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An application of skew frequency curves to river stages

Vernon A. C. Gevecker

AN APPLICATION OF SKEW FREQUENCY CURVES
TO
RIVER STAGES

BY

VERNON A. C. GEVECKER

A

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PREFACE

Floods have occurred in the past and will occur again in the future. References to floods are incorporated in some of the early writings of man. Unfortunately though man has recorded the event, he has seldom recorded facts whereby it is possible to determine the volume of flow or the height to which the water rose. Where man has recorded the height to which water rose, much data such as channel conditions, vegetation coverage on the flood plain, etc., remains vague or unknown.

It has been only in comparatively recent times with the advent of the steamboat that man has systematically recorded information regarding river stages and approached the problem in a logical manner. The period of record for most of the rivers of the United States, however, can be placed at well under one hundred years.

One hundred years of record for a phenomena with causes as complex as those causing floods is meager compared to the vast volume of unrecorded events. The engineer, however, must take recorded fact and make his predictions. This paper is written concerning records of less than one hundred years and a method used to establish flood levels upon which engineering structures could be designed.

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INTRODUCTION

The early history and development of the United States is closely linked with the rivers of this country. Along these natural means of access with the interior traveled the explorer, the hunter, and the trapper. These earliest individuals in the wilderness were nomads requiring no permanent home or shelter. They moved free and unhampered by equipment over the land. They came to see and took from the land only that which bountiful nature provided without interference and without the assistance of man.

As the explorer, hunter, and trapper extended the frontiers of this nation, settlers followed in their wake. These later arrivals demanded more protection and comfort than the hunter and trapper. They built permanent structures as a protection against the elements. These settlers also were not content to depend alone upon the bounty of nature. They cleared the land, tilled the soil, and turned the wilderness into a land of plenty.

As was to be expected, the early settlers chose the lands laying adjacent to the streams for there lay the richest soil. As the number of settlers increased the need for communities arose. These early communities likewise were founded on the banks of the streams to be near the farms they served and to be near the cheap and relatively rapid transportation by water.

The history of any community founded on the flood plain of any stream is incomplete without reference to the floods which passed through the valley and man's effort to protect himself from the ravages of these floods. Many of these communities upon the banks of the various rivers have been subject to floods almost since their founding.

EARLY HISTORY OF FLOODS IN MISSISSIPPI VALLEY

"In the Alluvial Valley the Mississippi River is an aggrading, or soil building stream. In time of flood the river goes out of its banks, dropping its load of sediment as it goes. This action is due to the slowing up of the waters as they leave the river's channel; and the larger share of this material settles on or near the edges of the stream. For this reason the banks are generally from 10 to 15 feet above the lowlands away from the river."(1) Sometimes the communities along the river were well situated on high ground but the farmlands along the banks were subject to inundation.

New Orleans, founded in 1717, is noteworthy in that it was the first community on the Mississippi River and was the place of origin of the vast levee system now extending up the Mississippi River and many of its tributaries.(2) "The site was chosen because of its fertility and proximity to the river, in spite of strenuous opposition based on the belief that the future town would be frequently overflowed."(3)

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(1) Mississippi River Commission. The Mississippi River. Prepared in the Office of the President, Mississippi River Commission, Vicksburg, June 1940. p. 9.

(2) ibid., p.9.

(3) ibid., pp. 9 - 10.

"Less than a year after the founding of the city the Mississippi overflowed its banks, and New Orleans was flooded to a depth of from six to twelve inches."(4) "De la Tour, the

(4) Asbury, Herbert. The French Quarter. Garden City Publishing Co., Garden City, New York, 1938. p. 8.

engineer whose opposition to the site was thus overruled, directed 'a dike or levee to be raised in front, more effectively to preserve the city from overflow.' The early levees were small embankments, yet they served the purpose of that day because flood waters, allowed access to the low ground away from the river, seldom were more than a few feet deep on the high ground along the river, which was the only ground then under cultivation. By 1727, the New Orleans levee on the city front had been completed to a length of 5,400 feet. Each planter was required to complete the levee along his own river front. By 1735 the levee line extended along both banks of the river from 30 miles above New Orleans to 12 miles below. During that year high stages caused many crevasses. In 1743, an ordinance required the inhabitants to complete their levees by January 1, 1744, under penalty of forfeiture of their lands to the French crown. By 1812, when Louisiana was admitted to the Union, the levees extended from the lowest settlements to Baton Rouge on the left bank, and to Pointe Coupee on the right bank.

The construction of levees by small groups often resulted in conflict of interests, at times situated on opposite sides of the river, at times on neighboring plantations. One reason for this was the steadily increasing flood heights that resulted from the confinement of the flood waters, which formerly could spread in a sheet over the full width of the valley, but which now began to threaten those whose levees were low. Recognizing this evil, gaps were left in the dike system through which excess flood waters could escape in the low, as yet unoccupied, parts of the valley. In spite of this, many of the early floods were noted for levee failures caused by overtopping, crevassing, and bank caving at points where no flood waters were intended to escape. Raising low places in the levees merely resulted in transferring the danger points elsewhere."[5]

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(5) Mississippi River Commission, op. cit., pp. 10 - 11.

In the years that followed the founding of the City of New Orleans, the dugout and the canoe gave way to the flatboat which in turn gave way to the keelboat. The invention and development of the steamboat eliminated the keelboat and placed demands for channel improvement for navigation purposes.

The first effective step by the federal government was an Act of Congress passed in 1879 creating a Mississippi

River Commission charged with the improvement of the river channel and the construction of flood protection works.[6]

(6) Mississippi River Commission, op. cit., p. 17.

As a result of the work of this commission, Congress passed the River and Harbor Act of 1881 and appropriated $1,000,000 for river work. Thus the first step towards coordination of all river work was started. Subsequent years saw great development in channel works and levees.

"In 1927 the most disastrous flood of record occurred. Flood levels attained unprecedented heights and losses exceeded those ever before experienced in the valley. For the first time in history, completed and seasoned levee works constructed to the standards of the Mississippi River Commission proved of inadequate height and therefore failed."[7]

(7) Mississippi River Commission, op. cit., pp. 18 - 19.

As a result of this flood and after much investigation, Congress passed the Flood Control Act of May 15, 1928. Among other provisions contained in this act were projects for "Construction, extension and repair of the levees between Cape Girardeau, Mo., and Head of Passes, La.,"[8] and "for

(8) Mississippi River Commission, op. cit., pp. 19 - 20.

the prosecution of certain flood control works on the Mississippi River between Rock Island, Ill., and Cape Girardeau, Mo." (9)

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(9) Mississippi River Commission, op. cit., p. 21.

PREVIOUS INVESTIGATIONS OF FLOOD FREQUENCIES

Prior to 1931 studies of frequencies of floods in the St. Louis Office, U. S. Engineer Department, were made at several gauge stations by selecting the single highest daily stage in each year and plotting these stages in order of magnitude, beginning with the smallest, on semilogarithmic paper. Stage, or gauge height, was plotted on the decimally divided ordinate and the time, in years, was plotted on the logarithmically divided abscissa. A smooth curve passing through these points was drawn by a draftsman using a spline. Selecting the stage height indicated at the twenty-fifth year as an example, the curve may be interpreted in the following manner. On an average over a long period of time the annual daily high stage indicated will be equaled or exceeded once in twenty-five years. Specifically this means that considering all the years of record and all the years of future observations we may expect under average conditions that one year out of twenty-five or four per cent of the time in years the single high stage indicated will at least be equaled and may be exceeded for one or more days of observation in that year and that in the other twenty-four years the maximum daily stage or stages may equal the indicated stage but in all probability all the daily stages in those years will be less than the indicated stage. Actually the sequence of flood and drouth years is not known. No cyclic trend has as yet been found. The indicated stage for the example used as illustration may actually be equaled or exceeded in four consecutive years and may not be equaled in any of the ninety-six or more following years.

The defects of using the single highest daily stage in any year of observation are several. First, and foremost, the method is subject somewhat to the skill of the draftsman in drawing the curve through the points. Any slight variation in this curve may change its slope such that when the curve is extrapolated, usually on a straight line, the annual daily high stage indicated for some year beyond the period of record may vary an appreciable amount. Second, the curve is based upon a single daily high stage in each year of record although in the twenty-five year period used as illustration there may be two or more daily stages in any one year exceeding all stages in the other twenty-four years. Third, although the annual daily high stage can be determined, no means are available to determine the number of days in any period the stage will equal or exceed any given stage of lesser amount.

In 1931 the St. Louis Office, U. S. Engineer Department, studied frequencies of floods by means of Allen Hazen Probability Paper. (10)(11)(12) In this method the same single

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(10) Hazen, Allen. Storage to be Provided in Impounding Reservoirs for Municipal Water Supply. American Society of Civil Engineers. Trans. Vol. 77, pp. 1549-1550. (1914)

(11) Hazen, Allen. Flood Flows, Discussion of. American Society of Civil Engineers. Trans. Vol. 77, pp. 627-631. (1914)

(12) Hall, L. Standish. The Probable Variations in Yearly Run-Off as Determined from a Study of California Streams. American Society of Civil Engineers. Trans. Vol. 84, pp. 208-212. (1921)

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highest daily stage in each year is also selected. These annual daily high stages are then plotted on paper with the stage, or gauge height, plotted on the decimally divided ordinate and the time, in years, plotted in order of magnitude, beginning with the largest, on the abscissa with divisions spaced in accordance with a probability distribution. This abscissa is divided to represent percentage of time from 0.01 to 99.99. The plotting point on the abscissa is determined from the equation

$$P = \frac{2m - 1}{2n}$$

where: P is a percentage of occurrence

m is the order of magnitude of the stage beginning with the largest

n is the total number of years of record.

A smooth curve was then drawn passing through these points by a draftsman using a spline. This curve was interpreted as follows: on the average over a long period of time a single annual daily high stage will be equaled or exceeded the percent of the time read from the curve for that stage. This type of plotting does not have as much extrapolation at either end, high or low, and therefore is not subject to the skill of the draftsman in as great a degree as the curve mentioned in the previous paragraphs. The curves plotted on probability paper, however, are subject to the second and third defects of the curves mentioned in the previous paragraphs.

Further investigation in the literature revealed that Mr. C. H. Pierce stated "It has been shown theoretically and verified experimentally that certain natural phenomena follow definite mathematical laws." (13) Mr. Pierce applied the

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(13) Pierce, C. H. The Probable Variations in Yearly Run-Off as Determined from a Study of California Streams, Discussion of. American Society of Civil Engineers. Trans. Vol. 84, p. 235. (1921)

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equation of the normal frequency curve

$$y = ke^{-h^2x^2},$$

where: k and h are constants and
e is the base of natural or Napierian logarithms,

to quantities of flow and computed frequency curves for a number of New England streams. These are symetrical curves and to appearance do not fit the observed data especially at the low flows.

Mr. H. Alden Foster suggested the use of nonsymetrical frequency curves. (14) He stipulated that the general shape

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(14) Foster, H. Alden. Theoretical Frequency Curves and their Application to Engineering Problems. American Society of Civil Engineers. Trans. Vol. 87, pp. 145-157. (1924)

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of the curve must generally follow the observed data and that the equation of the curve must satisfy the following conditions:

"(1) The curve must have a maximum point. Hence, for some finite value of x, dy/dx must equal zero.

(2) The curve will be tangent to the X-axis at one end, and possibly at both ends. Therefore, when y = 0, dy/dx must equal zero." (15)

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(15) ibid. p. 147.

These conditions are satisfied by the general and basic equation for skew frequency curves

$$\frac{dy}{dx} = \frac{y(x + a)}{f(x)}$$

where: f(x) is any function of x

y is the frequency

a is a constant.

This may be recognized as the basic equation in Pearson's theory of frequency.[16] Although these curves have been

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(16) Pearson, Karl. Tables for Statisticians and Biometricians. Cambridge University Press, London, 1914. p. lxi.

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known for some years their application to problems in hydrology has been very limited.

SKEW FREQUENCY CURVES

In the summer of 1931, the St. Louis Office, U. S. Engineer Department, began a study of the frequencies of river stage heights for a number of gauge locations. The intent was not only to study the maximum elevation that a river stage could be expected to reach in any given period of years but also to determine the number of times in any period of years that the elevation of the river stage would exceed certain predetermined elevations and the number of days the stage would remain above these elevations in each instance. The study was intended to furnish data for another study regarding the economics of raising and maintaining levees at the aforementioned predetermined elevations.

The two types of frequency curves mentioned on pages 8 to 11 inclusive having been based upon the single highest daily stage in each year of record gave only information regarding the maximum elevation of the highest stage that could be expected in any period of years. These curves could not give any information regarding the number of times in any period the elevation of the river stage would exceed any predetermined elevation and likewise could not give any information regarding the number of days the stage would remain above the predetermined elevation in each instance.

It was immediately obvious that any study which was to determine the total number of days that any stage was to be equaled or exceeded in a period of years would have to include, as observed data, all the daily stages for the period of record. It was also apparent that the distribution of the daily stages did not fall into any symetrical form such as a normal frequency distribution.

Investigations in the literature revealed the suggestions of Mr. H. Alden Foster recommending the use of Pearson's skew frequency curves.[17] The general and basic

(17) Foster, op. cit., p. 148.

equation in Pearson's theory of frequency utilizing skew frequency curves

$$\frac{1}{y}\frac{dy}{dx} = \frac{x - a}{f(x)}$$

where: y is the probability of a value

x is the frequency

a is a constant

f(x) is some function of x

can be expanded to Maclaurin's theorem to

$$\frac{1}{y}\frac{dy}{dx} = \frac{x - a}{c_0 + c_1 x + c_2 x^2 + \ldots\ldots c_n x^n + \ldots}$$

The constants a, c_0, c_1, c_2, ..., c_n, ... can be determined from the moment coefficients of the frequency distribution. These constants are functions of certain other constants β_1, β_2, β_3, ... If we stop our expansion at c_2, the differential equation becomes

$$\frac{1}{y}\frac{dy}{dx} = \frac{x - a}{c_0 + c_1 x + c_2 x^2}$$

We then need to compute only two other constants

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

and

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

where μ_2, μ_3, and μ_4 are the second, third, and fourth moment coefficients of the observed frequencies about the mean stage.

$$\mu_2 = \frac{\Sigma(\text{Second moment of observed frequencies about mean})}{\Sigma \text{observed frequencies}}$$

$$\mu_3 = \frac{\Sigma(\text{Third moment of observed frequencies about mean})}{\Sigma \text{observed frequencies}}$$

$$\mu_4 = \frac{\Sigma(\text{Fourth moment of observed frequencies about mean})}{\Sigma \text{observed frequencies}}$$

From these values and others which may be computed from them we can determine some facts regarding the curve which will fit our observed data.

β_2 is a measure of the flatness of the curve. For the normal distribution

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{\mu_4}{\sigma^4}$$

has the value of 3. Curves which have a value of β_2 greater than 3 have a sharper peak than the normal curve and curves which have a value of β_2 less than 3 have a flatter peak than the normal curve. This value β_2 - 3 is called kurtosis.

β_1 is a measure of the skewness or lack of symetry of the frequency distribution about the modal stage or the stage occuring with greatest frequency. If the skewness

$$sk = \frac{\sqrt{\beta_1}\,(\beta_2 + 3)}{2(5\beta_2 - 6\beta_1 - 9)}$$

is positive the modal stage will be to the left of the mean with the longer tail of the curve extending to the right and if the skewness, sk, is negative the modal stage will be to the right of the mean with the longer tail of the curve stretching to the left.

The mean stage, $\bar{x}$, is determined by taking a statistical average of all frequencies.

The standard deviation

$$\sigma = \mu_2^{1/2}$$

is a measure of the distribution of the observed frequencies about the mean stage. If the frequency distribution had zero skewness and the form of a normal curve approximately two thirds of the frequencies would lie in the area bounded by the curve and the ordinates mean plus the standard deviation and mean minus the standard deviation.

The distance from the mean stage to the modal stage

$$d = \frac{\sigma \sqrt{\beta_1} (\beta_2 + 3)}{2(5\beta_2 - 6\beta_1 - 9)}$$

can be computed from the foregoing values.

When all values have been computed, an inspection of a graph of the observed frequencies will indicate if the proposed curve apparently fits the observed data.

The Pearson curves are divided into seven general types, two of which are further divided into sub-types. By means of the constants β_1 and β_2 the criteria

$$K_1 = 2\beta_2 - 3\beta_1 - 6$$

and

$$K_2 = \frac{\beta_1 (\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)}$$

may be computed and the type curve with its form equation determined.

Once the form equation has been determined, the constants in that equation are evaluated from the constants enumerated on pages 16 to 18 inclusive. With the constants evaluated, the frequencies can be computed by substitution of values for the stage, x, and the ordinates to the frequency curve computed.

AN APPLICATION OF SKEW FREQUENCY CURVES

The Pearson type frequency curves were applied to river stage heights of several gauges in the St. Louis district having records varying from 37 to 70 years at the time of the study. Two of these locations, the St. Louis Gauge having a record of 70 years and the Cape Girardeau Gauge having a record of 43 years will be discussed herein.

The St. Louis Gauge was established January 1, 1861 and a continuous record of river stages exists from that date. During this period of record several stages exceeding 30.0 feet, bankful stage, were recorded and there is evidence that earlier floods would exceed the stages of recorded floods if they were forced to flow in the narrowed channel existant in 1930.

Since the installation of the gauge in 1861 many changes were wrought in the St. Louis District of the Mississippi River Valley. The channel was improved for navigation purposes by the installation of control works and the shortening of the river. Dredging took place and bars were removed. The river was narrowed in places and levees constructed to protect the communities and farmlands on the banks of the stream. All of these changes affected the river stages. The general effect of confining the flow has been to raise the stage, especially the flood flows. Any large volume of flow in 1861 would produce a water surface elevation on the gauge of a specified amount. This same volume of flow in 1930 would produce a greater surface elevation on the gauge.

With the knowledge that the same volume of flow would produce different readings on a gauge in different years, investigation was made to determine if the river stages could be converted to volumes of flow for the purpose of computing the frequency curves. This investigation revealed that rating curves were nonexistent for many years of the period of record. The only method by which flows could be evaluated was from empirical formulae which depended upon slope of the water surface, the roughness of the channel cross-section, the area of cross-section, and the wetted perimeter of the channel. Since this information was also lacking, the decision to use river stage heights was reached.

With the knowledge of the variability of volumes of flow for stage heights in different years it was realized that a single curve could not be drawn utilizing the stages for the entire seventy years. The period of record was therefore divided into seven intervals of ten years each, viz., 1861 - 1870, 1871 - 1880, 1881 - 1890, 1891 - 1900, 1901 - 1910, 1911 - 1920, and 1921 - 1930. It was hoped that the changes in ten years would be progressive and not cause too great a variation in gauge height for any volume of flow.

The elevation of the zero mark on the St. Louis Gauge was changed several times since its establishment. All readings over the years had to be corrected to read as of the gauge zero in 1930.

Once corrected, the stages in a ten year interval were sorted into one-foot class intervals. Each stage was placed in the interval of the foot mark preceeding the decimal point of the stage. The stage 15.79, for instance, was placed in the 15-foot interval which included all readings between 15.00 and 15.99 feet. The total number of frequencies in each class interval was then plotted on the ordinate opposite the abscissa of the center of the class interval. The total number of frequencies in the foregoing 15-foot interval were plotted opposite 15.50 feet. These plotted points of observed frequencies were connected by a series of straight lines. See figure 1, page 22 for an illustration. The resulting distribution of observed data would discourage any draftsman from attempting to pass a curve through these points. If, however, the observed data was accumulated from the lowest interval observed to the highest and these cumulated frequencies plotted on the cumulated frequency ordinates opposite their cell midpoints on the abscissa, the curve assumes a smoother appearance. A drafted curve passing through these last points would have a better appearance and be more pleasing to a visual inspection. A curve so drawn would be subject to the opinion of the draftsman and different men may vary the position of the curve.

In order to compute a frequency curve which would fit

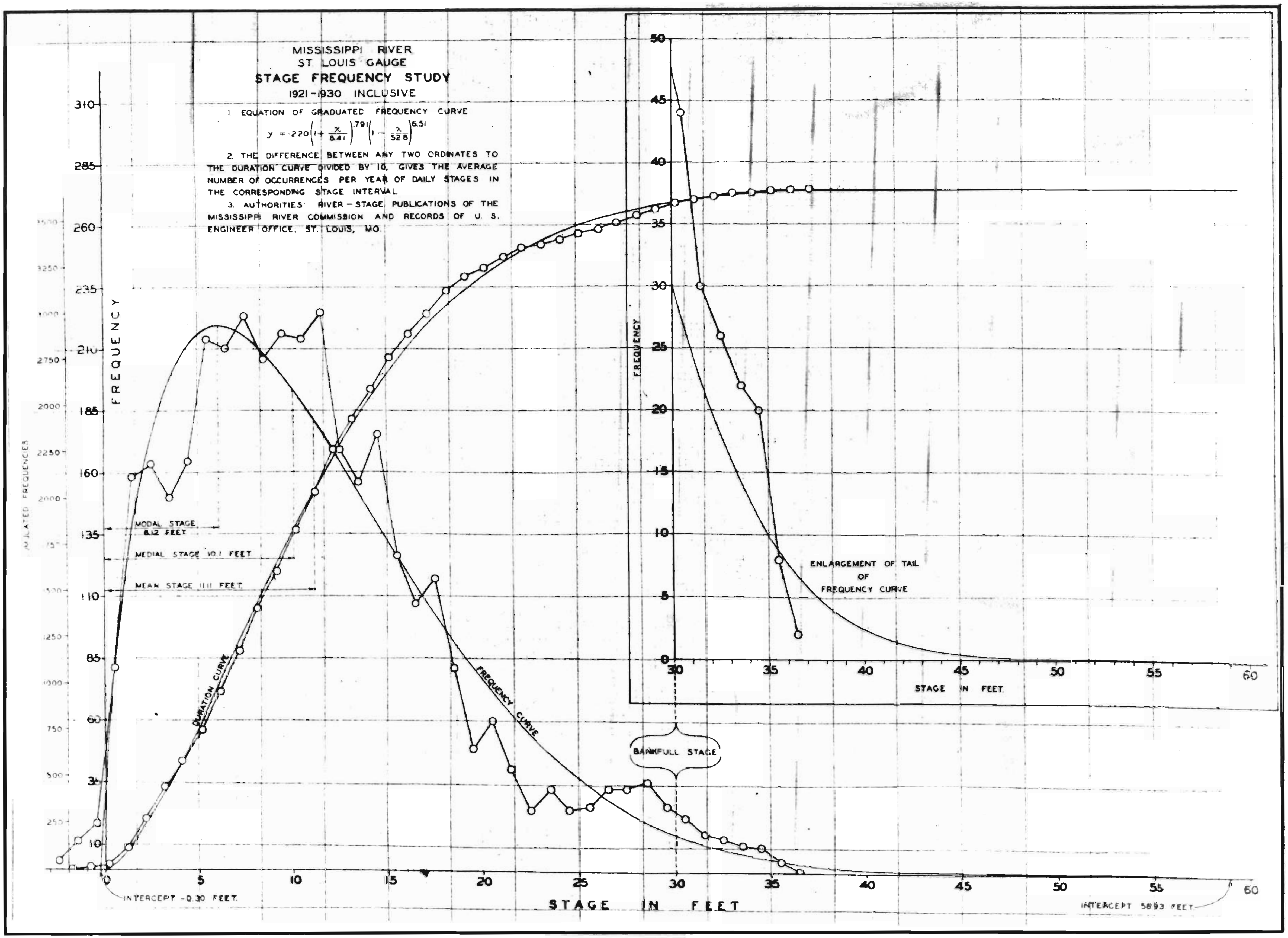

Figure 1: Type I_I Pearson Curve

the observed data for each ten year interval, the observed frequencies sorted as indicated in the previous paragraph would have to be tabulated as shown by table II, page 37 of appendix A. An arbitrary origin, $\overline{O}$, is chosen as near to the mean as can be estimated and the first, second, third, and fourth raw moments of the observed frequencies computed about this arbitrary origin. From the summation of the observed frequencies, first, second, third, and fourth raw moments of these frequencies, the raw moment coefficients are computed. The first raw moment coefficient is

$$\nu_1' = \frac{\Sigma fx}{\Sigma f}$$

where Σfx is the summation of the first raw moments about the arbitrary origin and is the distance from the mean stage to the arbitrary origin. The second, third, and fourth raw moment coefficients are

$$\nu_2' = \frac{\Sigma fx^2}{\Sigma f}$$

$$\nu_3' = \frac{\Sigma fx^3}{\Sigma f}$$

$$\nu_4' = \frac{\Sigma fx^4}{\Sigma f}$$

where Σfx^2, Σfx^3, and Σfx^4 are the summations of the second, third, and fourth raw moments respectively about the arbitrary origin.

The statistical mean, $\overline{x}$, is computed from the first raw moment coefficient

$$\bar{x} = \bar{o} + m\nu_1'$$

where m is the width of the cell interval in feet. The second, third, and fourth raw moment coefficients are adjusted to the mean stage

$$\mu_2 = \nu_2' - \nu_1'^2$$

$$\mu_3 = \nu_3' - 3\nu_1'\nu_2' + 2\nu_1'^3$$

$$\mu_4 = \nu_4' - 4\nu_1'\nu_3' + 6\nu_1'^2\nu_2' - 3\nu_1'^4$$

The second and fourth adjusted moment coefficients are then corrected for the distribution of the frequencies in the class intervals about their centers by application of Sheppard's corrections.

$${}_c\mu_2 = \mu_2 - \frac{1}{12}$$

$${}_c\mu_4 = \mu_4 - \frac{1}{2}\mu_2 + \frac{7}{240}$$

The mean stage, $\overline{x}$, the adjusted third moment coefficient, μ_3, the corrected second moment coefficient, ${}_c\mu_2$, and the corrected fourth moment coefficient, ${}_c\mu_4$, are then used to compute the constants β_1 and β_2, the skewness, sk, the standard deviation, σ, and the criteria, K_1, and K_2. From these values, the type curve with its form equation was determined.

H. Alden Foster implies that a Type III Pearson curve

$$y = y_o e^{-p\frac{x}{a}}\left(1 + \frac{x}{a}\right)^p$$

is the only type curve which will fit hydrological data.[18]

(18) Foster, op. cit., p. 148

The experience in the St. Louis District with data concerning the Mississippi River revealed that a Type I_I curve

$$y = y_o\left(1 + \frac{x}{a_1}\right)^{m_1}\left(1 - \frac{x}{a_2}\right)^{m_2}$$

figure 1, page 22, or a Type J_I curve

$$y = y_o\left(1 + \frac{x}{a_1}\right)^{m_1} \Big/ \left(1 - \frac{x}{a_2}\right)^{m_2}$$

figure 2, page 26, fitted all observed data regardless of length of period or location of gauge. It is conceivable that other types of Pearson curves may fit some other hydrological data. In these curves

> y is the frequency at any stage, x;
> y_o is the frequency at the modal stage; and
> p, a_1, a_2, m_1, m_2, are constants.

With the type curve and its form equation known, the statistics enumerated on page 16 ro page 18 inclusive are used to compute the constants of the form equations men-

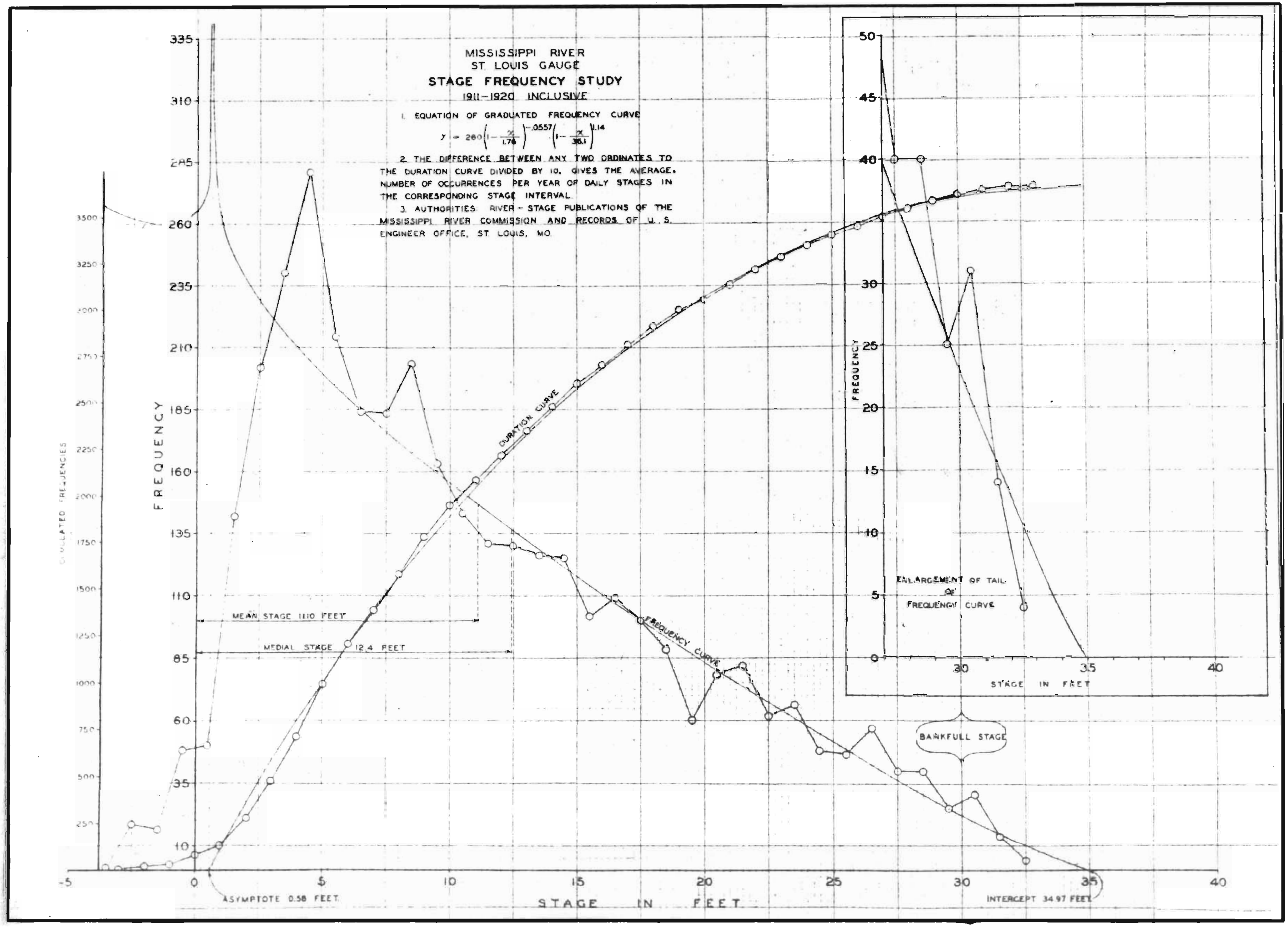

Figure 2: Type J_I Pearson Curve

tioned in the preceeding paragraph by means of the following equations:

Modal stage

$$x_{mode} = \bar{x} - d$$

Constants: a_1, a_2, m_1, m_2

$$r = \frac{6(\beta_2 - \beta_1 - 1)}{(3\beta_1 - 2\beta_2 + 6)}$$

$$\epsilon = \frac{r^2}{4 + \frac{\frac{1}{4}\beta_1(r+2)^2}{r+1}}$$

$$b^2 = \frac{\mu_2 r^2 (r+1)}{\epsilon}$$

$$r = m_1' + m_2'$$

$$\epsilon = m_1' m_2'$$

$$m'^2 - rm' + \epsilon - 0$$

$$m_1 = m_1' - 1$$

$$m_2 = m_2' - 1$$

$$a_1 + a_2 = b$$

$$\frac{a_1}{a_2} = \frac{m_1}{m_2}$$

Frequency at the modal stage

$$y_0 = \frac{N}{b} \frac{(m_1 + m_2 + 1)}{\frac{\Gamma(m_1 + 1)}{(e^{-m_1} m_1^{m_1})}} \sqrt{\frac{m_1 + m_2}{m_2}}\, e^{\frac{1}{12}\left(\frac{1}{m_1 + m_2} - \frac{1}{m_2}\right)}$$

With the constants in the form equation evaluated, the ordinates to the frequency curve can be most easily computed by a tabulation as shown in table III, page 45, appendix A.

A complete computation illustrative of the procedure involved for a Type I_I Pearson frequency curve for the St. Louis Gauge is included in appendix A, pages 37 to 48 inclusive. The data used was sorted into two foot class intervals and used for another study. It includes all observed stages for the 1/24 year period 1^h March 18 to 6^h April 12, 1861 to 1930 inclusive. Sufficient footnote references are given that ready reference may be made to source data and tables used in the computations.

A duration curve was computed for each curve prepared by summing the areas under the frequency curve by means of Simpson's Rule and plotting these cumulated frequencies on the abscissa opposite the correct ordinates.

The median or medial stage was selected from the duration curve such that fifty per cent of the computed frequencies would occur below and fifty per cent of the computed frequencies would occur above the selected stage.

The frequency curve for the Cape Girardeau Gauge, figure 3, page 29, is herein included as an illustration of a slight variation in the procedure described on previous pages. The observed data for the period of record, 1897 - 1933, was used in computation of the curve. This was done

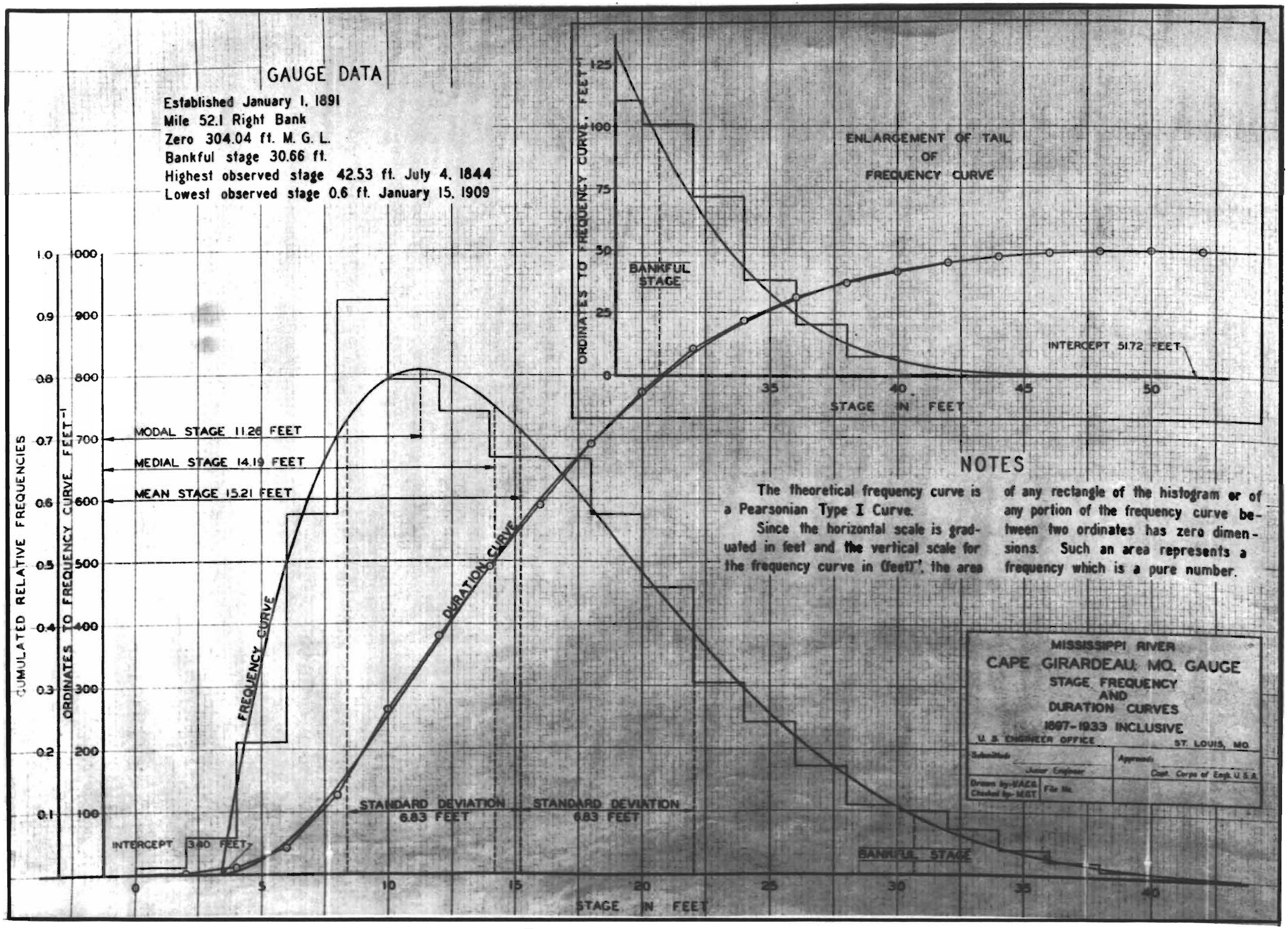

Figure 3: Pearson Curve with Histogram

because investigation disclosed that the channel conditions and banks were not materially altered in this reach of the river after the first date given above. The class interval or cell was taken as two feet instead of the one foot interval used in the St. Louis Gauge Study. The observed frequencies were plotted as a histogram instead of the type plotting used for the St. Louis Gauge.

SUMMARY

A summary of the results of the Study of the St. Louis Gauge is given in table I, page 32. It will be noted that the number of observations in each decade is either 3652 or 3653 dependent upon the number of leap years in the respective decades.

The modal stage, the stage with greatest number of occurrences, varies over a considerable range. The curves for two decades are modeless, Type J_I curves, figure 2, page 26. The observed frequencies are unequally distributed about the mean stage rising rapidly to a peak and decreasing gradually towards the high stages. No type frequency curve apparently can be made to fit this data very closely. The causes of the unusually high frequency at certain low stages is merely surmise. The cause is attributed by some to ice gorges which occurred at infrequent intervals and blocked the normal flow of the stream. No evidence was at hand to support this guess.

The medial stage showed a progressive variation in the computed frequencies but there is insufficient evidence upon which to state whether or not a trend exists.

The mean stages are scattered and exhibit no progressive changes in the period of record. The scattering is attri-

SUMMARY OF RESULTS OF PEARSONIAN STAGE FREQUENCY CURVES
ST. LOUIS GAUGE 1861 - 1930 INCLUSIVE

Decade	No. Daily Stages	Modal Stage	Medial Stage	Mean Stage	Stand. Dev.	Stand. Dev. Squared	% of Daily Stages over 30 feet. (b)
1861-70	3652	9.08	11.6	12.34	6.22	38.7260	.29
1871-80	3653	7.11	11.9	12.85	6.00	36.0555	.28
1881-90	3652	8.55	13.25	14.03	6.79	46.0506	1.03
1891-00	3652	--(a)	12.9	10.99	7.50	56.2449	1.57
1901-10	3652	10.57	12.5	13.15	7.42	55.0342	1.73
1911-20	3653	--(a)	12.4	11.10	7.84	61.4186	1.45
1921-30	3652	6.12	10.1	11.11	7.24	52.9126	1.70
TOTALS				85.57		346.4424	
Means				12.22		49.4918	
Square Root						7.0350	

(a) The Pearsonian Curves for these two decades were modeless or "J-shaped". See figure II, page 26.

(b) In other words, the last column gives 100 times the empirical probability of a single daily stage of 30 feet or over. Areas by Simpson's Rule.

TABLE I: SUMMARY OF RESULTS

buted to chance.

The standard deviations show an abrupt increase for the decade 1881 - 1890 which increase is continued in subsequent decades. This may be significant in that the first coordinated program for flood protection was begun in 1882 and extended in succeeding years causing a greater range between high and low stages.(19)

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(19) Mississippi River Commission, op, cit., p. 17.

The per cent of daily stages over 30 feet, bankful stage at St. Louis, shows an abrupt increase for the decade 1881 - 1890, the decade in which coordinated levee construction was started. This is significant in that it supports the contention of the hydraulic engineer in his statement that when a stream is confined between levees the added area of cross section required for flood flows is primarily gained by an increase in the elevation of the water surface.

The per cent of overbank stages varying from 1.03% to 1.70% in the last five decades of study indicates that stages of over 30.0 feet on the St. Louis Gauge may on the average be expected from 37 to 62 days in any decade. This in part answers the question on page 14 regarding the number of days the stage will be greater than 30.0 feet.

The curves in no way indicate the number of times the river stage may rise above bankful stage and fall again in any period. The curves in no way indicate the lengths in days a single rise can be expected to be greater than bankful stage. The answers to these problems must be ascertained by some other study.

An examination of figure 1, page 22, will show that the maximum computed high stage is in the neighborhood of 59 feet. This is some 15 feet greater than the estimated stage of the maximum flood which passed St. Louis in 1844. The possibility of such a stage is extremely remote as shown by the minute value of its ordinate and may be assumed to be impossible of attainment. We may say in the words of the insurance and legal fields such a stage would be "an act of God".

All curves were tested for goodness of fit by the Chi-Squared, χ^2, test. The tests indicated a poor fit primarily due to the long flat tail to the curve on the right side and steep slope to the curve on the left side.

CONCLUSIONS

From the apparent erratic results briefly described herein it would appear that the high volume of computations required in this investigation are not justified. The questions asked were not fully answered. The following conclusions, however, can be drawn from this work.

1. That hydrologic data are amenable to statistical analysis.

2. That volumes of flow be used as the variable rather than elevations of water surface. The volumes of flow may be converted to elevations when required by use of a rating curve for the gauge in question.

3. That the greater the period of record, the greater the number of observations used, the smoother will be the histogram of observed data and the more nearly will the computed curve conform to the observed data.

4. That the computed curves should be revised at frequent intervals in order that the smoothing effect of added observations be made available and more reliable frequency curves be on hand for current studies utilizing stage frequency relationships.

APPENDIX A

This appendix consists of a complete calculation and drawing of the frequency and duration curves illustrative of the procedure used on the study of the different gauges in the St. Louis District, U. S. Engineer Department. The data used for this illustration is part of a further study and consists only of those observations and fractions of observations occurring in that fractional part of the period occurring between 1^h March 18 to 6^h April 24, 1861 - 1930 inclusive.

It will be noted that the actual skewness was not computed. In its place a coefficient of skewness

$$K = \sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{\mu_3}{\sigma^3}$$

was used as a rough measure of this statistic. The actual skewness would be

sk = 0.345335

It will also be noted that the width of cell or class interval, m, is two feet. This means that the total frequency, N, must be divided by two in the computation of the modal frequency.

CALCULATION OF MOMENTS AND STATISTICS OF OBSERVED FREQUENCIES AND STAGES MISSISSIPPI RIVER ~ ST. LOUIS GAUGE 1861 TO 1930 INCLUSIVE FOR THE 1/24 YEAR PERIOD 1h MARCH 18 TO 6h APRIL 12

Stage Midpoint	Stage Boundary	Deviation from $\bar{O}$ X	f Observed Frequency	fX	fX^2	fX^3	fX^4
	-3.0						
-2.0	-1.0	-9					
0	1.0	-8					
2.0	3.0	-7					
4.0	5.0	-6	4.0000	-24.0000	144.0000	-864.0000	5184.0000
6.0	7.0	-5	37.1250	-185.6250	928.1250	-4640.6250	23203.1250
8.0	9.0	-4	71.5833	-286.3332	1145.3328	-4581.3312	18325.3248
10.0	11.0	-3	77.9583	-233.8749	701.6247	-2104.8741	6314.6223
12.0	13.0	-2	145.5417	-291.0834	582.1668	-1164.3336	2328.6672
14.0	15.0	-1	152.1667	-152.1667	152.1667	-152.1667	152.1667
16.0	17.0	0	162.8333	0.	0.	0.	0.
18.0	19.0	1	163.3333	163.3333	163.3333	163.3333	163.3333
20.0	21.0	2	68.1250	136.2500	272.5000	545.0000	1090.0000
22.0	23.0	3	68.8333	206.4999	619.4997	1858.4991	5575.4973
24.0	25.0	4	55.6250	222.5000	890.0000	3560.0000	14240.0000
26.0	27.0	5	38.7500	193.7500	968.7500	4843.7500	24218.7500
28.0	29.0	6	11.4583	68.7498	412.4988	2474.9928	14849.9568
30.0	31.0	7	2.2500	15.7500	110.2500	771.7500	5402.2500
32.0	33.0	8					
34.0	35.0	9					
36.0	37.0	10					
38.0	39.0	11					
40.0		12					
	41.0						
Σ (Summation)			1059.5832	-166.2502	7090.2478	709.9946	121047.6934
Divide by Σf				-0.156902	6.691544	0.670070	114.240858

TABLE II: COMPUTATION OF RAW MOMENT COEFFICIENTS

Raw moment coefficients:

$$\nu_1' = -0.156902$$
$$\nu_2' = 6.691544$$
$$\nu_3' = 0.670070$$
$$\nu_4' = 114.240858$$

$$\nu_1'^2 = 0.024618$$
$$\nu_1'^3 = -0.003863$$
$$\nu_1'^4 = 0.000606$$

$\overline{m}$ = units per cell = 2

$\overline{o}$ = arbitrary origin = 16.0 feet

$$\bar{x} = \text{mean stage} = \bar{o} + m\nu_1' = 16.0 - 2 \times 0.156902$$
$$= 16.0 - 0.313804$$
$$= 15.686196 \text{ feet}$$

Moment coefficients transferred to mean stage

$$\mu_2 = \nu_2' - \nu_1'^2 = 6.691544 - 0.024618$$
$$= 6.666926$$

$$\mu_3 = \nu_3' - 3\nu_1'\nu_2' + 2\nu_1'^3$$
$$= 0.670070 - 3(-0.156902)(6.691544) + 2(-0.003863)$$
$$= 3.812094$$

$$\mu_4 = \nu_4' - 4\nu_1'\nu_3' + 6\nu_1'^2\nu_2' - 3\nu_1'^4$$
$$= 114.240858 - 4(-0.156902)(0.670070) + 6(0.024618)(6.691544) + 3(0.000606)$$
$$= 115.647976$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{14.53206066}{296.33087349} = 0.049040$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{115.647976}{44.447902} = 2.601877$$

$$k = \sqrt{\beta_1} = \sqrt{0.049040} = 0.22145$$

$$\sigma = m\mu_2^{1/2} = 5.164078$$

Sheppard's corrections applied to μ_2 and μ_4 [20]

$$_c\mu_2 = \mu_2 - \frac{1}{12} = 6.666926 - 0.083333$$
$$= 6.583593$$

$$_c\mu_4 = \mu_4 - \frac{1}{2}\mu_2 + \frac{7}{240}$$
$$= 115.647976 - \frac{1}{2}(6.666926) + \frac{7}{240}$$
$$= 112.343680$$

(20) Rider, Paul R., "An Introduction to Modern Statistical Methods", John Wiley & Sons, New York, 1939, p. 22.

$$_c\beta_1 = \frac{\mu_3^2}{_c\mu_2^3} = \frac{14.53206066}{285.3572602} = 0.05092585$$

$$_c\beta_2 = \frac{_c\mu_4}{_c\mu_2^2} = \frac{112.343680}{43.343697} = 2.5919266$$

$$_ck = \sqrt{_c\beta_1} = \sqrt{0.05092585} = 0.225667$$

$$_c\sigma = m\, _c\mu_2^{1/2} = 5.131902$$

Criteria to determine type of Pearson Curve [21]

(21) Pearson, Karl, "Tables for Statisticians and Biometricians", Cambridge University Press, London, 1914, p. lxii.

$$K_1 = 2\beta_2 - 3\beta_1 - 6$$
$$= 2(2.5919266) - 3(0.05092585) - 6.0$$
$$= -0.96892435$$

$$K_2 = \frac{\beta_1(\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)}$$
$$= \frac{0.05092585(2.5919266 + 3.0)^2}{4(4 \times 2.5919266 - 3 \times 0.05092585)(-0.96892435)}$$
$$= \frac{1.5924332}{-39.5899732}$$
$$= -0.04022314$$

$K_2 < 0$, i.e. negative. Below $f = 0$. Type I curve. Limited range. [22]

(22) Ibid., p. lxiii.

Diagram XXXV, Type I curve.[23]

(23) Pearson, Karl, op. cit., p. 66.

Distance from mode to mean[24]

$$d = \frac{\sigma \sqrt{\beta_1}(\beta_2 + 3)}{2(5\beta_2 - 6\beta_1 - 9)}$$
$$= \frac{5.131902 \times 0.225667 (2.5919266 + 3.0)}{2(5 \times 2.5919266 - 6 \times 0.05092585 - 9.0)}$$
$$= 1.772271 \text{ feet}$$

(24) Pearson, Karl, op. cit. p. lxv.

Modal Stage:

$$x_{mode} = \bar{x} - d$$
$$= 15.686196 - 1.772271$$
$$= 13.913925 \text{ feet.}$$

Evaluation of constants of Type I curve:[25][26]

$$y = y_0\left(1 + \frac{x}{a_1}\right)^{m_1}\left(1 - \frac{x}{a_2}\right)^{m_2}$$

(25) Pearson, Karl, op. cit. p. lxix.

(26) Pearson, Karl, Philosopical Transactions of the Royal Society, London, Vol. 186 A, pp. 367-371, 1896.

$$r = \frac{6(\beta_2 - \beta_1 - 1)}{(3\beta_1 - 2\beta_2 + 6)}$$
$$= \frac{6(2.5919266 - 0.05092585 - 1.0)}{(3 \times 0.05092585 - 2 \times 2.5919266 + 6.0)}$$
$$= 9.5425447$$

$$e = \frac{r^2}{\dfrac{4 + \frac{1}{4}\beta_1(r+2)^2}{r+1}}$$

$$= \frac{9.5425477^2}{\dfrac{4 + \frac{1}{4}(0.05092585)(9.5425477 + 2.0)^2}{9.5425477 + 1.0}}$$

$$= 17.6613187$$

$$b^2 = \frac{\mu_2 r^2 (r+1)}{e}$$

$$= \frac{6.583593 \times 9.5425477(9.5425477 + 1.0)}{17.6613187}$$

$$= 357.860451$$

$$b = 18.917199$$

$$r = m_1' + m_2'$$

$$e = m_1' m_2'$$

m_1' and m_2' are roots of

$$m'^2 - rm' + e = 0$$

$$m_{1,2}' = \frac{r \pm \sqrt{r^2 - 4e}}{2}$$

$$= \frac{9.5425447 \pm \sqrt{9.5425447^2 - 4(17.6613187)}}{2}$$

$$= 2.5121309, \quad 7.0304138$$

$$m_1 = m_1' - 1$$
$$= 2.5121309 - 1.0$$
$$= 1.5121309$$

$$m_2 = m_2' - 1$$
$$= 7.0304138 - 1.0$$
$$= 6.0304138$$

$$\frac{a_1}{a_2} = \frac{m_1}{m_2}$$

$$a_1 + a_2 = b$$

$$a_1 = b \frac{m_1}{m_1 + m_2}$$

$$= 18.917199 \frac{1.5121309}{1.5121309 + 6.0304138}$$

$$= 3.792523$$

$$a_2 = b \frac{m_2}{m_1 + m_2}$$

$$= 18.917199 \frac{6.0304138}{1.5121309 + 6.0304138}$$

$$= 15.124675$$

$$y = y_0\left(1 + \frac{x}{3.792523}\right)^{1.5121309}\left(1 - \frac{x}{15.124675}\right)^{6.0304138}$$

Evaluation of y_0, frequency at modal stage: [27]

$$y_0 = \frac{N}{b} \frac{(m_1 + m_2 + 1)}{\frac{\Gamma(m_1+1)}{e^{-m_1} m_1^{m_1}}} \sqrt{\frac{m_1 + m_2}{m_2}} \; e^{\frac{1}{12}\left(\frac{1}{m_1+m_2} - \frac{1}{m_2}\right)}$$

..... (lxxix)

(27) Pearson, Karl, "Tables for Statisticians and Biometricians", Cambridge University Press, London, 1914, p. lxix.

N = Total frequency ÷ units per cell

$$\text{Let } Z = \frac{\Gamma(m_1 + 1)}{e^{-m_1} m^{m_1}} \qquad (28)$$

$$= \frac{\Gamma(1.5121309 + 1.0)}{e^{-1.5121309} \times 1.5121309^{1.5121309}}$$

$$= \frac{\Gamma(2.5121309)}{e^{-1.5121309} \times 1.5121309^{1.5121309}}$$

(28) Ibid. p. lxix.

$$\Gamma(2.5121309) = 1.5121309\ \Gamma(1.5121309)$$ [29]

(29) Ibid. p. lv.

$$\begin{aligned}\log \Gamma(2.5121309) &= \log 1.5121309 \\ &\quad + \log \Gamma(1.5121309) \\ &= 0.1795892 \\ &\quad + \bar{1}.9477669 \\ &= 0.1273561\end{aligned}$$ [30]

$$\Gamma(2.5121309) = 1.3407756$$

(30) Ibid. p. 60.

$$\begin{aligned}&\log \Gamma(m_1+1) &\rightarrow\ & 0.1273561 \\ &+\log e^{m_1} &\rightarrow\ & 0.6567101 \\ &-\log m_1^{m_1} &\rightarrow\ & -0.2715624 \\ \hline &\log Z &\rightarrow\ & 0.5125038\end{aligned}$$

$$Z = 3.254654$$

$$m_1 + m_2 + 1 = 8.5425447$$

$$\sqrt{\frac{m_1 + m_2}{m_2}} = 1.1183696$$

$$e^{\frac{1}{12}\left(\frac{1}{m_1+m_2} - \frac{1}{m_2}\right)} = e^{-0.002770404}$$

$$\begin{aligned}&\log 529.7916 &\rightarrow\ & 2.7241050 \\ &-\log 18.917199 &\rightarrow\ & -1.2768569 \\ &+\log 8.5425447 &\rightarrow\ & 0.9315873 \\ &-\log 3.254654 &\rightarrow\ & -0.5125038 \\ &+\log 1.1183696 &\rightarrow\ & 0.0485854 \\ &+\log e^{-0.002770404} &\rightarrow\ & -0.0012032 \\ \hline &\log y_0 &\rightarrow\ & 1.9137138\end{aligned}$$

$y_0 = 81.9811$, Origin at modal stage

Reduce equation and solve for frequencies, y at various stages.

$$y = 81.9811 \left(1 + \frac{x}{3.792523}\right)^{1.5121309} \left(1 - \frac{x}{15.124675}\right)^{6.0304138}$$

Where x is the deviation from the modal stage in working units

$$= 8.40046 \times 10^{-9} \left(3.792523 + x\right)^{1.5121309} \left(15.124675 - x\right)^{6.0304138}$$

$$\begin{aligned} x_L &= \text{Left intercept, } y = 0 \\ &= \bar{x} - d - 2a_1 \\ &= x_{mode} - 2a_1 \\ &= 13.913925 - 2 \times 3.792523 = 13.913925 - 7.585045 \\ &= 6.328880 \text{ feet} \end{aligned}$$

$$\begin{aligned} x_R &= \text{Right intercept, } y = 0 \\ &= \bar{x} - d + 2a_2 \\ &= x_{mode} + 2a_2 \\ &= 13.913925 + 2 \times 15.124675 = 13.913925 + 30.24935 \\ &= 44.163276 \text{ feet} \end{aligned}$$

$$\begin{aligned} R &= \text{Range} \\ &= 2a_2 + \bar{x} - d + 2a_1 - \bar{x} + d \\ &= 2(a_2 + a_1) \\ &= x_R - x_L \\ &= 44.163276 - 6.328880 \\ &= 37.834396 \text{ feet} \end{aligned}$$

MISSISSIPPI RIVER ~ ST. LOUIS GAUGE ~ 1861-1930 INCLUSIVE ~ 1/24 YEAR PERIOD

Stage Midpoint	Stage Boundary	Deviation from Modal Stage, feet Col. III	x Deviation in working units Col. IV	$a_1 + x$ Col. V	$\log(a_1 + x)$ Col. VI	$a_2 - x$ Col. VII	$\log(a_2 - x)$ Col. VIII	log 8.40046 x 10^{-9} Col. IX	$\log(a_1 + x)^{m_1}$ Col. X	$\log(a_2 - x)^{m_2}$ Col. XI	log y Col. XII	y Col. XIII
	-1.0											
0.0												
	+1.0											
2.0												
	3.0											
4.0												
	5.0											
6.0												
	7.0											
8.0		-5.913925	-2.956962	0.835561	$\bar{1}$.9219781	18.081637	1.2572378	$\bar{7}$.9243033	$\bar{1}$.8820207	7.5816642	1.3879882	24.4336
	9.0											
10.0		-3.913925	-1.956962	1.835561	0.2637688	17.081637	1.2325295	"	0.3988530	7.4326629	1.7558192	56.9927
	11.0											
12.0		-1.913925	-0.956962	2.835561	0.4526390	16.081637	1.2063303	"	0.6844494	7.2764709	1.8852236	76.7757
	13.0											
14.0		+0.086075	+0.043037	3.835561	0.5838238	15.081637	1.1784485	"	0.8828256	7.1065321	1.9136610	81.9712
	15.0											
16.0		2.086075	1.043037	4.835561	0.6844468	14.081637	1.1486531	"	1.0349732	6.9268535	1.8861260	76.9353
	17.0											
18.0		4.086075	2.043037	5.835561	0.7660825	13.081637	1.1166621	"	1.1584170	6.7339345	1.8166548	65.5625
	19.0											
20.0		6.086075	3.043037	6.835561	0.8347741	12.081637	1.0821258	"	1.2622877	6.5256664	1.7122574	51.5534
	21.0											
22.0		8.086075	4.043037	7.835561	0.8940701	11.081637	1.0446039	"	1.3519459	6.2993938	1.5696430	37.1230
	23.0											
24.0		10.086075	5.043037	8.835561	0.9462340	10.081637	1.0035311	"	1.4308291	6.0517078	1.4068408	25.5177
	25.0											
26.0		12.086075	6.043037	9.835561	0.9927991	9.081637	0.9581642	"	1.5012422	5.7781266	1.2036721	15.9835
	27.0											
28.0		14.086075	7.043037	10.835561	1.0348514	8.081637	0.9074994	"	1.5648308	5.4725969	0.9617310	9.1565
	29.0											
30.0		16.086075	8.043037	11.835561	1.0731889	7.081637	0.8501337	"	1.6228021	5.1266580	0.6737634	4.7180
	31.0											
32.0		18.086075	9.043037	12.835561	1.1084148	6.081637	0.7840206	"	1.6760683	4.7279686	0.3283402	2.1298
	33.0											
34.0		20.086075	10.043037	13.835561	1.1409968	5.081637	0.7060037	"	1.7253365	4.2574945	$\bar{1}$.9071343	0.8075
	35.0											
36.0		22.086075	11.043037	14.835561	1.1713040	4.081637	0.6108345	"	1.7711650	3.6835607	$\bar{1}$.3790284	0.2395
	37.0											
38.0		24.086075	12.043037	15.835561	1.1996334	3.081637	0.4887817	"	1.8140027	2.9475559	$\bar{2}$.6858619	4.851 x 10^{-2}
	39.0											
40.0		26.086075	13.043037	16.835561	1.2262275	2.081637	0.3184052	"	1.8542165	1.9201151	$\bar{3}$.6986349	4.996 x 10^{-3}
	41.0											
42.0		28.086075	14.043037	17.835561	1.2512867	1.081637	0.0340819	"	1.8921093	0.2055280	$\bar{4}$.0219406	1.051 x 10^{-4}
	43.0											
44.0		30.086075	15.043037	18.835561	1.2749785	0.081637	$\bar{2}$.9118924	$\bar{7}$.9243033	1.9279344	$\bar{7}$.4382633	$\bar{8}$.2905010	1.951 x 10^{-8}
	45.0											
7.0		-6.913925	-3.456962	0.335561	$\bar{1}$.5257730	18.581637	1.2690840	$\bar{7}$.9243033	$\bar{1}$.2829067	7.6531017	0.8603117	7.2497
13.0		-0.913925	-0.456962	3.335561	0.5231689	15.581637	1.1926131	"	0.7910999	7.1919505	1.9073537	80.7893
13.913925		0.	0.	3.792523	0.5789282	15.124675	1.1796861	"	0.8754152	7.1139955	1.9137440	81.9811
15.0		1.086075	0.543037	4.335561	0.6370452	14.581637	1.1638063	"	0.9632957	7.0182336	1.9058326	80.5068
15.686196		1.772271	0.886136	4.678659	0.6701214	14.238539	1.1534654	$\bar{7}$.9243033	1.0133113	6.9558737	1.8934883	78.2507

Table III: Calculation of frequencies for Pearson Type I Curve.

SAMPLE CALCULATION FOR STAGE = 20.0'

Stage midpoint = 20.0' . . . . (Col. I)

Stage Boundries = 19.00' to 20.99' . . . . (Col. II)

Deviation from Modal Stage, feet

= Stage midpoint − Modal Stage

= 20.0 − 13.913925

= 6.086075 feet . . . . (Col. III)

x, Deviation in working units

= Col. III ÷ 2

$= \frac{1}{2}(6.086075)$

= 3.043037 (where $m = 2$) . . . . (Col. VI)

y, Equation of the frequency curve

$$= y_0 \left(1 + \frac{x}{a_1}\right)^{m_1} \left(1 - \frac{x}{a_2}\right)^{m_2}$$

$$= \frac{y_0}{a_1^{m_1} a_2^{m_2}} \left(a_1 + x\right)^{m_1} \left(a_2 - x\right)^{m_2}$$

$a_1 = 3.792523$

$a_2 = 15.124675$

$m_1 = 1.5121309$

$m_2 = 6.0304138$

$y_0 = 81.9811$

$$\frac{y_0}{a_1^{m_1} a_2^{m_2}} = \frac{81.9811}{3.792523^{1.5121309} \times 15.124675^{6.0304138}}$$

$\log y_0 = 1.9137138$

$$\log a_1 = 0.5789282$$

$$\log a_2 = 1.1796861$$

$$\begin{aligned} \log y_0 &= 1.9137138 \\ m_1 \log a_1 &= -0.8754152 \\ m_2 \log a_2 &= -7.1139953 \\ \hline &\quad 3.9243033-10 \ldots \text{(Col. IX)} \end{aligned}$$

$$\text{antilog } \bar{7}.9243033 \rightarrow 8.40046 \times 10^{-9}$$

$$\begin{aligned} a_1 + x &= 3.792523 + \text{Col. IV} \\ &= 3.792523 + 3.043037 \\ &= 6.835561 \ldots\ldots \text{(Col. V)} \end{aligned}$$

$$\log(a_1 + x) \rightarrow 0.8347741 \ldots \text{(Col. VI)}$$

$$m_1 \log(a_1 + x) \rightarrow 1.2622877 \ldots \text{(Col. X)}$$

$$\begin{aligned} a_2 - x &= 15.124675 - \text{Col. IV} \\ &= 15.124675 - 3.043037 \\ &= 12.081637 \ldots\ldots \text{(Col. VII)} \end{aligned}$$

$$\log(a_2 - x) \rightarrow 1.0821258 \ldots \text{(Col. VIII)}$$

$$m_2 \log(a_2 - x) \rightarrow 6.5256664 \ldots \text{(Col. XI)}$$

y, frequency at stage = 20.0'

$$= \text{Col. IX} + \text{Col. X} + \text{Col. IX}$$

$$\begin{aligned} & \bar{7}.9243033 \\ m_1 \log(a_1 + x) \rightarrow\ & 1.2622877 \\ m_2 \log(a_2 - x) \rightarrow\ & 6.5256664 \\ \hline \log y \longrightarrow\ & 1.7122574 \ldots \text{(Col. XII)} \end{aligned}$$

$$y = 51.5534 \ldots\ldots \text{(Col. XIII)}$$

In like manner frequencies are computed for various stages between the right and left intercepts.

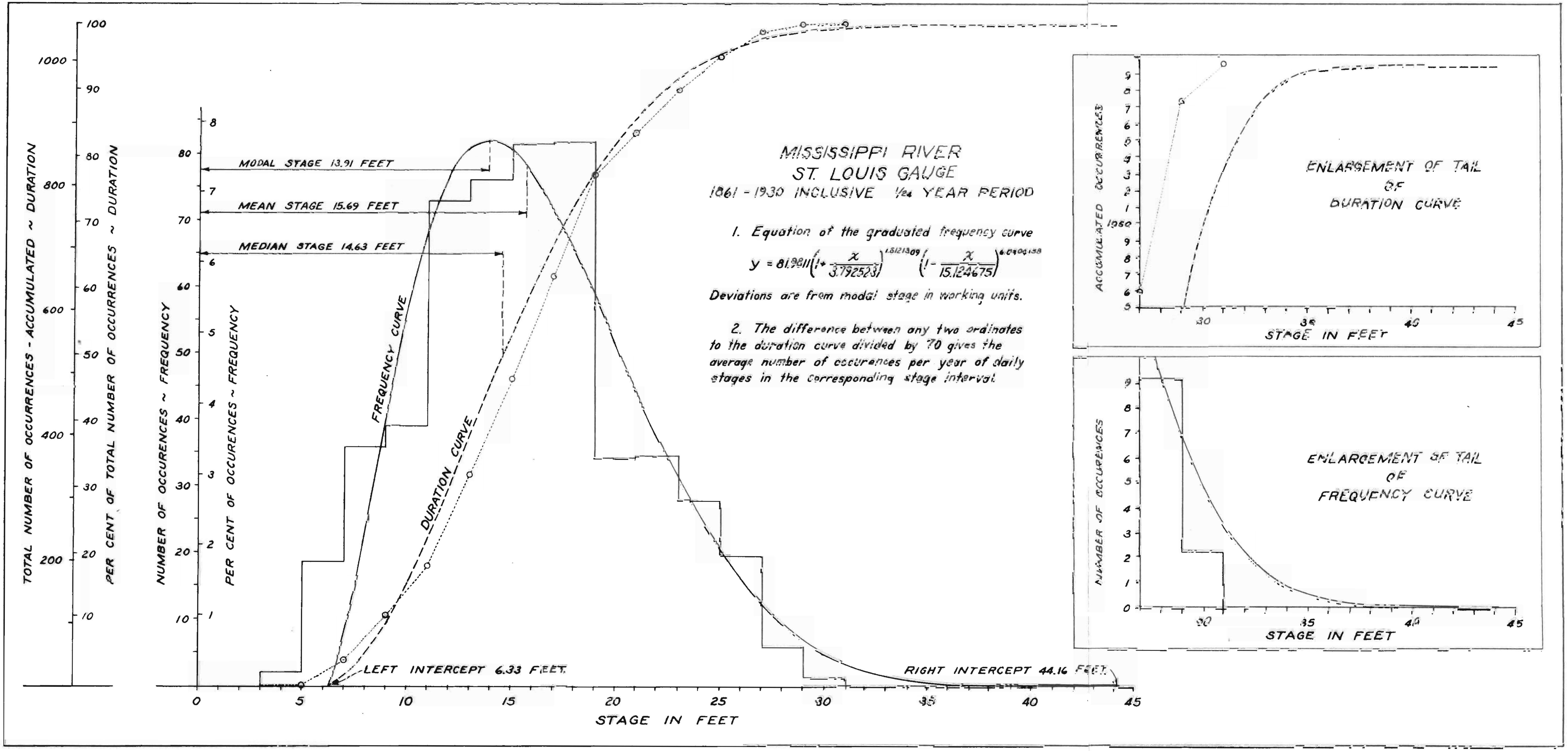

Fig 4: Frequency - Duration Curves

APPENDIX B

GLOSSARY

annual daily high stage: The gauge reading or stage at the regular time of observation on the single day when the water surface elevation was the highest during that year of record.

arbitrary origin: Any point which is used as a zero point, and from which all values on the scale are measured as deviations.

arithmetic mean: The sum of the observations in a statistical series divided by their number or frequency.

asymptote: A line which a curve approaches as a limit, when the curve is indefinitely extended.

bank caving: The failure of a levee due to the bank being undermined by the water washing against the bank and levee sliding into the river.

β_1 : The function μ_3^2/μ_2^3 which is used as a measure of skewness and a criterion of curve type.

β_2 : The function of μ_4/μ_2^2 which is often used as a measure of kurtosis and a criterion of curve type.

cell: The compartment formed by the intersection of a horizontal and perpendicular array. Also called class.

class interval: The width of a class; the distance between the upper and lower class boundaries; the distance between two consecutive actual class limits. Also called class, class size and group interval.

crevasse: A break in a levee occurring during a flood through which the waters flow inundating the land which the levee was intended to protect.

crevassing: The failure of a levee due to the opening of a fissure in the levee either by water pressure pushing out a portion of the levee or sliding the levee onto the land.

cumulated frequency curve: A graphic representation of a cumulative frequency distribution. The frequencies may be expressed in terms of either percentage or the actual numbers of observations.

cumulation: A summation in which the sum of the successive quantities of the series is obtained and recorded for each successive item or class of the series.

daily stage: The gauge reading or elevation of the water surface on a gauge for a single day as regularly observed at a stated time. viz., 7:00 A.M.

frequency: The number of observations or measures in one of the class intervals of a frequency distribution.

gauge, gage: An instrument used for measuring the height of the water surface. The instrument is divided in feet and decimals of a foot above some arbitrary zero point selected that the elevation of the water surface will in all probability be a positive or plus value at all stages. The elevation of the gauge zero will be referenced to some accepted datum plane.

gauge height: gauge reading, stage. The elevation of the water surface on a gauge.

gauge reading: gauge height, stage. The elevation of the water surface on a gauge.

gauge station: A location of a gauge usually named after some important landmark in the vicinity. viz., St. Louis Gauge at St. Louis, Missouri.

histogram: A graphic representation of a frequency distribution consisting of a series of rectangles of width proportional to the width of the class interval and proportional in area to the quantities represented.

kurtosis: The relative degree of flatness or peakedness in the region about the mode of a frequency curve as compared to the normal probability curve of the same variance.

mean: A calculated average.

median: That point on a scale of a frequency distribution below and above which just 50 per cent of the observations occur.

medial stage: Median, median stage.

modal stage: The stage or gauge reading at the mode.

mode: The value of the observation, measure, or score that occurs the greatest number of times.

moment: The product of a frequency and any power of its deviation from the point selected as the origin.

moment coefficient: μ_1 , μ_2, etc. The arithmetic mean of the deviations of the measures in a frequency distribution, each raised to the same power, the deviations being measured from the mean. The subscript of the symbol μ denotes the power to which the deviations are raised. Also called unit moment.

overtopping: The failure of a levee due to the water surface elevation being greater than the top of the levee itself.

probability paper: A graph paper so ruled that a cumulative frequency curve when plotted will be a straight line if the distribution is normal.

raw moment coefficient: ν_1', ν_2', ν_3' etc. The arithmetic mean of the deviations of the measures in a frequency distribution, each raised to the same power, the deviations being measured from the arbitrary origin. The subscript of the symbol ν' denotes the power to which the deviations are raised.

Sheppard's correction: A correction applied to the moments of the distributed variates, to correct for errors due to coarseness of grouping.

skew: Not symetrical; unbalanced; distorted; extending farther to one side of the mode than the other.

skew frequency curve: An unsymetrical frequency curve extending farther to one side of the mode than the other.

skewness: sk. The extent to which a frequency distribution or curve departs from a symetrical shape.

spline: A long narrow strip which can be bent to conform to curves of varying degrees of curvature. An aid in drafting.

stage: Gauge height, gauge reading. The elevation of the water surface on a gauge.

standard deviation: σ. The square root of the arithmetic mean of the squares of the deviations of the values of the variable from their arithmetic mean.

statistic: A value such as the number of observations, the arithmetic mean, the standard deviation, or any other measure which describes or characterizes a particular series of quantitative observations.

trend: The regular and persistent change in a variable during a long period of time.

twenty-five year flood: The flood, either volume or stage, which would be expected to occur on an average four per cent of the years of observation.

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VITA

Vernon Arthur Charles Gevecker was born January 19, 1909 in St. Louis, Missouri, the son of Harry and Louise (Meirose) Gevecker. In November 1932, he was married to Mildred Anna Twellman of St. Louis, Missouri. He has two children, a daughter, Carol Ann, and a son, Karl Richard.

His elementary education was received in the public schools of the City of St. Louis, Missouri. In September 1927 he entered Washington University and in September 1929 entered the Missouri School of Mines and Metallurgy receiving the degree of Bachelor of Science in Civil Engineering from the latter school in May 1931. In September 1934 he entered the Missouri School of Mines and Metallurgy and in September 1935 entered the California Institute of Technology for graduate studies receiving the degree of Master of Science in Civil Engineering from the latter school in June 1937.

He was employed by the St. Louis Office, U. S. Engineer Department, Corps of Engineers, U. S. Army for a total of four years between 1930 and 1935. Upon completion of graduate work he was employed at the St. Louis Plant, Proctor and Gamble Manufacturing Company for one year, 1937 to 1938. He came to the Missouri School of Mines and Metallurgy in September 1938 as an Instructor in Civil Engineering and remained in continuous employment at this school except for a period of military service in World War II. He served with the U. S. Army and Air Force for a period of five years between 1941 and 1946 and was discharged with the rank of Lieutenant Colonel, Corps of Engineers. He has served various summers with the Corps of Engineers, Department of the Army, on engineering works. His present occupation is Associate Professor of Civil Engineering at the Missouri School of Mines and Metallurgy, Rolla, Missouri.