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Rainfall-runoff studies for typical blocks in St. Louis

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Department:

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RAINFALL-RUNOFF STUDIES FOR TYPICAL BLOCKS IN ST. LOUIS

by

Frank Le Roy Flynt.

A

THESIS

submitted to the faculty of the

SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI

in partial fulfillment of the work required for the

DEGREE OF

CIVIL ENGINEER

1932

Approved by Joe B Butler

Professor of Civil Engineering

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS

INTRODUCTION

The following paper may be considered as a progress report on one phase of a study, now being made by the writer, of the rainfall and runoff records which have been collected over a long period of years under the direction of Mr. W. W. Horner, Chief Engineer of Sewers and Paving of the City of St. Louis. The mass of data which has been collected during this time is of considerable volume and even the information applicable to the single phase of the investigation discussed in the following pages, which is the runoff from typical city blocks, has not been fully worked up. This paper, therefore, is essentially an exposition of the working hypothesis which the writer has developed for use in studying the data in an effort to correlate the information derived therefrom into a comprehensive method of storm sewer design. No claim of finality is made for the formulas presented nor for the methods used in applying them. The process of developing these working formulas has been evolutionary and the results submitted are intended only to reflect the progress made up to the time of writing.

In the field of storm sewer design there have been no satisfactory formulas giving the relation between rainfall and runoff which would enable the designing engineer to be certain that his designs were, at the same time, both adequate and economical. All that could be done was to exercise the best judgment possible in the use of such formulas as were available and hope for the best. The formulas in general use are mainly rationalizations of fragmentary experimental data collected by many observers from many sources and modified to fit local conditions in the light of personal observation and experience. Owing to the lack of rational formulas dealing with the problem, many important factors have been ignored or overlooked in the collection of experimental data which has impaired the usefulness of the resulting working formulas. The lack of rational formulas is due to the extreme complexity of the laws governing the relation of rainfall to runoff. The reason for this complexity is the large number of variable factors affecting the relations of rainfall and runoff which makes the problem comparable to that of weather forecasting.

EXPERIMENTAL INSTALLATIONS IN ST. LOUIS

As a contribution to the solution of the problem, the City of St. Louis has since 1910 maintained a varying number of rain gauges at various points in the City. Most of these gauges are of the standard type, which measures only the total depth of rainfall, but, in addition there are several automatic recording gauges which provide information as to the rate of rainfall at any given time during the rain. Supplementing the City gauges, the records of the automatic tipping bucket gauges of the United States Weather Bureau and of St. Louis University are available for study.

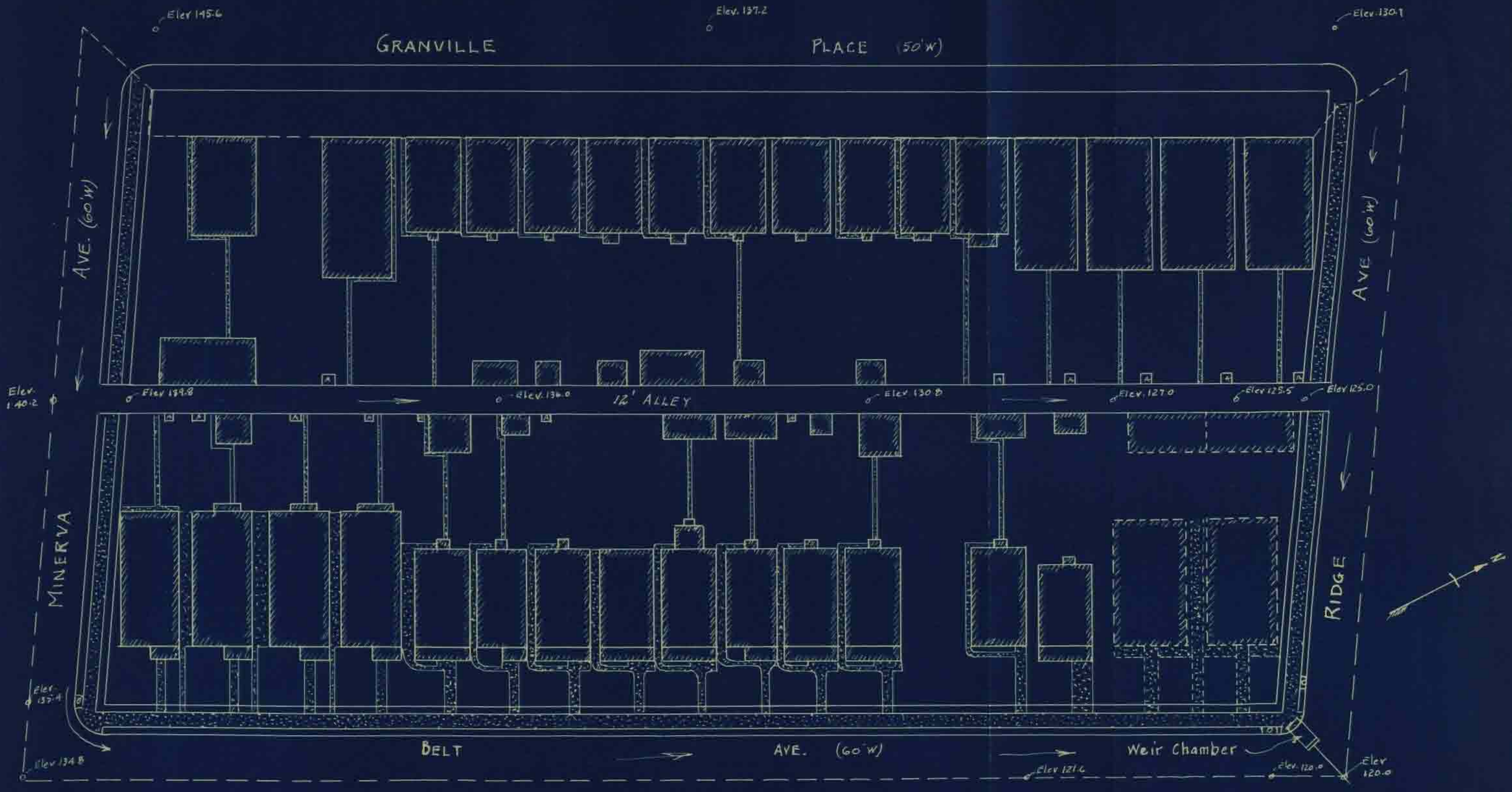
Under the supervision of Mr. Horner the use of automatic recording instruments has been extended to include methods of measuring the actual runoff from given areas. The Clarendon Sewer District was selected for special study and most of the recording apparatus is concentrated in this district. The Clarendon District, the boundaries of which are outlined in red on Plate 1 , is a fair sample of a built up residential section and was chosen for this work mainly because the trunk and lateral sewers seemed to be adequate and not subject to overcharge during heavy rains. Recording pressure gauges have been installed in pairs, one gauge at each end of selected lengths of sewer which give the elevation of the hydraulic grade at these points for any

PLATE I.

General Plan of the Sewerage Subdivisions of the City of St. Louis.

given time from which information, together with the size, shape, and roughness of the sewer cross-section, the rate of discharge can be computed by means of any of the "slope" formulas in general use. It is believed that these records extending as they do over a period of several years, will provide the most useful experimental data that have, so far, been collected.

The first step in the determination of the relation between rainfall and runoff, for use in the design of storm water sewers, is to ascertain the probable rate of runoff from the area tributary to a single inlet due to a given rate of rainfall. To this end two city blocks in the Clarendon District were chosen for special study. Complete surveys were made of these blocks and detailed maps made of them showing the size and location of all buildings, walks, and other structures which might affect the problem. One of these blocks, No. 3807 E-B, (see Plate 2.) lying south of Ridge Avenue and west of Belt Avenue, is located on a hillside and slopes decidedly to the north and east. It is almost completely built up with dwellings. In shape it is approximately rectangular, the length being about twice the width. Most of the houses in this block have the downspouts con-



Tributary Area 2.23 Acres.

MAP OF
CITY BLOCK 3807 E-B.
Scale 1" = 40'

PLATE 2

nected directly to the sewers and the water falling on the roofs does not pass through the weir chamber which measures only the water collected by the inlets. The other block, No. 4841, (see Plate 3.), lies north of Cates Avenue and east of Clarendon Avenue. It is very flat and is built up with dwellings. Its shape is rectangular, the length east and west being about three times the width. In this block few of the houses have the downspouts connected with the sewer but, as will be seen on the map, half of the houses drain into Cates Avenue and therefore the water from these roofs does not pass through the weir chamber. At each of these locations the runoff is measured by means of a 90° notch weir made of 3/8-in. steel plate installed at the lower end of a special weir chamber built on the inlet line between the catch basin and the sewer. A pressure bulb located in the weir chamber is connected with a Bristol recording gauge located above ground thus providing a continuous record of the head of water on the weir notch. Partly to avoid the expense of tipping bucket rain gauges and partly to insure an accurate time relation between the rainfall and the runoff records, Mr. Horner devised the ingenious scheme of adding another pen arm to the Bristol

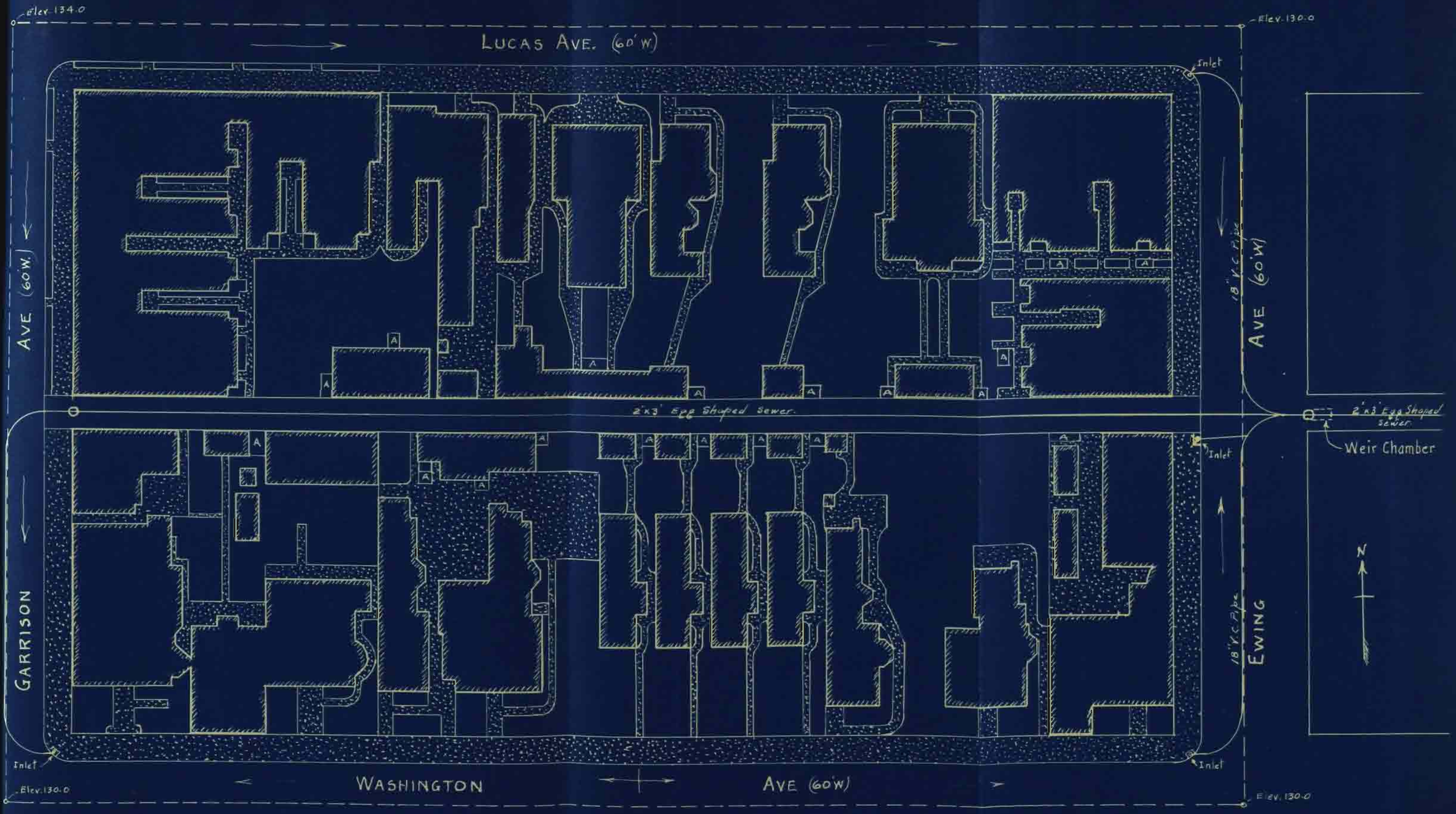
Tributary Area 3.66 Acres

MAP OF
CITY BLOCK 4841
Scale 1" = 40'

PLATE 3

gauge which records the rainfall on the same chart with the weir record. The rain pen is actuated by a home made rain gauge consisting of a pressure bulb made of two cast iron pipe caps screwed together with a short nipple. The top of the bulb is tapped for the connection of the 1/8-in. copper tube which transmits air pressure to the spring which actuates the pen arm. The bottom of the bulb is tapped for a 2½-in. pipe which after making a "U" bend, is carried up to a convenient height and fitted with a 12-in. funnel receiver for collecting the rain. A layer of oil covers the water in the bulb to prevent water vapor from reaching the pen arm spring. The side of the bulb is tapped for an ordinary faucet by means of which the water accumulated in the stand-pipe is drained off after each rain without disturbing the layer of oil.

In order to obtain the runoff from an entire block, a special installation was made at the block bounded by Ewing, Washington, Garrison, and Lucas Avenues. See map Plate 4 . A tipping bucket rain gauge was placed at the upper end of the block and in the sewer draining the block a weir chamber was built containing a rectangular weir. The head on the weir is



Tributary Area 4.36 Acres

MAP OF
CITY BLOCK 1012
Scale 1" = 40'

PLATE 4

measured by a pressure bulb connected with a recording gauge installed above ground. This block contains several large buildings and large areas of the yards are paved with brick or concrete making the percentage of impervious area comparatively large.

The rain pens at Belt and Ridge and at Cates and Clarendon are calibrated by introducing known quantities of water into the standpipe and noting the corresponding chart readings. From this data a calibration chart is made by means of which the chart readings may be converted into inches of accumulated rainfall. The pressure gauges at all three locations are calibrated by removing the pressure bulb from its bracket and immersing it in a vessel of water noting the chart readings corresponding to various depths of submergence.

DETAILED STUDY OF TYPICAL RECORDS

Plate 5 is a photostat of the chart showing the rainfall and runoff record of the rain of September 7, 1917 as recorded at Belt and Ridge. It will be seen that the rain pen moves from the outer circle of the chart toward the center under the influence of the pressure in the rain standpipe. The weir pen moves from the inner circle of the chart outward. The shaded rectangular areas in

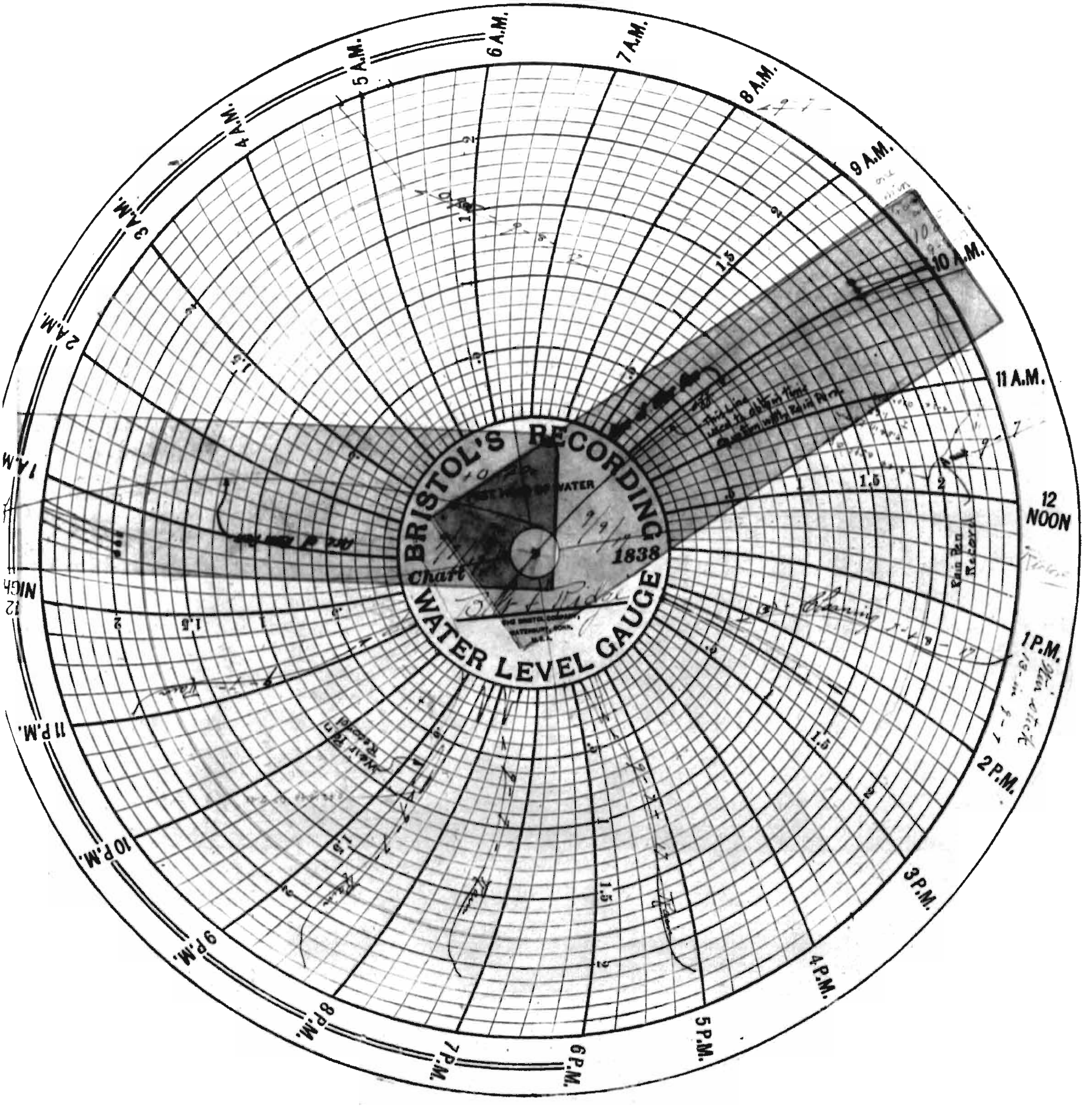

6 A.M.
7 A.M.
8 A.M.
9 A.M.
10 A.M.
11 A.M.
12 NOON
1 P.M.
2 P.M.
3 P.M.
4 P.M.
5 P.M.
6 P.M.
7 P.M.
8 P.M.
9 P.M.
10 P.M.
11 P.M.
12 NIGHT
1 A.M.
2 A.M.
3 A.M.
4 A.M.
5 A.M.
BRISTOL'S RECORDING WATER LEVEL GAUGE
1838
Chart
WATER
THE BRISTOL COMPANY, WATERBURY, CONN. U.S.A.
.5
1
1.5
2
This Line used to obtain Time Correction with Rain Rec.
Rain Rec. Record

PLATE 5

Time Equation
Rain Pen off - 12:48 AM
Weir Pen off - 10:06 A.M.
Equation 9 hrs 18 min
Clock 1 Min. Fast.

BELT AND RIDGE

Rain of Sept. 7, 1917

Area of Block 2.23 Acres

Rain Pen Time Apparent	Rain Pen Record	Total Rain-inches	Intensity Inches/Hr.	Actual Time	Weir Pen Record	Head on Weir Ft.	Cu. Ft. Per. Sec.	C.F.S. Per Acre	No. of Minute
11:09 AM	0.20	0.37	5.10	8:26 AM					1
10			5.10	27					2
11	0.30	0.54	4.20	28	0.06	0.18	0.04	0.02	3
12	0.34	0.61	1.20	29	0.50	0.63	0.81	0.36	4
13			1.20	8:30	0.90	1.03	2.72	1.22	5
14			1.20	31	0.92	1.05	2.84	1.28	6
11:15			1.20	32	0.80	0.93	2.14	0.96	7
16			1.20	33	0.70	0.83	1.60	0.72	8
17	0.40	0.71	2.70	34	0.60	0.73	1.16	0.52	9
18			2.70	8:35					10
19	0.45	0.80	4.80	36					11
11:20	0.50	0.88	3.60	37	0.54	0.67	0.95	0.43	12
21	0.53	0.94	1.50	38					13
22			1.50	39	0.70	0.83	1.60	0.72	14
23	0.56	0.99	0.75	8:40	0.87	1.00	2.53	1.14	15
24			0.75	41	0.89	1.02	2.65	1.19	16
11:25			0.76	42					17
26			0.75	43	0.80	0.93	2.14	0.96	18
27	0.59	1.04	0	44	0.70	0.83	1.60	0.72	19
28	0.59	1.04		8:45	0.60	0.73	1.16	0.52	20
				46					21
				47					22
				48					23
				49	0.50	0.63	0.81	0.36	24
				8:50	0.40	0.53	0.54	0.24	25
				51					26
				52	0.30	0.43	0.32	0.14	27
				53					28
				54					29
				8:55	0.20	0.33	0.16	0.07	30
				56					31
				57					32
				58					33
				59					34
				9:00	0.10	0.22	0.06	0.03	35

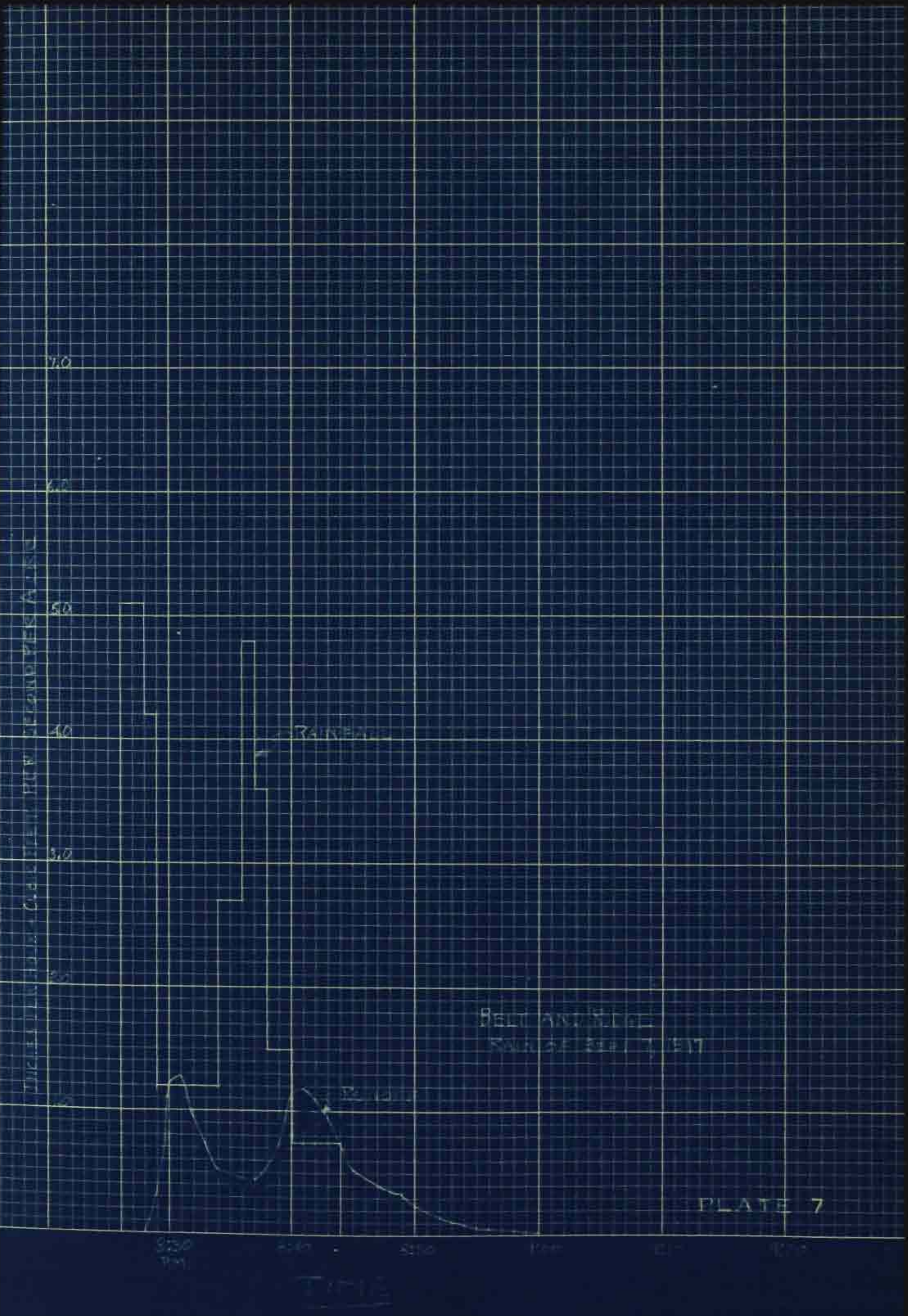


PLATE 7

the photostat are the two celluloid verniers used in reading the records of the two pens. On each strip of celluloid is scribed a line corresponding to the arc of the pen for which it was made. The three short lines near the outer circle constitute the vernier by means of which the record is read to single minutes. Before the attendant removes the chart, which is left on for two days, he swings the two pen arms simultaneously, describing two short arcs. These marks are used to obtain the time equation between the two pens. In Plate 5 the two verniers are placed on these two marks and it is seen that the weir pen, which is set to the true time scale was at 10:06 A.M. when the "off" mark was made. The rain pen arm, which is installed on the opposite side of the chart, does not give the true time reading but, by means of the vernier, the apparent time where the "off" mark crosses the outer circle is seen to be 12:48 A.M. This furnishes a time equation by means of which the apparent time of the rain pen may be converted into actual time. The apparent time for the rain pen is entered in the first column of the table shown in Plate 6 and the chart readings for the rain pen are recorded in the second column. Using the calibration chart the direct readings of the rain pen is converted into inches of total rainfall and entered in the third column. The increment of rainfall for each minute is multiplied by 60 giving the value of the intensity or rate of rainfall in inches per hour for that minute which result is entered in the fourth column. In the fifth column is recorded the actual time corresponding to the apparent time of the rain pen in column one. Since it is noted on the chart that the clock was one minute fast, this correction is made. By means of the weir pen vernier, the chart readings of the weir pen are measured and recorded in the sixth column. Then from the calibration chart the proper value of the head on the 90° notch weir is obtained and entered in column seven. These figures are converted into cubic feet per second from King's formula for 90° notch weirs, $Q = 2.52\, H^{2.47}$ and entered in column eight. The figures in this column are each divided by the number of acres in the area tributary to the inlet where the weir is installed thus giving the value of the runoff in terms of cubic feet per second per acre. It so happens that one inch of rainfall per hour is equal to 1.008 cubic feet per second per acre, therefore, in this study the unit of rainfall intensity and the unit rate of runoff are considered as equal. Plate 7 shows the rainfall-runoff graph of the rain of September 7, 1917 at Belt and Ridge made by plotting the values of Rainfall and Runoff, from columns four and nine of the table described above, on the same time scale.

Plate 8 is a photostat of the instrumental records at Cates and Clarendon of the rainfall and runoff for the same rain, September 7, 1917. The information in the table given in Plate 9 is derived from the instrumental records in exactly the same way as in the case of Belt and Ridge. Plate 10 shows the rainfall-runoff graph for the rain in question at Cates and Clarendon.

Plate 11 is a photostat of the instrumental records of the same rain at Ewing and Washington. In this case the total inches of rainfall up to any given minute can easily be read directly from the tipping bucket raingauge chart, each small step in the record representing 0.05 inch of rainfall. The rate of rainfall in inches per hour is determined as at the other locations, by multiplying the rainfall increment for each minute by 60. The rate of runoff is determined in the same manner as in the other locations, the only differences being that the

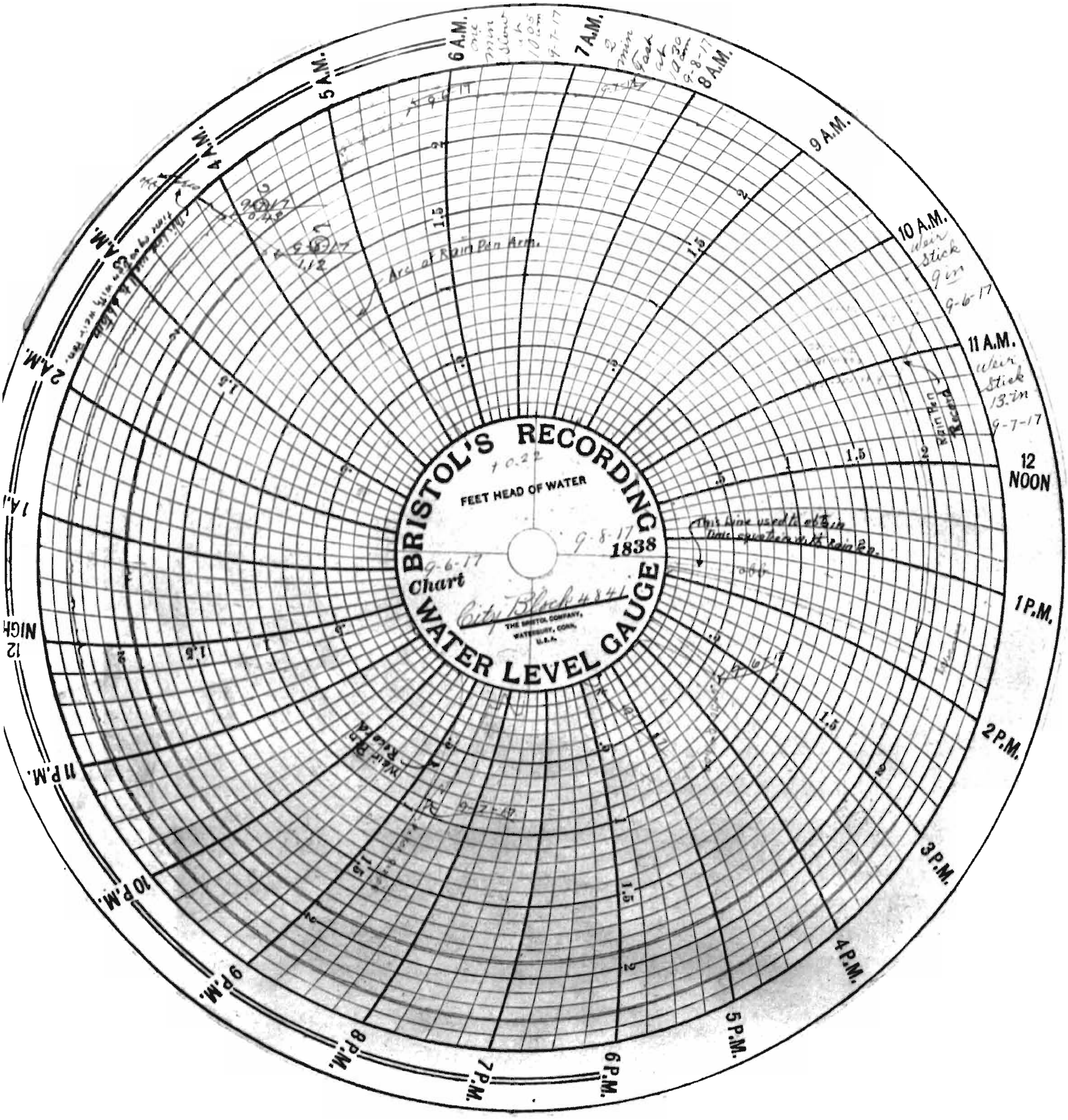

BRISTOL'S RECORDING WATER LEVEL GAUGE
FEET HEAD OF WATER
Chart
1838
THE BRISTOL COMPANY, WATERBURY, CONN. U.S.A.
City Block 4841
9-6-17
9-8-17
+0.22
12 NOON
1 P.M.
2 P.M.
3 P.M.
4 P.M.
5 P.M.
6 P.M.
7 P.M.
8 P.M.
9 P.M.
10 P.M.
11 P.M.
12 NIGHT
1 A.M.
2 A.M.
3 A.M.
4 A.M.
5 A.M.
6 A.M.
7 A.M.
8 A.M.
9 A.M.
10 A.M.
11 A.M.
.5
1
1.5
2
One min slow at 10:05 am 9-7-17
2 min fast at 10:30 am 9-8-17
Weir Stick 9 in 9-6-17
Weir Stick 13 in 9-7-17
Rain Pen Record
Weir Pen Record
This line used to obtain time equation with Rain Pen.
Arc of Rain Pen Arm.
This line used to obtain time equation with weir pen.
9-6-17
9-7-17
9-8-17
1.12
.06

PLATE 8

Time Equation
Rain Pen off 3:45AM
Weir Pen off 1:37 PM
Equation 9hrs 52 Min.
Clock on Time

CATES AND CLARENDON
Rain of Sept. 7, 1917

Area of Block 3.66 Acres.

Rain Pen Time Apparent	Rain Pen Record	Total Rainfall	Intensity Inches/Hr.	Actual Time	Weir Pen Record	Head on Weir	Cubic Feet Per Second	C.F.S Per Acre.	No of Minute
	0.16	0.31	0						
10:34 AM	0.16	0.31	4.20	8:26PM					1
35	0.20	0.38	2.10	27	0.30	0.40	0.27	0.08	2
36			2.10	28	0.40	0.50	0.46	0.13	3
37	0.24	0.45	2.20	29	0.50	0.60	0.72	0.20	4
38			2.20	8:30	0.60	0.71	1.09	0.30	5
39			2.20	31	0.70	0.81	1.50	0.41	6
10:40	0.30	0.56	1.35	32	0.80	0.91	2.01	0.55	7
41			1.35	33	0.83	0.94	2.19	0.60	8
42			1.35	34	0.80	0.91	2.01	0.55	9
43			1.35	8:35					10
44	0.35	0.65	5.40	36	0.76	0.87	1.80	0.49	11
10:45	0.40	0.74	2.40	37					12
46			2.40	38	0.80	0.91	2.01	0.55	13
47	0.44	0.82	1.40	39					14
48			1.40	8:40	0.89	1.00	2.53	0.69	15
49			1.40	41					16
10:50	0.48	0.89	1.20	42	0.90	1.01	2.59	0.71	17
51			1.20	43					18
52	0.50	0.93	0.75	44					19
53			0.75	8:45	0.89	1.00	2.53	0.69	20
54			0.75	46					21
10:55			0.75	47	0.80	0.91	2.01	0.55	22
56	0.53	0.98	0.30	48	0.70	0.81	1.50	0.41	23
57			0.30	49					24
58			0.30	8:50	0.60	0.71	1.09	0.30	25
59			0.30	51					26
11:00	0.54	1.00	0	52	0.50	0.60	0.72	0.20	27
				53					28
				54					29
				8:55	0.40	0.50	0.46	0.13	30
				56					31
				57					32
				58					33
				59					34
				9:00	0.30	0.40	0.27	0.08	35
				01					36
				02	0.20	0.30	0.12	0.03	37
				03					38
				04					39
				05					40
				0.6					41
									42
				9:12	0.10	0.20	0.05	0.01	43
				9:22PM	0.02			0	

PLATE 9.

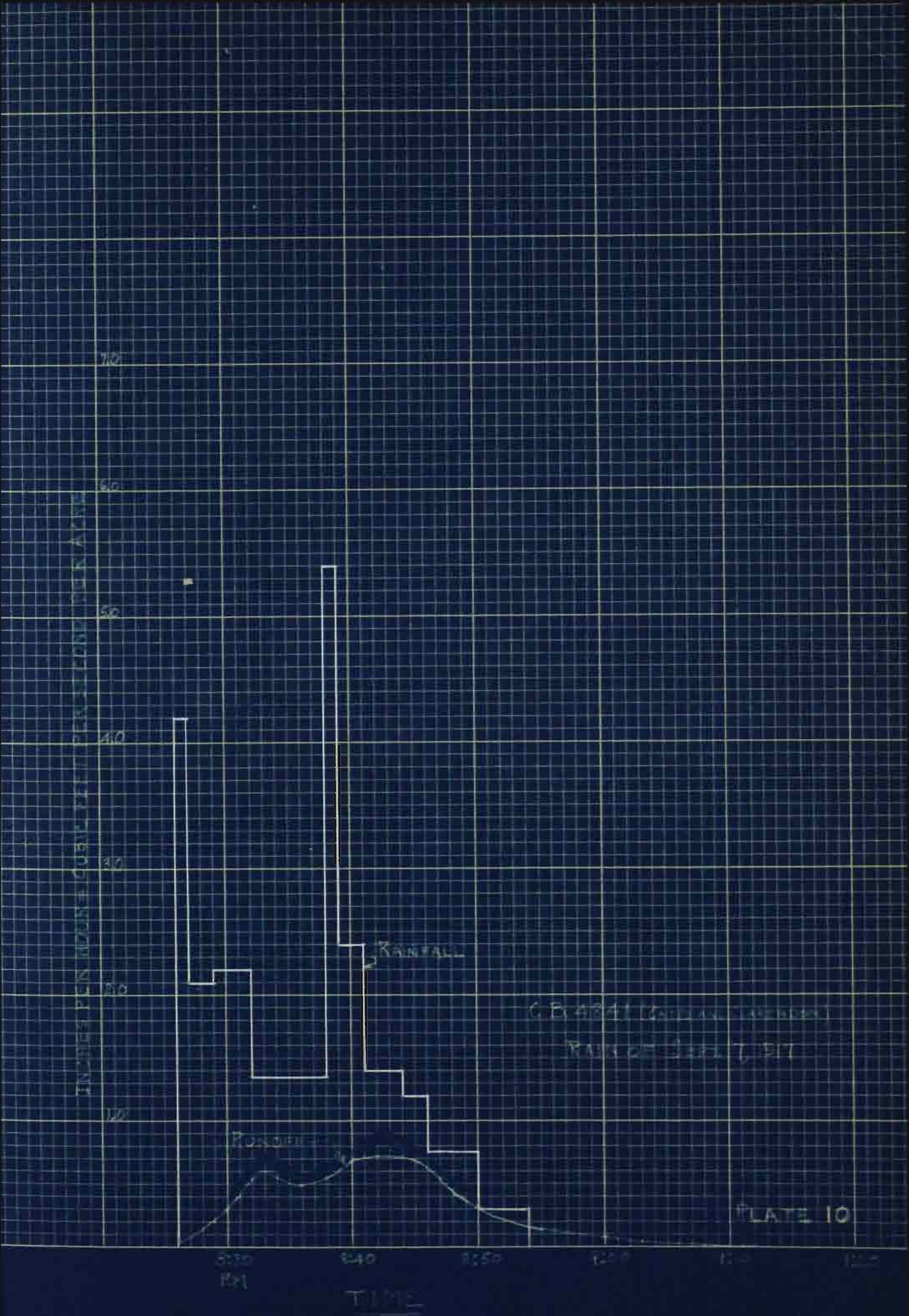
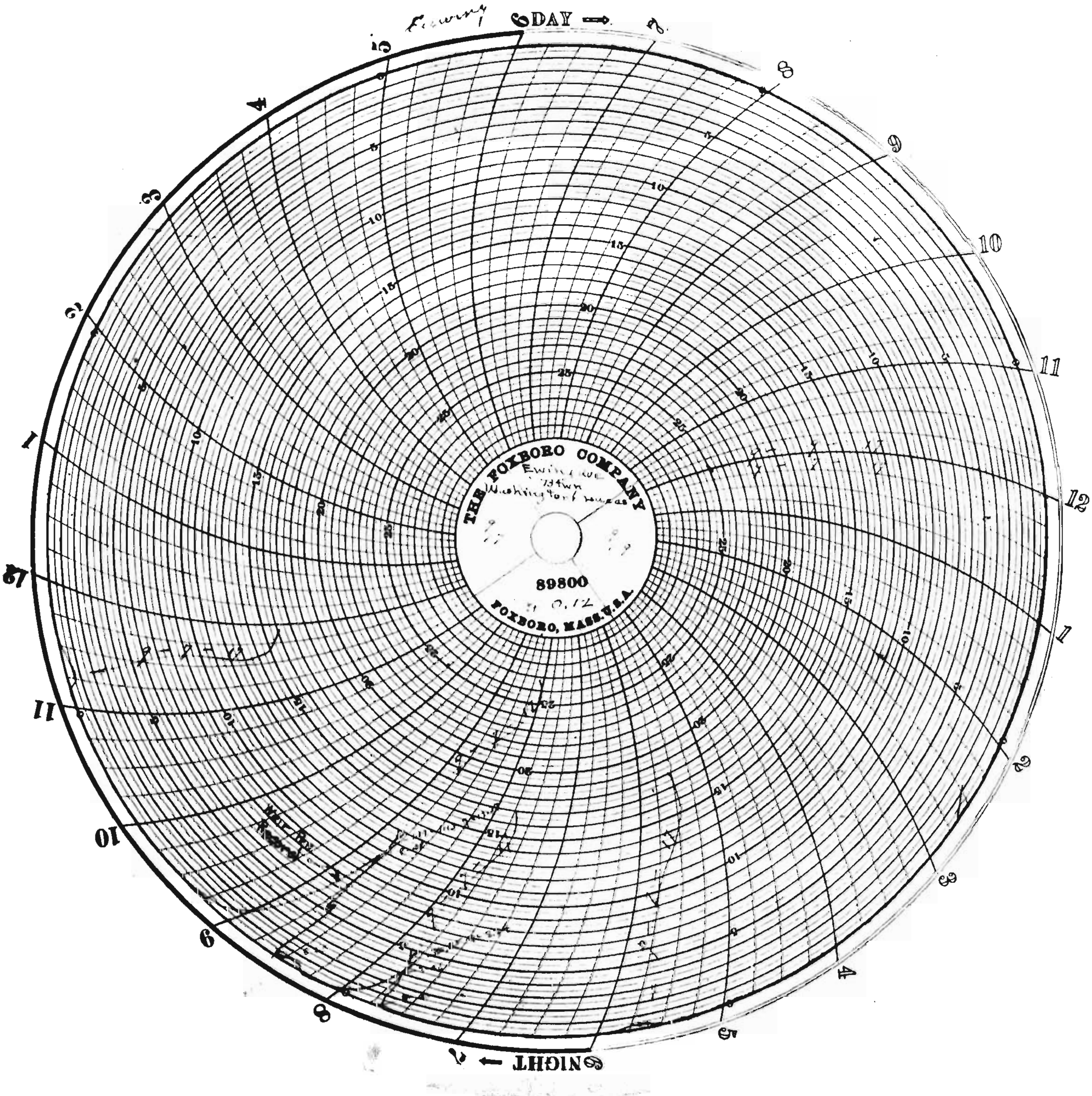
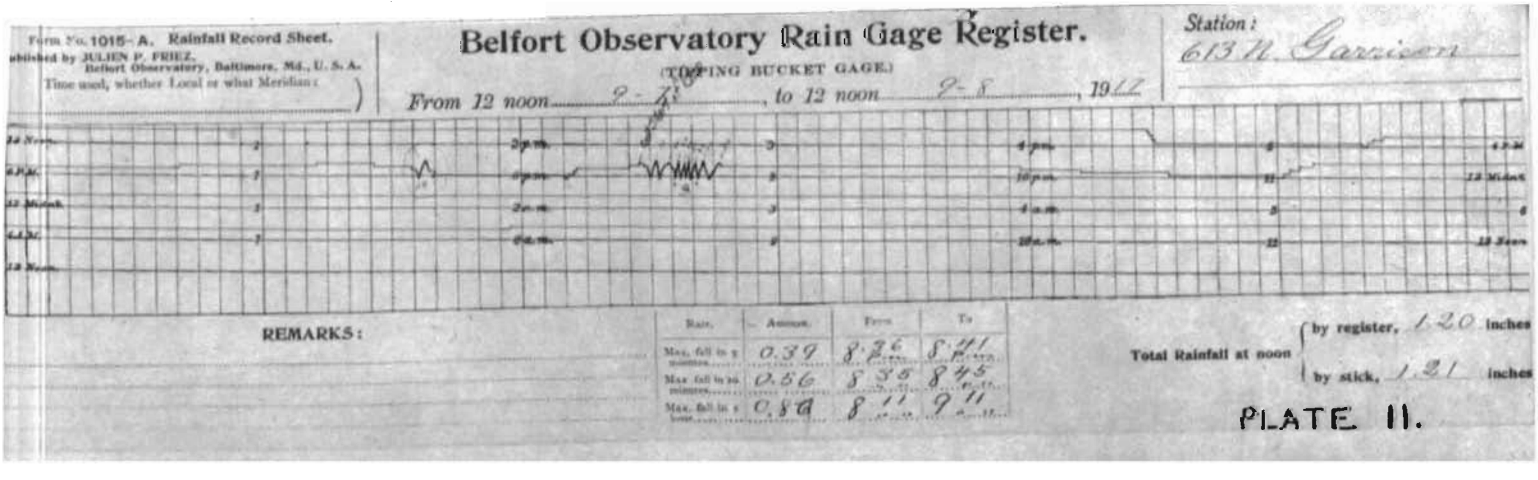
IN. PER HOUR & CUBIC FEET PER SECOND PER ACRE
7.0
6.0
5.0
4.0
3.0
2.0
1.0
RAINFALL
RUNOFF
C.B. 4641 ([illegible])
RAIN OF JULY 7, 1917
8:30
PM
8:40
8:50
TIME

PLATE 10





Form No. 1015-A. Rainfall Record Sheet.
Published by JULIEN P. FRIEZ,
Belfort Observatory, Baltimore, Md., U. S. A.
Time used, whether Local or what Meridian:

Belfort Observatory Rain Gage Register.

(TIPPING BUCKET GAGE.)

From 12 noon 9-7, to 12 noon 9-8, 1912

Station: 613 N. Garrison

REMARKS:

Rate.	Amount.	From	To
Max. fall in 5 minutes	0.39	8.36	8.41
Max fall in 10 minutes	0.56	8.35	8.45
Max. fall in 1 hour	0.86	8.11	9.11

Total Rainfall at noon { by register, 1.20 inches
by stick, 1.21 inches

PLATE II.

EWING AND WASHINGTON

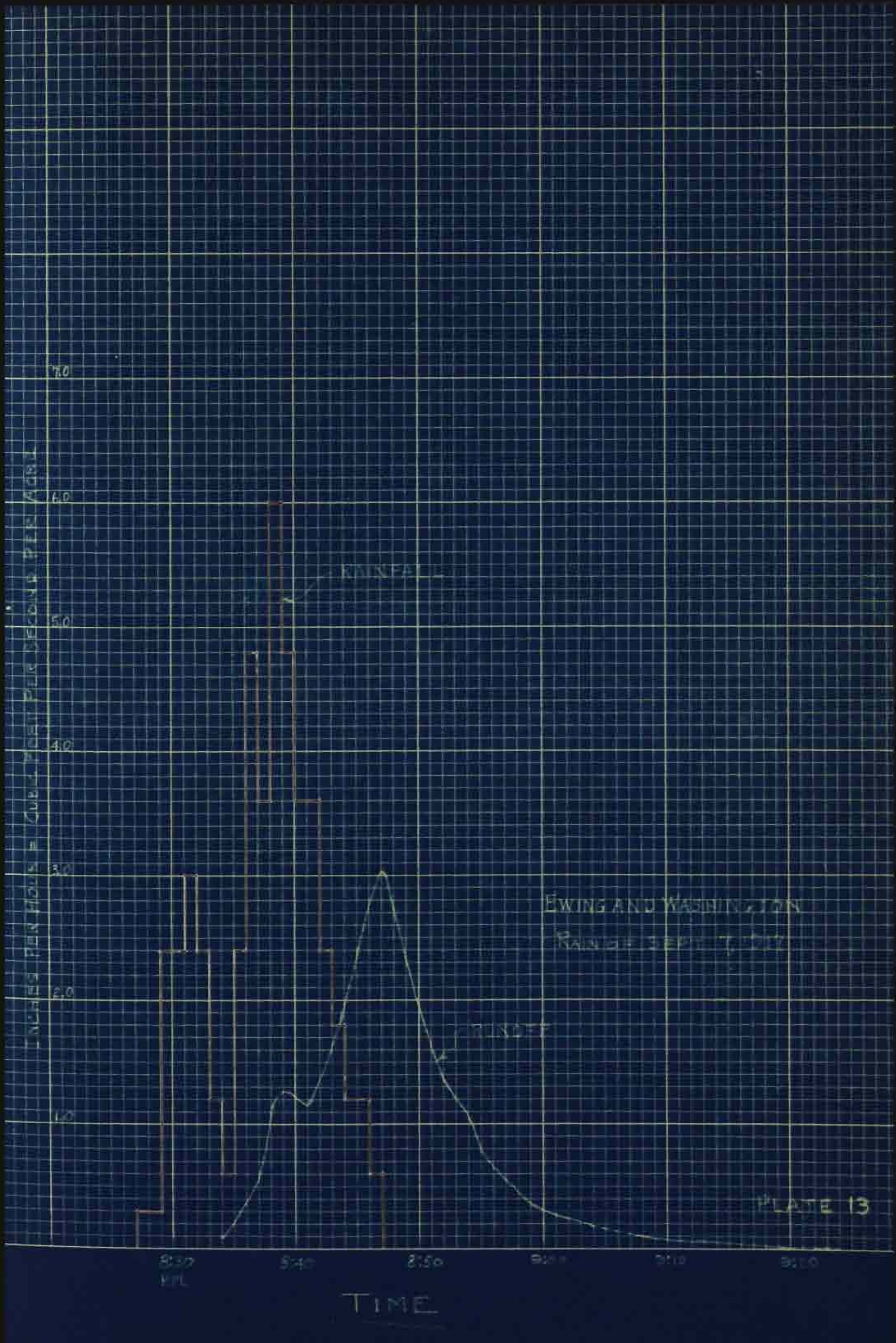
Clocks on time

RAIN OF SEPT. 7. 1917

AREA 4.36 ACRES.

Time	Total Rainfall	Intensity (I) Inches/Hour	Weir Pen Record	Head on Weir Ft.	Cubic Feet Per Second	C.F.S. (Q) Per Acre.	Σ I	Σ Q
8:27 PM.	0.00							
28		0.30					0.30	
29	0.01	0.30					0.60	
8:30	0.05	2.40					3.00	
31	0.09	2.40					5.40	
32	0.14	3.00					8.40	
33	0.18	2.40					10.80	
34	0.20	1.20	0.03	0.10	0.35	0.08	12.00	0.08
8:35	0.21	0.60				0.24	12.60	0.32
36	0.25	2.40				0.40	15.00	0.72
37	0.33	4.80	0.30	0.36	2.39	0.55	19.80	1.27
38	0.39	3.60	0.50	0.56	4.65	1.07	23.40	2.34
39	0.49	6.00	0.57	0.63	5.56	1.27	→C.M. 29.40	3.61
8:40	0.57	4.80				1.22	34.20	4.83
41	0.63	3.60	0.53	0.59	5.05	1.16	37.80	5.99
42	0.69	3.60				1.40	41.40	7.39
43	0.73	2.40	0.70	0.75	7.18	1.64	43.80	9.03
44	0.76	1.80	0.80	0.85	8.66	1.98	45.60	11.01
8:45	0.78	1.20	0.90	0.95	10.25	2.34	46.80	13.35
46	0.80	1.20				2.68	48.00	16.03
47	0.81	0.60	1.07	1.12	13.20	3.02	48.60	→C.M. 19.05
48	0.81		1.00	1.05	11.90	2.72		21.77
49			0.90	0.95	10.25	2.34		24.11
8:50			0.80	0.85	8.66	1.98		26.09
51			0.70	0.75	7.18	1.64		27.73
52			0.60	0.66	5.92	1.36		29.09
53						1.22		30.31
54			0.50	0.56	4.65	1.07		31.38
8:55			0.40	0.46	3.40	0.78		32.16
56						0.67		32.83
57			0.30	0.36	2.39	0.55		33.38
58						0.44		33.82
59			0.20	0.26	1.45	0.33		34.15
9:00						0.30		34.45
01						0.27		34.72
02						0.24		34.96
03						0.21		35.17
04			0.10	0.17	0.78	0.18		35.35
08			0.05	0.12	0.46	0.10		
8:24			0	0	0	0		35.87

PLATE 12



Ewing and Washington
Rain of Sept. 7, 1917

PLATE 13

recording instruments are slightly different in construction. The pressure gauge is manufactured by the Foxboro Company and the weir pen moves inward on the chart instead of outward as on the Bristol gauges. The weir is rectangular instead of triangular and therefore a different weir formula must be used in reducing the readings. Plate 12 gives the table showing the values of the chart readings and their reduction and Plate 13 is the rainfall-runoff graph for the rain of September 7, 1917 at this location.

In the rainfall-runoff graphs the rainfall intensity curve, being the differential of the accumulated rainfall curve, is plotted as a step curve; that is, the intensity is shown as constant for each minute instead of being continuously variable. This is done because the minute is the smallest time interval that can be read on the time scales of the instrumental records. Also, by plotting the rainfall intensity as a step curve, it is more easily distinguished from the runoff curve which is plotted as a continuous variable since the instrumental record gives the rate of runoff directly and differentiation is not necessary.

THEORETICAL ASPECTS OF RAINFALL-RUNOFF PROBLEM

The rainfall-runoff graphs afford an excellent picture of the relations of rainfall and runoff and it is these graphs that form the basis of the present study. Before attempting to analyze the information furnished by the graphs, it is well to consider the simplified conditions of a hypothetical rainfall upon a hypothetical ground surface. Such a study will help to fix in the mind the principal factors entering into the problem.

Assume a uniform rate of rainfall over a plane surface of uniform material and texture throughout. If the surface were perfectly horizontal and impervious and bounded by vertical walls, the depth of water accumulating on the plane in a unit of time, say inches per hour, would be equal to the rate of rainfall. If, however, the plane surface were pervious, the depth of water accumulating in a unit of time would be less than the rate of rainfall; the difference being the rate of absorption and evaporation. Consider, now, the case of a plane surface of pervious soil with a definite slope from the horizontal. If there has been no rainfall for a long time, the first drops of water to fall upon this surface will be greedily absorbed by the soil and, if the humidity is low, a considerable amount of the water will be evaporated. Before any of the water runs off from the place where it fell, it is necessary that a surface film of definite thickness shall have accumulated in order to overcome the force of surface tension which tends to hold the water in place. As the rain continues, the humidity of the atmosphere increases and evaporation ceases to be a major factor, but absorption will still continue although at a declining rate. The time required to build up a surface film of a thickness sufficient to insure runoff is, therefore, a function of the difference between the rate of rainfall and the rate of absorption and evaporation, all of which quantities may be expressed in the same units, say inches per hour. The time required to build up the critical thickness of surface film is an inverse function of the slope of the plane. If the rate of absorption be high and the rate of runoff low, it may require several minutes of rainfall before runoff begins. For instance a rainfall of one inch per hour would give a surface film only 1/60 of an inch in thickness in one minute even in the absence of absorption and evaporation. In the case of the hypothetical plane under consideration, the critical depth of surface film would be reached at the same time over all the area and runoff would everywhere begin simultaneously. In time the depth of water in transit, as well as its velocity, over any given elemental area, would increase with the distance of the elemental area below the starting point. It is probable that the absorption rate of the soil surface would vary with the depth of water overlying it, which would tend to balance, to some extent, the tendency of the absorption rate to decrease with the time.

In practice the area tributary to an inlet, instead of being a plane, has an irregular surface composed of various materials having wide ranges of slope and rate of absorption. There may be depressions of considerable area and depth which must be filled up before runoff from the entire contributing area becomes general. Then, too, the water on its way to the inlet, tends to collect in definite channels instead of flowing as a sheet, as in the case of the hypothetical plane. Except for the larger well defined channels which act as small storm water sewers, these channels do not follow the same course in every rain, nor even throughout the same rain, owing to accidental obstructions and changes in surface conditions. All of these considerations lead the writer to believe that it is futile to attempt to build up a rational formula for the runoff from inlet areas from experimental data collected in the study of small areas of uniform characteristics, although this data may be of value in other methods of attacking the problem. It seems that the most effective method of approach is to consider the entire inlet area as a unit and to use the records furnished by the instruments mentioned above to derive empirical formulas to fit the given conditions.

THE TIME FACTOR OR LAG

The first thing to attract the attention when examining a rainfall-runoff graph, such as those shown in Plates 7, 10 and 13, is that the shapes of the two curves are similar but that the runoff curve is always farther along the time scale than the rainfall curve. This time interval the writer designates as "Lag" in order to distinguish it from the similar terms "Inlet Time" and "Time of Concentration" which have often been used in discussions of rainfall and runoff and which have been defined as the time required for the rain falling upon the most distant portion of the inlet area to reach the inlet. The term "Lag", as used in this study, does not have this meaning but, of course, its value may be influenced to some extent by the "Inlet Time" as defined above. A study of the rains, so far plotted, shows that the lag varies between wide limits at each of the locations under observation. The variations appear to be quite erratic and the value of the lag seems to depend as much upon the characteristics of the rainfall curves as it does upon the physical characteristics of the inlet area; perhaps even more. In other words, the variation of the lag is as great or greater between different types of rains at the same location than it is between different locations with similar types of rains. The following hypothesis is an attempt to account for the erratic variation of the lag.

If we start with the inlet as a center and draw concentric circles, the radii of which increase in arithmetical progression, the zones between the circles, although of equal width, will increase in area as the distance from the inlet increases up to a certain point, depending upon the shape of the inlet area, after which the areas of the zones will tend to decrease with further increase of the distance. Assume that a rain is just beginning. The zone immediately surrounding the inlet begins to contribute a small amount of water almost at once. A short time later the next zone begins to contribute its water and so on throughout the series. If the rainfall continued indefinitely at a uniform rate, it would seem that the lag in this case would be the time for the water from the zone having the greatest area to reach the inlet, but it must be borne in mind that a greater percentage of the water falling on the distant areas is lost in transit due to absorption than in the case of the water from the adjacent areas because it is exposed to absorption for a longer time in its travel over the intervening ground. In the case of non-uniform rains, which includes practically all rains, the conditions are further complicated by the variations of the rate of absorption during the progress of a rain. Thus, a rain with a high intensity at the beginning, followed by other rainfall of lesser intensity, may result in the nearby or intermediate zones contributing most of the water entering the inlet at any given instant. If this same rain were turned end for end in time so that the period of high intensity followed the period of lesser intensity, the water from the more distant zones might predominate. In the first case, there would be a short lag and in the second, the lag would be longer. However, the problem of lag variation is not as simple as these examples would indicate. For instance, the water from a given period of high intensity during a rain does not all travel to the inlet at the same velocity. This water travels more or less in the form of a wave and this wave becomes longer and its crest flatter as it progresses to the inlet. Thus, the crest of a wave from a period of low intensity rainfall on a nearby zone may be higher than the crest of a wave due to a period of high intensity rainfall on a distant zone by the time each wave reaches the inlet. The rainfall-runoff graphs for some of the longer rains clearly indicate that the lag varies appreciably during the progress of the rain, but not always in the same way. The inability to predict the value of the lag from given conditions is due to the fact that its value depends upon a large number of factors, some of which are variables having contrary effects. Among these factors are: size, shape, slope and surface characteristics of the inlet area, the last mentioned often changing at least slightly during the course of a rain due to the filling of depressions, scouring of new channels, etc. Other factors are: the intensity and duration of rainfall, the distribution of the intensity of rainfall, throughout the rain and the amount of saturation resulting from previous rainfall occurring hours, days or even weeks before. Therefore, it appears that the erratic variation of the lag is the result of a delicate balance among the different factors determining its value. Although the lag cannot be accurately predicted from the rainfall curve alone, it can be measured if the corresponding runoff curve is available. If the rainfall curve has well defined peaks, the lag can be determined from the time interval between a peak on the rainfall curve and the corresponding peak on the runoff curve. When there are no prominent peaks, the mean lag for the entire rain may be found from the time interval between the center of mass of the rainfall curve and the center of mass of the runoff curve.

PERCENTAGE OF RUNOFF

Another striking feature of the rainfall-runoff graphs is that the peaks of the runoff curve are always considerably lower than the corresponding peaks of the rainfall curves. This effect has two distinct causes. First, the runoff is spread over a longer time than the rainfall causing it;therefore, the ordinates of the runoff curve must always be less than those of the rainfall curve, even if all the water falling as rain reaches the inlet as runoff which, of course, is not the case. Second, the area under the runoff curve is always less than the area under the rainfall curve since some of the water falling as rain is absorbed and evaporated, which statement introduces the topic "Percentage of Runoff". This phrase has often been used in discussions of rainfall and runoff, but there has been a diversity of opinion as to its meaning. It has been defined by various ratios, some of which are given below.

(A) The truly basic "Percentage of Runoff" may be defined by the ratio: $\frac{I-(a+e)}{I}$ where "I" is the instantaneous value of the rate or intensity of the rainfall on a small unit area and "a" and "e" are the instantaneous values of the rates of absorption and evaporation respectively for the same area. For example, if rain is falling on the area at the rate of "I" inches per hour, and at the same instant the soil is absorbing the water at the rate of "a" inches per hour, and evaporation is occurring at the rate of "e" inches per hour, the instantaneous value of the rate of runoff from the unit area must be $I-(a+e)$ inches per hour and the instantaneous value of the percentage of runoff is $100 \times \frac{I-(a+e)}{I}$

(B) The "Percentage of Runoff" has been defined as the ratio of the height of the runoff peaks to the height of the rainfall peaks corresponding to them, or it is the direct ratio of the rate of runoff to the rate of rainfall. This ratio includes the effect due to lag as well as the basic percentage given by (A).

(C) Sometimes the ratio $\frac{\Sigma Q \Delta t}{\Sigma I \Delta t}$ has been used to give the value of "Percentage of Runoff", "Q" being the instantaneous value of the rate of runoff and "I" being the instantaneous value of the rate of rainfall, Δt being a small unit of time. This value is the ratio of the area of the runoff curve for a given time to the area of the rainfall curve for the same time, or it is the ratio of the volume of water running off from a given area in a given time to the volume of water falling on the same area during the same time.

(D) The term "Percentage of Runoff", as used in the method of analysis developed in this study, is defined as the percentage of the volume of water falling upon the entire inlet area for a small unit of time Δt which ultimately reaches the inlet. This definition includes the basic percentage as given by (A) and in addition takes into account the losses due to absorption and evaporation in transit as well as the thin film of water left on the area at the end of the rain and also the water retained by depressions, all of which, of course, is ultimately either absorbed or evaporated. It does not include the effect due to lag which is covered by the factor "K" which will be discussed later.

DEVELOPMENT OF THE METHOD OF ANALYSIS

Plate 14 gives in condensed form the derivation of the working formulas used in the study of the rainfall-runoff graphs. Referring to this plate and using the notation given thereon, a more detailed description of the Unit Graph and its applications will now be given.

UNIT GRAPH FOR RAINFALL

The rectangle $OI\Delta t$ represents the volume of water falling upon a unit area, one acre, during a rainfall of constant intensity, "I" and continuing for a unit time, Δt or one minute. Since "I" represents cubic feet per second per acre, the actual volume in this case is 60 X I cubic feet, but as we are using one minute as the unit of time, we may say say 60 cubic feet is the corresponding unit volume. It will then be correct to say that the rectangle $OI\Delta t$ is the unit graph for a rainfall intensity of "I", and since its area is $I \times \Delta t$ or rate times unit time, its area is equal to "I". The unshaded portion of this rectangle, between "I" and "I'",

NOTATION

I = Rate or intensity of Rainfall. Inches per hour or CFS per Acre.

P = Percentage of water falling upon entire inlet area during unit time (Δt) which ultimately reaches the inlet as runoff.

Δt = Unit of time, in this study, one minute.

I' = The quantity or volume of water (CFS) falling upon unit area (Acre) of inlet area which ultimately reaches inlet = $P \times I$.

Q = Rate of Runoff or quantity of water (CFS per Acre) reaching the inlet at any given time (t).

y & y' = Respectively the ascending and descending instantaneous values of (Q) resulting from rainfall of unit duration on unit area.

K = Maximum value of (y) and (y').

t = Time measured from beginning of rainfall.

L = Lag or the value of (t) when (y & y') are maximum.

n & c = Arbitrary constants.

Equation of runoff curve, $y = K\left(\frac{t}{L}\right)^n$, $y' = \frac{K}{C^{(t-L)}}$

The area under the runoff curve is equal to :-

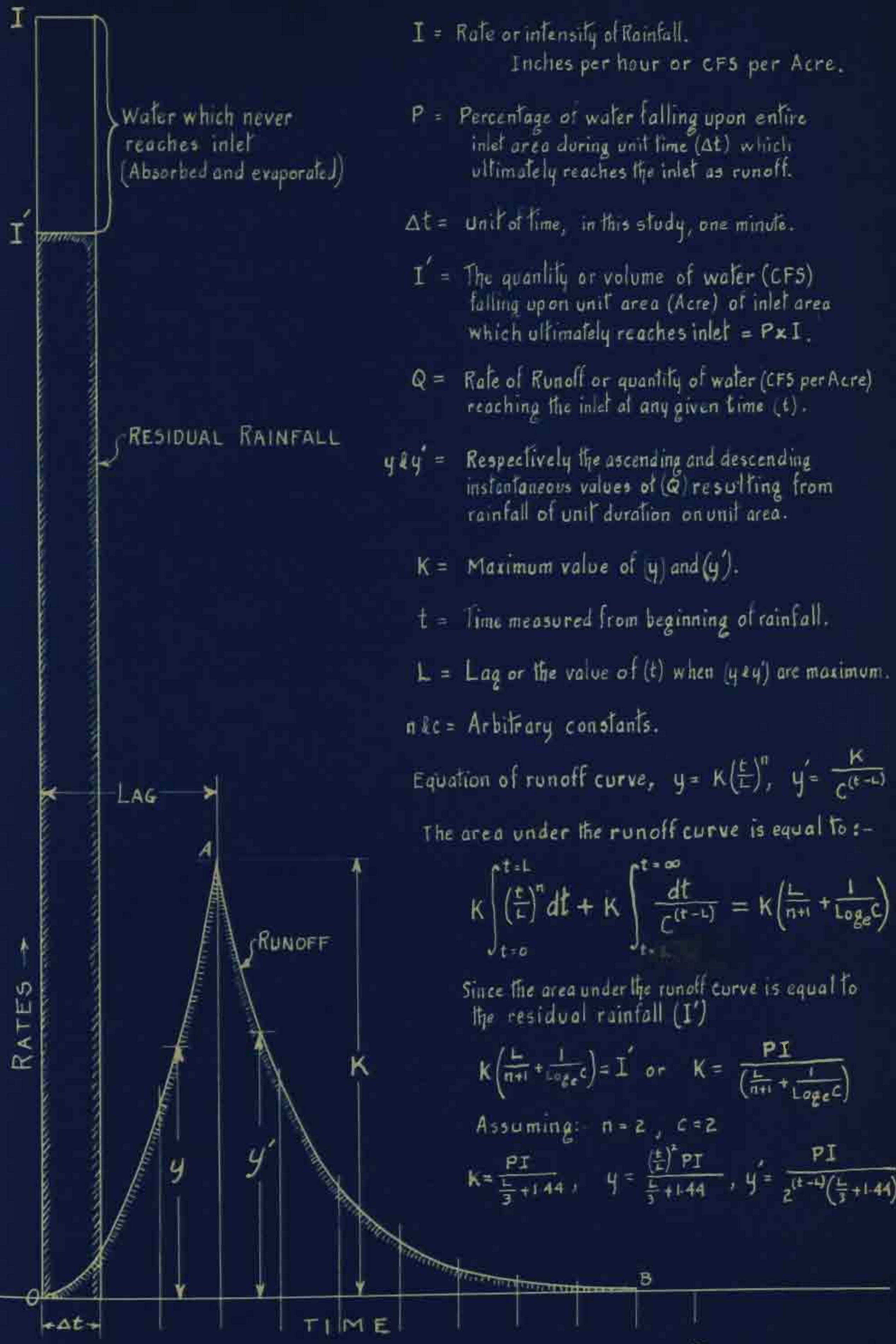
$$K\int_{t=0}^{t=L}\left(\frac{t}{L}\right)^n dt + K\int_{t=L}^{t=\infty}\frac{dt}{C^{(t-L)}} = K\left(\frac{L}{n+1} + \frac{1}{\operatorname{Log}_e C}\right)$$

Since the area under the runoff curve is equal to the residual rainfall (I')

$$K\left(\frac{L}{n+1} + \frac{1}{\operatorname{Log}_e C}\right) = I' \quad \text{or} \quad K = \frac{PI}{\left(\frac{L}{n+1} + \frac{1}{\operatorname{Log}_e C}\right)}$$

Assuming:- $n = 2$, $c = 2$

$$K = \frac{PI}{\frac{L}{3} + 1.44}, \quad y = \frac{\left(\frac{t}{L}\right)^2 PI}{\frac{L}{3} + 1.44}, \quad y' = \frac{PI}{2^{(t-L)}\left(\frac{L}{3} + 1.44\right)}$$



UNIT GRAPH REPRESENTING THEORETICAL RUNOFF RESULTING FROM RAINFALL ON UNIT AREA FOR UNIT TIME

PLATE 14

represents that portion of the water falling upon the unit area during the time Δt which never reaches the inlet, being absorbed or evaporated; but not necessarily absorbed or evaporated during the minute in which it fell.

The rectangle $OI'\Delta t$ represents the residual rainfall, or the volume of water falling during the minute in question which is available for runoff. As in the case of the total rainfall graph the area of the rectangle may be expressed by the rate, I' or since $P = \frac{I'}{I}$, it may be expressed as P X I.

UNIT RUNOFF GRAPH

The curve OAB is the unit runoff graph corresponding to the unit rainfall graph just described. It shows how the rate of runoff varies with the time, whereas the rainfall producing this runoff has a constant rate. The equations for the two branches of the unit runoff graph are purely empirical and were chosen because they seemed to best fit the conditions.

The equation for the branch OA is $y = K\left(\frac{t}{L}\right)^n$ (1)

The equation for the branch AB is $y' = \frac{K}{C^{(t-L)}}$ (2)

The equation for OA indicates that the rate of runoff increases as the n-th power of the time up to a maximum value K , which occurs when $t=L$. The equation for AB indicates that the rate of runoff decreases from the maximum value, K, along a curve which is asymptotic to the time axis. In practice, of course, this condition is not realized, but there is evidence that the runoff does continue for a comparatively long time after the rainfall ceases. This particular form of equation was chosen because the values of y' can be more easily computed from it than from the equation of a curve tangent to the time axis and at the same time it agrees very well with observed values of runoff rates.

The area under the curve OAB is, of course, equal to that of the rectangle $OI'\Delta t$. The area under the branch OA is given by the integral $\int_{t=0}^{t=L} y\,dt$ or, since $y = K\left(\frac{t}{L}\right)^n$, the integral may be written $K\int_{t=0}^{t=L}\left(\frac{t}{L}\right)^n dt$

The area under the branch AB is given by the integral $\int_{t=L}^{t=\infty} y'\,dt$ or since $y' = \frac{K}{C^{(t-L)}}$ the integral may be written $K\int_{t=L}^{t=\infty}\frac{dt}{C^{(t-L)}}$

The total area under the runoff curve equals $K\int_{t=0}^{t=L}\left(\frac{t}{L}\right)^n dt + K\int_{t=L}^{t=\infty}\frac{dt}{C^{(t-L)}}$

Integrating, we find the total area to be $K\left(\frac{L}{n+1}+\frac{1}{\log_e c}\right)$

Since the area of the runoff curve OAB is equal to that of the residual rainfall curve, $oI'\Delta t$, the area of which is $I'=PI$, then $K\left(\frac{L}{n+1}+\frac{1}{\log_e c}\right) = PI$.

From which $K= \dfrac{PI}{\frac{L}{n+1}+\frac{1}{\log_e c}}$ -------------------------(3)

Assuming that $n=2$ and $c=2$ we have $K = \dfrac{PI}{\frac{L}{3}+1.44}$ (4)

Substituting this value of K in equations (1) and (2)

$$y = \frac{\left(\frac{t}{L}\right)^2 PI}{\frac{L}{3}+1.44} \text{ -------------------------(5)}$$

$$y' = \frac{PI}{2^{(t-L)}\left(\frac{L}{3}+1.44\right)} \text{ -------------------------(6)}$$

USE OF UNIT GRAPH

Equations (5) and (6) show that for a given minute of rainfall we can compute the rate of runoff resulting from this rainfall for any subsequent minute provided the percentage of runoff "P", and the lag "L" is known.

Plate 15 shows a table giving the values for "K", "Y" and "Y' " which have been computed from equations (4), (5) and (6) for various values of "t" and for various lags but for unit value of I'. This plate also shows these values graphically in the form of unit graphs for various values of "L" and for unit value of I'. From the equations it will be seen that the values of "K", "Y" and "Y' " for any given value of I' may be found by multiplying the tabular values by the given value of I'.

As an illustration of the use of this table refer to Plate 16 for the case of a simple hypothetical rain of five minutes duration and with a lag of three minutes. Assume that, during the first minute, the rate of rainfall was two inches per hour; for the second minute, four inches per hour; for the third minute, six inches per hour; for the fourth minute, three inches per hour and for the fifth minute, one inch per hour. For the sake of simplicity, assume 100% runoff, or that all of the water reaches the inlet. Then P=100 and I'=I. Let the five vertical columns headed by the numbers from 1 to 5 in the table represent the five minutes of rainfall and the numbered horizontal lines represent the minutes of runoff. Using the table on Plate 15 ,find the values of y and y'

TABLE OF VALUES OF "K", "y" and y' FOR UNIT VALUE OF "I'"

t	LAG IN MINUTES											
	1	2	3	4	5	6	7	8	9	10	11	12
1	K 0.562	0.119	0.046	0.023	0.013	0.008	0.005	0.004	0.003	0.002	0.002	0.001
2	0.281	K 0.474	0.182	0.091	0.052	0.032	0.022	0.015	0.011	0.008	0.006	0.005
3	0.141	0.237	K 0.410	0.204	0.116	0.073	0.049	0.034	0.025	0.018	0.015	0.012
4	0.070	0.119	0.205	K 0.362	0.206	0.129	0.086	0.061	0.044	0.032	0.026	0.020
5	0.035	0.059	0.103	0.181	K 0.322	0.202	0.135	0.095	0.069	0.050	0.041	0.032
6	0.018	0.030	0.051	0.091	0.161	K 0.291	0.195	0.136	0.100	0.072	0.058	0.046
7	0.009	0.015	0.026	0.045	0.081	0.146	K 0.265	0.186	0.136	0.098	0.079	0.063
8	0.004	0.007	0.013	0.023	0.040	0.072	0.133	K 0.243	0.178	0.128	0.104	0.082
9	0.002	0.004	0.006	0.011	0.020	0.036	0.067	0.122	K 0.225	0.162	0.131	0.103
10	0.001	0.002	0.003	0.006	0.010	0.018	0.033	0.061	0.113	K 0.210	0.162	0.128
11	—	0.001	0.002	0.003	0.005	0.009	0.017	0.030	0.056	0.105	K 0.196	0.154
12		—	0.001	0.001	0.003	0.005	0.008	0.015	0.028	0.053	0.098	K 0.184
13			—	0.001	0.001	0.002	0.004	0.008	0.014	0.026	0.049	0.092
14				—	0.001	0.001	0.002	0.004	0.007	0.013	0.025	0.046
15					—	0.001	0.001	0.002	0.004	0.007	0.012	0.023
16						—	—	0.001	0.002	0.004	0.006	0.012
17								—	0.001	0.002	0.003	0.006
18									—	0.001	0.002	0.003
19										—	0.001	0.002
20											—	0.001

$$K = \frac{I'}{\frac{L}{3} + 1.44} \qquad y = K\left(\frac{t}{L}\right)^2 \qquad y' = \frac{K}{2^{(t-L)}}$$

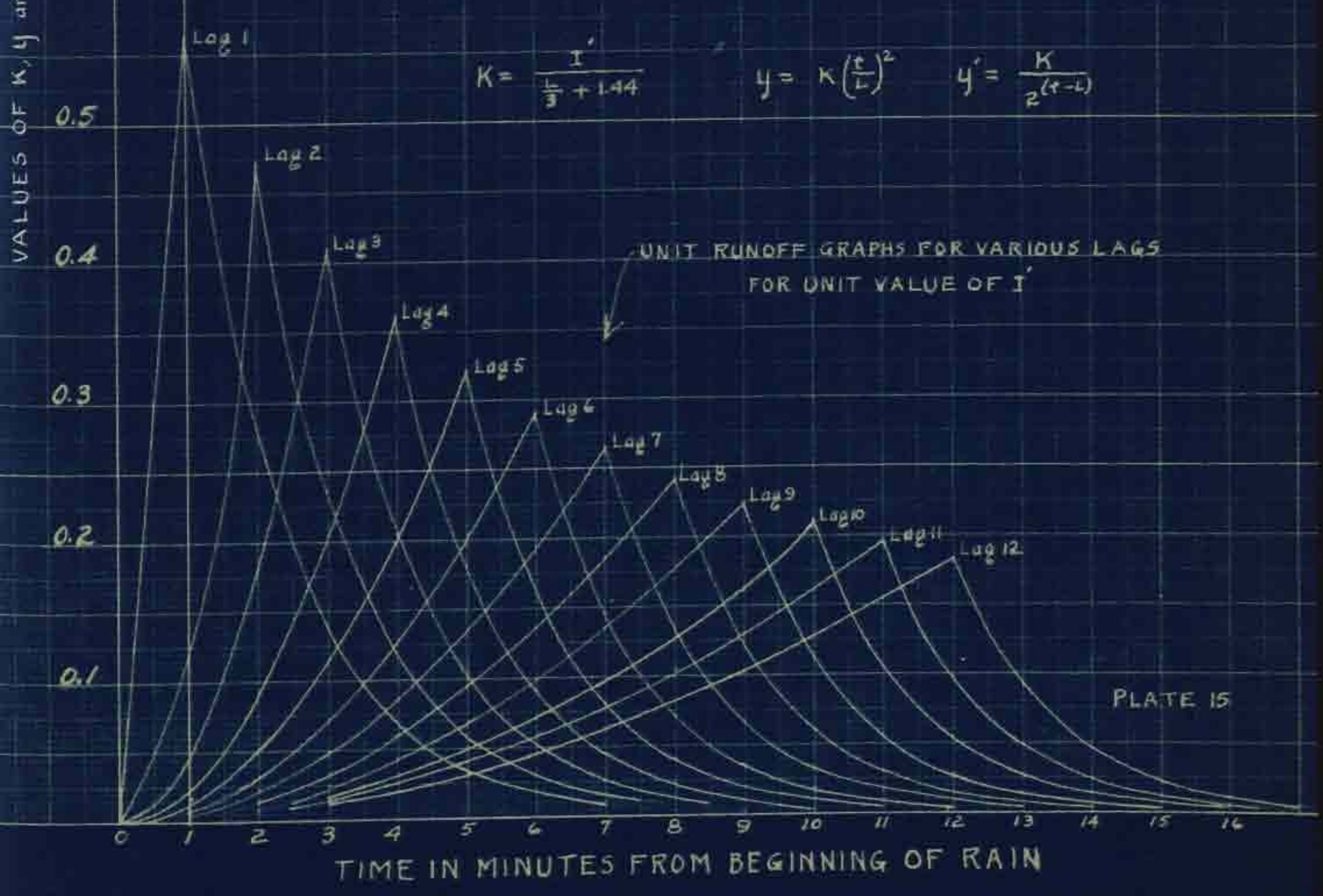
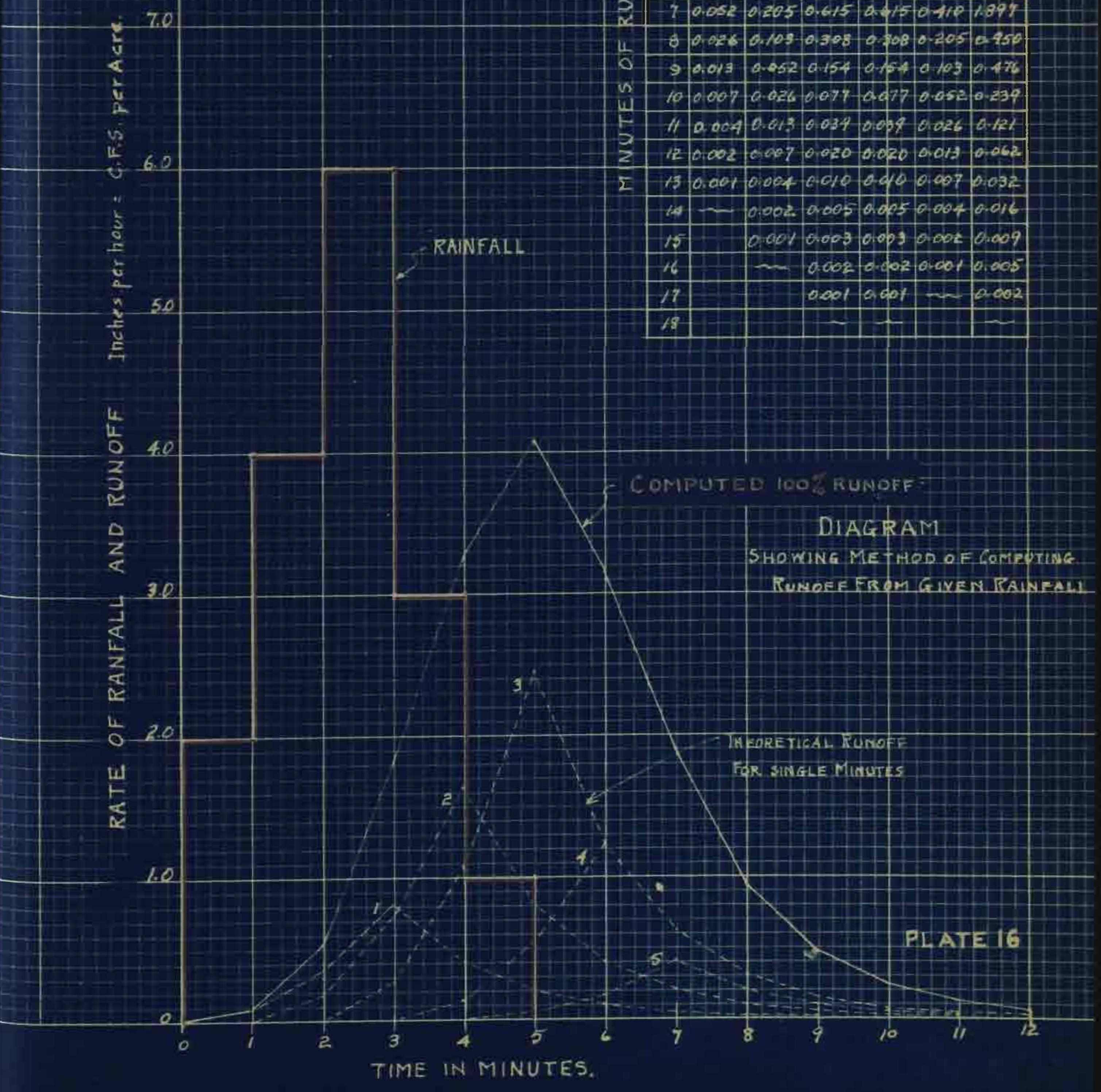


PLATE 15

MINUTES OF RAINFALL

t →	1	2	3	4	5	
I	2	4	6	3	1	Q
t ↓						
1	0.092					0.092
2	0.364	0.184				0.548
3	0.820	0.728	0.276			1.824
4	0.410	1.640	1.092	0.138		3.280
5	0.205	0.820	2.460	0.546	0.046	4.077
6	0.103	0.410	1.230	1.230	0.182	3.155
7	0.052	0.205	0.615	0.615	0.410	1.897
8	0.026	0.103	0.308	0.308	0.205	0.950
9	0.013	0.052	0.154	0.154	0.103	0.476
10	0.007	0.026	0.077	0.077	0.052	0.239
11	0.004	0.013	0.034	0.039	0.026	0.121
12	0.002	0.007	0.020	0.020	0.013	0.062
13	0.001	0.004	0.010	0.010	0.007	0.032
14	—	0.002	0.005	0.005	0.004	0.016
15		0.001	0.003	0.003	0.002	0.009
16		—	0.002	0.002	0.001	0.005
17			0.001	0.001	—	0.002
18			—	—		—

MINUTES OF RUNOFF



DIAGRAM

SHOWING METHOD OF COMPUTING RUNOFF FROM GIVEN RAINFALL

PLATE 16

by multiplying the values given under lag=3 by the rainfall rate for each minute, and record these values in the table in the vertical columns corresponding to that minute, bearing in mind that the runoff for each minute begins with that minute. The values of y and y' shown in any one of the vertical columns of the table when plotted as shown in dotted lines numbered 1, 2, 3, etc. represent the variation in the rate of runoff due to the rainfall in that minute considered as isolated from the rest of the rain. When the values of y and y' as shown in the table are added horizontally, the corresponding ordinates of these individual runoff graphs are summed up and the totals as shown in the column headed, Q, represent the rates of runoff, which, plotted as the line marked "Computed 100% Runoff" in the graph, represents the theoretical runoff resulting from the rainfall as given, assuming that all of the water falling on the area ultimately reaches the inlet.

THE 100 PERCENT RUNOFF GRAPH

The 100 Percent Runoff Graph is being used in the present study as a means of ascertaining the percentage of runoff, one of the two essential factors of the rainfall-runoff problem concerning which information is being sought, the other factor being the lag. Following is an outline of the method of using the "100 Percent Runoff Graph" to find the percentage of runoff.

Probably the best way to illustrate this method is to apply it to an actual rain. For example, the rain of September 7, 1917, as recorded by the instruments at Ewing and Washington, will be taken. The graph of the recorded rainfall and runoff at this location has been shown in Plate 13 . As stated on Page 19 there are two methods of determining the lag when both the rainfall and runoff are given. From an inspection of Plate 13 , it is evident that the lag, indicated by the time interval between the main peak of the rainfall curve and the main peak of the runoff curve, is about nine minutes. As a check on this method it is seen from the table on Plate 12 that the centers of mass of the two curves as found from a summation of the "I" and "Q" columns are between eight and nine minutes apart. Therefore, nine minutes is assumed as the correct lag for this particular storm and location. Plate 17 shows a "100 Percent Runoff" table made up for the rain in question according to the method used for the hypothetical rain shown on Plate 16 . On Plate 19 is shown the "100 Percent Runoff" curve obtained by

100 PER CENT RUNOFF TABLE
EWING AND WASHINGTON
RAIN OF SEPTEMBER 7 1917

t	8:28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	Q
100xI	30	30	240	240	300	240	120	60	240	480	360	600	480	360	360	240	180	120	120	60	
8:28	1																				0
29	3	1																			0
30	7	3	6																		0.02
31	13	7	26	6																	0.05
32	21	13	59	26	9																0.13
33	30	21	105	59	32	6															0.25
34	41	30	164	105	74	26	2														0.44
35	53	41	236	164	130	59	13	2													0.70
36	67	53	322	236	205	105	29	6	6												1.03
37	33	67	420	322	296	164	52	15	26	12											1.41
38	16	33	530	420	403	236	82	26	53	56	10										1.87
39	8	16	265	530	525	322	118	41	105	118	39	18									2.11
40	4	8	132	265	665	420	161	59	164	210	89	64	12								2.25
41	2	4	66	132	332	530	210	81	236	328	156	148	56	10							2.29
42	1	2	33	66	166	265	265	105	322	472	246	261	118	39	10						2.37
43	0	1	16	33	83	132	132	133	420	644	355	410	210	89	37	6					2.70
44		0	8	16	42	66	66	66	530	840	482	591	328	156	89	26	4				3.31
45			4	8	21	33	33	33	265	1060	630	805	472	246	156	59	19	2			3.85
46			2	4	10	16	16	16	132	530	799	1050	644	355	246	105	44	13	2		3.98
47			1	2	5	8	8	8	66	265	400	1330	840	482	355	164	78	29	13	2	4.06
48			0	1	3	4	4	4	33	132	200	665	1060	630	462	236	123	52	29	6	3.66
49				0	2	2	2	2	16	66	100	332	530	799	630	322	178	82	52	15	3.13
50					1	1	1	1	8	33	50	166	265	400	799	420	241	118	82	26	2.61
51					0	0	0	0	4	16	25	83	132	200	400	530	315	161	118	41	2.03
52									2	8	12	42	66	100	200	265	398	210	161	59	1.52
53									1	4	6	21	33	50	100	132	199	265	210	81	1.10
54									0	2	3	10	16	25	50	66	100	132	265	105	0.77
55										1	2	5	8	12	25	33	50	66	132	133	0.47
56										0	1	3	4	6	12	16	25	33	66	66	0.23
57											0	2	2	3	6	8	12	16	33	33	0.12
58												1	1	2	3	4	6	8	16	16	0.06
59												0	0	1	2	2	3	4	8	8	0.03
9:00														0	1	1	2	2	4	4	0.01
1															0	0	1	1	2	2	0
2																	0	0	1	1	0
3																			0	0	0

NOTE:-

To avoid decimal fractions, the values of the runoff ordinates in the table shown above are 1000 times the actual values. The horizontal summations must therefore be divided by 1000 in order to obtain the correct value of "Q".

PLATE 17

plotting the values of Q shown on Plate 17 on a copy of Plate 13 PLATE 19 showing the recorded rainfall and runoff for the September 7, 1917 storm at Ewing and Washington. It will be seen that the "100 Percent Runoff" curve is very similar in shape to the measured runoff curve, but, of course, the ordinates of the former are higher than those of the latter throughout most of the graph. It is found that the height of the main peak of the measured runoff curve is a little more than 74 percent of the height of the computed runoff curve. Furthermore, referring to the table on Plate 12 we find that the ratio of the area under the measured runoff curve to the area of the measured rainfall curve, as found by the summation of the "I" and "Q" columns of the table, amounts to about 0.74. Therefore, it may reasonably be assumed that for this particular rain at this particular location, the value of the lag is about nine minutes and that about 74 percent of the water falling as rain reached the sewer as runoff. As an additional check, a new table was made up, as shown in Plate 18 , using these values of L and P in equations (5) and (6), Page 25

and using a tentative approximate formula for determining the variation of percentage of runoff through-

RUNOFF TABLE (VARIABLE PERCENTAGE)

EWING AND WASHINGTON

SEPTEMBER 7, 1917

t	8:28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	Q
mod I	30	30	240	240	300	240	120	60	240	480	360	600	480	360	360	240	180	120	120	60	
P	29	35	40	44	47	50	53	56	58	59	62	63	65	67	68	70	71	73	74	75	
mod I'	5	11	96	106	141	120	64	34	139	283	222	378	312	241	244	168	128	86	89	45	
t																					0.00
8:28	0																				0.00
29	1	0																			0.00
30	2	1	3																		0.01
31	4	3	11	3																	0.02
32	6	5	24	12	4																0.05
33	8	8	42	27	15	4															0.10
34	11	11	66	47	35	13	2														0.19
35	14	15	96	73	62	30	7	1													0.30
36	18	20	131	106	97	53	16	4	4												0.45
37	9	25	171	144	141	83	28	9	15	9											0.63
38	5	13	216	188	192	120	44	15	35	31	7										0.87
39	3	7	108	258	251	163	64	24	61	71	25	11									1.03
40	2	4	54	119	318	214	87	34	96	125	56	42	9								1.16
41	1	2	27	60	159	270	119	46	139	186	98	95	34	7							1.25
42	0	1	14	30	80	135	144	61	189	283	154	166	78	27	7						1.37
43		0	7	15	40	68	72	77	247	386	222	260	137	60	27	5					1.62
44			4	8	20	34	36	39	312	532	303	378	215	106	61	18	4				2.07
45			2	4	10	17	18	20	156	640	395	512	312	166	108	42	14	3			2.42
46			1	2	5	9	9	10	78	320	500	670	425	241	168	74	32	10	3		2.56
47			0	1	3	5	5	5	39	160	250	850	555	328	244	116	56	22	10	1	2.65
48				0	2	3	3	3	20	80	125	425	700	430	342	168	88	39	22	5	2.46
49					1	2	2	2	10	40	63	213	350	542	435	228	128	61	39	11	2.13
50					0	1	1	1	5	20	32	107	175	271	550	300	174	88	61	20	1.81
51						0	0	0	3	10	16	54	88	136	275	380	228	120	89	31	1.43
52									2	5	8	27	44	68	138	190	288	156	121	45	1.09
53									1	3	4	14	22	34	69	95	144	198	158	61	0.80
54									0	2	2	7	11	17	35	48	72	99	200	80	0.57
55										1	1	4	6	9	18	24	36	50	100	101	0.35
56										0	0	2	3	5	9	12	18	25	50	51	0.18
57												1	2	3	5	6	9	12	25	26	0.09
58												0	1	2	3	3	5	6	12	13	0.05
59													0	1	2	2	3	3	6	7	0.02
9:00														0	1	1	2	2	3	4	0.01
1															0	0	1	1	2	2	0.01
2																	0	0	1	1	0
3																			0	0	0
4																					

See note on Plate 17

PLATE 18

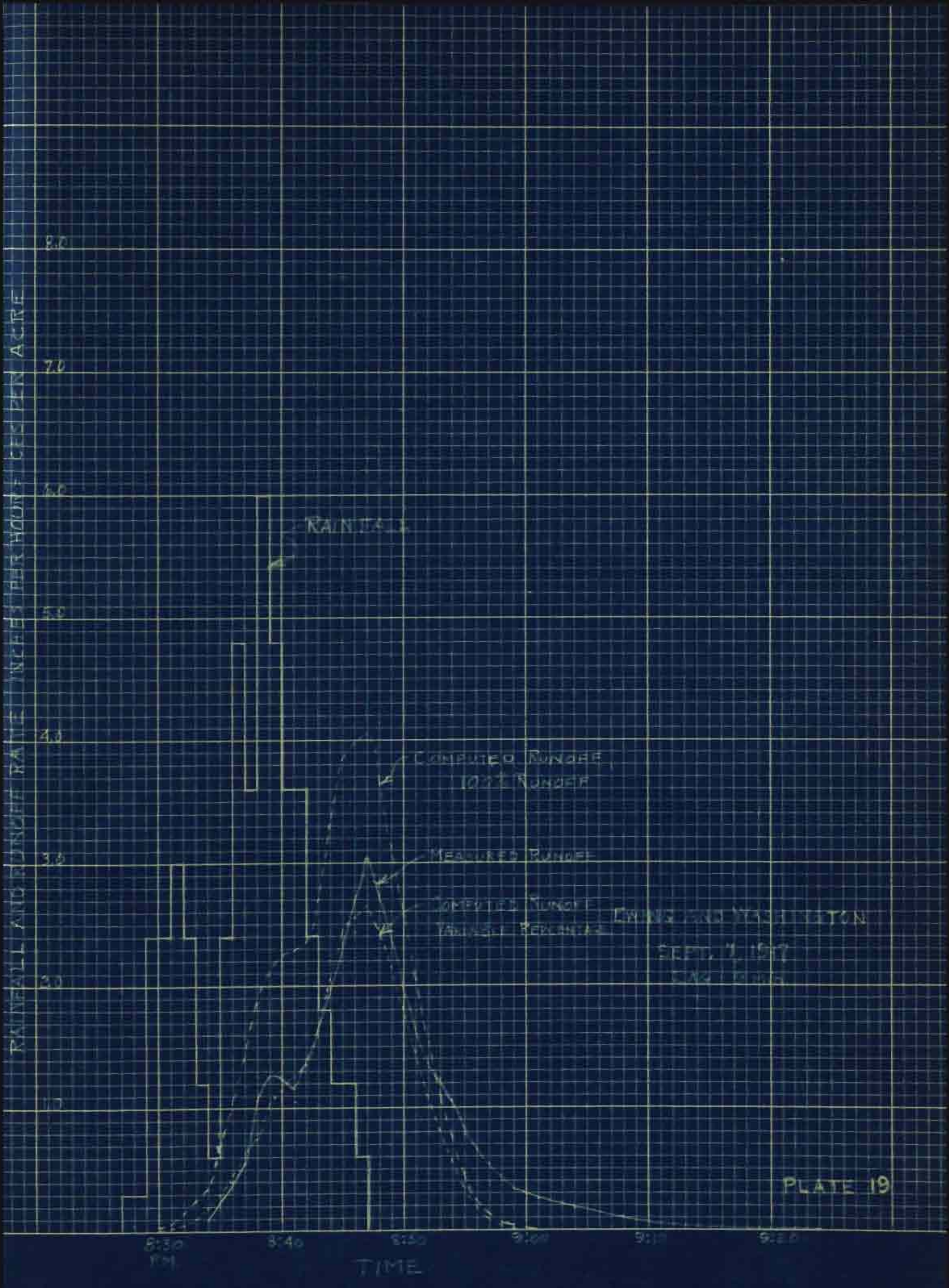

RAINFALL AND RUNOFF RATE INCHES PER HOUR; CFS PER ACRE
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
RAINFALL
COMPUTED RUNOFF
100% RUNOFF
MEASURED RUNOFF
COMPUTED RUNOFF
VARIABLE PERCENTAGE
OWINGS AND WASHINGTON
SEPT. 7, 1917
[illegible]
8:30
P.M.
8:40
8:50
9:00
9:10
9:20
TIME

PLATE 19

out the rain, a new runoff curve was plotted from the table in Plate 18 . It is seen that this computed runoff curve approximates very closely the measured runoff curve and the differences between the two curves may be rationally explained by painting out defects in the formulas used in computing the last mentioned runoff curve. In the first place, the choice of 2 as the value of the two constants n and c in the basic formulas for y and y' was purely arbitrary,and there is evidence that the value of n should be slightly greater than 2 and that the value of c should be slightly less than 2, but the work done thus far does not justify the assigning of more definite values to these constants at the present time. In the second place, the percentages used in table on Plate 18 to find the value of I' for each minute of the rain were determined from an approximate formula which assumes that the percentage of runoff varies as the cube root of the time from zero at the beginning of the rain to the final percentage at the end of the rain. The final percentage is difficult to obtain with accuracy, so, in this case the ratio of the total runoff to the total rainfall was used as the final percentage. The true value of the final percentage is probably somewhat higher than the value used. For simple rains of fairly uniform intensity this tentative formula, expressed as $P = 80\left(\frac{t}{D}\right)^{1/3}$ where D is the total duration of the rain and 80 is the final value of P, gives fairly good results, but it is quite evident that the percentage varies with the intensity of the rainfall as well as with the time. As yet no satisfactory formula has been devised to show this relationship.

DISCUSSION OF A FEW TYPICAL RAINS

Plate 20 shows the rainfall-runoff graphs for four typical rains at three locations. The first rain shown, June 27, 1921, is a typical short, sharp shower. There is some doubt as to the accuracy of the rainfall curve at Belt and Ridge since the rates of rainfall for these short rains of high intensity are difficult to measure, and any error in the rate for one minute would seriously affect the area of the rainfall curve, that is, the percentage of error would be greater than in the case of a long rain. The general direction of the storm is easily seen from the time when it appears at each of the three locations; 10:19 A.M. at Belt and Ridge, 10:23 at Cates and Clarendon, and 10:30 A.M. at Ewing and Washington. The second rain shown, August 2,1916, is also a short rain of high intensity, with a slightly longer duration than the rain of June 27, 1921. The intensity of this rainfall reaches high values at Belt and Ridge and at Cates and Clarendon, but these high values are always of short duration, the two consecutive minutes with an intensity of 12 inches per hour being quite unusual. The lower percentages of runoff for this rain as compared to that of June 27, 1921 is mainly due to the fact that the rain of June 27, 1921 was preceded by two small rains, one on June 26, 1921 and the other on June 25, 1921, whereas the nearest preceding rain to August 2, 1916 occurred on July 25, 1916 and this rainfall was very slight. Therefore, the rate of absorption was, no doubt, higher on August 2, 1916 than it was on June 27, 1921. In contrast to the high intensities at Belt and Ridge, and Cates and Clarendon, the rain at Ewing and Washington was insignificant. This type of storm usually covers a small area and in its travel, it no doubt passed directly over the first two locations and passed to one side of Ewing and Washington.

The two rains of April 25, 1921 and August 27, 1921 offer an interesting contrast. The April rain, in common with the majority of all rains, has its period of

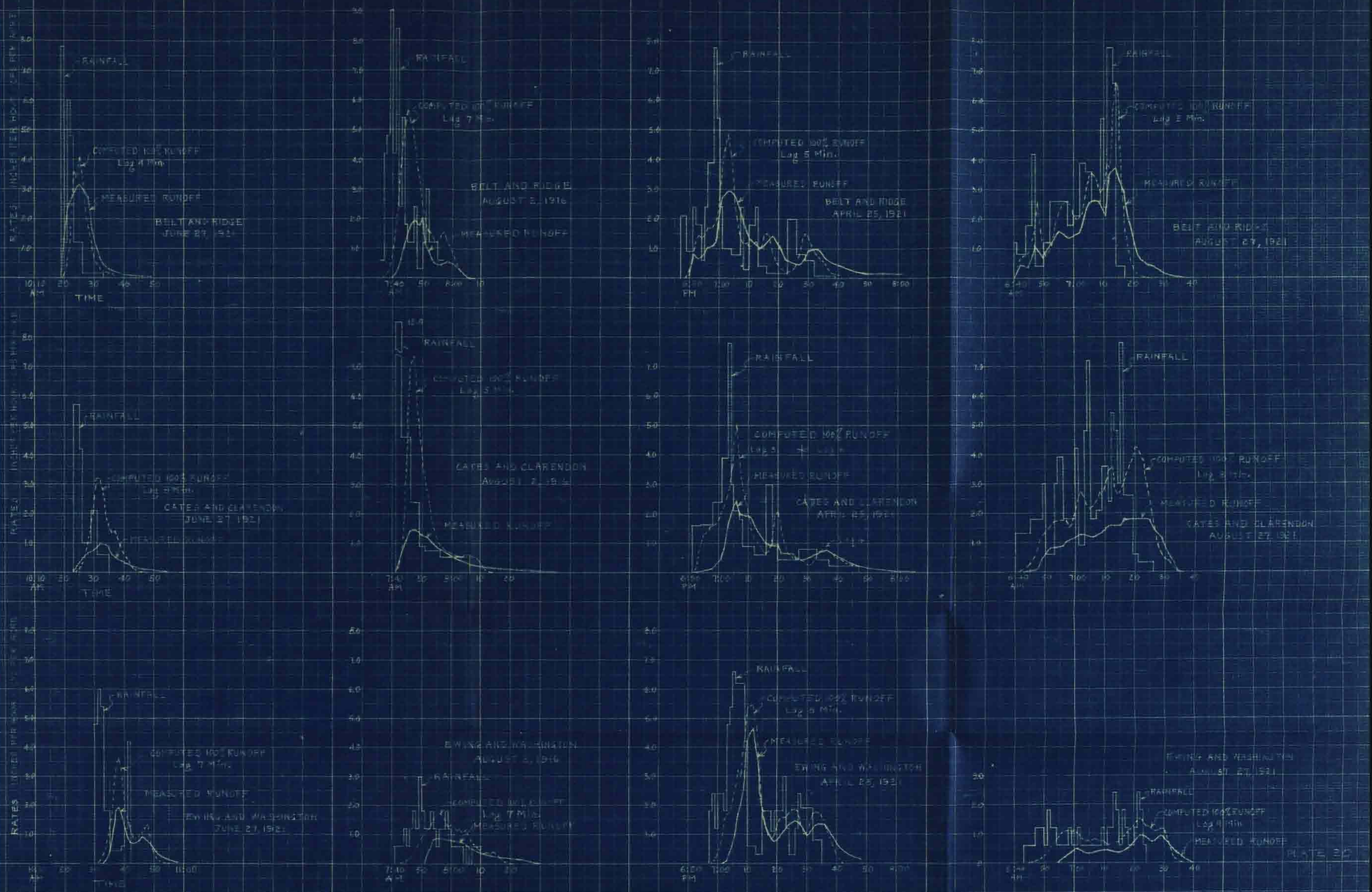

RATES
RAINFALL
COMPUTED 100% RUNOFF
Lag 4 Min.
MEASURED RUNOFF
BELT AND RIDGE
JUNE 27, 1921
10:10 AM
20
30
40
50
TIME
RAINFALL
COMPUTED 100% RUNOFF
Lag 7 Min.
BELT AND RIDGE
AUGUST 2, 1916
MEASURED RUNOFF
7:40 AM
50
8:00
10
RAINFALL
COMPUTED 100% RUNOFF
Lag 5 Min.
MEASURED RUNOFF
BELT AND RIDGE
APRIL 25, 1921
6:50 PM
7:00
10
20
30
40
50
8:00
RAINFALL
COMPUTED 100% RUNOFF
Lag 3 Min.
MEASURED RUNOFF
BELT AND RIDGE
AUGUST 27, 1921
6:40 AM
50
7:00
10
20
30
40
RATES
RAINFALL
COMPUTED 100% RUNOFF
Lag 8 Min.
CATES AND CLARENDON
JUNE 27, 1921
MEASURED RUNOFF
10:10 AM
20
30
40
50
TIME
10.4
RAINFALL
COMPUTED 100% RUNOFF
Lag 5 Min.
CATES AND CLARENDON
AUGUST 2, 1916
MEASURED RUNOFF
7:40 AM
50
8:00
10
20
RAINFALL
COMPUTED 100% RUNOFF
Lag 5
MEASURED RUNOFF
CATES AND CLARENDON
APRIL 25, 1921
6:50 PM
7:00
10
20
30
40
50
8:00
RAINFALL
COMPUTED 100% RUNOFF
Lag 8 Min.
MEASURED RUNOFF
CATES AND CLARENDON
AUGUST 27, 1921
6:40 AM
50
7:00
10
20
30
40
RATES
RAINFALL
COMPUTED 100% RUNOFF
Lag 7 Min.
MEASURED RUNOFF
EWING AND WASHINGTON
JUNE 27, 1921
10:40 AM
50
11:00
10
20
30
40
50
TIME
EWING AND WASHINGTON
AUGUST 2, 1916
RAINFALL
COMPUTED 100% RUNOFF
Lag 7 Min.
MEASURED RUNOFF
7:40 AM
50
8:00
10
20
RAINFALL
COMPUTED 100% RUNOFF
Lag 5 Min.
MEASURED RUNOFF
EWING AND WASHINGTON
APRIL 25, 1921
6:50 PM
7:00
10
20
30
40
50
8:00
EWING AND WASHINGTON
AUGUST 27, 1921
RAINFALL
COMPUTED 100% RUNOFF
Lag 8 Min.
MEASURED RUNOFF
6:40 AM
50
7:00
10
20
30
40

PLATE 30

highest intensity near the beginning, whereas the August rain has its period of highest intensity at the end. The latter type of rain is comparatively rare and nearly always occurs in the late summer, August or September. It is this type of rain which gives the highest rates of runoff at each of the locations under observation, which result might be expected when it is remembered that the percentage of runoff varies with the time. A comparison of the runoff curves for this rain at the two locations, Belt and Ridge, and Cates and Clarendon, calls attention to the fact that the latter location reacts more sluggishly to the variations in rainfall intensity than does the former location. This effect is due to the different shapes of the tributary areas. The block at Belt and Ridge is nearly square with a steep slope and the water moves over the area in the same general direction to the inlet. At Cates and Clarendon the area is very long and very flat. The alley paving acts as a well defined channel and the adjacent lots drain at right angles to the alley which in effect becomes a well defined water course. The period from 7:16 A.M. to 7:25 A.M. is of particular interest since the runoff does not follow the rainfall peak upward as it does at Belt and Ridge, but continues at a uniform level for nine minutes. This may be attributed to the large difference in lag at the two locations and to the fact that the flow in the alley channel is of such depth by the time that the last rainfall peak occurs it has little effect on the steady flow which persists for some time after the rain has ceased. The small precipitation on this date at Ewing and Washington again illustrates the limited extent of these high intensity storms.

CONCLUSIONS

As stated in the introduction to this paper, no claim of finality is made for the details of the method of rainfall-runoff analysis developed in the foregoing pages, in fact, it is doubtful if anyone will ever be able to say the last word on the subject of the relation of runoff to rainfall. It is a problem which, in the last analysis, will always be vitally affected by certain factors peculiar to the location under investigation.

It has been the aim of the writer to make the working formulas for use in the study of rainfall-runoff records as general in their application as possible, separating the major factors affecting the problem in order that the observed effects may be traced to their most logical causes. Summarizing the conclusions to be drawn from a study of the St. Louis rainfall-runoff records, we find that, given the rainfall record, the runoff due to this rainfall may be closely predicted if the two major factors, Lag and Percentage of Runoff, are known. It is very doubtful if the lag can ever be predicted accurately, due to its erratic variations, but there are indications that, with further study, it may be approximately determined from a knowledge of the physical characteristics of the drainage area and the characteristics of the rainfall. Fortunately, a small error in the assigned value of the lag will not seriously affect the value of the funoff as will be seen from the unit graphs for various lags on Plate 15 . It is believed that a little more study will enable us to predict the Percentage of Runoff very closely for any given conditions and, when this can be done with confidence, we will be ready to fit the unit graph formulas into the working formulas for sewer design, which can be done by starting with hypothetical rainfall graphs obtained from a probability - frequency study of the rainfall records available. The final design formulas can be checked against the records of the sewer gauges described at the beginning of this paper. Much work remains to be done before this result is accomplished, but, when Mr. Horner publishes the full account of the St. Louis rainfall-runoff experiments, of which this study forms only a small and incidental part, the engineer engaged in the design of storm sewers will have information at his disposal which will enable him to proceed with greater confidence than ever before.

BIBLIOGRAPHY

The writer has made no research into the literature of the Rainfall-Runoff problem, which is quite extensive, but so far has confined his work to a study of the local records. For this reason no comprehensive bibliography will be attempted. Subsequent to the development of the working hypothesis given in the foregoing pages, the attention of the writer has been directed to the following articles in which similar methods have been used. Since each of these articles is independent of the others, it would seem that the use of the Unit Graph is a logical method of approach to rainfall-runoff analysis. Following is a list of the articles mentioned:

"RAINFALL-RUNOFF ANALYSIS IN STORM SEWER DESIGN"
By John A. Rousculp
Engineering News-Record, Feb. 17, 1927.
In this article the unit graph used is obtained by a different method than the one used by the writer.

"STORM WATER DISCHARGE FROM SMALL AND LARGE AREAS"
By H. C. Granville
Indian Engineering, beginning in Issue of Feb. 21, 1931 and continuing for several months.
A very complete theoretical discussion of the rainfall-runoff problem.

"STREAMFLOW FROM RAINFALL BY UNIT-GRAPH METHOD"
By L. K. Sherman
Engineering News-Record, April 7, 1932
The unit-graph method applied to the runoff from large areas.