88)</sup>. The assumption is probably sound for silicon and germanium<sup>(64,78)</sup>; it is generally regarded as unsound for face centered metals other than perhaps aluminum<sup>(77)</sup>; and nobody has any clear ideas about the situation as regards metals with other crystal structures.

Attempts to apply statistical mechanics to kinks have so far been very limited. Efforts to apply the usual classical canonical-ensemble formulation founder quickly because the relevant phase-space integrals are highly divergent. Lothe and Hirth<sup>(54)</sup> treated a single kink by a very intricate procedure; even if one concedes the validity of their approach, he learns nothing about the behavior of systems of kinks. Ardell et al.<sup>(3)</sup> treated large assemblies of parallel kinks, but their assumption that kink interactions are wholly negligible led to a solution whose principal features derived from precisely those kink configurations for which their assumptions are probably least valid; for they assume not only that long-range kink interactions are negligible, but also that kinks can approach arbitrarily close to one another without interacting.

In the present investigation, a modified form of the Coulombic interaction potential of Kroupa and Brown<sup>(47)</sup> is used for nearest-neighbor interactions, and interactions between kinks not nearest neighbors are assumed not to interact at all. The modification made in the potential function is shown to be reasonable in light of the detailed structure of the lattice. The statistical approach used is that of constant-pressure ensembles<sup>(38,40,53,72,81)</sup>; it is shown that this leads to convergent partition functions which can, in principle, be applied to any kink system with a nearest-neighbor modified Coulomb potential function, though the computational problems for some types of systems are very severe (see Appendix C). Detailed studies are made of systems of physical interest, and the results obtained by this method are compared to those obtained by other approaches to the problems and, where possible, to experimental results. The "kink pressure" discussed by Southgate and Attard<sup>(78)</sup> is treated and shown to have a simple physical interpretation; it is further shown that this pressure depends strongly on the interaction potential and that estimates made disregarding this potential are seriously in error.

The basic purpose of this work is not to explain quantitatively the present experimental data; the present state of understanding of kink interactions and other phenomena in which dislocation core properties are important is too poor for this to be possible. The purpose is rather

to develop a formalism by which the kink interactions can be incorporated in the existing theories without destroying the desirable features of theories. It is felt that, when a better understanding of these interactions is available, the methods applied in this dissertation can be used to make quantitative predictions; at present, the predictions made are basically qualitative.



Figure 1 Parallel kinks on a dislocation line.

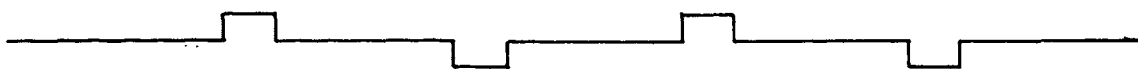


Figure 2 Kink pairs



## CHAPTER 2

## CONSTANT PRESSURE PARTITION FUNCTIONS

Constant pressure partition functions are essential for this investigation, because of the nature of the kink pair potential. In view of the rather limited use of these in most works on statistical mechanics, it appears desirable to explain their existence and properties. This discussion will to a considerable extent follow those of Hill<sup>(40)</sup>, Brown<sup>(21)</sup>, and Sack<sup>(72)</sup>, but will not go into the problems involved in formulating them consistently and without ad hoc assumptions. These problems, which are by no means trivial, are discussed by Bronw, Sack, Prigogine<sup>(68)</sup>, and Rushbrooke and Ursell<sup>(71)</sup>.

The theory of statistical mechanics can be regarded as built upon the concept of the micro canonical ensemble<sup>(40)</sup>, in which the volume, the number of particles of each species and the total energy are fixed. The link to thermodynamics then consists of the famous relationship:

$$(2.1) \quad S = k \ln W(E, V, N)$$

Here  $S$  is the entropy and  $W$  the number of possible configurations of the system consistent with the constraints. This can also be written:

$$(2.2) \quad \mathcal{S} = k \ln \Omega(E, V, N)$$

In (2.2)  $\Omega$  is a volume in phase space.

If one assumes  $\Omega$  to be normalized and the particles to be indistinguishable, one can use  $\Omega$  as an ordinary distribution, a cumulative distribution, or the differential of a cumulative distribution:

$$(2.3) \quad \Omega_d = \int_{E - \frac{\Delta E}{2}}^{E + \frac{\Delta E}{2}} d\Omega$$

$$\Omega_c = \int_{-\infty}^E d\Omega$$

$$\Omega' = \frac{d}{dE} \Omega_c$$

In any case, if the phase space is ergodic<sup>(40)</sup>, the probability of finding a given system in a given volume of phase space is, because of the normalization of  $\Omega$ , simply:

$$(2.4) \quad \rho = \Omega^{-1}$$

Now introduce the generalized ensemble: in the microcanonical ensemble three (or more) extensive variables are fixed: here  $N, E$ , and  $V$ . In many applications these are an inconvenient choice; for instance, very few processes are actually carried out at fixed internal energy rather than fixed temperature. The transformation to  $T$  as a variable rather than  $E$  leads to

the canonical ensemble, where  $N$ ,  $T$ , and  $V$  are fixed.

If one wishes to regard  $N$  as a variable and the chemical potential (or potentials)  $\mu_i$ <sup>(40)</sup> as fixed in systems belonging to the ensemble, the resulting ensemble is called a grand canonical ensemble. It is apparent that, at least formally, one can eliminate any extensive variable as an ensemble parameter and replace it with its conjugate intensive variable.

To do this, consider a very large number  $M$  of systems which are weakly coupled so that the values  $X_i$  for some extensive variable may vary and only their sum is fixed (e.g., one can consider a large number of constant-energy systems separated by thermally conducting walls and then separated from the rest of the universe by a thermal insulator). Thus, one develops the basic properties of the canonical and grand-canonical ensembles. For the canonical ensemble:

$$Q = \sum_j e^{-\beta E_j} \quad \text{or} \quad Q = \iint e^{-\beta E} d\tilde{q} d\tilde{p}$$

(2.5)  $A(N, T, V) = -kT \ln Q(N, T, V) \quad \beta = \frac{1}{kT}$

$A$  is the Helmholtz free energy.

For the grand canonical ensemble:

$$\Xi(\mu_i, T, V) = \sum_i Q(N_i, T, V) e^{\beta N_i \mu_i} = e^{\frac{PV}{kT}}$$

(2.6)

The derivation of a  $(T, p, N)$  ensemble - a constant pressure ensemble - from the canonical ensemble is no more difficult in principle, though the mechanics of its construction are somewhat awkward<sup>(21)</sup>. Its properties may be summarized:

$$W(T, p, N_i) = \int_0^{\infty} e^{-\beta p v} Q(N_i, T, v) dv$$

(2.7)

$$\frac{G}{kT} = -\ln W(T, p, N_i)$$

Once one has constructed this ensemble, there is no mechanical difficulty in constructing a  $(T, p, \mu)$  ensemble - a constant-pressure grand canonical ensemble - whose properties, formally, would be<sup>(72)</sup>:

$$\Psi(T, p, \mu_i) = \int_0^{\infty} \Xi(\mu_i, v, T) e^{-\beta p v} dv$$

$$(2.8) \quad \Delta = -[\Psi(T, p, \mu_i)]^{-1} \quad p \frac{\partial \Delta}{\partial p} = -1 \quad p \frac{\partial \Delta}{\partial T} = \frac{S}{V}$$

$$p \lambda_i \frac{\partial \Delta}{\partial \lambda_i} = kT \frac{N_i}{V} \quad \lambda_i = e^{\beta \mu_i}$$

However, as pointed out by Prigogine<sup>(68)</sup>, there is a very serious fundamental question concerning the constant-pressure grand canonical ensemble:  $T$ ,  $p$ , and  $\vec{\mu}$  are not independent; they are related by a nonholonomic constraint, the Gibbs-Duhem relation<sup>(40)</sup>

$$(2.9) \quad \sum_i N_i d\mu_i - v dp + S dT = 0$$

Sack<sup>(72)</sup> and Brown<sup>(21)</sup> discuss the problems raised by this. Since this investigation is not a fundamental study of ensembles but rather a study of their application to specific metallurgical problems, it seems appropriate not to discuss these problems in detail, but to point out that though the fundamental problems are not wholly resolved, the constant-pressure grand canonical partition function can be used to solve problems and that, when it is applied to problems whose solutions are already known, it gives the correct answers<sup>(72)</sup>. However, in the present investigation it appears to be inadequate to produce the required information, such as equilibrium kink populations. The means used to circumvent this obstacle are described in detail in Chapter 4.

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## CHAPTER 3

APPLICATION OF ENSEMBLES TO A  
RECTILINEAR KINK ASSEMBLY

The statistical mechanics of noninteracting particles, whether interchangeable or not, has proven to be a fairly simple matter<sup>(40)</sup>, but the construction of partition functions for systems of interacting particles is found to offer almost insuperable difficulties. However, a number of special techniques are known<sup>(38,46,53,67,81)</sup> for various types of one dimensional systems with nearest-neighbor interactions; the only ones among these which are readily applicable to kinks depend upon the properties of convolutions. These properties, which are treated in considerable detail in Doetsch<sup>(30)</sup>, will be discussed briefly.

The convolution of two functions  $f$  and  $g$  is defined by:

$$(3.1) \quad f * g = \int_0^{\infty} f(x) g(y-x) dx$$

It may also be defined by the following expression, if the functions are assumed to vanish for negative values of their arguments:

$$(3.2) \quad \int_0^y f(x) g(y-x) dx$$

It can be proven that, in general, the convolution operation is associative and commutative. Further, it can be shown that if  $\mathcal{L}(f)$  is defined by

$$(3.3) \quad \mathcal{L}(f) = \int_0^{\infty} e^{-st} f(t) dt$$

then the following relations

hold:

$$(3.4) \quad \begin{aligned} \mathcal{L}(f * g) &= \mathcal{L}(f) \cdot \mathcal{L}(g) \\ \mathcal{L}(f_1 * f_2 * \dots * f_m) &= \prod_{j=1}^m [\mathcal{L}(f_j)] \end{aligned}$$

Now with these results in mind, one can construct a constant-pressure canonical ensemble partition function for a rectilinear system of identical or nonidentical particles. This was first done by Takahasi<sup>(81)</sup> who did not, however, apply the method systematically to a system of physical interest. "Gursey<sup>(38)</sup> showed that the method led very neatly to a complete solution of the properties of a rectilinear hard-sphere gas. To show what can be done with the method if a problem has sufficiently simple properties, his solution is reproduced in Appendix A.

It is very unfortunate that this analysis is restricted to one-dimensional systems with nearest-neighbor interactions, and several methods have been developed to extend it. Byckling<sup>(25)</sup> has shown that a



very similar method can be used to study systems with interactions between more remote neighbors; unfortunately, his method leads to computational problems of great complexity. He reported in 1962 that computations were in progress on some imperfect gas problems (much simpler mathematically than kink problems). Three and a half years after publication of his paper the results of these computations had still not appeared. Several more or less successful efforts have been made to extend similar methods to certain two- and three-dimensional problems<sup>(4,5,49)</sup>, but the problems tractable by these methods are gas problems with no interesting analogues in dislocation theory. Essentially, the problem is that the simplicity of the method is due to its reliance on the transformation properties of the convolution, which have no analogues in multi-dimensional systems.

This method will now be illustrated by solution of the simplest interesting problem in the statistical mechanics of kink interactions: that of a large number of parallel kinks confined to an otherwise straight dislocation segment pinned at both ends, as illustrated in Figure 1.

Consider  $(N+2)$  kinks, numbered  $0, 1, 2, \dots, (N+1)$  and constrained to move only in the X-direction. Let the X-coordinate of the  $\underline{j}$ th kink be  $q_j$ . Suppose that by some pinning mechanism one kink is fixed with  $q = 0$ , and

another is fixed with  $q = L$ . Suppose that the interaction energy of the kinks can be written:

$$(3.5) \quad E = kT \sum_{j=0}^{N+1} \sum_{k=0}^{N+1} (\delta_{j,k+1} + \delta_{j,k-1}) f(q_j - q_k)$$

If the Hamiltonian of the system is separable,  $Q$ , the canonical-ensemble partition function, can be written:

$$(3.6) \quad Q = \frac{R}{(N+2)!} \sum_S \int_{R_S} \dots \int e^{-\beta E(q_1, \dots, q_N)} dq_1 \dots dq_{N-1} dq_N$$

Here  $R$  is the result of integration over the momentum coordinates and is independent of the interactions. The factor  $(N+2)!$  appears because the kinks are assumed indistinguishable; the sum  $S$  is taken over all orderings of the kinks on the  $X$ -axis. The region  $R_S$  is the part of  $q$ -space corresponding to the ordering of the  $q_i$  in term  $S$ . The terms in  $S$  differ only in the permutation of identical objects; hence one would expect the integral to be independent of the index  $S$ , and so one can rewrite (3.6) as:

$$(3.7) \quad Q = R \int_{R_1} \dots \int e^{-\beta E(q_1, \dots, q_N)} dq_1 \dots dq_{N-1} dq_N$$

In equation (3.7) the integration is carried out for a given ordering of the kinks on the  $X$ -axis: say,  $0, 1, 2 \dots, N+1$ . The integral can then be written in a more convenient form:

$$(3.8) \quad Q = R \int_0^L \int_0^{q_N} \int_0^{q_{N-1}} \dots \int_0^{q_2} e^{-\beta E(q_1, \dots, q_N)} dq_1 \dots dq_N$$

Using equation (3.5) one can rewrite this as:

$$(3.9) \quad Q = T \int_0^L e^{-f(L-q_N)} \int_0^{q_N} e^{-f(q_N-q_{N-1})} \dots \int_0^{q_2} e^{-f(q_1)-f(q_2-q_1)} dq_1 \dots dq_N$$

This is in the form of a multiple convolution, and it is this fact which permits a convenient solution of the problem. Referring to Sack's paper<sup>(72)</sup>, it is seen that:

$$(3.10) \quad e^{-\frac{G}{kT}} = \int_0^\infty e^{-sL} Q(L) dL \quad \text{where } s = \frac{P}{kT}$$

Inserting equation (3.9) into this and letting:

$$(3.11) \quad \int_0^\infty e^{-sx} e^{-f(x)} dx = F(s)$$

one sees that, if the coupling between internal modes of dislocations is neglected, permitting one to factor R as the product of N momentum integrations, there follows in the limit of large N:

$$(3.12) \quad e^{-\frac{G}{kT}} = D^N [F(s)]^N$$

By elementary manipulations,

$$(3.13) \quad \frac{G}{NkT} = -\ln D - \ln F(s)$$

Consider now the Coulombic potential developed by Kroupa (as discussed in Appendix A):

$$(3.14) \quad E_{ij} = \frac{A}{|q_i - q_j|}$$

Here the constant A depends upon whether the dislocation in question is screw, edge, or mixed. Using this pair potential, one finds that the function  $F(s)$ , which is of course also a function of  $T$ , is given by:

$$(3.15) \quad F(s) = \frac{z}{\sqrt{s}} \sqrt{\frac{A}{kT}} K_{-1}(z)$$

In this expression  $K_{-1}(z)$  is a Bessel's function of the third kind<sup>(86)</sup> and  $z$  is defined by:

$$(3.16) \quad z = z \sqrt{s} \sqrt{\frac{A}{kT}}$$

Making use of the relations between Bessel functions of different orders<sup>(86)</sup> one can also derive:

$$(3.17) \quad \frac{U_0}{N} = kT \frac{z}{2} \frac{K_0(z)}{K_1(z)}$$

$$(3.18) \quad \frac{L}{N} = \frac{kT}{P} + \frac{\sqrt{A}}{\sqrt{P}} \frac{K_0(z)}{K_1(z)}$$

In (3.17) the internal energy given is the internal energy of interaction only, and excludes contributions from internal modes.

It must be noted that, because of the one-dimensional nature of the problem, the "kink pressure" has the units of energy per unit length, as was observed by Southgate and Attard<sup>(78)</sup>. Equations (3.16), (3.17), (3.18) constitute the solution to the statistical problem; their application to specific physical problems will be discussed in the next chapter.

It remains to discuss the application of constant-pressure ensembles to systems containing antiparallel kinks (kinks of opposite sign). Before embarking on computations, one must consider two fundamental problems.

First, the present model for kink behavior is unrealistic in one respect; kinks are allowed to approach arbitrarily close to one another. This is, of course, not possible when one considers the crystal as a lattice of atoms: two kinks cannot be separated by distances of less than one lattice distance. This is of no importance in the treatment of parallel kinks because such configurations correspond to very high energy states and thus would contribute significantly to the partition functions only at very high kink densities, where the whole analysis would probably break down anyway, quite apart from the physical fact that in a crystal with a finite Peierls stress there is a bound to the kink density that will be tolerated without a complete rearrangement of the dislocation line. However, in the case of antiparallel kinks the closely spaced configurations are, at reasonable densities,

favorable energetically, and incorrect treatment of them will invalidate the whole analysis. It is therefore necessary to select a cutoff "recombination distance", which is the shortest distance by which two kinks can be separated and still retain their identities. It is intuitively apparent that this will be of the order of one atomic spacing, but it is not clear how one can evaluate it more closely. In this work, it will be taken equal to one atomic spacing; if a more accurate value becomes available, it can be incorporated into the analysis quite easily, as this parameter will be retained as an arbitrary constant in all equations down to the point at which numerical computations are necessary.

The second problem has to do with the type of problem treated. In the experimental work requiring theoretical analysis of parallel kink behavior, one generally is given a kink density and is interested in its theoretical properties<sup>(6,33,78)</sup>; on the other hand, in studying problems where antiparallel kink pairs are important<sup>(15,16,17,29,32,62,64,75,76,77)</sup> the real question a theory should answer is this: given a dislocation segment under fixed external conditions, what is the most probable number of kink pairs upon it? This question cannot be answered by just constructing functions analogous to those in reference (38), and the natural approach to the problem is the use of constant-pressure grand canonical partition functions<sup>(72)</sup>. However,

it appears that the best way to deal with this problem is by an approximate technique based on physical intuition and the reasonable assumption that kink pairs interact relatively weakly. This is contained in Chapter 5.

Assuming a Coulombic potential, it is possible to construct constant-pressure canonical partition functions for systems containing arbitrary numbers of positive and negative kinks; this calculation is outlined in Appendix C. However, this is difficult and not particularly relevant to either of the situations of physical interest as discussed previously; for, in the case of parallel kinks such partition functions are irrelevant, while in internal friction studies one generally assumes that equal numbers of kinks of each sign are nucleated and that they remain in closely spaced pairs. If one treats this situation, he arrives at a picture of typical kink configurations resembling Figure 2: the dislocation contains closely spaced kink pairs linked by a relatively strong interaction between the kinks in the pair but with only a weak interaction between different pairs.

Consider now this situation; let the Laplace transform function corresponding to "strong" interaction be  $F_1(s)$  and that corresponding to "weak" interaction by  $F_2(s)$ . Then one can write analogues of equations (3.8), (3.9) and (3.11).

$$(3.19) \quad Q = D_1^N D_2^N \int_0^L \int_0^{q_{1N}} \dots \int_0^{q_2} e^{-\beta E(q_1, \dots, q_N)} dq_1 \dots dq_N$$

$$(3.20) \quad Q = D_1^N D_2^N \int_0^L e^{-f_2(L - q_{1N})} \int_0^{q_{1N}} e^{-f_1(q_{2N} - q_{2N-1})} \dots \int_0^{q_2} e^{-f_1(q_1 - q_1) - f_2(q_1)} dq_1 \dots dq_N$$

$$(3.21) \quad e^{-\frac{G}{RT}} = D_1^N D_2^N [F_1(s)]^N [F_2(s)]^N$$

The properties of the functions  $F_1(s)$  and  $F_2(s)$  will be discussed in the chapter on the application of this analysis to specific problems. For the moment, attention will be centered on the question of physical importance: given the functions and equation (12), how can one determine the behavior of a dislocation containing kink pairs; and in particular, how can one determine the equilibrium density of kinks arising from thermal excitation?

Considering the elegance and simplicity of grand canonical formulations, it is unfortunate that the grand canonical constant-pressure formalism mentioned in the last chapter cannot be applied successfully to the present problem. However, the fact is that it cannot; and, in order to deal with the problem, it was necessary to make use of a nonrigorous treatment. This treatment is, however, physically reasonable and the weak spot in the argument lies in the identification of a parameter appearing only



as a multiplicative factor in the solution and which should, in any event, affect the solution only by a uniform scale factor of order unity.

Suppose one regards the kink pairs as independent entities (which is a good approximation at reasonable temperatures and stresses because of the large energy of kink formation) and associates with them a typical length, say ten lattice distances. That is, the dislocation will be thought of as made up of segments of this length, each of which either contains one kink pair or no kink pairs (the neglect of the possibility that such a segment might contain more than one kink pair will be seen to be physically reasonable); it is assumed that the probability of a given typical length  $L_0$ , containing a kink pair is small compared to unity. It must be pointed out that there is no way of saying that the correct value for the "typical length" is ten lattice distances rather than twenty, but it is intuitively reasonable that it should be on this general order of magnitude; certainly it will not, say, be as small as one lattice spacing.

Under these assumptions, one can regard the distribution of kink pairs on their typical lengths as being governed by a Maxwell-Boltzmann distribution; hence, the number of kinks per unit length of dislocation line should be given by:

$$(3.22) \quad \frac{N}{L} = L_0^{-1} \left( \sum_{j=0}^{\infty} e^{-\frac{j\mu}{kT}} \right) e^{-\frac{N\mu}{kT}}$$

Here  $L_0$  is the typical length and  $u$  the average internal energy of a kink pair. The evaluation of the average internal energy of interaction of a kink pair, for a given pressure, is easily carried out and, adding to this the kink-pair formation energy, one arrives at the average internal energy  $U$ . While the kink pressure appears to be an arbitrary parameter, it really is not so, because of its definition as an average energy derivative<sup>(40)</sup>. Hence one gets the kink pressure and the average internal energy of kink pairs by a self-consistent process which is easily carried out with modern computational equipment. The inclusion of external stresses in this calculation is easily carried out, as will be seen in Chapter 5. The details of the computation, and its application to internal friction, are also included in Chapter 5.

## CHAPTER 4

## SYSTEMS OF PARALLEL KINKS

The properties of systems containing only parallel kinks can be developed readily from (3.17) and (3.18):

$$(3.17) \quad \frac{u_0}{N} = kT \frac{z}{2} \frac{K_0(z)}{K_1(z)}$$

$$z = 2\sqrt{S} \sqrt{\frac{A}{kT}}$$

$$(3.18) \quad \frac{L}{N} = \frac{kT}{P} + \frac{\sqrt{A}}{\sqrt{P}} \frac{K_0(z)}{K_1(z)}$$

Using function tables in Abramowitz and Stegun<sup>(1)</sup>, these quantities have been evaluated for various kink pressures and densities in aluminum and silicon, and the results tabulated in Table 1 and Table 2. Screw kinks in pure edge dislocations, probably the most important type because of mobility consideration, have been assumed in these computations.

To get an idea of the relative size of the interaction effects, one may calculate the self energy of a kink in the same approximation, using the expression<sup>(47)</sup>

$$(4.1) \quad E_s^s \approx \frac{\mu b^2 a}{4\pi(1-\nu)} \ln 2$$

where  $a$  is the lattice spacing,  $b$  the Burgers vector,  $\mu$  the shear modulus, and  $\nu$  the

Poisson's ratio. Assuming  $b = a$  and using elastic data from Southgate's paper<sup>(79)</sup> for silicon and from Lothe's paper<sup>(54)</sup> for aluminum, one estimates:

$$\text{For Al, } E_s^S = 6.55 \times 10^{-13} \text{ erg}$$

$$\text{For Si, } E_s^S = 45.0 \times 10^{-13} \text{ erg}$$

The exact numerical values do not mean much, either here or in Tables 1 and 2; both the estimates of self energy and interaction energy are probably in error by as much as 25 per cent because of the various approximations used. It is important to note, however, that most of the approximations used (see Appendix A) will affect the self energies and interaction energies in about the same way so that one would expect comparisons of self energy to interaction energy in a given system to be better than either of the estimates taken by itself. It is of great interest to note from these tables that there are physically reasonable systems in which the interaction energies are as large as the self energies. Moreover, since interactions between parallel kinks which are not nearest neighbors will be, whatever their magnitude, almost certainly repulsive, these estimates of interaction energies are probably low. This indicates that workers<sup>(3,18,19,20,32)</sup> who disregard interactions altogether are probably significantly wrong, at least as far as energy estimates enter their calculations.

While this formalism has been developed to study

internal friction, it is of interest to note its application to the theory of work hardening and, in particular, to the so-called "forest dislocation" approach<sup>(6)</sup>. Consider a "forest" of parallel dislocations threading a slip plane. Suppose that the average unimpeded length of a "forest dislocation" is  $D'$ , and that the work hardening is taking place by formation of kinks in these forest dislocations as slip takes place in the slip plane<sup>(27)</sup>; suppose further that the kinks produced by intersection are mobile within the limits of the segments of average length  $D'$ , and are prevented from leaving these segments by some of the various possible types of restraints at the segment ends.

Qualitatively, one could then predict the course of work hardening as follows: initially, the kinks produced would be more or less independent, and would spread out through the segments. In this region, interaction effects would be slight and one would expect that considerable strain could take place at a stress mainly determined by the energy required to produce a kink, without taking interactions into account:

$$(4.2) \quad \left[ \left( \frac{\sigma_o}{\approx} \cdot \vec{b} \right) \cdot \vec{dl} \right] s = N_o E_s |\vec{dl}| s$$

In (4.2),  $s$  is the width of the crystal,  $dl$  is the length through which the slip dislocation moves,  $N_o$  is

the density of forest dislocations, and  $\sigma_0$  is the constant stress. After a certain time, the interaction energy would become significant, and instead of  $E_s^S$  one would have to use  $E_s^S + E_I$ , where  $E_I$  is the interaction energy per kink. This would then raise the required stress considerably; for, as the geometrical relationships entering into (4.2) would be about constant, one would have a stress given by:

$$(4.3) \quad \sigma = \sigma_0 \left( \frac{E_s^S + E_I}{E_s^S} \right)$$

Thus, at given temperature, one can see that this picture will correctly predict (qualitatively) the transition from the region in which deformation is approximately described by ideal plasticity to that in which work hardening is important. Because the internal energy of interaction of kinks will increase more rapidly than linearly with kink density, as may be seen from the tables, the work hardening curve is concave upward until the kink pressure is sufficient to break down the obstacles to kink motion, at which point the curve will have an inflection point. Because of the sharp increase in kink pressure with temperature if other experimental conditions are the same, one would expect the stress at which the inflection takes place to decrease sharply with temperature, and also the amount of strain before this "forest breakdown" (in the work-hardening region)

to decrease sharply with temperature. All these predictions are confirmed by examination of the data of Berner<sup>(14)</sup> on aluminum. Quantitative estimates cannot be made, because of the difficulty of estimating such parameters as  $D'$ , but the qualitative agreement is important, as it seems to cover points that, in the original forest work hardening theory, had to be dealt with by separate ad hoc assumptions. If one writes

$$(4.4) \quad \frac{\sigma}{\sigma_0} = \frac{E_s^s + E_I \left( \frac{N}{D'} \right) + E_{coll} \left( D', \frac{N}{D'} \right)}{E_s^s}$$

where  $E_{coll}$  is now the non-nearest neighbor contribution to kink interaction energies (which is neglected in the present treatment) one can ask of any treatment of work hardening: by how much can this ratio exceed unity? In the usual theory this is a very embarrassing question, because it is difficult to see how it can be more than 1.2 or so<sup>(27)</sup>. In the present treatment it can easily be seen to be as large as 2 in some cases. This is still lower than some observed data, but is a step in the right direction and, what is important, it arises naturally from the forest-dislocation treatment and not from any specially introduced hypothesis.

The question of the line tension of a dislocation line containing kinks has attracted considerable attention. Brailsford<sup>(18)</sup> has pointed out that, if kink

interactions are neglected, certain types of dislocation line segments have zero line tension; Eshelby<sup>(33)</sup> has, on the other hand, remarked that, if one takes these interactions into account, all dislocation segments should have some line tension even when they contain "built-in" kinks. It seems to be of interest to use this formulation to study line tension in dislocations containing kinks.

The simplest case, and that leading to the highest effective line tension, is that in which a previously straight dislocation line is "bowed out" by creating and moving kinks so as to give it an effective radius of curvature. This is illustrated in Figure 5.

To analyze the situation in Figure 5 one must first assume the separation  $L'$  to be much greater than the "kink lengths"  $L$ , and assume that interaction across  $L'$  contributes little to the energy of the situation. Referring to the figure, one sees that the total length of the kinked dislocation, neglecting the actual length of the kinks, is given by:

$$(4.5) \quad L_T = L' + 2 \sqrt{L^2 + (Nb)^2}$$

Noting that the original length is just  $L' + 2L$ , the change in length is given by:



$$(4.6) \quad \Delta L = 2L \left[ \sqrt{1 + b^2 N^2 L^{-2}} - 1 \right]$$

Assuming that  $Nb(L)^{-1}$  is small,

$$(4.7) \quad \Delta L = \frac{b^2 N^2}{L}$$

Recalling also that the energy required to create this situation is:

$$(4.8) \quad \Delta E = 2N (E_s^J + E_I)$$

One can estimate the effective line tension as:

$$(4.9) \quad \frac{\Delta E}{\Delta L} = \frac{2L}{N} \frac{(E_s^J + E_I)}{b^2}$$

Values of this energy derivative for various physically interesting situations are included in Table C. Recalling that the flexible-string approximation<sup>(27)</sup> gives a line tension of about  $\frac{1}{2} \mu b^2$ , which is about  $10^{-4}$  erg  $\text{cm}^{-1}$  in aluminum and  $6 \times 10^{-4}$  erg  $\text{cm}^{-1}$  in silicon, one notes that in this case the kink-model line tension is quite large. It is also worth noting that the interaction contribution to the line tension is quite large in silicon but relatively small in aluminum.

The other possible line-tension problem of interest is that of a dislocation line containing built-in kinks such as are illustrated in Figure 1, which changes its

shape due to kink redistribution under an external stress. It is impossible to generalize in these cases, but one might say that, where the basic effect of the applied stress is to redistribute already existing kinks, the effective line tension of the relevant dislocation lines in aluminum should be small. In silicon, however, as can be seen from Tables B and C, there would be a considerable tendency of the kinks to resist redistribution and a correspondingly larger line tension.

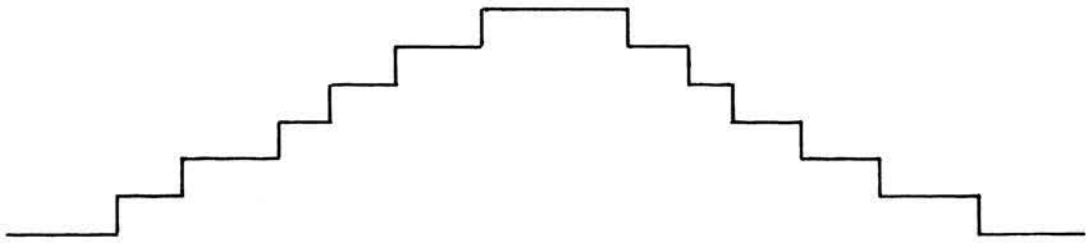


Figure 5 Dislocation Bowing under applied stress.

TABLE A  
KINK INTERACTION ENERGIES IN ALUMINUM  
FOR SCREW KINKS IN AN EDGE DISLOCATION

The following physical parameters are assumed:

$$\mu = .28 \times 10^{12} \text{ erg cm}^{-3}$$

$$\nu = .287 = \text{Poisson's Ratio}$$

$$b = a = 2.86 \times 10^{-7} \text{ cm}$$

$$A = \frac{a^2 b^2 \mu}{8} - \frac{1-2\nu}{1-\nu} = .445 \times 10^{-20} \text{ erg cm}$$

at 10<sup>0</sup>K:

P, dynes	$\frac{L}{N} \times 10^3, \text{ cm}$	Nearest neighbor distances per kink	$\frac{E_I}{N}, \text{ ergs}$
10 <sup>-7</sup>	22.4	7.9	2.08 × 10 <sup>-14</sup>
5 × 10 <sup>-8</sup>	30.0	10.5	1.45 × 10 <sup>-14</sup>
10 <sup>-8</sup>	77.4	27.0	6.34 × 10 <sup>-15</sup>
5 × 10 <sup>-9</sup>	121.9	42.6	4.41 × 10 <sup>-15</sup>
10 <sup>-9</sup>	321	112	1.83 × 10 <sup>-15</sup>
10 <sup>-10</sup>	1842	645	5.89 × 10 <sup>-16</sup>

At 100<sup>o</sup>K

P, dynes	$\frac{L}{N} \times 10^3$ , cm	Nearest Neighbor distances per kink	$\frac{E_I}{N}$ , ergs
$10^{-6}$	7.73	2.71	$6.35 \times 10^{-14}$
$5 \times 10^{-7}$	11.6	4.04	$4.38 \times 10^{-14}$
$10^{-7}$	32.2	11.3	$1.83 \times 10^{-14}$
$5 \times 10^{-8}$	52.2	18.2	$1.23 \times 10^{-14}$
$10^{-8}$	197	69	$4.67 \times 10^{-15}$
$10^{-9}$	1475	516	$9.50 \times 10^{-16}$

At 300<sup>o</sup>K

P, dynes	$\frac{L}{N} \times 10^3$ , cm	Nearest neighbor distances per kink	$\frac{E_I}{N}$ , ergs
$10^{-6}$	9.34	3.4	$5.86 \times 10^{-14}$
$5 \times 10^{-7}$	15.0	5.26	$3.93 \times 10^{-14}$
$10^{-7}$	53.2	18.6	$1.79 \times 10^{-14}$
$5 \times 10^{-8}$	113	40	$9.14 \times 10^{-15}$
$10^{-8}$	444	156	$3.01 \times 10^{-15}$
$10^{-9}$	4200	1470	$1.18 \times 10^{-15}$

TABLE B

KINK INTERACTION ENERGIES IN SILICON  
FOR SCREW KINKS IN AN EDGE DISLOCATION

The following physical parameters are assumed:

$$\mu = .7 \times 10^{12} \text{ erg cm}^{-2}$$

$$\nu = .3 = \text{Poisson's Ratio}$$

$$b = a = 4.0 \times 10^{-8} \text{ cm}$$

$$A = \frac{a^2 b^2 \mu}{8} - \frac{1-2\nu}{1-\nu} = 4.08 \times 10^{-20} \text{ erg cm}$$

At 10°K

P, dynes	$\frac{L}{N} \times 10^3$ , cm	Nearest neighbor distances per kink	$\frac{E_I}{N}$ , ergs
$10^{-6}$	20.4	5.1	$2.02 \times 10^{-13}$
$5 \times 10^{-7}$	28.9	7.2	$1.42 \times 10^{-13}$
$10^{-7}$	65.0	16.2	$6.40 \times 10^{-14}$
$5 \times 10^{-8}$	93.0	23.2	$2.68 \times 10^{-14}$
$10^{-8}$	337	84.3	$1.99 \times 10^{-14}$
$10^{-9}$	1990	498	$6.04 \times 10^{-15}$

At 100°K

P, dynes	$\frac{L}{N} \times 10^6$ , cm	Nearest neighbor distances per kink	$\frac{E_I}{N}$ , ergs
$10^{-5}$	6.58	1.5	$6.40 \times 10^{-12}$
$5 \times 10^{-6}$	9.35	2.5	$4.52 \times 10^{-12}$
$10^{-6}$	21.6	5.4	$2.02 \times 10^{-12}$
$5 \times 10^{-7}$	31.4	7.8	$1.43 \times 10^{-12}$
$10^{-7}$	78.0	19.5	$6.42 \times 10^{-13}$
$10^{-8}$	340	85	$2.07 \times 10^{-13}$

At 1000°K

P, dynes	$\frac{L}{N} \times 10^6$ , cm	Nearest neighbor distances per kink	$\frac{E_I}{N}$ , ergs
$10^{-5}$	7.80	2	$6.39 \times 10^{-12}$
$5 \times 10^{-6}$	11.82	3	$4.53 \times 10^{-12}$
$10^{-6}$	33.6	8.4	$1.99 \times 10^{-12}$
$5 \times 10^{-7}$	55.5	13.9	$1.39 \times 10^{-12}$
$10^{-7}$	205	50	$6.16 \times 10^{-13}$
$10^{-8}$	1550	390	$3.56 \times 10^{-13}$

TABLE C

## ENERGY DERIVATIVES IN KINK CONFIGURATIONS

Silicon at 100 <sup>o</sup> K		$E_s = 45 \times 10^{-13}$ erg	
$\frac{L}{N}$ , cm	$9.35 \times 10^{-8}$	$21.6 \times 10^{-8}$	$78 \times 10^{-8}$
$\frac{dE}{dL}$ , ergs cm <sup>-1</sup>	$10.5 \times 10^{-4}$	$17.6 \times 10^{-4}$	$50.1 \times 10^{-4}$
Silicon at 1000 <sup>o</sup> K			
$\frac{L}{N}$ , cm	$11.8 \times 10^{-8}$	$55.5 \times 10^{-8}$	$205 \times 10^{-8}$
$\frac{dE}{dL}$ , ergs cm <sup>-1</sup>	$13.2 \times 10^{-4}$	$40.9 \times 10^{-4}$	$131 \times 10^{-4}$
Aluminum at 100 <sup>o</sup> K		$E_s = 6.55 \times 10^{-13}$ erg	
$\frac{L}{N}$ , cm	$11.6 \times 10^{-8}$	$52.2 \times 10^{-8}$	$197 \times 10^{-8}$
$\frac{dE}{dL}$ , ergs cm <sup>-1</sup>	$19.8 \times 10^{-5}$	$85.7 \times 10^{-5}$	$318 \times 10^{-5}$
Aluminum at 300 <sup>o</sup> K			
$\frac{L}{N}$ , cm	$15.0 \times 10^{-8}$	$53.2 \times 10^{-8}$	$113 \times 10^{-8}$
$\frac{dE}{dL}$ , ergs cm <sup>-1</sup>	$25.6 \times 10^{-5}$	$87.6 \times 10^{-5}$	$183.6 \times 10^{-5}$



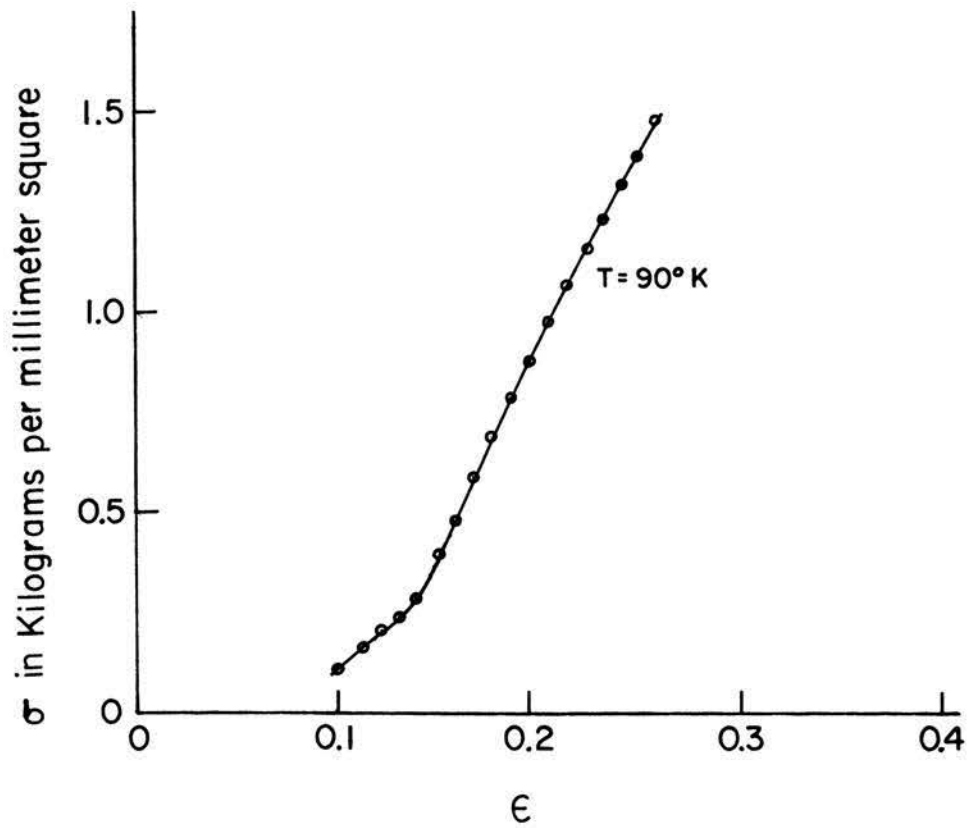


Figure 3 Work hardening of 99.99 percent Aluminum  
(Taken from Berner (reference 14))

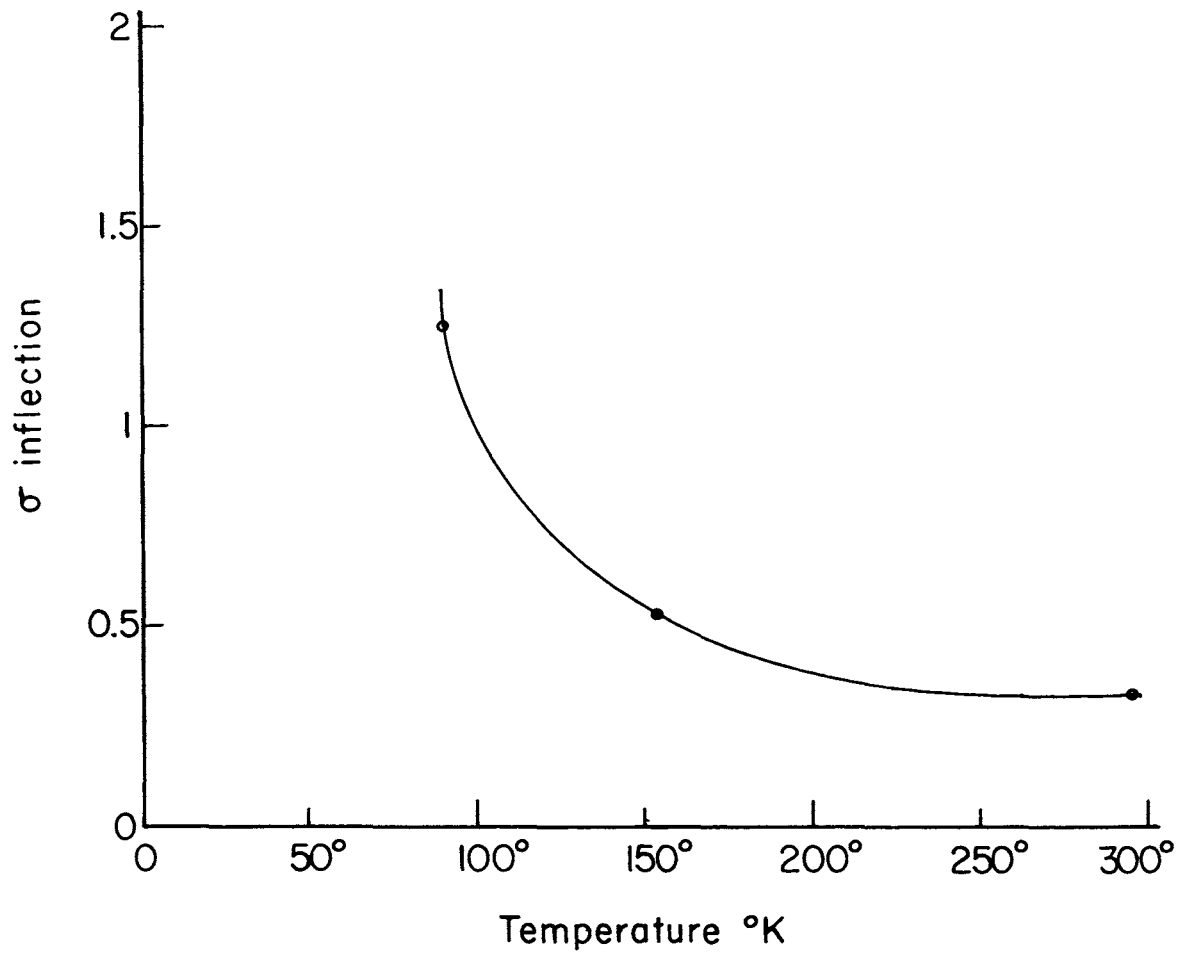


Figure 4

## CHAPTER 5

## KINK PAIRS

Suppose a pair of unlike kinks is subjected to an applied stress  $\sigma$ . Then, if  $B = (kT)^{-1}b^2\sigma$ , one can derive the forces on the kinks from a potential:

$$(5.1) \quad \begin{aligned} f_1(x) &= -Ax^{-1} - BkTx & x > \epsilon \\ f_1(x) &= \infty & x < \epsilon \end{aligned}$$

Suppose also that the "weak interaction" mentioned in Chapter 3 actually corresponds to no interaction at all. Then one can treat the kinks as independent entities, and determine the various additive properties of the system (e.g., energy terms) by simply summing over the kink pairs present in the system of interest.

Now if  $H_1(x, y)$  are defined as in Appendix D, the transform function is given by:

$$(5.2) \quad F_1(s) = \int_{\epsilon}^{\infty} e^{\frac{A}{kT} \frac{1}{x}} e^{-(s-B)x} dx = \epsilon H_0\left(\epsilon(s-B), \frac{A}{\epsilon kT}\right)$$

Referring to equation (3.10), one sees that:

$$(5.3) \quad G = -NkT \ln[\epsilon D] - NkT \ln H_0\left(\epsilon(s-B), \frac{A}{\epsilon kT}\right)$$

Using thermodynamic identities, one can derive an expression for the internal energy of interaction of one

kink pair:

$$(5.4) \quad u_{\text{I}} = -NkT\gamma \frac{H_{-1}(\epsilon(s-B), \gamma)}{H_0(\epsilon(s-B), \gamma)} \quad \text{where } \gamma = \frac{A}{\epsilon kT}$$

To proceed with the solution, consider the probability that a dislocation of unit length contains  $N$  kink pairs.

This is:

$$(5.5) \quad P = \frac{e^{-N \left[ \frac{u_0}{kT} - \gamma \frac{H_{-1}(\epsilon(s-B), \gamma)}{H_0(\epsilon(s-B), \gamma)} \right]}}{L_0 \sum_{j=0}^{\infty} e^{-j \left[ \frac{u_0}{kT} - \gamma \frac{H_{-1}(\epsilon(s-B), \gamma)}{H_0(\epsilon(s-B), \gamma)} \right]}}$$

Here,  $U_0$  is twice the self energy of a single kink and  $L_0$  is the typical length discussed in Chapter 3. In this equation the sum should really not be taken from zero to infinity, but rather to some large number which is the total number of distinguishable kinks that can be contained on the dislocation line. But the exponential functions should be rather small because of  $U_0$ , so the difference should be very slight.

The question arises: how does the population of kinks affect the internal friction? It can only do so when kink pairs collapse, for the in-phase motion of the kinks in a conservative field will contribute nothing. The average number of kinks is easily evaluated; letting:

$$(5.6) \quad \chi = e^{-\frac{u_0}{kT}} e^{\gamma \frac{H_{-1}(\epsilon(s-\beta), \gamma)}{H_0(\epsilon(s-\beta), \gamma)}}$$

and recalling that:

$$(5.7) \quad \sum_{N=0}^{\infty} \chi^N = \frac{1}{1-\chi} \quad \text{if } |\chi| < 1$$

one sees that:

$$(5.8) \quad \frac{\bar{N}}{L} = \frac{1}{L_0} \frac{\sum_{N=0}^{\infty} N \chi^N}{\sum_{N=0}^{\infty} \chi^N} = \frac{\chi}{L_0} (1+\chi)$$

The convergence of all the series used here is guaranteed by the condition  $|\chi| < 1$ ; if this condition did not hold, the dislocation would not be stable in the crystal but would propagate to the surface by kink generation.

The method used to evaluate internal friction contributions is as follows: by a self-consistent pressure calculation and equation (5.8), one can calculate the excess kink pair population for given stress over the zero-stress value. This, multiplied by four times the self energy of a kink, gives the energy decrement by this mechanism per cycle from positive to negative stress and return. This calculation was done for aluminum and silicon. Decrements and populations for aluminum are contained in Table D, for silicon in Table E.

It must be noted that this contribution to internal friction is independent of frequency at low frequency

and hence cannot be identified with the Bordoni peak. Thus, if one attributes the Bordoni peak to kink behavior, it must be ascribed to the response function for the kink distribution to internal stress, a nonequilibrium property which is outside the scope of this work.

Table F gives transform functions for screw kinks in aluminum and silicon for various physically interesting values of the parameters; it is thought that these may be applicable to other physically interesting computations.

## CHAPTER 6

## SUMMARY

In this work, it has appeared that, for systems of parallel, "built-in" kinks, elastic interactions are probably quite important and must be taken into consideration in calculation of such properties as line tension; and second, that in the treatment of pairs of antiparallel, "thermal" kinks, elastic interactions are almost wholly irrelevant except at temperatures so low that the thermal excitation of kink pairs can be wholly disregarded as far as observable effects are concerned.

The first conclusion is hardly surprising; despite the conjectures of Ardell<sup>(3)</sup>, Brailsford<sup>(18)</sup>, and others, a picture of dislocation behavior based on complete neglect of kink interactions contains fundamental inadequacies as pointed out by Eshelby<sup>(33)</sup>. The only surprise is that the effect is not greater, and it might well be greater, were one able to develop and use a more realistic potential (as was pointed out, this cannot be done because of the present lack of knowledge about the dislocation core).

That the interactions are of so little importance in the case of "thermal" kinks is a distinct surprise and suggest that, in spite of the crudeness of their

assumptions, the work of Schiller and Seeger<sup>(76)</sup> is probably quite close to the truth; from this work, one can visualize as a logical and physically reasonable approximation the ad hoc assumption they were obliged to make regarding interactions. Even if the potential function used here is seriously in error, this will still not change this result; as may be seen from Table D, it would take a change of several orders of magnitude to make a material difference to the interaction contribution. While the Kroupa-Brown potential is approximate, it is unreasonable to suppose that it is so much in error. Besides, one can see heuristically from the development that the errors made in developing the potential should have resulted in overestimating rather than underestimating the interactions.

One can construct a heuristic argument to account for the insignificance of the interaction term in the treatment of kink pairs: imagine a pair of unlike kinks, more or less tied to each other, and undergoing a fairly small pressure from outside. Suppose one wished to raise the average interaction energy of this kink pair by changing the external parameters of the system; he could either increase the pressure to force them closer together, or lower the temperature to reduce the thermal excitation which keeps them apart. In either case, one can see that the changes in external parameters would be likely to make the kinks recombine, recovering their



elastic self-energy. Thus, to produce an interaction energy large in comparison with the self energy, one would have to go to stress or temperature conditions where the equilibrium population of kinks would be very small.

A discussion of the relevance of the values obtained for the logarithmic decrements (Table D columns 4,5,6) seems called for. It will be noted that to secure the actual logarithmic decrement from the numbers in these columns, one must multiply by the length in centimeters of suitable dislocation lines per cubic centimeter of material. Assuming a dislocation density of  $10^8 \text{ cm}^{-2}$  (which is reasonable for a deformed metal) and that  $10^{-2}$  of the dislocation length is of suitable orientation and type to participate in these processes (which seems reasonable), one sees that friction from this source will probably be quite small in silicon, but it may be detectable in aluminum.

It is interesting to compare the results obtained here with those secured by other investigators. Past work on the Bordoni peak<sup>(15,16,17,29,59,75,76)</sup> has shown fairly conclusively that one of two processes is involved: either a change in equilibrium kink densities with applied stress (as discussed in Chapter 5) or a relaxation-time phenomenon associated with kink movement. That is, the peak arises either as a static statistical-mechanical effect, or as a truly dynamic

effect, having to do with rate of approach to equilibrium rather than with the equilibrium itself. It has been generally thought<sup>(75,76)</sup> that this peak was due to a relaxation time phenomenon, and this is confirmed by the present calculation.

The dependence of the logarithmic decrement upon strain amplitude is a matter of interest. Investigating the behavior of kinks in connection with internal friction, Suzuki and Elbaum<sup>(80)</sup> predicted a rather peculiar dependence upon strain amplitude, illustrated in Figure 6. This is totally different from that predicted in this work (Figure 7). However, Suzuki and Elbaum assumed that one knew that at zero stress the kinks would lie on certain specified points, and would not move very far from them on application of a stress. This is a wholly different approach from the present one, in which not only are the locations of the kinks treated as unknown variables but even the numbers present. The present work suggests that the assumptions made in the work of Suzuki and Elbaum are not well founded.

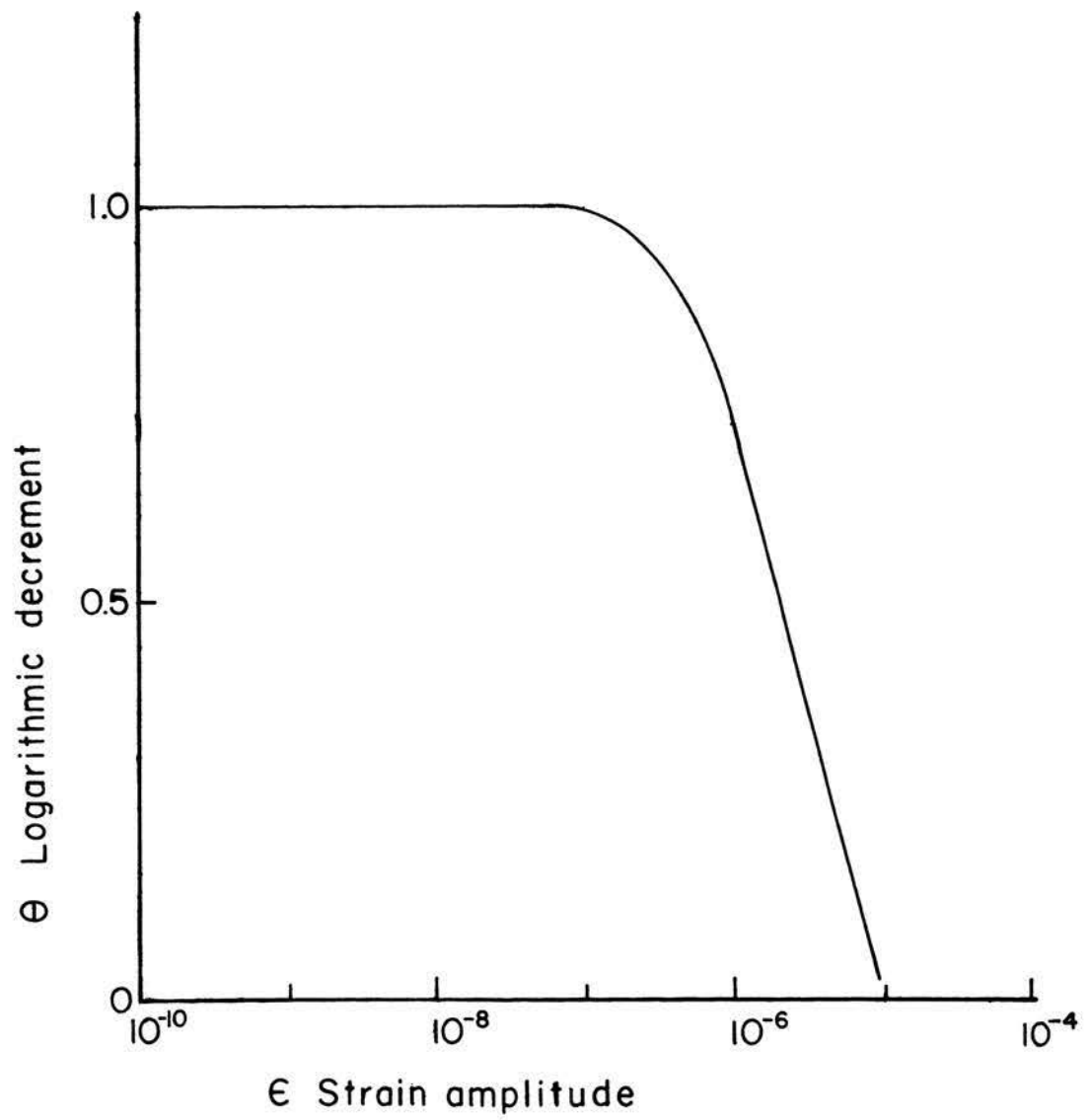
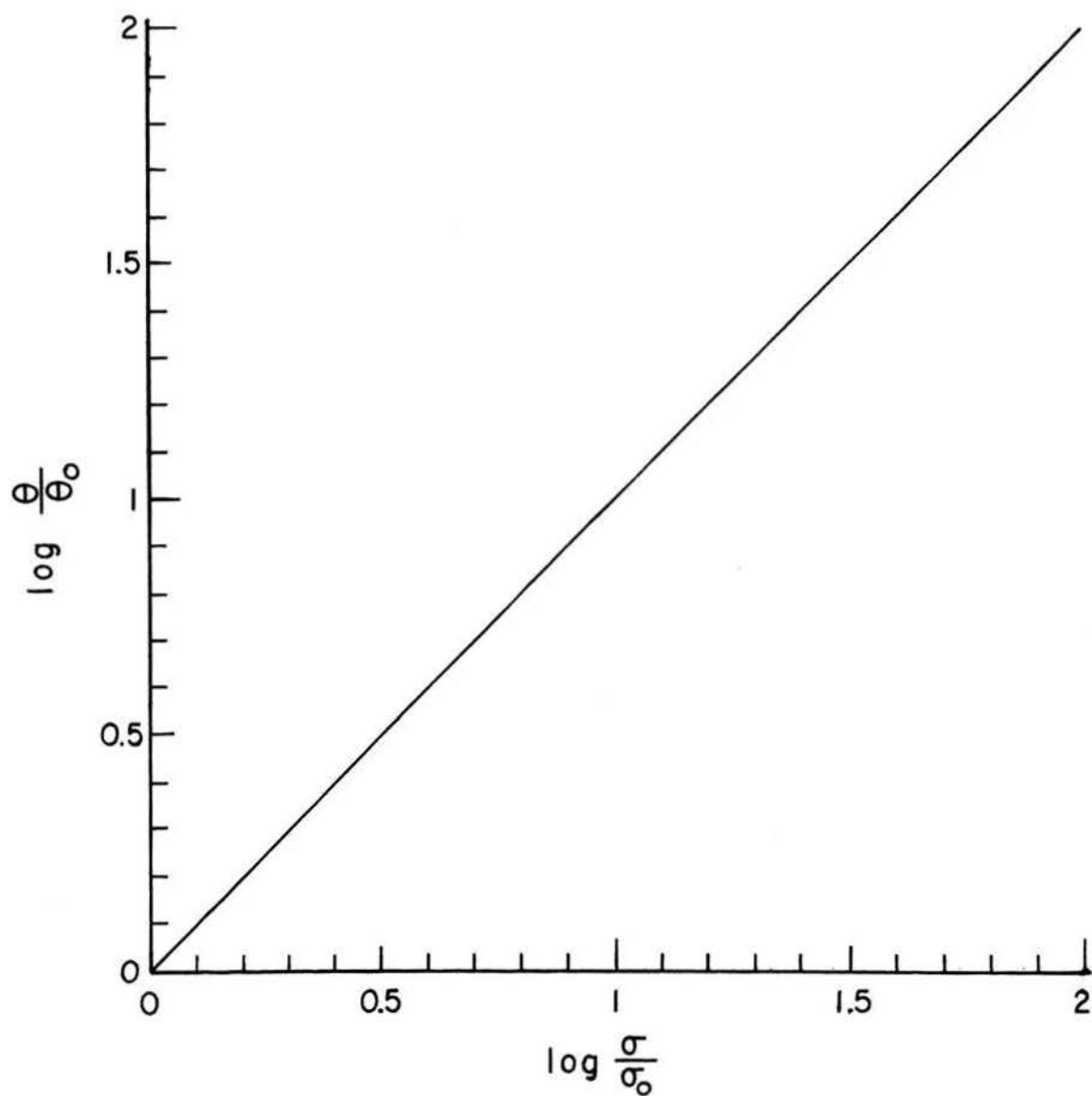


Figure 6 (Taken from Suzuki and Elbaum (reference 80))



$$\theta_0 = \theta (\sigma = 10^{-10} \mu)$$

$$\sigma = 10^{-10} \mu$$

$\frac{\theta}{\theta_0}$  Relative logarithmic decrement

$\frac{\sigma}{\sigma_0}$  Relative stress amplitude

Figure 7 This curve represents data on Aluminum for 100°K to 500°K to about  $\pm 5\%$ .

## CHAPTER 7

## CONCLUSIONS

The problem of kink interactions has been rather thoroughly solved in the case of systems of parallel kinks, making no approximations except the use of the Coulombic nearest neighbor potential. In the case of kink pair behavior, an approximate solution has been obtained based on physically reasonable assumptions.

In the former case, it is found that interactions are often physically important; in the latter, they are in general not so, at least as far as equilibrium properties are concerned. The results are discussed in the light of previous work and previous conjectures regarding interaction terms. It is found that, in general, this approach leads to improvements in the theoretical explanation of experimental data, and to confirmation of some of the more generally accepted conjectures on the subject of kink interactions.

The basic purpose of this work was to demonstrate the possibility of including kink interactions within the framework of dislocation theory by the use of constant pressure ensembles. This has been accomplished and qualitative checks made on the work. Quantitative calculations must await a better knowledge of dislocation core effects.

## Appendix A

THE CALCULATION OF KROUPA AND BROWN<sup>(47)</sup>

Consider an infinite straight dislocation line lying along the Z-axis and having a Burger's vector  $b$ . Suppose  $b$  is resolved into three components:  $b_s$  in the Z-direction,  $b_e$  in the X-direction, and  $b_p$  in the Y-direction. A kink pair will be produced in this dislocation by bringing up a long rectangular dislocation loop,  $r$  long by  $a$  high, where  $r > a$ , and affixing it to the line.

The energetic analysis of the problem will be this: let the energy of the straight dislocation line be  $E_1$ , that of the loop  $E_2$ . Let  $E_{12}$  be the extra energy needed to create the loop because of the presence of the straight dislocation, and to bring it up into its final position. The "kink energy" will now be  $E_k = E_2 + E_{12}$ . Suppose that this can be resolved into two factors:  $E_k = 2E_s + E_I(r)$ , where  $E_I(r)$  vanishes as  $r \rightarrow \infty$ . Then  $E_s$  can be identified as a "kink creation energy" and  $E_I$  as a "kink interaction energy". It is not obvious that this analysis can be made, but it will be shown that in fact it can.

For the resolved Burgers vector one can write  $E_s = E_s^p + E_s^e + E_s^s$  and  $E_1 = E_1^p + E_1^e + E_1^s$ . The self-energy of the loop can be calculated as (69):

$$\begin{aligned}
(A.1) \quad E_2^P &= \frac{\mu (b_p)^2}{2\pi(1-\nu)} \left[ a \ln \frac{2a}{c} + r \ln \frac{2r}{c} - \frac{a}{2} \ln \frac{\sqrt{a^2+r^2} + a}{\sqrt{a^2+r^2} - a} \right. \\
&\quad \left. - \frac{r}{2} \ln \frac{\sqrt{a^2+r^2} + r}{\sqrt{a^2+r^2} - r} + 2\sqrt{a^2+r^2} - 2(a+r) \right] \\
E_2^E &= \frac{\mu (b_E)^2}{2\pi} \left\{ a \left[ \ln \frac{2a}{c} - \frac{1}{2} \ln \frac{\sqrt{a^2+r^2} + a}{\sqrt{a^2+r^2} - a} + \frac{\sqrt{a^2+r^2}}{a} - \frac{a+r}{a} \right] \right. \\
&\quad \left. + \frac{r}{1-\nu} \left[ \ln \frac{2r}{c} - \frac{1}{2} \ln \frac{\sqrt{a^2+r^2} + r}{\sqrt{a^2+r^2} - r} + \frac{\sqrt{a^2+r^2}}{r} - \frac{a+r}{r} \right] \right\} \\
E_2^S &= \frac{\mu (b_s)^2}{2\pi} \left\{ r \left[ \ln \frac{2r}{c} - \frac{1}{2} \ln \frac{\sqrt{a^2+r^2} + r}{\sqrt{a^2+r^2} - r} + \frac{\sqrt{a^2+r^2}}{r} - \frac{a+r}{r} \right] \right. \\
&\quad \left. + \frac{a}{1-\nu} \left[ \ln \frac{2a}{c} - \frac{1}{2} \ln \frac{\sqrt{a^2+r^2} + a}{\sqrt{a^2+r^2} - a} + \frac{\sqrt{a^2+r^2}}{a} - \frac{a+r}{a} \right] \right\}
\end{aligned}$$

Here  $c$  is the "core radius" of the dislocation.

The excess energy of the loop due to the field of the straight dislocation can also be evaluated as a sum of three terms:

$$\begin{aligned}
(A.2) \quad E_{12}^P &= \frac{-\mu (b_p)^2}{2\pi(1-\nu)} r \ln \frac{a}{c} \\
E_{12}^E &= \frac{-\mu (b_E)^2}{2\pi(1-\nu)} r \ln \frac{a}{c} \\
E_{12}^S &= \frac{-\mu (b_s)^2}{2\pi(1-\nu)} r \ln \frac{a}{c}
\end{aligned}$$

Adding the contributions and performing the suggested decomposition,

$$(A.3) \quad E_s^p = \frac{\mu (b_p)^2 a}{4\pi(1-\nu)} \left[ \ln \frac{2a}{c} - 2 \right]$$

$$E_s^E = \frac{\mu (b_E)^2 a}{4\pi} \left[ \ln \frac{2a}{c} - \frac{2-\nu}{1-\nu} \right]$$

$$E_s^S = \frac{\mu (b_S)^2 a}{4\pi(1-\nu)} \left[ \ln \frac{2a}{c} - 2 + \nu \right]$$

Also,

$$(A.4) \quad E_I^p = \frac{-\mu (b_p)^2}{8\pi(1-\nu)} \frac{a^2}{r}$$

$$E_I^S = \frac{-\mu (b_S)^2}{8\pi(1-\nu)} (1+\nu) \frac{a^2}{r}$$

$$E_I^E = \frac{-\mu (b_E)^2}{8\pi(1-\nu)} (1-2\nu) \frac{a^2}{r}$$

Here it has been assumed that  $a/r$  is less than unity; this does not present a serious problem, as it is hard to imagine a problem of physical interest where this would not hold. But it is also assumed that  $a/c$  is at least 4, and this is a serious objection; it means that, while the potential will be reasonable for crystals with strong directional bonding, where one would expect a very narrow, sharply defined dislocation, one cannot apply the potential to, say, copper, where one has reason to believe that the dislocations are big. Eshelby<sup>(23)</sup>, however, feels that the potential is actually better than the derivation makes it appear and that its application to problems not falling under these restrictions ought to be considered. Seeger and Schiller<sup>(58)</sup> claim to have developed this potential without these restrictions; however, their paper does not present



their computation in sufficient detail to permit one to judge its value.

This method cannot be applied directly to antiparallel kinks; however, by considering a dislocation plus two semi-infinite loops, Kroupa and Brown arrive at an interaction potential:

$$(A.5) \quad E_{\mathbf{I}} = \frac{\mu}{8\pi(1-\nu)} \left[ (b_p)^2 + (1+\nu)(b_s)^2 + (1-2\nu)(b_E)^2 \right] \frac{a^2}{r}$$

## Appendix B

## "GURSEY'S SOLUTION TO THE HARD-SPHERE GAS PROBLEM"

Consider  $N$  identical particles constrained to move along a line of length  $L$ . The classical canonical-ensemble partition function of the system is

$$(B.1) \quad Z(T, L, N) = \frac{1}{h^N N!} \int_0^L \dots \int_0^L \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} e^{-\frac{H}{kT}} dp_1 \dots dp_N dx_1 \dots dx_N$$

Suppose that the Hamiltonian of the system is separable:

$$(B.2) \quad H = \sum_{i=1}^N \frac{p_i^2}{2m} + U(x_1, \dots, x_N) + U'$$

where  $U'$  is a constant (energy zero adjustment) and  $U$  is a symmetric function of its arguments. Integrating over momentum and noting that

$$\int_0^L \dots \int_0^L \varphi(x_1, \dots, x_N) dx_1 \dots dx_N = N! \int_0^L \int_0^{x_N} \dots \int_0^{x_2} \varphi(x_1, \dots, x_N) dx_1 \dots dx_N$$

$$\varphi = e^{-\frac{U}{kT}}$$

we arrive at the following expression:

$$(B.3) \quad Z(T, L, N) = \left( \frac{2\pi m kT}{h^2} \right)^{\frac{N}{2}} Q$$

$$Q = \int_0^L \int_0^{x_N} \dots \int_0^{x_2} e^{-\frac{U}{kT}} dx_1 \dots dx_{N-1} dx_N$$

The following assumptions will be made:

- 1)  $\lim_{x \rightarrow 0} E(x) = 0$
- 2) The particles are held together by two particles located at  $x = 0$ ,  $x = L$ .
- 3) There is some finite cut-off in the potential (this condition can be relaxed but need not be for the present problem).
- 4) Particles which are not nearest neighbors do not interact.

Then if

$$U(x_1, \dots, x_N) = kT \left[ f(L-x_N) + f(x_N-x_{N-1}) + \dots + f(x_1) \right]$$

one has:

$$(B.4) \quad Q = \int_0^L e^{-f(L-x_N)} \int_0^{x_N} e^{-f(x_N-x_{N-1})} \int_0^{x_{N-1}} e^{-f(x_{N-1}-x_{N-2})} \dots \\ \dots \int_0^{x_2} e^{-f(x_2-x_1)} e^{-f(x_1)} dx_1 \dots dx_{N-2} dx_{N-1} dx_N$$

Referring to Doetsch (30) one sees:

$$(B.5) \quad \int_0^\infty e^{-sL} Q(L) dL = \varphi(s) = \left[ \mathcal{L}(e^{-f(L)}) \right]^{N+1} \quad s = \frac{p}{kT}$$

This expression can be inverted (again according to Doetsch) as:

$$(B.6) \quad Q(L) = \int_{c-i\infty}^{c+i\infty} \left[ \mathcal{L}(e^{-f(L)}) \right]^{N+1} e^{sL} ds$$

Now consider:

$$\int_{c-i\infty}^{c+i\infty} e^{sL} \varphi(L) dL, \quad \text{Re}(c) > \text{Re}(\text{any pole of } \varphi(L))$$

Carlsaw and Jaeger<sup>(26)</sup> have shown that, if

$|Q(L)| < cr^{-k}$ ,  $k > 0$  when  $L = Re^{i\theta}$ ,  $-\pi \leq \theta \leq \pi$ , then this integral is equal to:

$$\frac{1}{2\pi i} \oint \varphi(L) e^{sL} dL$$

where the contour contains all the poles of the function  $Q(L)$ .

This is all one needs to construct a partition function for the hard-sphere gas. If the hard sphere radius is  $\sigma$ , then the transform function is given by:

$$\int_0^\sigma 0 \cdot dL + \int_\sigma^\infty e^{-sL} dL = s^{-1} e^{-\sigma s}$$

Then it follows:

$$(B.7) \quad Q(L) = \frac{1}{2\pi i} \oint e^{Ls} s^{-(N+1)} e^{-(N+1)\sigma s} ds$$

This obviously has one pole of order  $N + 1$  at  $s = 0$ .

As is known, the residue is:

$$\frac{1}{N!} \left\{ \frac{d^N}{ds^N} s^{N+1} \frac{e^{s(L - (N+1)\sigma)}}{s^{N+1}} \right\}_{s=0} = \frac{[L - \sigma(N+1)]^N}{N!}$$

So the partition function is:

$$(B.8) \quad Z(T, L, N) = \left( \frac{2\pi m k T}{h^2} \right)^{\frac{N}{2}} \frac{[L - \sigma(N+1)]^N}{N!}$$

This agrees with solutions by other methods<sup>(84)</sup>:

## Appendix C

### SOLUTION FOR ARBITRARY NUMBERS OF POSITIVE AND NEGATIVE KINKS

In this appendix, a Laplace-transform procedure is applied to obtain a fundamental thermodynamic equation of state for a mixture of two different kinds of objects (which will be identified as kinks of different signs) constrained to move on a line (identified as the dislocation containing them), and interacting with a Laplace-transformable interaction energy function (corresponding to the Kroupa-Brown potential); it is assumed that only nearest neighbors interact.

Consider  $(N+2)$  impenetrable objects located on the X-axis and constrained to move in the X-direction only. These will be numbered serially in order of increasing X-coordinate:  $0, 1, 2, \dots, N, N+1$ . The zeroth object is considered to be fixed at  $x = 0$ , the  $(N+1)$ th at  $x = L$ ; the coordinate of the  $i$ th object will be denoted by  $x_i$ . The objects are of two types: there is a fraction  $q$  of objects of type 1 and a fraction  $(1-q)$  of objects of type 2;  $q$  is such that  $(N+2)q$  is an integer. It is assumed that the potential energy of interaction between objects numbered  $i$  and  $j$  is zero unless  $|i-j| = 1$ , in which case it is  $kTf_{mn}(|x_i - x_j|)$ , where atom  $i$  is of type  $m$  and atom  $j$  of type  $n$ . It is assumed that  $f_{mn}(x) = f_{nm}(x)$ .

The total potential energy of the system is given by

$$(C.1) \quad E = kT \sum_{j=1}^{N+1} f_{mn}(x_j - x_{j-1})$$

Furthermore, if  $D_m$  is the internal partition function of an object of type  $m$ , and  $m_m$  is its (generalized) mass, and if  $L_m = h^{-1}(2\pi m_m kT)^{\frac{1}{2}}$ , the partition function of the system is given by:

$$(C.2) \quad Q = \left(\frac{D_1}{L_1^3}\right)^{q'N} \left(\frac{D_2}{L_2^3}\right)^{(1-q')N} \frac{Z}{[q(N+2)]! [(1-q)(N+2)]!}$$

where  $q' = qN(N+2)^{-1}$  and

$$(C.3) \quad Z = \sum_S \int_0^L \int_0^L \cdots \int_0^L e^{-\frac{E}{kT}} dx_1 \cdots dx_{N-1} dx_N$$

The sum over  $S$  denotes a sum over all possible orderings of the  $(N+2)$  objects on the  $X$ -axis.

In each term of the sum over  $S$ , the position of each object, say that numbered  $j$ , is restricted by the requirement  $x_{j-1} < x_j < x_{j+1}$ ; for, if this were not so, the state in question would correspond to a different ordering of atoms, and hence a different term in the sum. Hence (C.3) can be rewritten

$$(C.4) \quad Z = \sum_S \int_0^L \int_0^{x_N} \cdots \int_0^{x_2} e^{-\frac{E}{kT}} dx_1 \cdots dx_{N-1} dx_N$$

Furthermore, it can be seen that all states in  $S$  differing only by the interchange of identical atoms will make equal contributions, so the sum over  $S$  may be replaced by a sum over  $S'$ , the distinguishable states of the system, with each term in  $S'$  counted  $[q(N+2)]! [(1-q)(N+2)]!$  times. Then (C.2) may be written

$$(C.5) \quad Q = \left(\frac{D_1}{L_1^3}\right)^{q'N} \left(\frac{D_2}{L_2^3}\right)^{(1-q')N} \sum_{S'} \int_0^L \int_0^{x_N} \dots \int_0^{x_2} e^{-\frac{E}{kT}} dx_1 \dots dx_{N-1} dx_N$$

Using the expression (C.1) for the energy of the system, this leads to:

$$(C.6) \quad Q = \left(\frac{D_1}{L_1^3}\right)^{q'N} \left(\frac{D_2}{L_2^3}\right)^{(1-q')N} \sum_{S'} \int_0^L e^{-f_{mn}(L-x_N)} \int_0^{x_N} e^{-f_{mn}(x_N-x_{N-1})} \dots \int_0^{x_2} e^{-f_{mn}(x_2-x_1)} e^{-f_{mn}(x_1)} dx_1 \dots dx_{N-1} dx_N$$

where each pair of  $m$  and  $n$  depends, of course, on the term in  $S'$  under consideration (the parameterization is omitted for the sake of brevity and clarity in the expressions).

Equation (C.6) is in the form of a sum over multiple convolutions<sup>(30)</sup>. As was noted in Chapter 3, if

$$(C.7) \quad \mathcal{L}(f) = \int_0^\infty e^{-sL} f(L) dL$$

Then

$$(C.8) \quad \mathcal{L}(f * g) = \mathcal{L}(f) \cdot \mathcal{L}(g)$$

where  $f * g$  denotes the convolution of the functions  $f$  and  $g$ . This can be shown to hold also for multiple convolutions.

As noted in Chapter 2, it can be shown that

$$(C.9) \quad e^{-\frac{G}{kT}} = \int_0^{\infty} e^{-sL} Q(L) dL$$

Before applying this to Equation (C.6), it seems desirable to establish some notation. Let the total number of nearest-neighbor pairs in which both objects are of type 1 be  $N_{11}$ ; likewise define  $N_{22}$ ; and let  $N_{12}$  be the number of nearest-neighbor bonds between unlike objects. It is easily seen that, if the total number of objects is large enough so that end effects can be ignored,

$$(C.10) \quad N_{11} + \frac{N_{12}}{2} = qN \quad N_{22} + \frac{N_{12}}{2} = (1-q)N$$

For convenience, let

$$\mathcal{L}(e^{-f_{mn}}) = F_{mn}$$

Taking the Laplace transform of (C.6) and using the notation just established,

$$(C.11) \quad e^{-\frac{G}{kT}} = \left(\frac{D_1}{L_1}\right)^{qN} \left(\frac{D_2}{L_2}\right)^{(1-q)N} \sum_{s'} F_{11}^{N_{11}(s')} F_{22}^{N_{22}(s')} F_{12}^{N_{12}(s')}$$



For sufficiently long chains, (C.10) will hold, and  $q'$  will be arbitrarily close to  $q$ . Then one can write

$$(C.12) \quad e^{-\frac{G}{kT}} = \left(\frac{D_1 F_{11}}{L_1^3}\right)^{qN} \left(\frac{D_2 F_{22}}{L_2^3}\right)^{(1-q)N} \sum_{s'} \left(\frac{F_{12}^2}{F_{11} F_{22}}\right)^{\frac{N_{12}(s')}{2}}$$

In the limit of an infinite system, this sum may be approximated by its largest term, according to the method of steepest descents<sup>(40)</sup>. This process is exactly analogous to the one-dimensional Ising problem, which is discussed in detail in Hill's book<sup>(40)</sup>. For completeness' sake, this analysis is outlined in Appendix F.

It is shown that the maximum term corresponds to

$$(C.13) \quad t = \frac{N_{12}}{N} = (1 - e^{-2B})^{-1} \left[ 1 - \left\{ 1 - 4q(1-q)(1 - e^{-2B}) \right\}^{\frac{1}{2}} \right]$$

$$\text{where } B = \ln \frac{F_{12}}{\sqrt{F_{11} F_{22}}}$$

And, in the limit of large  $N$ ,

$$(C.14) \quad e^{-\frac{G}{kT}} = \left(\frac{D_1 F_{11}}{L_1^3}\right)^{qN} \left(\frac{D_2 F_{22}}{L_2^3}\right)^{(1-q)N} g(tN) e^{tNB}$$

In (C.14)  $g(tN)$  is the multiplicity of the maximum-term state.

Inserting the value of  $g(tN)$  from the one-dimensional Ising treatment, one has:

$$(C.15) \quad \frac{G}{NkT} = -q \ln \left( \frac{D_1 F_{11}}{L_1^2} \right) - (1-q) \ln \left( \frac{D_2 F_{22}}{L_2^2} \right) - Bt + q \ln \left( 1 - \frac{t}{2q} \right) \\ + (1-q) \ln \left( 1 - \frac{t}{2(1-q)} \right) + \frac{t}{2} \ln \left( \frac{t^2}{[2q-t][2(1-q)-t]} \right)$$

Letting  $s = (kT)^{-1}p$ ,  $p$  being the (one-dimensional) pressure, one has for the volume:

$$(C.16) \quad \frac{L}{N} = - \frac{q}{F_{11}} \left( \frac{\partial F_{11}}{\partial s} \right)_T - \frac{(1-q)}{F_{22}} \left( \frac{\partial F_{22}}{\partial s} \right)_T - B \left( \frac{\partial t}{\partial s} \right)_T - t \left( \frac{\partial B}{\partial s} \right)_T \\ + \left( \frac{\partial t}{\partial s} \right)_T \left[ \ln t - \frac{1}{2} \ln \left\{ (2q-t)(2[1-q]-t) \right\} \right]$$

Equations (C.15) and (C.16) constitute the final solution to the problem for arbitrary kink populations. Considering the complexity of these equations, it is fortunate that the general problem treated in this appendix appears to be of little physical interest.

## APPENDIX D

## PROPERTIES OF TRANSFORM FUNCTIONS

In this work, the following types of integrals, referred to as "Transform Functions", are used:

$$(D.1) \int_0^{\infty} e^{-sr} e^{-\frac{A}{kT} r^{-1}} dr$$

$$(D.2) \int_{\epsilon}^{\infty} e^{-sr} e^{\frac{A}{kT} r^{-1}} dr \quad s = \frac{P}{kT}$$

Integrals of the form (D.1) are readily expressed as modified Bessel functions of the third kind<sup>(86)</sup>. However, integrals of the form (D.2) cannot, apparently, be expressed in terms of a finite number of tabulated functions. In this appendix the properties of these integrals will be developed.

If one lets:

$$(D.3) \quad \frac{A}{\epsilon kT} = y \quad s\epsilon = x \quad \frac{r}{\epsilon} = t$$

then it is possible to express (D.2) as:

$$(D.4) \quad \epsilon \int_1^{\infty} e^{-xt} e^{\frac{y}{t}} dt$$

Therefore, rather than study the transform functions themselves, the functions studied will be the  $H_k$  functions defined by:

$$(D.5) \quad H_k(x, y) = \int_1^{\infty} t^k e^{-xt} e^{\frac{y}{t}} dt$$

These functions cannot, apparently, be evaluated in closed form. However, certain important derivatives can be secured by use of the chain rule:

$$(D.6) \quad \left. \frac{\partial H_k(x, y)}{\partial P} \right|_T = -\frac{x}{P} H_{k+1}(x, y)$$

$$(D.7) \quad \left. \frac{\partial H_k(x, y)}{\partial T} \right|_P = \frac{x}{T} H_{k+1}(x, y) - \frac{y}{T} H_{k-1}(x, y)$$

These derivatives are just those needed to evaluate the thermodynamic properties of kink distributions.

More interesting properties can be developed by regarding the  $H_k$  as functions of  $x$  only, with  $y$  as a fixed parameter. Integrating by parts, if  $k \neq -1$ ,

$$(D.8) \quad H_k(x, y) = \frac{-e^{y-x}}{k+1} + \frac{x}{k+1} H_{k+1}(x, y) + \frac{y}{k+1} H_{k-1}(x, y)$$

Also by noting that:

$$(D.9) \quad \left. \frac{\partial H_k(x, y)}{\partial x} \right|_y = -H_{k+1}(x, y)$$

one can see that the  $H_k$  functions for  $k \neq -1$  must satisfy:

$$(D.10) \quad \left[ \frac{d^2}{dx^2} + \frac{k+2}{x} \frac{d}{dx} + \frac{y}{x} \right] H_k(x, y) = \frac{e^{y-x}}{x}$$

Unfortunately, this is not an especially tractable differential equation (especially considering the boundary conditions), though it is clearly Fuchsian about  $x = 0$ .

Another approach to these functions was suggested by Y. L. Luke<sup>(58)</sup>. Letting:

$$(D.11) \quad xy = -\frac{z^2}{4} \quad xt = \frac{1}{t} z e^\omega \quad \kappa = -(1+\nu)$$

one arrives at the representation:

$$(D.12) \quad H_\kappa(x, y) = \left(\frac{2x}{z}\right)^\nu \int_{\ln \frac{2x}{z}}^{\infty} e^{-\nu\omega} e^{-z \cosh \omega} d\omega$$

Now suppose one considers the behavior of functions

$$(D.13) \quad B(z) = \int_a^{\infty} e^{-\nu\omega} e^{-z \cosh \omega} d\omega$$

Obviously, if  $a = -\infty$ ,  $B(z) = \frac{1}{2}K_\nu(z)$ , where  $K_\nu$  is again the modified Bessel function of the third kind.

The B functions satisfy a differential equation:

$$(D.14) \quad z^2 \frac{d^2 B}{dz^2} + z \frac{dB}{dz} - (z^2 + \nu^2) B = e^{-a\nu} e^{-z \cosh a}$$

A solution of this may be got by the method of variation of parameters, in the form:

$$(D.15) \quad B(z) = u(z) I_\nu(z) + v(z) K_\nu(z)$$

Then the problem can be solved if one can conveniently evaluate such integrals as:

$$(D.16) \quad \int z^\alpha e^{-z \cosh a} I_\nu(z) dz$$

Unfortunately, these again are somewhat intractable.

It appears from these efforts that, even if one could get integrals of the form (D.2) in closed form, they would probably be rather intricate and not especially well suited to numerical manipulations.

Accordingly, a straightforward numerical approach was used and no effort was made to simplify or solve the problem directly. The numerical treatment will be discussed in Appendix E, and the results tabulated in Table F.

## APPENDIX E

## NUMERICAL EVALUATION OF TRANSFORM FUNCTIONS

The method used to obtain transform functions  $H_k(x,y)$  is that of Gauss-Laguerre quadrature, as described by Hildebrand<sup>(39)</sup>. The zeroes and weighting factors used were those of Lowan<sup>(56)</sup>.

The quadrature used was a fifteen-point one; the reason for this choice was the nonavailability of accurate tabulations of zeroes and weighting factors for still higher order quadrature. The results were checked by reworking the calculations using fourteen point quadrature and the results were found to agree quite well, in most cases to six decimal places. Gauss-Laguerre quadrature was chosen because this type of quadrature gives higher accuracy in the evaluation of integrals having the form of Laplace transforms than do other methods involving the same amount of computational effort (this is discussed in detail in Hildebrand's book).

The results of this computation are contained in Tables F-1 through F-6. Should values of  $H_k(x,y)$  be desired for other values of  $k$ , they may be generated by the recursion relation developed in Appendix D.

## Appendix F

## EXACT ISING SOLUTION OF A ONE-DIMENSIONAL LATTICE

Consider a one-dimensional crystal containing two kinds of atoms, type 1 and type 2. Let the energy of interaction of a pair of atoms be identically zero unless they are nearest neighbors. Let a nearest-neighbor pair of atoms, both of type  $i$ , have interaction energy  $E_{ii}$ , and let a mixed pair have the interaction energy  $E_{12}$ .

If the lattice has  $N_{11}$  pairs of type 1 atoms nearest neighbor to one another,  $N_{22}$  pairs of type 2 atoms, and  $N_{12}$  mixed pairs, the total interaction energy of the system can be written:

$$E = N_{11}E_{11} + N_{22}E_{22} + N_{12}E_{12}.$$

Since  $2N_{11} + N_{12} = 2N_1$  and  $2N_{22} + N_{12} = 2N_2$ , where  $N_i$  is the number of atoms of type  $i$ , the energy can be written:

$$E = N_1E_{11} + N_2E_{22} + JN_{12}, \text{ where } J = E_{12} - \frac{1}{2}(E_{11} + E_{22}).$$

The Canonical ensemble partition function  $Q$  is given by:

$$Q = \sum e^{-\frac{E}{kT}}$$

This can be rewritten as a sum over  $N_{12}$ , letting  $g(N_{12})$  be the number of configurations of the system corresponding



to a given  $N_{12}$ , since all these will make equal contributions to the sum:

$$(F.1) \quad Q = \sum_{N_{12}} g(N_{12}) e^{-\frac{N_1 E_{11}}{\kappa T}} e^{-\frac{N_2 E_{22}}{\kappa T}} e^{-N_{12} \frac{J}{\kappa T}}$$

It is now desired to evaluate  $g(N_{12})$ . Suppose first of all that  $N$  is so large that one need not worry about the evenness or oddness of it. Consider the possible arrangements of the atoms beginning with a type 1. If the arrangement is thought of as a series of bunches of atoms of the same type, separated by unlike-neighbor bonds,  $\frac{1}{2}(N_{12} + 1)$  of these will contain atoms of type 1. If  $K = \frac{1}{2}(N_{12} + 1)$ , the number of ways of arranging the type 1 atoms in these groupings is

$$\frac{(N_1 - 1)!}{(N_1 - K)! (K - 1)!}$$

The number of ways of arranging the type 2 atoms in the other groupings is

$$\frac{(N_2 - 1)!}{(N_2 - K)! (K - 1)!}$$

Accordingly, using the binomial notation, one can write:

$$(F.2) \quad g(N_1, N_2, N_{12}) = 2 \binom{N_1 - 1}{K - 1} \binom{N_2 - 1}{K - 1}$$

Where the factor 2 enters because of the possibility of starting the chain with either a type 1 or a type 2 atom.

Accordingly,

$$(F.3) \quad Q = 2 e^{-\frac{N_1 E_{11} + N_2 E_{22}}{kT}} \sum_{N_{12}} \binom{N_1-1}{k-1} \binom{N_2-1}{k-1} e^{-\frac{N_{12} J}{kT}}$$

As  $N$  becomes large, the partition function should become asymptotically equal to its largest term.<sup>(40)</sup> If  $N_{12}$  is regarded as a continuous variable, the derivative of the largest term with respect to it should equal zero; alternatively, since the largest term will clearly be non-vanishing, the derivative of the logarithm of the largest term should vanish.

Thus, for the value of  $N_{12}$  giving the largest term,

$$(F.4) \quad \frac{J}{kT} - \frac{\partial}{\partial N_{12}} \binom{N_1-1}{k-1} - \frac{\partial}{\partial N_{12}} \binom{N_2-1}{k-1} = 0$$

Using Sterling's approximation, one can reduce this to

$$e^{\frac{J}{kT}} = \frac{\sqrt{N_1 - k} \sqrt{N_2 - k}}{k - 1}$$

Neglecting unity in comparison to the large numbers, one gets for the number of different types of bonds corresponding to the largest term,

$$(F.5) \quad \frac{\sqrt{N_{11}^* N_{22}^*}}{N_{12}^*} = \frac{1}{2} e^{\frac{J}{kT}}$$

Here the asterisk denotes properties of the maximum-term state.

It is now possible to write an asymptotic form for the partition function for a large system:

$$(F.6) \quad Q = e^{-\frac{N_1 E_{11} + N_2 E_{22}}{kT}} q(N_{12}^*) e^{-N_{12}^* \frac{J}{kT}}$$

$$\text{This leads to: } E = kT^2 \frac{\partial \ln Q}{\partial T} = N_1 E_{11} + N_2 E_{22}$$

$N_{12}^*$  can be written:

$$\frac{N_{12}^*}{N} = \frac{-\gamma + 1}{1 - B} \quad \text{where} \quad B = e^{\frac{2J}{kT}}$$

$$\gamma = \sqrt{1 - 4 \frac{N_1 N_2}{N^2} (1 - B)}$$

Likewise  $C_v$  can be obtained by taking a derivative:

$$(F.7) \quad \frac{C_v}{N} = J \left[ \frac{1 - \gamma}{(1 - B)^2} \frac{\partial B}{\partial T} - \frac{1}{1 - \gamma} \frac{\partial \gamma}{\partial T} \right]$$

$$\text{Where } \frac{\partial B}{\partial T} = -\frac{2J}{kT^2} B \quad \frac{\partial \gamma}{\partial T} = \frac{2}{\gamma} \frac{N_1 N_2}{N^2} B \frac{\partial B}{\partial T}$$

TABLE D  
RESULTS FOR ALUMINUM

The data are in six columns: the first column contains the temperature in degrees Kelvin; the second, the equilibrium kink-pair population (per centimeter) calculated with complete neglect of interaction effects; the third, the equilibrium kink-pair population calculated using the Kroupa-Brown potential; and the last three, respectively, the logarithmic decrements per centimeter of dislocation in a cubic centimeter of material, for resolved shear stress amplitudes of  $10^{-6}\mu$ ,  $10^{-8}\mu$ , and  $10^{-10}\mu$ . The decrements are negative because they correspond to loss of energy.

1.00000E+02 6.08636E-15 6.08638E-15-3.32480E-24-3.29513E-26-3.37285E-28  
1.10000E+02 4.69423E-13 4.69424E-13-1.89230E-22-1.90925E-24-1.94561E-26  
1.20000E+02 1.75483E-11 1.75484E-11-5.49425E-21-5.49660E-23-5.58319E-25  
1.30000E+02 3.75803E-10 3.75803E-10-9.27381E-20-9.26004E-22-9.37948E-24  
1.40000E+02 5.19492E-09 5.19494E-09-1.01587E-18-1.02475E-20-1.03597E-22  
1.50000E+02 5.05960E-08 5.05961E-08-7.91172E-18-8.11391E-20-8.18992E-22  
1.60000E+02 3.70754E-07 3.70755E-07-5.06256E-17-4.90008E-19-4.93838E-21  
1.70000E+02 2.14929E-06 2.14929E-06-2.38238E-16-2.36742E-18-2.38414E-20  
1.80000E+02 1.02495E-05 1.02495E-05-8.89545E-16-9.50767E-18-9.56920E-20  
1.90000E+02 4.14666E-05 4.14667E-05-3.27176E-15-3.27027E-17-3.28925E-19  
2.00000E+02 1.45880E-04 1.45880E-04-9.88965E-15-9.86610E-17-9.91489E-19  
2.10000E+02 4.55265E-04 4.55266E-04-2.69106E-14-2.66056E-16-2.67147E-18  
2.20000E+02 1.28114E-03 1.28115E-03-6.42185E-14-6.50747E-16-6.53539E-18  
2.30000E+02 3.29504E-03 3.29505E-03-1.47423E-13-1.46645E-15-1.47042E-17  
2.40000E+02 7.83313E-03 7.83314E-03-3.06108E-13-3.06814E-15-3.07550E-17  
2.50000E+02 1.73749E-02 1.73749E-02-6.02798E-13-6.02421E-15-6.03370E-17  
2.60000E+02 3.62490E-02 3.62491E-02-1.05054E-12-1.11611E-14-1.11876E-16  
2.70000E+02 7.16165E-02 7.16166E-02-2.02328E-12-1.97130E-14-1.97327E-16  
2.80000E+02 1.34774E-01 1.34774E-01-3.02744E-12-3.32346E-14-3.32891E-16  
2.90000E+02 2.42808E-01 2.42808E-01-5.52131E-12-5.39465E-14-5.39712E-16  
3.00000E+02 4.20605E-01 4.20606E-01-7.84893E-12-8.42347E-14-8.44369E-16  
3.10000E+02 7.03223E-01 7.03224E-01-1.27608E-11-1.27760E-13-1.27926E-15  
3.20000E+02 1.13857E+00 1.13857E+00-2.06034E-11-1.88080E-13-1.88280E-15  
3.30000E+02 1.79037E+00 1.79037E+00-2.56874E-11-2.70003E-13-2.69926E-15  
3.40000E+02 2.74135E+00 2.74135E+00-3.87829E-11-3.77302E-13-3.77854E-15  
3.50000E+02 4.09649E+00 4.09649E+00-5.65123E-11-5.17222E-13-5.17574E-15  
3.60000E+02 5.98644E+00 5.98644E+00-7.84893E-11-6.93845E-13-6.94988E-15  
3.70000E+02 8.57075E+00 8.57076E+00-8.65503E-11-9.16161E-13-9.16431E-15  
3.80000E+02 1.20411E+01 1.20411E+01-1.23930E-10-1.18973E-12-1.18841E-14  
3.90000E+02 1.66242E+01 1.66242E+01-1.44903E-10-1.51665E-12-1.51764E-14  
4.00000E+02 2.25847E+01 2.25847E+01-2.11921E-10-1.90729E-12-1.91084E-14  
4.10000E+02 3.02270E+01 3.02270E+01-2.06752E-10-2.37305E-12-2.37473E-14  
4.20000E+02 3.98978E+01 3.98978E+01-2.69106E-10-2.90859E-12-2.91574E-14  
4.30000E+02 5.19870E+01 5.19871E+01-4.59983E-10-3.54187E-12-3.54016E-14  
4.40000E+02 6.69293E+01 6.69294E+01-4.28123E-10-4.25554E-12-4.25383E-14  
4.50000E+02 8.52043E+01 8.52043E+01-5.44192E-10-5.06517E-12-5.06191E-14  
4.60000E+02 1.07336E+02 1.07336E+02-6.14264E-10-5.97883E-12-5.96982E-14  
4.70000E+02 1.33895E+02 1.33895E+02-6.01194E-10-6.97385E-12-6.98127E-14  
4.80000E+02 1.65494E+02 1.65495E+02-5.88669E-10-8.08440E-12-8.10029E-14  
4.90000E+02 2.02790E+02 2.02790E+02-7.68875E-10-9.30338E-12-9.33029E-14  
5.00000E+02 2.46478E+02 2.46478E+02-1.50699E-09-1.06808E-11-1.06734E-13  
5.10000E+02 2.97295E+02 2.97295E+02-1.10808E-09-1.21335E-11-1.21313E-13  
5.20000E+02 3.56012E+02 3.56012E+02-1.26790E-09-1.37295E-11-1.37047E-13  
5.30000E+02 4.23437E+02 4.23437E+02-1.42169E-09-1.54253E-11-1.53946E-13  
5.40000E+02 5.00407E+02 5.00407E+02-1.74420E-09-1.71804E-11-1.71996E-13  
5.50000E+02 5.87787E+02 5.87787E+02-2.05499E-09-1.91285E-11-1.91210E-13  
5.60000E+02 6.86468E+02 6.86468E+02-1.17733E-09-2.11248E-11-2.11552E-13

5.70000E+02 7.97362E+02 7.97362E+02-1.32192E-09-2.31997E-11-2.33012E-13  
 5.80000E+02 9.21400E+02 9.21400E+02-2.92305E-09-2.55766E-11-2.55578E-13  
 5.90000E+02 1.05952E+03 1.05952E+03-3.19278E-09-2.79368E-11-2.79193E-13  
 6.00000E+02 1.21270E+03 1.21270E+03-3.13957E-09-3.04538E-11-3.03847E-13  
 6.10000E+02 1.38189E+03 1.38189E+03-4.63215E-09-3.31971E-11-3.29500E-13  
 6.20000E+02 1.56806E+03 1.56806E+03-4.55744E-09-3.58518E-11-3.56057E-13  
 6.30000E+02 1.77219E+03 1.77219E+03-2.99006E-09-3.84223E-11-3.83551E-13  
 6.40000E+02 1.99524E+03 1.99524E+03-2.94334E-09-4.12068E-11-4.11892E-13  
 6.50000E+02 2.23820E+03 2.23820E+03-2.89806E-09-4.40506E-11-4.41042E-13  
 6.60000E+02 2.50201E+03 2.50201E+03-4.28123E-09-4.70935E-11-4.70964E-13  
 6.70000E+02 2.78762E+03 2.78762E+03-4.21733E-09-5.01863E-11-5.01567E-13  
 6.80000E+02 3.09598E+03 3.09598E+03-4.15531E-09-5.31880E-11-5.32822E-13  
 6.90000E+02 3.42801E+03 3.42802E+03-9.55522E-09-5.66488E-11-5.64713E-13  
 7.00000E+02 3.78462E+03 3.78462E+03-9.41871E-09-6.00106E-11-5.97119E-13  
 7.10000E+02 4.16670E+03 4.16670E+03-5.30632E-09-6.30125E-11-6.30019E-13  
 7.20000E+02 4.57510E+03 4.57510E+03-5.23262E-09-6.64542E-11-6.63313E-13  
 7.30000E+02 5.01068E+03 5.01069E+03-6.45117E-09-6.98017E-11-6.96985E-13  
 7.40000E+02 5.47427E+03 5.47427E+03-7.63679E-09-7.31859E-11-7.31045E-13  
 7.50000E+02 5.96664E+03 5.96665E+03-7.53497E-09-7.64799E-11-7.65352E-13  
 7.60000E+02 6.48858E+03 6.48859E+03-8.67513E-09-7.96873E-11-7.99872E-13  
 7.70000E+02 7.04083E+03 7.04083E+03-8.56247E-09-8.35452E-11-8.34535E-13  
 7.80000E+02 7.62409E+03 7.62409E+03-9.66022E-09-8.65797E-11-8.69347E-13  
 7.90000E+02 8.23905E+03 8.23905E+03-1.07301E-08-9.03720E-11-9.04208E-13  
 8.00000E+02 8.88637E+03 8.88637E+03-1.05960E-08-9.41871E-11-9.39093E-13  
 8.10000E+02 9.56666E+03 9.56666E+03-1.04652E-08-9.67453E-11-9.74023E-13  
 8.20000E+02 1.02805E+04 1.02805E+04-1.14862E-08-9.99303E-11-1.00883E-12  
 8.30000E+02 1.10284E+04 1.10285E+04 0.00000E-99-1.03265E-10-1.04366E-12  
 8.40000E+02 1.18111E+04 1.18111E+04 0.00000E-99-1.07642E-10-1.07833E-12  
 8.50000E+02 1.26289E+04 1.26289E+04 0.00000E-99-1.09700E-10-1.11262E-12  
 8.60000E+02 1.34823E+04 1.34823E+04-1.09519E-08-1.14995E-10-1.14700E-12  
 8.70000E+02 1.43717E+04 1.43717E+04-1.08261E-08-1.18004E-10-1.18091E-12  
 8.80000E+02 1.52976E+04 1.52976E+04-2.14061E-08-1.23085E-10-1.21469E-12  
 8.90000E+02 1.62603E+04 1.62603E+04-2.11656E-08-1.25935E-10-1.24813E-12  
 9.00000E+02 1.72602E+04 1.72602E+04-1.04652E-08-1.27675E-10-1.28115E-12  
 9.10000E+02 1.82976E+04 1.82976E+04-2.07004E-08-1.32483E-10-1.31396E-12  
 9.20000E+02 1.93727E+04 1.93727E+04-2.04754E-08-1.35138E-10-1.34616E-12  
 9.30000E+02 2.04858E+04 2.04858E+04-2.02553E-08-1.38748E-10-1.37796E-12  
 9.40000E+02 2.16371E+04 2.16371E+04-2.00398E-08-1.41280E-10-1.40960E-12  
 9.50000E+02 2.28269E+04 2.28269E+04-2.97433E-08-1.44750E-10-1.44056E-12  
 9.60000E+02 2.40552E+04 2.40552E+04-1.96223E-08-1.46186E-10-1.47098E-12  
 9.70000E+02 2.53222E+04 2.53222E+04-2.91300E-08-1.50505E-10-1.50107E-12  
 9.80000E+02 2.66281E+04 2.66281E+04-2.88328E-08-1.53775E-10-1.53073E-12  
 9.90000E+02 2.79728E+04 2.79729E+04-2.85415E-08-1.56978E-10-1.55979E-12  
 1.00000E+03 2.93566E+04 2.93566E+04-2.82561E-08-1.58234E-10-1.58827E-12

## TABLE E

## RESULTS FOR SILICON

The data are in six columns: the first column contains the temperature in degrees Kelvin; the second, the equilibrium kink-pair population (per centimeter) calculated with complete neglect of kink interactions; the third, the equilibrium kink-pair population calculated using the Kroupa-Brown potential; and the last three, respectively, logarithmic decrements per centimeter of dislocation in a cubic centimeter of material, for resolved shear stress amplitudes of  $10^{-6}\mu$ ,  $10^{-8}\mu$ , and  $10^{-10}\mu$ . The decrements are negative because they correspond to loss of energy.

6.60000E+02 8.83944E-16 8.83948E-16-1.99918E-25-2.03534E-27-2.08490E-29  
6.70000E+02 1.84755E-15 1.84756E-15-4.07716E-25-4.06639E-27-4.16217E-29  
6.80000E+02 3.77878E-15 3.77879E-15-8.03441E-25-7.95671E-27-8.13658E-29  
6.90000E+02 7.57006E-15 7.57009E-15-1.52757E-24-1.52562E-26-1.55902E-28  
7.00000E+02 1.48670E-14 1.48671E-14-2.87159E-24-2.86938E-26-2.93042E-28  
7.10000E+02 2.86480E-14 2.86481E-14-5.29933E-24-5.29897E-26-5.40793E-28  
7.20000E+02 5.42064E-14 5.42067E-14-9.52086E-24-9.61356E-26-9.80590E-28  
7.30000E+02 1.00790E-13 1.00790E-13-1.69451E-23-1.71499E-25-1.74831E-27  
7.40000E+02 1.84292E-13 1.84293E-13-3.02980E-23-3.01065E-25-3.06711E-27  
7.50000E+02 3.31594E-13 3.31595E-13-5.11978E-23-5.20225E-25-5.29781E-27  
7.60000E+02 5.87479E-13 5.87482E-13-8.98585E-23-8.85903E-25-9.01556E-27  
7.70000E+02 1.02547E-12 1.02548E-12-1.47261E-22-1.48667E-24-1.51241E-26  
7.80000E+02 1.76464E-12 1.76465E-12-2.44491E-22-2.46143E-24-2.50251E-26  
7.90000E+02 2.99516E-12 2.99517E-12-4.01240E-22-4.02056E-24-4.08634E-26  
8.00000E+02 5.01695E-12 5.01697E-12-6.47490E-22-6.48714E-24-6.58821E-26  
8.10000E+02 8.29712E-12 8.29716E-12-1.03083E-21-1.03347E-23-1.04925E-25  
8.20000E+02 1.35545E-11 1.35546E-11-1.63424E-21-1.62732E-23-1.65146E-25  
8.30000E+02 2.18830E-11 2.18831E-11-2.51497E-21-2.53298E-23-2.56992E-25  
8.40000E+02 3.49283E-11 3.49284E-11-3.95765E-21-3.90150E-23-3.95564E-25  
8.50000E+02 5.51403E-11 5.51405E-11-5.85147E-21-5.94303E-23-6.02455E-25  
8.60000E+02 8.61290E-11 8.61293E-11-9.01976E-21-8.96342E-23-9.08258E-25  
8.70000E+02 1.33160E-10 1.33161E-10-1.36259E-20-1.33889E-22-1.35588E-24  
8.80000E+02 2.03846E-10 2.03847E-10-1.96209E-20-1.97936E-22-2.00498E-24  
8.90000E+02 3.09082E-10 3.09083E-10-2.95350E-20-2.90224E-22-2.93777E-24  
9.00000E+02 4.64332E-10 4.64333E-10-4.12331E-20-4.21609E-22-4.26657E-24  
9.10000E+02 6.91350E-10 6.91353E-10-6.06037E-20-6.07311E-22-6.14356E-24  
9.20000E+02 1.02049E-09 1.02049E-09-8.68362E-20-8.67521E-22-8.77334E-24  
9.30000E+02 1.49377E-09 1.49377E-09-1.21926E-19-1.22895E-21-1.24287E-23  
9.40000E+02 2.16889E-09 2.16889E-09-1.72718E-19-1.72855E-21-1.74714E-23  
9.50000E+02 3.12451E-09 3.12452E-09-2.36005E-19-2.41214E-21-2.43764E-23  
9.60000E+02 4.46707E-09 4.46709E-09-3.24818E-19-3.34214E-21-3.37643E-23  
9.70000E+02 6.33962E-09 6.33964E-09-4.56965E-19-4.59835E-21-4.64396E-23  
9.80000E+02 8.93307E-09 8.93310E-09-6.36379E-19-6.28148E-21-6.34394E-23  
9.90000E+02 1.25005E-08 1.25006E-08-8.85056E-19-8.53037E-21-8.60917E-23  
1.00000E+03 1.73756E-08 1.73756E-08-1.18545E-18-1.15040E-20-1.16085E-22



## APPENDIX G

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TABLE F-1  
TRANSFORM FUNCTIONS FOR ALUMINUM AT 50°K.

The tabulated values are as follows:

$\delta$	$H_{-1}(\delta, \gamma)$	$H_0(\delta, \gamma)$	$H_1(\delta, \gamma)$
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The parameter  $\epsilon$  is taken equal to one nearest neighbor spacing. The parameter  $\gamma$  is that appropriate to this material and temperature, according to the work of Kroupa and Brown.



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0.2860000E-18	0.3318229E+01	0.3496503E+19	0.1222553E+38
0.5720000E-18	0.3318228E+01	0.1748252E+19	0.3056383E+37
0.8580000E-18	0.3318228E+01	0.1165501E+19	0.1358393E+37
0.1144000E-17	0.3318229E+01	0.8741258E+18	0.7640960E+36
0.1430000E-17	0.3318229E+01	0.6993007E+18	0.4890214E+36
0.1716000E-17	0.3318229E+01	0.5827506E+18	0.3395982E+36
0.2002000E-17	0.3318229E+01	0.4995005E+18	0.2495007E+36
0.2288000E-17	0.3318229E+01	0.4370629E+18	0.1910240E+36
0.2574000E-17	0.3318229E+01	0.3885004E+18	0.1509325E+36
0.2860000E-17	0.3318229E+01	0.3496503E+18	0.1222553E+36
0.5720000E-17	0.3318228E+01	0.1748252E+18	0.3056383E+35
0.8580000E-17	0.3318228E+01	0.1165501E+18	0.1358393E+35
0.1144000E-16	0.3318229E+01	0.8741258E+17	0.7640960E+34
0.1430000E-16	0.3318229E+01	0.6993007E+17	0.4890214E+34
0.1716000E-16	0.3318229E+01	0.5827506E+17	0.3395982E+34
0.2002000E-16	0.3318229E+01	0.4995005E+17	0.2495007E+34
0.2288000E-16	0.3318229E+01	0.4370629E+17	0.1910240E+34
0.2574000E-16	0.3318229E+01	0.3885004E+17	0.1509325E+34
0.2860000E-16	0.3318229E+01	0.3496503E+17	0.1222553E+34
0.5720000E-16	0.3318228E+01	0.1748252E+17	0.3056383E+33
0.8580000E-16	0.3318228E+01	0.1165501E+17	0.1358393E+33
0.1144000E-15	0.3318229E+01	0.8741258E+16	0.7640960E+32
0.1430000E-15	0.3318229E+01	0.6993007E+16	0.4890214E+32
0.1716000E-15	0.3318229E+01	0.5827506E+16	0.3395982E+32
0.2002000E-15	0.3318229E+01	0.4995005E+16	0.2495007E+32
0.2288000E-15	0.3318229E+01	0.4370629E+16	0.1910240E+32
0.2574000E-15	0.3318229E+01	0.3885004E+16	0.1509325E+32
0.2860000E-15	0.3318229E+01	0.3496503E+16	0.1222553E+32
0.5720000E-15	0.3318228E+01	0.1748252E+16	0.3056383E+31
0.8580000E-15	0.3318228E+01	0.1165501E+16	0.1358393E+31
0.1144000E-14	0.3318229E+01	0.8741258E+15	0.7640960E+30
0.1430000E-14	0.3318229E+01	0.6993007E+15	0.4890214E+30
0.1716000E-14	0.3318229E+01	0.5827506E+15	0.3395982E+30
0.2002000E-14	0.3318229E+01	0.4995005E+15	0.2495007E+30
0.2288000E-14	0.3318229E+01	0.4370629E+15	0.1910240E+30
0.2574000E-14	0.3318229E+01	0.3885004E+15	0.1509325E+30
0.2860000E-14	0.3318229E+01	0.3496503E+15	0.1222553E+30
0.5720000E-14	0.3318228E+01	0.1748252E+15	0.3056383E+29
0.8580000E-14	0.3318228E+01	0.1165501E+15	0.1358393E+29
0.1144000E-13	0.3318229E+01	0.8741258E+14	0.7640960E+28
0.1430000E-13	0.3318229E+01	0.6993007E+14	0.4890214E+28
0.1716000E-13	0.3318229E+01	0.5827506E+14	0.3395982E+28
0.2002000E-13	0.3318229E+01	0.4995005E+14	0.2495007E+28
0.2288000E-13	0.3318229E+01	0.4370629E+14	0.1910240E+28
0.2574000E-13	0.3318229E+01	0.3885004E+14	0.1509325E+28
0.2860000E-13	0.3318229E+01	0.3496503E+14	0.1222553E+28

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0.5720000E-13	0.3318228E+01	0.1748252E+14	0.3056383E+27	
0.8580000E-13	0.3318228E+01	0.1165501E+14	0.1358393E+27	
0.1144000E-12	0.3318229E+01	0.8741258E+13	0.7640960E+26	
0.1430000E-12	0.3318229E+01	0.6993007E+13	0.4890214E+26	
0.1716000E-12	0.3318229E+01	0.5827506E+13	0.3395982E+26	
0.2002000E-12	0.3318229E+01	0.4995005E+13	0.2495007E+26	
0.2288000E-12	0.3318229E+01	0.4370629E+13	0.1910240E+26	
0.2574000E-12	0.3318229E+01	0.3885004E+13	0.1509325E+26	
0.2860000E-12	0.3318229E+01	0.3496503E+13	0.1222553E+26	
0.5720000E-12	0.3318228E+01	0.1748252E+13	0.3056383E+25	
0.8580000E-12	0.3318228E+01	0.1165501E+13	0.1358393E+25	
0.1144000E-11	0.3318229E+01	0.8741258E+12	0.7640960E+24	
0.1430000E-11	0.3318229E+01	0.6993007E+12	0.4890214E+24	
0.1716000E-11	0.3318229E+01	0.5827506E+12	0.3395982E+24	
0.2002000E-11	0.3318229E+01	0.4995005E+12	0.2495007E+24	
0.2288000E-11	0.3318229E+01	0.4370629E+12	0.1910240E+24	
0.2574000E-11	0.3318229E+01	0.3885004E+12	0.1509325E+24	
0.2860000E-11	0.3318229E+01	0.3496503E+12	0.1222553E+24	
0.5720000E-11	0.3318228E+01	0.1748252E+12	0.3056383E+23	
0.8580000E-11	0.3318228E+01	0.1165501E+12	0.1358393E+23	
0.1144000E-10	0.3318229E+01	0.8741258E+11	0.7640960E+22	
0.1430000E-10	0.3318229E+01	0.6993007E+11	0.4890214E+22	
0.1716000E-10	0.3318229E+01	0.5827506E+11	0.3395982E+22	
0.2002000E-10	0.3318229E+01	0.4995005E+11	0.2495007E+22	
0.2288000E-10	0.3318229E+01	0.4370629E+11	0.1910240E+22	
0.2574000E-10	0.3318229E+01	0.3885004E+11	0.1509325E+22	
0.2860000E-10	0.3318229E+01	0.3496503E+11	0.1222553E+22	
0.5720000E-10	0.3318228E+01	0.1748252E+11	0.3056383E+21	
0.8580000E-10	0.3318228E+01	0.1165501E+11	0.1358393E+21	
0.1144000E-09	0.3318229E+01	0.8741258E+10	0.7640960E+20	
0.1430000E-09	0.3318229E+01	0.6993007E+10	0.4890214E+20	
0.1716000E-09	0.3318229E+01	0.5827506E+10	0.3395982E+20	
0.2002000E-09	0.3318229E+01	0.4995005E+10	0.2495007E+20	
0.2288000E-09	0.3318229E+01	0.4370629E+10	0.1910240E+20	
0.2574000E-09	0.3318229E+01	0.3885004E+10	0.1509325E+20	
0.2860000E-09	0.3318229E+01	0.3496503E+10	0.1222553E+20	
0.5720000E-09	0.3318229E+01	0.1748252E+10	0.3056383E+19	
0.8580000E-09	0.3318229E+01	0.1165501E+10	0.1358393E+19	
0.1144000E-08	0.3318229E+01	0.8741259E+09	0.7640960E+18	
0.1430000E-08	0.3318229E+01	0.6993007E+09	0.4890214E+18	
0.1716000E-08	0.3318230E+01	0.5827506E+09	0.3395982E+18	
0.2002000E-08	0.3318230E+01	0.4995005E+09	0.2495007E+18	
0.2288000E-08	0.3318230E+01	0.4370630E+09	0.1910240E+18	
0.2574000E-08	0.3318230E+01	0.3885004E+09	0.1509325E+18	
0.2860000E-08	0.3318230E+01	0.3496504E+09	0.1222553E+18	
0.5720000E-08	0.3318232E+01	0.1748252E+09	0.3056384E+17	

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0.8580000E-08	0.3318233E+01	0.1165502E+09	0.1358393E+17
0.1144000E-07	0.3318235E+01	0.8741265E+08	0.7640962E+16
0.1430000E-07	0.3318237E+01	0.6993013E+08	0.4890216E+16
0.1716000E-07	0.3318239E+01	0.5827513E+08	0.3395983E+16
0.2002000E-07	0.3318240E+01	0.4995012E+08	0.2495008E+16
0.2288000E-07	0.3318242E+01	0.4370636E+08	0.1910241E+16
0.2574000E-07	0.3318244E+01	0.3885011E+08	0.1509326E+16
0.2860000E-07	0.3318245E+01	0.3496510E+08	0.1222554E+16
0.5720000E-07	0.3318261E+01	0.1748259E+08	0.3056387E+15
0.8580000E-07	0.3318277E+01	0.1165508E+08	0.1358395E+15
0.1144000E-06	0.3318294E+01	0.8741331E+07	0.7640979E+14
0.1430000E-06	0.3318311E+01	0.6993080E+07	0.4890230E+14
0.1716000E-06	0.3318327E+01	0.5827579E+07	0.3395995E+14
0.2002000E-06	0.3318343E+01	0.4995078E+07	0.2495018E+14
0.2288000E-06	0.3318360E+01	0.4370703E+07	0.1910250E+14
0.2574000E-06	0.3318376E+01	0.3885077E+07	0.1509334E+14
0.2860000E-06	0.3318393E+01	0.3496577E+07	0.1222561E+14
0.5720000E-06	0.3318556E+01	0.1748325E+07	0.3056422E+13
0.8580000E-06	0.3318719E+01	0.1165575E+07	0.1358419E+13
0.1144000E-05	0.3318884E+01	0.8741996E+06	0.7641157E+12
0.1430000E-05	0.3319048E+01	0.6993745E+06	0.4890372E+12
0.1716000E-05	0.3319211E+01	0.5828244E+06	0.3396114E+12
0.2002000E-05	0.3319375E+01	0.4995743E+06	0.2495120E+12
0.2288000E-05	0.3319539E+01	0.4371368E+06	0.1910338E+12
0.2574000E-05	0.3319703E+01	0.3885742E+06	0.1509413E+12
0.2860000E-05	0.3319866E+01	0.3497242E+06	0.1222632E+12
0.5720000E-05	0.3321505E+01	0.1748991E+06	0.3056778E+11
0.8580000E-05	0.3323144E+01	0.1166241E+06	0.1358656E+11
0.1144000E-04	0.3324785E+01	0.8748656E+05	0.7642934E+10
0.1430000E-04	0.3326427E+01	0.7000407E+05	0.4891794E+10
0.1716000E-04	0.3328069E+01	0.5834907E+05	0.3397298E+10
0.2002000E-04	0.3329713E+01	0.5002408E+05	0.2496136E+10
0.2288000E-04	0.3331357E+01	0.4378035E+05	0.1911227E+10
0.2574000E-04	0.3333002E+01	0.3892412E+05	0.1510203E+10
0.2860000E-04	0.3334648E+01	0.3503913E+05	0.1223344E+10
0.5720000E-04	0.3351161E+01	0.1755679E+05	0.3060339E+09
0.8580000E-04	0.3367767E+01	0.1172946E+05	0.1361032E+09
0.1144000E-03	0.3384468E+01	0.8815890E+04	0.7660780E+08
0.1430000E-03	0.3401263E+01	0.7067816E+04	0.4906088E+08
0.1716000E-03	0.3418154E+01	0.5902493E+04	0.3409224E+08
0.2002000E-03	0.3435140E+01	0.5070173E+04	0.2506369E+08
0.2288000E-03	0.3452223E+01	0.4445977E+04	0.1920192E+08
0.2574000E-03	0.3469402E+01	0.3960532E+04	0.1518181E+08
0.2860000E-03	0.3486678E+01	0.3572213E+04	0.1230532E+08
0.5720000E-03	0.3664900E+01	0.1825810E+04	0.3096708E+07
0.8580000E-03	0.3853416E+01	0.1244976E+04	0.1385572E+07

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C CZ\*ZZ6184ZZMT06 M B MCNEIL 08/30/65 FORAD 0180 010 000 00

0.1144000E-02	0.4052771E+01	0.9555862E+03	0.7847094E+06
0.1430000E-02	0.4263532E+01	0.7828193E+03	0.5057012E+06
0.1716000E-02	0.4486298E+01	0.6684025E+03	0.3536605E+06
0.2002000E-02	0.4721688E+01	0.5873638E+03	0.2616979E+06
0.2288000E-02	0.4970353E+01	0.5272181E+03	0.2018264E+06
0.2574000E-02	0.5232973E+01	0.4810313E+03	0.1606539E+06
0.2860000E-02	0.5510255E+01	0.4446438E+03	0.1311153E+06
0.5720000E-02	0.9268838E+01	0.2999561E+03	0.3565954E+05
0.8580000E-02	0.1553849E+02	0.2846518E+03	0.1760109E+05
0.1144000E-01	0.2571321E+02	0.3163813E+03	0.1130717E+05
0.1430000E-01	0.4179340E+02	0.3843560E+03	0.8557768E+04
0.1716000E-01	0.6657671E+02	0.4914480E+03	0.7309461E+04
0.2002000E-01	0.1038845E+03	0.6462832E+03	0.6871526E+04
0.2288000E-01	0.1588237E+03	0.8616166E+03	0.6973371E+04
0.2574000E-01	0.2380840E+03	0.1154125E+04	0.7508038E+04
0.2860000E-01	0.3502672E+03	0.1544644E+04	0.8444542E+04
0.5720000E-01	0.7309294E+04	0.1929066E+05	0.5162410E+05
0.8580000E-01	0.5577194E+05	0.1165105E+06	0.2439822E+06
0.1144000E-00	0.2360730E+06	0.4287598E+06	0.7793456E+06
0.1430000E-00	0.6882651E+06	0.1137587E+07	0.1880993E+07
0.1716000E-00	0.1562288E+07	0.2412149E+07	0.3725285E+07
0.2002000E-00	0.2974352E+07	0.4361190E+07	0.6395918E+07
0.2288000E-00	0.4981961E+07	0.7014551E+07	0.9878060E+07
0.2574000E-00	0.7580558E+07	0.1032982E+08	0.1407831E+08
0.2860000E+00	0.1071439E+08	0.1421185E+08	0.1885374E+08
0.5720000E+00	0.5000172E+08	0.5818201E+08	0.6771748E+08
0.8580000E+00	0.6820756E+08	0.7571079E+08	0.8407988E+08
0.1144000E+01	0.6603763E+08	0.7159219E+08	0.7767361E+08
0.1430000E+01	0.5591818E+08	0.5980455E+08	0.6402840E+08
0.1716000E+01	0.4451661E+08	0.4721278E+08	0.5013724E+08
0.2002000E+01	0.3435300E+08	0.3623597E+08	0.3827904E+08
0.2288000E+01	0.2607227E+08	0.2740074E+08	0.2884368E+08
0.2574000E+01	0.1960588E+08	0.2055213E+08	0.2158093E+08
0.2860000E+01	0.1466624E+08	0.1534545E+08	0.1608440E+08
0.5720000E+01	0.7577352E+06	0.7867784E+06	0.8181790E+06
0.8580000E+01	0.3914722E+05	0.4048267E+05	0.4191377E+05
0.1144000E+02	0.2043457E+04	0.2106461E+04	0.2173508E+04
0.1430000E+02	0.1075714E+03	0.1106012E+03	0.1138074E+03
0.1716000E+02	0.5702278E+01	0.5850234E+01	0.6006084E+01
0.2002000E+02	0.3040439E+00	0.3113613E+00	0.3190395E+00
0.2288000E+02	0.1629253E-01	0.1665832E-01	0.1704088E-01
0.2574000E+02	0.8768227E-03	0.8952766E-03	0.9145216E-03
0.2860000E+02	0.4736645E-04	0.4830487E-04	0.4928109E-04
0.5720000E+02	0.1149780E-16	0.1164210E-16	0.1179005E-16
0.8580000E+02	0.3212120E-29	0.3241709E-29	0.3271844E-29
0.1144000E+03	0.9649001E-42	0.9719263E-42	0.9790551E-42

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0.1430000E+03	0.3030606E-54	0.3048859E-54	0.3067331E-54
0.1716000E+03	0.9810751E-67	0.9861136E-67	0.9912038E-67
0.2002000E+03	0.3246162E-79	0.3260694E-79	0.3275356E-79
0.2288000E+03	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+03	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2860000E+03	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.5720000E+03	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8580000E+03	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1144000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1430000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1716000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2002000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2288000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2860000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.5720000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8580000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1144000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1430000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1716000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2002000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2288000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2860000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.5720000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8580000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1144000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1430000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1716000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2002000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2288000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2860000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.5720000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8580000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1144000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1430000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1716000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2002000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2288000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2860000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.5720000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8580000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1144000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1430000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00

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0.1716000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2002000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2288000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2860000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.5720000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8580000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1144000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1430000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1716000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2002000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2288000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2860000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.5720000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8580000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1144000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1430000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1716000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2002000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2288000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2860000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.5720000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8580000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1144000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1430000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1716000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2002000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2288000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00

STOP END OF PROGRAM AT STATEMENT 0102 + 01 LINES.

TABLE F-2  
TRANSFORM FUNCTIONS FOR ALUMINUM AT 100°K.

The tabulated values are as follows:

$\delta$	$H_{-1}(\delta, \gamma)$	$H_0(\delta, \gamma)$	$H_1(\delta, \gamma)$
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The parameter  $\epsilon$  is taken equal to one nearest neighbor spacing. The parameter  $\gamma$  is that appropriate to this material and temperature, according to the work of Kroupa and Brown.

C	CZ*ZZ6128ZZRT06	R. B. MCNEIL	08/15/65	FORAD	0100 005 000
	0.1144000E-17	0.3318229E+01	0.8741258E+18		0.7640960E+36
	0.1430000E-17	0.3318229E+01	0.6993007E+18		0.4890214E+36
	0.1716000E-17	0.3318229E+01	0.5827506E+18		0.3395982E+36
	0.2002000E-17	0.3318229E+01	0.4995005E+18		0.2495007E+36
	0.2288000E-17	0.3318229E+01	0.4370629E+18		0.1910240E+36
	0.2574000E-17	0.3318229E+01	0.3885004E+18		0.1509325E+36
	0.2860000E-17	0.3318229E+01	0.3496503E+18		0.1222553E+36
	0.5720000E-17	0.3318228E+01	0.1748252E+18		0.3056383E+35
	0.8580000E-17	0.3318228E+01	0.1165501E+18		0.1358393E+35
	0.1144000E-16	0.3318229E+01	0.8741258E+17		0.7640960E+34
	0.1430000E-16	0.3318229E+01	0.6993007E+17		0.4890214E+34
	0.1716000E-16	0.3318229E+01	0.5827506E+17		0.3395982E+34
	0.2002000E-16	0.3318229E+01	0.4995005E+17		0.2495007E+34
	0.2288000E-16	0.3318229E+01	0.4370629E+17		0.1910240E+34
	0.2574000E-16	0.3318229E+01	0.3885004E+17		0.1509325E+34
	0.2860000E-16	0.3318229E+01	0.3496503E+17		0.1222553E+34
	0.5720000E-16	0.3318228E+01	0.1748252E+17		0.3056383E+33
	0.8580000E-16	0.3318228E+01	0.1165501E+17		0.1358393E+33
	0.1144000E-15	0.3318229E+01	0.8741258E+16		0.7640960E+32
	0.1430000E-15	0.3318229E+01	0.6993007E+16		0.4890214E+32
	0.1716000E-15	0.3318229E+01	0.5827506E+16		0.3395982E+32
	0.2002000E-15	0.3318229E+01	0.4995005E+16		0.2495007E+32
	0.2288000E-15	0.3318229E+01	0.4370629E+16		0.1910240E+32
	0.2574000E-15	0.3318229E+01	0.3885004E+16		0.1509325E+32
	0.2860000E-15	0.3318229E+01	0.3496503E+16		0.1222553E+32
	0.5720000E-15	0.3318228E+01	0.1748252E+16		0.3056383E+31
	0.8580000E-15	0.3318228E+01	0.1165501E+16		0.1358393E+31
	0.1144000E-14	0.3318229E+01	0.8741258E+15		0.7640960E+30
	0.1430000E-14	0.3318229E+01	0.6993007E+15		0.4890214E+30
	0.1716000E-14	0.3318229E+01	0.5827506E+15		0.3395982E+30
	0.2002000E-14	0.3318229E+01	0.4995005E+15		0.2495007E+30
	0.2288000E-14	0.3318229E+01	0.4370629E+15		0.1910240E+30
	0.2574000E-14	0.3318229E+01	0.3885004E+15		0.1509325E+30
	0.2860000E-14	0.3318229E+01	0.3496503E+15		0.1222553E+30
	0.5720000E-14	0.3318228E+01	0.1748252E+15		0.3056383E+29
	0.8580000E-14	0.3318228E+01	0.1165501E+15		0.1358393E+29
	0.1144000E-13	0.3318229E+01	0.8741258E+14		0.7640960E+28
	0.1430000E-13	0.3318229E+01	0.6993007E+14		0.4890214E+28
	0.1716000E-13	0.3318229E+01	0.5827506E+14		0.3395982E+28
	0.2002000E-13	0.3318229E+01	0.4995005E+14		0.2495007E+28
	0.2288000E-13	0.3318229E+01	0.4370629E+14		0.1910240E+28
	0.2574000E-13	0.3318229E+01	0.3885004E+14		0.1509325E+28
	0.2860000E-13	0.3318229E+01	0.3496503E+14		0.1222553E+28
	0.5720000E-13	0.3318228E+01	0.1748252E+14		0.3056383E+27
	0.8580000E-13	0.3318228E+01	0.1165501E+14		0.1358393E+27
	0.1144000E-12	0.3318229E+01	0.8741258E+13		0.7640960E+26



C	CZ*ZZ6128ZZMT06	M B MCNEIL	08/15/65 FORMO	0100 005 000 0
	0.1430000E-12	0.3318229E+01	0.6993007E+13	0.4890214E+26
	0.1716000E-12	0.3318229E+01	0.5827506E+13	0.3395982E+26
	0.2002000E-12	0.3318229E+01	0.4995005E+13	0.2495007E+26
	0.2288000E-12	0.3318229E+01	0.4370629E+13	0.1910240E+26
	0.2574000E-12	0.3318229E+01	0.3885004E+13	0.1509325E+26
	0.2860000E-12	0.3318229E+01	0.3496503E+13	0.1222553E+26
	0.5720000E-12	0.3318228E+01	0.1748252E+13	0.3056383E+25
	0.8580000E-12	0.3318228E+01	0.1165501E+13	0.1358393E+25
	0.1144000E-11	0.3318229E+01	0.8741258E+12	0.7640960E+24
	0.1430000E-11	0.3318229E+01	0.6993007E+12	0.4890214E+24
	0.1716000E-11	0.3318229E+01	0.5827506E+12	0.3395982E+24
	0.2002000E-11	0.3318229E+01	0.4995005E+12	0.2495007E+24
	0.2288000E-11	0.3318229E+01	0.4370629E+12	0.1910240E+24
	0.2574000E-11	0.3318229E+01	0.3885004E+12	0.1509325E+24
	0.2860000E-11	0.3318229E+01	0.3496503E+12	0.1222553E+24
	0.5720000E-11	0.3318228E+01	0.1748252E+12	0.3056383E+23
	0.8580000E-11	0.3318228E+01	0.1165501E+12	0.1358393E+23
	0.1144000E-10	0.3318229E+01	0.8741258E+11	0.7640960E+22
	0.1430000E-10	0.3318229E+01	0.6993007E+11	0.4890214E+22
	0.1716000E-10	0.3318229E+01	0.5827506E+11	0.3395982E+22
	0.2002000E-10	0.3318229E+01	0.4995005E+11	0.2495007E+22
	0.2288000E-10	0.3318229E+01	0.4370629E+11	0.1910240E+22
	0.2574000E-10	0.3318229E+01	0.3885004E+11	0.1509325E+22
	0.2860000E-10	0.3318229E+01	0.3496503E+11	0.1222553E+22
	0.5720000E-10	0.3318228E+01	0.1748252E+11	0.3056383E+21
	0.8580000E-10	0.3318228E+01	0.1165501E+11	0.1358393E+21
	0.1144000E-09	0.3318229E+01	0.8741258E+10	0.7640960E+20
	0.1430000E-09	0.3318229E+01	0.6993007E+10	0.4890214E+20
	0.1716000E-09	0.3318229E+01	0.5827506E+10	0.3395982E+20
	0.2002000E-09	0.3318229E+01	0.4995005E+10	0.2495007E+20
	0.2288000E-09	0.3318229E+01	0.4370629E+10	0.1910240E+20
	0.2574000E-09	0.3318229E+01	0.3885004E+10	0.1509325E+20
	0.2860000E-09	0.3318229E+01	0.3496503E+10	0.1222553E+20
	0.5720000E-09	0.3318229E+01	0.1748252E+10	0.3056383E+19
	0.8580000E-09	0.3318229E+01	0.1165501E+10	0.1358393E+19
	0.1144000E-08	0.3318229E+01	0.8741259E+09	0.7640960E+18
	0.1430000E-08	0.3318229E+01	0.6993007E+09	0.4890214E+18
	0.1716000E-08	0.3318229E+01	0.5827506E+09	0.3395982E+18
	0.2002000E-08	0.3318229E+01	0.4995005E+09	0.2495007E+18
	0.2288000E-08	0.3318230E+01	0.4370629E+09	0.1910240E+18
	0.2574000E-08	0.3318229E+01	0.3885004E+09	0.1509325E+18
	0.2860000E-08	0.3318230E+01	0.3496503E+09	0.1222553E+18
	0.5720000E-08	0.3318230E+01	0.1748252E+09	0.3056384E+17
	0.8580000E-08	0.3318231E+01	0.1165501E+09	0.1358393E+17
	0.1144000E-07	0.3318232E+01	0.8741261E+08	0.7640960E+16
	0.1430000E-07	0.3318232E+01	0.6993010E+08	0.4890215E+16

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	0.1716000E-07	0.3318234E+01	0.5827509E+08		0.3395983E+16
	0.2002000E-07	0.3318234E+01	0.4995008E+08		0.2495008E+16
	0.2288000E-07	0.3318235E+01	0.4370632E+08		0.1910240E+16
	0.2574000E-07	0.3318236E+01	0.3885007E+08		0.1509326E+16
	0.2860000E-07	0.3318237E+01	0.3496507E+08		0.1222554E+16
	0.5720000E-07	0.3318244E+01	0.1748255E+08		0.3056385E+15
	0.8580000E-07	0.3318252E+01	0.1165505E+08		0.1358394E+15
	0.1144000E-06	0.3318260E+01	0.8741294E+07		0.7640969E+14
	0.1430000E-06	0.3318267E+01	0.6993042E+07		0.4890222E+14
	0.1716000E-06	0.3318275E+01	0.5827541E+07		0.3395988E+14
	0.2002000E-06	0.3318283E+01	0.4995040E+07		0.2495013E+14
	0.2288000E-06	0.3318291E+01	0.4370665E+07		0.1910245E+14
	0.2574000E-06	0.3318298E+01	0.3885039E+07		0.1509330E+14
	0.2860000E-06	0.3318306E+01	0.3496539E+07		0.1222557E+14
	0.5720000E-06	0.3318383E+01	0.1748288E+07		0.3056403E+13
	0.8580000E-06	0.3318461E+01	0.1165537E+07		0.1358406E+13
	0.1144000E-05	0.3318539E+01	0.8741622E+06		0.7641058E+12
	0.1430000E-05	0.3318617E+01	0.6993370E+06		0.4890293E+12
	0.1716000E-05	0.3318694E+01	0.5827869E+06		0.3396048E+12
	0.2002000E-05	0.3318772E+01	0.4995369E+06		0.2495064E+12
	0.2288000E-05	0.3318849E+01	0.4370993E+06		0.1910289E+12
	0.2574000E-05	0.3318927E+01	0.3885368E+06		0.1509369E+12
	0.2860000E-05	0.3319005E+01	0.3496867E+06		0.1222593E+12
	0.5720000E-05	0.3319780E+01	0.1748616E+06		0.3056580E+11
	0.8580000E-05	0.3320556E+01	0.1165866E+06		0.1358524E+11
	0.1144000E-04	0.3321333E+01	0.8744904E+05		0.7641947E+10
	0.1430000E-04	0.3322110E+01	0.6996653E+05		0.4891004E+10
	0.1716000E-04	0.3322886E+01	0.5831152E+05		0.3396640E+10
	0.2002000E-04	0.3323663E+01	0.4998652E+05		0.2495571E+10
	0.2288000E-04	0.3324440E+01	0.4374277E+05		0.1910733E+10
	0.2574000E-04	0.3325218E+01	0.3888653E+05		0.1509764E+10
	0.2860000E-04	0.3325996E+01	0.3500153E+05		0.1222948E+10
	0.5720000E-04	0.3333780E+01	0.1751905E+05		0.3058359E+09
	0.8580000E-04	0.3341584E+01	0.1169159E+05		0.1359710E+09
	0.1144000E-03	0.3349405E+01	0.8777876E+04		0.7650848E+08
	0.1430000E-03	0.3357246E+01	0.7029664E+04		0.4898130E+08
	0.1716000E-03	0.3365104E+01	0.5864202E+04		0.3402581E+08
	0.2002000E-03	0.3372981E+01	0.5031741E+04		0.2500667E+08
	0.2288000E-03	0.3380877E+01	0.4407405E+04		0.1915194E+08
	0.2574000E-03	0.3388791E+01	0.3921819E+04		0.1513732E+08
	0.2860000E-03	0.3396724E+01	0.3533358E+04		0.1226521E+08
	0.5720000E-03	0.3477079E+01	0.1785505E+04		0.3076323E+07
	0.8580000E-03	0.3559326E+01	0.1203157E+04		0.1371755E+07
	0.1144000E-02	0.3643498E+01	0.9121904E+03		0.7741700E+06
	0.1430000E-02	0.3729626E+01	0.7377790E+03		0.4971229E+06
	0.1716000E-02	0.3817742E+01	0.6216483E+03		0.3463852E+06

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0.2002000E-02	0.3907881E+01	0.5388233E+03	0.2553491E+06
0.2288000E-02	0.4000073E+01	0.4768163E+03	0.1961688E+06
0.2574000E-02	0.4094352E+01	0.4286901E+03	0.1555305E+06
0.2860000E-02	0.4190753E+01	0.3902820E+03	0.1264161E+06
0.5720000E-02	0.5278887E+01	0.2202033E+03	0.3276528E+05
0.8580000E-02	0.6617189E+01	0.1673030E+03	0.1514316E+05
0.1144000E-01	0.8241829E+01	0.1442181E+03	0.8888157E+04
0.1430000E-01	0.1019015E+02	0.1335136E+03	0.5958227E+04
0.1716000E-01	0.1250022E+02	0.1294075E+03	0.4351885E+04
0.2002000E-01	0.1521034E+02	0.1294448E+03	0.3377704E+04
0.2288000E-01	0.1835863E+02	0.1324162E+03	0.2744551E+04
0.2574000E-01	0.2198257E+02	0.1376640E+03	0.2312249E+04
0.2860000E-01	0.2611860E+02	0.1448041E+03	0.2006395E+04
0.5720000E-01	0.1023072E+03	0.2923494E+03	0.1210656E+04
0.8580000E-01	0.2530760E+03	0.5494271E+03	0.1398510E+04
0.1144000E-00	0.4750916E+03	0.8839486E+03	0.1787004E+04
0.1430000E-00	0.7487762E+03	0.1259971E+04	0.2231799E+04
0.1716000E-00	0.1049922E+04	0.1645292E+04	0.2672719E+04
0.2002000E-00	0.1356838E+04	0.2015951E+04	0.3079125E+04
0.2288000E-00	0.1653077E+04	0.2356485E+04	0.3436154E+04
0.2574000E-00	0.1927629E+04	0.2658383E+04	0.3738320E+04
0.2860000E+00	0.2174039E+04	0.2918163E+04	0.3985672E+04
0.5720000E+00	0.3122999E+04	0.3694148E+04	0.4423207E+04
0.8580000E+00	0.2718369E+04	0.3087746E+04	0.3548578E+04
0.1144000E+01	0.2106370E+04	0.2349786E+04	0.2651085E+04
0.1430000E+01	0.1574322E+04	0.1739243E+04	0.1942084E+04
0.1716000E+01	0.1161946E+04	0.1275807E+04	0.1414895E+04
0.2002000E+01	0.8537376E+03	0.9332662E+03	0.1029730E+04
0.2288000E+01	0.6263786E+03	0.6823375E+03	0.7497410E+03
0.2574000E+01	0.4594661E+03	0.4990366E+03	0.5463848E+03
0.2860000E+01	0.3371247E+03	0.3652076E+03	0.3986021E+03
0.5720000E+01	0.1568508E+02	0.1669658E+02	0.1784912E+02
0.8580000E+01	0.7598400E+00	0.8003687E+00	0.8454586E+00
0.1144000E+02	0.3779133E-01	0.3951872E-01	0.4141042E-01
0.1430000E+02	0.1915071E-02	0.1991854E-02	0.2074987E-02
0.1716000E+02	0.9842375E-04	0.1019437E-03	0.1057217E-03
0.2002000E+02	0.5114681E-05	0.5279925E-05	0.5456066E-05
0.2288000E+02	0.2681752E-06	0.2760811E-06	0.2844613E-06
0.2574000E+02	0.1416533E-07	0.1454950E-07	0.1495482E-07
0.2860000E+02	0.7528951E-09	0.7718067E-09	0.7916805E-09
0.5720000E+02	0.1666349E-21	0.1690553E-21	0.1715464E-21
0.8580000E+02	0.4469128E-34	0.4514905E-34	0.4561623E-34
0.1144000E+03	0.1311806E-46	0.1322190E-46	0.1332739E-46
0.1430000E+03	0.4059182E-59	0.4085371E-59	0.4111898E-59
0.1716000E+03	0.1300417E-71	0.1307498E-71	0.1314657E-71
0.2002000E+03	0.4269779E-84	0.4289893E-84	0.4310197E-84



TABLE F-3  
TRANSFORM FUNCTIONS FOR ALUMINUM AT 300°K.

The tabulated values are as follows:

$\delta$	$H_{-1}(\delta, \gamma)$	$H_0(\delta, \gamma)$	$H_1(\delta, \gamma)$
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The parameter  $\epsilon$  is taken equal to one nearest neighbor spacing. The parameter  $\gamma$  is that appropriate to this material and temperature, according to the work of Kroupa and Brown.

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0.2860000E-18	0.3318229E+01	0.3496503E+19	0.1222553E+38
0.5720000E-18	0.3318228E+01	0.1748252E+19	0.3056383E+37
0.8580000E-18	0.3318228E+01	0.1165501E+19	0.1358393E+37
0.1144000E-17	0.3318229E+01	0.8741258E+18	0.7640960E+36
0.1430000E-17	0.3318229E+01	0.6993007E+18	0.4890214E+36
0.1716000E-17	0.3318229E+01	0.5827506E+18	0.3395982E+36
0.2002000E-17	0.3318229E+01	0.4995005E+18	0.2495007E+36
0.2288000E-17	0.3318229E+01	0.4370629E+18	0.1910240E+36
0.2574000E-17	0.3318229E+01	0.3885004E+18	0.1509325E+36
0.2860000E-17	0.3318229E+01	0.3496503E+18	0.1222553E+36
0.5720000E-17	0.3318228E+01	0.1748252E+18	0.3056383E+35
0.8580000E-17	0.3318228E+01	0.1165501E+18	0.1358393E+35
0.1144000E-16	0.3318229E+01	0.8741258E+17	0.7640960E+34
0.1430000E-16	0.3318229E+01	0.6993007E+17	0.4890214E+34
0.1716000E-16	0.3318229E+01	0.5827506E+17	0.3395982E+34
0.2002000E-16	0.3318229E+01	0.4995005E+17	0.2495007E+34
0.2288000E-16	0.3318229E+01	0.4370629E+17	0.1910240E+34
0.2574000E-16	0.3318229E+01	0.3885004E+17	0.1509325E+34
0.2860000E-16	0.3318229E+01	0.3496503E+17	0.1222553E+34
0.5720000E-16	0.3318228E+01	0.1748252E+17	0.3056383E+33
0.8580000E-16	0.3318228E+01	0.1165501E+17	0.1358393E+33
0.1144000E-15	0.3318229E+01	0.8741258E+16	0.7640960E+32
0.1430000E-15	0.3318229E+01	0.6993007E+16	0.4890214E+32
0.1716000E-15	0.3318229E+01	0.5827506E+16	0.3395982E+32
0.2002000E-15	0.3318229E+01	0.4995005E+16	0.2495007E+32
0.2288000E-15	0.3318229E+01	0.4370629E+16	0.1910240E+32
0.2574000E-15	0.3318229E+01	0.3885004E+16	0.1509325E+32
0.2860000E-15	0.3318229E+01	0.3496503E+16	0.1222553E+32
0.5720000E-15	0.3318228E+01	0.1748252E+16	0.3056383E+31
0.8580000E-15	0.3318228E+01	0.1165501E+16	0.1358393E+31
0.1144000E-14	0.3318229E+01	0.8741258E+15	0.7640960E+30
0.1430000E-14	0.3318229E+01	0.6993007E+15	0.4890214E+30
0.1716000E-14	0.3318229E+01	0.5827506E+15	0.3395982E+30
0.2002000E-14	0.3318229E+01	0.4995005E+15	0.2495007E+30
0.2288000E-14	0.3318229E+01	0.4370629E+15	0.1910240E+30
0.2574000E-14	0.3318229E+01	0.3885004E+15	0.1509325E+30
0.2860000E-14	0.3318229E+01	0.3496503E+15	0.1222553E+30
0.5720000E-14	0.3318228E+01	0.1748252E+15	0.3056383E+29
0.8580000E-14	0.3318228E+01	0.1165501E+15	0.1358393E+29
0.1144000E-13	0.3318229E+01	0.8741258E+14	0.7640960E+28
0.1430000E-13	0.3318229E+01	0.6993007E+14	0.4890214E+28
0.1716000E-13	0.3318229E+01	0.5827506E+14	0.3395982E+28
0.2002000E-13	0.3318229E+01	0.4995005E+14	0.2495007E+28
0.2288000E-13	0.3318229E+01	0.4370629E+14	0.1910240E+28
0.2574000E-13	0.3318229E+01	0.3885004E+14	0.1509325E+28
0.2860000E-13	0.3318229E+01	0.3496503E+14	0.1222553E+28

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	0.5720000E-13	0.3318228E+01	0.1748252E+14	0.3056383E+27
	0.8580000E-13	0.3318228E+01	0.1165501E+14	0.1358393E+27
	0.1144000E-12	0.3318229E+01	0.8741258E+13	0.7640960E+26
	0.1430000E-12	0.3318229E+01	0.6993007E+13	0.4890214E+26
	0.1716000E-12	0.3318229E+01	0.5827506E+13	0.3395982E+26
	0.2002000E-12	0.3318229E+01	0.4995005E+13	0.2495007E+26
	0.2288000E-12	0.3318229E+01	0.4370629E+13	0.1910240E+26
	0.2574000E-12	0.3318229E+01	0.3885004E+13	0.1509325E+26
	0.2860000E-12	0.3318229E+01	0.3496503E+13	0.1222553E+26
	0.5720000E-12	0.3318228E+01	0.1748252E+13	0.3056383E+25
	0.8580000E-12	0.3318228E+01	0.1165501E+13	0.1358393E+25
	0.1144000E-11	0.3318229E+01	0.8741258E+12	0.7640960E+24
	0.1430000E-11	0.3318229E+01	0.6993007E+12	0.4890214E+24
	0.1716000E-11	0.3318229E+01	0.5827506E+12	0.3395982E+24
	0.2002000E-11	0.3318229E+01	0.4995005E+12	0.2495007E+24
	0.2288000E-11	0.3318229E+01	0.4370629E+12	0.1910240E+24
	0.2574000E-11	0.3318229E+01	0.3885004E+12	0.1509325E+24
	0.2860000E-11	0.3318229E+01	0.3496503E+12	0.1222553E+24
	0.5720000E-11	0.3318228E+01	0.1748252E+12	0.3056383E+23
	0.8580000E-11	0.3318228E+01	0.1165501E+12	0.1358393E+23
	0.1144000E-10	0.3318229E+01	0.8741258E+11	0.7640960E+22
	0.1430000E-10	0.3318229E+01	0.6993007E+11	0.4890214E+22
	0.1716000E-10	0.3318229E+01	0.5827506E+11	0.3395982E+22
	0.2002000E-10	0.3318229E+01	0.4995005E+11	0.2495007E+22
	0.2288000E-10	0.3318229E+01	0.4370629E+11	0.1910240E+22
	0.2574000E-10	0.3318229E+01	0.3885004E+11	0.1509325E+22
	0.2860000E-10	0.3318229E+01	0.3496503E+11	0.1222553E+22
	0.5720000E-10	0.3318228E+01	0.1748252E+11	0.3056383E+21
	0.8580000E-10	0.3318228E+01	0.1165501E+11	0.1358393E+21
	0.1144000E-09	0.3318229E+01	0.8741258E+10	0.7640960E+20
	0.1430000E-09	0.3318229E+01	0.6993007E+10	0.4890214E+20
	0.1716000E-09	0.3318229E+01	0.5827506E+10	0.3395982E+20
	0.2002000E-09	0.3318229E+01	0.4995005E+10	0.2495007E+20
	0.2288000E-09	0.3318229E+01	0.4370629E+10	0.1910240E+20
	0.2574000E-09	0.3318229E+01	0.3885004E+10	0.1509325E+20
	0.2860000E-09	0.3318229E+01	0.3496503E+10	0.1222553E+20
	0.5720000E-09	0.3318228E+01	0.1748252E+10	0.3056383E+19
	0.8580000E-09	0.3318228E+01	0.1165501E+10	0.1358393E+19
	0.1144000E-08	0.3318229E+01	0.8741258E+09	0.7640960E+18
	0.1430000E-08	0.3318229E+01	0.6993007E+09	0.4890214E+18
	0.1716000E-08	0.3318229E+01	0.5827506E+09	0.3395982E+18
	0.2002000E-08	0.3318229E+01	0.4995005E+09	0.2495007E+18
	0.2288000E-08	0.3318229E+01	0.4370629E+09	0.1910240E+18
	0.2574000E-08	0.3318229E+01	0.3885004E+09	0.1509325E+18
	0.2860000E-08	0.3318229E+01	0.3496503E+09	0.1222553E+18
	0.5720000E-08	0.3318229E+01	0.1748252E+09	0.3056383E+17

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0.8580000E-08	0.3318229E+01	0.1165501E+09	0.1358393E+17
0.1144000E-07	0.3318230E+01	0.8741260E+08	0.7640960E+16
0.1430000E-07	0.3318230E+01	0.6993008E+08	0.4890214E+16
0.1716000E-07	0.3318230E+01	0.5827506E+08	0.3395982E+16
0.2002000E-07	0.3318230E+01	0.4995005E+08	0.2495007E+16
0.2288000E-07	0.3318230E+01	0.4370630E+08	0.1910240E+16
0.2574000E-07	0.3318231E+01	0.3885005E+08	0.1509325E+16
0.2860000E-07	0.3318231E+01	0.3496504E+08	0.1222554E+16
0.5720000E-07	0.3318232E+01	0.1748253E+08	0.3056384E+15
0.8580000E-07	0.3318234E+01	0.1165502E+08	0.1358393E+15
0.1144000E-06	0.3318237E+01	0.8741269E+07	0.7640963E+14
0.1430000E-06	0.3318239E+01	0.6993018E+07	0.4890217E+14
0.1716000E-06	0.3318241E+01	0.5827516E+07	0.3395984E+14
0.2002000E-06	0.3318243E+01	0.4995016E+07	0.2495009E+14
0.2288000E-06	0.3318245E+01	0.4370640E+07	0.1910241E+14
0.2574000E-06	0.3318247E+01	0.3885015E+07	0.1509327E+14
0.2860000E-06	0.3318249E+01	0.3496515E+07	0.1222555E+14
0.5720000E-06	0.3318268E+01	0.1748263E+07	0.3056389E+13
0.8580000E-06	0.3318289E+01	0.1165512E+07	0.1358397E+13
0.1144000E-05	0.3318309E+01	0.8741372E+06	0.7640993E+12
0.1430000E-05	0.3318330E+01	0.6993121E+06	0.4890240E+12
0.1716000E-05	0.3318350E+01	0.5827620E+06	0.3396004E+12
0.2002000E-05	0.3318370E+01	0.4995119E+06	0.2495026E+12
0.2288000E-05	0.3318390E+01	0.4370743E+06	0.1910256E+12
0.2574000E-05	0.3318410E+01	0.3885118E+06	0.1509340E+12
0.2860000E-05	0.3318430E+01	0.3496617E+06	0.1222566E+12
0.5720000E-05	0.3318631E+01	0.1748366E+06	0.3056449E+11
0.8580000E-05	0.3318832E+01	0.1165616E+06	0.1358436E+11
0.1144000E-04	0.3319034E+01	0.8742405E+05	0.7641289E+10
0.1430000E-04	0.3319235E+01	0.6994154E+05	0.4890477E+10
0.1716000E-04	0.3319436E+01	0.5828653E+05	0.3396201E+10
0.2002000E-04	0.3319638E+01	0.4996152E+05	0.2495195E+10
0.2288000E-04	0.3319839E+01	0.4371777E+05	0.1910404E+10
0.2574000E-04	0.3320041E+01	0.3886151E+05	0.1509471E+10
0.2860000E-04	0.3320242E+01	0.3497651E+05	0.1222685E+10
0.5720000E-04	0.3322254E+01	0.1749400E+05	0.3057041E+09
0.8580000E-04	0.3324268E+01	0.1166650E+05	0.1358831E+09
0.1144000E-03	0.3326282E+01	0.8752750E+04	0.7644250E+08
0.1430000E-03	0.3328295E+01	0.7004500E+04	0.4892847E+08
0.1716000E-03	0.3330309E+01	0.5839001E+04	0.3398176E+08
0.2002000E-03	0.3332322E+01	0.5006503E+04	0.2496888E+08
0.2288000E-03	0.3334336E+01	0.4382129E+04	0.1911886E+08
0.2574000E-03	0.3336351E+01	0.3896506E+04	0.1510789E+08
0.2860000E-03	0.3338365E+01	0.3508009E+04	0.1223871E+08
0.5720000E-03	0.3358512E+01	0.1759779E+04	0.3062980E+07
0.8580000E-03	0.3378671E+01	0.1177050E+04	0.1362797E+07



C	CZ*ZZ6188ZZMT06	M B MCNEIL	08/31/65	FORMO	0180 010 00
0.1144000E-02	0.3398840E+01	0.8856964E+03	0.7674039E+06		
0.1430000E-02	0.3419019E+01	0.7108928E+03	0.4916716E+06		
0.1716000E-02	0.3439206E+01	0.5943641E+03	0.3418099E+06		
0.2002000E-02	0.3459401E+01	0.5111354E+03	0.2513992E+06		
0.2288000E-02	0.3479603E+01	0.4487191E+03	0.1926876E+06		
0.2574000E-02	0.3499811E+01	0.4001777E+03	0.1524134E+06		
0.2860000E-02	0.3520023E+01	0.3613488E+03	0.1235901E+06		
0.5720000E-02	0.3722285E+01	0.1867300E+03	0.3124091E+05		
0.8580000E-02	0.3924268E+01	0.1286528E+03	0.1404182E+05		
0.1144000E-01	0.4125299E+01	0.9970456E+02	0.7989270E+04		
0.1430000E-01	0.4324773E+01	0.8240276E+02	0.5172783E+04		
0.1716000E-01	0.4522153E+01	0.7091992E+02	0.3634725E+04		
0.2002000E-01	0.4716965E+01	0.6275858E+02	0.2702444E+04		
0.2288000E-01	0.4908793E+01	0.5667007E+02	0.2094193E+04		
0.2574000E-01	0.5097280E+01	0.5196078E+02	0.1675007E+04		
0.2860000E-01	0.5282115E+01	0.4821460E+02	0.1373610E+04		
0.5720000E-01	0.6890349E+01	0.3171562E+02	0.3905296E+03		
0.8580000E-01	0.8035659E+01	0.2628709E+02	0.1980475E+03		
0.1144000E-00	0.8776542E+01	0.2338269E+02	0.1267241E+03		
0.1430000E-00	0.9205948E+01	0.2139770E+02	0.9164482E+02		
0.1716000E-00	0.9408030E+01	0.1984618E+02	0.7132034E+02		
0.2002000E-00	0.9448704E+01	0.1854159E+02	0.5821044E+02		
0.2288000E-00	0.9376786E+01	0.1739980E+02	0.4909072E+02		
0.2574000E-00	0.9227579E+01	0.1637754E+02	0.4238356E+02		
0.2860000E+00	0.9026352E+01	0.1544975E+02	0.3723715E+02		
0.5720000E+00	0.6411343E+01	0.9227881E+01	0.1576953E+02		
0.8580000E+00	0.4379144E+01	0.5895635E+01	0.8868557E+01		
0.1144000E+01	0.3013819E+01	0.3901760E+01	0.5473919E+01		
0.1430000E+01	0.2094103E+01	0.2638300E+01	0.3536999E+01		
0.1716000E+01	0.1466576E+01	0.1810001E+01	0.2348718E+01		
0.2002000E+01	0.1033592E+01	0.1254845E+01	0.1588334E+01		
0.2288000E+01	0.7321753E+00	0.8769567E+00	0.1088280E+01		
0.2574000E+01	0.5208601E+00	0.6167696E+00	0.7530865E+00		
0.2860000E+01	0.3718615E+00	0.4360321E+00	0.5252123E+00		
0.5720000E+01	0.1445335E-01	0.1603239E-01	0.1797717E-01		
0.8580000E+01	0.6328083E-03	0.6843405E-03	0.7445713E-03		
0.1144000E+02	0.2943256E-04	0.3135476E-04	0.3353429E-04		
0.1430000E+02	0.1421161E-05	0.1498847E-05	0.1585182E-05		
0.1716000E+02	0.7040796E-07	0.7372205E-07	0.7735250E-07		
0.2002000E+02	0.3554473E-08	0.3701495E-08	0.3860815E-08		
0.2288000E+02	0.1820497E-09	0.1887690E-09	0.1959887E-09		
0.2574000E+02	0.9431089E-11	0.9745447E-11	0.1008092E-10		
0.2860000E+02	0.4931278E-12	0.5081162E-12	0.5240214E-12		
0.5720000E+02	0.1002477E-24	0.1018723E-24	0.1035498E-24		
0.8580000E+02	0.2599824E-37	0.2628589E-37	0.2657991E-37		
0.1144000E+03	0.7494568E-50	0.7557535E-50	0.7621560E-50		

C	CZ*ZZ6188ZZMT06	M B MCNEIL	08/31/65 FORMO	0180 010 01
0.1430000E+03	0.2293038E-62	0.2308568E-62	0.2324308E-62	
0.1716000E+03	0.7289497E-75	0.7330851E-75	0.7372675E-75	
0.2002000E+03	0.2380000E-87	0.2391616E-87	0.2403346E-87	
0.2288000E+03	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2574000E+03	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2860000E+03	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.5720000E+03	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.8580000E+03	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1144000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1430000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1716000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2002000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2288000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2574000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2860000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.5720000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.8580000E+04	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1144000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1430000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1716000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2002000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2288000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2574000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2860000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.5720000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.8580000E+05	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1144000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1430000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1716000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2002000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2288000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2574000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2860000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.5720000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.8580000E+06	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1144000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1430000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1716000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2002000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2288000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2574000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.2860000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.5720000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.8580000E+07	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1144000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00	
0.1430000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00	

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C CZ\*ZZ6188ZZMT06 M B MCNEIL 08/31/65 FORM0 0180 010 00

0.1716000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2002000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2288000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2860000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.5720000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8580000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1144000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1430000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1716000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2002000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2288000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2860000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.5720000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8580000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1144000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1430000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1716000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2002000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2288000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2860000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.5720000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8580000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1144000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1430000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1716000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2002000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2288000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2574000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00

STOP END OF PROGRAM AT STATEMENT 0102 + 01 LINES.

TABLE F-4  
TRANSFORM FUNCTIONS FOR SILICON AT 50°K.

The tabulated values are as follows:

$$\delta \quad H_{-1}(\delta, \gamma) \quad H_0(\delta, \gamma) \quad H_1(\delta, \gamma)$$

The parameter  $\epsilon$  is taken equal to one nearest neighbor spacing. The parameter  $\gamma$  is that appropriate to this material and temperature, according to the work of Kroupa and Brown.

CZ*ZZ6179ZZMT06	M B MCNEIL	08/27/65 FORMD	0180 010 000 000
0.4000000E-18	0.3318228E+01	0.2500000E+19	0.6249999E+37
0.8000000E-18	0.3318228E+01	0.1250000E+19	0.1562500E+37
0.1200000E-17	0.3318229E+01	0.8333333E+18	0.6944444E+36
0.1600000E-17	0.3318229E+01	0.6250000E+18	0.3906250E+36
0.2000000E-17	0.3318229E+01	0.5000000E+18	0.2500000E+36
0.2400000E-17	0.3318229E+01	0.4166666E+18	0.1736111E+36
0.2800000E-17	0.3318229E+01	0.3571428E+18	0.1275510E+36
0.3200000E-17	0.3318228E+01	0.3125000E+18	0.9765623E+35
0.3600000E-17	0.3318228E+01	0.2777778E+18	0.7716047E+35
0.4000000E-17	0.3318228E+01	0.2500000E+18	0.6249999E+35
0.8000000E-17	0.3318228E+01	0.1250000E+18	0.1562500E+35
0.1200000E-16	0.3318229E+01	0.8333333E+17	0.6944444E+34
0.1600000E-16	0.3318229E+01	0.6250000E+17	0.3906250E+34
0.2000000E-16	0.3318229E+01	0.5000000E+17	0.2500000E+34
0.2400000E-16	0.3318229E+01	0.4166666E+17	0.1736111E+34
0.2800000E-16	0.3318229E+01	0.3571428E+17	0.1275510E+34
0.3200000E-16	0.3318228E+01	0.3125000E+17	0.9765623E+33
0.3600000E-16	0.3318228E+01	0.2777778E+17	0.7716047E+33
0.4000000E-16	0.3318228E+01	0.2500000E+17	0.6249999E+33
0.8000000E-16	0.3318228E+01	0.1250000E+17	0.1562500E+33
0.1200000E-15	0.3318229E+01	0.8333333E+16	0.6944444E+32
0.1600000E-15	0.3318229E+01	0.6250000E+16	0.3906250E+32
0.2000000E-15	0.3318229E+01	0.5000000E+16	0.2500000E+32
0.2400000E-15	0.3318229E+01	0.4166666E+16	0.1736111E+32
0.2800000E-15	0.3318229E+01	0.3571428E+16	0.1275510E+32
0.3200000E-15	0.3318228E+01	0.3125000E+16	0.9765623E+31
0.3600000E-15	0.3318228E+01	0.2777778E+16	0.7716047E+31
0.4000000E-15	0.3318228E+01	0.2500000E+16	0.6249999E+31
0.8000000E-15	0.3318228E+01	0.1250000E+16	0.1562500E+31
0.1200000E-14	0.3318229E+01	0.8333333E+15	0.6944444E+30
0.1600000E-14	0.3318229E+01	0.6250000E+15	0.3906250E+30
0.2000000E-14	0.3318229E+01	0.5000000E+15	0.2500000E+30
0.2400000E-14	0.3318229E+01	0.4166666E+15	0.1736111E+30
0.2800000E-14	0.3318229E+01	0.3571428E+15	0.1275510E+30
0.3200000E-14	0.3318228E+01	0.3125000E+15	0.9765623E+29
0.3600000E-14	0.3318228E+01	0.2777778E+15	0.7716047E+29
0.4000000E-14	0.3318228E+01	0.2500000E+15	0.6249999E+29
0.8000000E-14	0.3318228E+01	0.1250000E+15	0.1562500E+29
0.1200000E-13	0.3318229E+01	0.8333333E+14	0.6944444E+28
0.1600000E-13	0.3318229E+01	0.6250000E+14	0.3906250E+28
0.2000000E-13	0.3318229E+01	0.5000000E+14	0.2500000E+28
0.2400000E-13	0.3318229E+01	0.4166666E+14	0.1736111E+28
0.2800000E-13	0.3318229E+01	0.3571428E+14	0.1275510E+28
0.3200000E-13	0.3318228E+01	0.3125000E+14	0.9765623E+27
0.3600000E-13	0.3318228E+01	0.2777778E+14	0.7716047E+27
0.4000000E-13	0.3318228E+01	0.2500000E+14	0.6249999E+27

CZ\*ZZ6179ZZMT06 M B MCNEIL 08/27/65 FORMO 0180 010 000 000

0.8000000E-13	0.3318228E+01	0.1250000E+14	0.1562500E+27
0.1200000E-12	0.3318229E+01	0.8333333E+13	0.6944444E+26
0.1600000E-12	0.3318229E+01	0.6250000E+13	0.3906250E+26
0.2000000E-12	0.3318229E+01	0.5000000E+13	0.2500000E+26
0.2400000E-12	0.3318229E+01	0.4166666E+13	0.1736111E+26
0.2800000E-12	0.3318229E+01	0.3571428E+13	0.1275510E+26
0.3200000E-12	0.3318228E+01	0.3125000E+13	0.9765623E+25
0.3600000E-12	0.3318228E+01	0.2777778E+13	0.7716047E+25
0.4000000E-12	0.3318228E+01	0.2500000E+13	0.6249999E+25
0.8000000E-12	0.3318228E+01	0.1250000E+13	0.1562500E+25
0.1200000E-11	0.3318229E+01	0.8333333E+12	0.6944444E+24
0.1600000E-11	0.3318229E+01	0.6250000E+12	0.3906250E+24
0.2000000E-11	0.3318229E+01	0.5000000E+12	0.2500000E+24
0.2400000E-11	0.3318229E+01	0.4166666E+12	0.1736111E+24
0.2800000E-11	0.3318229E+01	0.3571428E+12	0.1275510E+24
0.3200000E-11	0.3318228E+01	0.3125000E+12	0.9765623E+23
0.3600000E-11	0.3318228E+01	0.2777778E+12	0.7716047E+23
0.4000000E-11	0.3318228E+01	0.2500000E+12	0.6249999E+23
0.8000000E-11	0.3318228E+01	0.1250000E+12	0.1562500E+23
0.1200000E-10	0.3318229E+01	0.8333333E+11	0.6944444E+22
0.1600000E-10	0.3318229E+01	0.6250000E+11	0.3906250E+22
0.2000000E-10	0.3318229E+01	0.5000000E+11	0.2500000E+22
0.2400000E-10	0.3318229E+01	0.4166666E+11	0.1736111E+22
0.2800000E-10	0.3318229E+01	0.3571428E+11	0.1275510E+22
0.3200000E-10	0.3318228E+01	0.3125000E+11	0.9765623E+21
0.3600000E-10	0.3318228E+01	0.2777778E+11	0.7716047E+21
0.4000000E-10	0.3318228E+01	0.2500000E+11	0.6249999E+21
0.8000000E-10	0.3318228E+01	0.1250000E+11	0.1562500E+21
0.1200000E-09	0.3318229E+01	0.8333333E+10	0.6944444E+20
0.1600000E-09	0.3318229E+01	0.6250000E+10	0.3906250E+20
0.2000000E-09	0.3318229E+01	0.5000000E+10	0.2500000E+20
0.2400000E-09	0.3318229E+01	0.4166667E+10	0.1736111E+20
0.2800000E-09	0.3318229E+01	0.3571428E+10	0.1275510E+20
0.3200000E-09	0.3318228E+01	0.3125000E+10	0.9765623E+19
0.3600000E-09	0.3318228E+01	0.2777778E+10	0.7716047E+19
0.4000000E-09	0.3318228E+01	0.2500000E+10	0.6249999E+19
0.8000000E-09	0.3318229E+01	0.1250000E+10	0.1562500E+19
0.1200000E-08	0.3318230E+01	0.8333334E+09	0.6944444E+18
0.1600000E-08	0.3318230E+01	0.6250001E+09	0.3906250E+18
0.2000000E-08	0.3318231E+01	0.5000001E+09	0.2500000E+18
0.2400000E-08	0.3318231E+01	0.4166667E+09	0.1736111E+18
0.2800000E-08	0.3318231E+01	0.3571429E+09	0.1275510E+18
0.3200000E-08	0.3318231E+01	0.3125001E+09	0.9765623E+17
0.3600000E-08	0.3318231E+01	0.2777779E+09	0.7716048E+17
0.4000000E-08	0.3318232E+01	0.2500001E+09	0.6250000E+17
0.8000000E-08	0.3318236E+01	0.1250001E+09	0.1562500E+17

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0.1200000E-07	0.3318240E+01	0.8333344E+08	0.6944446E+16
0.1600000E-07	0.3318244E+01	0.6250012E+08	0.3906252E+16
0.2000000E-07	0.3318248E+01	0.5000012E+08	0.2500002E+16
0.2400000E-07	0.3318252E+01	0.4166678E+08	0.1736112E+16
0.2800000E-07	0.3318256E+01	0.3571440E+08	0.1275511E+16
0.3200000E-07	0.3318259E+01	0.3125012E+08	0.9765635E+15
0.3600000E-07	0.3318262E+01	0.2777790E+08	0.7716058E+15
0.4000000E-07	0.3318266E+01	0.2500012E+08	0.6250008E+15
0.8000000E-07	0.3318305E+01	0.1250012E+08	0.1562504E+15
0.1200000E-06	0.3318343E+01	0.8333453E+07	0.6944474E+14
0.1600000E-06	0.3318382E+01	0.6250121E+07	0.3906273E+14
0.2000000E-06	0.3318420E+01	0.5000121E+07	0.2500018E+14
0.2400000E-06	0.3318459E+01	0.4166788E+07	0.1736126E+14
0.2800000E-06	0.3318497E+01	0.3571550E+07	0.1275523E+14
0.3200000E-06	0.3318534E+01	0.3125121E+07	0.9765738E+13
0.3600000E-06	0.3318572E+01	0.2777899E+07	0.7716151E+13
0.4000000E-06	0.3318611E+01	0.2500121E+07	0.6250091E+13
0.8000000E-06	0.3318994E+01	0.1250122E+07	0.1562546E+13
0.1200000E-05	0.3319377E+01	0.8334549E+06	0.6944752E+12
0.1600000E-05	0.3319761E+01	0.6251217E+06	0.3906481E+12
0.2000000E-05	0.3320144E+01	0.5001217E+06	0.2500185E+12
0.2400000E-05	0.3320527E+01	0.4167884E+06	0.1736265E+12
0.2800000E-05	0.3320910E+01	0.3572646E+06	0.1275642E+12
0.3200000E-05	0.3321293E+01	0.3126217E+06	0.9766779E+11
0.3600000E-05	0.3321676E+01	0.2778995E+06	0.7717076E+11
0.4000000E-05	0.3322059E+01	0.2501218E+06	0.6250923E+11
0.8000000E-05	0.3325897E+01	0.1251219E+06	0.1562962E+11
0.1200000E-04	0.3329739E+01	0.8345529E+05	0.6947529E+10
0.1600000E-04	0.3333586E+01	0.6262203E+05	0.3908564E+10
0.2000000E-04	0.3337439E+01	0.5012210E+05	0.2501852E+10
0.2400000E-04	0.3341297E+01	0.4178884E+05	0.1737655E+10
0.2800000E-04	0.3345161E+01	0.3583653E+05	0.1276834E+10
0.3200000E-04	0.3349028E+01	0.3137231E+05	0.9777207E+09
0.3600000E-04	0.3352903E+01	0.2790016E+05	0.7726347E+09
0.4000000E-04	0.3356782E+01	0.2512246E+05	0.6259271E+09
0.8000000E-04	0.3395873E+01	0.1262316E+05	0.1567147E+09
0.1200000E-03	0.3435506E+01	0.8457191E+04	0.6975503E+08
0.1600000E-03	0.3475690E+01	0.6374567E+04	0.3929601E+08
0.2000000E-03	0.3516431E+01	0.5125282E+04	0.2518727E+08
0.2400000E-03	0.3557738E+01	0.4292669E+04	0.1751756E+08
0.2800000E-03	0.3599617E+01	0.3698158E+04	0.1288953E+08
0.3200000E-03	0.3642077E+01	0.3252464E+04	0.9883550E+07
0.3600000E-03	0.3685126E+01	0.2905982E+04	0.7821135E+07
0.4000000E-03	0.3728772E+01	0.2628952E+04	0.6344817E+07
0.8000000E-03	0.4199858E+01	0.1386807E+04	0.1611158E+07
0.1200000E-02	0.4740120E+01	0.9787398E+03	0.7277830E+06

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0.1600000E-02	0.5359334E+01	0.7798283E+03	0.4163585E+06
0.2000000E-02	0.6068554E+01	0.6651564E+03	0.2712189E+06
0.2400000E-02	0.6880276E+01	0.5931502E+03	0.1918649E+06
0.2800000E-02	0.7808591E+01	0.5460543E+03	0.1437290E+06
0.3200000E-02	0.8869386E+01	0.5150524E+03	0.1123181E+06
0.3600000E-02	0.1008055E+02	0.4953073E+03	0.9068303E+05
0.4000000E-02	0.1146219E+02	0.4839783E+03	0.7515072E+05
0.8000000E-02	0.4126637E+02	0.6453984E+03	0.2601166E+05
0.1200000E-01	0.1407801E+03	0.1334164E+04	0.2069748E+05
0.1600000E-01	0.4445415E+03	0.3126821E+04	0.2713677E+05
0.2000000E-01	0.1298344E+04	0.7443279E+04	0.4642190E+05
0.2400000E-01	0.3525472E+04	0.1732139E+05	0.8808730E+05
0.2800000E-01	0.8956050E+04	0.3889572E+05	0.1714493E+06
0.3200000E-01	0.2141491E+05	0.8395979E+05	0.3314186E+06
0.3600000E-01	0.4846279E+05	0.1741846E+06	0.6281219E+06
0.4000000E-01	0.1043117E+06	0.3477636E+06	0.1161373E+07
0.8000000E-01	0.3039971E+08	0.6585686E+08	0.1426729E+09
0.1200000E-00	0.9945752E+09	0.1767925E+10	0.3142614E+10
0.1600000E-00	0.1036543E+11	0.1641028E+11	0.2598037E+11
0.2000000E-00	0.5517726E+11	0.8091963E+11	0.1186719E+12
0.2400000E-00	0.1915749E+12	0.2660560E+12	0.3694943E+12
0.2800000E-00	0.4985980E+12	0.6647522E+12	0.8862765E+12
0.3200000E-00	0.1058962E+13	0.1367742E+13	0.1766559E+13
0.3600000E-00	0.1938641E+13	0.2441116E+13	0.3073829E+13
0.4000000E+00	0.3173931E+13	0.3914316E+13	0.4827416E+13
0.8000000E+00	0.2697562E+14	0.3012243E+14	0.3363660E+14
0.1200000E+01	0.4128627E+14	0.4450154E+14	0.4796888E+14
0.1600000E+01	0.3976049E+14	0.4209514E+14	0.4457086E+14
0.2000000E+01	0.3192406E+14	0.3344249E+14	0.3503893E+14
0.2400000E+01	0.2353242E+14	0.2448582E+14	0.2548423E+14
0.2800000E+01	0.1659246E+14	0.1718728E+14	0.1780936E+14
0.3200000E+01	0.1141501E+14	0.1178790E+14	0.1217791E+14
0.3600000E+01	0.7742759E+13	0.7978468E+13	0.8225178E+13
0.4000000E+01	0.5208137E+13	0.5358401E+13	0.5515816E+13
0.8000000E+01	0.8857851E+11	0.9065934E+11	0.9284060E+11
0.1200000E+02	0.1484596E+10	0.1516186E+10	0.1549164E+10
0.1600000E+02	0.2506421E+08	0.2555335E+08	0.2606209E+08
0.2000000E+02	0.4258879E+06	0.4335693E+06	0.4415341E+06
0.2400000E+02	0.7276284E+04	0.7398335E+04	0.7524561E+04
0.2800000E+02	0.1249001E+03	0.1268588E+03	0.1288800E+03
0.3200000E+02	0.2152727E+01	0.2184432E+01	0.2217086E+01
0.3600000E+02	0.3723675E-01	0.3775383E-01	0.3828546E-01
0.4000000E+02	0.6461500E-03	0.6546388E-03	0.6633534E-03
0.8000000E+02	0.1802050E-20	0.1817484E-20	0.1833184E-20
0.1200000E+03	0.5704155E-38	0.5740471E-38	0.5777250E-38
0.1600000E+03	0.1931684E-55	0.1941477E-55	0.1951369E-55





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0.2400000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2800000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3200000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3600000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.4000000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8000000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1200000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1600000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2000000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2400000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2800000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3200000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3600000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.4000000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8000000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1200000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1600000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2000000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2400000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2800000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3200000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3600000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.4000000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8000000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1200000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1600000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2000000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2400000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2800000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3200000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3600000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00

STOP END OF PROGRAM AT STATEMENT 0102 + 01 LINES.

TABLE F-5  
TRANSFORM FUNCTIONS FOR SILICON AT 100°K.

The tabulated values are as follows:

$$\delta \quad H_{-1}(\delta, \gamma) \quad H_0(\delta, \gamma) \quad H_1(\delta, \gamma)$$

The parameter  $\epsilon$  is taken equal to one nearest neighbor spacing. The parameter  $\gamma$  is that appropriate to this material and temperature, according to the work of Kroupa and Brown.

5.2

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0.4000000E-18	0.3318228E+01	0.2500000E+19	0.6249999E+37
0.8000000E-18	0.3318228E+01	0.1250000E+19	0.1562500E+37
0.1200000E-17	0.3318229E+01	0.8333333E+18	0.6944444E+36
0.1600000E-17	0.3318229E+01	0.6250000E+18	0.3906250E+36
0.2000000E-17	0.3318229E+01	0.5000000E+18	0.2500000E+36
0.2400000E-17	0.3318229E+01	0.4166666E+18	0.1736111E+36
0.2800000E-17	0.3318229E+01	0.3571428E+18	0.1275510E+36
0.3200000E-17	0.3318228E+01	0.3125000E+18	0.9765623E+35
0.3600000E-17	0.3318228E+01	0.2777778E+18	0.7716047E+35
0.4000000E-17	0.3318228E+01	0.2500000E+18	0.6249999E+35
0.8000000E-17	0.3318228E+01	0.1250000E+18	0.1562500E+35
0.1200000E-16	0.3318229E+01	0.8333333E+17	0.6944444E+34
0.1600000E-16	0.3318229E+01	0.6250000E+17	0.3906250E+34
0.2000000E-16	0.3318229E+01	0.5000000E+17	0.2500000E+34
0.2400000E-16	0.3318229E+01	0.4166666E+17	0.1736111E+34
0.2800000E-16	0.3318229E+01	0.3571428E+17	0.1275510E+34
0.3200000E-16	0.3318228E+01	0.3125000E+17	0.9765623E+33
0.3600000E-16	0.3318228E+01	0.2777778E+17	0.7716047E+33
0.4000000E-16	0.3318228E+01	0.2500000E+17	0.6249999E+33
0.8000000E-16	0.3318228E+01	0.1250000E+17	0.1562500E+33
0.1200000E-15	0.3318229E+01	0.8333333E+16	0.6944444E+32
0.1600000E-15	0.3318229E+01	0.6250000E+16	0.3906250E+32
0.2000000E-15	0.3318229E+01	0.5000000E+16	0.2500000E+32
0.2400000E-15	0.3318229E+01	0.4166666E+16	0.1736111E+32
0.2800000E-15	0.3318229E+01	0.3571428E+16	0.1275510E+32
0.3200000E-15	0.3318228E+01	0.3125000E+16	0.9765623E+31
0.3600000E-15	0.3318228E+01	0.2777778E+16	0.7716047E+31
0.4000000E-15	0.3318228E+01	0.2500000E+16	0.6249999E+31
0.8000000E-15	0.3318228E+01	0.1250000E+16	0.1562500E+31
0.1200000E-14	0.3318229E+01	0.8333333E+15	0.6944444E+30
0.1600000E-14	0.3318229E+01	0.6250000E+15	0.3906250E+30
0.2000000E-14	0.3318229E+01	0.5000000E+15	0.2500000E+30
0.2400000E-14	0.3318229E+01	0.4166666E+15	0.1736111E+30
0.2800000E-14	0.3318229E+01	0.3571428E+15	0.1275510E+30
0.3200000E-14	0.3318228E+01	0.3125000E+15	0.9765623E+29
0.3600000E-14	0.3318228E+01	0.2777778E+15	0.7716047E+29
0.4000000E-14	0.3318228E+01	0.2500000E+15	0.6249999E+29
0.8000000E-14	0.3318228E+01	0.1250000E+15	0.1562500E+29
0.1200000E-13	0.3318229E+01	0.8333333E+14	0.6944444E+28
0.1600000E-13	0.3318229E+01	0.6250000E+14	0.3906250E+28
0.2000000E-13	0.3318229E+01	0.5000000E+14	0.2500000E+28
0.2400000E-13	0.3318229E+01	0.4166666E+14	0.1736111E+28
0.2800000E-13	0.3318229E+01	0.3571428E+14	0.1275510E+28
0.3200000E-13	0.3318228E+01	0.3125000E+14	0.9765623E+27
0.3600000E-13	0.3318228E+01	0.2777778E+14	0.7716047E+27
0.4000000E-13	0.3318228E+01	0.2500000E+14	0.6249999E+27

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0.8000000E-13	0.3318228E+01	0.1250000E+14	0.1562500E+27				
0.1200000E-12	0.3318229E+01	0.8333333E+13	0.6944444E+26				
0.1600000E-12	0.3318229E+01	0.6250000E+13	0.3906250E+26				
0.2000000E-12	0.3318229E+01	0.5000000E+13	0.2500000E+26				
0.2400000E-12	0.3318229E+01	0.4166666E+13	0.1736111E+26				
0.2800000E-12	0.3318229E+01	0.3571428E+13	0.1275510E+26				
0.3200000E-12	0.3318228E+01	0.3125000E+13	0.9765623E+25				
0.3600000E-12	0.3318228E+01	0.2777778E+13	0.7716047E+25				
0.4000000E-12	0.3318228E+01	0.2500000E+13	0.6249999E+25				
0.8000000E-12	0.3318228E+01	0.1250000E+13	0.1562500E+25				
0.1200000E-11	0.3318229E+01	0.8333333E+12	0.6944444E+24				
0.1600000E-11	0.3318229E+01	0.6250000E+12	0.3906250E+24				
0.2000000E-11	0.3318229E+01	0.5000000E+12	0.2500000E+24				
0.2400000E-11	0.3318229E+01	0.4166666E+12	0.1736111E+24				
0.2800000E-11	0.3318229E+01	0.3571428E+12	0.1275510E+24				
0.3200000E-11	0.3318228E+01	0.3125000E+12	0.9765623E+23				
0.3600000E-11	0.3318228E+01	0.2777778E+12	0.7716047E+23				
0.4000000E-11	0.3318228E+01	0.2500000E+12	0.6249999E+23				
0.8000000E-11	0.3318228E+01	0.1250000E+12	0.1562500E+23				
0.1200000E-10	0.3318229E+01	0.8333333E+11	0.6944444E+22				
0.1600000E-10	0.3318229E+01	0.6250000E+11	0.3906250E+22				
0.2000000E-10	0.3318229E+01	0.5000000E+11	0.2500000E+22				
0.2400000E-10	0.3318229E+01	0.4166666E+11	0.1736111E+22				
0.2800000E-10	0.3318229E+01	0.3571428E+11	0.1275510E+22				
0.3200000E-10	0.3318228E+01	0.3125000E+11	0.9765623E+21				
0.3600000E-10	0.3318228E+01	0.2777778E+11	0.7716047E+21				
0.4000000E-10	0.3318228E+01	0.2500000E+11	0.6249999E+21				
0.8000000E-10	0.3318228E+01	0.1250000E+11	0.1562500E+21				
0.1200000E-09	0.3318229E+01	0.8333333E+10	0.6944444E+20				
0.1600000E-09	0.3318229E+01	0.6250000E+10	0.3906250E+20				
0.2000000E-09	0.3318229E+01	0.5000000E+10	0.2500000E+20				
0.2400000E-09	0.3318229E+01	0.4166667E+10	0.1736111E+20				
0.2800000E-09	0.3318229E+01	0.3571429E+10	0.1275510E+20				
0.3200000E-09	0.3318229E+01	0.3125000E+10	0.9765623E+19				
0.3600000E-09	0.3318228E+01	0.2777778E+10	0.7716048E+19				
0.4000000E-09	0.3318229E+01	0.2500000E+10	0.6249999E+19				
0.8000000E-09	0.3318230E+01	0.1250000E+10	0.1562500E+19				
0.1200000E-08	0.3318231E+01	0.8333335E+09	0.6944444E+18				
0.1600000E-08	0.3318232E+01	0.6250002E+09	0.3906250E+18				
0.2000000E-08	0.3318233E+01	0.5000002E+09	0.2500000E+18				
0.2400000E-08	0.3318233E+01	0.4166668E+09	0.1736111E+18				
0.2800000E-08	0.3318234E+01	0.3571431E+09	0.1275510E+18				
0.3200000E-08	0.3318234E+01	0.3125002E+09	0.9765625E+17				
0.3600000E-08	0.3318234E+01	0.2777780E+09	0.7716049E+17				
0.4000000E-08	0.3318236E+01	0.2500002E+09	0.6250001E+17				
0.8000000E-08	0.3318244E+01	0.1250002E+09	0.1562501E+17				

CZ*ZZ6176ZZMT06	M	B	MCNEIL	08/27/65	FORMO	0180 010 000 000
0.1200000E-07	0.3318252E+01	0.8333357E+08	0.6944450E+16			
0.1600000E-07	0.3318260E+01	0.6250024E+08	0.3906254E+16			
0.2000000E-07	0.3318267E+01	0.5000024E+08	0.2500003E+16			
0.2400000E-07	0.3318275E+01	0.4166691E+08	0.1736114E+16			
0.2800000E-07	0.3318283E+01	0.3571452E+08	0.1275513E+16			
0.3200000E-07	0.3318290E+01	0.3125024E+08	0.9765646E+15			
0.3600000E-07	0.3318297E+01	0.2777802E+08	0.7716068E+15			
0.4000000E-07	0.3318306E+01	0.2500024E+08	0.6250017E+15			
0.8000000E-07	0.3318383E+01	0.1250024E+08	0.1562509E+15			
0.1200000E-06	0.3318462E+01	0.8333577E+07	0.6944505E+14			
0.1600000E-06	0.3318540E+01	0.6250244E+07	0.3906296E+14			
0.2000000E-06	0.3318618E+01	0.5000243E+07	0.2500037E+14			
0.2400000E-06	0.3318695E+01	0.4166910E+07	0.1736142E+14			
0.2800000E-06	0.3318773E+01	0.3571672E+07	0.1275536E+14			
0.3200000E-06	0.3318850E+01	0.3125244E+07	0.9765854E+13			
0.3600000E-06	0.3318928E+01	0.2778022E+07	0.7716253E+13			
0.4000000E-06	0.3319006E+01	0.2500244E+07	0.6250183E+13			
0.8000000E-06	0.3319784E+01	0.1250244E+07	0.1562592E+13			
0.1200000E-05	0.3320563E+01	0.8335777E+06	0.6945060E+12			
0.1600000E-05	0.3321342E+01	0.6252445E+06	0.3906712E+12			
0.2000000E-05	0.3322120E+01	0.5002445E+06	0.2500370E+12			
0.2400000E-05	0.3322899E+01	0.4169113E+06	0.1736419E+12			
0.2800000E-05	0.3323679E+01	0.3573875E+06	0.1275774E+12			
0.3200000E-05	0.3324457E+01	0.3127447E+06	0.9767936E+11			
0.3600000E-05	0.3325237E+01	0.2780225E+06	0.7718103E+11			
0.4000000E-05	0.3326017E+01	0.2502448E+06	0.6251849E+11			
0.8000000E-05	0.3333829E+01	0.1252451E+06	0.1563425E+11			
0.1200000E-04	0.3341663E+01	0.8357871E+05	0.6950619E+10			
0.1600000E-04	0.3349520E+01	0.6274567E+05	0.3910884E+10			
0.2000000E-04	0.3357400E+01	0.5024596E+05	0.2503709E+10			
0.2400000E-04	0.3365302E+01	0.4191291E+05	0.1739203E+10			
0.2800000E-04	0.3373227E+01	0.3596082E+05	0.1278162E+10			
0.3200000E-04	0.3381174E+01	0.3149683E+05	0.9788839E+09			
0.3600000E-04	0.3389146E+01	0.2802489E+05	0.7736693E+09			
0.4000000E-04	0.3397140E+01	0.2524741E+05	0.6268590E+09			
0.8000000E-04	0.3478366E+01	0.1275032E+05	0.1571841E+09			
0.1200000E-03	0.3561972E+01	0.8586628E+04	0.7007032E+08			
0.1600000E-03	0.3648031E+01	0.6506324E+04	0.3953429E+08			
0.2000000E-03	0.3736615E+01	0.5259412E+04	0.2537936E+08			
0.2400000E-03	0.3827798E+01	0.4429226E+04	0.1767888E+08			
0.2800000E-03	0.3921655E+01	0.3837197E+04	0.1302890E+08			
0.3200000E-03	0.4018270E+01	0.3394040E+04	0.1000647E+08			
0.3600000E-03	0.4117720E+01	0.3050153E+04	0.7931273E+07			
0.4000000E-03	0.4220092E+01	0.2775777E+04	0.6444749E+07			
0.8000000E-03	0.5424650E+01	0.1563715E+04	0.1665603E+07			
0.1200000E-02	0.7033445E+01	0.1193337E+04	0.7677057E+06			

CZ*ZZ6176ZZMT06	M	B	MCNEIL	08/27/65	FORMO	0180 010 000 000
0.1600000E-02			0.9180918E+01		0.1041696E+04	0.4496235E+06
0.2000000E-02			0.1204465E+02		0.9863690E+03	0.3010970E+06
0.2400000E-02			0.1585859E+02		0.9889154E+03	0.2201230E+06
0.2800000E-02			0.2093026E+02		0.1035521E+04	0.1715190E+06
0.3200000E-02			0.2766288E+02		0.1122320E+04	0.1405167E+06
0.3600000E-02			0.3658396E+02		0.1250692E+04	0.1200505E+06
0.4000000E-02			0.4838196E+02		0.1425589E+04	0.1064183E+06
0.8000000E-02			0.7399368E+03		0.9555582E+04	0.1444496E+06
0.1200000E-01			0.9410462E+04		0.8277295E+05	0.7404997E+06
0.1600000E-01			0.9943652E+05		0.6795486E+06	0.4653656E+07
0.2000000E-01			0.8884582E+06		0.5033750E+07	0.2852855E+08
0.2400000E-01			0.6827230E+07		0.3337070E+08	0.1631210E+09
0.2800000E-01			0.4579829E+08		0.1984181E+09	0.8596430E+09
0.3200000E-01			0.2717149E+09		0.1064001E+10	0.4166502E+10
0.3600000E-01			0.1442119E+10		0.5179924E+10	0.1860570E+11
0.4000000E-01			0.6916399E+10		0.2305025E+11	0.7681949E+11
0.8000000E-01			0.7950122E+15		0.1722273E+16	0.3731041E+16
0.1200000E-00			0.1090116E+19		0.1937752E+19	0.3444480E+19
0.1600000E-00			0.1463478E+21		0.2316940E+21	0.3668118E+21
0.2000000E-00			0.4997798E+22		0.7329466E+22	0.1074895E+23
0.2400000E-00			0.7125733E+23		0.9896093E+23	0.1374352E+24
0.2800000E-00			0.5626601E+24		0.7501621E+24	0.1000148E+25
0.3200000E-00			0.2924715E+25		0.3777524E+25	0.4879000E+25
0.3600000E-00			0.1118946E+26		0.1408964E+26	0.1774151E+26
0.4000000E+00			0.3397077E+26		0.4189511E+26	0.5166796E+26
0.8000000E+00			0.6628597E+28		0.7401722E+28	0.8265019E+28
0.1200000E+01			0.3351406E+29		0.3612000E+29	0.3892856E+29
0.1600000E+01			0.6056101E+29		0.6409279E+29	0.6783055E+29
0.2000000E+01			0.7157804E+29		0.7491767E+29	0.7841315E+29
0.2400000E+01			0.6838519E+29		0.7104471E+29	0.7380779E+29
0.2800000E+01			0.5793875E+29		0.5987144E+29	0.6186886E+29
0.3200000E+01			0.4560895E+29		0.4694218E+29	0.4831480E+29
0.3600000E+01			0.3423331E+29		0.3512531E+29	0.3604106E+29
0.4000000E+01			0.2488150E+29		0.2546763E+29	0.2606814E+29
0.8000000E+01			0.5547602E+27		0.5625076E+27	0.5704249E+27
0.1200000E+02			0.9870759E+25		0.9990816E+25	0.1011369E+26
0.1600000E+02			0.1729059E+24		0.1748824E+24	0.1769044E+24
0.2000000E+02			0.3029690E+22		0.3062688E+22	0.3096416E+22
0.2400000E+02			0.5317888E+20		0.5373294E+20	0.5429871E+20
0.2800000E+02			0.9350704E+18		0.9444109E+18	0.9539406E+18
0.3200000E+02			0.1646879E+17		0.1662680E+17	0.1678789E+17
0.3600000E+02			0.2904933E+15		0.2931749E+15	0.2959066E+15
0.4000000E+02			0.5131181E+13		0.5176823E+13	0.5223285E+13
0.8000000E+02			0.1609105E-04		0.1619615E-04	0.1630264E-04
0.1200000E+03			0.5421412E-22		0.5449431E-22	0.5477741E-22
0.1600000E+03			0.1908851E-39		0.1917018E-39	0.1925254E-39





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0.2400000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2800000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3200000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3600000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.4000000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8000000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1200000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1600000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2000000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2400000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2800000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3200000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3600000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.4000000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8000000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1200000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1600000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2000000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2400000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2800000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3200000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3600000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.4000000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8000000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1200000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1600000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2000000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2400000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2800000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3200000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3600000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00

STOP END OF PROGRAM AT STATEMENT 0102 + 01 LINES.

TABLE F-6  
TRANSFORM FUNCTIONS FOR SILICON AT 300°K.

The tabulated values are as follows:

$$\delta \quad H_{-1}(\delta, \gamma) \quad H_0(\delta, \gamma) \quad H_1(\delta, \gamma)$$

The parameter  $\epsilon$  is taken equal to one nearest neighbor spacing. The parameter  $\gamma$  is that appropriate to this material and temperature, according to the work of Kroupa and Brown.

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0.4000000E-18	0.3318228E+01	0.2500000E+19	0.6249999E+37
0.8000000E-18	0.3318228E+01	0.1250000E+19	0.1562500E+37
0.1200000E-17	0.3318229E+01	0.8333333E+18	0.6944444E+36
0.1600000E-17	0.3318229E+01	0.6250000E+18	0.3906250E+36
0.2000000E-17	0.3318229E+01	0.5000000E+18	0.2500000E+36
0.2400000E-17	0.3318229E+01	0.4166666E+18	0.1736111E+36
0.2800000E-17	0.3318229E+01	0.3571428E+18	0.1275510E+36
0.3200000E-17	0.3318228E+01	0.3125000E+18	0.9765623E+35
0.3600000E-17	0.3318228E+01	0.2777778E+18	0.7716047E+35
0.4000000E-17	0.3318228E+01	0.2500000E+18	0.6249999E+35
0.8000000E-17	0.3318228E+01	0.1250000E+18	0.1562500E+35
0.1200000E-16	0.3318229E+01	0.8333333E+17	0.6944444E+34
0.1600000E-16	0.3318229E+01	0.6250000E+17	0.3906250E+34
0.2000000E-16	0.3318229E+01	0.5000000E+17	0.2500000E+34
0.2400000E-16	0.3318229E+01	0.4166666E+17	0.1736111E+34
0.2800000E-16	0.3318229E+01	0.3571428E+17	0.1275510E+34
0.3200000E-16	0.3318228E+01	0.3125000E+17	0.9765623E+33
0.3600000E-16	0.3318228E+01	0.2777778E+17	0.7716047E+33
0.4000000E-16	0.3318228E+01	0.2500000E+17	0.6249999E+33
0.8000000E-16	0.3318228E+01	0.1250000E+17	0.1562500E+33
0.1200000E-15	0.3318229E+01	0.8333333E+16	0.6944444E+32
0.1600000E-15	0.3318229E+01	0.6250000E+16	0.3906250E+32
0.2000000E-15	0.3318229E+01	0.5000000E+16	0.2500000E+32
0.2400000E-15	0.3318229E+01	0.4166666E+16	0.1736111E+32
0.2800000E-15	0.3318229E+01	0.3571428E+16	0.1275510E+32
0.3200000E-15	0.3318228E+01	0.3125000E+16	0.9765623E+31
0.3600000E-15	0.3318228E+01	0.2777778E+16	0.7716047E+31
0.4000000E-15	0.3318228E+01	0.2500000E+16	0.6249999E+31
0.8000000E-15	0.3318228E+01	0.1250000E+16	0.1562500E+31
0.1200000E-14	0.3318229E+01	0.8333333E+15	0.6944444E+30
0.1600000E-14	0.3318229E+01	0.6250000E+15	0.3906250E+30
0.2000000E-14	0.3318229E+01	0.5000000E+15	0.2500000E+30
0.2400000E-14	0.3318229E+01	0.4166666E+15	0.1736111E+30
0.2800000E-14	0.3318229E+01	0.3571428E+15	0.1275510E+30
0.3200000E-14	0.3318228E+01	0.3125000E+15	0.9765623E+29
0.3600000E-14	0.3318228E+01	0.2777778E+15	0.7716047E+29
0.4000000E-14	0.3318228E+01	0.2500000E+15	0.6249999E+29
0.8000000E-14	0.3318228E+01	0.1250000E+15	0.1562500E+29
0.1200000E-13	0.3318229E+01	0.8333333E+14	0.6944444E+28
0.1600000E-13	0.3318229E+01	0.6250000E+14	0.3906250E+28
0.2000000E-13	0.3318229E+01	0.5000000E+14	0.2500000E+28
0.2400000E-13	0.3318229E+01	0.4166666E+14	0.1736111E+28
0.2800000E-13	0.3318229E+01	0.3571428E+14	0.1275510E+28
0.3200000E-13	0.3318228E+01	0.3125000E+14	0.9765623E+27
0.3600000E-13	0.3318228E+01	0.2777778E+14	0.7716047E+27
0.4000000E-13	0.3318228E+01	0.2500000E+14	0.6249999E+27

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0.8000000E-13	0.3318228E+01	0.1250000E+14	0.1562500E+27
0.1200000E-12	0.3318229E+01	0.8333333E+13	0.6944444E+26
0.1600000E-12	0.3318229E+01	0.6250000E+13	0.3906250E+26
0.2000000E-12	0.3318229E+01	0.5000000E+13	0.2500000E+26
0.2400000E-12	0.3318229E+01	0.4166666E+13	0.1736111E+26
0.2800000E-12	0.3318229E+01	0.3571428E+13	0.1275510E+26
0.3200000E-12	0.3318228E+01	0.3125000E+13	0.9765623E+25
0.3600000E-12	0.3318228E+01	0.2777778E+13	0.7716047E+25
0.4000000E-12	0.3318228E+01	0.2500000E+13	0.6249999E+25
0.8000000E-12	0.3318228E+01	0.1250000E+13	0.1562500E+25
0.1200000E-11	0.3318229E+01	0.8333333E+12	0.6944444E+24
0.1600000E-11	0.3318229E+01	0.6250000E+12	0.3906250E+24
0.2000000E-11	0.3318229E+01	0.5000000E+12	0.2500000E+24
0.2400000E-11	0.3318229E+01	0.4166666E+12	0.1736111E+24
0.2800000E-11	0.3318229E+01	0.3571428E+12	0.1275510E+24
0.3200000E-11	0.3318228E+01	0.3125000E+12	0.9765623E+23
0.3600000E-11	0.3318228E+01	0.2777778E+12	0.7716047E+23
0.4000000E-11	0.3318228E+01	0.2500000E+12	0.6249999E+23
0.8000000E-11	0.3318228E+01	0.1250000E+12	0.1562500E+23
0.1200000E-10	0.3318229E+01	0.8333333E+11	0.6944444E+22
0.1600000E-10	0.3318229E+01	0.6250000E+11	0.3906250E+22
0.2000000E-10	0.3318229E+01	0.5000000E+11	0.2500000E+22
0.2400000E-10	0.3318229E+01	0.4166666E+11	0.1736111E+22
0.2800000E-10	0.3318229E+01	0.3571428E+11	0.1275510E+22
0.3200000E-10	0.3318228E+01	0.3125000E+11	0.9765623E+21
0.3600000E-10	0.3318228E+01	0.2777778E+11	0.7716047E+21
0.4000000E-10	0.3318228E+01	0.2500000E+11	0.6249999E+21
0.8000000E-10	0.3318228E+01	0.1250000E+11	0.1562500E+21
0.1200000E-09	0.3318229E+01	0.8333333E+10	0.6944444E+20
0.1600000E-09	0.3318229E+01	0.6250000E+10	0.3906250E+20
0.2000000E-09	0.3318229E+01	0.5000000E+10	0.2500000E+20
0.2400000E-09	0.3318229E+01	0.4166666E+10	0.1736111E+20
0.2800000E-09	0.3318229E+01	0.3571428E+10	0.1275510E+20
0.3200000E-09	0.3318228E+01	0.3125000E+10	0.9765623E+19
0.3600000E-09	0.3318228E+01	0.2777778E+10	0.7716047E+19
0.4000000E-09	0.3318228E+01	0.2500000E+10	0.6249999E+19
0.8000000E-09	0.3318229E+01	0.1250000E+10	0.1562500E+19
0.1200000E-08	0.3318229E+01	0.8333333E+09	0.6944444E+18
0.1600000E-08	0.3318230E+01	0.6250000E+09	0.3906250E+18
0.2000000E-08	0.3318230E+01	0.5000001E+09	0.2500000E+18
0.2400000E-08	0.3318230E+01	0.4166667E+09	0.1736111E+18
0.2800000E-08	0.3318230E+01	0.3571429E+09	0.1275510E+18
0.3200000E-08	0.3318230E+01	0.3125000E+09	0.9765623E+17
0.3600000E-08	0.3318230E+01	0.2777778E+09	0.7716048E+17
0.4000000E-08	0.3318230E+01	0.2500001E+09	0.6249999E+17
0.8000000E-08	0.3318233E+01	0.1250001E+09	0.1562500E+17

C	CZ*ZZ6173ZZMT06	M	B	MCNEIL	08/27/65	FORMO	0180 010 000 <sup>6. Y</sup> 00
	0.1200000E-07			0.3318236E+01		0.8333341E+08	0.6944445E+16
	0.1600000E-07			0.3318239E+01		0.6250008E+08	0.3906251E+16
	0.2000000E-07			0.3318241E+01		0.5000008E+08	0.2500001E+16
	0.2400000E-07			0.3318244E+01		0.4166675E+08	0.1736112E+16
	0.2800000E-07			0.3318246E+01		0.3571436E+08	0.1275511E+16
	0.3200000E-07			0.3318248E+01		0.3125008E+08	0.9765631E+15
	0.3600000E-07			0.3318250E+01		0.2777785E+08	0.7716055E+15
	0.4000000E-07			0.3318253E+01		0.2500008E+08	0.6250005E+15
	0.8000000E-07			0.3318278E+01		0.1250008E+08	0.1562502E+15
	0.1200000E-06			0.3318304E+01		0.8333413E+07	0.6944464E+14
	0.1600000E-06			0.3318330E+01		0.6250080E+07	0.3906265E+14
	0.2000000E-06			0.3318355E+01		0.5000080E+07	0.2500012E+14
	0.2400000E-06			0.3318380E+01		0.4166747E+07	0.1736121E+14
	0.2800000E-06			0.3318405E+01		0.3571509E+07	0.1275519E+14
	0.3200000E-06			0.3318430E+01		0.3125081E+07	0.9765700E+13
	0.3600000E-06			0.3318455E+01		0.2777859E+07	0.7716117E+13
	0.4000000E-06			0.3318480E+01		0.2500081E+07	0.6250061E+13
	0.8000000E-06			0.3318733E+01		0.1250081E+07	0.1562531E+13
	0.1200000E-05			0.3318985E+01		0.8334143E+06	0.6944650E+12
	0.1600000E-05			0.3319238E+01		0.6250810E+06	0.3906404E+12
	0.2000000E-05			0.3319490E+01		0.5000810E+06	0.2500123E+12
	0.2400000E-05			0.3319742E+01		0.4167477E+06	0.1736214E+12
	0.2800000E-05			0.3319994E+01		0.3572239E+06	0.1275598E+12
	0.3200000E-05			0.3320246E+01		0.3125811E+06	0.9766396E+11
	0.3600000E-05			0.3320498E+01		0.2778589E+06	0.7716735E+11
	0.4000000E-05			0.3320751E+01		0.2500811E+06	0.6250617E+11
	0.8000000E-05			0.3323276E+01		0.1250811E+06	0.1562809E+11
	0.1200000E-04			0.3325803E+01		0.8341453E+05	0.6946507E+10
	0.1600000E-04			0.3328332E+01		0.6258123E+05	0.3907797E+10
	0.2000000E-04			0.3330864E+01		0.5008126E+05	0.2501238E+10
	0.2400000E-04			0.3333398E+01		0.4174796E+05	0.1737143E+10
	0.2800000E-04			0.3335934E+01		0.3579561E+05	0.1276395E+10
	0.3200000E-04			0.3338471E+01		0.3133135E+05	0.9773367E+09
	0.3600000E-04			0.3341011E+01		0.2785916E+05	0.7722933E+09
	0.4000000E-04			0.3343554E+01		0.2508141E+05	0.6256196E+09
	0.8000000E-04			0.3369105E+01		0.1258172E+05	0.1565603E+09
	0.1200000E-03			0.3394881E+01		0.8415357E+04	0.6965169E+08
	0.1600000E-03			0.3420885E+01		0.6332326E+04	0.3921818E+08
	0.2000000E-03			0.3447118E+01		0.5082632E+04	0.2512475E+08
	0.2400000E-03			0.3473582E+01		0.4249605E+04	0.1746524E+08
	0.2800000E-03			0.3500281E+01		0.3654676E+04	0.1284450E+08
	0.3200000E-03			0.3527213E+01		0.3208557E+04	0.9843977E+07
	0.3600000E-03			0.3554383E+01		0.2861646E+04	0.7785810E+07
	0.4000000E-03			0.3581792E+01		0.2584182E+04	0.6312889E+07
	0.8000000E-03			0.3869481E+01		0.1337415E+04	0.1594477E+07
	0.1200000E-02			0.4183342E+01		0.9241687E+03	0.7161347E+06

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C CZ\*ZZ6173ZZMT06 M B MCNEIL 08/27/65 FORMO 0180 010 000 01

0.1600000E-02	0.4525577E+01	0.7194529E+03	0.4071858E+06
0.2000000E-02	0.4898550E+01	0.5982801E+03	0.2634950E+06
0.2400000E-02	0.5304797E+01	0.5189960E+03	0.1850727E+06
0.2800000E-02	0.5747042E+01	0.4637564E+03	0.1375697E+06
0.3200000E-02	0.6228194E+01	0.4236465E+03	0.1066015E+06
0.3600000E-02	0.6751375E+01	0.3937205E+03	0.8527923E+05
0.4000000E-02	0.7319922E+01	0.3710173E+03	0.6996546E+05
0.8000000E-02	0.1641814E+02	0.3160544E+03	0.2042880E+05
0.1200000E-01	0.3591556E+02	0.3962040E+03	0.1136979E+05
0.1600000E-01	0.7537773E+02	0.5834065E+03	0.8798676E+04
0.2000000E-01	0.1511330E+03	0.9180291E+03	0.8549480E+04
0.2400000E-01	0.2897154E+03	0.1474310E+04	0.9740882E+04
0.2800000E-01	0.5323897E+03	0.2363059E+04	0.1227318E+05
0.3200000E-01	0.9408498E+03	0.3740320E+04	0.1635440E+05
0.3600000E-01	0.1604118E+04	0.5818379E+04	0.2238149E+05
0.4000000E-01	0.2646606E+04	0.8878170E+04	0.3091085E+05
0.8000000E-01	0.1064505E+06	0.2307123E+06	0.5007636E+06
0.1200000E-00	0.1010989E+07	0.1797328E+07	0.3196192E+07
0.1600000E-00	0.4520634E+07	0.7157454E+07	0.1133369E+08
0.2000000E-00	0.1300517E+08	0.1907364E+08	0.2797604E+08
0.2400000E-00	0.2828700E+08	0.3928656E+08	0.5456697E+08
0.2800000E-00	0.5098232E+08	0.6797558E+08	0.9063860E+08
0.3200000E-00	0.8051205E+08	0.1039949E+09	0.1343354E+09
0.3600000E-00	0.1154794E+09	0.1454211E+09	0.1831385E+09
0.4000000E+00	0.1541063E+09	0.1900715E+09	0.2344475E+09
0.8000000E+00	0.4645617E+09	0.5190137E+09	0.5799833E+09
0.1200000E+01	0.4804806E+09	0.5186930E+09	0.5602306E+09
0.1600000E+01	0.3785634E+09	0.4020193E+09	0.4272757E+09
0.2000000E+01	0.2704002E+09	0.2846078E+09	0.2998819E+09
0.2400000E+01	0.1853266E+09	0.1940533E+09	0.2034450E+09
0.2800000E+01	0.1246428E+09	0.1300930E+09	0.1359664E+09
0.3200000E+01	0.8307436E+08	0.8652340E+08	0.9024365E+08
0.3600000E+01	0.5512349E+08	0.5732645E+08	0.5970332E+08
0.4000000E+01	0.3649676E+08	0.3791256E+08	0.3943980E+08
0.8000000E+01	0.5831986E+06	0.6021597E+06	0.6224149E+06
0.1200000E+02	0.9459174E+04	0.9728543E+04	0.1001384E+05
0.1600000E+02	0.1555954E+03	0.1595459E+03	0.1637029E+03
0.2000000E+02	0.2587748E+01	0.2647095E+01	0.2709231E+01
0.2400000E+02	0.4342111E-01	0.4432985E-01	0.4527740E-01
0.2800000E+02	0.7339402E-03	0.7480731E-03	0.7627597E-03
0.3200000E+02	0.1248230E-04	0.1270497E-04	0.1293569E-04
0.3600000E+02	0.2134088E-06	0.2169556E-06	0.2206219E-06
0.4000000E+02	0.3665246E-08	0.3722280E-08	0.3781108E-08
0.8000000E+02	0.9613415E-26	0.9705126E-26	0.9798595E-26
0.1200000E+03	0.2957058E-43	0.2977441E-43	0.2998106E-43
0.1600000E+03	0.9848508E-61	0.9901681E-61	0.9955427E-61



C CZ\*ZZ6173ZZMT06 M B MCNEIL 08/27/65 FORM0 0180 010 000 000 <sup>67</sup>

0.2400000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2800000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3200000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3600000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.4000000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8000000E+08	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1200000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1600000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2000000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2400000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2800000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3200000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3600000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.4000000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8000000E+09	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1200000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1600000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2000000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2400000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2800000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3200000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3600000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.4000000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.8000000E+10	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1200000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.1600000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2000000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2400000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.2800000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3200000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00
0.3600000E+11	0.0000000E-00	0.0000000E-00	0.0000000E-00

STOP END OF PROGRAM AT STATEMENT 0102 + 01 LINES.



## VITA

Michael Brewer McNeil, the son of Mr. and Mrs. Brewer T. McNeil, was born in Houston, Texas, July 26, 1938. He attended St. John's School and Kinkaid Preparatory School in Houston, Texas, graduating from the latter in June 1955. He received a Bachelor of Arts degree from Rice University in June, 1959; spent the year 1959-60 at the University of Birmingham (U.K.); and returned to Rice University as a graduate student, receiving a Master of Arts degree in June, 1962. He worked at the Texas Instruments, Incorporated laboratories in Dallas, Texas until August, 1964, when he enrolled in the University of Missouri at Rolla as a candidate for a Doctor of Philosophy degree with a major in Metallurgical engineering.