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# CONTROL OF A PENDULUM USING HEDGE ALGEBRAS CONTAINING ACTUATOR SATURATION

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## ABSTRACT

In this study, the control problem of a pendulum using hedge-algebras-based fuzzy controller (HAC) containing actuator saturation is presented. In HAC, linguistic values of linguistic terms are obtained through semantically quantifying mappings (SQMs) based on several fuzzy parameters of each linguistic variable without using any fuzzy set and inherent order relationships between linguistic values are always ensured. Hence, the design of a HAC leads to determining parameters of SQMs. Numerical results of HAC are compared with those of an analogical conventional fuzzy controller (FC) in order to show advantages of the proposed method.

*Keywords*: pendulum, fuzzy control, hedge algebras; actuator saturation.

## 1. INTRODUCTION

Fuzzy set theory, first published by Zadeh in 1965, is a useful mathematical tool to model uncertain data and it has widely applied been in practice. As a typical unstable system, pendulum systems are often used as a benchmark for verifying the performance and effectiveness of a control method because of the simplicities of the structure. Recently, a lot of researches on stabilization control of pendulum systems have been introduced [1 - 10].

Hedge Algebras (HA) theory [11 - 17], first introduced in 1990, demonstrated that linguistic values can form an algebraic structure. It is a complete hedge algebra structure with the main property of which, inherent semantic order of linguistic values is always guaranteed. It is even a rich enough algebraic structure, and therefore, it can describe completely reasoning processes. Applications of HA in vibration control of structures with remarkable results [1, 18 - 20] have provided a new approach in control problem of vibrating structures.

The issue of actuator saturation is very important in real control because any actuation mechanisms are subjected to inherent physical limitations. Therefore, the limitation of actuator should be considered in control in general and in vibration control of structures in practice [21] and this is the research purpose of the paper. A pendulum structure is considered for verifying the control performance of HAC with the influence of actuator saturation.

## 2. PROBLEM UNDER CONSIDERATION

Consider a pendulum of length $l$ containing a concentrated mass $m$ at its end. A spring (of constant stiffness $k$) and a damper (of constant damping factor $c$) connect the pendulum to a fixed base as shown in Figure 1.

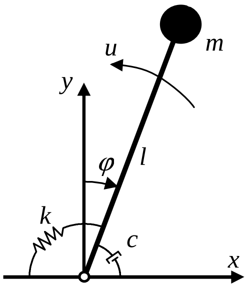

*Figure 1.* Pendulum model.

When the pendulum is deflected away from the stability in the vertical position with initial conditions including rotation angle $\varphi(0)$ and angular velocity $\dot{\varphi}(0)$, control moment $u$ applied at the base of the pendulum brings the pendulum to stability position, that is $\varphi \to 0$ and $\dot{\varphi} \to 0$. Vibration equation of the pendulum with the control moment $u$ subjected to the limitation of the actuator can be expressed as follows:

$$ml^2\ddot{\varphi} + c\dot{\varphi} + k\varphi = \text{sat}(u) \tag{1}$$

In which, sat($u$) depends on the limitation $u_{\text{lim}}$ of the actuator [21]:

$$\text{sat}(u) = \begin{cases} u_{\text{lim}} & \text{if} \quad u \ge u_{\text{lim}} \\ u & \text{if} \quad -u_{\text{lim}} \le u \le u_{\text{lim}} \\ -u_{\text{lim}} & \text{if} \quad u \le -u_{\text{lim}} \end{cases} \tag{2}$$

Control diagram of the pendulum is presented in Figure 2, where, $[x_1, x_2]^T = [\varphi, \dot{\varphi}]^T$.

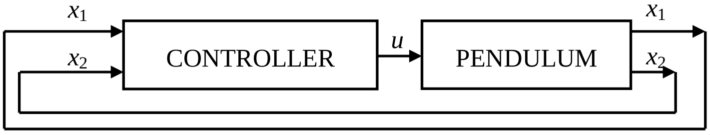

*Figure 2.* Control diagram.

It is assumed that the reference domains of the state variables and the control variable are given by: $-a_0 \le x_1 \le a_0$, $-b_0 \le x_2 \le b_0$ and $-c_0 \le u \le c_0$.

## 3. CONTROLLER DESIGN

The idea and basic formulas of HAs theory based on definitions, theorems, and propositions in [11-17] were summarized and presented in [1, 18-20].

Consider a HA structure $AX = (X, G, C, H, \le)$ of linguistic variable $X$ with primary terms $G = \{Negative, Positive\}$; $C = \{\boldsymbol{0}, \boldsymbol{W}, \boldsymbol{1}\}$ with $\boldsymbol{0}$, $\boldsymbol{W}$ and $\boldsymbol{1}$ are specific constants called *absolutely*

*Negative*, *neutral* and *absolutely Positive*, respectively; $H = \{Very, Little\}$ is the set of hedges; and $\leq$ is a partially ordering relation on $X$. Therefore, $p = q = 1$, where $p$ and $q$ are the number of positive and negative hedges, respectively, and as a result, semantically quantifying mapping (SQM) values $\phi$ of all linguistic values of the linguistic variable $X$ are determined through only two independent fuzziness parameters, which are $0 < fm(c^-)$ and $\mu(h^-) < 1$, where $fm(c^-)$ and $\mu(h^-)$ are fuzziness measure of negative primary terms and negative hedges, respectively. Several typical linguistic values with SQM values of $X$ are presented in Table 1 in the case of $fm(c^-) = \mu(h^-) = 0.5$. In which, *Ne*, *Po*, *V* and *L* stand for *Negative*, *Positive*, *Very* and *Little*, respectively.

*Table 1.* Several typical linguistic values with SQM values.

| Linguistic value | *Ne* | *LNe* | ***W*** | *LPo* | *Po* |
|---|---|---|---|---|---|
| SQM value $\phi$ | 0.25 | 0.375 | 0.5 | 0.625 | 0.75 |

Hence, when using HA to describe linguistic variables, inherent semantic order of linguistic values is always guaranteed.

Next, designing steps of the controller HAC in order to control the pendulum to vertical stability position are presented. Configuration of HAC with two state variables $x_1$, $x_2$ and control variable $u$ is shown in Figure 3 [20].

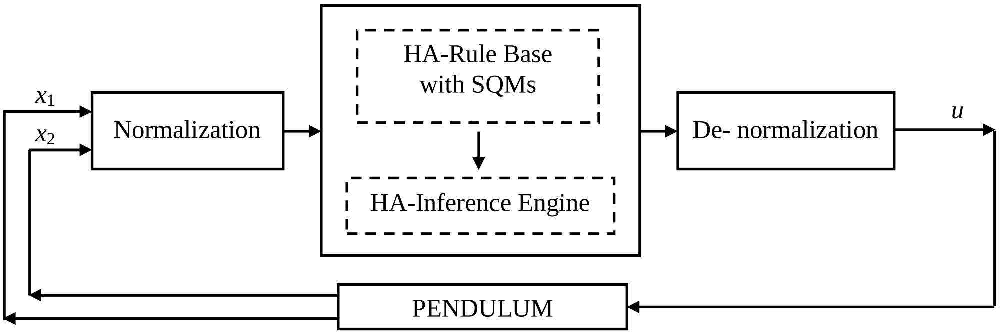

*Figure 3.* Configuration of HAC.

Although linguistic variables $x_1$, $x_2$ and $u$ under consideration are different, their hedge algebra are here defined with a similar HA structure (AX) as shown in Example 1 in the case of $fm(c^-) = \mu(h^-) = 0.5$. The components of HAC are defined as follows:

* HA-Rule Base with SQMs: a typical fuzzy rule base stored in the form of if-then rules with SQM values is used for HAC as arranged in Table 2.

*Table 2.* HA-Rule Base with SQMs.

| $x_2$ / $x_1$ | *LNe*: 0.375 | ***W***: 0.5 | *LPo*: 0.625 |
|---|---|---|---|
| *LNe*: 0.375 | *Ne*: 0.25 | *LNe*: 0.375 | ***W***: 0.5 |
| ***W***: 0.5 | *LNe*: 0.375 | ***W***: 0.5 | *LPo*: 0.625 |
| *LPo*: 0.625 | ***W***: 0.5 | *LPo*: 0.625 | *Po*: 0.75 |

* Normalization: the reference domain of a linguistic variable $X$, given in the form of an interval [a, b], must be transformed to the SQM domain, given in an interval [0, 1], by a 1-1 mapping $g_X : [a, b] \rightarrow [0, 1]$. Normalizations for the state variables are shown in Figure 4.

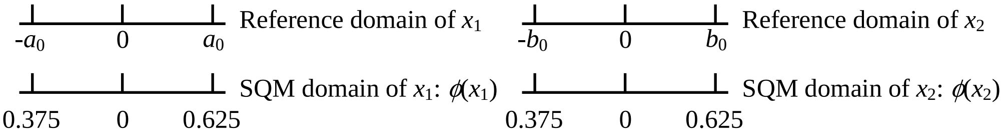

*Figure 4*. Normalizations for the state variables.

* HA-Inference Engine: semantically quantifying surface (SQS), established through grid nodes representing the HA rule base (Table 2) as shown in Figure 5, is used as HA-Inference Engine in order to determine SQM values of the control moment $u$ from given SQM values of the input variables ($x_1$ and $x_2$) [19].

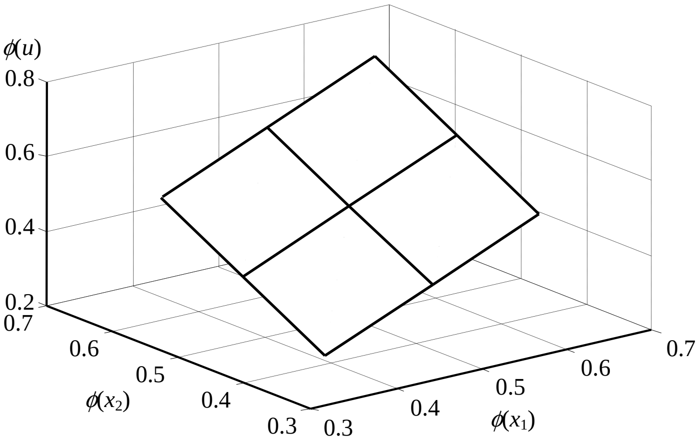

*Figure 5*. HA-Inference Engine – SQS.

* De-normalization: to convert SQM values to real values of the control moment $u$, the inverted mapping $g_X^{-1}$ of $g_X$ must be performed as shown in Figure 6.

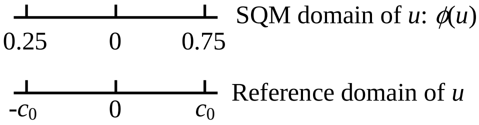

*Figure 6*. De-normalization of the control moment *u*.

Via the designing steps of HAC, several properties of HAC could be found as follows:

* Components and operation principle of HAC are similar to those of a conventional fuzzy controller (FC) in vibration control of structures with two input state variables $x_1$, $x_2$ and one output control variable $u$. The similar FC is presented in Appendix 1.

* Inherent order relationships among linguistic values of a linguistic variable are closely guaranteed without using any fuzzy sets.

* HA-Rule Base with SQMs can be clearly represented by a semantically quantifying surface (SQS).

* The steps of normalization, inference and de-normalization of HAC are very simple when only using linear interpolations.

* Therefore, it can be asserted that the controller HAC is simple in establishment, coherent in implementation and efficient in computation time [20].

## 4. NUMERICAL SIMULATIONS

In this section, the results of numerical simulations in control of the pendulum using HAC containing actuator saturation are presented. Study cases are shown in Table 3.

*Table 3*. Study cases.

| Study cases | $l$, m | $m$, kg | $c$, Nms | $k$, Nm |
|---|---|---|---|---|
| Case 1 | 1.5 | 1.8 | 0.05 | 2 |
| Case 2 | 1.5 | 1.8×1.2 | 0.05 | 2×1.2 |
| Case 3 | 1.5 | 1.8×1.2 | 0.05 | 2×0.8 |
| Case 4 | 1.5 | 1.8×0.8 | 0.05 | 2×1.2 |
| Case 5 | 1.5 | 1.8×0.8 | 0.05 | 2×0.8 |

The initial conditions of the pendulum are given as: $[x_1(0), x_2(0)]^T = [\varphi(0), \dot{\varphi}(0)]^T = [0.6 \text{ rad}, 1 \text{ rad/s}]^T$. In the uncontrolled case, vibration of the pendulum is presented in Figure 7.

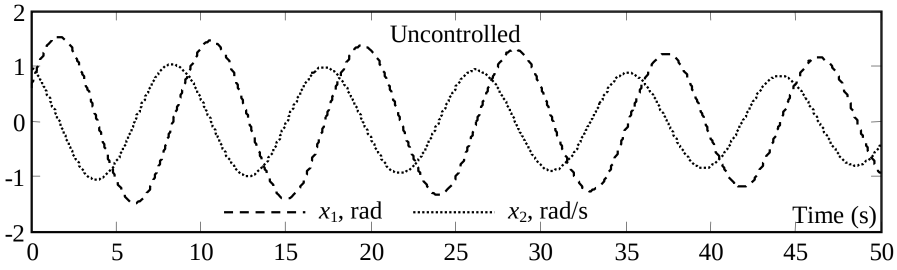

*Figure 7*. Vibration of the pendulum in the uncontrolled case.

In order to verify the control performance of HAC, needed times to bring the pendulum from un-equilibrium (non-zero initial conditions) to equilibrium in vertical position are computed for the different study cases and limitations ($u_{\text{lim}}$) of the actuator. In which, the equilibrium in vertical position of the pendulum is subjected to following conditions: $x_1 \leq 0.001$ rad and $x_2 \leq 0.001$ rad/s. Simulation results are shown in Figure 8. It can be found from Figure 8 that:

- Both of the controllers HAC and FC meet the control purpose when bringing the pendulum to equilibrium in vertical position for the different study cases and limitations ($u_{\text{lim}}$) of the actuator. It demonstrates the robustness capacity of the controllers.

- The limitation of the actuator is proportional to the control performance of the controllers.

- In all the study cases for different values of $u_{\text{lim}}$, the control performance of HAC is higher than those of FC. When decreasing *m* and *k* by 20% (Case 4), the needed time to bring the pendulum to equilibrium in vertical position is the shortest and vice versa for Case 3.

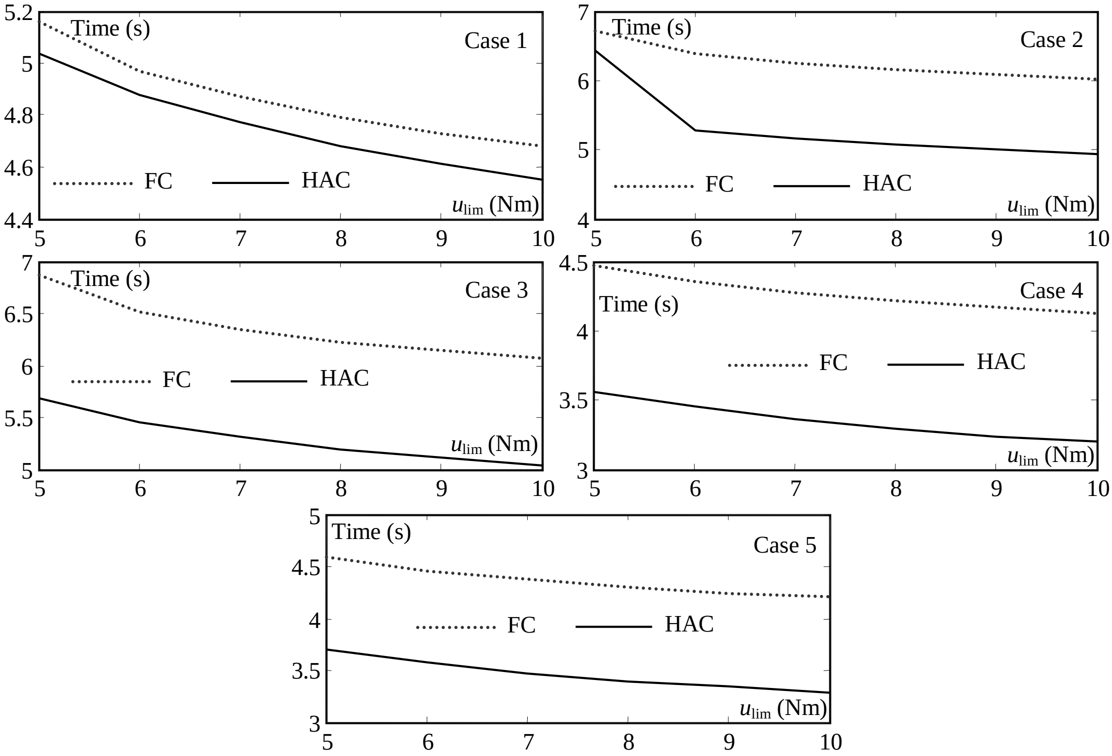

*Figure 8*. Needed times (s) to bring the pendulum to equilibrium in vertical position.

Time responses of the rotational angle, the angular velocity and the control moment for Case 1 with $u_{\text{lim}}$ = 8 Nm, Case 3 with $u_{\text{lim}}$ = 5 Nm and Case 4 with $u_{\text{lim}}$ = 10 Nm are presented in Figures 9-17, respectively.

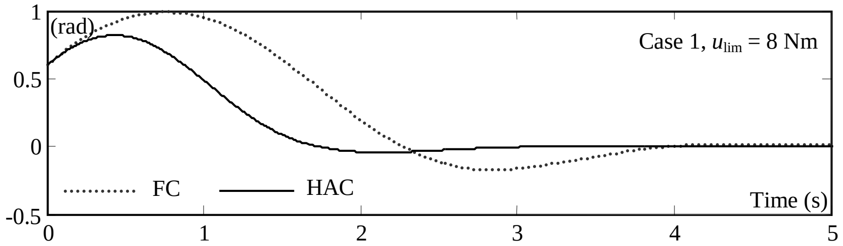

*Figure 9*. Time response (s) of the rotational angle $x_1$ (rad) of the pendulum, Case 1 with $u_{\text{lim}}$ = 8 Nm.

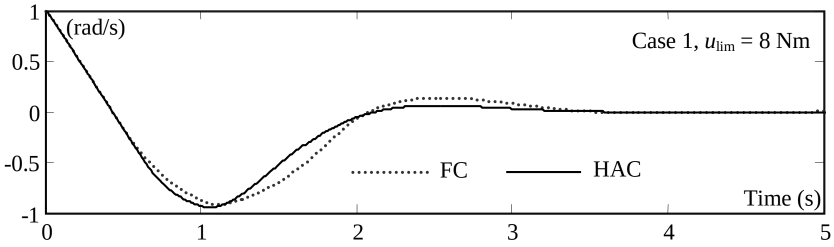

*Figure 10*. Time response (s) of the angular velocity $x_2$ (rad/s) of the pendulum, Case 1 with $u_{\text{lim}}$ = 8 Nm.

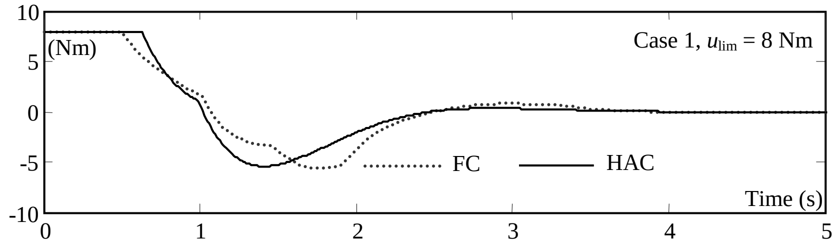

*Figure 11*. Time response (s) of the control moment $u$ (Nm), Case 1 with $u_{\text{lim}}$ = 8 Nm.

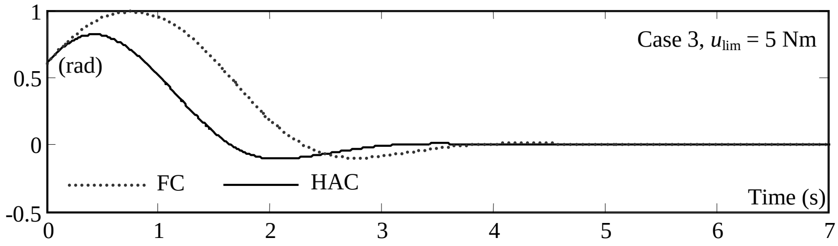

*Figure 12*. Time response (s) of the rotational angle $x_1$ (rad) of the pendulum, Case 3 with $u_{\text{lim}}$ = 5 Nm.

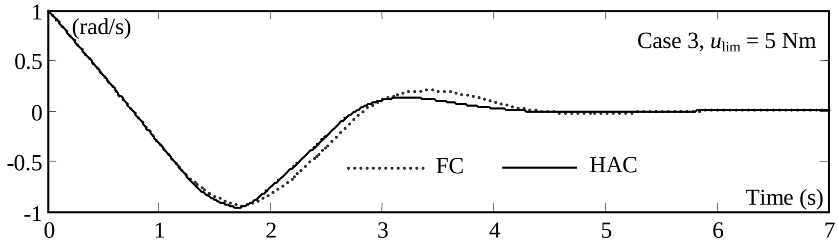

*Figure 13*. Time response (s) of the angular velocity $x_2$ (rad/s) of the pendulum, Case 3 with $u_{\text{lim}}$ = 5 Nm.

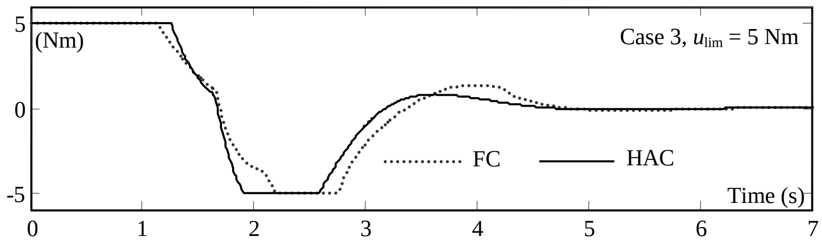

*Figure 14*. Time response (s) of the control moment $u$ (Nm), Case 3 with $u_{\text{lim}}$ = 5 Nm.

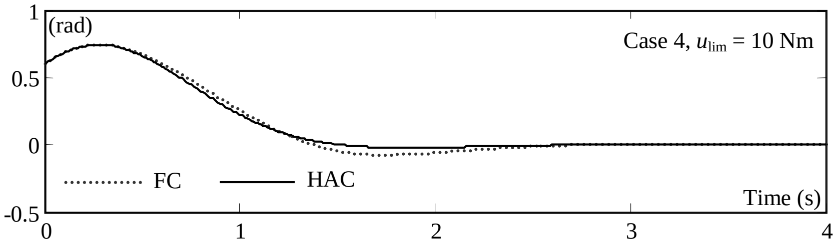

*Figure 15*. Time response (s) of the rotational angle $x_1$ (rad) of the pendulum, Case 4 with $u_{\lim}$ = 10 Nm.

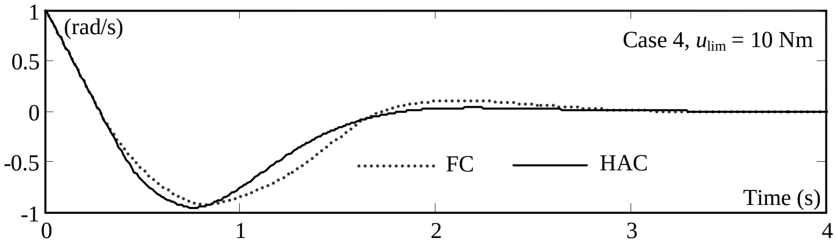

*Figure 16*. Time response (s) of the angular velocity $x_2$ (rad/s) of the pendulum, Case 4 with $u_{\lim}$ = 10 Nm.

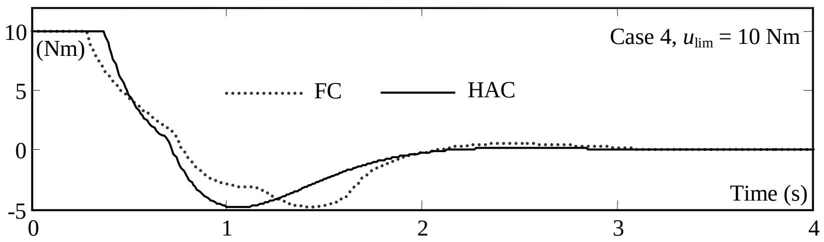

*Figure 17*. Time response (s) of the control moment $u$ (Nm), Case 4 with $u_{\lim}$ = 10 Nm.

To verify the computational performance of the controller HAC, computational time (CPU time) of HAC is measured and presented in Table 4. It can be found that CPU time of HAC is very much reduced by over 85% in comparison with that of the analogical FC. In which, CPU times are measured when running computer programs on the machine ASUS U46E with 8GB RAM, the OS is Windows 7 Home Premium, the programming language is Matlab 7.6.0, the total time of simulation is 8 s and the time step size is 0.01 s. It is a significant benefit of HAC in real control because computation time of output control forces is one of the main factors constituting input time delay of a controller.

*Table 4*. Computational time - CPU time (s).

| Controller | FC | HAC | Reduction ratio of HAC, % |
|---|---|---|---|
| CPU time | 4.13 | 0.61 | 85.23 |

## 5. CONCLUSION

In this study, the control problem of a pendulum using hedge-algebras-based fuzzy controller (HAC) containing actuator saturation is presented. It can be found from the numerical results that, HAC has robustness capacity, higher performance in control when comparing with that of analogical FC, and especially, CPU time of HAC is much reduced in comparison with that of FC (about 85 %). These show advantages of HAC in control in general and in structural control in particular.

Considering different problems in the field of control, such as input time delay, neuro-fuzzy systems, adaptive and sliding mode control and so on, into the proposed controller for different types of structures, such as active suspension systems, smart structures, flexible robots and so on, will provide further and interested researches in fuzzy control of structures.

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## APPENDIX 1

In this section, designing steps of the conventional fuzzy controller (FC) which is similar to the controller HAC are presented. Control diagram of analogical FC with two input state variables $x_1$, $x_2$ and one output control variable $u$ is shown Figure 18.

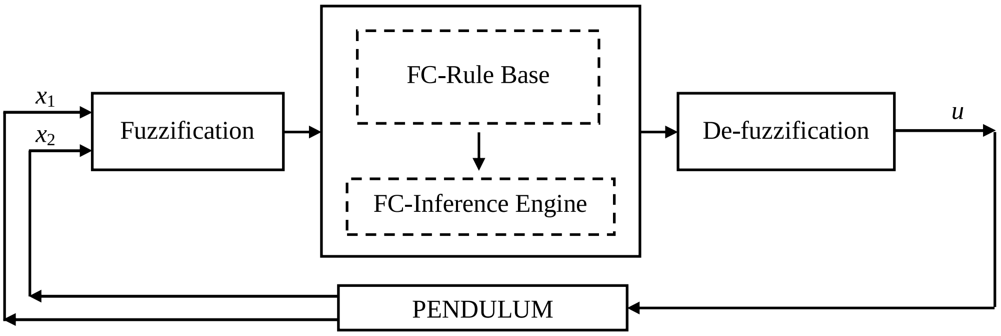

*Figure 18*. Control diagram of FC.

FC-Rule Base is arranged in Table 5. Fuzzification of the linguistic variables is presented in Figure 19, where $Z$ stands for the linguistic value "Zero". Mamdani method and centre gravity method are used as inference engine and de-fuzzification method of the controller FC.

*Table 5*. FC-Rule Base.

| $x_1$ \ $x_2$ | *LNe* | *Z* | *LPo* |
|---|---|---|---|
| *LNe* | *Ne* | *LNe* | *Z* |
| *Z* | *LNe* | *Z* | *LPo* |
| *LPo* | *Z* | *LPo* | *Po* |

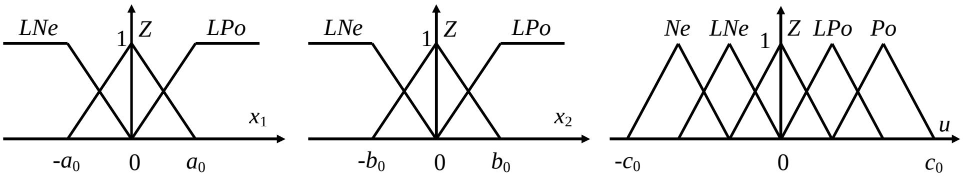

*Figure 19*. Fuzzification of the linguistic variables.

# TÓM TẮT

## ĐIỀU KHIỂN CON LẮC SỬ DỤNG ĐẠI SỐ GIA TỬ CÓ KỂ ĐẾN GIỚI HẠN CỦA MÁY KÍCH ĐỘNG

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Trong nghiên cứu này, bài toán điều khiển con lắc sử dụng bộ điều khiển mờ dựa trên đại số gia tử (HAC) có kể đến giới hạn của máy kích động (actuator saturation) được trình bày. Trong HAC, các giá trị ngôn ngữ của các biến ngôn ngữ thu được nhờ các ánh xạ ngữ nghĩa định lượng (SQMs) dựa trên một số tham số mờ của mỗi biến ngôn ngữ mà không cần sử dụng các tập mờ và quan hệ thứ tự ngữ nghĩa vốn có giữa các giá trị ngôn ngữ luôn được đảm bảo. Vì vậy, việc thiết kế một bộ điều khiển HAC dẫn tới bài toán xác định các tham số của SQMs. Các kết quả mô phỏng số của HAC được so sánh với các kết quả khi sử dụng một bộ điều khiển mờ truyền thống (FC) tương ứng nhằm thể hiện các lợi thế của HAC.

*Từ khóa*: con lắc, điều khiển mờ, đại số gia tử, giới hạn của máy kích động.