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Robustness of Multiple Imputation under Missing at Random (MAR) Mechanism: A Simulation Study

Priyanka Garg

ROBUSTNESS OF MULTIPLE IMPUTATION UNDER MISSING AT RANDOM (MAR) MECHANISM: A SIMULATION STUDY

by

PRIYANKA GARG

(Under the Direction of Robert L. Vogel)

ABSTRACT

Missing data is an unavoidable issue in controlled clinical trials and public health research and practice. Presence of missing data and applying inappropriate methods of analysis generates biased estimates and reduces power of study. It is very important for investigators to use appropriate methods of analysis to deal with missing data in order to maintain internal (power of study) and external (generalization of sample results to larger population) validity of study. The focus of this dissertation is to compare different methods to deal with missing data in controlled clinical trials and public health research and practice. In addition, this dissertation also discusses that current approaches to deal with missing data might not produce valid inferences and may affect internal and external validity of results. Furthermore, emphasis is put on demonstrating how well multiple imputation works to deal with missing data under Missing at Random (MAR) mechanism with monotonic and non-monotonic missing data patterns for a range of percent missing under both normal and non-normal distributions. The results of this dissertation showed that multiple imputation is an efficient technique to obtain valid inferences compared to single imputation methods. In addition estimates obtained from multiple imputation also preserve the internal validity of study.

Key Words: Multiple Imputation (MI) Method, Missing at Random (MAR), Monotone Missing Data Pattern, Non-Monotone Missing Data Pattern, Sensitivity, Specificity.

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1. INTRODUCTION

1.1 Background

Missing data and incomplete data are a common occurrence in clinical, social and institutional research. Regardless of how carefully the researcher designs the experiment or survey, missing data problems usually exist. When missingness is injected into a study or research design, it impairs the validity of the study assumptions. There are two main concerns: the internal validity of the study which manifests itself in the power of study; and the external validity of study which manifests itself in the generalization of results (Croninge and Douglas, 2005). The internal validity refers to researcher's confidence that the observed effect is due to variables under study. The external validity refers to the extent of generalization of results of the sample which was studied, to the population. The external validity is reliant on internal validity. The presence of the missing data and the absence of an appropriate method to deal with the missing data may result in losing information and producing biased estimates. For example, subject discontinuation in the treatment arm due to any reason such as adverse event, may not provide correct comparison between placebo and treatment arm. The mean of the treatment arm would be different than it should have been which may, in turn, lead to results in the wrong comparison of the treatment arm with the placebo arm. This may affect the internal validity of a study. The researchers would not be able to generalize the results to the population once the internal validity of the study is compromised. The work in this dissertation will focus on investigating the internal validity of the study with the presence of missing data by generating unbiased estimates among different distributions such as Normal distribution, Cauchy distribution, t-distribution and Chi-square distribution.

Due to missing data, investigators may face many problems during data analysis and interpretation of results. Power and variability are associated with the sample size of the study. If the records with missing values are dropped from analysis, the number of records available is reduced which further leads to reduced statistical power. In addition, subjects who drop out from the study may have extreme outliers. Not including the dropout subjects in the analysis may result in underestimation of the variability which may further lead to producing narrow confidence interval (CHMP, 2010). Moreover, a high number of cases with missing values may result in producing biased estimates. For instance, suppose we randomly draw a sample from a population which includes responders (N_1) and non-responders (N_2), and we do not have any information about the non-responders. Suppose researchers in this example do not make any attempt to gather information for non-respondents. Ultimately, we will have respondents (n_1) from the sample (n). The sample mean ($\bar{x}$) will be calculated based on sample respondents (n_1). The mean estimate of respondents ($\bar{X}_1$) would be equal to mean estimate for total sample size (n) (i.e. $E(\bar{x}) = \bar{X}_1$) and the bias would be as follows:

$$B(\bar{x}) = \bar{X}_1 - \bar{X} = (N_2 / N)(\bar{X}_1 - \bar{X}_2)$$

Where, N is total number of records in the population, N_1 is total number of responders in the population, N_2 is total number of non-responders in the population, $\bar{X}_1$ is mean of responders, $\bar{X}_2$ is mean of non-responders, and $\bar{X}$ is mean of total population.

The bias due to missing cases is independent of sample size, so increasing sample size would not help in reducing bias. The solution to reduce the bias is to reduce the proportion of non-respondents to total population (N_2/N).

Missing data in a study can be resolved using many different approaches, but each approach may offer a different conclusion. For instance, list-wise deletion methods result in a loss of power in the statistical analysis. King et al. (2001) performed a content analysis of American Political Science Review, American Journal of Political Science, and British Journal of Political Science, and found that the most common method of analysis includes list-wise deletion. List-wise deletion is the only method which uses observations (rows) that have complete data. If there are missing values in the particular observation, this method deletes the entire observation from the study. In addition, they also estimated that the list-wise deletion method was adopted by approximately 94% of the articles published between 1993 and 1997. Furthermore, King et al. (2001) with their content analysis were able to estimate that the list-wise deletion method during 1993-1997 was responsible for reducing sample size by approximately one third on average. Reducing sample size generally leads to increase variability that ultimately reduces the power of study. Power is the probability of a test to reject the null hypothesis when the null hypothesis is not true. Generally, increasing sample size results in increased precision of estimation and power of tests. If we draw a sample from population then the variance of the sample mean is σ^2 / n, where σ^2 is population variance and n is sample size. Increase in sample size (n) would decrease the ratio and ultimately the standard error. Reducing standard error results in increasing the probability of a correct conclusion which is associated with an increased power of test.

Other approaches to treat missing data in a study include mean substitution, and multiple imputation. Mean substitution is the method which imputes the missing values with the mean value based on the observed values of the variable. This may result in underestimation of the variance (Cohen et al., 2003; Croninge and Douglas, 2005; Tsikriktsis, 2005). Multiple imputations impute each missing values multiple times and combines all the parameters of analysis into a single point. This method helps to reduce bias in estimates of all parameters. There are a few more methods to deal with missing data which are discussed in section 1.4.

1.2 Evolution of Missing Data Estimation Method

One of the earliest and most common approaches to data analysis in the presence of missing data is to delete any case with missing values and use the remaining data in the analysis. Inferences made about the attribute with missing data were performed without the help of the non-missing observed variables. Rubin (1976) developed inferential methodology for missing data. Rubin (1976) proposed imputing missing values multiple times to have multiple data sets and perform analysis of each data set to have multiple estimates. Rubin (1976) further suggested combining the parameters for all analysis to have a single point estimate. The estimates are combined to reflect within imputation and between imputation variability (Rubin, 1976; Marwala, 2009). The Expectation Maximization (EM) algorithm was developed by Dempster, Laird, and Rubin (1977) for missing data estimation. Since then, researchers have been applying different methods to analyze the missing data in different scenarios such as case deletion, pair-wise deletion, simple-rule prediction, mean substitution, hot-deck imputation, cold-deck imputation, imputation using regression, regression-based nearest neighbor hot-decking, tree-based imputation, and stochastic imputation (Marwala, 2009).

Little and Rubin (1987) identified some issues regarding case deletion and the single imputation methods. Case deletion methods may reduce statistical power and single imputation (mean imputation) may result in underestimating the variance of estimates. As a result of the above methods, Rubin (1987) developed the multiple imputation method to deal with missing data analysis.

Since the development of the multiple imputation method, there have been many other methods developed. Some of the methods developed are semi-hidden Markov models (Yu and Kobayashi, 2003), fuzzy approaches (Gabrys, 2002; Nelwamondo and Marwala, 2007b), and genetic algorithms (Junninen et al., 2004; Abdella, 2005; Abdella and Marwala, 2005). Researchers are currently working on the development of methods to analyze the robustness of missing data estimation methods such as sensitivity analysis of missing data estimation results. In addition, researchers are working on different useful and robust approaches for missing data analysis such as computational intelligence techniques and optimization techniques (Dhlamini, Nelwamondo, and Marwala, 2006; Nelwamondo, Mohamed, and Marwala, 2007; Nelwamondo and Marwala, 2007a, 2008; Marwala, 2009).

1.3 Missing Data Mechanisms

Before attempting to resolve issues raised due to missing data by applying multiple imputations or any other imputation method, it is very important to understand the mechanisms in which missing data occurs. There are two missing mechanisms which are referred to as *ignorable* missing mechanism and another referred to as *non-ignorable* missing mechanism. The ignorable missing mechanism appears when the probability of observing a missing data item is independent of the value of that data item. To the contrary, the non-ignorable missing data mechanism is when the probability of observing the missing data item is dependent on the value of that data item. The ignorable missing data mechanism is followed by Missing Completely at Random (MCAR), and Missing at Random (MAR) while the non-ignorable missing data mechanism is followed by Missing Not at Random (MNAR) (Little and Rubin, 1987; Little and Rubin 2002; Graham, et al., 2003; Wayman, 2003).

1.3.1 Missing Completely at Random

Missing completely at random (MCAR) arises when a subject with incomplete observations are a random subset of the complete sample of subjects (Rubin, 1976). The MCAR mechanism means that the missing value is independent of observed and unobserved observations but it may be associated with observed covariates (Molenberghs and Kenward, 2007). Under the MCAR mechanism, the probability that an observation is missing is not related to any other variable. In other words, the missingness does not depend on observed variables in analytical model. In addition, under the missing completely at random mechanism the subjects with missing, as well as non-missing observations, are a random sample from the source population. Loosing blood samples or a patient questionnaire accidently are examples of MCAR because it is not related to any other patient's characteristics (Greenland and Finkle, 1995; Donders et al., 2006).

Though MCAR is a strong assumption, it is usually not satisfied in practical applications (Raghunathan, 2004). In MCAR, the subjects with non-missing and missing data are not distinct. This means that the missing observations are independent of both the observed data and the missing data. The mathematical expression for MCAR can be written in terms of conditional probability as follows (Little and Rubin, 1987):

$$P(M \mid Y_o, Y_m) = P(M)$$

where, M indicates missing value, Y_o are observed values, Y_m are missing values and P(.) indicates a probability.

From the above expression, it is evident that neither Y_o nor Y_m would be able to predict the missing value as MCAR is defined as the conditional probability of M given Y_o and Y_m which equals the probability of M. Analysis of complete cases would be an appropriate approach to conclude any findings under the MCAR missing mechanism. Donders et al. (2006) illustrated that single and multiple imputation also result in unbiased estimate if missing mechanism is MCAR.

1.3.2 Missing at Random

When the missing observations in the data are independent of the missing variables themselves, yet possibly dependent on other observed variables, then the mechanism is known as Missing at Random (MAR). Under the MAR mechanism, the cases with missing data differ from cases with non-missing data. (Little and Rubin, 1987; Marwala, 2009). The difference of missing and non-missing values can be determined by dividing the interest variable into missing and non-missing groups. If the means of two groups are statistically significant from each other for other variables of interest, it implies that missing mechanism is MAR (Little and Rubin, 1987; Tsikritis, 2005). Unlike MCAR, the missing data is predictable from other observed variables. Therefore, the mathematical expression for MAR can be written as follows (Little and Rubin, 1987):

$$P(M \mid Y_o, Y_m) = P(M \mid Y_o)$$

where, Y_o are observed values and Y_m are missing values. M indicates missing value indicator and is equal to 1 if Y is observed and 0 if Y is missing.

The above expression clearly indicates that the missing data may be dependent on observed data which may include covariates, but is independent of the actual missing values. The work in this dissertation is focused on the MAR missing data mechanism.

1.3.3 Missing Not at Random

Under the MNAR mechanism the missing value may be dependent on both the observed values and the missing values of the variable itself, as well as, other variables in analytical model (Fielding et al., 2008; Croninge et al., 2005). When the mechanism is MNAR, the missingness of data is non-ignorable (i.e. the probability of observing missing data item is dependent of the value of that data item) (Molenbergh et al., 2004). There is no clear method available for dealing with potential bias associated with MNAR, so it has the potential threat to the external validity of study (Croninge et al., 2005). Conclusively under the MNAR mechanism the probability of missing depends on the variables which have missing value. Also, unlike testing for MCAR vs MAR as described earlier there is no way to test for MAR vs MNAR.

1.4 Strategies to Manage Missing Data

Historically, researchers have been using different methods to analyze the missing data. These methods include case deletion, list-wise deletion, pair-wise deletion, mean substitution, hot-deck imputation, cold-deck imputation, and imputation using regression (Marwala, 2009). In addition, single imputation using the EM algorithm, multiple-imputation, and full information maximum likelihood approaches are some of the modern alternatives when working with missing values (Acock, 2005).

1.4.1 Case Deletion

One of the most commonly used methods is the case deletion method. In this method, the cases with missing data are deleted from the study and the analysis is performed on the remaining data. Different methods have been applied to delete cases with missingness such as list-wise deletion and pair-wise deletion (Marwala, 2009). Both list-wise and pair-wise deletion are common default options found in popular statistical analysis packages such as SPSS, SAS, and Stata. The case deletion method may lead to biased estimates of both parameters and their standard errors. For instance, in a study in which some of the sample involves obese patients, the obese patients may be more likely not to report their weight. This was the case in the Baker County Cancer Screening Telephone Survey (Vogel, 2006). If we perform the case deletion method, it would omit patients who are obese and would therefore result in estimates that are downwardly biased, since part of the population is not adequately represented.

Cook (1977) developed a new measure based on confidence ellipsoids. The determination of the least squares estimate of the parameter vector in full rank linear regression models to judge the contribution of each data points (Cook, 1977). Davidian and Giltinan (1995) showed that Cook's approach is difficult to apply in recently developed models such as Generalized Linear Mixed Models (GLMM) due to the complexity of the observed-data likelihood function that usually involves interchangeable integrals. Therefore, Zhu et. al., (2001) tried to generalize Cook's (1977) approach and developed case deletion measures to check their global influence on general models with missing data. Zhu et al., (2001) illustrated that simplifying Cook's approach can help in computing maximum likelihood estimates for missing data more efficiently (Xu et al., 2006).

1.4.2 List-Wise Deletion

Consider data as expressed in a spreadsheet where the columns represent variables and the rows represent observations. List-wise deletion is the only method which uses observations (rows) that have complete data. If there are missing values in a particular observation, this method deletes the entire row from the study (Croninge and Douglas 2005).

The list-wise deletion method is especially inappropriate when the amount of missing data is large. In this situation, the list-wise deletion method results in a loss of power in the statistical analysis (Croninge and Douglas, 2005). The list-wise deletion method poses two other major statistical problems: there will be loss of information and it assumes that the unobserved values are not important, thus ignored (Marwala, 2009). In addition, operations management researchers suggested that list-wise deletion is one of the least accurate methods to deal with missing values (Tsikritis, 2005).

1.4.3 Pair-Wise Deletion

The pair-wise deletion method can be used to analyze data and get unbiased estimates when the missing data is MCAR. The pair-wise deletion method has several advantages over the list-wise deletion method, such as using all available data which helps in preserving all information as well as retaining statistical power for analysis (Croninge and Douglas, 2005). Therefore, a pair-wise deletion method is also known as "available-case analysis". If the variable which has missing observations is not used in the analysis, the pair-wise deletion method can be used for complete data analysis. Several researchers have suggested that pair-wise deletion is better than the list-wise deletion method (Marwala, 2009). On the other hand, Allison (2002) suggested some disadvantages of pair-wise deletion method. The pair-wise deletion method sometimes may not result in a positive definite covariance structure. The pattern of missingness is responsible for the senseless covariance structures. The greater chance of sample size variation across the pairs of variables would result in decreasing the likelihood to produce a positive definite covariance structure (Allison 2002). The formula below for sample covariance will explain the covariance problem:

$$Cov(x, y) = \frac{\sum (x_i - \hat{\mu}_x)(y_i - \hat{\mu}_y)}{n - 1}$$

where, $\hat{\mu}_x$ is the mean estimate of variable x, $\hat{\mu}_y$ is the mean estimate of variable y. The pair-wise deletion method uses different sample size of complete cases on both variables (x and y) and computes the covariance. For example, $\hat{\mu}_x$ is computed from the available data on x variable and $\hat{\mu}_y$ is computed based on the available data on y variable. The researchers find same issue while computing the denominator of correlation coefficient, ρ:

$$\rho = \frac{Cov(x, y)}{\sqrt{\sigma_x^2 \sigma_y^2}}$$

σ_x^2 is computed from the available data on x variable and σ_y^2 is computed based on the available data on y variable which may leads to problem of producing correlation greater than one. The reason for this problem is that the elements in correlation computation are not consistent with one another which produce non positive definite matrices. This leads to additional problems of estimation in multivariable data analysis (i.e. regression models) (Enders, 2010).

In addition, the pair-wise deletion method results in the actual sample size on which summary statistics are calculated to be between the minimum and the maximum number of cases for variables, thus computing ambiguous standard errors (Croninge and Douglas, 2005). For standard error calculation, the sample size is the main key component. In the pair-wise deletion method there may be different sample sizes for covariance matrix calculation. Thus, there is no straightforward way to compute the standard error which leads to either an underestimated or overd standard error (Enders, 2010).

1.4.4 Mean Substitution

The mean substitution method imputes the missing values with the mean value based on the observed values of the attribute or variable. Many researchers suggest that mean substitution is not a good approach for dealing with missing data as it systematically underestimates the variance of the estimators and provides biased estimates of the median (Little and Rubin, 1989; Allison, 2002; Rubin et al., 2007).

Suppose we have the following data:

Group	A	A	A	A	A	A	A	A	A	A	A	A
Diastolic BP	90	89	88	92	95	93	85	89	90	95	86	82

The mean and median of complete data would be 89.5 and standard deviation would be 3.942. If we randomly delete 3 observations, then data would be as follows:

Group	A	A	A	A	A	A	A	A	A	A	A	A
Diastolic BP	90	89	.	92	95	93	85	.	90	.	86	82

Complete case analysis deletes the records with missing observation, so the mean of the data with 3 missing observation is 89.11 and median is 90.0, and standard deviation is 4.136. In mean imputation, we impute the missing observations with mean of observed data for that variable. After mean imputation, the new mean, median and standard deviation with imputed data is 89.11, 89.11 and 3.527, respectively. The above example depicts that mean substitution produces downwardly biased estimates of the standard error.

1.4.5 Hot / Cold-Deck Imputation

In the hot deck imputation method, the missing values are replaced by similar non-missing cases that share the same values in matching variables from the same data set. There are two main steps for this imputation method. First, data are divided into clusters of observed data. Second, missing values are replaced with the non-missing value within the same cluster. Cold deck imputation method is same as hot deck imputation method and the only difference is that the data source for imputation of missing value must be other than the current data set (Marwala, 2009).

The example of hot/cold deck imputation will be explained in the following table.

The first table below has missing values.

Sex	Age	Weight	Race
Male	54	.	AA
Female	55	156	AA
Male	51	175	AA
Male	53	180	AA
Male	59	187	AA
Male	60	190	AA
Female	52	176	AA
Female	58	.	AA
Female	60	190	AA

* AA-African American

The second table below represents the cluster of weights based on the first table.

Race	Sex	Age			
		21-30	31-40	41-50	51-60
AA	Male	.,.,.,.	.,.,.,.	.,.,.,.	175, 180, 190, 187
	Female	.,.,.,.	.,.,.,	.,.,.,.	176, 156, 190, 195

There are missing values in the first table. The hot deck imputation method will randomly select the value from the cluster of matching group based on age, sex, and race. Then, it imputes the missing value with randomly selected value. For example, the first table has missing value of weight for an African American male with age 54. The hot deck imputation method will randomly select a weight from the cluster of African American males with age group of 51-60 and impute the missing value of first table. That is, one of the four values 175, 180, 190, 187 will be randomly selected with equal probability as a surrogate for the missing value.

In the case of cold deck imputation, the known information of prior research/survey with similar characteristics is found to have most appropriate value and the missing value of first table is imputed.

1.4.6 Linear Regression Imputation

In the linear regression imputation method, a regression model is constructed from the non-missing data. Based on the regression model, predicted values are used to impute the missing values. As the missing values which are being imputed and used in the regression model are conditional upon the predicted values, linear regression imputation is considered a conditional approach (Rubin et al., 2007; Little and Rubin, 1989).

Researchers found some disadvantages associated with the linear regression imputation method. The main disadvantage that Little and Rubin (2002) mentioned was having poor predicting power in case the regression model did not provide a good fit. In addition, linear regression imputation assumes the percentage of variance explained to be 100%, or the coefficient of determination is 1, which leads to an underestimate of the variability (Rubin et al., 2007; Little and Rubin, 1989; Enders, 2001). Rubin and Little (1987) stated that regression imputation procedure works well with only a monotonic missing data pattern (Graham et al., 1994). For instance, if the data has variables X_1, X_2,….., X_n and the variable X_y is missing for a certain observation which indicates missing for the consequent variables X_z, z>y then this type of missing pattern is referred to as monotone missing pattern.

1.4.7 Multiple Imputation

Multiple imputation was originally proposed by Rubin (1976). The objective was to develop a practical and useful procedure for missing values in incomplete data. Multiple imputation imputes missing values by using an appropriate model multiple times and combines the parameters for all analysis to have a single point. For monotone missing pattern, a regression model may be used to impute the missing values whereas under non-monotone missing pattern, the Markov Chain Monte Carlo (MCMC) method is used for the imputation. Multiple imputation also introduces random variation which enhances the possibility to reduce bias in estimates of all parameters. In addition, multiple imputation provides a more accurate estimate of the standard error and thus ultimately helps to preserve the original available data distribution (Little and Rubin, 1989; Landerman, Land and Pieper, 1997; Allison, 2002).

Researchers in public health use enhanced data or secondary datasets for the analysis. Missing data is a major problem with survey related data set. When the survey is sent out to the group of people and by any reason if it does not return then MI cannot help. However, MI works well to impute the missing data item for an observation. In public data sets such as the National Survey of Families and Household (NSFH), the General Social Survey (GSS), the Survey of Income and Program Participation (SIPP) missing information is a major problem (Acock, 2005).

In longitudinal studies with monotone missing data pattern or a non-longitudinal setting with non-monotone missing data pattern MI works well with the missing data item for an observation. In a clinical trials setting, if an individual is supposed to come in every week and if an individual could not make it due to any reason, then multiple imputation works well to impute the missing data item for the particular individual.

This dissertation explores the characteristics of multiple imputation under monotonic and non-monotonic missing data patterns over a wide range of probability distributions and a wide range of percent of data missing. In this dissertation, I will examine the robustness of multiple imputation under a variety of situations that would normally occur in complex sample surveys and other research designs found in public health.

2. LITERATURE REVIEW

Missing data can be treated by using different techniques such as case-deletion, list-wise deletion, mean substitution or multiple imputation. Anderson et al. (1983) suggested that a common practice to deal with missing data is either to delete the observation with the missing values or impute the missing value with a predicted value. Little and Rubin (1987) proposed that maximum likelihood is an accepted method to deal with missing values. However, Little and Rubin (1989) show that if the sample size is small then the maximum likelihood estimation method could lead to a biased estimate. Cohen and Cohen (1983) suggested that dropping the subjects depend on the presence of the missing value and on the dependent or independent variable. Cohen and Cohen (1983) stated that if the missing value is present on a dependent variable it is reasonable to drop the subject, but if the missing value is present on an independent variable then it is good to investigate the proportion of missing and non-missing. In addition, at the same time the researcher should also investigate the effect of missingness on the result or power of analysis.

Orme and Reis (1991) and Fairclough (1998) have shown that if the percentage of missing values is very high then using list-wise deletion or pairwise deletion methods may conclude a biased estimate and a wrong comparison of two treatment groups. In addition, the authors also discussed that if the percentage of missing values on one or several independent variables is low, then different methods of analysis may produce inconsistent results. Moreover, using different statistical software's (SAS or SPSS) default option may also produce different results as they use different methods to deal with missing data.

Malhotra (1987) and Stumpf (1978) have found that if the analysis has been conducted by the list-wise deletion method then it may result in loss of data. Cohen and Cohen (1983), Gilley and Leone (1991), and Rubin et al. (2007) suggested that the loss of data may result in less statistical power to detect the statistical difference. Furthermore, loss of data may subsequently produce a biased estimate and may reduce the precision of the parameters (Cohen and Cohen, 1983; Donner, 1982; Little and Rubin; 1989, Orme and Reis; 1991).

Rubin (1987) and Schafer (1999) proposed that the multiple imputation method is the most accepted method to deal with the missing data problem. Dragset (2009) discussed that under MCAR and MAR assumption, multiple imputation provides unbiased estimators and standard errors. Even under the MNAR assumption, multiple imputation is considered to be a most effective technique (Rassler et al., 2008). Schafer (1997) suggested that a small number of imputations provide efficient estimates of standard error. Schafer (1999) further explained that in almost all cases, a maximum of ten imputations is sufficient, but an in depth investigation is required for problems with a higher percentage of missing values (Horton and Lipsitz; 2001). However, several investigators suggested that the number of imputations depends on the percentage of missing values in the dataset (Schafer and Graham, 2002; Graham et al., 2007; Spratt et al., 2010).

Another approach to impute the data with missing values is the multivariate imputation by chained equation (MICE). Buuren and Oudshoorn (2000) released the package for MICE as S-PLUS library. Royston (2004) developed the package for MICE in STATA. After the development of MICE in STATA some users converted MICE into R software (R Development Core Team, 2011; Buuren and Groothuis-Oudshoorn, 2011). In MICE, the Gibbs sampler approach is used to create multiple imputation. In the Gibbs sampling approach, the conditional sampling is applied to the distribution of missing values based on the distribution of other variables. The MICE approach is considered a flexible approach because it gives flexibility to the researcher having a multivariate structure on the data. The researcher may be able to specify the full spectrum of conditional imputation models. For instance, researchers can use the logistic regression model to impute missing values for dichotomous variables and the linear regression model to impute missing values for continuous variables (Heymans et al., 2007; Farhangfar et al., 2008). Buuren and Oudshoorn (1999) suggested that specifying all conditional models may not be easy. If the imputation model has too many variables it may lead to multicollinearity problems. In addition, MICE requires comprehensive computational skills (Buuren et al., 2005).

3. METHOD

3.1 Multiple Imputation

Multiple Imputation (MI) replaces each missing value with the two or more possible values (Molenberghs et al., 2007). In addition, it introduces random variation which enhances the possibility to have unbiased estimates of all parameters. Multiple imputation is one step ahead of maximum likelihood (ML) estimates. Multiple imputation has features of ML estimates (i.e. summarizing likelihood by averaging over a predictive distribution for the missing values) along with uncertainty created by imputation. It generates various sets of data based on feasible models in order to replace the missing values. Hence, it provides multiple completed data sets for analysis. The underlying statistical reasoning in multiple imputation is that an average of the completed-data likelihood over unknown missing values can be used to estimate observed-data likelihood. In other words, both analyses (i.e. likelihood-based analysis and analysis from "observed-data" likelihood) are approximately equal and variation across the different datasets signifies the imputation uncertainty (He, 2010).

3.2 Procedure for Analysis

As described in Rubin (1987), there are four different stages involved in multiple imputation (MI).

a) *Look at the data and determine pattern*: The first step is to look at the data and based on the available information determine the pattern of missingness. The missing data pattern can be monotonic or non-monotonic. Based on the missing data pattern, the second step can be determined.

b) *Imputation*: The second step of multiple imputation is imputing multiple values for each missing value to create M complete datasets. This step randomly selects the value to fill the missing value from the predictive distribution of missing data given observed data.

c) *Analysis*: The third step is analyzing each completed dataset separately to generate M sets of estimates. Every set of estimates may differ slightly from each other.

d) *Pooling*: The fourth step is to combine all sets of estimates to generate an overall estimate and calculate the variation among parameter estimates.

3.3 Theoretical Support/Validation for Multiple Imputation

The average estimate of β is

$$\hat{\beta}^* = \frac{1}{M}\sum_{m=1}^{M}\hat{\beta}^m$$

and, an estimate of the covariance matrix of $\hat{\beta}^*$ is

$$V = W + \left(\frac{M+1}{M}\right)B$$

Where, the within-imputation covariance matrix,

$$W = \frac{1}{M}\sum_{m=1}^{M}V^m ,$$

And the between-imputation covariance matrix of $\hat{\beta}^m$ is,

$$B = \frac{1}{M-1}\sum_{m=1}^{M}\left(\hat{\beta}^m - \hat{\beta}^*\right)\left(\hat{\beta}^m - \hat{\beta}^*\right)^T$$

The theoretical validation for multiple imputation is shown below,

For any random variable X & Y

$$E[E(Y \mid X)] = E(Y) \qquad (3.1)$$

$$VAR(Y) = E[VAR(Y \mid X)] + VAR[E(Y \mid X)] \tag{3.2}$$

Joint probability density function

$$f(x,y) = f_1(x) * f_2(y \mid x) = g_1(x \mid y) * g_2(y) \tag{3.3}$$

Where, $f_2(y \mid x)$ and $g_1(x \mid y)$ provide the conditional distributions of y given x and of x given y respectively. $f_1(x)$ and $g_2(y)$ provide the marginal distributions for x and y respectively.

Given a continuous multivariate density function $f(x_1, x_2, .,., x_n)$,

The marginal density of x_1 is

$$f_1(x_1) = \underbrace{\int_{-\infty}^{+\infty} \ldots\ldots \int_{-\infty}^{+\infty}}_{n-1} f(x_1, x_2, \ldots., x_n) dx_2, dx_3, \ldots\ldots, dx_n \tag{3.4}$$

Suppose we have a problem that involves two parameters γ_1 and γ_2 and some data y. The joint posterior distribution density function for γ_1 and γ_2 given data y is $f(\gamma_1, \gamma_2 \mid y)$ and from equation 3.3,

$$f(\gamma_1, \gamma_2 \mid y) = f(\gamma_1 \mid y) * f(\gamma_2 \mid \gamma_1, y) \tag{3.5}$$

Consider γ_1 to be a nuisance parameter and then the marginal distribution of γ_2 from equation 3.4 is:

$$f(\gamma_2 \mid y) = \int_{-\infty}^{+\infty} f(\gamma_1, \gamma_2 \mid y) d\gamma_1 \tag{3.6}$$

From equation 3.1 and 3.2, the mean and variance of γ_2 can be written as follow,

$$E(\gamma_2 \mid y) = E_{\gamma_1}(E_{\gamma_2}(\gamma_2 \mid \gamma_1, y)) \tag{3.7}$$

$$VAR(\gamma_2 \mid y) = E_{\gamma_1}[VAR_{\gamma_2}(\gamma_2 \mid \gamma_1, y)] + VAR_{\gamma_1}[E_{\gamma_2}(\gamma_2 \mid \gamma_1, y)] \tag{3.8}$$

The expected value and variance can be approximated using empirical moments and we let γ_1^m, m= 1, 2…, M, be draws from the marginal posterior distribution of γ_2,

$$E(\gamma_2 \mid y) \approx \frac{1}{M}\sum_{m=1}^{M}(E_{\gamma_2}(\gamma_2 \mid \gamma_{1,}^m, y)) = \overline{\gamma_2} \tag{3.9}$$

$$VAR(\gamma_2 \mid y) = \frac{1}{M}\sum_{m=1}^{M} VAR_{\gamma_2}(\gamma_2 \mid \gamma_1^m, y) + \frac{1}{M-1}\sum_{m=1}^{M}(E_{\gamma_2}(\gamma_2 \mid \gamma_1^m, y) - \overline{\gamma_2})^2 \tag{3.10}$$

Finally, we use γ_2 to represent the parameters of the *substantive model* and γ_1 to represent the *missing data*.

3.3 Advantage and Disadvantage of Multiple Imputation

Several authors have mentioned that multiple imputation is more efficient for missing data imputation and analysis than for single imputation. According to Rubin (1987), multiple imputation has two advantages which are the same as single imputation. First, multiple imputation has the ability to allow the use of the data collector's knowledge to impute missing data. Second, it allows the use of methods requiring complete data for analysis. In addition to the two advantages showed with single imputation, multiple imputation has four additional advantages (Rubin, 1987; Little and Rubin, 1989). First, efficient point estimates and variances are produced when researchers use the multiple imputation method. Second, multiple imputation introduces random variation which enhances the possibility to have unbiased estimates of all parameters. Third, investigators are enabled to examine sensitivity of inferences to various models for non-response by use of multiple random imputation from numerous models. Little and Rubin (1989), Landerman et al. (1997), Faris et al. (2002) suggested a fourth advantage of multiple imputation. They claim it produces unbiased estimates of the standard error and thus ultimately helps to preserve original available data distribution.

With all these advantages, multiple imputation also poses some disadvantages. Analysis of multiple data sets is time consuming and requires statistical expertise. Eventually the analysis of multiple imputed data sets would cost more than analyzing single data (Rubin, 1987; Pigott, 2001; Faris et al., 2002). However, use of efficient statistical software and statistical programs help in implementing multiple imputation methods more efficiently (Rubin, 1987).

In this dissertation, the data will be created using simulation. The advantage of simulation to introduce the missing values is that the true value is known which allows the researcher to compare the real estimates and the imputed estimates in terms of precision. Simulation also allows researchers to study the problem with different scenarios such as different percentage of missing values. In addition, using simulation permits researchers to control the experimental conditions. Simulation also has some disadvantages such as it is randomly based therefore it may be less accurate than any mathematical model. Moreover, simulating complex models may require extra computer time and special skills to run a model.

In this dissertation, monotonic and non-monotonic missing patterns with an extensive range of percent of data missing will be created using simulation, and the effect of the multiple imputation method will be explored on the precision of the estimates. Monotonic missing data patterns usually occur in longitudinal studies whereas non-monotonic missing data patterns usually occur in non-longitudinal studies such as cross sectional studies and surveys. This dissertation will also explore the effect of multiple imputation on the normal distribution as well as non-normal distributions such as the Cauchy distribution, t-distribution, and Chi-square distribution by exploring the effect of different degrees of freedom as non-normal data converge to normality. The change in the variability will be measured for both normal distribution and non-normal distributions such as Cauchy distribution, t-distribution, Chi-square distribution. Overall in this dissertation, the robustness of the missing data under the MAR mechanism is measured for normal distribution and non-normal distributions such as Cauchy distribution, t-distribution, and Chi-square distribution. The following chapters explore the results of normal distribution, Cauchy distribution, t-distribution, and Chi-square distribution under monotonic and non-monotonic missing patterns.

4. RESULTS OF MONOTONE MISSING DATA PATTERN

4.1 Simulation

We conducted simulation studies and sensitivity analysis with different percentages of missingness. We simulated 1,000 samples for each of four sample sizes: 100, 500, 1000, and 5000. For consistency we generated the same random number for each variable (X_1, X_2, X_3, Z_1, U_1, U_2, U_3) in our sensitivity analysis comparing different distributions: Normal distribution, Cauchy distribution, t-distribution, Chi-square distribution. In the simulations we assume the data is missing at random (MAR) with a missing data rate of approximately 10%, 15%, 20%, and 25%. The SAS multiple imputation procedure (Proc MI) creates multiple data sets based on the number of imputations and incorporates within and between subject variability. According to Rubin (1996), multiple imputation with a set of three to five different imputations will provide acceptable results. In our study each missing value was replaced with a set of five and ten imputations. There were three covariates created X_1, X_2, X_3 and the response variable Z_1 for each distribution. Random uniform variables (U_1, U_2, and U3) are generated to determine which X_i are to be deleted. The same X_i are deleted for each monotonic simulation policy. The values for each policy are different as they come from different distributions, but the position in the data matrix remains the same. This allows for a more valid comparison of policies.

The monotone missing data pattern was simulated using approximately 10, 15, 20, and 25 percent of missing on the response variable (Z_1). To generate approximately 10% of missing data on Z_1 we used a sequential simulation technique; if $U_1 < 0.03$ then X_2 is missing, creating 3% missing on X_2; if $U_2 < 0.03$ or X_2 is missing then X_1 is missing, creating 6% missing on X_1; finally if $U_3 < 0.03$ or X_1 is missing, then Z_1 is missing, creating approximately 10% missing on Z_1. The same technique was used to generate approximately 15, 20, and 25 percent of missing data in the original data, using 0.05, 0.07, and 0.09 as cutoffs for U_1 - U_3 respectively.

The selection of model depends on the missing data pattern. Regression method is a parametric method used under the assumption of multivariate normality. Depending on the number of imputation *m* completed datasets created, the analysis is conducted on each completed data sets. Using PROC MIANALYZE, valid statistical inferences can be generated. These valid inferences are then combined to provide one result from *m* different analyses.

The purpose of this dissertation is to check how the deviation from normality impacts multiple imputation. The Standard Normal distribution, N (0, 1) is used as the gold standard to compare results of non-normal distributions such as the Cauchy distribution, the t-distribution with different degrees of freedom, and the Chi-square distribution with different degrees of freedom. MI incorporates within and between variability; therefore we expect the imputed variance to be greater than the true variance. However, there may be a few instances with smaller imputed variance and larger true variance. We have recorded the number of instances in which the ratio of observed variance and imputed variance greater than 1, 1.05 and 1.1.

In this dissertation we are showing how well MI works under monotone missing data pattern or non-monotone missing data pattern. Under this chapter, the results of monotone missing data are discussed. To support the results we also show how many instances reject the null hypothesis under each of the following methods: available data, mean substitution, single regression imputation, and multiple imputation. Moreover, we provide the results of sensitivity and specificity to assure that the instances rejecting the null hypothesis are actually the same as the original/full data. Sensitivity is defined as the number of true positives divided by the sum of the true positives and the false negatives. Specificity is defined as the true negatives divided by the sum of the true negatives and false positives. We applied this epidemiological principal to compare various imputation results relative to the true results. The next chapter will focus on the results of non-monotone missing data pattern.

4.2 Results of Different Distributions

4.2.1 Results of Normal Distribution

From Table 1 the sample size of 500 with approximately 10% of the data missing and five imputations, the mean difference (True-Imputed) is 0.000038 (CI: -0.00093, 0.00101) and the geometric average ratio of variance (True/Imputed) is 0.8987 (CI: 0.8945, 0.9030). Less than three percent of the simulations have ratio of observed to imputed variances greater than one. The ratio of observed variance to imputed variance exceeds 1.05 about 0.1% and the ratio of observed variance to imputed variance does not exceed 1.10, providing evidence that multiple imputation is a conservative method under the conditions of this simulation. Using the same sample size, n=500 and a 10% missing data rate but with ten imputations, the mean difference (True-Imputed) is 0.000085 (CI: -0.00084, 0.00101) and the geometric average of the ratio of variances (True/Imputed) is 0.9044 (CI: 0.9015, 0.9027). The percentage of variance ratios exceeding one is 0.6. There was no instance with ratio of observed and imputed variances greater than 1.05 or 1.1 which supports the idea that multiple imputation provides a conservative method for imputing data under the Normal Distribution.

For a sample size of 500 with approximately 15% missing data and ten imputations, the mean difference is 0.000598 (CI: -0.00060, 0.00180) and the geometric average of the ratio of variances is 0.8441 (CI: 0.8400, 0.8483). Only 0.4% of the simulations have a ratio of observed to imputed variance greater than one and there were no simulations providing a ratio of variance greater than 1.05. With approximately 20% missing and ten imputations, the mean difference is -0.00050 (CI: -0.00195, 0.000941) and the geometric average of the ratio of variances is 0.7817

(CI: 0.7764, 0.7870). In this case, 0.2% of the simulations have a ratio of observed to imputed variances greater than one. There was no instance with ratio of observed to imputed variances greater than 1.05.

Table 1: Normal distribution with Monotonic Missing data Pattern

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.000732	-0.00147 (-0.1499)	0.00293 (0.1234)	0.8872	0.8813 (0.4578)	0.8931 (1.1447)	0.081	0.018	0.002
100		10	0.000788	-0.00128 (-0.1392)	0.00285 (0.1155)	0.8925	0.8880 (0.6069)	0.8970 (1.1244)	0.051	0.011	0.004
100	15	5	0.000150	-0.00281 (-0.1504)	0.00311 (0.1751)	0.8297	0.8221 (0.4568)	0.8374 (1.2758)	0.069	0.022	0.004
100		10	0.00110	-0.00172 (-0.1319)	0.00393 (0.1459)	0.8250	0.8192 (0.5173)	0.8309 (1.1310)	0.031	0.008	0.001
100	20	5	0.00117	-0.00238 (-0.1841)	0.00472 (0.1842)	0.7576	0.7486 (0.3162)	0.7667 (1.1250)	0.037	0.013	0.004
100		10	0.000269	-0.00312 (-0.1619)	0.00366 (0.1708)	0.7659	0.7593 (0.4383)	0.7725 (1.1359)	0.018	0.007	0.003
100	25	5	0.00221	-0.00179 (-0.1898)	0.00621 (0.2539)	0.7012	0.6913 (0.2334)	0.7112 (1.3545)	0.029	0.009	0.005
100		10	0.00104	-0.00286 (-0.1818)	0.00494 (0.2202)	0.7109	0.7034 (0.3507)	0.7186 (1.1147)	0.015	0.006	0.001
500	10	5	0.000038	-0.00093 (-0.0534)	0.00101 (0.0434)	0.8987	0.8945 (0.6389)	0.9030 (1.0745)	0.026	0.001	0
500		10	0.000085	-0.00084 (-0.0526)	0.00101 (0.0450)	0.9044	0.9015 (0.7196)	0.9072 (1.0447)	0.006	0	0
500	15	5	0.000451	-0.00084 (-0.0680)	0.00174 (0.0599)	0.8390	0.8329 (0.5194)	0.8452 (1.0342)	0.024	0	0
500		10	0.000598	-0.00060 (-0.0619)	0.00180 (0.0534)	0.8441	0.8400 (0.5947)	0.8483 (1.0239)	0.004	0	0
500	20	5	-0.00015	-0.00167 (-0.0829)	0.00136 (0.0843)	0.7733	0.7660 (0.4425)	0.7807 (1.0225)	0.006	0	0
500		10	-0.00050	-0.00195 (-0.0748)	0.000941 (0.0867)	0.7817	0.7764 (0.5064)	0.7870 (1.0069)	0.002	0	0
500	25	5	-0.00034	-0.00209 (-0.0973)	0.00141 (0.1107)	0.7183	0.7094 (0.3166)	0.7274 (1.0303)	0.008	0	0
500		10	0.000076	-0.00159 (-0.1028)	0.00174 (0.0909)	0.7332	0.7274 (0.4322)	0.7391 (1.0572)	0.001	0.001	0

With approximately 25% missing and ten imputations, the mean difference is 0.000076 (CI: -0.00159, 0.00174) and the geometric average of the ratio of variances is 0.7332 (CI: 0.7274, 0.7391). Under the conditions of this model, 0.1% of the simulations have a ratio of observed to imputed variances greater than one. Only 0.1% of the instances have a ratio of observed variance to imputed variance larger than 1.05. There was no instance with a ratio of observed variance to imputed variance greater than 1.1.

Table 1 continues: Normal distribution with Monotonic Missing data Pattern

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00019	-0.00086 (-0.0452)	0.000483 (0.0324)	0.8972	0.8932 (0.6955)	0.9011 (1.0199)	0.01	0	0
1000		10	-0.00004	-0.00068 (-0.0394)	0.000597 (0.0289)	0.9077	0.9051 (0.7514)	0.9103 (1.0128)	0.002	0	0
1000	15	5	-0.00003	-0.00090 (-0.0495)	0.000831 (0.0426)	0.8383	0.8326 (0.5263)	0.8440 (1.0226)	0.009	0	0
1000		10	-0.00022	-0.00105 (-0.0505)	0.000608 (0.0395)	0.8460	0.8420 (0.5653)	0.8499 (1.0080)	0.002	0	0
1000	20	5	-0.00096	-0.00200 (-0.0560)	0.000082 (0.0471)	0.7831	0.7757 (0.3521)	0.7906 (1.0524)	0.005	0.001	0
1000		10	-0.00072	-0.00171 (-0.0542)	0.000270 (0.0439)	0.7942	0.7890 (0.4774)	0.7994 (0.9852)	0	0	0
1000	25	5	-0.00015	-0.00135 (-0.0622)	0.00104 (0.0517)	0.7282	0.7200 (0.3852)	0.7364 (1.0231)	0.005	0	0
1000		10	-0.00056	-0.00173 (-0.0651)	0.000613 (0.0543)	0.7339	0.7282 (0.4424)	0.7396 (0.9838)	0	0	0
5000	10	5	0.000208	-0.00008 (-0.0148)	0.000494 (0.0141)	0.8970	0.8932 (0.6898)	0.9009 (1.0060)	0.006	0	0
5000		10	0.000065	-0.00021 (-0.0134)	0.000340 (0.0122)	0.9045	0.9020 (0.7529)	0.9070 (0.9983)	0	0	0
5000	15	5	0.000118	-0.00027 (-0.0187)	0.000508 (0.0187)	0.8337	0.8278 (0.5172)	0.8396 (0.9950)	0	0	0
5000		10	0.000074	-0.00030 (-0.0198)	0.000449 (0.0178)	0.8444	0.8404 (0.6139)	0.8484 (0.9836)	0	0	0
5000	20	5	0.000196	-0.00028 (-0.0213)	0.000667 (0.0336)	0.7851	0.7778 (0.4219)	0.7924 (1.0060)	0.001	0	0
5000		10	0.000253	-0.00020 (-0.0221)	0.000703 (0.0226)	0.7912	0.7864 (0.4684)	0.7961 (0.9757)	0	0	0
5000	25	5	0.000294	-0.00025 (-0.0283)	0.000835 (0.0270)	0.7263	0.7179 (0.3714)	0.7347 (1.0064)	0.002	0	0
5000		10	0.000163	-0.00035 (-0.0271)	0.000672 (0.0279)	0.7378	0.7324 (0.4931)	0.7432 (0.9528)	0	0	0

With a sample size of 100 and the percent of missing data increasing from 10% to 25% in increments of 5%, the mean difference and the geometric average ratio of variance is attenuated. The geometric average is decreasing with an increase in the percent missing. However, the geometric average increases as sample size increases for each fixed level of missing data. The worst situation occurs with a 25% missing data rate in which the geometric mean is 0.73 with five imputations. On the other hand, with a 5% missing data rate, the geometric mean hovers about 0.90. Overall, we can say by increasing the sample size with different percent of missingness the arithmetic mean is close to zero and the geometric mean ranges from 0.73 to 0.91. As the sample size increases, the percent count which is defined as the ratio of the observed variance to imputed variance being greater than 1, 1.05 and 1.1 approaches zero. This implies that multiple imputation under the Normal Distribution is a conservative imputation method in the sense that it does not underestimate the variance which would cause an over estimation of statistically significant tests of hypotheses.

4.2.2 Results of Cauchy Distribution

The Cauchy distribution is a symmetric distribution with heavy tails. It does not have a mean but samples taken from the Cauchy distribution will allow a sample mean and variance to be computed. In our study we have incorporated the Cauchy distribution as one of the distributions to check for the deviation from normality using the multiple imputation method. In Table 2 for a sample size of 500 with approximately 10% missing data and five imputations, the mean difference (True-Imputed) is -0.0604 (CI: -0.2679, 0.1470) and the geometric average of the ratio of the observed variance to the imputed variance is 0.9321 (CI: 0.8956, 0.9701). The percent count is counting the number of instances that have a ratio of observed variance and imputed variance greater than 1.0, 1.05 and 1.1 respectively. Of the simulations performed, 11.7% of the simulations have an average ratio of the variances greater than one. In addition, 9.7% of the simulations produce results that exceed 1.05 and 9.2% of the simulated results exceed 1.1. Using the same sample size, n=500 and 10% missing data, but with ten imputations, the mean difference is -0.1313 (CI: -0.3169, 0.0544) and the geometric average of the ratio of the observed variance to the imputed variance is 0.9364 (CI: 0.8990, 0.9753). Under these conditions, 11.6% of the simulations have an average ratio of the variances greater than one. Also, 10.3% and 8.8% of the ratios of observed variance to imputed variance are greater than 1.05 and 1.1 respectively.

Table 2: Cauchy distribution with Monotonic Missing data Pattern

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.0256	-0.3283 (-96.1665)	0.3795 (101.7)	0.8814	0.8439 (0.000121)	0.9205 (2397.3)	0.109	0.086	0.081
100		10	0.0362	-0.2577 (-68.3899)	0.3302 (73.2637)	0.8875	0.8494 (0.000164)	0.9273 (2111.5)	0.11	0.09	0.078
100	15	5	0.0115	-0.3955 (-90.9471)	0.4185 (89.6336)	0.8572	0.8047 (0.000224)	0.9132 (2279.3)	0.143	0.137	0.13
100		10	-0.2095	-0.5543 (-113.1)	0.1353 (30.4161)	0.8688	0.8162 (0.000652)	0.9247 (2697.5)	0.153	0.134	0.127
100	20	5	0.2553	-1.3579 (-220.8)	1.8685 (731.3)	0.8429	0.7800 (0.000136)	0.9108 (14589.0)	0.186	0.171	0.169
100		10	-0.6303	-1.7147 (-341.4)	0.4542 (316.5)	0.8486	0.7857 (0.000213)	0.9165 (11302.3)	0.184	0.173	0.17
100	25	5	0.5821	-0.4569 (-164.3)	1.6211 (300.2)	0.8027	0.7352 (0.000307)	0.8765 (22830.0)	0.201	0.195	0.184
100		10	0.3923	-1.0009 (-369.5)	1.7854 (461.7)	0.8073	0.7381 (0.000057)	0.8828 (23455.7)	0.209	0.195	0.185
500	10	5	-0.0604	-0.2679 (-70.4726)	0.1470 (45.1267)	0.9321	0.8956 (0.0559)	0.9701 (14530.0)	0.117	0.097	0.092
500		10	-0.1313	-0.3169 (-70.5545)	0.0544 (37.8704)	0.9364	0.8990 (0.00824)	0.9753 (15071.1)	0.116	0.103	0.088
500	15	5	-0.1059	-0.3583 (-70.3420)	0.1464 (33.4427)	0.9382	0.8873 (0.00171)	0.9919 (12187.9)	0.175	0.158	0.146
500		10	-0.1020	-0.3509 (-70.3193)	0.1468 (54.6617)	0.9392	0.8884 (0.00335)	0.9929 (12998.7)	0.167	0.156	0.144
500	20	5	-0.5602	-1.0148 (-92.1917)	-0.1056 (106.7)	0.9172	0.8554 (0.00122)	0.9835 (11913.0)	0.207	0.192	0.179
500		10	-0.2822	-0.6022 (-70.5661)	0.0377 (50.9887)	0.9358	0.8745 (0.00403)	1.0015 (8346.0)	0.199	0.187	0.176
500	25	5	-0.0824	-0.5896 (-135.1)	0.4247 (97.5532)	0.9110	0.8405 (0.000512)	0.9875 (8620.9)	0.225	0.217	0.21
500		10	-0.2239	-0.6865 (-115.4)	0.2387 (81.9705)	0.9294	0.8584 (0.000738)	1.0063 (9166.9)	0.227	0.216	0.206

With a sample size of 500, approximately 15% of the data missing and using ten imputations, the mean difference is -0.1020 (CI: -0.3509, 0.1468) and the geometric average of the ratio of variances is 0.9392 (CI: 0.8884, 0.9929). Under these conditions, 16.7% of the simulated results have an average ratio of variances greater than one, whereas, 15% and 14.4% of simulated results provide ratios of the variances greater than 1.05 and 1.10 respectively. With a sample size of n=500, approximately 20% of the data missing and ten imputations, the mean difference is -0.2822 (CI: -0.6022, 0.0377) and the geometric average of the ratio of the variances is 0.9358 (CI: 0.8745, 1.0015). Almost 20% of the simulated results have an average ratio of variances greater than one. In addition, 18.7% and 17.6% of simulated results produced ratios of the observed variance to the imputed variance in excess of 1.05 and 1.10 respectively.

Finally, with a sample size of n=500, approximately 25% of the data missing and ten imputations, the mean difference is -0.2239 (CI: -0.6865, 0.2387) and the geometric average of the ratio of the observed variance to the imputed variance is 0.9294 (CI: 0.8485, 1.0063). With an increase in the percentage of missing data, we also see an increase in an overestimation of the variance due to imputation. Of the simulated results, 22.7% of the simulations have an average ratio of the variances greater than one. In addition, 21.6% and 20.6% of ratios of the observed variances to the imputed variances exceed 1.05 and 1.10 respectively.

Table 2 provides the summary of the simulated results for the Cauchy Distribution with sample sizes of 100, 500, 1000 and 5000 and the percentage of missing data ranging from 10% to 25% with increments of 5%. Included in Table 2 are the mean difference and the geometric average of the ratio of observed variance to the imputed variance. The number of simulated results with the observed variance greater than the imputed variance by at least 5% and 10% is increasing as the percentage of missing data increases. In addition, the results are mixed with respect to the number of imputations. That is, using ten imputations rather than five imputations do not guarantee a reduction in the percentage of times the imputed variance is an underestimate of the true variance. Also, as the sample size increases, the number of simulations with an observed variance greater than imputed variance is increasing regardless of the amount of missing data. This suggests that when working with a symmetric, yet heavy tailed distribution, the chances of underestimating the true variance with multiple imputation is relatively high as sample size increases.

Table 2 continues: Cauchy distribution with Monotonic Missing data Pattern

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.0940	-0.2602 (-48.3381)	0.0722 (18.6433)	0.9123	0.8799 (0.0162)	0.9460 (7075.2)	0.1	0.087	0.082
1000		10	-0.0786	-0.1996 (-35.1939)	0.0424 (14.6012)	0.9228	0.8902 (0.0293)	0.9566 (6532.4)	0.108	0.091	0.085
1000	15	5	-0.1732	-0.4326 (-94.4528)	0.0863 (27.1632)	0.9122	0.8657 (0.00707)	0.9611 (5819.8)	0.16	0.156	0.143
1000		10	-0.0596	-0.2393 (-43.1517)	0.1201 (23.1687)	0.9205	0.8740 (0.0112)	0.9695 (7089.0)	0.164	0.156	0.141
1000	20	5	-0.0970	-0.3632 (-42.7572)	0.1692 (49.3410)	0.8985	0.8454 (0.0102)	0.9549 (6178.7)	0.195	0.186	0.17
1000		10	-0.1047	-0.3773 (-71.0262)	0.1680 (55.3226)	0.9023	0.8492 (0.0149)	0.9587 (6011.9)	0.188	0.176	0.171
1000	25	5	-0.2519	-0.5756 (-60.6591)	0.0718 (66.6588)	0.8981	0.8353 (0.00994)	0.9656 (3442.3)	0.224	0.211	0.201
1000		10	-0.2303	-0.5515 (-84.5325)	0.0908 (41.9382)	0.9100	0.8469 (0.0103)	0.9778 (5333.2)	0.217	0.204	0.194
5000	10	5	0.1485	-0.0355 (-14.2569)	0.3326 (75.6047)	0.9448	0.9147 (0.3754)	0.9759 (921.7)	0.136	0.115	0.101
5000		10	0.0571	-0.0712 (-24.1414)	0.1854 (30.0949)	0.9520	0.9217 (0.3681)	0.9833 (1073.0)	0.136	0.119	0.101
5000	15	5	0.0229	-0.2612 (-95.2121)	0.3070 (66.4965)	0.9468	0.9023 (0.0327)	0.9934 (3387.9)	0.168	0.154	0.144
5000		10	0.0931	-0.1693 (-47.5025)	0.3555 (78.7339)	0.9571	0.9117 (0.0406)	1.0048 (3891.0)	0.172	0.159	0.146
5000	20	5	0.00746	-0.5809 (-177.8)	0.5958 (167.4)	0.9447	0.8865 (0.000253)	1.0067 (3807.6)	0.214	0.201	0.191
5000		10	0.2373	-0.3017 (-133.3)	0.7762 (155.1)	0.9453	0.8881 (0.000271)	1.0062 (3408.7)	0.207	0.193	0.18
5000	25	5	0.1103	-0.3855 (-95.3305)	0.6060 (132.2)	0.9345	0.8687 (0.000214)	1.0053 (3140.4)	0.221	0.212	0.205
5000		10	-0.0782	-0.7154 (-222.6)	0.5591 (130.5)	0.9568	0.8903 (0.000775)	1.0284 (3114.1)	0.23	0.217	0.207

Overall, we can say by increasing the sample size with fixed percentage of missing data, the arithmetic mean and the variance are changing. As the sample size increases from 100 to 5000, the number of simulations with a ratio of observed variance to imputed variance being more than 1, 1.05, and 1.1 is increasing, yet the geometric average approaches one with an estimated geometric average of 0.96 with a sample size of 5000. While comparing the results of Cauchy distribution with normal distribution, which is considered the "Gold Standard", the number of simulated results providing an underestimate of the true variance is greater with the Cauchy distribution than that of the Normal Distribution. Although the Cauchy distribution is a symmetric distribution, it also has heavy tails and therefore multiple imputation may not work as well as it might with other symmetric yet non-normal distributions.

4.2.3 Results of t-Distribution

Simulation studies were conducted for the t-distribution with various degrees of freedom (df) ranging from 2 df to 30 df to demonstrate how a symmetric distribution behaves with respect to multiple imputation as one moves away from normality. This can be demonstrated by observing the behavior of the t-distribution as the degrees of freedom are decreased from 30 df to 2 df. Results of the t-distribution with 30 df are shown in Table 3 and the t-distribution for the other degrees of freedom are found in tables in appendix A.

The t-distribution is a symmetric distribution that asymptotically approaches the Normal Distribution as the degrees of freedom increase. We have used the t-distribution as one of the distributions to check for deviation from normality using multiple imputation with different percentages of missing data. Different degrees of freedom have been used to measure the change in the results as the t-distribution asymptotically approaches the normal distribution. We would expect the results to improve as the degrees of freedom increase.

For the t-distribution with 30 df and a sample size of 500 with approximately 10% missing data and five imputations, the mean difference is 0.000050 (CI: -0.00094, 0.00104) and the geometric average of the ratio of the observer variance to the imputed variance is 0.8954 (CI:

0.8912, 0.8997). Under the conditions of this simulation, 2% of the simulated results have an average ratio of variances greater than one. Also, 0.1% of the simulations produce a ratio of observed variance to imputed variance greater than 1.05. There were no simulated results producing a ratio of the variances greater than 1.1. Using the same conditions, only with ten imputations rather than five imputations, the mean difference is 0.000011 (CI: -0.00092, 0.00095) and the geometric average of the ratio of the observed variance to the imputed variance is 0.9046 (CI: 0.9016, 0.9075). Only 1.1% of the simulated results yielded an average ratio of variances greater than 1 and there were no simulated results producing a ratio of variances greater than 1.05.

For a sample size of 500 with approximately 15% of the data missing and ten imputations, the mean difference is 0.00039 (CI: -0.00085, 0.00163) and the geometric average of the ratio of variances is 0.8400 (CI: 0.8357, 0.8445). The simulations produced an average ratio greater than one at a rate of 0.3% and an average ratio of the variances greater than 1.05 at a rate of 0.1%. There were no simulated results with an average ratio of the variances greater than 1.1. Increasing the percentage of missing data to 20% and using ten imputations, the mean difference is 0.00068 (CI: -0.00078, 0.00214) and the geometric average of the ratio of the variances is 0.7851 (CI: 0.7800, 0.7902). There was a slight increase in the percentage of simulated results that produced an average ratio of variances greater than one. Under these conditions the rate was 0.4%. There were no simulated results with an average ratio of variances greater than 1.05. Finally, increasing the percentage of missing data to 25% missing and using ten imputations, the mean difference is 0.00016 (CI: -0.6865, 0.2387) and the geometric average of the ratio of the variances is 0.7962 (CI: 0.7304, 0.7420). There were no simulated results producing a ratio of variances greater than one.

Table 3: t-distribution (df = 30) with Monotonic Missing data Pattern

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00049	-0.00268 (-0.1044)	0.00170 (0.1423)	0.8853	0.8794 (0.5378)	0.8912 (1.2186)	0.079	0.018	0.006
100		10	-0.00003	-0.00215 (-0.1171)	0.00210 (0.1252)	0.8936	0.8892 (0.6733)	0.8981 (1.0818)	0.062	0.011	0
100	15	5	0.00163	-0.00117 (-0.1574)	0.00443 (0.1580)	0.8209	0.8134 (0.4496)	0.8285 (1.2578)	0.053	0.016	0.005
100		10	0.000908	-0.00181 (-0.1457)	0.00363 (0.1465)	0.8267	0.8209 (0.5178)	0.8325 (1.2223)	0.025	0.013	0.004
100	20	5	0.00397	0.000460 (-0.2038)	0.00747 (0.1764)	0.7421	0.7331 (0.3083)	0.7513 (1.2959)	0.027	0.013	0.007
100		10	0.00302	-0.00027 (-0.1825)	0.00631 (0.1666)	0.7700	0.7629 (0.3021)	0.7772 (1.2934)	0.022	0.011	0.004
100	25	5	0.00133	-0.00266 (-0.2105)	0.00532 (0.2057)	0.7014	0.6913 (0.2546)	0.7115 (1.1806)	0.032	0.012	0.005
100		10	0.00173	-0.00209 (-0.2208)	0.00556 (0.1858)	0.7026	0.6946 (0.3336)	0.7107 (1.1008)	0.012	0.002	0.001
500	10	5	0.000050	-0.00094 (-0.0518)	0.00104 (0.0585)	0.8954	0.8912 (0.6461)	0.8997 (1.0841)	0.02	0.001	0
500		10	0.000011	-0.00092 (-0.0513)	0.000946 (0.0601)	0.9046	0.9016 (0.7366)	0.9075 (1.0205)	0.011	0	0
500	15	5	0.000105	-0.00120 (-0.0804)	0.00141 (0.0770)	0.8328	0.8265 (0.4892)	0.8392 (1.0501)	0.02	0.001	0
500		10	0.000387	-0.00085 (-0.0686)	0.00163 (0.0636)	0.8400	0.8357 (0.6227)	0.8445 (1.0515)	0.003	0.001	0
500	20	5	0.000651	-0.00090 (-0.0917)	0.00220 (0.0740)	0.7761	0.7686 (0.3939)	0.7838 (1.0370)	0.007	0	0
500		10	0.000681	-0.00078 (-0.0820)	0.00214 (0.0675)	0.7851	0.7800 (0.5311)	0.7902 (1.0228)	0.004	0	0
500	25	5	0.000803	-0.00098 (-0.1332)	0.00259 (0.0867)	0.7278	0.7192 (0.3076)	0.7365 (1.1055)	0.012	0.001	0.001
500		10	0.000164	-0.00155 (-0.1041)	0.00188 (0.0954)	0.7362	0.7304 (0.4738)	0.7420 (0.9728)	0	0	0

Table 3, provides a summary of the t-distribution with 30 degrees of freedom for sample sizes: 100, 500, 1000 and 5000, in addition to various percentages of missing data. The percentage of missing data ranges from 10% to 25% in increments of 5%. The other attributes summarized in Table 3 include the mean difference between the observed mean and the imputed mean along with a 95% confidence interval and the geometric average of the ratio of the observed variance and imputed variance. For a fixed sample size, the geometric average is decreasing as the percentage of missing data increases. However, the geometric mean is fairly

Table 3 continues: t-distribution (df = 30) with Monotonic Missing data Pattern

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00063	-0.00133 (-0.0382)	0.000069 (0.0381)	0.8978	0.8938 (0.6347)	0.9018 (1.0238)	0.009	0	0
1000		10	-0.00071	-0.00137 (-0.0372)	-0.00004 (0.0327)	0.9040	0.9013 (0.6901)	0.9068 (1.0127)	0.002	0	0
1000	15	5	-0.00070	-0.00162 (-0.0475)	0.000217 (0.0552)	0.8341	0.8280 (0.5254)	0.8401 (1.0497)	0.009	0	0
1000		10	-0.00048	-0.00134 (-0.0485)	0.000381 (0.0371)	0.8424	0.8385 (0.6037)	0.8464 (1.0282)	0.001	0	0
1000	20	5	-0.00065	-0.00173 (-0.0605)	0.000436 (0.0568)	0.7735	0.7661 (0.4631)	0.7810 (1.0163)	0.005	0	0
1000		10	-0.00041	-0.00145 (-0.0669)	0.000624 (0.0480)	0.7957	0.7907 (0.5016)	0.8008 (0.9959)	0	0	0
1000	25	5	-0.00056	-0.00186 (-0.0783)	0.000740 (0.0643)	0.7307	0.7222 (0.3364)	0.7394 (1.0459)	0.002	0	0
1000		10	-0.00112	-0.00238 (-0.0674)	0.000135 (0.0685)	0.7382	0.7322 (0.4825)	0.7441 (0.9707)	0	0	0
5000	10	5	-0.00008	-0.00040 (-0.0205)	0.000239 (0.0160)	0.8961	0.8920 (0.6083)	0.9003 (0.9989)	0	0	0
5000		10	-0.00009	-0.00040 (-0.0193)	0.000222 (0.0154)	0.9048	0.9023 (0.7348)	0.9074 (0.9978)	0	0	0
5000	15	5	-0.00004	-0.00046 (-0.0185)	0.000370 (0.0229)	0.8424	0.8368 (0.5356)	0.8480 (0.9955)	0	0	0
5000		10	-0.00005	-0.00044 (-0.0207)	0.000347 (0.0205)	0.8512	0.8475 (0.6369)	0.8549 (0.9697)	0	0	0
5000	20	5	0.000043	-0.00045 (-0.0229)	0.000534 (0.0241)	0.7816	0.7744 (0.3553)	0.7889 (1.0107)	0.003	0	0
5000		10	0.000035	-0.00044 (-0.0226)	0.000511 (0.0301)	0.7917	0.7869 (0.5381)	0.7965 (0.9705)	0	0	0
5000	25	5	-0.00011	-0.00067 (-0.0255)	0.000459 (0.0276)	0.7298	0.7213 (0.3183)	0.7384 (0.9953)	0	0	0
5000		10	-0.00014	-0.00068 (-0.0277)	0.000401 (0.0286)	0.7389	0.7333 (0.4638)	0.7444 (0.9483)	0	0	0

consistent regardless of sample size. For example, the geometric mean is consistently 0.9 with only 5% of the data missing regardless of sample size and in the range of 0.73 with 25% of the data missing. On the other hand, multiple imputation appears to be conservative in the sense that it rarely provides a ratio of the variance greater than one. As sample size increases, the percentage of ratios of observed variance to imputed variance greater than one is decreasing. With sample sizes of 1000 and 5000, there were no observed simulated results that produced a variance ratio greater than 1.05, thus, there is evidence to support the notion that multiple imputation does not underestimate the variance and allow the null hypothesis to be falsely rejected.

Based on the results provided in the Table 3 and those in Appendix A, we can say that the results are improving as the degrees of freedom increases. As the sample size increases, the number of simulated results with the ratio of observed variance to the imputed variance being greater than 1, 1.05, and 1.1 is decreasing. On comparison of the results of the t-distribution with 30 degrees of freedom (Table 3) with that of the normal distribution (Table 1), we can conclude that the results are almost identical. As the degrees of freedom increase from 2 to 30 with an increase in sample size and various percentages of missing data, the number of simulations with a ratio of observed variance to imputed variance greater than one goes to zero. Based on these results we can say that multiple imputation appears to work well with a distribution such as t-distribution and its performance increases as degrees of freedom and sample size increase.

4.2.4 Results of Chi-Square Distribution

To explore the behavior of multiple imputation as we move away from symmetry to a skewed distribution. Simulation studies were conducted based on the Chi-square distribution using various different degrees of freedom. In this study, we used the following degrees of freedom: 2, 4, 6, 8, 10, 15, 20, 25, 30, 40 and 50. Results of the simulations based on the chi-square distribution with 50 df are provided in Table 4 and the results using the Chi-square distribution with the other degrees of freedom are found in the tables located in Appendix B.

The Chi-square distribution is an asymmetric distribution. However, as the degrees of freedom increase, it asymptotically approaches the normal distribution. The Chi-square distribution is used as one of the distributions to test for the appropriateness of using multiple imputation when the data is not normally distributed. Different degrees of freedom have been used to measure the change in the results.

Table 4: Chi-Square distribution (df = 50) with Monotonic Missing data Pattern

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00712	-0.0295 (-1.3158)	0.0152 (1.4387)	0.8857	0.8798 (0.4159)	0.8916 (1.2469)	0.083	0.029	0.009
100		10	-0.00537	-0.0267 (-1.1046)	0.0159 (1.2842)	0.8939	0.8893 (0.6065)	0.8986 (1.1439)	0.062	0.013	0.004
100	15	5	0.00654	-0.0227 (-1.4945)	0.0357 (2.1911)	0.8162	0.8083 (0.4247)	0.8242 (1.1286)	0.054	0.018	0.003
100		10	0.00952	-0.0184 (-1.5443)	0.0375 (2.1232)	0.8287	0.8227 (0.4626)	0.8347 (1.3008)	0.031	0.005	0.004
100	20	5	-0.00187	-0.0364 (-1.7737)	0.0326 (2.5726)	0.7600	0.7511 (0.3406)	0.7689 (1.2311)	0.035	0.013	0.006
100		10	0.0112	-0.0224 (-2.0322)	0.0448 (2.2416)	0.7742	0.7675 (0.4027)	0.7811 (1.1376)	0.024	0.008	0.002
100	25	5	0.000653	-0.0409 (-2.0854)	0.0422 (2.8124)	0.6963	0.6859 (0.2518)	0.7068 (1.2027)	0.036	0.019	0.009
100		10	-0.00607	-0.0449 (-2.2059)	0.0328 (2.1667)	0.7162	0.7086 (0.3705)	0.7239 (1.2296)	0.022	0.01	0.004
500	10	5	0.000172	-0.00930 (-0.4751)	0.00964 (0.4776)	0.8976	0.8933 (0.5730)	0.9020 (1.0525)	0.032	0.001	0
500		10	-0.00051	-0.00962 (-0.4643)	0.00859 (0.4838)	0.9026	0.8997 (0.6523)	0.9056 (1.0301)	0.007	0	0
500	15	5	-0.00370	-0.0163 (-0.8180)	0.00892 (0.5953)	0.8368	0.8306 (0.4981)	0.8431 (1.0619)	0.015	0.001	0
500		10	-0.00114	-0.0132 (-0.5818)	0.0109 (0.5668)	0.8473	0.8431 (0.5938)	0.8516 (1.0397)	0.006	0	0
500	20	5	-0.00741	-0.0227 (-0.8511)	0.00786 (0.7483)	0.7711	0.7634 (0.4355)	0.7789 (1.1405)	0.012	0.002	0.001
500		10	0.00316	-0.0112 (-0.7947)	0.0176 (0.6755)	0.7876	0.7823 (0.5217)	0.7929 (1.0221)	0.005	0	0
500	25	5	0.00335	-0.0138 (-1.1860)	0.0205 (0.8220)	0.7267	0.7183 (0.3075)	0.7353 (1.0310)	0.009	0	0
500		10	0.000525	-0.0163 (-0.7499)	0.0173 (0.8970)	0.7307	0.7246 (0.4419)	0.7368 (1.0207)	0.002	0	0

For a Chi-square distribution with 50 df, a sample size of 500 with approximately 10% missing data and using five imputations, the mean difference (True-Imputed) is 0.00017 (CI: -0.0093, 0.00964) and the geometric average of the ratio of observed variance to the imputed variance is 0.8976 (CI: 0.8933, 0.9020). Based on the simulations, 3.2% of the simulation results have a ratio of the observed variance to the imputed variance greater than one with 0.1% of the simulated results producing a variance ratio greater than 1.05 and no simulated results greater than 1.1. Under the same conditions as above but with ten imputations rather than five, the mean difference is -0.0005 (CI: -0.0096, 0.0086) and the geometric average of the ratio of observed variance to the imputed variance is 0.9026 (CI: 0.8997, 0.9056). With ten imputations, the simulated results produced a ratio of observed variance to imputed variance greater than one at a rate of 0.7% with no simulated results producing a variance ratio greater than 1.05.

With a sample size of 500 and increasing the percentage to approximately 15% missing and using ten imputations, the mean difference is -0.00114 (CI: -0.0132, 0.0109). The geometric average of the ratio of the variances is 0.8473 (CI: 0.8431, 0.8516). There was a slight increase in the percentage of variance ratios greater than one. The rate of variance ratios greater than one was 0.6% with no simulated results producing a variance ratio greater than 1.05. Increasing the percentage of missing data to approximately 20% and using ten imputations, the mean difference is 0.00316 (CI: -0.0112, 0.0176) and the geometric average of the ratio of the observed variance to the imputed variance is 0.7876 (CI: 0.7823, 0.7929). The percentage of simulated results producing a ratio of the observed variance to the imputed variance is 0.5% with no simulated results producing a ratio of variances greater than 1.5.

Finally at the maximum percentage of missing data 25%, and using ten imputations, the mean difference is 0.00053 (CI: -0.0163, 0.0173) and the geometric average of the ratio of the observed variance to the imputed variance is 0.7307 (CI: 0.7246, 0.7368). Surprisingly, the simulated results produced only 0.2% of the variance ratios greater than one and no variance ratios greater than 1.05.

Table 4 continues: Chi-Square distribution (df = 50) with Monotonic Missing data Pattern

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000345	-0.00622 (-0.2916)	0.00691 (0.3116)	0.9000	0.8959 (0.5822)	0.9041 (1.0232)	0.019	0	0
1000		10	0.000200	-0.00606 (-0.3034)	0.00646 (0.3259)	0.9038	0.9010 (0.7336)	0.9066 (1.0168)	0.002	0	0
1000	15	5	0.000721	-0.00790 (-0.3873)	0.00934 (0.4644)	0.8408	0.8348 (0.4876)	0.8469 (1.0226)	0.007	0	0
1000		10	0.00170	-0.00658 (-0.3726)	0.00997 (0.4500)	0.8483	0.8444 (0.6487)	0.8522 (1.0094)	0.001	0	0
1000	20	5	0.000895	-0.00945 (-0.5138)	0.0112 (0.5955)	0.7882	0.7810 (0.4156)	0.7955 (1.0197)	0.006	0	0
1000		10	-0.00283	-0.0128 (-0.4986)	0.00711 (0.6196)	0.7934	0.7887 (0.5411)	0.7982 (0.9740)	0	0	0
1000	25	5	-0.00540	-0.0177 (-0.5755)	0.00688 (0.6382)	0.7328	0.7245 (0.3772)	0.7413 (1.0551)	0.008	0.001	0
1000		10	-0.00415	-0.0158 (-0.6221)	0.00752 (0.7769)	0.7325	0.7268 (0.5099)	0.7383 (1.0042)	0.001	0	0
5000	10	5	-0.00280	-0.00577 (-0.1491)	0.000178 (0.1462)	0.8983	0.8943 (0.6490)	0.9023 (1.0072)	0.006	0	0
5000		10	-0.00226	-0.00515 (-0.1504)	0.000630 (0.1474)	0.9057	0.9031 (0.7455)	0.9083 (0.9924)	0	0	0
5000	15	5	-0.00325	-0.00730 (-0.2238)	0.000793 (0.1962)	0.8413	0.8356 (0.3925)	0.8471 (1.0043)	0.002	0	0
5000		10	-0.00242	-0.00630 (-0.2286)	0.00147 (0.1865)	0.8456	0.8417 (0.6051)	0.8494 (0.9695)	0	0	0
5000	20	5	-0.00521	-0.0100 (-0.2778)	-0.00042 (0.2422)	0.7882	0.7809 (0.4059)	0.7955 (1.0107)	0.002	0	0
5000		10	-0.00155	-0.00626 (-0.2688)	0.00315 (0.2285)	0.7896	0.7850 (0.5625)	0.7943 (0.9888)	0	0	0
5000	25	5	-0.00571	-0.0112 (-0.3069)	-0.00021 (0.2578)	0.7308	0.7225 (0.3355)	0.7392 (0.9969)	0	0	0
5000		10	-0.00527	-0.0107 (-0.2443)	0.000123 (0.2661)	0.7434	0.7380 (0.4939)	0.7489 (0.9468)	0	0	0

Table 4, provides a summary of the Chi-square distribution with 50 degrees of freedom for sample sizes: 100, 500, 1000 and 5000, in addition to various percentages of missing data. The percentage of missing data ranges from 10% to 25% in increments of 5%. The other attributes summarized in Table 4 include the mean difference between the observed mean and the imputed mean along with a 95% confidence interval and the geometric average of the ratio of the observed variance and imputed variance. For a fixed sample size, the geometric average is decreasing as the percentage of missing data increases. However, the geometric mean is fairly consistent regardless of sample size. For example, the geometric mean is consistently in the range of 0.88 to 0.90 with only 5% of the data missing regardless of sample size and in the range of 0.71 to 0.73 with 25% of the data missing. On the other hand, multiple imputation appears to be conservative in the sense that it rarely provides a ratio of the variance greater than 1.05. As sample size increases, the percentage of ratios of observed variance to imputed variance greater than one is decreasing. With sample sizes of 1000 and 5000, there were no observed simulated results that produced a variance ratio greater than 1.05, thus, there is evidence to support the notion that multiple imputation does not underestimate the variance and allow the null hypothesis to be falsely rejected

Based on the results provided in the Table 4 and Appendix B, we can say that the results are improving as the degrees of freedom increases. As the sample size increases, the percentage of variance ratios exceeding 1, 1.05 and 1.1 is decreasing. If the sample size is large, the chi-square distribution asymptotically approaches the normal distribution with mean n and variance $2n$. On comparison of the results of the Chi-square distribution with 50 degrees of freedom (Table 4) with that of the normal distribution (Table 1), we can conclude that the results are almost identical. In addition, as the degrees of freedom increase from 2 to 50 with an increase in sample size and various percentages of missing data, the number of simulations with a ratio of observed variance to imputed variance greater than one goes to zero. Based on these results we can say that multiple imputation appears to work well with a distribution such as Chi-square distribution and its performance increases as degrees of freedom and sample size increase.

4.3 Analysis of Tests of Hypotheses

4.3.1 Significance at α=0.05 for Monotone Missing data Pattern

Under the monotonic missing data pattern for the Normal Distribution (Table 5), the results based on the hypothesis: H_0: $\mu=0$ verses H_a: $\mu\neq0$ are compared with different types of missing data methods such as: full data which is considered the gold standard, available data, mean substitution, single regression imputation, and multiple imputation. With a sample size of 100 with 10% of the data missing, the full data analysis rejected the null hypothesis 62 times at an alpha level of 0.05. In comparison, under the same simulation conditions, using the available data, the null was rejected 54 times; using mean substitution, the null was rejected 79 times and with single regression imputation, the null was rejected 74 times. The multiple imputation with five imputations rejected the null 56 times and with ten imputations it rejected the null 54 times. For sample size of 5000 and 15% of the data missing, the full data rejected the null 42 times at the alpha level of 0.05. In comparison, under the same simulation conditions, using the available data, the null was rejected 51 times, using mean substitution, the null was rejected 74 times and with single regression imputation, the null was rejected 95. However, multiple imputation with 5 and 10 imputations rejected the null 52 and 50 times respectively.

Overall, we can see that the number of instances that reject the null hypothesis is greater using the mean substitution method and the single regression imputation method. This is most likely due to an underestimate of the variance as neither method adds variability to the data. Because the variance is underestimated, the test statistic is over-estimated. However, the number of instances rejecting the null hypothesis using multiple imputation method is approximately the same as that found by using the full data results. Therefore, it appears that multiple imputation works well compared to available data, mean substitution and single regression methods with the normal distribution in terms of type I error.

Table 5: Significance P-values for Normal Distribution

N	% Miss	N(0,1) Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	62	54	79	74	56 (54)
	15	62	57	91	83	57 (61)
	20	62	61	106	111	65 (59)
	25	62	56	115	122	60 (51)
500	10	52	50	73	76	50 (49)
	15	52	54	98	81	52 (57)
	20	52	47	107	112	54 (50)
	25	52	45	118	128	59 (49)
1000	10	55	55	71	70	57 (57)
	15	55	50	83	88	50 (50)
	20	55	52	100	108	48 (51)
	25	55	51	114	120	53 (52)
5000	10	42	50	64	69	50 (47)
	15	42	51	74	95	52 (50)
	20	42	43	90	103	49 (46)
	25	42	46	108	126	55 (54)

Under the monotonic missing data pattern with the t-distribution with 2 df (Table 6), the results are provided for testing the hypothesis (H_0: $\mu=0$ and H_a: $\mu\neq0$) comparing the different types of missing data methods: full data, available data, mean substitution, single regression imputation, and multiple imputation. With a sample size of 100 and 10% of the data missing, the full data analysis rejected the null hypothesis 45 times at an alpha level of 0.05. In comparison, under the same simulation conditions and using the available data for the analysis, the null hypothesis was rejected 36 times; using mean substitution, the null was rejected 65 times, and with single regression imputation, the null was rejected 63 times. The multiple imputation with five imputations rejected the null 36 times and with ten imputations it rejected the null 37 times. With a sample size of 5000 with 15% of the data missing, the full data analysis rejected the null 45 times. On the other hand, the available data analysis rejected the null 41 times, mean substitution rejected the null 84 times and single regression imputation rejected the null 92 times. However, multiple imputation with five and ten imputations rejected the null 37 and 45 times respectively.

Table 6: Significance P-values for t-Distribution with 2df

N	% Miss	t-dist with 2df Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	45	36	65	63	36 (37)
	15	45	39	82	91	37 (42)
	20	45	41	98	104	42 (37)
	25	45	43	128	133	41 (43)
500	10	42	39	50	61	44 (37)
	15	42	39	69	76	38 (39)
	20	42	46	91	111	41 (40)
	25	42	38	107	120	41 (40)
1000	10	36	28	62	66	41 (34)
	15	36	31	75	76	33 (33)
	20	36	30	90	105	41 (41)
	25	36	38	98	109	48 (38)
5000	10	45	49	70	76	51 (49)
	15	45	41	84	92	37 (45)
	20	45	47	102	112	46 (47)
	25	45	42	116	134	51 (46)

Based on the results of Table 6 and Appendix C, we can say that the number of simulated results that rejected the null hypothesis is greatest using either the mean substitution method or the single regression imputation method. Both the mean substitution method and the regression single imputation methods systematically underestimate variance because missing values are replaced with mean values, adding no variability while increasing the degrees of freedom. However, number of simulated results that reject the null hypothesis using the multiple imputation method is approximately the same as found using the full data analysis results. Therefore, we can say that multiple imputation appears to work well compared to other imputation methods such as available data, mean substitution and single regression methods for t-distribution with 2 df.

Under the monotonic missing data pattern for the Chi-square distribution with 2 df (Table 7), the results based on the hypothesis H_0: μ=2 and H_a: $\mu \neq 2$ are comparing the different types of missing data methods: full data analysis, available data analysis, mean substitution, single regression imputation, and multiple imputation. Observing the results of Table 7 with sample size of 100 and 10% of the data missing, the full data analysis rejected the null hypothesis 46

Table 7: Significance P-values for Chi-Square Distribution with 2 df

N	% Miss	Chi-Sqr dist with 2 df Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	46	53	79	72	51 (51)
	15	46	59	90	100	54 (54)
	20	46	70	114	125	66 (65)
	25	46	69	126	127	69 (68)
500	10	70	62	88	85	65 (62)
	15	70	61	100	106	61 (59)
	20	70	57	121	118	66 (60)
	25	70	60	143	136	66 (62)
1000	10	58	52	74	77	53 (53)
	15	58	54	93	101	61 (57)
	20	58	63	107	128	66 (62)
	25	58	56	127	129	60 (55)
5000	10	42	40	61	72	43 (34)
	15	42	41	79	76	46 (38)
	20	42	43	92	97	48 (42)
	25	42	46	111	132	46 (46)

times with a significance level 0.05. The available data analysis rejected the null 53 times, mean substitution rejected the null 79 times and single regression imputation rejected the null 72 times. However, multiple imputation with 5 and 10 imputations rejected the null hypothesis 51 times. With a sample size of 5000 and 15% of the data missing, the full data analysis rejected the null hypothesis 42 times, whereas using the available data analysis rejected the null 41 times, mean substitution rejected the null 79 times and single regression imputation rejected the null 76 times. Multiple imputation with five and ten imputations rejected the null 46 and 38 times respectively.

Once again, based on the results of Table 7 and Appendix C, we can say that the number of simulated results rejecting the null hypothesis is greatest using either the mean substitution method or the single regression imputation method. As in the previous simulations this is most likely due to an underestimation of the variance when using those methods. However, the number of simulated results rejecting the null hypothesis using the multiple imputation method is approximately the same as that of the full data analysis results. Therefore, it appears that multiple imputation works well compared to the available data analysis method, mean substitution imputation and single regression imputation for the chi-square distribution with 2 df.

4.3.2 Sensitivity and Specificity Results with Monotone Missing Data Pattern

The results of sensitivity and specificity are presented to assure that the simulations that rejected the null hypothesis are actually same as those of the full data analysis. Sensitivity measures the proportion of actual positives which are correctly identified as such. Specificity measures the proportion of negatives which are correctly identified. We provide the sensitivity and specificity of each imputation method with respect to rejecting the null hypothesis under each condition. The analysis based on the full data set is the gold standard used in this analysis. Based on the Normal distribution (Table 8) with a sample size of 100 and 10% of the data missing, the available data analysis has sensitivity of 98.1 and specificity of 81.48. The mean substitution has sensitivity of 99.24 and specificity of 69.62. The single imputation has sensitivity of 98.49 and specificity of 64.86. The multiple imputation with five imputations has sensitivity of 97.99 and specificity of 76.79. Multiple imputation with ten imputations has sensitivity of 98.20 and specificity of 83.33. When the sample size is increased to 500 with 15% of the data missing, the available data analysis has sensitivity of 97.89 and specificity of 59.26. The mean substitution has sensitivity of 99.56 and specificity of 48.98. The single regression imputation has sensitivity of 98.37 and specificity of 45.68. Multiple imputation with five imputations has sensitivity of 97.89 and specificity of 61.54. Multiple imputation with ten imputations has sensitivity of 97.99 and specificity of 57.89.

With a sample size of 5000 and with 25% of the data missing, the available data analysis has sensitivity of 98.74 and specificity of 65.22. The mean substitution method has sensitivity of 99.78 and specificity of 37.04. The single imputation method has sensitivity of 99.20 and specificity of 27.78. Multiple imputation with five imputations has sensitivity of 98.94 and specificity of 58.18. Multiple imputation with ten imputations has sensitivity of 98.63 and specificity of 53.70.

Table 8: Sensitivity and Specificity for Normal Distribution

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	98.10	81.48	99.24	69.62	98.49	64.86	97.99 (98.20)	76.79 (83.33)
	15	97.78	71.93	99.12	59.34	97.71	49.40	97.77 (97.98)	71.93 (70.49)
	20	97.45	62.30	98.99	50.00	98.20	41.44	97.65 (97.45)	61.54 (64.41)
	25	97.25	64.29	98.87	45.22	97.50	32.79	97.13 (96.94)	58.33 (64.71)
500	10	98.21	70.00	99.14	60.27	98.38	48.68	98.32 (98.11)	72.00 (69.39)
	15	97.89	59.26	99.56	48.98	98.37	45.68	97.89 (97.99)	61.54 (57.89)
	20	97.27	55.32	99.11	41.12	98.20	32.14	97.36 (97.47)	50.00 (56.00)
	25	96.65	44.44	99.32	38.98	97.94	26.56	97.02 (96.74)	40.68 (42.86)
1000	10	98.84	80.00	99.25	67.61	98.71	61.43	98.73 (98.73)	75.44 (75.44)
	15	98.11	74.00	98.91	54.22	98.25	44.32	98.42 (98.00)	80.00 (72.00)
	20	98.10	71.15	99.22	48.00	98.43	37.96	97.48 (97.79)	64.58 (66.67)
	25	97.89	68.63	99.10	41.23	98.30	33.33	97.57 (97.26)	60.38 (55.77)
5000	10	99.26	70.00	99.57	59.38	99.46	53.62	98.95 (99.06)	64.00 (70.21)
	15	99.05	64.71	99.68	52.70	99.23	36.84	99.16 (99.16)	65.38 (68.00)
	20	98.64	67.44	99.67	43.33	99.33	34.95	98.84 (98.64)	63.27 (63.04)
	25	98.74	65.22	99.78	37.04	99.20	27.78	98.94 (98.63)	58.18 (53.70)

**Results of Multiple Imputation are based on computation.*
Reference group is Full data [N (0,1)]

From the results of multiple imputation under the Normal distribution (Table 8), we can see that with a sample size of 100 and 25% of the data missing with five imputations, 97.1% of the tests that should not be rejected, are not rejected. In addition, 58.3% of the tests that should be rejected were rejected. With sample size of 500 and 25% of the data missing with five imputations, 97% of the tests that should not be rejected are not rejected and 40.7% of the tests that should be rejected are rejected. With sample size of 1000 and 25% of the data missing with five imputations, 97.6% of the tests that should not be rejected are not rejected. On the other hand, 60.4% of the tests that should be rejected were rejected. Finally using a sample size of 5000 and with 25% of the data missing with five imputations, 98.9% of the tests that should not be rejected are not rejected and 58.2% of the tests that should be rejected were rejected. Therefore, multiple imputation, when compared to the other imputation techniques, outperforms those techniques with respect to specificity under normality. All methods provide equally high sensitivity.

The t-distribution with 2 degrees of freedom (Table 9) with a sample size of 100 and 10% of the data is missing, with the full data analysis as the gold standard, indicates that the available data analysis yields a sensitivity of 98.34 and specificity of 80.56. The mean substitution method has sensitivity of 99.25 and specificity of 58.46. The single regression imputation method has sensitivity of 98.72 and specificity of 52.38. The multiple imputation with five imputations has sensitivity of 98.03 and specificity of 72.22. The multiple imputation with ten imputations has sensitivity of 98.13 and specificity of 72.97. When the sample size is increased to 500 and 15% of the data is missing, the available data analysis provides sensitivity of 98.23 and specificity of 64.10. The mean substitution imputation method has sensitivity of 99.36 and specificity of 52.17. The single regression imputation method provides sensitivity of 98.05 and specificity of 31.58. Multiple imputation with five imputations has sensitivity of 98.34 and specificity of 68.42. Multiple imputation with ten imputations has sensitivity of 98.23 and specificity of 64.10.

With the maximum sample size of 5000 and 25% of the data missing, the available data analysis provides sensitivity of 97.50 and specificity of 50.00. The mean substitution imputation method yields sensitivity of 99.66 and specificity of 36.21. The single regression imputation method has sensitivity of 98.04 and specificity of 20.90. Multiple imputation with five imputations has sensitivity of 97.68 and specificity of 45.10. Multiple imputation with ten imputations has sensitivity of 97.59 and specificity of 47.83.

Table 9: Sensitivity and Specificity for t-Distribution with 2 df

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	98.34	80.56	99.25	58.46	98.72	52.38	98.03 (98.13)	72.22 (72.97)
	15	98.02	66.67	99.56	50.00	98.79	37.36	97.72 (98.23)	62.16 (66.67)
	20	97.92	60.98	99.78	43.88	98.66	31.73	97.81 (97.20)	57.14 (48.65)
	25	97.70	53.49	99.77	33.59	97.35	16.54	97.81 (97.18)	58.54 (41.86)
500	10	98.75	76.92	99.47	74.00	98.62	47.54	98.85 (98.65)	70.45 (78.38)
	15	98.23	64.10	99.36	52.17	98.05	31.58	98.34 (98.23)	68.42 (64.10)
	20	98.12	52.17	99.45	40.66	98.20	23.42	97.81 (98.23)	51.22 (62.50)
	25	98.03	60.53	99.66	36.45	98.15	21.67	97.81 (97.71)	51.22 (50.00)
1000	10	98.36	71.43	98.36	48.39	99.14	42.42	98.75 (98.45)	58.54 (61.76)
	15	98.04	54.84	99.46	41.33	98.27	26.32	98.04 (98.35)	51.52 (60.61)
	20	98.04	56.67	99.23	32.22	98.55	21.90	97.91 (98.02)	39.02 (41.46)
	25	98.23	50.00	99.34	30.61	98.32	19.27	98.21 (98.02)	39.58 (44.74)
5000	10	98.53	63.27	99.36	55.71	98.59	42.11	98.63 (98.84)	62.75 (69.39)
	15	98.13	65.85	99.45	47.62	98.90	38.04	97.51 (97.80)	56.76 (53.33)
	20	98.01	55.32	99.56	40.20	98.31	26.79	97.69 (97.59)	50.00 (46.81)
	25	97.50	50.00	99.66	36.21	98.04	20.90	97.68 (97.59)	45.10 (47.83)

From the results of multiple imputation under t-distribution (Table 9), we can see that with a sample size of 100 and 25% of the data missing with five imputations, 97.8% of the tests that should not be rejected are not rejected and 58.5% of the tests that should be rejected were rejected. With a sample size of 500 and 25% missing data with 5 imputations, 97.8% of tests that should not be rejected are not rejected and 51.2% of the tests that should be rejected were rejected. With a sample size of 1000 and 25% of the data missing with five imputations, 98.2% of the tests that should not be rejected are not rejected while 39.6% of the tests that should be rejected were rejected. Finally, with a sample size of 5000 and 25% of the data missing with five imputations, 97.7% of the tests that should not be rejected are not rejected and 45.1% of the tests that should be rejected were rejected.

Examination of the results (from Table 9 and Appendix C) show that all of the methods perform about the same with respect to sensitivity which is to be expected since the simulations were designed to reject only 5% of the hypotheses, and the number of simulations were large. On the other hand, when considering the specificity, we see that multiple imputation did better than both the mean substitution method and the single imputation method but not as well as the available data analysis method. One explanation for this is that the data are "missing at random" and therefore the available data analysis was a fairly good representation of the full data, as the available data analysis is a valid method of analysis under MAR. The major drawback to the available data analysis method is a loss of degrees of freedom. However with sample sizes of 100, 500, 1000, and 5000 with at most 25% of the data missing, this is not a severe problem.

Looking at the Chi-square distribution with 2 df (Table 10) with a sample size of 100 and 10% of the data missing, the available data analysis provides sensitivity of 99.05 and specificity of 69.81 with the full data analysis being used as the gold standard. The mean substitution imputation method has sensitivity of 99.78 and specificity of 55.70. The single regression imputation has sensitivity of 99.03 and specificity of 51.39. The multiple imputation with five imputations has sensitivity of 99.26 and specificity of 76.47. The multiple imputation with ten imputations has sensitivity of 98.95 and specificity of 70.59. Increasing the sample size to 500 and having 15% of the data missing, the available data analysis has sensitivity of 97.12 and specificity of 70.49. The mean substitution imputation method has sensitivity of 99.11 and specificity of 62.00. The single regression imputation has sensitivity of 97.99 and specificity of 49.06. Multiple imputation with five imputations has sensitivity of 96.70 and specificity of 63.93. Multiple imputation with ten imputations has sensitivity of 97.02 and specificity of 71.19.

Table 10: Sensitivity and Specificity for Chi-square with 2 df

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	99.05	69.81	99.78	55.70	99.03	51.39	99.26 (98.95)	76.47 (70.59)
	15	99.04	62.71	99.45	45.56	99.11	38.00	98.73 (98.52)	62.96 (59.26)
	20	99.03	52.86	99.77	38.60	98.63	27.20	98.50 (98.61)	48.48 (50.77)
	25	99.03	53.62	99.89	35.71	98.85	28.35	98.60 (98.71)	47.83 (50.00)
500	10	98.19	85.48	99.56	75.00	98.25	63.53	98.07 (98.08)	80.00 (83.87)
	15	97.12	70.49	99.11	62.00	97.99	49.06	96.70 (97.02)	63.93 (71.19)
	20	96.92	71.93	98.75	48.76	96.71	34.75	97.11 (96.81)	65.15 (66.67)
	25	96.91	68.33	99.42	45.45	97.45	35.29	96.25 (96.70)	53.03 (62.90)
1000	10	98.31	80.77	99.46	71.62	98.27	54.55	98.20 (98.20)	77.36 (77.36)
	15	97.46	62.96	98.79	50.54	98.11	40.59	97.34 (97.24)	54.10 (56.14)
	20	97.33	52.38	98.66	42.99	98.05	32.03	97.11 (97.12)	46.97 (50.00)
	25	96.93	51.79	99.20	40.16	97.70	29.46	97.23 (96.40)	53.33 (43.64)
5000	10	98.75	75.00	99.25	57.38	99.25	48.61	98.64 (98.45)	67.44 (79.41)
	15	98.44	65.85	99.46	46.84	98.16	32.89	98.11 (98.34)	52.17 (68.42)
	20	98.01	53.49	99.23	38.04	98.45	28.87	97.79 (98.12)	43.75 (57.14)
	25	97.90	47.83	99.66	35.14	98.73	23.48	97.59 (98.11)	41.30 (52.17)

With a sample size of 5000 and 25% of the data missing, the available data analysis has sensitivity of 97.90 and specificity of 47.83. The mean substitution method has sensitivity of 99.66 and specificity of 35.14. The single regression imputation method has sensitivity of 98.73

and specificity of 23.48. Multiple imputation with five imputations has sensitivity of 97.59 and specificity of 41.30. Multiple imputation with ten imputations has sensitivity of 98.11 and specificity of 52.17.

From the results of multiple imputation under Chi-square distribution (Table 10), given a sample size of 100 and 25% of the data is missing with five imputations, 98.6% of the tests that should not be rejected are not rejected and 47.8% of the tests that should be rejected are rejected. With sample size of 500 subject to 25% missing data with five imputations, 96.3% of the tests that should not be rejected are not rejected while 53.0% of the tests that should be rejected are rejected. Increasing the sample size to 1000 and allowing 25% of the data to be missing with five imputations, 97.2% of the tests that should not be rejected are not rejected while 53.3% of the tests that should be rejected are rejected. With sample size of 5000 and 25% of the data missing with five imputations, 97.6% of the tests that should not be rejected are not rejected and only 41.3% of the tests that should be rejected are rejected.

Examination of the results (from Table 10 and Appendix C) show that all of the methods perform about the same with respect to sensitivity which is to be expected since the simulations were designed to reject only 5% of the hypotheses, and the number of simulations were large. On the other hand, when considering the specificity, we see that multiple imputation did better than both the mean substitution method and the single imputation method but not as well as the available data analysis method. One explanation for this is that the data are "missing at random" and therefore the available data analysis was a fairly good representation of the full data, as the available data analysis is a valid method of analysis under MAR. The major drawback to the available data analysis method is a loss of degrees of freedom. However with sample sizes of 100, 500, 1000, and 5000 with at most 25% of the data missing, this is not a severe problem.

5. RESULTS OF NON-MONOTONE MISSING DATA PATTERN

5.1 Simulation

In this chapter we explore the behavior of multiple imputation under the same conditions as chapter four with the exception that the missing data pattern is not monotonic but random. We conducted simulation studies and sensitivity analysis with different percentages of missingness in the data. We simulated 1,000 samples for each of four sample sizes; 100, 500, 1000, and 5000. For consistency we generated the same random number for each variable: X_1, X_2, X_3, Z_1, U_1, U_2, U_3 in our sensitivity analysis that compares the following distributions: Normal distribution, Cauchy distribution, t-distribution, Chi-square distribution. In the simulations we assume data is missing at random (MAR) with a missing data rate of approximately 10%, 15%, 20%, and 25%. The SAS multiple imputation procedure (Proc MI) is used to create multiple data sets based on the number of imputations requested and incorporates the "within" and "between" subject variability to obtain an estimate of the variance. As in chapter 4, each missing value was replaced with set of five and ten imputations. There were three covariates created X_1, X_2, X_3 and the response variable Z_1 for each distribution. Random uniform variables U_1, U_2, and U3 are generated to determine which values of X_i are deleted. The same X_i are deleted for each non-monotonic simulation policy. The values for each policy are different as they come from different distributions, but the position in the data matrix remains the same. This allows for a more valid comparison of policies.

The non-monotone missing data pattern was simulated using approximately 10, 15, 20, and 25 percent of missing on the response variable Z_1, as well as the covariates X_1 and X_2. To generate approximately 10% of missing data on Z_1 and the two covariates X_1 and X_2 we generate three uniform random variables, U_1, U_2, and U_3. If $U_1 < 0.10$ then the value for X_1 is deleted, creating approximately 10% of the data to be missing for variable X_1; if $U_2 < 0.10$ then the value for X_2 is deleted, creating approximately 10% of the data to be missing for X_2; finally if $U_3 < 0.10$ then the value for Z_1 is deleted, creating approximately 10% of the data to be missing for Z_1. The same technique was used to generate approximately 15, 20, and 25 percent of missing in the original data, using 0.15, 0.20, and 0.25 as cutoffs for U_1, U_2, and U_3.

The selection of the model depends on the missing data pattern. For a non-monotonic missing data pattern, the Markov Chain Monte Carlo (MCMC) method is used under the assumption of multivariate normality. Letting *m* denote the number of imputations, *m* completed datasets are created and an analysis is conducted on each of the completed data sets. Using PROC MIANALYZE, valid statistical estimates of parameters can be generated. The estimates are combined to provide one result from the *m* different analyses. Multiple imputation incorporates the "within" and "between" variability, therefore we expect the imputed variance to be greater than the true variance. However, there may be a few instances with a smaller imputed variance than the true variance. As part of the evaluation of multiple imputation in this chapter, we recorded the number of simulated results in which the ratio of the observed variance to the imputed variance greater than 1, 1.05 and 1.1.

In this chapter we will discuss the results of a non-monotone missing data pattern under the assumption of normality as well as under the assumption of non-normal distributions such as the Cauchy distribution, the t-distribution with various degrees of freedom, and the Chi-square distribution with various degrees of freedom. To support the results we show how many simulated results reject the null hypothesis under each method of analysis which includes: available data analysis, mean substitution imputation, single regression imputation, and multiple imputation. Moreover, we compute the sensitivity and specificity of each method to assure that the simulated results that reject the null hypothesis are actually same as the analysis on the full data.

5.2 Results of Different Distributions

5.2.1 Results of Normal Distribution

From Table 11 with a sample size of 500 with approximately 10% of the data missing and five imputations, the difference of the observed mean and the imputed mean is 0.000235 (CI: -0.00076, 0.00123) and the geometric average of the ratio of observed variance to the imputed variance is 0.8822 (CI: 0.8774, 0.8871). On examination of the variance ratios, 2.3% of the simulated results have a ratio of the observed variance to the imputed variance greater than one. There were no simulated results that produced a variance ratio greater than 1.05. Under the same conditions with respect to sample size and percentage of data missing but using ten imputations, the mean difference is 0.000242 (CI: -0.00075, 0.00123) and the geometric average of the ratio of the variances is 0.8902 (CI: 0.8870, 0.8934). With an increase of imputations, only 1.2% of the simulated results have a ratio of observed variance to imputed variance greater than one, with no simulated results of the variance ratio greater than 1.05.

For a sample size of 500 with approximately 15% missing data and ten imputations, the mean difference is -0.00001 (CI: -0.00124, 0.00121) and the geometric average of the ratio of the variances is 0.8400 (CI: 0.8359, 0.8442). Under these conditions, only 0.2% of the simulated results produce a ratio of the observed variance to the imputed variance that is greater than one. Increasing the amount of missing data to 20% and using ten imputations, the mean difference is -0.00033 (CI: -0.00180, 0.00114) and the geometric average of the ratio of the variances is 0.7826 (CI: 0.7773, 0.7881). With 20% missing data and ten imputations, 0.1% of the simulated results have a ratio of the observed variance to the imputed variance that is greater than one. There were no simulated results with a ratio of the observed variance to the imputed variance greater than 1.05 or 1.1.

Finally, with approximately 25% of the data missing and using ten imputations, the mean difference is -0.000098 (CI: -0.00270, 0.000731) and the geometric average of the ratio of the variances is 0.7350 (CI: 0.7292, 0.7410). With 25% missing data and ten imputations, 0.2% of the simulated results have a ratio of the observed variance to the imputed variance greater than one, with no simulated results producing a ratio of the variances greater than 1.05 or 1.1.

Table 11: Normal distribution with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.000323	-0.00196 (-0.1119)	0.00261 (0.1465)	0.8776	0.8718 (0.5926)	0.8834 (1.1213)	0.066	0.012	0.002
100		10	-0.00010	-0.00229 (-0.1103)	0.00208 (0.1393)	0.8829	0.8782 (0.6315)	0.8876 (1.1387)	0.06	0.007	0.003
100	15	5	0.000050	-0.00290 (-0.1765)	0.00300 (0.1246)	0.8222	0.8147 (0.3849)	0.8297 (1.1344)	0.044	0.012	0.002
100		10	-0.00005	-0.00281 (-0.1971)	0.00271 (0.1236)	0.8267	0.8210 (0.5267)	0.8325 (1.0963)	0.019	0.003	0
100	20	5	-0.00226	-0.00571 (-0.2148)	0.00119 (0.1705)	0.7659	0.7573 (0.3420)	0.7746 (1.2211)	0.034	0.014	0.005
100		10	-0.00179	-0.00502 (-0.2049)	0.00145 (0.1941)	0.7704	0.7637 (0.4585)	0.7771 (1.1379)	0.013	0.003	0.002
100	25	5	-0.00183	-0.00577 (-0.2563)	0.00211 (0.2045)	0.7042	0.6945 (0.2721)	0.7140 (1.2152)	0.023	0.012	0.007
100		10	-0.00243	-0.00614 (-0.2490)	0.00129 (0.2101)	0.7165	0.7093 (0.3912)	0.7239 (1.1202)	0.011	0.003	0.002
500	10	5	0.000235	-0.00076 (-0.0571)	0.00123 (0.0561)	0.8822	0.8774 (0.5532)	0.8871 (1.0442)	0.023	0	0
500		10	0.000242	-0.00075 (-0.0514)	0.00123 (0.0516)	0.8902	0.8870 (0.6583)	0.8934 (1.0315)	0.012	0	0
500	15	5	0.000616	-0.00068 (-0.0662)	0.00191 (0.0645)	0.8272	0.8211 (0.5522)	0.8334 (1.0540)	0.016	0.002	0
500		10	-0.00001	-0.00124 (-0.0598)	0.00121 (0.0603)	0.8400	0.8359 (0.5965)	0.8442 (1.0053)	0.002	0	0
500	20	5	0.000102	-0.00142 (-0.0800)	0.00163 (0.0744)	0.7766	0.7689 (0.2899)	0.7845 (1.0297)	0.013	0	0
500		10	-0.00033	-0.00180 (-0.0780)	0.00114 (0.0690)	0.7826	0.7773 (0.4926)	0.7881 (1.0331)	0.001	0	0
500	25	5	-0.00007	-0.00186 (-0.0894)	0.00173 (0.0821)	0.7269	0.7186 (0.3589)	0.7352 (1.0443)	0.006	0	0
500		10	-0.00098	-0.00270 (-0.0806)	0.000731 (0.0696)	0.7350	0.7292 (0.4215)	0.7410 (1.0158)	0.002	0	0

Table 11 continues: Normal distribution with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000110	-0.00062 (-0.0327)	0.000839 (0.0431)	0.8858	0.8814 (0.6421)	0.8902 (1.0188)	0.013	0	0
1000		10	0.000168	-0.00053 (-0.0333)	0.000866 (0.0391)	0.8881	0.8850 (0.7078)	0.8911 (1.0031)	0.002	0	0
1000	15	5	-0.00034	-0.00126 (-0.0445)	0.000579 (0.0475)	0.8308	0.8246 (0.4899)	0.8370 (1.0359)	0.009	0	0
1000		10	-0.00006	-0.00094 (-0.0441)	0.000815 (0.0555)	0.8350	0.8306 (0.5731)	0.8394 (0.9987)	0	0	0
1000	20	5	-0.00016	-0.00124 (-0.0633)	0.000923 (0.0538)	0.7774	0.7701 (0.4089)	0.7848 (1.0338)	0.005	0	0
1000		10	-0.00033	-0.00138 (-0.0453)	0.000713 (0.0518)	0.7864	0.7815 (0.5186)	0.7914 (0.9791)	0	0	0
1000	25	5	-0.00041	-0.00164 (-0.0601)	0.000823 (0.0724)	0.7330	0.7246 (0.3305)	0.7416 (1.0213)	0.002	0	0
1000		10	-0.00054	-0.00177 (-0.0552)	0.000679 (0.0691)	0.7352	0.7296 (0.4406)	0.7409 (1.0421)	0.001	0	0
5000	10	5	0.000132	-0.00019 (-0.0158)	0.000458 (0.0171)	0.8828	0.8785 (0.6303)	0.8871 (1.0043)	0.002	0	0
5000		10	0.000063	-0.00025 (-0.0179)	0.000371 (0.0146)	0.8891	0.8863 (0.7286)	0.8919 (0.9866)	0	0	0
5000	15	5	0.000021	-0.00039 (-0.0229)	0.000436 (0.0219)	0.8329	0.8268 (0.4958)	0.8389 (1.0077)	0.001	0	0
5000		10	-0.00005	-0.00045 (-0.0226)	0.000350 (0.0204)	0.8401	0.8362 (0.6521)	0.8439 (0.9808)	0	0	0
5000	20	5	-0.00014	-0.00062 (-0.0270)	0.000346 (0.0237)	0.7749	0.7675 (0.3724)	0.7823 (1.0112)	0.001	0	0
5000		10	-0.00007	-0.00053 (-0.0260)	0.000392 (0.0232)	0.7851	0.7802 (0.5137)	0.7901 (0.9744)	0	0	0
5000	25	5	-0.00057	-0.00113 (-0.0259)	-0.00002 (0.0256)	0.7238	0.7156 (0.3511)	0.7321 (1.0016)	0.001	0	0
5000		10	-0.00037	-0.00090 (-0.0263)	0.000163 (0.0247)	0.7377	0.7321 (0.4758)	0.7432 (0.9616)	0	0	0

With a sample size of 100 and the percentage of missing data increasing from 10% to 25% in increments of 5%, the mean difference and the geometric average of the ratio of variance is attenuated. The geometric average is decreasing with an increase in the percent of missing data. However, as in the case with a monotonic missing data pattern, as sample size increases, the geometric average increase for each fixed level of missing data. The worst case occurs with a sample size of 100 and 25% of the data missing using five imputations in which the geometric mean is 0.704. On the other hand, with only 5% of the data missing, the geometric mean hovers about 0.88. As the sample size increases, the number of simulated results being greater than 1, 1.05 and 1.1 approaches zero. This suggests that multiple imputation applied under the condition of normality is a conservative imputation method in the sense that it does not underestimate the variance. However, the imputed variance tends to be slightly greater than the true variance hence, it is less likely to reject a null hypothesis.

5.2.2 Results of Cauchy Distribution

As in Chapter 4, we examine the Cauchy Distribution to explore how multiple imputation with a non-monotonic missing data pattern will behave under various sample sizes and percentage of missing data. In Table 12 for a sample size of 500 with approximately 10% missing data and five imputations, the mean difference is 2.6828 (CI: -2.7269, 8.0925) and the geometric average of the ratio of observed variance to the imputed variance is 0.9821 (CI: 0.9356, 1.0310). Under these conditions, 15.7% of the simulated results produced an average ratio of the variances greater than one. At the same time, 13.7% of ratio of the observed variances to the imputed variance is greater than 1.05 and 12.6% are greater than 1.1. Maintaining the sample size at n=500 and the percent of missing data at10% but increasing the number of imputations to ten, the mean difference is 0.7012 (CI: -0.6149, 2.0172) and the geometric average of the ratio of the variances is 0.9948 (CI: 0.9475, 1.0444). With the increase in the number of imputations, the percentage of simulated results exceeding one is reduced to 15.4%. The percentage of simulated results exceeding 1.05 is 13.7% and the percentage of results exceeding 1.10 is 12.5%.

Maintaining a sample size of n=500 and using ten imputations while increasing the percentage of missing to 15% results in mean difference of 0.5506 (CI: -0.1674, 1.2686) and a geometric average of the ratio of the variances of 0.9424 (CI: 0.8926, 0.9949). As expected, with an increase in the percentage of missing data, the results of the simulations demonstrate an increase in the number of times the imputed variance is smaller than the observed variance. In this case, 17.8% of the simulated results produced a ratio of the variances greater than one. In addition, 16.3% of the simulated results produced a variance ratio greater than 1.05 while 15% of the simulated results exceeded 1.10. By increasing the percentage of missing data to 20% and using ten imputations, the mean difference is 0.9237 (CI: -0.7344, 2.5819) and the geometric average of the ratio of the variances is 0.9539 (CI: 0.8932, 1.0188). Again, the percentage of simulated results that produce ratio of variances is greater than one increase. In this case, 19.8% of the simulated results exceed one while 18.4% exceed 1.05 and 17.9% exceed 1.10. Finally, with approximately 25% missing data and ten imputations, the mean difference is 1.6098 (CI: -0.7698, 3.9887) and the geometric average of the ratio of the variances is 0.9583 (CI: 0.8852, 1.0374). The simulated results yielded 21.3% of the variance ratios greater than one with 20% greater than 1.05 and 19% greater than 1.10.

Table 12 provides a summary of the simulated results for the Cauchy distribution with a non-monotonic missing data pattern for sample sizes of 100, 500, 1000 and 5000 and the percentage of missing data ranging from 10% to 25% in increments of 5%. Included in Table 12 are the mean difference and the geometric average of the ratio of the observed variance to the imputed variance. The number of simulated results with the observed variance greater than the imputed variance by at least 5% and 10% is increasing as the percentage of missing data increases over fixed sample sizes. In addition, the results for the Cauchy distribution are mixed with respect to the number of imputations, in that increasing the number of imputations does not generally mean a reduction in the percentage of times the imputed variance is an underestimate of the true variance. Finally as sample size increases, the number of simulations with an observed variance greater than an imputed variance is increasing regardless of the amount of missing data. This suggests that when working with a symmetric, yet heavy tailed distribution under a non-monotonic missing data pattern, the chances of underestimating the true variance with multiple imputation is relatively high in the presence of large sample sizes.

Table 12: Cauchy distribution with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00236	-0.2641 (-43.5579)	0.2594 (73.3740)	0.9529	0.9077 (0.0673)	1.0004 (3620.5)	0.154	0.138	0.132
100		10	-0.0166	-0.2682 (-35.8336)	0.2350 (72.9365)	0.9560	0.9109 (0.0617)	1.0033 (3534.4)	0.151	0.137	0.129
100	15	5	0.0980	-0.2686 (-53.8396)	0.4645 (105.1)	0.9120	0.8617 (0.1340)	0.9652 (36903.4)	0.164	0.157	0.146
100		10	0.1573	-0.2083 (-53.6927)	0.5228 (105.0)	0.9218	0.8713 (0.0990)	0.9752 (39737.5)	0.171	0.157	0.148
100	20	5	0.000077	-0.4666 (-68.0972)	0.4667 (105.2)	0.8972	0.8379 (0.1602)	0.9608 (29903.6)	0.176	0.171	0.166
100		10	0.1518	-0.2824 (-59.2767)	0.5860 (105.0)	0.8996	0.8407 (0.0779)	0.9627 (37129.8)	0.183	0.172	0.166
100	25	5	0.3666	-0.2442 (-84.7957)	0.9773 (155.1)	0.8971	0.8299 (0.0995)	0.9698 (33955.4)	0.209	0.198	0.190
100		10	0.3998	-0.1423 (-62.9542)	0.9420 (132.0)	0.8937	0.8273 (0.1340)	0.9655 (31514.2)	0.209	0.198	0.191
500	10	5	2.6828	-2.7269 (-57.0867)	8.0925 (2754.6)	0.9821	0.9356 (0.1298)	1.0310 (1348.3)	0.157	0.137	0.126
500		10	0.7012	-0.6149 (-61.7098)	2.0172 (663.8)	0.9948	0.9475 (0.1176)	1.0444 (1373.7)	0.154	0.137	0.125
500	15	5	0.1182	-0.2272 (-111.5)	0.4635 (88.8946)	0.9327	0.8831 (0.1738)	0.9851 (1184.3)	0.184	0.162	0.151
500		10	0.5506	-0.1674 (-32.6024)	1.2686 (336.1)	0.9424	0.8926 (0.1712)	0.9949 (1040.6)	0.178	0.163	0.150
500	20	5	1.5488	-1.5653 (-153.8)	4.6629 (1573.2)	0.9337	0.8739 (0.1830)	0.9976 (1066.0)	0.197	0.185	0.176
500		10	0.9237	-0.7344 (-151.7)	2.5819 (819.4)	0.9539	0.8932 (0.1901)	1.0188 (1317.8)	0.198	0.184	0.179
500	25	5	3.3572	-2.4179 (-153.1)	9.1322 (2925.4)	0.9450	0.8721 (0.2156)	1.0240 (52944.7)	0.208	0.201	0.192
500		10	1.6094	-0.7698 (-153.2)	3.9887 (1173.5)	0.9583	0.8852 (0.1979)	1.0374 (37468.0)	0.213	0.200	0.190

Table 12 continues: Cauchy distribution with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.2505	-0.0816 (-43.7969)	0.5827 (134.8)	0.9230	0.8922 (0.2715)	0.9548 (315.4)	0.139	0.124	0.114
1000		10	1.0691	-1.2397 (-216.5)	3.3778 (1151.6)	0.9265	0.8957 (0.2711)	0.9583 (320.0)	0.142	0.126	0.11
1000	15	5	0.3559	-0.1186 (-87.1447)	0.8304 (128.1)	0.8823	0.8454 (0.2737)	0.9208 (421.1)	0.150	0.139	0.131
1000		10	1.0847	-0.2599 (-24.2309)	2.4293 (555.5)	0.8907	0.8537 (0.2691)	0.9292 (375.3)	0.144	0.136	0.126
1000	20	5	1.0288	-0.6003 (-84.4395)	2.6580 (803.2)	0.8806	0.8325 (0.2255)	0.9314 (981.6)	0.188	0.173	0.166
1000		10	2.0465	-0.9771 (-76.6214)	5.0700 (1506.5)	0.8925	0.8444 (0.2255)	0.9435 (1066.7)	0.181	0.171	0.162
1000	25	5	0.4770	-1.8728 (-943.8)	2.8269 (605.7)	0.9025	0.8353 (0.2187)	0.9751 (1907891)	0.208	0.196	0.187
1000		10	0.4507	-1.9484 (-943.9)	2.8498 (709.5)	0.9155	0.8477 (0.2165)	0.9889 (2282659)	0.202	0.187	0.175
5000	10	5	-1.9059	-6.4097 (-2287.2)	2.5978 (98.2141)	0.9679	0.9174 (0.1476)	1.0211 (25640292)	0.132	0.113	0.104
5000		10	-2.0198	-6.5184 (-2287.2)	2.4788 (95.1723)	0.9790	0.9282 (0.1522)	1.0327 (25445600)	0.134	0.119	0.107
5000	15	5	-1.9673	-6.4782 (-2287.2)	2.5436 (159.8)	0.9554	0.8969 (0.1918)	1.0177 (20065452)	0.16	0.147	0.137
5000		10	-2.0399	-6.5456 (-2287.3)	2.4659 (151.5)	0.9583	0.8994 (0.1823)	1.0210 (23236764)	0.162	0.147	0.138
5000	20	5	-1.6512	-6.2051 (-2287.5)	2.9027 (295.8)	0.9866	0.9122 (0.1991)	1.0671 (37266929)	0.193	0.181	0.175
5000		10	-1.7570	-6.2845 (-2287.6)	2.7706 (164.0)	0.9920	0.9172 (0.1952)	1.0729 (34237189)	0.193	0.176	0.166
5000	25	5	-1.7940	-6.3839 (-2287.7)	2.7959 (323.7)	0.9795	0.8989 (0.2167)	1.0674 (46024974)	0.219	0.207	0.2
5000		10	-1.9627	-6.5028 (-2287.8)	2.5774 (156.8)	0.9949	0.9135 (0.2091)	1.0835 (35370772)	0.218	0.206	0.197

Overall, we can say by increasing the sample size with different percent of missing the arithmetic mean and the variance are changing. As the sample size increases from 100 to 5000, the number of simulations with a ratio of observed variance to imputed variance being greater than 1, 1.05, and 1.1 is increasing, yet the geometric average approaches one with an estimated geometric average of 0.99 with a sample size of 5000. While comparing the results of Cauchy distribution with Normal distribution, which is considered the "Gold Standard", the number of simulated results providing an under estimate of the true variance is greater with the Cauchy

Distribution than that of the Normal distribution. Although the Cauchy distribution is a symmetric distribution, it also has heavy tails and therefore, multiple imputation may not work as well as it might with other symmetric yet Non-Normal distributions.

5.2.3 Results of t-Distribution

Simulation studies were conducted for t-distribution with various degrees of freedom ranging from 2 df to 30 df to demonstrate the behavior of multiple imputation on a symmetric distribution with a non-monotonic missing data pattern as that distribution departs from normality. This is easily demonstrated by observing the behavior of the t-distribution as the degrees of freedom are decreased from 30 df to 2 df. Results of t-distribution with 30 df are shown in Table 13 and the t-distribution for the other degrees of freedom are in tables found in Appendix D.

We have used t-distribution as one of the distribution to check for deviation from normality using multiple imputation with different percent of missing. Various degrees of freedom have been used to measure the change in the results as the t-distribution asymptotically approaches the Normal distribution. We would expect the results to improve as the degrees of freedom increase.

For the t-distribution with 30 df and a sample size of 500 with approximately 10% missing data and five imputations, the mean difference is 0.000431 (CI: -0.00064, 0.00150) and the geometric average of the ratio of the observed variance to the imputed variance is 0.8809 (CI: 0.8759, 0.8860). Under the conditions of this simulation, 2.9% of the simulated results have an average ratio of variances greater than 1. There were no simulated results with an average ratio of variance greater than 1.05.

Table 13: t-distribution (df = 30) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.000405	-0.00200 (-0.1202)	0.00281 (0.1392)	0.8800	0.8739 (0.5504)	0.8861 (1.1968)	0.101	0.039	0.013
100		10	0.00106	-0.00130 (-0.1279)	0.00342 (0.1423)	0.8888	0.8838 (0.5595)	0.8938 (1.1830)	0.069	0.022	0.006
100	15	5	-0.00015	-0.00317 (-0.1916)	0.00286 (0.1874)	0.8252	0.8174 (0.4192)	0.8329 (1.2114)	0.071	0.024	0.014
100		10	-0.00010	-0.00304 (-0.1618)	0.00284 (0.1818)	0.8302	0.8240 (0.3600)	0.8364 (1.1341)	0.044	0.012	0.001
100	20	5	0.00109	-0.00265 (-0.2155)	0.00483 (0.1966)	0.7652	0.7565 (0.2990)	0.7740 (1.2666)	0.049	0.015	0.003
100		10	-0.00033	-0.00390 (-0.1754)	0.00324 (0.1874)	0.7763	0.7698 (0.4872)	0.7829 (1.0981)	0.021	0.007	0
100	25	5	0.000102	-0.00407 (-0.2165)	0.00427 (0.2376)	0.7189	0.7087 (0.3462)	0.7291 (1.3697)	0.05	0.03	0.015
100		10	0.000213	-0.00386 (-0.2075)	0.00428 (0.2440)	0.7234	0.7155 (0.3140)	0.7313 (1.1460)	0.016	0.005	0.001
500	10	5	0.000431	-0.00064 (-0.0590)	0.00150 (0.0612)	0.8809	0.8759 (0.5235)	0.8860 (1.0419)	0.029	0	0
500		10	-0.00001	-0.00103 (-0.0556)	0.00101 (0.0514)	0.8918	0.8886 (0.7111)	0.8950 (1.0429)	0.011	0	0
500	15	5	-0.00122	-0.00254 (-0.0767)	0.000095 (0.0779)	0.8288	0.8227 (0.5090)	0.8350 (1.0445)	0.012	0	0
500		10	-0.00060	-0.00186 (-0.0620)	0.000663 (0.0765)	0.8359	0.8316 (0.6030)	0.8403 (1.0061)	0.003	0	0
500	20	5	0.000461	-0.00114 (-0.0805)	0.00206 (0.0873)	0.7777	0.7700 (0.4023)	0.7854 (1.0857)	0.01	0.001	0
500		10	0.000048	-0.00147 (-0.0796)	0.00156 (0.0710)	0.7884	0.7833 (0.5229)	0.7936 (1.0096)	0.002	0	0
500	25	5	3.25E-6	-0.00186 (-0.0945)	0.00187 (0.0987)	0.7259	0.7172 (0.2829)	0.7347 (1.0487)	0.008	0	0
500		10	-0.00041	-0.00225 (-0.0872)	0.00142 (0.0911)	0.7343	0.7282 (0.4241)	0.7404 (0.9790)	0	0	0

Using the same sample size of n=500 and 10% percent of the data missing data with an increase to ten imputations, the mean difference is -0.00001 (CI: -0.00103, 0.00101) and the geometric average of the ratio of the variances is 0.8918 (CI: 0.8886, 0.8950). With the additional five imputations, the percentage of simulated results exceeding one is 1.1% with no simulated results reaching the 1.05 threshold. With a sample size of n=500 and 15% of the data missing, based on ten imputations the mean difference is -0.00060 (CI: -0.00186, 0.000663) and the geometric average of the ratio of the variances is 0.8359 (CI: 0.8316, 0.8403). Under the conditions, the percentage of simulated results exceeding one is 0.3% with no simulated results achieving a value of 1.05. Increasing the percentage of missing data to 20% missing with ten imputations provides a mean difference of 0.000048 (CI: -0.00147, 0.00156) and a geometric average of the ratio of the variances of 0.7884 (CI: 0.7833, 0.7936). Under these conditions, the percentage of simulated results that are greater than one is reduced to 0.25 with no simulated results greater than 1.05. Finally at the maximum percentage of missing data, 25%, and ten imputations, the mean difference is -0.00041 (CI: -0.00225, 0.00142) and the geometric average of the ratio of the variances is 0.7343 (CI: 0.7282, 0.7404). There were no simulated results in which the ratio of the variances was greater than one.

Table 13 provides a summary of the t-distribution with 30 degrees of freedom for sample sizes: 100, 500, 1000, and 5000 in addition to various percentages of missing data. The percentage of missing data ranges from a low of 10% up to 25% in increments of 5%. Other attributes summarized in Table 13 include the difference between the observed mean and the imputed mean along with a 95% confidence interval and the geometric average of the ratio of the observed variance to the imputed variance. For a fixed sample size, the geometric average is decreasing as the percentage of missing data increases. Regardless, the geometric mean is consistent over sample size for a fixed level of missing data. For example, the geometric mean is consistently about 0.88 with 5% of the data missing and about 0.72 with 25% of the data missing. On the other hand, multiple imputation with a non-monotonic missing pattern appears to be conservative in the sense that variance ratios rarely appear to be greater than one when the sample size exceeds 100. As sample size increases, the percentage of ratios of observed variance to imputed variance being greater than one is decreasing. With sample sizes of 1000 and 5000, there were no simulated results that produced a variance ratio greater than 1.05. Due to these results, there is evidence to support the notion that multiple imputation does not underestimate the variance often which would allow the null hypothesis to be falsely rejected.

Table 13 continues: t-distribution (df = 30) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00029	-0.00105 (-0.0440)	0.000473 (0.0438)	0.8890	0.8846 (0.5789)	0.8935 (1.0284)	0.015	0	0
1000		10	0.000174	-0.00054 (-0.0387)	0.000891 (0.0404)	0.8913	0.8882 (0.6695)	0.8945 (1.0128)	0.003	0	0
1000	15	5	-0.00043	-0.00135 (-0.0553)	0.000481 (0.0570)	0.8288	0.8227 (0.4760)	0.8349 (1.0204)	0.008	0	0
1000		10	-0.00006	-0.00096 (-0.0485)	0.000841 (0.0476)	0.8411	0.8371 (0.5944)	0.8452 (0.9887)	0	0	0
1000	20	5	-0.00004	-0.00114 (-0.0670)	0.00105 (0.0567)	0.7805	0.7730 (0.4558)	0.7881 (1.0404)	0.011	0	0
1000		10	0.000222	-0.00084 (-0.0519)	0.00128 (0.0520)	0.7851	0.7799 (0.5046)	0.7903 (1.0096)	0.002	0	0
1000	25	5	-0.00042	-0.00173 (-0.0594)	0.000884 (0.0945)	0.7214	0.7131 (0.3104)	0.7299 (0.9907)	0	0	0
1000		10	-0.00052	-0.00178 (-0.0628)	0.000736 (0.0773)	0.7370	0.7311 (0.3930)	0.7429 (0.9740)	0	0	0
5000	10	5	0.000223	-0.00011 (-0.0151)	0.000554 (0.0174)	0.8835	0.8791 (0.5954)	0.8880 (1.0042)	0.003	0	0
5000		10	0.000117	-0.00020 (-0.0153)	0.000438 (0.0154)	0.8924	0.8897 (0.7488)	0.8951 (0.9916)	0	0	0
5000	15	5	0.000390	-0.00002 (-0.0220)	0.000802 (0.0207)	0.8243	0.8181 (0.4609)	0.8306 (1.0061)	0.001	0	0
5000		10	0.000380	-0.00002 (-0.0216)	0.000780 (0.0193)	0.8406	0.8369 (0.6555)	0.8443 (0.9824)	0	0	0
5000	20	5	0.000344	-0.00016 (-0.0250)	0.000847 (0.0286)	0.7807	0.7737 (0.4168)	0.7877 (1.0178)	0.001	0	0
5000		10	0.000297	-0.00018 (-0.0230)	0.000777 (0.0250)	0.7902	0.7853 (0.4718)	0.7951 (0.9660)	0	0	0
5000	25	5	0.000156	-0.00043 (-0.0309)	0.000746 (0.0285)	0.7204	0.7120 (0.2882)	0.7289 (1.0142)	0.002	0	0
5000		10	0.000282	-0.00028 (-0.0308)	0.000843 (0.0322)	0.7384	0.7330 (0.4199)	0.7438 (0.9620)	0	0	0

Based on the results provided in Table 13 and those in Appendix E, we can say that the results of multiple imputation with a non-monotonic missing data pattern improve as the degrees of freedom increase. As sample size increases, the number of simulated results with the ratio of observed variance to imputed variance being greater than 1, 1.05, and 1.1 is decreasing. On comparison of the results of the t-distribution with 30 degrees of freedom (Table 13) and that of the Normal distribution (Table 11) we can conclude that the results are almost identical. As the degrees of freedom increase from 2 to 30 and as the sample size increases from 100 to 5000, the number of simulated results producing a ratio of the observed variance to the imputed variance greater than one approaches zero regardless of the percentage of missing data.

5.2.4 Results of Chi-Square Distribution

To explore the behavior of multiple imputation with a non-monotonic missing data pattern as we move away from symmetry to a skewed distribution, simulation studies were performed using the Chi-square distribution with various degrees of freedom. By allowing the degrees of freedom to increase from two degrees of freedom to 50 degrees of freedom, we can explore how multiple imputation behaves as we move from a skewed distribution to one that is symmetric and asymptotically approaching the normal distribution. In this study, we used the following degrees of freedom: 2, 4, 6, 8, 10, 15, 20, 25, 30, 40 and 50. Results of the simulations based on the Chi-square distribution with 50 df are provided in Table 14 and the results using the Chi-square distribution with the remaining degrees of freedom are found in the tables in Appendix E.

For a Chi-square distribution with 50 df, a sample size of 500 with approximately 10% missing data and using five imputations, the mean difference is -0.00447 (CI: -0.0145, 0.00559) and the geometric average of the ratio of the observed variance to the imputed variance is 0.8856 (CI: 0.8809, 0.8903). Based on the simulations, 2.8% of the simulated results have a ratio of observed variance to imputed variance greater than one, with 0.2% of simulated results producing a ratio of variances greater than 1.05 and no results greater than 1.1. Using the same conditions as just mentioned except with ten imputations, the mean difference is -0.00005 (CI: -0.00966, 0.00957) and the geometric average of the ratio of the variances is 0.8874 (CI: 0.8841, 0.8908). With an increase in the number of imputations, the simulated results for the ratio of the variance exceeding one is 0.4%. There were no simulated results producing a variance ratio greater than 1.05.

Table 14: Chi-Square distribution (df = 50) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.0309	0.00778 (-1.1331)	0.0541 (1.3415)	0.8804	0.8740 (0.4732)	0.8869 (1.1732)	0.091	0.031	0.009
100		10	0.0262	0.00473 (-0.9990)	0.0476 (1.2574)	0.8835	0.8786 (0.6210)	0.8885 (1.2165)	0.06	0.019	0.004
100	15	5	0.0356	0.00658 (-1.7538)	0.0647 (1.7091)	0.8238	0.8160 (0.4363)	0.8317 (1.1795)	0.074	0.027	0.012
100		10	0.0402	0.0124 (-1.6236)	0.0679 (1.8059)	0.8286	0.8224 (0.4444)	0.8349 (1.2025)	0.046	0.014	0.004
100	20	5	0.0385	0.00411 (-1.5691)	0.0729 (1.9699)	0.7713	0.7621 (0.3107)	0.7807 (1.4091)	0.047	0.027	0.008
100		10	0.0459	0.0127 (-1.9283)	0.0791 (1.7155)	0.7804	0.7737 (0.4330)	0.7871 (1.1533)	0.025	0.013	0.004
100	25	5	0.0435	0.00474 (-2.4194)	0.0823 (1.9921)	0.7186	0.7084 (0.2383)	0.7288 (1.3611)	0.038	0.017	0.007
100		10	0.0287	-0.00946 (-1.9038)	0.0669 (2.1918)	0.7223	0.7149 (0.3962)	0.7299 (1.2468)	0.02	0.007	0.002
500	10	5	-0.00447	-0.0145 (-0.5458)	0.00559 (0.4694)	0.8856	0.8809 (0.6369)	0.8903 (1.0667)	0.028	0.002	0
500		10	-0.00005	-0.00966 (-0.5475)	0.00957 (0.4967)	0.8874	0.8841 (0.6747)	0.8908 (1.0441)	0.004	0	0
500	15	5	-0.00127	-0.0142 (-0.7792)	0.0117 (0.7512)	0.8286	0.8222 (0.4829)	0.8350 (1.0654)	0.013	0.001	0
500		10	0.000989	-0.0113 (-0.6756)	0.0133 (0.6467)	0.8340	0.8296 (0.5866)	0.8384 (1.0139)	0.002	0	0
500	20	5	0.00285	-0.0129 (-0.7661)	0.0186 (0.9836)	0.7722	0.7643 (0.3032)	0.7802 (1.0565)	0.013	0.001	0
500		10	0.00285	-0.0119 (-0.6977)	0.0176 (0.8615)	0.7819	0.7765 (0.4405)	0.7874 (1.0115)	0.001	0	0
500	25	5	0.00670	-0.0111 (-1.1478)	0.0245 (1.0706)	0.7232	0.7145 (0.3156)	0.7320 (1.1317)	0.011	0.005	0.002
500		10	0.00588	-0.0110 (-1.0200)	0.0228 (0.9456)	0.7396	0.7339 (0.4453)	0.7454 (1.0374)	0.001	0	0

With a sample size of 500, increasing the percentage of missing to 15% and using ten imputations, the mean difference is 0.000989 (CI: -0.0113, 0.0133) and the geometric average of the ratio of the variances is 0.8340 (CI: 0.8296, 0.8384). Under these conditions, 0.2% of the simulated results have a variance of ratio greater than 1. There were no simulated results with a variance ratio greater than 1.05. Increasing the percentage of missing data to 20% and using ten imputations, the mean difference is 0.00285 (CI: -0.0119, 0.0176) and the geometric average of the ratio of the variances is 0.7819 (CI: 0.7765, 0.7874). This set of simulations produced 0.1% of the simulated variance ratios to exceed one, with no simulated variance ratios exceeding 1.05. Finally with approximately 25% of the data missing and using ten imputations, the mean difference is 0.00588 (CI: -0.0110, 0.0228) and the geometric average of the ratio of variances is 0.7396 (CI: 0.7339, 0.7454). Again the percentage of simulated variance ratio declined. For this set of simulations, only 0.1% of the simulated results produced a ratio of variances greater than one. There were no simulated results with a ratio of the observed variance to the imputed variance greater than 1.05.

Table 14 provides a summary of the simulated results for a Chi-square distribution with 50 degrees of freedom for sample sizes: 100, 500, 1000 and 5000, in addition to various percentages of missing data under the non-monotonic missing data pattern paradigm. The percentage of missing data ranges from 10% to 25% in increments of 5%. The other attributes summarized in Table 14 include the mean difference between the mean of the observed data and the mean of the imputed data along with a 95% confidence interval and the geometric mean of the ratio of the observed variance and imputed variance. For fixed sample sizes, the geometric mean is decreasing as the percentage of missing data increases, implying multiple imputation becomes more conservative as the amount of missing data increases. As previously observed, the geometric mean is fairly consistent regardless of the sample size. For example, the geometric mean is consistently in the vicinity of 0.87 with only 5% of the data missing regardless of the sample size and about 0.73 with 25% of the data missing. On the other hand, multiple imputations rarely underestimate the variance in any appreciable fashion as simulated ratios rarely reach the 1.05 level. As sample size increases, the percentage of ratios of observed variance to imputed variance being greater than one is decreasing. With sample sizes of 1000 and 5000 there were no observed simulations that produced a variance ratio greater than 1.05, again providing evidence that multiple imputation does not routinely underestimate the true variance and allow the null hypothesis to be falsely rejected.

Table 14 continues: Chi-Square distribution (df = 50) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000568	-0.00665 (-0.3294)	0.00778 (0.3317)	0.8833	0.8788 (0.5964)	0.8878 (1.0166)	0.01	0	0
1000		10	0.000587	-0.00627 (-0.3044)	0.00744 (0.3494)	0.8900	0.8869 (0.6753)	0.8930 (0.9990)	0	0	0
1000	15	5	-0.00302	-0.0120 (-0.4343)	0.00594 (0.4509)	0.8312	0.8251 (0.5057)	0.8372 (1.0387)	0.009	0	0
1000		10	0.000362	-0.00806 (-0.3853)	0.00879 (0.3814)	0.8377	0.8337 (0.6243)	0.8417 (1.0097)	0.002	0	0
1000	20	5	-0.00616	-0.0166 (-0.5167)	0.00432 (0.5598)	0.7838	0.7767 (0.4002)	0.7911 (1.0195)	0.008	0	0
1000		10	0.00287	-0.00731 (-0.5399)	0.0131 (0.4918)	0.7878	0.7828 (0.5086)	0.7928 (0.9759)	0	0	0
1000	25	5	0.00357	-0.00847 (-0.6455)	0.0156 (0.5979)	0.7257	0.7173 (0.3071)	0.7342 (1.0093)	0.001	0	0
1000		10	0.00671	-0.00515 (-0.6563)	0.0186 (0.6015)	0.7375	0.7315 (0.4457)	0.7435 (0.9709)	0	0	0
5000	10	5	-0.00246	-0.00578 (-0.1630)	0.000862 (0.1613)	0.8825	0.8780 (0.5716)	0.8870 (1.0046)	0.003	0	0
5000		10	-0.00172	-0.00489 (-0.1546)	0.00145 (0.1775)	0.8910	0.8881 (0.7210)	0.8938 (0.9840)	0	0	0
5000	15	5	-0.00213	-0.00629 (-0.2023)	0.00204 (0.2116)	0.8367	0.8310 (0.4874)	0.8425 (1.0006)	0.001	0	0
5000		10	-0.00196	-0.00587 (-0.1783)	0.00196 (0.1987)	0.8395	0.8355 (0.6021)	0.8434 (0.9978)	0	0	0
5000	20	5	-0.00086	-0.00586 (-0.2388)	0.00414 (0.2344)	0.7771	0.7698 (0.4280)	0.7845 (1.0046)	0.002	0	0
5000		10	-0.00084	-0.00559 (-0.2358)	0.00390 (0.2298)	0.7897	0.7849 (0.5420)	0.7945 (0.9567)	0	0	0
5000	25	5	-0.00036	-0.00604 (-0.2902)	0.00533 (0.2484)	0.7296	0.7214 (0.3475)	0.7378 (0.9857)	0	0	0
5000		10	0.001000	-0.00446 (-0.2833)	0.00646 (0.2561)	0.7365	0.7309 (0.4362)	0.7421 (0.9451)	0	0	0

Based on the results provided in the Table 14 and Appendix F, we can say that the results are improving as the degrees of freedom increases. As the sample size increases, the percentage of variance ratios exceeding 1, 1.05 and 1.1 is decreasing. If the sample size is large, the chi-square distribution asymptotically approaches the Normal distribution with mean *n* and variance *2n*. A Chi-square distribution with 50 df converges to take on the shape of the Normal distribution. On comparison of the results of the Chi-square distribution with 50 degrees of freedom (Table 14) with that of the Normal Distribution (Table 11), we can conclude that the results are remarkably similar. In addition, as the degrees of freedom increase from 2 to 50 with an increase in sample size and various percentages of missing data, the number of simulations with a ratio of the observed variance to the imputed variance being greater than one goes to zero. Based on these results, we can conclude that multiple imputation appears to work well with a distribution such as the Chi-square distribution and its performance increases as degrees of freedom and sample size increases.

5.3 Analysis of Tests of Hypotheses

5.3.1. Significance at α=0.05 for Non-Monotone Missing data Pattern

Table 15: Significance P-values Normal distribution

N	% Miss	N(0,1) Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	46	44	67	59	48 (43)
	15	46	46	76	78	51 (51)
	20	46	48	93	96	41 (49)
	25	46	51	108	125	49 (46)
500	10	48	48	66	70	50 (49)
	15	48	44	85	77	48 (44)
	20	48	47	104	104	50 (45)
	25	48	43	114	119	56 (50)
1000	10	38	40	66	69	40 (41)
	15	38	45	84	81	42 (46)
	20	38	45	96	102	52 (44)
	25	38	42	117	130	49 (41)
5000	10	52	41	73	72	47 (44)
	15	52	42	82	81	48 (45)
	20	52	41	88	103	43 (45)
	25	52	41	104	115	53 (43)

Under the non-monotonic missing data pattern for the Normal distribution (Table 15), the results based on the hypothesis: H_0: μ=0 verses H_a: μ≠0 are compared with different types of missing data methods such as: full data which is considered the gold standard, available data analysis, mean substitution, single regression imputation, and multiple imputation. With a sample size of 100 and 10% of the data missing, the full data analysis rejected the null hypothesis 46 times at an alpha level of 0.05. In comparison, under the same simulation conditions, using the available data analysis, the null hypothesis was rejected 44 times; using mean substitution, the null hypothesis was rejected 67 times and with single regression imputation, the null hypothesis was rejected 59 times. The multiple imputation with five imputations rejected the null hypothesis 48 times and with ten imputations it rejected the null hypothesis 43 times. For sample size of 5000 with 15% of the data missing, the full data analysis rejected the null hypothesis 52 times at the alpha level of 0.05. In comparison, under the same simulation conditions, using the available data analysis, the null hypothesis was rejected 42 times; using mean substitution, the null hypothesis was rejected 82 times and with single regression imputation, the null hypothesis was rejected 81. However, multiple imputation with five and ten imputations rejected the null hypothesis 48 and 45 times respectively.

Overall, we can see that the number of simulated results that reject the null hypothesis is greatest using either the mean substitution method or the single regression imputation method. This is most likely due to an underestimate of the variance, as neither method adds variability to the data but it does add degrees of freedom which causes an underestimate the sample variability. Because the variance is underestimated, the test statistic is over-estimated, which in turn creates a smaller p-value. However, number of simulated results rejecting the null hypothesis using the multiple imputation method is approximately the same as that found by using the full data analysis results. Therefore, it appears that multiple imputation provides approximately the same number of rejections as the full data analysis and performs better than the available data analysis, mean substitution and single regression methods in terms of type I error when applied to the Normal distribution.

Table 16: Significance P-values t-Distribution with 2df

N	% Miss	t-dist with 2df Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	45	39	69	64	39 (41)
	15	45	31	82	73	31 (35)
	20	45	39	94	101	48 (39)
	25	45	41	116	136	49 (45)
500	10	42	46	66	73	47 (46)
	15	42	42	78	89	44 (46)
	20	42	49	89	88	44 (47)
	25	42	46	117	118	45 (46)
1000	10	36	40	66	79	40 (42)
	15	36	37	85	99	40 (39)
	20	36	38	104	109	46 (41)
	25	36	43	126	165	50 (48)
5000	10	45	46	76	66	47 (46)
	15	45	43	91	90	54 (46)
	20	45	41	98	112	49 (41)
	25	45	39	120	135	47 (50)

Under the non-monotonic missing data pattern with the t-distribution with 2 df (Table 16), results are provided for testing the hypothesis: H_0: $\mu=0$ and H_a: $\mu\neq0$ comparing different types of missing data methods: full data analysis, available data analysis, mean substitution, single regression imputation, and multiple imputation. With a sample size of 100 and 10% of the data missing, the full data analysis rejected the null hypothesis 45 times at an alpha level of 0.05. In comparison, under the same simulation conditions, using the available data analysis, the null hypothesis was rejected 39 times; using mean substitution, the null hypothesis was rejected 69 times, and with single regression imputation, the null hypothesis was rejected 64 times. The multiple imputation procedure with five imputations rejected the null hypothesis 39 times and with ten imputations it rejected the null hypothesis 41 times. For sample size of 5000 and 15% of the data missing, the full data analysis rejected the null hypothesis 45 times. On the other hand, the available data analysis rejected the null hypothesis 43 times, the mean substitution method rejected the null hypothesis 91 times and single regression imputation rejected the null hypothesis 90 times. However, multiple imputation with five and ten imputations rejected the null hypothesis 54 and 46 times respectively.

Based on the simulated results (from Table 16 and Appendix G), the mean substitution method and the single regression imputation method reject the greatest number of null hypotheses at a rate much greater than what is expected at alpha=0.05. Again, this is primarily due to the way in which these methods underestimate the sample variance. Both the mean substitution method and the regression single imputation methods systematically underestimate variance because missing values are replaced with mean values that cannot add any variability to the data, yet at the same time these methods increase the degrees of freedom. However, the number of simulated results rejecting the null hypothesis with the multiple imputation method is approximately the same as found using the full data analysis results. Therefore, with respect to rejecting the expected number of hypotheses at alpha=0.05, multiple imputation appears to work well compared to the other imputation methods such as mean substitution and single regression methods for t-distribution with 2 df. The results are mixed when the multiple imputation method is compared to the available data analysis method with respect to the number of hypotheses rejected with alpha=0.05.

Under the non- monotonic missing data pattern for the Chi-square distribution with 2 df (Table 17), the results based on the hypothesis H_0: $\mu=2$ verses H_a: $\mu\neq2$) are comparing the different types of missing data methods: full data analysis, available data analysis, mean substitution, single regression imputation, and multiple imputation.

Table 17: Significance P-values Chi-Square distribution with 2 df

N	% Miss	Chi-Sqr dist with 2df Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	46	51	74	80	58 (49)
	15	46	54	94	99	58 (52)
	20	46	55	100	122	55 (51)
	25	46	54	127	136	57 (54)
500	10	70	61	85	87	63 (61)
	15	70	57	101	97	59 (56)
	20	70	57	122	117	67 (53)
	25	70	55	145	160	69 (66)
1000	10	58	56	83	84	57 (57)
	15	58	51	91	94	54 (48)
	20	58	51	103	116	52 (50)
	25	58	44	129	137	55 (55)
5000	10	42	36	58	60	35 (32)
	15	42	37	69	77	36 (36)
	20	42	38	93	109	46 (40)
	25	42	43	119	109	59 (43)

For sample size of 100 with 10% of the data missing, the full data analysis rejected the null hypothesis 46 times with alpha set to the 0.05 level. The available data analysis rejected the null hypothesis 51 times, the mean substitution method rejected the null hypothesis 74 times and single regression imputation rejected the null hypothesis 80 times. However, multiple imputation with five and ten imputations rejected the null hypothesis 58 and 49 times respectively. Increasing the sample size to 5000 with 15% of the data missing, the full data analysis rejected the null hypothesis 42 times, whereas using the available data analysis, the null hypothesis was rejected only 37 times, the mean substitution method rejected the null hypothesis 69 times and single regression imputation rejected the null hypothesis 77 times. Multiple imputation with five and ten imputations rejected the null hypothesis 36 times.

Once again, we can say based on the results of Table 17 and Appendix G that the number of simulated results rejecting the null hypothesis is greatest using either the mean substitution method or the single regression imputation method. As in the previous simulations, this is primarily due to the addition of data values that do not add any variability to the data being analyzed. These in turn results in an underestimation of the sample variance that results in an overestimate of the test statistic and finally a smaller p-value than what is expected given the full data. However, the number of simulated results rejecting the null hypothesis using multiple imputation method is approximately the same as that of the full data analysis results. Therefore, it appears that multiple imputation works well compared to the available data analysis method, mean substitution imputation and single regression imputation for the chi-square distribution with 2 df.

5.3.2 Sensitivity and Specificity Results with Non-Monotone Missing data Pattern

The results of sensitivity and specificity are presented to assure that the simulations that rejected the null hypothesis are actually same as the full data analysis. We provide the sensitivity and specificity of the available data analysis and each imputation method with respect to rejecting the null hypothesis under each condition. The analysis based on the full data is the gold standard used in this analysis. Based on the Normal distribution (Table 18) with sample size of 100 and 10% of the data missing, the available data analysis has sensitivity of 98.33 and specificity of 68.18. The mean substitution method has sensitivity of 99.25 and specificity of 58.21. The single imputation method has sensitivity of 98.62 and specificity of 55.93. The multiple imputation with five imputations has sensitivity of 98.63 and specificity of 68.75. Multiple imputation with ten imputations has sensitivity of 98.33 and specificity of 69.77. When the sample size is increased to n= 500 with 15% of the data missing, the available data analysis has sensitivity of 98.33 and specificity of 72.73; whereas, the mean substitution has sensitivity of 99.56 and specificity of 51.76; the single regression imputation has sensitivity of 98.59 and specificity of 45.45; and multiple imputation with five imputations has sensitivity of 98.0 and specificity of 60.42. Finally, multiple imputation with ten imputations has sensitivity of 98.33 and specificity of 72.73. With a sample size of 5000 and 25% of the data missing, the available data analysis has sensitivity of 97.40 and specificity of 65.85. The mean substitution method has sensitivity of 99.33 and specificity of 44.23. The single imputation method has sensitivity of 98.08 and specificity of 30.43. Multiple imputation with five imputations has sensitivity of 97.57 and specificity of 54.72 and multiple imputation with ten imputations has sensitivity of 96.97 and specificity of 53.49.

Table 18: Sensitivity and Specificity for Normal distribution

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	98.33	68.18	99.25	58.21	98.62	55.93	98.63 (98.33)	68.75 (69.77)
	15	98.01	58.70	99.35	52.63	98.16	37.18	98.42 (98.42)	62.00 (68.78)
	20	97.80	52.08	99.45	44.09	98.45	33.33	97.39 (98.00)	51.22 (55.10)
	25	97.58	45.10	98.99	34.26	98.17	24.00	97.37 (97.59)	42.86 (50.00)
500	10	98.43	68.75	99.47	65.15	99.03	55.71	98.42 (98.42)	66.00 (67.35)
	15	98.33	72.73	99.56	51.76	98.59	45.45	98.00 (98.33)	60.42 (72.73)
	20	98.32	68.09	99.55	42.31	98.10	29.81	97.89 (98.12)	56.00 (66.67)
	25	97.91	65.12	99.66	39.47	97.85	24.37	97.99 (97.68)	51.79 (52.00)
1000	10	98.54	60.00	99.36	48.48	98.71	37.68	98.33 (98.44)	55.00 (56.10)
	15	98.54	53.33	99.35	38.10	98.80	33.33	98.43 (98.53)	54.76 (52.17)
	20	98.54	53.33	99.56	35.42	98.89	27.45	98.52 (98.54)	46.15 (54.55)
	25	97.91	42.86	99.66	29.91	98.28	17.69	97.69 (98.02)	32.65 (46.34)
5000	10	98.02	80.49	99.46	64.38	98.49	52.78	98.11 (98.33)	72.34 (81.82)
	15	97.91	76.19	99.24	54.88	98.15	43.21	97.48 (97.91)	58.33 (71.11)
	20	97.40	65.85	99.23	51.14	98.33	35.92	97.28 (97.38)	60.47 (60.00)
	25	97.40	65.85	99.33	44.23	98.08	30.43	97.57 (96.97)	54.72 (53.49)

**Results of Multiple Imputation are based on computation.*
Reference group is Full data [N (0,1)]

From the results of multiple imputation under Normal distribution (Table 18), we can see that with a sample size of 100 and 25% of the data missing with five imputations, 97.4% of the tests that should not be rejected, are not rejected. In addition, 42.9% of the tests that should be rejected were rejected. With sample size of 500 and 25% of the data missing with five imputations, 98% of the tests that should not be rejected are not rejected and 51.8% of the tests that should be rejected are rejected. With sample size of 1000 and 25% of the data missing with five imputations, 97.7% of the tests that should not be rejected are not rejected. On the other hand, 32.7% of the tests that should be rejected were rejected. Finally using a sample size of 5000 and with 25% of the data missing and five imputations, 97.6% of the tests that should not be rejected are not rejected and 54.7% of the tests that should be rejected were rejected. Therefore, multiple imputation, when compared to the other imputation techniques, outperforms those techniques with respect to specificity under normality.

Examination of the t-distribution with 2 degrees of freedom (Table 19), a sample size of 100 and 10% of the data missing, with the full data analysis as the gold standard, shows that if we use the available data analysis, we get a sensitivity of 98.34 and specificity of 78.36. The mean substitution method has sensitivity of 99.46 and specificity of 57.97. The single regression imputation method has sensitivity of 98.51 and specificity of 48.44. The multiple imputation with five imputations has sensitivity of 98.02 and specificity of 66.67. The multiple imputation with ten imputations has sensitivity of 98.44 and specificity of 73.17. When the sample size is increased to 500 with 15% of the data missing, the available data analysis has sensitivity of 98.54 and specificity of 66.67. The mean substitution imputation method has sensitivity of 99.46 and specificity of 47.44. The single regression imputation method provides sensitivity of 98.90 and specificity of 35.96. Multiple imputation with five imputations has sensitivity of 98.22 and specificity of 56.82. Multiple imputation with ten imputations has sensitivity of 98.85 and specificity of 67.39.

Table 19: Sensitivity and Specificity for t-Distribution with 2 df

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	98.34	78.36	99.46	57.97	98.51	48.44	98.02 (98.44)	66.67 (73.17)
	15	97.63	70.97	99.13	45.12	97.95	35.62	97.42 (97.82)	64.52 (68.57)
	20	97.82	61.54	99.01	38.30	98.44	30.69	97.58 (97.81)	45.83 (61.54)
	25	97.60	53.66	99.21	32.76	97.69	18.38	97.37 (97.59)	40.82 (48.89)
500	10	99.06	71.74	99.68	59.09	98.92	43.84	98.64 (98.95)	61.70 (69.57)
	15	98.54	66.67	99.46	47.44	98.90	35.96	98.22 (98.85)	56.82 (67.39)
	20	98.74	61.22	99.56	42.70	98.74	31.82	98.33 (98.43)	59.09 (57.45)
	25	97.91	47.83	99.66	33.33	98.30	22.88	97.91 (97.90)	48.89 (47.83)
1000	10	99.06	67.50	99.47	46.97	99.02	34.18	98.75 (98.85)	60.00 (59.52)
	15	98.44	56.76	99.78	40.00	99.00	27.27	98.33 (98.65)	50.00 (58.97)
	20	98.34	52.63	99.55	30.77	98.88	23.85	98.22 (98.33)	41.30 (48.78)
	25	98.02	39.53	99.31	23.81	98.56	14.55	98.11 (98.00)	36.00 (35.42)
5000	10	98.32	63.04	99.24	50.00	98.40	45.45	98.11 (98.43)	57.45 (65.22)
	15	97.70	53.49	98.90	38.46	97.69	26.67	97.99 (97.90)	48.15 (54.35)
	20	97.81	58.54	99.00	36.73	98.09	25.00	98.00 (97.91)	53.06 (60.98)
	25	97.40	51.28	99.21	31.67	98.04	20.74	97.48 (97.58)	44.68 (44.00)

With the maximum sample size of 5000 and 25% of the data missing, the available data analysis provides sensitivity of 97.40 and specificity of 51.28. The mean substitution imputation method yields sensitivity of 99.21 and specificity of 31.67. The single regression imputation method has sensitivity of 98.04 and specificity of 20.74. Multiple imputation with five imputations has sensitivity of 97.48 and specificity of 44.68. Multiple imputation with ten imputations has sensitivity of 97.58 and specificity of 44.

From the results of multiple imputation under t-distribution (Table 19), we can see that with a sample size of 100 and 25% of the data missing with five imputations, 97.4% of the tests that should not be rejected are not rejected and 40.8% of the tests that should be rejected were rejected. With a sample size of 500 and 25% missing data with five imputations, 97.9% of tests that should not be rejected are not rejected and 48.9% of the tests that should be rejected were rejected. With a sample size of 1000 and 25% of the data missing with five imputations, 98.1% of the tests that should not be rejected are not rejected while 36% of the tests that should be rejected were rejected. Finally, with a sample size of 5000 and 25% of the data missing with five imputations, 97.5% of the tests that should not be rejected are not rejected and 44.7% of the tests that should be rejected were rejected. As in Chapter 4, examination of the results shown in Table 19 and Appendix G that all of the methods perform about the same with respect to sensitivity which is to be expected since the simulations were designed to reject only 5% of the hypotheses, and the number of simulations were large. On the other hand, when considering the specificity, we see that multiple imputation did better than both the mean substitution method and the single imputation method but not as well as the available data analysis method. Just as in the case with monotonic missing data patterns, the non-monotonic missing data pattern is simulated under the assumption of "missing at random" and therefore the available data analysis provides a fairly good representation of the full data. The major drawback to the available data analysis method under the non-monotonic missing data pattern is the same as in the monotonic missing data pattern, a loss of degrees of freedom in the analysis due to the missingness. However with sample sizes of 100, 500, 1000, and 5000 with at most 25% of the data missing, this is not a severe problem.

Looking at the Chi-square distribution with 2 df (Table 20) with a sample size of 100 and 10% of the data missing, the available data analysis provides a measure of sensitivity equal to 98.84 and specificity of 68.63 with the full data analysis used as the gold standard. The mean substitution imputation method has sensitivity of 99.57 and specificity of 56.76 whereas the single regression imputation has sensitivity of 98.91 and specificity of 45. The multiple imputation method with five imputations has sensitivity of 98.73 and specificity of 58.62. The multiple imputation method with ten imputations has sensitivity of 98.53 and specificity of 65.31. Increasing the sample size to 500 and allowing 15% of the data to be missing, the available data analysis has sensitivity of 96.82 and specificity of 70.18, the mean substitution imputation method has sensitivity of 99 and specificity of 60.40; and the single regression imputation has sensitivity of 97.45 and specificity of 48.45. On the other hand, multiple imputation with five imputations has sensitivity of 96.60 and specificity of 64.41 and Multiple imputation with ten imputations has sensitivity of 96.61 and specificity of 67.86.

With a sample size of 5000 and 25% of the data missing, the available data analysis has sensitivity of 97.60 and specificity of 44.19. The mean substitution has sensitivity of 99.66 and specificity of 32.77, with the single regression imputation method having sensitivity of 97.64 and specificity of 19.27. Multiple imputation with five imputations has sensitivity of 97.98 and specificity of 38.98 and increasing to ten imputations provides sensitivity of 97.28 and specificity of 37.21.

From the results of multiple imputation under Chi-square distribution (Table 20), given a sample size of 100 and a missing data rate of 25%, using five imputations, 97.8% of the tests that should not be rejected are not rejected and 43.9% of the tests that should be rejected are rejected. With sample size of 500 subject to 25% of the data missing and five imputations, 95.9% of the tests that should not be rejected are not rejected while 46.4% of the tests that should be rejected are rejected. Increasing the sample size to 1000 and allowing 25% of the data to be missing with five imputations, 97.1% of the tests that should not be rejected are not rejected while 56.4% of

Table 20: Sensitivity and Specificity for Chi-square with 2 df

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	98.84	68.63	99.57	56.76	98.91	45.00	98.73 (98.53)	58.62 (65.31)
	15	98.63	61.11	99.67	45.74	98.67	34.34	98.73 (98.31)	58.62 (57.69)
	20	98.41	56.36	99.33	40.00	98.63	27.87	97.99 (98.21)	49.09 (56.86)
	25	98.41	57.41	99.31	31.50	98.50	24.26	97.77 (97.89)	43.86 (48.15)
500	10	97.34	73.77	98.91	70.59	97.48	54.02	97.33 (97.34)	71.43 (73.77)
	15	96.82	70.18	99.00	60.40	97.45	48.45	96.60 (96.61)	64.41 (67.86)
	20	96.39	63.16	98.97	50.00	97.28	39.32	96.68 (96.30)	58.21 (66.04)
	25	95.87	56.36	99.18	43.45	98.10	33.75	95.92 (96.04)	46.38 (50.00)
1000	10	98.62	80.36	99.45	63.86	99.13	59.52	98.41 (98.52)	75.44 (77.19)
	15	97.47	66.67	99.34	57.14	98.23	44.68	97.67 (97.58)	66.67 (72.92)
	20	97.37	64.71	99.11	48.54	97.62	31.90	97.26 (97.37)	61.54 (66.00)
	25	96.86	63.64	99.43	41.09	97.68	27.74	97.14 (97.04)	56.36 (54.55)
5000	10	98.34	72.22	99.26	60.34	98.51	46.67	98.26 (98.14)	71.43 (75.00)
	15	97.92	59.46	98.71	43.48	97.94	29.87	97.82 (98.13)	59.33 (66.67)
	20	97.61	50.00	98.90	34.41	98.43	25.69	97.90 (97.60)	47.83 (47.50)
	25	97.60	44.19	99.66	32.77	97.64	19.27	97.98 (97.28)	38.98 (37.21)

the tests that should be rejected are rejected. Finally with a sample size of 5000 and 25% of the data missing with five imputations, 98 % of the tests that should not be rejected are not rejected and only 39% of the tests that should be rejected are rejected.

Examination of the results (in Table 20 and Appendix G) of these simulations, show that all of the methods perform about the same with respect to sensitivity which is to be expected since the simulations were designed to reject only 5% of the hypotheses, and there were a large number of simulations for each set of conditions. On the other hand, when considering the specificity, we see that multiple imputation did better than both the mean substitution method and the single imputation method but not as well as the available data analysis method. As was the case in the monotonic missing pattern analysis, an explanation for this is that the data are "missing at random" and therefore the available data analysis was a fairly good representation of the full data, as the available data analysis is a valid method of analysis under MAR. The major drawback to the available data analysis method is a loss of degrees of freedom. However with sample sizes of 100, 500, 1000, and 5000 with at most 25% of the data missing, this is not a severe problem.

CONCLUSION

As stated by van Buuren (2012), "The goal of multiple imputation is to obtain statistically valid inferences from incomplete data. Though this goal is ambitious, it is achievable." Many researchers who use imputation methods attempt to achieve an even more ambitious goal. That is, some researchers believe that imputation methods are designed to re-create the lost data. Lost data may have severe consequences in public health research because the absence of data due to non-response or dropout complicates the generalizability of the study findings to a larger population. In addition, standard statistical software frequently excludes records with missing values from the analysis which in turn reduces the power of the study.

It is not the purpose of this dissertation to determine how well multiple imputation re-creates lost data. Rather, the purpose of this dissertation is to determine how well multiple imputation provides valid inferences. All simulations and measures of performance were designed in an attempt to assess how well multiple imputation works in providing valid statistical inferences under various conditions. Those conditions include a range of percent of missing data from 10% to 25% by escalation of 5% and sample sizes ranging from 100 to 5000 with under both the normal distribution and non-normal distributions.

When imputing data, there are two general concerns that involve the estimation of the variance of a parameter: underestimation of the variance and overestimation of the variance. As indicated by Rubin et al. (2007), imputation methods that underestimate the variance will provide invalid inferences that may result in the errant rejection of a valid hypothesis. For example, underestimated variance is the major flaw in most predictive single imputation methods such as "mean substitution" or single regression imputation. Predictive single imputation methods such as “mean substitution” and single regression imputation allows for an abundance of errant rejections of the null hypothesis due to an underestimation of the variance. On the other hand, imputation methods that consistently overestimate the variance are also biased but they are conservative and the overestimation represents the penalty that must be paid for incomplete information. The major penalty is a reduction in statistical power of the tests of hypotheses and therefore failing to reject a hypothesis that should be rejected. Not using a suitable approach for the statistical analysis of incomplete data generates biased estimates that may affect the external validity of the study in the context of generalization of the results to larger population.

Work in this dissertation compares inferences drawn from multiple imputation to the inferences drawn from the complete data under the assumption of MAR. Sensitivity and specificity techniques, which are generally found in epidemiology, are used to examine these inferences and to determine the validity of various analytic approaches including multiple imputation. Sensitivity and specificity ensure that the simulation results rejecting the null hypothesis are actually the same as those from the analysis of the complete data.

This dissertation was centered on the two aspects of missing data patterns: monotonic missing data patterns and non-monotonic missing data patterns with a range of the percent of missing data from 10% to 25% in increments of 5%. The purpose of evaluating the multiple imputation method on monotonic missing data is that monotonic missing data pattern frequently occurs in clinical trial settings that are longitudinal in nature, such as repeated measures studies. The Normal distribution was used as the "gold standard" to evaluate considered departures from Normality by examining non-normal distributions such as Cauchy distribution, t-distribution and Chi-square distribution with various degrees of freedom. Simulation results based on a monotonic missing data pattern for sample sizes of 100, 500, 1000 and 5000 and missing data ranging from 10% to 25% with increments of 5% indicated that multiple imputation works well when compared to other single imputation methods such as mean substitution and single regression imputation. Multiple imputation also performed well against the available data analysis. This is important as, maximum likelihood methods are always valid methods of analysis under the assumption of MAR. If multiple imputation methods provided inferior results to the available data analysis using maximum likelihood, then there would be little point in using multiple imputation under the assumption of MAR and normality as maximum likelihood methods are less time consuming. In addition, multiple imputation helps to retain statistical power of the study whereas analysis of available data using maximum likelihood methods loses power because of a reduction in sample size. In addition, maximum likelihood methods require that a probability distribution be specified in order to estimate parameters of interest. In this dissertation, we studied the impact of various distributions on multiple imputation assuming that the comparable maximum likelihood estimation would be based on the Normal distribution. This type of analysis is important as many researchers apply statistical tests assuming normality regardless of the distribution from which the data was derived. As the distributions moved from normality to symmetric and heavy tailed or to asymmetry, multiple imputation analysis appears to be robust. Being robust is important because it tells us that multiple imputation will provide valid inferences even when the researcher applies statistical techniques that require normality to data that is not normally distributed. Under the Normal distribution, the ratio of the observed variance to imputed variance greater than 1, 1.05 and 1.1 approaches zero as the sample size increased from 100 to 5000. This is important because these ratios measure the true variance to the imputed variance. The larger these ratios are, the more multiple imputation has underestimated the variance and in turn increased the probability of a false rejection of the null hypothesis. By having these ratios approach zero as sample size increases suggests that multiple imputation will not underestimate the variance as sample sizes become sufficiently large. Underestimation of the variance would increase the value of the Wald type statistic, thereby, inflating the alpha level. It is clear that multiple imputation under the Normal distribution is a conservative imputation method.

However for the Cauchy distribution, multiple imputation did not work as well as with other symmetric distributions such as the t-distribution. While the Cauchy distribution is a symmetric distribution, it has heavy tails and multiple imputation tends to underestimate the true variance when the data come from a heavy tailed distribution even when the imputations come from the same Cauchy distribution. This result is to be expected as the Cauchy distribution is far from normal. The analysis based on maximum likelihood using the available data and assuming a normal distribution also fared poorly–which is to be expected due to the gross miss-specification of the distribution. As the Cauchy distribution is a pathological case, it would be unreasonable to believe any method of analysis assuming normality would work well.

The ratio of observed variance to imputed variance that is greater than one goes to zero under the t-distribution and the Chi-square distribution by increasing the sample size and degrees of freedom over all levels of the percent of missing data in this study. Multiple imputation did not underestimate the variance which indicates that multiple imputation is a conservative imputation method under t-distribution and Chi-square distribution as the degrees of freedom increase.

The second part of the dissertation focused on non-monotonic missing data patterns, which occur in both longitudinal and non-longitudinal studies. As in the case of monotonic missing data patterns, this dissertation considered four sample sizes: 100, 500, 1000, and 5000 with a percent of missing data ranging from 10% to 25% by increments of 5%. Multiple imputation uses the MCMC model to impute missing values under non-monotonic missing data pattern. In most statistical packages, MCMC methods assumed a multivariate normal distribution and uses Gibbs sampling to obtain imputed values.

The results of the simulations indicate that multiple imputation under non-monotonic missing data pattern provides valid inferences compare to mean substitution imputation, single regression imputation and available data analysis. Under a non-monotonic missing data pattern a similar simulation analysis was performed as with the monotonic missing data pattern. Simulations were performed under the assumption of the Normal distribution, t- distribution, and Chi-square distribution by increasing degrees of freedom. Like the case of monotonic missing data patterns, multiple imputation for non-monotonic missing data patterns did not underestimate the variance in a systematic or biased fashion. Therefore multiple imputation provided evidence of being a conservative method for non-monotonic missing data pattern.

In public health research, most data are collected by surveys. Missing information is a serious problem with the use of surveys because dropping a case may lead to reduced power and provide conclusions that investigators may not be able to generalize to the larger target population. For example, socioeconomic status is a determining factor of many health outcomes. In the past, several studies have been conducted to evaluate the role of income in health outcomes. However, investigators always face challenges in obtaining complete information about income (Kim et al., 2007; Lannin et al., 1998; Banks et al., 2006). In surveys, high earning and low earning individuals generally do not want to disclose their income for a variety of reasons. Therefore it is expected that survey data will have missing information for income variables. This lack of information reduces the analyzable population sample and results in obtaining biased and invalid inferences. In addition, missing information would also restrict the generalizability of conclusion (Ryder et al. 2011). For instance, if low-income people tend to have a higher proportion of missing data compared to middle income or high-income individuals, investigators would not be able to generalize results to a low-income larger population since the majority of information is missing in the low-income subgroup. Work in this dissertation showed that multiple imputation can be applied in a way to effectively deal with missing information in public health practice and research to obtain valid inferences.

According to Peugh & Enders (2004) and Bodner (2006), many investigators do not discuss the methods used to handle missing data. In these cases, the method adopted for dealing with missing values is generally the complete case analysis, which is also known as list-wise deletion. Another popular method that is used is called the available case analysis, also referred to as case-wise deletion (Langkamp et al., 2010). As previously mentioned, under the assumption of MAR, maximum likelihood methods of analysis are valid, even if there is a loss of power. However, the use of complete case analysis or available case analysis methods were not recommended by American Psychological Association Task Force on Statistical Inference because these methods drop cases which lead to reduce sample size and ultimately statistical power of analysis (Wilkinson, 1999; Langkamp et al., 2010).

Based on the outcomes of this dissertation, it is evident that in surveys where missing information could have monotonic or non-monotonic missing data patterns, multiple imputation works well because it retains all original available information, while augmenting the original available data multiple times with imputed values. Multiple imputation provides valid inferences under MAR as outlined in chapter 3 as well as accounting for both within and between subject variability which reduces the possibility of biased estimates of all parameters. Moreover, even for clinical trial settings when the measurement is recorded repeatedly over the time on a subject and if subject has missing information for any visit, multiple imputation also provides valid statistical inferences. Conclusively, public health practice can be improved by imputing missing information in survey data by using multiple imputation techniques. We highly suggest that use of multiple imputation techniques as a valid alternative to other data analysis techniques that involve incomplete data.

Work in this dissertation assumed a MAR type of missing data mechanism under monotonic and non-monotonic missing data patterns. The results indicate that multiple imputation would be appropriate to use under the MAR assumption which ultimately helps to reduce the bias in estimates of all parameters while retaining all of the original available data and its distribution. Even though both MLE and Bayesian methods are valid and unbiased under the MAR, MLE requires specification of the distribution and Bayesian analysis requires priors and distributional assumptions. Since the basic distributional assumption would be normality, MLE and Bayesian methods would be biased when the normality assumption is violated. In addition, MLE could also lead to biased estimate when the sample size is small. Conclusively based on the results of this dissertation, it is evident that the multiple imputation method provides unbiased estimates under normal as well as non-normal distributions such as t-distribution, Chi-square distribution with various degrees of freedom.

There are several limitations to our work. First, the work was limited to the MAR type of missing data mechanism. Yet some of the non-responses in public health surveys or clinical trials might have MNAR missing data mechanism, therefore one area for future study could be to examine the impact of MNAR type of missing data mechanisms mixed with MAR data missing mechanisms. This could be examined using various mixture percentages of MNAR and MAR with both monotonic and non-monotonic missing data patterns. Secondly, in this dissertation we assessed only the effect of multiple imputation on continuous variables. Public health practice or clinical trial research may consist of ordinal or nominal variables, therefore the another scope of future work may include application of multiple imputation to discrete variables such as ordinal or nominal variables under MAR, MNAR and a mixture of MAR and MNAR. Thirdly, the work in this dissertation demonstrates that it is plausible to use the multiple imputation method under MAR type of assumption. However, it is very important to specify the imputation model appropriately in order to reduce possibility of bias. Future research may require recognizing and assessing the impact of bias due to an inadequately specified imputation model was employed. This has been recognized when attempting to impute binary and ordinal data using MCMC under the assumption of multivariate normality. Currently there is work on using a full condition specification model that uses an approach of imputation using chained equations (ICE). This approach is advanced by the MICE Project (Buuren and Oudshoorn, 2000). There are several reported problems with this approach such as inefficient algorithms and difficulties with processing larger number of variables when the sample size is small.

Finally the purpose of this dissertation was to evaluate the robustness of multiple imputation under MAR mechanism. We evaluated multiple imputation techniques with various percent of missing data under both monotonic and non-monotonic missing data patterns with normal and non-normal distributions. Sensitivity and specificity analysis results confirmed that simulations rejecting the null hypothesis were actually the same in both multiple imputation and full data analysis. Therefore, multiple imputation proved to work well compared to other methods discussed in this dissertation. Furthermore, we recommend that multiple imputation is an effective technique to deal with missing data in public health research and clinical trials because it provides valid inferences.

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APPENDICES

MONOTONE MISSING DATA PATTERN

APPENDIX A t-DISTRIBUTION WITH DIFFERENT DEGREES OF FREEDOM

Table 1: t-distribution (df = 2) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.00206	-0.00418 (-0.6053)	0.00829 (0.8906)	0.8951	0.8817 (0.3568)	0.9087 (12.1115)	0.12	0.084	0.071
100		10	0.00517	-0.00076 (-0.4804)	0.0111 (0.8392)	0.8976	0.8848 (0.2525)	0.9106 (10.1510)	0.11	0.085	0.067
100	15	5	0.00987	-0.00014 (1.6836)	0.0199 (2.2858)	0.8229	0.8043 (0.1600)	0.8419 (89.2209)	0.119	0.094	0.078
100		10	0.00768	-0.00195 (-1.6262)	0.0173 (2.2775)	0.8391	0.8208 (0.1230)	0.8579 (83.7440)	0.114	0.1	0.086
100	20	5	0.00416	-0.00833 (-1.7116)	0.0167 (2.2458)	0.7812	0.7601 (0.1561)	0.8028 (96.9012)	0.135	0.113	0.096
100		10	0.00670	-0.00533 (-1.6291)	0.0187 (2.2648)	0.7920	0.7715 (0.1497)	0.8131 (84.4490)	0.13	0.111	0.09
100	25	5	0.00368	-0.0102 (-1.6320)	0.0175 (2.2608)	0.7401	0.7167 (0.1486)	0.7642 (104.7)	0.136	0.119	0.108
100		10	0.000624	-0.0131 (-1.8446)	0.0143 (2.2515)	0.7377	0.7156 (0.1861)	0.7605 (81.1378)	0.128	0.112	0.097
500	10	5	0.000154	-0.00277 (-0.1872)	0.00308 (0.3741)	0.8954	0.8865 (0.5562)	0.9043 (7.7211)	0.091	0.062	0.053
500		10	0.000054	-0.00279 (-0.1545)	0.00290 (0.5100)	0.9005	0.8917 (0.6203)	0.9093 (8.7256)	0.098	0.064	0.043
500	15	5	0.00109	-0.00310 (-0.2716)	0.00528 (0.6132)	0.8479	0.8337 (0.4516)	0.8624 (33.8325)	0.112	0.082	0.056
500		10	0.00173	-0.00247 (-0.2990)	0.00594 (0.7076)	0.8503	0.8370 (0.5432)	0.8637 (31.4496)	0.11	0.083	0.061
500	20	5	0.00284	-0.00249 (-0.5360)	0.00817 (0.7512)	0.7865	0.7711 (0.3606)	0.8023 (27.4675)	0.112	0.089	0.069
500		10	0.00183	-0.00312 (-0.3027)	0.00679 (0.6669)	0.7895	0.7752 (0.4801)	0.8041 (32.2882)	0.097	0.077	0.061
500	25	5	0.00303	-0.00325 (-0.8198)	0.00930 (0.7789)	0.7330	0.7162 (0.2414)	0.7501 (28.5458)	0.112	0.094	0.075
500		10	0.00220	-0.00334 (-0.4065)	0.00773 (0.6287)	0.7451	0.7295 (0.3642)	0.7609 (22.7368)	0.1	0.087	0.07

Table 1 continues: t-distribution (df = 2) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00021	-0.00221 (-0.1074)	0.00180 (0.1753)	0.8974	0.8893 (0.6330)	0.9055 (4.0608)	0.104	0.062	0.046
1000		10	-0.00077	-0.00267 (-0.1199)	0.00113 (0.1878)	0.9041	0.8960 (0.7048)	0.9122 (4.6264)	0.1	0.07	0.053
1000	15	5	0.000648	-0.00208 (-0.1701)	0.00338 (0.3001)	0.8402	0.8281 (0.4530)	0.8524 (16.5555)	0.106	0.077	0.059
1000		10	0.00136	-0.00126 (-0.1811)	0.00399 (0.3011)	0.8506	0.8394 (0.5238)	0.8619 (15.3948)	0.094	0.068	0.052
1000	20	5	0.000719	-0.00283 (-0.3393)	0.00427 (0.3076)	0.7894	0.7753 (0.2939)	0.8037 (16.2968)	0.104	0.08	0.064
1000		10	0.000990	-0.00249 (-0.3745)	0.00447 (0.4933)	0.7997	0.7872 (0.4881)	0.8124 (13.4079)	0.089	0.075	0.065
1000	25	5	0.00198	-0.00224 (-0.5145)	0.00620 (0.4233)	0.7372	0.7224 (0.3020)	0.7522 (10.2682)	0.111	0.083	0.068
1000		10	0.000739	-0.00311 (-0.3400)	0.00458 (0.3093)	0.7477	0.7345 (0.3861)	0.7612 (10.2314)	0.093	0.08	0.071
5000	10	5	-0.00004	-0.00103 (-0.0542)	0.000954 (0.1213)	0.9011	0.8940 (0.6234)	0.9083 (6.8089)	0.103	0.062	0.045
5000		10	-0.00010	-0.00105 (-0.0559)	0.000852 (0.1216)	0.9069	0.9003 (0.7287)	0.9135 (8.0015)	0.086	0.05	0.038
5000	15	5	-0.00011	-0.00139 (-0.1249)	0.00118 (0.0971)	0.8378	0.8278 (0.4919)	0.8480 (8.6894)	0.094	0.071	0.048
5000		10	0.000448	-0.00074 (-0.0686)	0.00164 (0.1194)	0.8503	0.8412 (0.5396)	0.8595 (7.6958)	0.088	0.059	0.048
5000	20	5	0.00116	-0.00040 (-0.1039)	0.00272 (0.1920)	0.7956	0.7827 (0.3255)	0.8088 (16.8897)	0.103	0.074	0.047
5000		10	0.00114	-0.00038 (-0.1125)	0.00265 (0.1954)	0.7932	0.7815 (0.5005)	0.8051 (16.6051)	0.076	0.061	0.049
5000	25	5	0.00130	-0.00061 (-0.1131)	0.00321 (0.1697)	0.7395	0.7257 (0.3044)	0.7536 (11.0205)	0.09	0.069	0.057
5000		10	0.00122	-0.00059 (-0.1161)	0.00304 (0.1801)	0.7499	0.7374 (0.4542)	0.7627 (18.8390)	0.074	0.063	0.051

Table 2: t-distribution (df = 4) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00072	-0.00385 (-0.1718)	0.00242 (0.2081)	0.8890	0.8818 (0.5347)	0.8963 (2.0597)	0.11	0.059	0.039
100		10	-0.00024	-0.00320 (-0.1949)	0.00272 (0.1682)	0.8979	0.8917 (0.6480)	0.9042 (2.0083)	0.097	0.052	0.033
100	15	5	-0.00196	-0.00597 (-0.2391)	0.00206 (0.2302)	0.8275	0.8182 (0.4269)	0.8368 (2.1062)	0.096	0.054	0.036
100		10	-0.00149	-0.00538 (-0.2140)	0.00241 (0.2234)	0.8292	0.8214 (0.4766)	0.8370 (2.4769)	0.074	0.045	0.033
100	20	5	-0.00125	-0.00615 (-0.2751)	0.00365 (0.2372)	0.7680	0.7573 (0.2563)	0.7788 (2.6338)	0.092	0.062	0.042
100		10	-0.00105	-0.00590 (-0.2819)	0.00381 (0.2310)	0.7715	0.7628 (0.4071)	0.7803 (2.3916)	0.053	0.04	0.03
100	25	5	-0.00278	-0.00861 (-0.3275)	0.00306 (0.4944)	0.7038	0.6921 (0.2484)	0.7158 (2.4704)	0.069	0.054	0.04
100		10	-0.00335	-0.00890 (-0.2796)	0.00221 (0.3986)	0.7200	0.7104 (0.3438)	0.7298 (2.0156)	0.057	0.038	0.025
500	10	5	0.000427	-0.00092 (-0.0651)	0.00177 (0.0900)	0.8965	0.8917 (0.6394)	0.9013 (1.5116)	0.061	0.023	0.01
500		10	0.000536	-0.00076 (-0.0654)	0.00183 (0.0656)	0.9050	0.9015 (0.6891)	0.9086 (1.5064)	0.049	0.011	0.006
500	15	5	0.000093	-0.00171 (-0.1170)	0.00190 (0.1089)	0.8358	0.8289 (0.4200)	0.8427 (1.4516)	0.056	0.024	0.012
500		10	0.00117	-0.00051 (-0.0811)	0.00286 (0.1172)	0.8419	0.8368 (0.6183)	0.8470 (1.4798)	0.037	0.017	0.007
500	20	5	-0.00090	-0.00310 (-0.1173)	0.00130 (0.1525)	0.7779	0.7698 (0.4172)	0.7860 (1.6420)	0.034	0.021	0.01
500		10	-0.00057	-0.00268 (-0.1443)	0.00155 (0.1466)	0.7893	0.7833 (0.4957)	0.7953 (1.4889)	0.022	0.009	0.007
500	25	5	-0.00147	-0.00402 (-0.2122)	0.00108 (0.1681)	0.7339	0.7246 (0.3582)	0.7433 (1.5221)	0.03	0.012	0.005
500		10	-0.00180	-0.00423 (-0.1461)	0.000644 (0.1631)	0.7334	0.7264 (0.4145)	0.7404 (1.4862)	0.022	0.012	0.008

Table 2 continues: t-distribution (df = 4) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000190	-0.00077 (-0.0525)	0.00115 (0.0477)	0.9013	0.8969 (0.6104)	0.9057 (1.1465)	0.054	0.016	0.005
1000		10	-0.00024	-0.00113 (-0.0460)	0.000656 (0.0541)	0.9031	0.8998 (0.7142)	0.9063 (1.2746)	0.028	0.006	0.004
1000	15	5	-0.00042	-0.00166 (-0.0606)	0.000813 (0.0695)	0.8451	0.8388 (0.5520)	0.8513 (1.2265)	0.046	0.011	0.007
1000		10	-0.00040	-0.00158 (-0.0550)	0.000781 (0.0544)	0.8432	0.8387 (0.6182)	0.8477 (1.1640)	0.026	0.009	0.003
1000	20	5	-0.00080	-0.00232 (-0.0890)	0.000711 (0.0909)	0.7795	0.7717 (0.4603)	0.7873 (1.2735)	0.031	0.012	0.003
1000		10	-0.00074	-0.00218 (-0.0866)	0.000699 (0.0904)	0.7853	0.7797 (0.5116)	0.7909 (1.2116)	0.011	0.002	0.001
1000	25	5	-0.00235	-0.00411 (-0.0761)	-0.00059 (0.1138)	0.7311	0.7224 (0.3209)	0.7399 (1.1615)	0.021	0.012	0.003
1000		10	-0.00154	-0.00324 (-0.0898)	0.000165 (0.1302)	0.7378	0.7317 (0.4342)	0.7441 (1.1375)	0.01	0.003	0.001
5000	10	5	0.000125	-0.00031 (-0.0321)	0.000559 (0.0190)	0.8956	0.8915 (0.6430)	0.8997 (1.0317)	0.013	0	0
5000		10	3.862E-6	-0.00041 (-0.0256)	0.000414 (0.0203)	0.9063	0.9036 (0.7553)	0.9090 (1.0949)	0.007	0.001	0
5000	15	5	-0.00002	-0.00058 (-0.0288)	0.000544 (0.0294)	0.8362	0.8300 (0.3754)	0.8425 (1.0641)	0.015	0.002	0
5000		10	-0.00016	-0.00069 (-0.0267)	0.000365 (0.0285)	0.8470	0.8430 (0.6159)	0.8511 (1.0300)	0.005	0	0
5000	20	5	-0.00031	-0.00098 (-0.0358)	0.000359 (0.0338)	0.7809	0.7735 (0.4369)	0.7884 (1.0218)	0.007	0	0
5000		10	-0.00039	-0.00101 (-0.0371)	0.000230 (0.0290)	0.7866	0.7816 (0.5656)	0.7916 (1.0157)	0.002	0	0
5000	25	5	-0.00078	-0.00157 (-0.0449)	0.000013 (0.0419)	0.7319	0.7234 (0.3405)	0.7404 (1.0362)	0.005	0	0
5000		10	-0.00046	-0.00120 (-0.0445)	0.000290 (0.0385)	0.7419	0.7363 (0.4902)	0.7476 (0.9980)	0	0	0

Table 3: t-distribution (df = 6) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.000202	-0.00247 (-0.1824)	0.00288 (0.1979)	0.8872	0.8810 (0.5375)	0.8935 (1.5479)	0.907	0.036	0.022
100		10	-0.00007	-0.00258 (-0.1682)	0.00243 (0.2091)	0.8980	0.8930 (0.6561)	0.9030 (1.4974)	0.093	0.038	0.013
100	15	5	0.00145	-0.00189 (-0.1409)	0.00479 (0.1959)	0.8256	0.8175 (0.4109)	0.8339 (1.7505)	0.086	0.043	0.021
100		10	0.000323	-0.00294 (-0.1679)	0.00359 (0.1810)	0.8314	0.8247 (0.3652)	0.8382 (1.6251)	0.064	0.036	0.022
100	20	5	0.000851	-0.00330 (-0.2344)	0.00500 (0.2739)	0.7604	0.7503 (0.3014)	0.7707 (1.6941)	0.076	0.044	0.027
100		10	0.000402	-0.00363 (-0.2110)	0.00443 (0.2799)	0.7672	0.7594 (0.3902)	0.7750 (1.5562)	0.051	0.032	0.018
100	25	5	-0.00106	-0.00606 (-0.2944)	0.00395 (0.2940)	0.7011	0.6904 (0.2750)	0.7120 (1.8958)	0.051	0.03	0.018
100		10	0.000818	-0.00394 (-0.2315)	0.00558 (0.3104)	0.7154	0.7069 (0.3292)	0.7240 (1.8012)	0.031	0.023	0.013
500	10	5	-0.00037	-0.00157 (-0.0645)	0.000837 (0.0550)	0.8967	0.8921 (0.5743)	0.9013 (1.4561)	0.05	0.006	0.002
500		10	-0.00031	-0.00146 (-0.0576)	0.000835 (0.0502)	0.9052	0.9020 (0.6992)	0.9084 (1.3837)	0.032	0.003	0.002
500	15	5	-0.00064	-0.00218 (-0.0894)	0.000901 (0.0791)	0.8400	0.8338 (0.5041)	0.8462 (1.3203)	0.028	0.004	0.001
500		10	-0.00060	-0.00211 (-0.0818)	0.000905 (0.0713)	0.8453	0.8408 (0.5683)	0.8498 (1.3607)	0.011	0.003	0.002
500	20	5	-0.00117	-0.00303 (-0.0910)	0.000700 (0.1018)	0.7778	0.7701 (0.4013)	0.7855 (1.2374)	0.019	0.006	0.001
500		10	-0.00024	-0.00208 (-0.0882)	0.00160 (0.0906)	0.7870	0.7814 (0.5004)	0.7926 (1.3747)	0.007	0.002	0.001
500	25	5	-0.00110	-0.00337 (-0.1185)	0.00118 (0.1115)	0.7260	0.7173 (0.3649)	0.7348 (1.2354)	0.023	0.01	0.002
500		10	-0.00107	-0.00325 (-0.1679)	0.00112 (0.0986)	0.7289	0.7226 (0.4171)	0.7353 (1.1211)	0.005	0.001	0.001

Table 3 continues: t-distribution (df = 6) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00025	-0.00107 (-0.0474)	0.000573 (0.0433)	0.8974	0.8931 (0.5897)	0.9018 (1.2283)	0.022	0.001	0.001
1000		10	-0.00028	-0.00108 (-0.0391)	0.000511 (0.0427)	0.9054	0.9025 (0.7267)	0.9083 (1.1596)	0.007	0.001	0.001
1000	15	5	-0.00002	-0.00107 (-0.0588)	0.00102 (0.0529)	0.8342	0.8280 (0.4799)	0.8406 (1.0823)	0.022	0.001	0
1000		10	-0.00030	-0.00133 (-0.0592)	0.000731 (0.0590)	0.8448	0.8408 (0.6458)	0.8488 (1.2533)	0.004	0.001	0.001
1000	20	5	-0.00088	-0.00222 (-0.0956)	0.000458 (0.0634)	0.7817	0.7745 (0.4611)	0.7890 (1.1258)	0.015	0.001	0.001
1000		10	-0.00087	-0.00213 (-0.0805)	0.000387 (0.0741)	0.7861	0.7810 (0.5226)	0.7913 (1.0271)	0.002	0	0
1000	25	5	-0.00100	-0.00258 (-0.0852)	0.000577 (0.0956)	0.7356	0.7272 (0.3542)	0.7441 (1.1406)	0.012	0.003	0.001
1000		10	-0.00124	-0.00273 (-0.0895)	0.000257 (0.0992)	0.7396	0.7337 (0.4014)	0.7455 (1.0063)	0.001	0	0
5000	10	5	0.000153	-0.00021 (-0.0211)	0.000518 (0.0181)	0.8991	0.8952 (0.6665)	0.9030 (1.0400)	0.005	0	0
5000		10	0.000180	-0.00017 (-0.0183)	0.000530 (0.0170)	0.9063	0.9038 (0.7417)	0.9089 (1.0142)	0.002	0	0
5000	15	5	0.000307	-0.00016 (-0.0250)	0.000776 (0.0217)	0.8442	0.8388 (0.5869)	0.8497 (1.0181)	0.007	0	0
5000		10	0.000177	-0.00028 (-0.0248)	0.000635 (0.0212)	0.8494	0.8456 (0.6627)	0.8532 (0.9810)	0	0	0
5000	20	5	0.000073	-0.00051 (-0.0258)	0.000659 (0.0330)	0.7812	0.7741 (0.3745)	0.7885 (1.0110)	0.001	0	0
5000		10	-0.00006	-0.00062 (-0.0309)	0.000508 (0.0293)	0.7910	0.7861 (0.5430)	0.7960 (1.0177)	0.001	0	0
5000	25	5	-0.00031	-0.00098 (-0.0336)	0.000364 (0.0361)	0.7327	0.7246 (0.3546)	0.7409 (1.0153)	0.003	0	0
5000		10	-0.00004	-0.00069 (-0.0303)	0.000602 (0.0328)	0.7414	0.7358 (0.4431)	0.7471 (0.9638)	0	0	0

Table 4: t-distribution (df = 8) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00061	-0.00322 (-0.1500)	0.00200 (0.1443)	0.8895	0.8835 (0.5501)	0.8956 (1.2440)	0.087	0.036	0.017
100		10	-0.00095	-0.00340 (-0.1372)	0.00150 (0.1221)	0.8949	0.8901 (0.6248)	0.8996 (1.2816)	0.081	0.026	0.007
100	15	5	0.000916	-0.00236 (-0.1778)	0.00419 (0.1968)	0.8213	0.8135 (0.3973)	0.8292 (1.2971)	0.068	0.023	0.012
100		10	-0.00068	-0.00375 (-0.1585)	0.00239 (0.1585)	0.8298	0.8236 (0.4867)	0.8360 (1.4266)	0.04	0.018	0.01
100	20	5	-0.00099	-0.00494 (-0.2514)	0.00296 (0.1978)	0.7655	0.7563 (0.3381)	0.7747 (1.4369)	0.057	0.033	0.02
100		10	-0.00102	-0.00483 (-0.1790)	0.00278 (0.1874)	0.7646	0.7576 (0.4216)	0.7717 (1.5363)	0.029	0.017	0.004
100	25	5	-0.00115	-0.00572 (-0.2403)	0.00342 (0.2660)	0.7117	0.7014 (0.2602)	0.7223 (1.5017)	0.051	0.036	0.021
100		10	0.00126	-0.00308 (-0.2096)	0.00560 (0.2852)	0.7125	0.7046 (0.3426)	0.7205 (1.2837)	0.028	0.016	0.009
500	10	5	-0.00052	-0.00160 (-0.0558)	0.000559 (0.0512)	0.8978	0.8936 (0.6136)	0.9021 (1.0974)	0.039	0.007	0
500		10	-0.00018	-0.00119 (-0.0594)	0.000836 (0.0507)	0.9014	0.8982 (0.7196)	0.9047 (1.0361)	0.016	0	0
500	15	5	-0.00044	-0.00191 (-0.0802)	0.00102 (0.0794)	0.8344	0.8283 (0.4738)	0.8406 (1.0761)	0.027	0.004	0
500		10	-0.00075	-0.00212 (-0.0717)	0.000622 (0.0943)	0.8452	0.8409 (0.6031)	0.8494 (1.0565)	0.007	0.001	0
500	20	5	-0.00193	-0.00370 (-0.0999)	-0.00015 (0.0798)	0.7733	0.7652 (0.3418)	0.7815 (1.1160)	0.019	0.003	0.001
500		10	-0.00174	-0.00341 (-0.0822)	-0.00006 (0.0779)	0.7888	0.7833 (0.5139)	0.7942 (1.0248)	0.005	0	0
500	25	5	-0.00118	-0.00319 (-0.0963)	0.000822 (0.0981)	0.7299	0.7214 (0.3391)	0.7386 (1.1126)	0.011	0.002	0.001
500		10	-0.00109	-0.00299 (-0.1006)	0.000813 (0.0863)	0.7305	0.7242 (0.4419)	0.7368 (1.0831)	0.004	0.001	0

Table 4 continues: t-distribution (df = 8) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00007	-0.00082 (-0.0463)	0.000690 (0.0348)	0.8994	0.8954 (0.6055)	0.9034 (1.0507)	0.02	0.001	0
1000		10	0.000160	-0.00058 (-0.0430)	0.000903 (0.0415)	0.9028	0.8999 (0.7114)	0.9056 (1.0231)	0.008	0	0
1000	15	5	-0.00050	-0.00150 (-0.0521)	0.000502 (0.0567)	0.8371	0.8311 (0.5083)	0.8431 (1.0238)	0.013	0	0
1000		10	-0.00064	-0.00160 (-0.0493)	0.000322 (0.0468)	0.8455	0.8413 (0.6241)	0.8496 (1.0206)	0.005	0	0
1000	20	5	-0.00040	-0.00162 (-0.0697)	0.000820 (0.0619)	0.7786	0.7713 (0.4402)	0.7860 (1.0379)	0.011	0	0
1000		10	-0.00068	-0.00186 (-0.0692)	0.000491 (0.0646)	0.7901	0.7851 (0.5433)	0.7951 (1.0026)	0.001	0	0
1000	25	5	0.000163	-0.00126 (-0.0878)	0.00159 (0.0820)	0.7254	0.7169 (0.3149)	0.7339 (1.0703)	0.007	0.001	0
1000		10	-0.00034	-0.00171 (-0.0679)	0.00103 (0.0704)	0.7345	0.7286 (0.4220)	0.7405 (1.0152)	0.001	0	0
5000	10	5	0.000028	-0.00031 (-0.0147)	0.000368 (0.0164)	0.9004	0.8965 (0.6493)	0.9042 (1.0097)	0.006	0	0
5000		10	-0.00005	-0.00037 (-0.0173)	0.000269 (0.0146)	0.9057	0.9033 (0.7586)	0.9081 (0.9916)	0	0	0
5000	15	5	0.000021	-0.00042 (-0.0189)	0.000457 (0.0212)	0.8394	0.8337 (0.5166)	0.8451 (1.0037)	0.002	0	0
5000		10	0.000128	-0.00029 (-0.0227)	0.000550 (0.0199)	0.8480	0.8442 (0.5525)	0.8518 (1.0020)	0.001	0	0
5000	20	5	0.000112	-0.00044 (-0.0302)	0.000669 (0.0293)	0.7787	0.7710 (0.4139)	0.7864 (1.0007)	0.001	0	0
5000		10	-0.00003	-0.00055 (-0.0235)	0.000487 (0.0271)	0.7895	0.7850 (0.5541)	0.7941 (0.9839)	0	0	0
5000	25	5	0.000481	-0.00016 (-0.0270)	0.00112 (0.0305)	0.7285	0.7202 (0.3856)	0.7369 (1.0028)	0.002	0	0
5000		10	0.000390	-0.00022 (-0.0284)	0.000996 (0.0283)	0.7422	0.7367 (0.4946)	0.7477 (0.9673)	0	0	0

Table 5: t-distribution (df = 10) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.000300	-0.00210 (-0.1468)	0.00270 (0.1385)	0.8936	0.8878 (0.5113)	0.8995 (1.2353)	0.106	0.038	0.014
100		10	0.000468	-0.00182 (-0.1352)	0.00276 (0.1400)	0.8961	0.8913 (0.5963)	0.9009 (1.2341)	0.075	0.027	0.013
100	15	5	0.000285	-0.00291 (-0.2158)	0.00348 (0.2060)	0.8203	0.8123 (0.4477)	0.8284 (1.6160)	0.064	0.027	0.008
100		10	0.00111	-0.00196 (-0.2087)	0.00419 (0.1911)	0.8342	0.8278 (0.4536)	0.8407 (1.5954)	0.057	0.016	0.01
100	20	5	-0.00019	-0.00416 (-0.2295)	0.00379 (0.2227)	0.7589	0.7495 (0.3220)	0.7684 (1.2943)	0.056	0.03	0.012
100		10	0.00243	-0.00127 (-0.2409)	0.00613 (0.1770)	0.7731	0.7658 (0.4447)	0.7804 (1.4851)	0.036	0.015	0.009
100	25	5	0.00149	-0.00295 (-0.2627)	0.00593 (0.2848)	0.7085	0.6982 (0.3172)	0.7190 (1.3257)	0.039	0.021	0.015
100		10	0.00252	-0.00180 (-0.2429)	0.00684 (0.2449)	0.7104	0.7024 (0.7186)	0.7186 (1.4079)	0.026	0.014	0.009
500	10	5	0.000164	-0.00086 (-0.0517)	0.00119 (0.0563)	0.8998	0.8954 (0.6338)	0.9043 (1.0815)	0.046	0.007	0
500		10	0.000243	-0.00073 (-0.0542)	0.00121 (0.0474)	0.9038	0.9008 (0.7113)	0.9069 (1.0298)	0.015	0	0
500	15	5	0.000440	-0.00089 (-0.0680)	0.00177 (0.0617)	0.8397	0.8336 (0.5007)	0.8459 (1.1254)	0.025	0.002	0.001
500		10	-0.00011	-0.00137 (-0.0743)	0.00115 (0.0608)	0.8466	0.8425 (0.5749)	0.8507 (1.0498)	0.004	0	0
500	20	5	0.000157	-0.00145 (-0.0894)	0.00176 (0.0795)	0.7761	0.7682 (0.3572)	0.7840 (1.0919)	0.016	0.002	0
500		10	-0.00015	-0.00168 (-0.0875)	0.00137 (0.0678)	0.7921	0.7867 (0.5066)	0.7975 (1.0517)	0.003	0.001	0
500	25	5	0.000294	-0.00162 (-0.1017)	0.00221 (0.0935)	0.7329	0.7241 (0.3437)	0.7418 (1.0860)	0.018	0.003	0
500		10	-0.00015	-0.00196 (-0.0987)	0.00165 (0.0860)	0.7364	0.7304 (0.4000)	0.7424 (1.0036)	0.001	0	0

Table 5 continues: t-distribution (df = 10) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000118	-0.00062 (-0.0338)	0.000851 (0.0429)	0.8953	0.8909 (0.6015)	0.8996 (1.0389)	0.019	0	0
1000		10	0.000199	-0.00049 (-0.0385)	0.000891 (0.0377)	0.9030	0.9003 (0.7637)	0.9058 (1.0355)	0.005	0	0
1000	15	5	0.000341	-0.00059 (-0.0412)	0.00128 (0.0420)	0.8398	0.8338 (0.5427)	0.8458 (1.0335)	0.012	0	0
1000		10	0.000113	-0.00078 (-0.0471)	0.00101 (0.0439)	0.8453	0.8414 (0.6063)	0.8493 (1.0092)	0.003	0	0
1000	20	5	0.000144	-0.00097 (-0.0610)	0.00126 (0.0641)	0.7817	0.7740 (0.3847)	0.7894 (1.0919)	0.017	0.001	0
1000		10	0.000160	-0.00090 (-0.0571)	0.00122 (0.0502)	0.7875	0.7823 (0.5144)	0.7927 (1.0010)	0.001	0	0
1000	25	5	0.000194	-0.00113 (-0.0703)	0.00151 (0.0607)	0.7235	0.7148 (0.3655)	0.7323 (1.0483)	0.008	0	0
1000		10	0.000098	-0.00115 (-0.0665)	0.00135 (0.0604)	0.7397	0.7338 (0.4645)	0.7456 (1.0093)	0.001	0	0
5000	10	5	4.95E-6	-0.00032 (-0.0135)	0.000326 (0.0182)	0.8968	0.8928 (0.6243)	0.9008 (1.0174)	0.002	0	0
5000		10	-0.00010	-0.00041 (-0.0155)	0.000205 (0.0154)	0.9044	0.9020 (0.7479)	0.9069 (0.9915)	0	0	0
5000	15	5	-0.00017	-0.00060 (-0.0228)	0.000256 (0.0237)	0.8379	0.8322 (0.5302)	0.8436 (1.0063)	0.001	0	0
5000		10	-0.00017	-0.00056 (-0.0194)	0.000219 (0.0191)	0.8453	0.8413 (0.6357)	0.8493 (0.9814)	0	0	0
5000	20	5	-0.00024	-0.00075 (-0.0241)	0.000260 (0.0300)	0.7844	0.7771 (0.4647)	0.7918 (1.0098)	0.002	0	0
5000		10	-0.00030	-0.00077 (-0.0240)	0.000180 (0.0231)	0.7886	0.7838 (0.5135)	0.7934 (0.9824)	0	0	0
5000	25	5	-0.00030	-0.00088 (-0.0299)	0.000275 (0.0301)	0.7209	0.7125 (0.3502)	0.7294 (1.0185)	0.004	0	0
5000		10	-0.00031	-0.00088 (-0.0316)	0.000252 (0.0318)	0.7378	0.7323 (0.4748)	0.7434 (0.9581)	0	0	0

Table 6: t-distribution (df = 15) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00248	-0.00478 (-0.1263)	-0.00017 (0.1274)	0.8882	0.8824 (0.5689)	0.8941 (1.2121)	0.086	0.031	0.016
100		10	-0.00249	-0.00472 (-0.1194)	-0.00026 (0.1355)	0.8947	0.8901 (0.6198)	0.8994 (1.2189)	0.067	0.02	0.009
100	15	5	-0.00121	-0.00437 (-0.1809)	0.00195 (0.1758)	0.8237	0.8159 (0.4366)	0.8316 (1.6168)	0.06	0.028	0.008
100		10	-0.00049	-0.00342 (-0.1438)	0.00244 (0.1984)	0.8318	0.8258 (0.5176)	0.8380 (1.2431)	0.042	0.011	0.005
100	20	5	-0.00075	-0.00449 (-0.2126)	0.00299 (0.2267)	0.7568	0.7478 (0.3435)	0.7660 (1.4800)	0.053	0.022	0.007
100		10	0.00128	-0.00232 (-0.1939)	0.00488 (0.2301)	0.7692	0.7622 (0.4002)	0.7762 (1.3451)	0.021	0.009	0.004
100	25	5	0.00253	-0.00165 (-0.2252)	0.00671 (0.2119)	0.6923	0.6818 (0.2484)	0.7029 (1.2962)	0.031	0.018	0.013
100		10	0.00271	-0.00128 (-0.2109)	0.00671 (0.2357)	0.7072	0.6989 (0.3229)	0.7155 (1.2691)	0.024	0.015	0.005
500	10	5	0.000808	-0.00025 (-0.0527)	0.00186 (0.0611)	0.8965	0.8921 (0.6368)	0.9008 (1.0520)	0.024	0.001	0
500		10	0.000715	-0.00027 (-0.0551)	0.00170 (0.0544)	0.9053	0.9024 (0.7331)	0.9082 (1.0516)	0.009	0.001	0
500	15	5	0.000669	-0.00069 (-0.0906)	0.00203 (0.0621)	0.8359	0.8297 (0.4624)	0.8421 (1.0848)	0.018	0.002	0
500		10	0.000747	-0.00053 (-0.0710)	0.00202 (0.0721)	0.8501	0.8459 (0.6085)	0.8542 (1.0247)	0.003	0	0
500	20	5	0.00158	-0.00003 (-0.0888)	0.00319 (0.1002)	0.7791	0.7709 (0.3464)	0.7873 (1.0561)	0.019	0.001	0
500		10	0.00123	-0.00033 (-0.0912)	0.00278 (0.0804)	0.7882	0.7830 (0.4946)	0.7935 (1.0162)	0.002	0	0
500	25	5	0.00144	-0.00046 (-0.0902)	0.00333 (0.0955)	0.7284	0.7199 (0.3178)	0.7370 (1.0792)	0.013	0.001	0
500		10	0.00130	-0.00042 (-0.0886)	0.00302 (0.0940)	0.7387	0.7328 (0.4169)	0.7446 (1.0460)	0.002	0	0

Table 6 continues: t-distribution (df = 15) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000105	-0.00063 (-0.0345)	0.000840 (0.0379)	0.8965	0.8924 (0.6114)	0.9006 (1.0380)	0.025	0	0
1000		10	0.000118	-0.00061 (-0.0427)	0.000842 (0.0333)	0.9070	0.9041 (0.6926)	0.9099 (1.0145)	0.004	0	0
1000	15	5	0.000243	-0.00075 (-0.0502)	0.00124 (0.0605)	0.8356	0.8296 (0.5342)	0.8415 (1.0348)	0.015	0	0
1000		10	-0.00002	-0.00096 (-0.0474)	0.000926 (0.0539)	0.8456	0.8418 (0.6332)	0.8495 (0.9925)	0	0	0
1000	20	5	-0.00002	-0.00121 (-0.0483)	0.00116 (0.0571)	0.7849	0.7779 (0.3945)	0.7920 (1.0502)	0.009	0.001	0
1000		10	0.000382	-0.00076 (-0.0576)	0.00152 (0.0571)	0.7858	0.7810 (0.4961)	0.7907 (0.9887)	0	0	0
1000	25	5	0.00113	-0.00026 (-0.0756)	0.00252 (0.0620)	0.7256	0.7173 (0.3425)	0.7340 (1.0540)	0.004	0.001	0
1000		10	0.000560	-0.00075 (-0.0604)	0.00187 (0.0743)	0.7423	0.7367 (0.4296)	0.7480 (0.9851)	0	0	0
5000	10	5	-3.75E-6	-0.00033 (-0.0145)	0.000318 (0.0167)	0.9017	0.8977 (0.6269)	0.9056 (1.0114)	0.003	0	0
5000		10	-0.00005	-0.00036 (-0.0155)	0.000258 (0.0216)	0.9048	0.9022 (0.7382)	0.9073 (0.9910)	0	0	0
5000	15	5	0.000083	-0.00036 (-0.0202)	0.000523 (0.0279)	0.8362	0.8306 (0.5402)	0.8420 (1.0043)	0.002	0	0
5000		10	0.000170	-0.00024 (-0.0214)	0.000577 (0.0269)	0.8481	0.8443 (0.6057)	0.8519 (0.9878)	0	0	0
5000	20	5	0.000440	-0.00007 (-0.0253)	0.000955 (0.0272)	0.7882	0.7810 (0.4218)	0.7955 (1.0089)	0.004	0	0
5000		10	0.000253	-0.00024 (-0.0236)	0.000748 (0.0354)	0.7935	0.7890 (0.5355)	0.7980 (0.9629)	0	0	0
5000	25	5	-0.00007	-0.00067 (-0.0315)	0.000534 (0.0316)	0.7248	0.7165 (0.3406)	0.7333 (0.9933)	0	0	0
5000		10	0.000207	-0.00037 (-0.0287)	0.000784 (0.0296)	0.7418	0.7363 (0.4588)	0.7473 (0.9540)	0	0	0

Table 7: t-distribution (df =20) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00115	-0.00340 (-0.1263)	0.00111 (0.1349)	0.8854	0.8796 (0.5558)	0.8912 (1.2597)	0.08	0.025	0.008
100		10	-0.00144	-0.00355 (-0.1180)	0.000672 (0.1394)	0.8942	0.8896 (0.6035)	0.8989 (1.2135)	0.066	0.019	0.007
100	15	5	-0.00034	-0.00335 (-0.1690)	0.00266 (0.1761)	0.8275	0.8197 (0.4311)	0.8352 (1.2161)	0.072	0.025	0.01
100		10	-0.00030	-0.00326 (-0.1544)	0.00267 (0.1720)	0.8314	0.8253 (0.4911)	0.8375 (1.1569)	0.041	0.017	0.008
100	20	5	-0.00120	-0.00486 (-0.2839)	0.00247 (0.2402)	0.7627	0.7536 (0.3331)	0.7719 (1.3151)	0.045	0.018	0.007
100		10	-0.00074	-0.00427 (-0.2617)	0.00280 (0.1920)	0.7696	0.7623 (0.4454)	0.7769 (1.2468)	0.028	0.014	0.006
100	25	5	-0.00053	-0.00470 (-0.2714)	0.00364 (0.2321)	0.7005	0.6901 (0.3017)	0.7111 (1.3153)	0.028	0.014	0.01
100		10	0.000602	-0.00340 (-0.3098)	0.00460 (0.2039)	0.7089	0.7011 (0.3555)	0.7167 (1.1823)	0.016	0.007	0.003
500	10	5	0.000017	-0.00097 (-0.0466)	0.00101 (0.0591)	0.8968	0.8925 (0.5706)	0.9011 (1.0691)	0.034	0.001	0
500		10	0.000068	-0.00087 (-0.0406)	0.00101 (0.0557)	0.9019	0.8989 (0.7387)	0.9048 (1.0617)	0.011	0.001	0
500	15	5	0.000508	-0.00082 (-0.0756)	0.00184 (0.0770)	0.8363	0.8304 (0.3911)	0.8424 (1.0574)	0.013	0.002	0
500		10	-0.00016	-0.00145 (-0.0684)	0.00113 (0.0560)	0.8462	0.8421 (0.5957)	0.8503 (1.0205)	0.003	0	0
500	20	5	-0.00091	-0.00248 (-0.0783)	0.000664 (0.0866)	0.7787	0.7709 (0.3731)	0.7867 (1.0375)	0.015	0	0
500		10	-0.00051	-0.00203 (-0.0822)	0.00101 (0.0791)	0.7888	0.7837 (0.5203)	0.7940 (1.0416)	0.004	0	0
500	25	5	0.000146	-0.00173 (-0.1107)	0.00202 (0.0885)	0.7238	0.7155 (0.3633)	0.7322 (1.0362)	0.011	0	0
500		10	-0.00021	-0.00203 (-0.0996)	0.00161 (0.0766)	0.7406	0.7346 (0.3457)	0.7467 (1.0514)	0.002	0.001	0

Table 7 continues: t-distribution (df =20) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000152	-0.00055 (-0.0382)	0.000856 (0.0411)	0.8945	0.8902 (0.5621)	0.8988 (1.0333)	0.015	0	0
1000		10	0.000178	-0.00050 (-0.0351)	0.000855 (0.0327)	0.9031	0.9003 (0.7549)	0.9058 (1.0061)	0.006	0	0
1000	15	5	0.000270	-0.00069 (-0.0523)	0.00123 (0.0478)	0.8397	0.8336 (0.4877)	0.8460 (1.0279)	0.007	0	0
1000		10	0.000042	-0.00086 (-0.0494)	0.000946 (0.0388)	0.8472	0.8433 (0.6178)	0.8512 (1.0026)	0.003	0	0
1000	20	5	-0.00013	-0.00125 (-0.0585)	0.000990 (0.0589)	0.7851	0.7777 (0.3789)	0.7925 (1.0539)	0.005	0.001	0
1000		10	0.000020	-0.00107 (-0.0515)	0.00111 (0.0627)	0.7886	0.7837 (0.4884)	0.7935 (1.0086)	0.001	0	0
1000	25	5	0.000115	-0.00120 (-0.0857)	0.00143 (0.0631)	0.7307	0.7222 (0.3367)	0.7393 (1.0229)	0.005	0	0
1000		10	2.679E-6	-0.00128 (-0.0810)	0.00129 (0.0699)	0.7410	0.7354 (0.4973)	0.7467 (0.9563)	0	0	0
5000	10	5	-0.00003	-0.00036 (-0.0148)	0.000294 (0.0160)	0.9009	0.8970 (0.6508)	0.9049 (1.0147)	0.004	0	0
5000		10	-0.00005	-0.00036 (-0.0168)	0.000254 (0.0169)	0.9080	0.9055 (0.7662)	0.9104 (0.9942)	0	0	0
5000	15	5	-0.00016	-0.00058 (-0.0211)	0.000263 (0.0247)	0.8410	0.8353 (0.4891)	0.8468 (1.0071)	0.003	0	0
5000		10	-2.39E-6	-0.00041 (-0.0180)	0.000403 (0.0257)	0.8505	0.8467 (0.6542)	0.8543 (0.9877)	0	0	0
5000	20	5	0.000145	-0.00037 (-0.0266)	0.000658 (0.0276)	0.7843	0.7773 (0.4512)	0.7914 (1.0078)	0.002	0	0
5000		10	0.000167	-0.00032 (-0.0230)	0.000656 (0.0243)	0.7930	0.7883 (0.4981)	0.7976 (0.9650)	0	0	0
5000	25	5	-0.00003	-0.00063 (-0.0256)	0.000560 (0.0266)	0.7255	0.7171 (0.3498)	0.7339 (1.0085)	0.002	0	0
5000		10	-0.00007	-0.00064 (-0.0238)	0.000498 (0.0319)	0.7348	0.7292 (0.4219)	0.7405 (0.9518)	0	0	0

Table 8: t-distribution (df = 25) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.00221	0.000011 (-0.1144)	0.00441 (0.1466)	0.8882	0.8826 (0.4818)	0.8939 (1.1718)	0.088	0.027	0.003
100		10	0.00189	-0.00018 (-0.1100)	0.00397 (0.1272)	0.8886	0.8841 (0.5420)	0.8931 (1.1522)	0.052	0.011	0.003
100	15	5	0.00206	-0.00088 (-0.1436)	0.00500 (0.1482)	0.8216	0.8140 (0.3364)	0.8291 (1.2254)	0.07	0.014	0.005
100		10	0.00224	-0.00054 (-0.1400)	0.00502 (0.1787)	0.8264	0.8204 (0.4247)	0.8325 (1.1576)	0.037	0.007	0.003
100	20	5	0.000955	-0.00257 (-0.1997)	0.00448 (0.1819)	0.7641	0.7552 (0.3393)	0.7732 (1.1765)	0.042	0.017	0.008
100		10	0.00163	-0.00173 (-0.1706)	0.00499 (0.2083)	0.7671	0.7602 (0.4594)	0.7741 (1.1441)	0.016	0.007	0.002
100	25	5	0.000743	-0.00352 (-0.2489)	0.00501 (0.1969)	0.6944	0.6843 (0.2687)	0.7046 (1.2090)	0.031	0.017	0.009
100		10	0.00150	-0.00252 (-0.2771)	0.00552 (0.2405)	0.7085	0.7011 (0.4138)	0.7160 (1.1290)	0.01	0.005	0.002
500	10	5	0.000518	-0.00046 (-0.0609)	0.00150 (0.0494)	0.8955	0.8910 (0.5577)	0.9001 (1.0551)	0.035	0.001	0
500		10	0.000597	-0.00036 (-0.0529)	0.00156 (0.0538)	0.9033	0.9004 (0.7146)	0.9062 (1.0349)	0.012	0	0
500	15	5	0.000646	-0.00064 (-0.0697)	0.00194 (0.0650)	0.8325	0.8262 (0.4927)	0.8388 (1.0631)	0.012	0.002	0
500		10	0.000765	-0.00046 (-0.0573)	0.00199 (0.0738)	0.8442	0.8399 (0.5765)	0.8485 (1.0438)	0.003	0	0
500	20	5	0.000627	-0.00088 (-0.0726)	0.00213 (0.0964)	0.7822	0.7747 (0.3761)	0.7898 (1.0490)	0.015	0	0
500		10	-0.00018	-0.00162 (-0.0898)	0.00127 (0.1009)	0.7859	0.7805 (0.5005)	0.7913 (1.0363)	0.002	0	0
500	25	5	-0.00047	-0.00224 (-0.0970)	0.00129 (0.0928)	0.7221	0.7136 (0.3267)	0.7308 (1.0751)	0.013	0.002	0
500		10	-0.00081	-0.00252 (-0.1269)	0.000912 (0.0946)	0.7311	0.7250 (0.4358)	0.7373 (1.0288)	0.002	0	0

Table 8 continues: t-distribution (df = 25) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000174	-0.00051 (-0.0300)	0.000857 (0.0416)	0.9000	0.8961 (0.6911)	0.9038 (1.0384)	0.017	0	0
1000		10	0.000423	-0.00022 (-0.0395)	0.00107 (0.0329)	0.9050	0.9024 (0.7536)	0.9076 (0.9994)	0	0	0
1000	15	5	0.000307	-0.00063 (-0.0527)	0.00125 (0.0473)	0.8376	0.8317 (0.5326)	0.8435 (1.0409)	0.007	0	0
1000		10	0.000412	-0.00048 (-0.0532)	0.00130 (0.0494)	0.8468	0.8429 (0.6078)	0.8508 (0.9886)	0	0	0
1000	20	5	0.000271	-0.00081 (-0.0587)	0.00136 (0.0649)	0.7812	0.7741 (0.4182)	0.7883 (1.0266)	0.009	0	0
1000		10	0.000150	-0.00089 (-0.0643)	0.00119 (0.0510)	0.7896	0.7846 (0.4623)	0.7947 (0.9961)	0	0	0
1000	25	5	-0.00006	-0.00132 (-0.0810)	0.00121 (0.0585)	0.7365	0.7284 (0.3564)	0.7447 (1.0302)	0.001	0	0
1000		10	-0.00050	-0.00172 (-0.0843)	0.000721 (0.0687)	0.7425	0.7370 (0.4548)	0.7480 (0.9627)	0	0	0
5000	10	5	-0.00023	-0.00053 (-0.0165)	0.000067 (0.0135)	0.9000	0.8961 (0.6279)	0.9039 (1.0078)	0.066	0	0
5000		10	-0.00012	-0.00041 (-0.0135)	0.000170 (0.0123)	0.9052	0.9026 (0.7405)	0.9078 (0.9971)	0	0	0
5000	15	5	-0.00005	-0.00045 (-0.0192)	0.000348 (0.0181)	0.8359	0.8299 (0.4863)	0.8419 (1.0062)	0.001	0	0
5000		10	-0.00008	-0.00047 (-0.0166)	0.000297 (0.0160)	0.8486	0.8449 (0.6446)	0.8523 (0.9979)	0	0	0
5000	20	5	-0.00023	-0.00070 (-0.0200)	-0.00070 (0.0256)	0.7830	0.7759 (0.4357)	0.7901 (1.0066)	0.002	0	0
5000		10	-0.00022	-0.00068 (-0.0263)	0.000244 (0.0240)	0.7943	0.7895 (0.5369)	0.7991 (0.9869)	0	0	0
5000	25	5	-0.00054	-0.00112 (-0.0270)	0.000043 (0.0320)	0.7344	0.7264 (0.3303)	0.7424 (1.0010)	0.001	0	0
5000		10	-0.00030	-0.00085 (-0.0245)	0.000241 (0.0262)	0.7406	0.7348 (0.4666)	0.7464 (0.9723)	0	0	0

APPENDIX B CHI-SQUARE DISTRIBUTION WITH DIFFERENT DEGREES OF FREEDOM

Table 1: Chi-Square distribution (df = 2) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00192	-0.00628 (-0.1946)	0.00244 (0.2680)	0.8896	0.8826 (0.4942)	0.8967 (1.6347)	0.123	0.074	0.044
100		10	-0.00109	-0.00513 (-0.1763)	0.00294 (0.2556)	0.8952	0.8893 (0.6360)	0.9011 (1.5958)	0.103	0.061	0.035
100	15	5	-0.00442	-0.0102 (-0.3373)	0.00135 (0.3457)	0.8246	0.8153 (0.4439)	0.8339 (2.4931)	0.114	0.08	0.052
100		10	-0.00426	-0.00977 (-0.2835)	0.00126 (0.3160)	0.8372	0.8294 (0.4478)	0.8450 (3.1186)	0.088	0.058	0.04
100	20	5	-0.00307	-0.0100 (-0.4144)	0.00390 (0.3510)	0.7616	0.7510 (0.3296)	0.7724 (2.4247)	0.088	0.055	0.035
100		10	-0.00311	-0.00987 (-0.4133)	0.00364 (0.3477)	0.7778	0.7687 (0.3457)	0.7870 (2.6331)	0.08	0.056	0.038
100	25	5	-0.00259	-0.0108 (-0.4277)	0.00563 (0.5394)	0.7154	0.7032 (0.2895)	0.7278 (2.6823)	0.085	0.053	0.047
100		10	-0.00190	-0.00952 (-0.4541)	0.00572 (0.5200)	0.7199	0.7095 (0.3402)	0.7303 (3.0445)	0.068	0.053	0.038
500	10	5	-0.00024	-0.00218 (-0.0981)	0.00169 (0.1006)	0.8995	0.8950 (0.5930)	0.9041 (1.1121)	0.068	0.022	0.003
500		10	-0.00079	-0.00264 (-0.0847)	0.00105 (0.1003)	0.9023	0.8987 (0.7225)	0.9059 (1.1735)	0.048	0.016	0.005
500	15	5	-0.00069	-0.00322 (-0.1373)	0.00184 (0.1393)	0.8398	0.8335 (0.5005)	0.8462 (1.1191)	0.05	0.015	0.003
500		10	-0.00031	-0.00276 (-0.1201)	0.00214 (0.1377)	0.8414	0.8365 (0.5249)	0.8463 (1.1071)	0.026	0.01	0.001
500	20	5	-0.00131	-0.00439 (-0.1453)	0.00176 (0.1516)	0.7718	0.7635 (0.4021)	0.7803 (1.1798)	0.031	0.009	0.005
500		10	-0.00063	-0.00357 (-0.1614)	0.00230 (0.1436)	0.7901	0.7842 (0.4718)	0.7960 (1.1595)	0.016	0.007	0.003
500	25	5	-0.00104	-0.00447 (-0.2024)	0.00239 (0.1755)	0.7219	0.7128 (0.3318)	0.7311 (1.3367)	0.03	0.015	0.004
500		10	-0.00113	-0.00449 (-0.1686)	0.00223 (0.1997)	0.7352	0.7286 (0.4315)	0.7419 (1.1687)	0.012	0.005	0.001

Table 1 continues: Chi-Square distribution (df = 2) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00018	-0.00152 (-0.0645)	0.00117 (0.0744)	0.8999	0.8956 (0.6165)	0.9042 (1.1082)	0.042	0.006	0.001
1000		10	-0.00006	-0.00137 (-0.0585)	0.00124 (0.0661)	0.9028	0.8998 (0.7281)	0.9058 (1.0568)	0.017	0.001	0
1000	15	5	0.000240	-0.00153 (-0.0948)	0.00201 (0.1088)	0.8367	0.8308 (0.5221)	0.8426 (1.1038)	0.022	0.005	0.001
1000		10	0.000281	-0.00144 (-0.0705)	0.00200 (0.1151)	0.8440	0.8397 (0.6158)	0.8483 (1.0653)	.008	.001	0
1000	20	5	0.000249	-0.00185 (-0.1307)	0.00235 (0.1184)	0.7763	0.7689 (0.4107)	0.7839 (1.1067)	0.017	0.005	0.001
1000		10	0.000206	-0.00180 (-0.1324)	0.00221 (0.1153)	0.7919	0.7866 (0.4924)	0.7973 (1.1034)	0.008	0.001	0.001
1000	25	5	0.000791	-0.00167 (-0.1284)	0.00326 (0.1507)	0.7245	0.7156 (0.2929)	0.7335 (1.0780)	0.019	0.005	0
1000		10	0.000243	-0.00213 (-0.1445)	0.00261 (0.1452)	0.7348	0.7285 (0.4601)	0.7413 (1.0590)	0.006	0.001	0
5000	10	5	-0.00013	-0.00072 (-0.0267)	0.000464 (0.0322)	0.8975	0.8934 (0.6896)	0.9016 (1.0103)	0.009	0	0
5000		10	-0.00019	-0.00075 (-0.0270)	0.000373 (0.0310)	0.9064	0.9038 (0.7574)	0.9090 (1.0092)	0.001	0	0
5000	15	5	-0.00047	-0.00123 (-0.0363)	0.000290 (0.0343)	0.8357	0.8300 (0.5201)	0.8415 (1.0177)	0.009	0	0
5000		10	-0.00041	-0.00115 (-0.0383)	0.000331 (0.0396)	0.8487	0.8451 (0.6322)	0.8524 (0.9892)	0	0	0
5000	20	5	-0.00050	-0.00145 (-0.0447)	0.000438 (0.0530)	0.7823	0.7748 (0.3511)	0.7900 (1.0030)	0.004	0	0
5000		10	-0.00036	-0.00127 (-0.0445)	0.000545 (0.0481)	0.7895	0.7847 (0.5405)	0.7944 (0.9832)	0	0	0
5000	25	5	-0.00058	-0.00166 (0.0612)	0.000502 (0.0567)	0.7266	0.7184 (0.3794)	0.7349 (1.0156)	0.002	0	0
5000		10	-0.00062	-0.00165 (-0.0511)	0.000418 (0.0527)	0.7405	0.7347 (0.4677)	0.7462 (0.9643)	0	0	0

Table 2: Chi-Square distribution (df = 4) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism												
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00289	-0.00900 (-0.3245)	0.00322 (0.4082)	0.8832	0.8768 (0.4786)	0.8897 (1.4310)	0.097	0.055	0.031
100		10	-0.00681	-0.0126 (-0.3392)	-0.00097 (0.3463)	0.8908	0.8856 (0.6312)	0.8960 (1.4133)	0.086	0.036	0.018
100	15	5	-0.00806	-0.0161 (-0.3692)	8.681E-6 (0.5281)	0.8193	0.8109 (0.4152)	0.8277 (1.4697)	0.079	0.047	0.023
100		10	-0.00456	-0.0122 (-0.3416)	0.00305 (0.4811)	0.8314	0.8246 (0.4658)	0.8381 (1.4414)	0.063	0.031	0.015
100	20	5	-0.00069	-0.0103 (-0.4498)	0.00890 (0.5344)	0.7645	0.7546 (0.3681)	0.7745 (1.5980)	0.094	0.053	0.03
100		10	0.000078	-0.00935 (-0.4927)	0.00951 (0.5645)	0.7713	0.7633 (0.3716)	0.7793 (1.3393)	0.047	0.032	0.019
100	25	5	0.00257	-0.00914 (-0.7822)	0.0143 (0.5775)	0.7082	0.6972 (0.2062)	0.7194 (1.4641)	0.069	0.04	0.028
100		10	0.00544	-0.00552 (-0.5816)	0.0164 (0.6205)	0.7187	0.7099 (0.2716)	0.7277 (1.6633)	0.042	0.027	0.015
500	10	5	-0.00129	-0.00394 (-0.1643)	0.00137 (0.1349)	0.8921	0.8876 (0.6287)	0.8966 (1.0879)	0.048	0.009	0
500		10	-0.00227	-0.00483 (-0.1435)	0.000278 (0.1279)	0.9051	0.9019 (0.7100)	0.9083 (1.0838)	0.031	0.005	0
500	15	5	-0.00195	-0.00542 (-0.1994)	0.00153 (0.1937)	0.8369	0.8308 (0.5070)	0.8430 (1.1457)	0.038	0.004	0.001
500		10	-0.00266	-0.00594 (-0.1712)	0.000620 (0.1994)	0.8376	0.8331 (0.6106)	0.8421 (1.0968)	0.009	0.002	0
500	20	5	-0.00254	-0.00681 (-0.2330)	0.00174 (0.2282)	0.7765	0.7688 (0.4330)	0.7843 (1.1541)	0.019	0.01	0.003
500		10	-0.00114	-0.00516 (-0.2057)	0.00289 (0.1970)	0.7902	0.7846 (0.4874)	0.7959 (1.0534)	0.007	0.001	0
500	25	5	-0.00205	-0.00710 (-0.2792)	0.00300 (0.2616)	0.7275	0.7187 (0.2901)	0.7365 (1.0668)	0.014	0.004	0
500		10	-0.00072	-0.00540 (-0.2598)	0.00396 (0.2351)	0.7339	0.7278 (0.4408)	0.7399 (1.0509)	0.002	0.001	0

Table 2 continues: Chi-Square distribution (df = 4) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00096	-0.00283 (-0.0874)	0.000903 (0.1134)	0.8953	0.8910 (0.6199)	0.8997 (1.0876)	0.023	0.003	0
1000		10	-0.00081	-0.00259 (-0.0818)	0.000970 (0.0922)	0.9036	0.9008 (0.7397)	0.9065 (1.0213)	.004	0	0
1000	15	5	0.00222	-0.00465 (-0.1154)	0.000217 (0.1092)	0.8377	0.8320 (0.4865)	0.8435 (1.1188)	0.019	0.003	0.001
1000		10	-0.00146	-0.00381 (-0.1135)	0.000886 (0.1247)	0.8455	0.8415 (0.6223)	0.8495 (1.0090)	.001	0	0
1000	20	5	-0.00166	-0.00457 (-0.1426)	0.00125 (0.1724)	0.7834	0.7760 (0.4275)	0.7908 (1.0424)	0.011	0	0
1000		10	-0.00013	-0.00296 (-0.1674)	0.00270 (0.1556)	0.7897	0.7847 (0.5541)	0.7947 (1.0082)	.001	0	0
1000	25	5	0.000091	-0.00337 (-0.1708)	0.00355 (0.1864)	0.7258	0.7171 (0.2899)	0.7346 (1.0569)	0.01	0.001	0
1000		10	0.000431	-0.00289 (-0.1509)	0.00375 (0.2097)	0.7385	0.7322 (0.4553)	0.7448 (1.0249)	0.003	0	0
5000	10	5	-0.00072	-0.00157 (-0.0394)	0.000121 (0.0414)	0.8984	0.8945 (0.6728)	0.9023 (1.0147)	0.007	0	0
5000		10	-0.00106	-0.00185 (-0.0402)	-0.00026 (0.0395)	0.9040	0.9013 (0.7445)	0.9066 (1.0022)	0.003	0	0
5000	15	5	-0.00114	-0.00225 (-0.0527)	-0.00003 (0.0639)	0.8356	0.8297 (0.5151)	0.8415 (1.0167)	0.002	0	0
5000		10	-0.00136	-0.00241 (-0.0558)	-0.00031 (0.0538)	0.8488	0.8452 (0.6683)	0.8524 (0.9825)	0	0	0
5000	20	5	-0.00118	-0.00254 (-0.0695)	-0.00254 (0.0690)	0.7871	0.7797 (0.3939)	0.7946 (1.0037)	0.001	0	0
5000		10	-0.00085	-0.00215 (-0.0642)	0.000455 (0.0683)	0.7920	0.7871 (0.4967)	0.7970 (0.9761)	0	0	0
5000	25	5	-0.00098	-0.00252 (-0.0778)	0.000566 (0.0903)	0.7249	0.7165 (0.3467)	0.7333 (1.0159)	0.002	0	0
5000		10	-0.00113	-0.00265 (-0.0755)	0.000389 (0.0850)	0.7399	0.7341 (0.4552)	0.7458 (0.9703)	0	0	0

Table 3: Chi-Square distribution (df = 6) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00284	-0.0103 (-0.4883)	0.00458 (0.4587)	0.8845	0.8785 (0.4891)	0.8906 (1.4565)	0.083	0.039	0.025
100		10	-0.00005	-0.00739 (-0.3716)	0.00729 (0.4445)	0.8923	0.8872 (0.6140)	0.8975 (1.5189)	0.075	0.037	0.02
100	15	5	-0.00218	-0.0120 (-0.7041)	0.00766 (0.8577)	0.8207	0.8126 (0.4626)	0.8288 (1.4998)	0.072	0.036	0.021
100		10	-0.00275	-0.0123 (-0.6431)	0.00677 (0.5726)	0.8299	0.8235 (0.5538)	0.8363 (1.3056)	0.057	0.032	0.015
100	20	5	-0.00632	-0.0179 (-0.6009)	0.00521 (0.6021)	0.7608	0.7511 (0.3090)	0.7706 (1.3561)	0.059	0.034	0.018
100		10	-0.00604	-0.0171 (-0.6968)	0.00501 (0.6176)	0.7656	0.7580 (0.4285)	0.7733 (1.3239)	0.043	0.023	0.013
100	25	5	-0.00509	-0.0190 (-0.8011)	0.00887 (0.7482)	0.6970	0.6862 (0.2426)	0.7080 (1.3786)	0.056	0.033	0.021
100		10	-0.00773	-0.0208 (-0.7904)	0.00537 (0.6614)	0.7102	0.7022 (0.3791)	0.7183 (1.3533)	0.026	0.02	0.013
500	10	5	0.000549	-0.00262 (-0.1341)	0.00371 (0.1641)	0.8987	0.8941 (0.6057)	0.9033 (1.0893)	0.061	.007	0
500		10	0.00128	-0.00175 (-0.1250)	0.00431 (0.1853)	0.9060	0.9029 (0.7220)	0.9092 (1.0636)	0.03	.003	0
500	15	5	0.000641	-0.00360 (-0.2349)	0.00489 (0.1982)	0.8382	0.8321 (0.5006)	0.8444 (1.1007)	0.023	.005	.001
500		10	0.00286	-0.00124 (-0.1716)	0.00696 (0.2076)	0.8453	0.8409 (0.6080)	0.8498 (1.0729)	0.012	.002	0
500	20	5	-0.00199	-0.00720 (-0.2656)	0.00322 (0.2905)	0.7771	0.7694 (0.3549)	0.7848 (1.0466)	0.017	0	0
500		10	-0.00129	-0.00629 (-0.2393)	0.00372 (0.2719)	0.7843	0.7789 (0.5306)	0.7897 (1.1212)	0.005	0.001	0.001
500	25	5	-0.00026	-0.00640 (-0.3308)	0.00588 (0.4260)	0.7340	0.7255 (0.3106)	0.7426 (1.0974)	0.014	0.004	0
500		10	0.00113	-0.00461 (-0.2836)	0.00687 (0.3521)	0.7360	0.7299 (0.4344)	0.7422 (0.9984)	0	0	0

Table 3 continues: Chi-Square distribution (df = 6) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.00106	-0.00125 (-0.1066)	0.00338 (0.1449)	0.9002	0.8961 (0.6456)	0.9043 (1.0660)	0.025	.002	0
1000		10	0.000930	-0.00127 (-0.0873)	0.00313 (0.1034)	0.9043	0.9015 (0.6487)	0.9071 (1.0268)	.002	0	0
1000	15	5	-0.00016	-0.00326 (-0.1641)	0.00293 (0.1684)	0.8343	0.8283 (0.4991)	0.8402 (1.0856)	.009	.001	0
1000		10	0.00179	-0.00114 (-0.1256)	0.00473 (0.1581)	0.8459	0.8419 (0.6261)	0.8500 (0.9999)	0	0	0
1000	20	5	-0.00098	-0.00477 (-0.1918)	0.00280 (0.2251)	0.7732	0.7656 (0.4180)	0.7809 (1.0411)	0.007	0	0
1000		10	-0.00037	-0.00397 (-0.1885)	0.00324 (0.1903)	0.7876	0.7826 (0.4830)	0.7927 (1.0030)	0.001	0	0
1000	25	5	-0.00021	-0.00454 (-0.2261)	0.00411 (0.2234)	0.7263	0.7178 (0.3374)	0.7350 (1.0577)	0.01	0.002	0
1000		10	-0.00066	-0.00479 (-0.2193)	0.00347 (0.2106)	0.7394	0.7336 (0.4805)	0.7453 (0.9770)	0	0	0
5000	10	5	0.000407	-0.00065 (-0.0462)	0.00146 (0.0490)	0.8995	0.8956 (0.6658)	0.9035 (1.0119)	.006	0	0
5000		10	0.000379	-0.00063 (-0.0445)	0.00139 (0.0478)	0.9037	0.9012 (0.7093)	0.9063 (0.9929)	0	0	0
5000	15	5	0.000066	-0.00127 (-0.0714)	0.00140 (0.0613)	0.8417	0.8361 (0.5059)	0.8473 (1.0077)	.002	0	0
5000		10	0.000915	-0.00039 (-0.0576)	0.00222 (0.0728)	0.8446	0.8408 (0.6282)	0.8485 (1.0012)	.001	0	0
5000	20	5	0.000548	-0.00108 (-0.0849)	0.00218 (0.0886)	0.7901	0.7831 (0.4396)	0.7971 (1.0058)	0.002	0	0
5000		10	-0.00008	-0.00165 (-0.0801)	0.00149 (0.0799)	0.7943	0.7896 (0.5297)	0.7990 (0.9760)	0	0	0
5000	25	5	-0.00030	-0.00213 (-0.0798)	0.00154 (0.1013)	0.7251	0.7170 (0.3656)	0.7332 (0.9957)	0	0	0
5000		10	0.000017	-0.00175 (-0.0811)	0.00179 (0.0904)	0.7374	0.7316 (0.4825)	0.7433 (0.9600)	0	0	0

Table 4: Chi-Square distribution (df = 8) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00622	-0.0145 (-0.4186)	0.00203 (0.5638)	0.8885	0.8824 (0.4762)	0.8946 (1.3836)	0.094	0.034	0.021
100		10	-0.00689	-0.0147 (-0.3719)	0.000905 (0.5317)	0.8957	0.8906 (0.5205)	0.9008 (1.5818)	0.077	0.029	0.016
100	15	5	-0.0158	-0.0266 (-0.5886)	-0.00487 (0.5802)	0.8193	0.8116 (0.4013)	0.8271 (1.3998)	0.068	0.029	0.012
100		10	-0.0177	-0.0281 (-0.6248)	-0.00727 (0.5884)	0.8270	0.8207 (0.4863)	0.8334 (1.3337)	0.042	0.023	0.012
100	20	5	-0.0101	-0.0239 (-0.7369)	0.00366 (0.6996)	0.7599	0.7504 (0.3490)	0.7695 (1.3363)	0.059	0.036	0.018
100		10	-0.0127	-0.0258 (-0.6385)	0.000412 (0.8413)	0.7633	0.7558 (0.3850)	0.7709 (1.4843)	0.025	0.011	0.004
100	25	5	-0.00655	-0.0231 (-0.8720)	0.0100 (0.8144)	0.7032	0.6925 (0.2489)	0.7141 (1.4344)	0.053	0.035	0.018
100		10	-0.00805	-0.0236 (-0.7959)	0.00749 (0.9029)	0.7194	0.7114 (0.3198)	0.7275 (1.4274)	0.029	0.017	0.009
500	10	5	0.000522	-0.00327 (-0.1706)	0.00432 (0.2541)	0.8968	0.8925 (0.5894)	0.9011 (1.1002)	0.033	0.003	0.001
500		10	0.000945	-0.00269 (-0.1571)	0.00458 (0.2046)	0.9029	0.8999 (0.7389)	0.9059 (1.0361)	0.017	0	0
500	15	5	-0.00032	-0.00533 (-0.2489)	0.00468 (0.2568)	0.8410	0.8347 (0.5251)	0.8473 (1.0836)	0.03	0.002	0
500		10	-0.00267	-0.00750 (-0.2366)	0.00215 (0.2369)	0.8413	0.8369 (0.6171)	0.8457 (1.0360)	0.007	0	0
500	20	5	-0.00224	-0.00830 (-0.3859)	0.00382 (0.2674)	0.7814	0.7736 (0.3441)	0.7893 (1.1306)	0.016	0.004	0.001
500		10	-0.00064	-0.00662 (-0.3571)	0.00535 (0.2818)	0.7877	0.7824 (0.4783)	0.7930 (1.0671)	0.003	0.001	0
500	25	5	0.000847	-0.00635 (-0.3623)	0.00804 (0.3421)	0.7232	0.7145 (0.3234)	0.7320 (1.0855)	0.017	0.003	0
500		10	0.000608	-0.00635 (-0.4196)	0.00757 (0.3009)	0.7389	0.7326 (0.4195)	0.7454 (1.1609)	0.004	0.001	0.001

Table 4 continues: Chi-Square distribution (df = 8) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism												
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000501	-0.00220 (-0.1269)	0.00320 (0.1580)	0.8980	0.8937 (0.6497)	0.9022 (1.0442)	0.029	0	0
1000		10	0.000265	-0.00228 (-0.1507)	0.00281 (0.1265)	0.9054	0.9025 (0.7334)	0.9082 (1.0121)	0.004	0	0
1000	15	5	-0.00048	-0.00401 (-0.1669)	0.00305 (0.1923)	0.8357	0.8296 (0.4917)	0.8418 (1.0363)	0.015	0	0
1000		10	-0.00132	-0.00466 (-0.1621)	0.00202 (0.1645)	0.8400	0.8360 (0.6180)	0.8440 (1.0006)	0.001	0	0
1000	20	5	-0.00084	-0.00510 (-0.2330)	0.00341 (0.1906)	0.7816	0.7743 (0.4270)	0.7890 (1.0893)	0.009	0.002	0
1000		10	-0.00187	-0.00589 (-0.1938)	0.00215 (0.2135)	0.7954	0.7906 (0.5598)	0.8003 (1.0367)	0.002	0	0
1000	25	5	0.000573	-0.00429 (-0.2613)	0.00544 (0.2361)	0.7301	0.7220 (0.3311)	0.7383 (1.0593)	0.005	0.001	0
1000		10	-0.00089	-0.00561 (-0.2535)	0.00383 (0.2402)	0.7338	0.7277 (0.4338)	0.7400 (0.9884)	0	0	0
5000	10	5	-0.00029	-0.00151 (-0.0573)	0.000934 (0.0745)	0.8963	0.8922 (0.6367)	0.9004 (1.0129)	0.009	0	0
5000		10	-0.00076	-0.00193 (-0.0576)	0.000404 (0.0674)	0.9064	0.9039 (0.7791)	0.9089 (1.0043)	0.001	0	0
5000	15	5	-0.00020	-0.00174 (-0.0794)	0.00133 (0.0678)	0.8432	0.8376 (0.5162)	0.8490 (1.0084)	0.001	0	0
5000		10	-0.00048	-0.00198 (-0.0774)	0.00102 (0.0703)	0.8489	0.8451 (0.6488)	0.8527 (0.9904)	0	0	0
5000	20	5	-0.00119	-0.00306 (-0.0967)	0.000683 (0.0943)	0.7850	0.7780 (0.4578)	0.7921 (0.9965)	0	0	0
5000		10	-0.00099	-0.00281 (-0.0971)	0.000836 (0.0894)	0.7934	0.7886 (0.5682)	0.7982 (0.9700)	0	0	0
5000	25	5	-0.00129	-0.00347 (-0.1184)	0.000897 (0.0983)	0.7315	0.7231 (0.3214)	0.7401 (1.0121)	0.001	0	0
5000		10	-0.00121	-0.00333 (-0.1161)	0.000904 (0.0954)	0.7356	0.7298 (0.4675)	0.7414 (0.9846)	0	0	0

Table 5: Chi-Square distribution (df = 10) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.000193	-0.00913 (-0.5137)	0.00951 (0.5605)	0.8914	0.8856 (0.5386)	0.8973 (1.2768)	0.098	0.03	0.011
100		10	-0.00159	-0.0104 (-0.3970)	0.00725 (0.4535)	0.8932	0.8884 (0.6426)	0.8979 (1.2872)	0.074	0.021	.009
100	15	5	-0.00603	-0.0186 (-0.6974)	0.00652 (0.5243)	0.8201	0.8122 (0.3647)	0.8280 (1.3218)	0.061	0.029	0.012
100		10	-0.00201	-0.0138 (-0.5093)	0.00974 (0.6102)	0.8275	0.8213 (0.5306)	0.8337 (1.2035)	0.047	0.019	.008
100	20	5	-0.00363	-0.0185 (-0.9065)	0.0113 (0.8842)	0.7539	0.7443 (0.2940)	0.7635 (1.3528)	0.043	0.029	0.016
100		10	0.00256	-0.0120 (-0.6941)	0.0171 (0.7246)	0.7717	0.7643 (0.4160)	0.7791 (1.2702)	0.027	0.014	.005
100	25	5	0.00576	-0.0120 (-0.8680)	0.0235 (0.8487)	0.6978	0.6879 (0.2737)	0.7078 (1.3439)	0.031	0.017	.008
100		10	0.00289	-0.0138 (-0.9018)	0.0195 (0.8446)	0.7134	0.7058 (0.3380)	0.7211 (1.2681)	0.016	.006	.002
500	10	5	0.00184	-0.00244 (-0.1922)	0.00611 (0.2180)	0.8972	0.8927 (0.5626)	0.9018 (1.0643)	0.041	.002	0
500		10	0.00305	-0.00101 (-0.1835)	0.00711 (0.2406)	0.9012	0.8981 (0.7196)	0.9043 (1.0590)	0.014	.001	0
500	15	5	0.00221	-0.00340 (-0.3172)	0.00781 (0.3578)	0.8348	0.8286 (0.4231)	0.8410 (1.0618)	0.013	.003	0
500		10	0.00184	-0.00351 (-0.3288)	0.00720 (0.2962)	0.8471	0.8429 (0.4808)	0.8514 (1.0323)	.006	0	0
500	20	5	0.00260	-0.00393 (-0.3667)	0.00912 (0.2989)	0.7732	0.7657 (0.3975)	0.7808 (1.0808)	0.013	.002	0
500		10	0.000828	-0.00559 (-0.2963)	0.00725 (0.3642)	0.7844	0.7790 (0.4900)	0.7898 (1.0817)	.006	.002	0
500	25	5	0.00555	-0.00203 (-0.3740)	0.0131 (0.4039)	0.7188	0.7100 (0.3232)	0.7277 (1.0764)	.007	.001	0
500		10	0.00507	-0.00229 (-0.3842)	0.0124 (0.4381)	0.7307	0.7246 (0.4451)	0.7368 (1.0457)	.001	0	0

Table 5 continues: Chi-Square distribution (df = 10) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000829	-0.00212 (-0.1289)	0.00377 (0.1449)	0.8934	0.8891 (0.6535)	0.8977 (1.0540)	0.017	.001	0
1000		10	0.000159	-0.00263 (-0.1414)	0.00295 (0.1515)	0.9033	0.9005 (0.7102)	0.9061 (1.0417)	.004	0	0
1000	15	5	-0.00034	-0.00413 (-0.1953)	0.00345 (0.1856)	0.8363	0.8303 (0.5033)	0.8424 (1.0386)	0.01	0	0
1000		10	-0.00051	-0.00412 (-0.1733)	0.00309 (0.1630)	0.8429	0.8389 (0.6479)	0.8469 (1.0110)	.001	0	0
1000	20	5	-0.00121	-0.00590 (-0.2137)	0.00349 (0.2439)	0.7826	0.7755 (0.3241)	0.7897 (1.0283)	.006	0	0
1000		10	-0.00138	-0.00585 (-0.2194)	0.00309 (0.2379)	0.7873	0.7823 (0.5353)	0.7923 (0.9801)	0	0	0
1000	25	5	0.000568	-0.00475 (-0.2860)	0.00588 (0.2790)	0.7283	0.7197 (0.3409)	0.7370 (1.0276)	0.01	0	0
1000		10	-0.00153	-0.00682 (-0.2460)	0.00376 (0.3256)	0.7371	0.7314 (0.4217)	0.7429 (0.9980)	0	0	0
5000	10	5	0.000384	-0.00096 (-0.0752)	0.00173 (0.0674)	0.8959	0.8920 (0.6290)	0.8998 (1.0031)	.003	0	0
5000		10	0.000254	-0.00104 (-0.0697)	0.00155 (0.0638)	0.9033	0.9007 (0.7312)	0.9059 (0.9948)	0	0	0
5000	15	5	-0.00057	-0.00232 (-0.0935)	0.00117 (0.0896)	0.8364	0.8307 (0.5221)	0.8421 (1.0205)	.003	0	0
5000		10	-0.00062	-0.00231 (-0.0823)	0.00106 (0.0888)	0.8463	0.8426 (0.6162)	0.8501 (0.9795)	0	0	0
5000	20	5	-0.00142	-0.00359 (-0.1050)	0.000755 (0.1123)	0.7836	0.7763 (0.3967)	0.7908 (1.0130)	.001	0	0
5000		10	-0.00080	-0.00285 (-0.0978)	0.00125 (0.0961)	0.7907	0.7857 (0.5192)	0.7958 (0.9602)	0	0	0
5000	25	5	-0.00078	-0.00329 (-0.1178)	0.00174 (0.1239)	0.7328	0.7243 (0.3421)	0.7414 (0.9963)	0	0	0
5000		10	-0.00239	-0.00478 (-0.1163)	-0.00001 (0.1013)	0.7453	0.7398 (0.4854)	0.7509 (0.9699)	0	0	0

Table 6: Chi-Square distribution (df = 15) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.00305	-0.00842 (-0.5516)	0.0145 (0.7022)	0.8864	0.8803 (0.5033)	0.8925 (1.3368)	0.092	0.036	0.011
100		10	0.00209	-0.00902 (-0.5351)	0.0132 (0.6667)	0.8973	0.8926 (0.6148)	0.9020 (1.2620)	0.081	0.026	0.014
100	15	5	-0.00234	-0.0178 (-0.8959)	0.0131 (0.7992)	0.8268	0.8192 (0.3640)	0.8346 (1.2784)	0.067	0.027	0.012
100		10	-0.00064	-0.0153 (-0.7094)	0.0140 (0.6997)	0.8297	0.8236 (0.5081)	0.8358 (1.2316)	0.042	0.02	.009
100	20	5	0.0109	-0.00770 (-0.7933)	0.0294 (1.0922)	0.7696	0.7604 (0.3413)	0.7789 (1.4682)	0.046	0.02	.008
100		10	0.00454	-0.0129 (-0.9539)	0.0219 (0.8041)	0.7740	0.7668 (0.4576)	0.7812 (1.2866)	0.029	0.016	0.011
100	25	5	0.00836	-0.0129 (-1.1861)	0.0296 (1.2025)	0.7113	0.7012 (0.2440)	0.7216 (1.4637)	0.05	0.032	0.017
100		10	0.00281	-0.0173 (-1.0690)	0.0230 (1.2453)	0.7165	0.7087 (0.3315)	0.7244 (1.2805)	0.024	0.01	.008
500	10	5	0.00309	-0.00206 (-0.2089)	0.00823 (0.2870)	0.8985	0.8942 (0.6169)	0.9028 (1.0584)	0.032	.001	0
500		10	0.00487	-0.00008 (-0.2577)	0.00982 (0.2564)	0.9050	0.9021 (0.7177)	0.9080 (1.0854)	0.012	.001	0
500	15	5	0.00515	-0.00169 (-0.2853)	0.0120 (0.4341)	0.8347	0.8285 (0.5306)	0.8409 (1.0630)	0.021	.001	0
500		10	0.00728	0.000664 (-0.3056)	0.0139 (0.3945)	0.8477	0.8435 (0.5992)	0.8518 (1.0293)	.007	0	0
500	20	5	0.00449	-0.00383 (-0.4878)	0.0128 (0.4915)	0.7802	0.7728 (0.4182)	0.7877 (1.0466)	0.012	0	0
500		10	0.00767	-0.00029 (-0.4049)	0.0156 (0.5283)	0.7918	0.7865 (0.4690)	0.7972 (1.0106)	.003	0	0
500	25	5	0.0118	0.00240 (-0.4753)	0.0212 (0.5832)	0.7218	0.7131 (0.3463)	0.7306 (1.0746)	0.011	.003	0
500		10	0.0102	0.00127 (-0.4183)	0.0192 (0.5651)	0.7416	0.7356 (0.4312)	0.7476 (0.9877)	0	0	0

Table 6 continues: Chi-Square distribution (df = 15) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism												
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000335	-0.00334 (-0.1880)	0.00401 (0.1781)	0.8944	0.8901 (0.6568)	0.8988 (1.0400)	0.025	0	0
1000		10	0.00221	-0.00137 (-0.1812)	0.00579 (0.1693)	0.9019	0.8992 (0.7347)	0.9047 (1.0223)	.004	0	0
1000	15	5	0.000725	-0.00424 (-0.2694)	0.00569 (0.2624)	0.8348	0.8287 (0.4536)	0.8411 (1.0302)	0.011	0	0
1000		10	0.00174	-0.00304 (-0.2348)	0.00652 (0.2244)	0.8462	0.8422 (0.5626)	0.8503 (1.0101)	.001	0	0
1000	20	5	0.00206	-0.00387 (-0.3225)	0.00799 (0.4035)	0.7790	0.7712 (0.3837)	0.7868 (1.0837)	0.01	.002	0
1000		10	0.00341	-0.00240 (-0.3041)	0.00922 (0.3297)	0.7915	0.7866 (0.5255)	0.7965 (1.0082)	.001	0	0
1000	25	5	0.00392	-0.00306 (-0.3952)	0.0109 (0.3760)	0.7279	0.7197 (0.3653)	0.7363 (1.0159)	.004	0	0
1000		10	0.00383	-0.00285 (-0.3624)	0.0105 (0.3981)	0.7393	0.7334 (0.4528)	0.7452 (0.9888)	0	0	0
5000	10	5	-0.00041	-0.00206 (-0.0941)	0.00125 (0.0746)	0.8974	0.8936 (0.6195)	0.9013 (1.0077)	.004	0	0
5000		10	-0.00046	-0.00202 (-0.0909)	0.00109 (0.0827)	0.9064	0.9040 (0.7571)	0.9089 (0.9972)	0	0	0
5000	15	5	-0.00056	-0.00270 (-0.1177)	0.00159 (0.1281)	0.8405	0.8346 (0.5255)	0.8463 (1.0099)	.002	0	0
5000		10	-0.00074	-0.00284 (-0.0976)	0.00137 (0.1105)	0.8474	0.8435 (0.6297)	0.8513 (0.9960)	0	0	0
5000	20	5	0.000736	-0.00179 (-0.1114)	0.00326 (0.1409)	0.7859	0.7789 (0.4328)	0.7930 (1.0049)	.001	0	0
5000		10	0.000087	-0.00237 (-0.1209)	0.00254 (0.1143)	0.7903	0.7854 (0.5505)	0.7952 (0.9876)	0	0	0
5000	25	5	0.000935	-0.00205 (-0.1562)	0.00392 (0.1634)	0.7328	0.7244 (0.3511)	0.7414 (1.0123)	.003	0	0
5000		10	0.000458	-0.00239 (-0.1716)	0.00331 (0.1435)	0.7447	0.7390 (0.4916)	0.7503 (0.9654)	0	0	0

Table 7: Chi-Square distribution (df = 20) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.00775	-0.00552 (-0.7395)	0.0210 (0.7837)	0.8919	0.8859 (0.5533)	0.8978 (1.3671)	0.09	0.035	0.017
100		10	0.00906	-0.00407 (-0.7796)	0.0222 (0.8978)	0.8945	0.8896 (0.6130)	0.8993 (1.2483)	0.08	0.018	.007
100	15	5	0.0157	-0.00196 (-0.7805)	0.0333 (0.9710)	0.8237	0.8155 (0.3129)	0.8319 (1.2551)	0.072	0.028	0.013
100		10	0.0165	-0.00077 (-0.7921)	0.0337 (0.8927)	0.8327	0.8265 (0.5213)	0.8389 (1.2084)	0.037	0.019	.007
100	20	5	0.00913	-0.0132 (-1.2176)	0.0314 (1.3117)	0.7558	0.7463 (0.3212)	0.7654 (1.2706)	0.049	0.03	0.018
100		10	0.00939	-0.0118 (-1.0533)	0.0306 (1.2181)	0.7696	0.7623 (0.4059)	0.7770 (1.2333)	0.025	0.013	.009
100	25	5	0.0203	-0.00566 (-1.4794)	0.0462 (1.5033)	0.7092	0.6993 (0.2486)	0.7193 (1.2817)	0.031	0.021	0.011
100		10	0.0192	-0.00516 (-1.6415)	0.0436 (1.4746)	0.7170	0.7096 (0.3690)	0.7246 (1.1610)	0.013	.006	.004
500	10	5	0.00140	-0.00465 (-0.3608)	0.00745 (0.2834)	0.8934	0.8889 (0.5779)	0.8980 (1.0436)	0.027	0	0
500		10	-0.00038	-0.00618 (-0.2928)	0.00542 (0.2979)	0.9016	0.8986 (0.7120)	0.9045 (1.0478)	0.01	0	0
500	15	5	0.00119	-0.00691 (-0.3806)	0.00929 (0.4813)	0.8363	0.8303 (0.4912)	0.8424 (1.0955)	0.026	.001	0
500		10	-0.00369	-0.0113 (-0.4598)	0.00392 (0.4228)	0.8435	0.8391 (0.5931)	0.8479 (1.0370)	.005	0	0
500	20	5	-0.00084	-0.0105 (-0.5029)	0.00883 (0.5291)	0.7760	0.7683 (0.2927)	0.7837 (1.1139)	.009	.002	.001
500		10	-0.00256	-0.0118 (-0.5214)	0.00670 (0.4392)	0.7894	0.7841 (0.5530)	0.7946 (1.0179)	.001	0	0
500	25	5	0.000804	-0.0102 (-0.6958)	0.0118 (0.5605)	0.7228	0.7136 (0.2958)	0.7321 (1.0517)	.005	.001	0
500		10	0.00419	-0.00637 (-0.6265)	0.0147 (0.5849)	0.7375	0.7316 (0.4589)	0.7435 (1.0000)	.001	0	0

Table 7 continues: Chi-Square distribution (df = 20) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.00119	-0.00303 (-0.2419)	0.00542 (0.2103)	0.8960	0.8918 (0.6616)	0.9002 (1.0211)	0.019	0	0
1000		10	0.000529	-0.00338 (-0.2343)	0.00444 (0.2100)	0.9037	0.9010 (0.7391)	0.9064 (1.0226)	.006	0	0
1000	15	5	0.00135	-0.00420 (-0.2454)	0.00690 (0.3094)	0.8332	0.8269 (0.4193)	0.8396 (1.0431)	0.013	0	0
1000		10	0.00270	-0.00254 (-0.2425)	0.00794 (0.3570)	0.8455	0.8415 (0.6380)	0.8495 (1.0173)	.005	0	0
1000	20	5	0.00275	-0.00399 (-0.3762)	0.00949 (0.3760)	0.7833	0.7761 (0.4164)	0.7905 (1.0397)	.005	0	0
1000		10	0.00220	-0.00427 (-0.3298)	0.00867 (0.3653)	0.7886	0.7837 (0.4887)	0.7935 (1.0105)	.001	0	0
1000	25	5	0.00435	-0.00337 (-0.4200)	0.0121 (0.4148)	0.7316	0.7231 (0.2829)	0.7402 (1.0663)	.006	.001	0
1000		10	0.00489	-0.00256 (-0.4314)	0.0123 (0.3908)	0.7338	0.7280 (0.4791)	0.7395 (1.0011)	.001	0	0
5000	10	5	-0.00028	-0.00215 (-0.0868)	0.00159 (0.0940)	0.8992	0.8952 (0.5854)	0.9033 (1.0120)	0.002	0	0
5000		10	-0.00028	-0.00207 (-0.0815)	0.00151 (0.0982)	0.9038	0.9012 (0.7597)	0.9064 (0.9963)	0	0	0
5000	15	5	0.000683	-0.00186 (-0.1187)	0.00322 (0.1278)	0.8417	0.8358 (0.4799)	0.8475 (1.0035)	0.003	0	0
5000		10	0.00161	-0.00080 (-0.1198)	0.00403 (0.1262)	0.8491	0.8453 (0.6470)	0.8528 (1.0002)	0.001	0	0
5000	20	5	0.00190	-0.00117 (-0.1477)	0.00497 (0.1605)	0.7782	0.7708 (0.3424)	0.7857 (1.0109)	0.002	0	0
5000		10	0.000485	-0.00238 (-0.1540)	0.00335 (0.1465)	0.7967	0.7921 (0.5770)	0.8013 (0.9714)	0	0	0
5000	25	5	0.00290	-0.00058 (-0.1588)	0.00637 (0.1928)	0.7274	0.7193 (0.3773)	0.7357 (1.0067)	0.001	0	0
5000		10	0.00259	-0.00067 (-0.1536)	0.00586 (0.1678)	0.7412	0.7357 (0.4837)	0.7468 (0.9599)	0	0	0

Table 8: Chi-Square distribution (df = 25) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism												
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00032	-0.0153 (-0.7467)	0.0147 (0.9668)	0.8909	0.8849 (0.5324)	0.8970 (1.3705)	0.095	0.03	0.014
100		10	0.000173	-0.0145 (-0.7293)	0.0148 (0.9336)	0.8906	0.8859 (0.5996)	0.8953 (1.2653)	0.061	0.018	0.007
100	15	5	-0.00220	-0.0221 (-1.0363)	0.0177 (0.9538)	0.8200	0.8122 (0.3567)	0.8279 (1.2929)	0.058	0.02	0.007
100		10	-0.00502	-0.0244 (-1.0344)	0.0143 (0.8764)	0.8280	0.8222 (0.4726)	0.8339 (1.1415)	0.035	0.012	0.002
100	20	5	0.000523	-0.0237 (-1.4192)	0.0247 (1.3140)	0.7567	0.7479 (0.3017)	0.7656 (1.2183)	0.03	0.013	0.005
100		10	0.000569	-0.0225 (-1.1671)	0.0236 (1.3354)	0.7697	0.7626 (0.3721)	0.7769 (1.2119)	0.025	0.013	0.005
100	25	5	0.000162	-0.0284 (-1.4534)	0.0288 (1.5351)	0.7041	0.6940 (0.2283)	0.7143 (1.2666)	0.033	0.017	0.009
100		10	-0.00307	-0.0304 (-1.3573)	0.0242 (1.3626)	0.7135	0.7054 (0.3273)	0.7216 (1.1947)	0.018	0.006	0.002
500	10	5	-0.00333	-0.00983 (-0.3133)	0.00318 (0.4789)	0.8959	0.8916 (0.6054)	0.9001 (1.0678)	0.027	0.002	0
500		10	-0.00269	-0.00879 (-0.3198)	0.00341 (0.4044)	0.9017	0.8986 (0.6750)	0.9049 (1.0212)	0.011	0	0
500	15	5	-0.00101	-0.00949 (-0.5350)	0.00747 (0.4986)	0.8317	0.8255 (0.4437)	0.8379 (1.0416)	0.016	0	0
500		10	-0.00480	-0.0128 (-0.5415)	0.00321 (0.4782)	0.8463	0.8422 (0.6319)	0.8504 (1.0231)	0.007	0	0
500	20	5	-0.00124	-0.0115 (-0.5262)	0.00903 (0.4955)	0.7724	0.7649 (0.3784)	0.7800 (1.0934)	0.012	0.001	0
500		10	0.00177	-0.00821 (-0.5405)	0.0117 (0.5476)	0.7855	0.7802 (0.5357)	0.7909 (1.0072)	0.001	0	0
500	25	5	-0.00519	-0.0172 (-0.6880)	0.00687 (0.6075)	0.7278	0.7190 (0.3094)	0.7366 (1.1559)	0.014	0.003	0.002
500		10	-0.00037	-0.0118 (-0.5445)	0.0110 (0.5830)	0.7378	0.7318 (0.4450)	0.7440 (0.9734)	0	0	0

Table 8 continues: Chi-Square distribution (df = 25) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00354	-0.00812 (-0.2261)	0.00104 (0.3210)	0.8975	0.8932 (0.6178)	0.9018 (1.0413)	0.022	0	0
1000		10	-0.00236	-0.00674 (-0.2153)	0.00202 (0.2983)	0.9049	0.9022 (0.7607)	0.9075 (1.0189)	0.003	0	0
1000	15	5	-0.00328	-0.00935 (-0.3220)	0.00280 (0.3958)	0.8401	0.8342 (0.5090)	0.8460 (1.0620)	0.016	0.001	0
1000		10	-0.00441	-0.0102 (-0.2747)	0.00138 (0.3136)	0.8486	0.8446 (0.6277)	0.8526 (1.0035)	0.001	0	0
1000	20	5	-0.00380	-0.0115 (-0.3676)	0.00388 (0.5071)	0.7839	0.7764 (0.4031)	0.7914 (1.0552)	0.012	0.001	0
1000		10	-0.00373	-0.0109 (-0.3655)	0.00349 (0.4576)	0.7903	0.7853 (0.5205)	0.7953 (0.9980)	0	0	0
1000	25	5	-0.00163	-0.0106 (-0.5498)	0.00731 (0.4579)	0.7321	0.7237 (0.3010)	0.7406 (1.0312)	0.006	0	0
1000		10	-0.00109	-0.00946 (-0.4144)	0.00729 (0.4503)	0.7362	0.7305 (0.3645)	0.7420 (0.9779)	0	0	0
5000	10	5	-0.00091	-0.00306 (-0.1174)	0.00123 (0.1234)	0.8999	0.8961 (0.6623)	0.9037 (1.0053)	0.001	0	0
5000		10	-0.00048	-0.00253 (-0.1168)	0.00158 (0.1130)	0.9045	0.9019 (0.6930)	0.9071 (0.9947)	0	0	0
5000	15	5	0.000204	-0.00262 (-0.1457)	0.00303 (0.1496)	0.8426	0.8371 (0.4749)	0.8482 (1.0003)	0.001	0	0
5000		10	-0.00016	-0.00278 (-0.1310)	0.00246 (0.1408)	0.8484	0.8446 (0.6275)	0.8522 (0.9769)	0	0	0
5000	20	5	-0.00008	-0.00343 (-0.1719)	0.00327 (0.1665)	0.7833	0.7759 (0.4198)	0.7909 (1.0054)	0.001	0	0
5000		10	-0.00035	-0.00353 (-0.1576)	0.00284 (0.1528)	0.7903	0.7854 (0.5451)	0.7953 (0.9718)	0	0	0
5000	25	5	0.000589	-0.00328 (-0.1764)	0.00445 (0.2113)	0.7308	0.7225 (0.3626)	0.7391 (1.0362)	0.002	0	0
5000		10	-0.00008	-0.00379 (-0.1736)	0.00362 (0.2389)	0.7378	0.7323 (0.4864)	0.7433 (0.9434)	0	0	0

Table 9: Chi-Square distribution (df = 30) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.00638	-0.00990 (-0.9166)	0.0227 (0.8334)	0.8822	0.8763 (0.4726)	0.8881 (1.1401)	0.091	0.03	0.008
100		10	0.00711	-0.00852 (-0.8238)	0.0227 (0.9123)	0.8945	0.8900 (0.6723)	0.8991 (1.2063)	0.06	0.019	0.005
100	15	5	0.0107	-0.0119 (-1.3730)	0.0332 (1.3843)	0.8219	0.8143 (0.4025)	0.8296 (1.4541)	0.048	0.026	0.011
100		10	0.0112	-0.0102 (-1.0915)	0.0326 (1.2191)	0.8301	0.8241 (0.5060)	0.8362 (1.2697)	0.04	0.014	0.006
100	20	5	0.0172	-0.00912 (-1.7644)	0.0436 (1.5703)	0.7602	0.7509 (0.2583)	0.7697 (1.2323)	0.048	0.022	0.01
100		10	0.0103	-0.0149 (-1.4002)	0.0356 (1.3560)	0.7674	0.7604 (0.4194)	0.7745 (1.1899)	0.023	0.012	0.007
100	25	5	0.0154	-0.0166 (-1.7121)	0.0473 (2.1817)	0.7016	0.6911 (0.2828)	0.7122 (1.3447)	0.044	0.017	0.009
100		10	0.00500	-0.0247 (-1.6882)	0.0347 (1.4758)	0.7157	0.7081 (0.4059)	0.7234 (1.2883)	0.01	0.004	0.003
500	10	5	-0.00387	-0.0112 (-0.4228)	0.00346 (0.3773)	0.8982	0.8938 (0.6177)	0.9026 (1.0570)	0.031	0.002	0
500		10	-0.00636	-0.0134 (-0.3414)	0.000695 (0.4008)	0.9029	0.8999 (0.7275)	0.9058 (1.0160)	0.011	0	0
500	15	5	-0.00464	-0.0145 (-0.5917)	0.00524 (0.4724)	0.8384	0.8321 (0.5329)	0.8447 (1.0509)	0.023	0.001	0
500		10	-0.00719	-0.0164 (-0.4534)	0.00206 (0.4481)	0.8420	0.8379 (0.6007)	0.8462 (1.0253)	0.002	0	0
500	20	5	-0.00808	-0.0197 (-0.6634)	0.00352 (0.6754)	0.7766	0.7692 (0.4206)	0.7841 (1.0494)	0.006	0	0
500		10	-0.00815	-0.0192 (-0.6299)	0.00289 (0.5522)	0.7831	0.7779 (0.5042)	0.7883 (1.0437)	0.003	0	0
500	25	5	-0.00662	-0.0202 (-0.8544)	0.00698 (0.7108)	0.7285	0.7201 (0.3101)	0.7371 (1.1054)	0.011	0.002	0.001
500		10	-0.0123	-0.0254 (-0.7143)	0.000858 (0.6712)	0.7275	0.7212 (0.4168)	0.7337 (0.9871)	0	0	0

Table 9 continues: Chi-Square distribution (df = 30) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00240	-0.00760 (-0.2730)	0.00280 (0.2933)	0.9011	0.8971 (0.6551)	0.9052 (1.0313)	0.016	0	0
1000		10	-0.00417	-0.00417 (-0.2791)	0.000807 (0.2568)	0.9045	0.9017 (0.7329)	0.9073 (1.0060)	0.002	0	0
1000	15	5	-0.00548	-0.0122 (-0.3191)	0.00123 (0.3179)	0.8371	0.8313 (0.5249)	0.8429 (1.0379)	0.006	0	0
1000		10	-0.00544	-0.0119 (-0.4347)	0.00104 (0.3036)	0.8466	0.8427 (0.5903)	0.8506 (0.9945)	0	0	0
1000	20	5	-0.00584	-0.0140 (-0.4569)	0.00228 (0.3799)	0.7851	0.7775 (0.4139)	0.7927 (1.0269)	0.005	0	0
1000		10	-0.00629	-0.0143 (-0.4260)	0.00176 (0.3923)	0.7898	0.7848 (0.5272)	0.7948 (0.9820)	0	0	0
1000	25	5	-0.00366	-0.0131 (-0.4658)	0.00578 (0.4474)	0.7296	0.7211 (0.3046)	0.7382 (1.0085)	0.003	0	0
1000		10	-0.00853	-0.0180 (-0.5130)	0.000932 (0.5099)	-0.00853	-0.0180 (-0.5130)	0.000932 (0.5099)	0	0	0
5000	10	5	-0.00025	-0.00254 (-0.1201)	0.00203 (0.1057)	0.8964	0.8923 (0.6572)	0.9005 (1.0039)	0.003	0	0
5000		10	0.000360	-0.00181 (-0.1277)	0.00253 (0.1030)	0.9068	0.9043 (0.7570)	0.9093 (0.9872)	0	0	0
5000	15	5	-0.00044	-0.00355 (-0.1652)	0.00267 (0.1407)	0.8390	0.8334 (0.5191)	0.8447 (1.0043)	0.003	0	0
5000		10	0.000998	-0.00190 (-0.1450)	0.00389 (0.1339)	0.8478	0.8440 (0.6081)	0.8516 (0.9920)	0	0	0
5000	20	5	0.000727	-0.00299 (-0.1982)	0.00444 (0.1965)	0.7811	0.7738 (0.4187)	0.7885 (1.0223)	0.002	0	0
5000		10	-0.00052	-0.00412 (-0.1886)	0.00309 (0.1878)	0.7912	0.7866 (0.5459)	0.7958 (0.9544)	0	0	0
5000	25	5	0.00153	-0.00277 (-0.2509)	0.00584 (0.2357)	0.7270	0.7185 (0.3070)	0.7357 (1.0018)	0.001	0	0
5000		10	-0.00033	-0.00448 (-0.2071)	0.00381 (0.2190)	0.7447	0.7392 (0.4575)	0.7503 (0.9630)	0	0	0

Table 10: Chi-Square distribution (df = 40) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.00605	-0.0136 (-0.9577)	0.0257 (1.0993)	0.8869	0.8808 (0.4376)	0.8931 (1.2330)	0.089	0.026	0.007
100		10	0.0100	-0.00852 (-1.0095)	0.0286 (0.9696)	0.8942	0.8895 (0.5550)	0.8989 (1.2057)	0.07	0.022	0.006
100	15	5	-0.0103	-0.0355 (-1.5437)	0.0149 (1.5569)	0.8194	0.8117 (0.4263)	0.8272 (1.1511)	0.065	0.018	0.009
100		10	-0.00240	-0.0265 (-1.1476)	0.0217 (1.7540)	0.8334	0.8277 (0.5305)	0.8392 (1.1569)	0.032	0.009	0.002
100	20	5	0.00239	-0.0289 (-1.7601)	0.0337 (1.6879)	0.7629	0.7534 (0.2777)	0.7725 (1.2304)	0.044	0.022	0.008
100		10	0.00345	-0.0262 (-1.4146)	0.0331 (1.7608)	0.7721	0.7649 (0.4385)	0.7793 (1.2317)	0.027	0.011	0.004
100	25	5	0.00237	-0.0332 (-1.7376)	0.0380 (2.3438)	0.7079	0.6977 (0.2498)	0.7182 (1.2510)	0.039	0.019	0.008
100		10	-0.00290	-0.0367 (-1.7629)	0.0309 (2.2559)	0.7144	0.7065 (0.3655)	0.7225 (1.3786)	0.023	0.012	0.003
500	10	5	0.00664	-0.00178 (-0.4354)	0.0151 (0.5464)	0.8949	0.8907 (0.6051)	0.8993 (1.0441)	0.026	0	0
500		10	0.00936	0.00137 (-0.4248)	0.0174 (0.5077)	0.9031	0.9001 (0.7279)	0.9061 (1.0285)	0.012	0	0
500	15	5	0.00278	-0.00831 (-0.5187)	0.0139 (0.4853)	0.8337	0.8274 (0.5119)	0.8401 (1.0649)	0.02	0.001	0
500		10	0.00153	-0.00903 (-0.4533)	0.0121 (0.4576)	0.8444	0.8404 (0.6025)	0.8485 (1.0158)	0.005	0	0
500	20	5	-0.00360	-0.0172 (-0.6810)	0.00998 (0.6122)	0.7781	0.7704 (0.3797)	0.7860 (1.1172)	0.008	0.002	0.001
500		10	-0.00350	-0.0166 (-0.6978)	0.00964 (0.7149)	0.7888	0.7836 (0.5159)	0.7940 (1.0282)	0.005	0	0
500	25	5	-0.00136	-0.0165 (-0.6928)	0.0138 (0.7356)	0.7238	0.7150 (0.3065)	0.7326 (1.0448)	0.016	0	0
500		10	-0.00252	-0.0173 (-0.8218)	0.0123 (0.7462)	0.7310	0.7251 (0.4715)	0.7369 (0.9766)	0	0	0

Table 10 continues: Chi-Square distribution (df = 40) with Monotonic Missing data Pattern:

Results of 1000 Simulations from MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.00391	-0.00193 (-0.3797)	0.00976 (0.3035)	0.9007	0.8968 (0.6772)	0.9046 (1.0224)	0.016	0	0
1000		10	0.00355	-0.00207 (-0.3171)	0.00917 (0.3114)	0.9041	0.9014 (0.7149)	0.9068 (1.0199)	0.001	0	0
1000	15	5	-0.00518	-0.0131 (-0.4064)	0.00276 (0.4057)	0.8329	0.8268 (0.4792)	0.8390 (1.0380)	0.009	0	0
1000		10	-0.00403	-0.0117 (-0.3699)	0.00364 (0.3690)	0.8413	0.8372 (0.6110)	0.8453 (0.9753)	0	0	0
1000	20	5	-0.00257	-0.0122 (-0.5304)	0.00705 (0.5330)	0.7778	0.7703 (0.4419)	0.7854 (1.0171)	0.004	0	0
1000		10	-0.00359	-0.0128 (-0.4498)	0.00562 (0.4160)	0.7907	0.7858 (0.5152)	0.7956 (0.9833)	0	0	0
1000	25	5	-0.00803	-0.0190 (-0.5836)	0.00297 (0.4794)	0.7262	0.7177 (0.3016)	0.7348 (1.0242)	0.003	0	0
1000		10	-0.00052	-0.0110 (-0.5178)	0.00998 (0.5237)	0.7298	0.7240 (0.4731)	0.7357 (0.9725)	0	0	0
5000	10	5	0.000119	-0.00253 (-0.1306)	0.00277 (0.1388)	0.8949	0.8909 (0.6212)	0.8990 (1.0053)	0.003	0	0
5000		10	0.000089	-0.00247 (-0.1593)	0.00265 (0.1327)	0.9046	0.9022 (0.7642)	0.9071 (0.9999)	0	0	0
5000	15	5	-0.00091	-0.00448 (-0.1646)	0.00266 (0.1696)	0.8420	0.8362 (0.4674)	0.8477 (1.0030)	0.002	0	0
5000		10	-0.00186	-0.00522 (-0.1909)	0.00149 (0.1746)	0.8517	0.8481 (0.6382)	0.8553 (0.9895)	0	0	0
5000	20	5	-0.00083	-0.00507 (-0.2592)	0.00341 (0.2235)	0.7795	0.7721 (0.4145)	0.7869 (1.0047)	0.001	0	0
5000		10	-0.00161	-0.00567 (-0.2361)	0.00244 (0.1892)	0.7885	0.7835 (0.5330)	0.7935 (0.9988)	0	0	0
5000	25	5	-0.00036	-0.00529 (-0.2823)	0.00456 (0.2593)	0.7236	0.7154 (0.3634)	0.7319 (0.9930)	0	0	0
5000		10	-0.00049	-0.00524 (-0.2485)	0.00426 (0.2501)	0.7430	0.7373 (0.4551)	0.7487 (0.9696)	0	0	0

APPENDIX C t-DISTRIBUTION AND CHI-SQUARE DISTRIBUTION WITH 10 AND 30 DFs

Table 1: Significance P-values for t-Distribution with 10 df with Monotonic Missing data Pattern:

N	% Miss	t-dist with 10 df Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	50	51	70	78	44 (45)
	15	50	46	91	93	51 (50)
	20	50	47	114	108	52 (51)
	25	50	50	120	131	49 (50)
500	10	49	40	59	59	45 (40)
	15	49	37	67	74	43 (34)
	20	49	41	98	116	46 (44)
	25	49	38	112	117	44 (51)
1000	10	39	34	53	65	35 (37)
	15	39	39	73	70	42 (38)
	20	39	37	82	95	39 (39)
	25	39	37	109	126	37 (42)
5000	10	56	50	69	67	49 (47)
	15	56	42	84	83	46 (41)
	20	56	44	90	105	45 (45)
	25	56	47	101	114	33 (47)

Table 2: Significance P-values for t-Distribution with 30 df with Monotonic Missing data Pattern:

N	% Miss	t-dist with 30 df Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	49	48	74	78	47 (49)
	15	49	55	94	90	58 (47)
	20	49	51	105	101	56 (55)
	25	49	50	117	129	55 (50)
500	10	48	56	76	73	52 (56)
	15	48	56	84	86	48 (52)
	20	48	59	99	105	56 (65)
	25	48	55	117	142	56 (58)
1000	10	55	52	79	80	57 (54)
	15	55	55	93	92	57 (56)
	20	55	54	113	103	51 (55)
	25	55	52	121	123	55 (53)
5000	10	52	54	73	78	53 (55)
	15	52	51	92	69	55 (54)
	20	52	48	97	65	50 (49)
	25	52	57	121	85	51 (58)

Table 3: Significance P-values for Chi-Square Distribution with 10 df with Monotonic Missing data Pattern:

N	% Miss	Chi-Sqr dist with 10 df Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	49	50	61	55	65 (66)
	15	49	55	79	65	58 (58)
	20	49	54	95	67	54 (49)
	25	49	49	117	73	54 (43)
500	10	46	51	70	59	48 (50)
	15	46	54	81	65	53 (55)
	20	46	49	85	58	50 (48)
	25	46	46	107	67	49 (41)
1000	10	47	51	85	57	51 (47)
	15	47	44	94	55	50 (48)
	20	47	52	110	73	55 (50)
	25	47	52	125	62	57 (54)
5000	10	60	70	86	70	63 (68)
	15	60	59	98	74	61 (59)
	20	60	64	121	78	65 (64)
	25	60	67	136	92	61 (66)

Table 4: Significance P-values for Chi-Square Distribution with 30 df with Monotonic Missing data Pattern:

N	% Miss	Chi-Sqr dist with 30 df Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	47	45	72	54	51 (56)
	15	47	56	102	75	59 (61)
	20	47	56	115	74	59 (53)
	25	47	61	134	87	59 (57)
500	10	48	47	70	55	47 (46)
	15	48	41	80	60	40 (43)
	20	48	43	97	58	48 (45)
	25	48	45	107	77	49 (42)
1000	10	45	40	65	48	42 (46)
	15	45	33	79	52	32 (37)
	20	45	33	98	58	36 (33)
	25	45	36	110	58	35 (37)
5000	10	49	53	69	61	51 (50)
	15	49	55	86	69	56 (53)
	20	49	54	105	64	57 (54)
	25	49	50	119	78	57 (53)

Table 5: Sensitivity and Specificity for t-Distribution with 10 df with Monotonic Missing data Pattern:

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	98.42	68.63	99.57	65.71	99.02	52.56	97.70 (98.32)	63.64 (75.56)
	15	98.12	69.57	99.34	88.00	98.79	41.49	97.68 (98.11)	54.90 (64.00)
	20	97.69	59.57	99.21	37.72	97.54	25.93	97.15 (97.58)	44.25 (52.94)
	25	97.27	48.00	99.21	35.83	98.16	68.00	96.42 (97.26)	32.65 (48.00)
500	10	98.13	77.50	99.15	69.49	98.41	57.63	98.32 (98.02)	73.33 (75.00)
	15	97.51	67.57	99.14	61.19	98.06	41.89	97.70 (97.41)	62.79 (70.59)
	20	97.81	57.14	99.22	42.86	98.31	69.39	97.38 (97.59)	52.17 (59.09)
	25	97.40	63.16	98.99	35.71	97.85	25.64	97.59 (97.47)	59.09 (49.02)
1000	10	98.45	70.59	99.05	56.60	98.72	41.54	98.13 (98.34)	58.33 (62.16)
	15	98.13	53.85	99.35	45.21	98.17	31.43	97.91 (97.92)	45.24 (50.00)
	20	98.24	59.46	99.13	37.80	98.12	23.16	98.02 (97.92)	51.28 (48.72)
	25	97.82	48.65	99.22	29.36	98.40	19.84	97.61 (97.70)	43.24 (40.48)
5000	10	98.32	80.00	99.36	72.46	98.39	61.19	98.11 (98.01)	77.55 (78.72)
	15	97.60	78.57	99.13	57.14	98.26	48.19	97.59 (97.39)	71.74 (75.61)
	20	97.28	68.18	99.12	53.33	97.77	34.29	97.17 (97.38)	64.44 (68.89)
	25	97.27	63.83	99.22	48.51	97.63	30.70	96.38 (97.27)	63.64 (63.83)

Table 6: Sensitivity and Specificity for t-Distribution with 30 df with Monotonic Missing data Pattern:

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	98.43	70.83	98.92	52.70	98.70	47.44	98.01 (98.21)	63.83 (65.31)
	15	98.20	58.18	99.12	43.62	98.35	37.78	97.98 (98.32)	51.72 (70.21)
	20	97.58	50.98	99.00	38.10	98.00	30.69	97.56 (97.67)	46.43 (49.09)
	25	97.16	44.00	99.10	35.04	98.17	25.58	97.25 (97.26)	41.82 (46.00)
500	10	99.05	69.64	99.68	59.21	98.92	52.05	98.84 (99.05)	71.15 (69.64)
	15	98.62	62.50	99.45	51.19	99.23	47.67	98.00 (98.52)	60.42 (65.38)
	20	98.62	59.32	99.22	41.41	98.55	33.33	98.31 (98.93)	57.14 (58.46)
	25	98.10	54.55	99.66	38.46	99.07	28.17	97.46 (97.88)	42.86 (48.28)
1000	10	98.10	71.15	98.92	56.96	98.15	47.50	98.09 (98.10)	64.91 (68.52)
	15	97.36	54.55	98.79	47.31	97.69	36.96	97.03 (97.35)	47.37 (53.57)
	20	96.62	42.59	98.99	40.71	97.77	33.98	96.94 (97.04)	50.98 (49.09)
	25	96.84	48.08	98.98	38.02	97.38	26.02	96.61 (96.73)	41.82 (45.28)
5000	10	98.25	70.37	99.57	65.75	99.13	56.41	98.52 (98.62)	71.70 (70.91)
	15	98.11	66.67	99.23	48.91	98.50	55.07	97.99 (98.20)	60.00 (64.81)
	20	97.69	62.50	99.12	45.36	97.33	41.54	97.58 (97.58)	58.00 (59.18)
	25	97.67	52.63	99.32	38.02	98.14	41.18	97.37 (97.56)	52.94 (50.00)

Table 7: Sensitivity and Specificity for Chi-square with 10 df with Monotonic Missing data Pattern:

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	99.05	80.00	99.57	73.77	99.26	76.36	98.82 (98.72)	58.46 (56.06)
	15	98.62	65.45	99.57	56.96	98.40	52.31	98.73 (98.62)	63.79 (62.07)
	20	98.20	59.26	99.34	45.26	98.07	46.27	98.10 (97.79)	57.41 (57.14)
	25	97.69	55.10	99.66	39.32	97.73	38.36	97.57 (97.57)	48.15 (48.15)
500	10	98.95	70.59	99.68	61.43	99.04	62.71	98.74 (98.84)	70.83 (70.00)
	15	98.84	64.81	99.78	54.32	99.04	56.92	98.94 (98.94)	67.92 (65.45)
	20	98.53	65.31	99.67	50.59	98.41	53.45	98.63 (98.53)	66.00 (66.67)
	25	98.43	67.39	99.66	40.19	98.39	46.27	98.32 (97.81)	61.22 (60.98)
1000	10	98.42	62.75	99.23	47.06	98.30	54.39	98.31 (98.01)	60.78 (59.57)
	15	98.12	65.91	99.45	44.68	98.20	54.55	98.42 (98.32)	64.00 (64.58)
	20	98.00	53.85	99.66	40.00	98.27	42.47	98.20 (98.11)	54.55 (58.00)
	25	97.68	48.08	99.43	33.60	97.97	45.16	97.67 (97.78)	43.86 (48.15)
5000	10	99.14	74.29	99.67	66.28	98.92	71.43	98.83 (99.14)	77.78 (76.47)
	15	98.41	76.27	99.45	56.12	98.16	58.11	98.19 (98.30)	70.49 (74.58)
	20	97.65	59.38	99.66	47.11	98.27	57.14	97.75 (97.54)	60.00 (57.81)
	25	97.75	58.21	99.54	41.18	98.13	46.74	97.87 (97.75)	65.57 (59.09)

Table 8: Sensitivity and Specificity for Chi-square with 30 df with Monotonic Missing data Pattern:

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	98.64	75.56	99.46	58.33	98.84	66.67	98.74 (99.05)	68.63 (67.86)
	15	98.20	53.57	99.44	41.18	98.16	40.00	98.30 (98.51)	52.54 (54.10)
	20	98.20	53.57	99.66	38.26	98.16	40.54	97.98 (97.99)	47.46 (52.83)
	25	98.30	50.82	99.65	32.84	98.69	40.23	98.19 (98.41)	50.85 (56.14)
500	10	98.85	78.72	99.68	64.29	99.15	72.73	98.74 (98.74)	76.60 (78.26)
	15	98.12	73.17	99.46	53.75	98.72	60.00	97.92 (98.22)	70.00 (72.09)
	20	97.81	62.79	99.34	43.30	97.98	50.00	97.69 (97.91)	54.17 (62.22)
	25	97.59	55.56	99.33	39.25	97.72	35.06	97.58 (97.49)	51.02 (57.14)
1000	10	98.44	75.00	99.57	63.08	98.53	64.58	98.64 (98.53)	76.19 (67.39)
	15	98.14	81.82	99.57	51.90	98.42	57.69	97.93 (98.13)	78.13 (72.97)
	20	97.83	72.73	99.33	39.80	98.09	46.55	97.72 (97.52)	63.89 (63.64)
	25	97.20	50.00	99.44	36.36	98.30	50.00	96.89 (96.99)	42.86 (43.24)
5000	10	9926	79.25	99.57	65.22	99.04	65.57	98.95 (99.05)	76.47 (80.00)
	15	98.73	67.27	99.56	52.33	99.03	57.97	98.83 (98.52)	67.86 (66.04)
	20	98.20	59.26	99.55	42.86	98.40	53.13	98.41 (98.20)	59.65 (59.26)
	25	97.89	58.00	99.66	38.66	98.70	47.44	98.09 (97.99)	54.39 (56.60)

APPENDIX D PERECNT COUNT COMPARISON BETWEEN DIFFERENT DISTRIBUTIONS

% Count $\geq$1.0 for Cauchy distribution, t-distribution, and Normal distribution

N=100:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.109	0.12	0.11	0.907	0.087	0.106	0.086	0.08	0.088	0.079	0.081
	10	0.11	0.11	0.097	0.093	0.081	0.075	0.067	0.066	0.052	0.062	0.051
15	5	0.143	0.119	0.096	0.086	0.068	0.064	0.06	0.072	0.07	0.053	0.069
	10	0.153	0.114	0.074	0.064	0.04	0.057	0.042	0.041	0.037	0.025	0.031
20	5	0.186	0.135	0.092	0.076	0.057	0.056	0.053	0.045	0.042	0.027	0.037
	10	0.184	0.13	0.053	0.051	0.029	0.036	0.021	0.028	0.016	0.022	0.018
25	5	0.201	0.136	0.069	0.051	0.051	0.039	0.031	0.028	0.031	0.032	0.029
	10	0.209	0.128	0.057	0.031	0.028	0.026	0.024	0.016	0.01	0.012	0.015

N=500:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.117	0.091	0.061	0.05	0.039	0.046	0.024	0.034	0.035	0.02	0.026
	10	0.116	0.098	0.049	0.032	0.016	0.015	0.009	0.011	0.012	0.011	0.006
15	5	0.175	0.112	0.056	0.028	0.027	0.025	0.018	0.013	0.012	0.02	0.024
	10	0.167	0.11	0.037	0.011	0.007	0.004	0.003	0.003	0.003	0.003	0.004
20	5	0.207	0.112	0.034	0.019	0.019	0.016	0.019	0.015	0.015	0.007	0.006
	10	0.199	0.097	0.022	0.007	0.005	0.003	0.002	0.004	0.002	0.004	0.002
25	5	0.225	0.112	0.03	0.023	0.011	0.018	0.013	0.011	0.013	0.012	0.008
	10	0.227	0.1	0.022	0.005	0.004	0.001	0.002	0.002	0.002	0	0.001

N=1000:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.1	0.104	0.054	0.022	0.02	0.019	0.025	0.015	0.017	0.009	0.01
	10	0.108	0.1	0.028	0.007	0.008	0.005	0.004	0.006	0	0.002	0.002
15	5	0.16	0.106	0.046	0.022	0.013	0.012	0.015	0.007	0.007	0.009	0.009
	10	0.164	0.094	0.026	0.004	0.005	0.003	0	0.003	0	0.001	0.002
20	5	0.195	0.104	0.031	0.015	0.011	0.017	0.009	0.005	0.009	0.005	0.005
	10	0.188	0.089	0.011	0.002	0.001	0.001	0	0.001	0	0	0
25	5	0.224	0.111	0.021	0.012	0.007	0.008	0.004	0.005	0.001	0.002	0.005
	10	0.217	0.093	0.01	0.001	0.001	0.001	0	0	0	0	0

N=5000:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.136	0.103	0.013	0.005	0.006	0.002	0.003	0.004	0.066	0	0.006
	10	0.136	0.086	0.007	0.002	0	0	0	0	0	0	0
15	5	0.168	0.094	0.015	0.007	0.002	0.001	0.002	0.003	0.001	0	0
	10	0.172	0.088	0.005	0	0.001	0	0	0	0	0	0
20	5	0.214	0.103	0.007	0.001	0.001	0.002	0.004	0.002	0.002	0.003	0.001
	10	0.207	0.076	0.002	0.001	0	0	0	0	0	0	0
25	5	0.221	0.09	0.005	0.003	0.002	0.004	0	0.002	0.001	0	0.002
	10	0.23	0.074	0	0	0	0	0	0	0	0	0

% Count $\geq$1.05 for Cauchy distribution, t-distribution, and Normal distribution

N=100:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.086	0.084	0.059	0.036	0.036	0.038	0.031	0.025	0.027	0.018	0.018
	10	0.09	0.085	0.052	0.038	0.026	0.027	0.02	0.019	0.011	0.011	0.011
15	5	0.137	0.094	0.054	0.043	0.023	0.027	0.028	0.025	0.014	0.016	0.022
	10	0.134	0.1	0.045	0.036	0.018	0.016	0.011	0.017	0.007	0.013	0.008
20	5	0.171	0.113	0.062	0.044	0.033	0.03	0.022	0.018	0.017	0.013	0.013
	10	0.173	0.111	0.04	0.032	0.017	0.015	0.009	0.014	0.007	0.011	0.007
25	5	0.195	0.119	0.054	0.03	0.036	0.021	0.018	0.014	0.017	0.012	0.009
	10	0.195	0.112	0.038	0.023	0.016	0.014	0.015	0.007	0.005	0.002	0.006

N=500:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.097	0.062	0.023	0.006	0.007	0.007	0.001	0.001	0.001	0.001	0.001
	10	0.103	0.064	0.011	0.003	0	0	0.001	0.001	0	0	0
15	5	0.158	0.082	0.024	0.004	0.004	0.002	0.002	0.002	0.002	0.001	0
	10	0.156	0.083	0.017	0.003	0.001	0	0	0	0	0.001	0
20	5	0.192	0.089	0.021	0.006	0.003	0.002	0.001	0	0	0	0
	10	0.187	0.077	0.009	0.002	0	0.001	0	0	0	0	0
25	5	0.217	0.094	0.012	0.01	0.002	0.003	0.001	0	0.002	0.001	0
	10	0.216	0.087	0.012	0.001	0.001	0	0	0.001	0	0	0.001

N=1000:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.087	0.062	0.016	0.001	0.001	0	0	0	0	0	0
	10	0.091	0.07	0.006	0.001	0	0	0	0	0	0	0
15	5	0.156	0.077	0.011	0.001	0	0	0	0	0	0	0
	10	0.156	0.068	0.009	0.001	0	0	0	0	0	0	0
20	5	0.186	0.08	0.012	0.001	0	0.001	0.001	0.001	0	0	0.001
	10	0.176	0.075	0.002	0	0	0	0	0	0	0	0
25	5	0.211	0.083	0.012	0.003	0.001	0	0.001	0	0	0	0
	10	0.204	0.08	0.003	0	0	0	0	0	0	0	0

N=5000:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.115	0.062	0	0	0	0	0	0	0	0	0
	10	0.119	0.05	0.001	0	0	0	0	0	0	0	0
15	5	0.154	0.071	0.002	0	0	0	0	0	0	0	0
	10	0.159	0.059	0	0	0	0	0	0	0	0	0
20	5	0.201	0.074	0	0	0	0	0	0	0	0	0
	10	0.193	0.061	0	0	0	0	0	0	0	0	0
25	5	0.212	0.069	0	0	0	0	0	0	0	0	0
	10	0.217	0.063	0	0	0	0	0	0	0	0	0

% Count $\geq$1.1 for Cauchy distribution, t-distribution, and Normal distribution

N=100:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.081	0.071	0.039	0.022	0.017	0.014	0.016	0.008	0.003	0.006	0.002
	10	0.078	0.067	0.033	0.013	0.007	0.013	0.009	0.007	0.003	0	0.004
15	5	0.13	0.078	0.036	0.021	0.012	0.008	0.008	0.01	0.005	0.005	0.004
	10	0.127	0.086	0.033	0.022	0.01	0.01	0.005	0.008	0.003	0.004	0.001
20	5	0.169	0.096	0.042	0.027	0.02	0.012	0.007	0.007	0.008	0.007	0.004
	10	0.17	0.09	0.03	0.018	0.004	0.009	0.004	0.006	0.002	0.004	0.003
25	5	0.184	0.108	0.04	0.018	0.021	0.015	0.013	0.01	0.009	0.005	0.005
	10	0.185	0.097	0.025	0.013	0.009	0.009	0.005	0.003	0.002	0.001	0.001

N=500:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.092	0.053	0.01	0.002	0	0	0	0	0	0	0
	10	0.088	0.043	0.006	0.002	0	0	0	0	0	0	0
15	5	0.146	0.056	0.012	0.001	0	0.001	0	0	0	0	0
	10	0.144	0.061	0.007	0.002	0	0	0	0	0	0	0
20	5	0.179	0.069	0.01	0.001	0.001	0	0	0	0	0	0
	10	0.176	0.061	0.007	0.001	0	0	0	0	0	0	0
25	5	0.21	0.075	0.005	0.002	0.001	0	0	0	0	0.001	0
	10	0.206	0.07	0.008	0.001	0	0	0	0	0	0	0

N=1000:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.082	0.046	0.005	0.001	0	0	0	0	0	0	0
	10	0.085	0.053	0.004	0.001	0	0	0	0	0	0	0
15	5	0.143	0.059	0.007	0	0	0	0	0	0	0	0
	10	0.141	0.052	0.003	0.001	0	0	0	0	0	0	0
20	5	0.17	0.064	0.003	0.001	0	0	0	0	0	0	0
	10	0.171	0.065	0.001	0	0	0	0	0	0	0	0
25	5	0.201	0.068	0.003	0.001	0	0	0	0	0	0	0
	10	0.194	0.071	0.001	0	0	0	0	0	0	0	0

N=5000:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.101	0.045	0	0	0	0	0	0	0	0	0
	10	0.101	0.038	0	0	0	0	0	0	0	0	0
15	5	0.144	0.048	0	0	0	0	0	0	0	0	0
	10	0.146	0.048	0	0	0	0	0	0	0	0	0
20	5	0.191	0.047	0	0	0	0	0	0	0	0	0
	10	0.18	0.049	0	0	0	0	0	0	0	0	0
25	5	0.205	0.057	0	0	0	0	0	0	0	0	0
	10	0.207	0.051	0	0	0	0	0	0	0	0	0

% Count ≥1.0 for Chi-square distribution and Normal distribution

N=100:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.123	0.097	0.083	0.094	0.098	0.092	0.09	0.095	0.091	0.089	0.083	0.081
	10	0.103	0.086	0.075	0.077	0.074	0.081	0.08	0.061	0.06	0.07	0.062	0.051
15	5	0.114	0.079	0.072	0.068	0.061	0.067	0.072	0.058	0.048	0.065	0.054	0.069
	10	0.088	0.063	0.057	0.042	0.047	0.042	0.037	0.035	0.04	0.032	0.031	0.031
20	5	0.088	0.094	0.059	0.059	0.043	0.046	0.049	0.03	0.048	0.044	0.035	0.037
	10	0.08	0.047	0.043	0.025	0.027	0.029	0.025	0.025	0.023	0.027	0.024	0.018
25	5	0.085	0.069	0.056	0.053	0.031	0.05	0.031	0.033	0.044	0.039	0.036	0.029
	10	0.068	0.042	0.026	0.029	0.016	0.024	0.013	0.018	0.01	0.023	0.022	0.015

N=500:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.068	0.048	0.061	0.033	0.041	0.032	0.027	0.027	0.031	0.026	0.032	0.026
	10	0.048	0.031	0.03	0.017	0.014	0.012	0.01	0.011	0.011	0.012	0.007	0.006
15	5	0.05	0.038	0.023	0.03	0.013	0.021	0.026	0.016	0.023	0.02	0.015	0.024
	10	0.026	0.009	0.012	0.007	0.006	0.007	0.005	0.007	0.002	0.005	0.006	0.004
20	5	0.031	0.019	0.017	0.016	0.013	0.012	0.009	0.012	0.006	0.008	0.012	0.006
	10	0.016	0.007	0.005	0.003	0.006	0.003	0.001	0.001	0.003	0.005	0.005	0.002
25	5	0.03	0.014	0.014	0.017	0.007	0.011	0.005	0.014	0.011	0.016	0.009	0.008
	10	0.012	0.002	0	0.004	0.001	0	0.001	0	0	0	0.002	0.001

N=1000:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.042	0.023	0.025	0.029	0.017	0.025	0.019	0.022	0.016	0.016	0.019	0.01
	10	0.017	0.004	0.002	0.004	0.004	0.004	0.006	0.003	0.002	0.001	0.002	0.002
15	5	0.022	0.019	0.009	0.015	0.01	0.011	0.013	0.016	0.006	0.009	0.007	0.009
	10	0.008	0.001	0	0.001	0.001	0.001	0.005	0.001	0	0	0.001	0.002
20	5	0.017	0.011	0.007	0.009	0.006	0.01	0.005	0.012	0.005	0.004	0.006	0.005
	10	0.008	0.001	0.001	0.002	0	0.001	0.001	0	0	0	0	0
25	5	0.019	0.01	0.01	0.005	0.01	0.004	0.006	0.006	0.003	0.003	0.008	0.005
	10	0.006	0.003	0	0	0	0	0.001	0	0	0	0.001	0

N=5000:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.009	0.007	0.006	0.009	0.003	0.004	0.002	0.001	0.003	0.003	0.006	0.006
	10	0.001	0.003	0	0.001	0	0	0	0	0	0	0	0
15	5	0.009	0.002	0.002	0.001	0.003	0.002	0.003	0.001	0.003	0.002	0.002	0
	10	0	0	0.001	0	0	0	0.001	0	0	0	0	0
20	5	0.004	0.001	0.002	0	0.001	0.001	0.002	0.001	0.002	0.001	0.002	0.001
	10	0	0	0	0	0	0	0	0	0	0	0	0
25	5	0.002	0.002	0	0.001	0	0.003	0.001	0.002	0.001	0	0	0.002
	10	0	0	0	0	0	0	0	0	0	0	0	0

% Count ≥1.05 for Chi-square distribution and Normal distribution

N=100:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.074	0.055	0.039	0.034	0.03	0.036	0.035	0.03	0.03	0.026	0.029	0.018
	10	0.061	0.036	0.037	0.029	0.021	0.026	0.018	0.018	0.019	0.022	0.013	0.011
15	5	0.08	0.047	0.036	0.029	0.029	0.027	0.028	0.02	0.026	0.018	0.018	0.022
	10	0.058	0.031	0.032	0.023	0.019	0.02	0.019	0.012	0.014	0.009	0.005	0.008
20	5	0.055	0.053	0.034	0.036	0.029	0.02	0.03	0.013	0.022	0.022	0.013	0.013
	10	0.056	0.032	0.023	0.011	0.014	0.016	0.013	0.013	0.012	0.011	0.008	0.007
25	5	0.053	0.04	0.033	0.035	0.017	0.032	0.021	0.017	0.017	0.019	0.019	0.009
	10	0.053	0.027	0.02	0.017	0.006	0.01	0.006	0.006	0.004	0.012	0.01	0.006

N=500:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.022	0.009	0.007	0.003	0.002	0.001	0	0.002	0.002	0	0.001	0.001
	10	0.016	0.005	0.003	0	0.001	0.001	0	0	0	0	0	0
15	5	0.015	0.004	0.005	0.002	0.003	0.001	0.001	0	0.001	0.001	0.001	0
	10	0.01	0.002	0.002	0	0	0	0	0	0	0	0	0
20	5	0.009	0.01	0	0.004	0.002	0	0.002	0.001	0	0.002	0.002	0
	10	0.007	0.001	0.001	0.001	0.002	0	0	0	0	0	0	0
25	5	0.015	0.004	0.004	0.003	0.001	0.003	0.001	0.003	0.002	0	0	0
	10	0.005	0.001	0	0.001	0	0	0	0	0	0	0	0.001

N=1000:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.006	0.003	0.002	0	0.001	0	0	0	0	0	0	0
	10	0.001	0	0	0	0	0	0	0	0	0	0	0
15	5	0.005	0.003	0.001	0	0	0	0	0.001	0	0	0	0
	10	0.001	0	0	0	0	0	0	0	0	0	0	0
20	5	0.005	0	0	0.002	0	0.002	0	0.001	0	0	0	0.001
	10	0.001	0	0	0	0	0	0	0	0	0	0	0
25	5	0.005	0.001	0.002	0.001	0	0	0.001	0	0	0	0.001	0
	10	0.001	0	0	0	0	0	0	0	0	0	0	0

N=5000:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
15	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
20	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
25	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0

% Count ≥1.1 for Chi-square distribution and Normal distribution

N=100:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.044	0.031	0.025	0.021	0.011	0.011	0.017	0.014	0.008	0.007	0.009	0.002
	10	0.035	0.018	0.02	0.016	0.009	0.014	0.007	0.007	0.005	0.006	0.004	0.004
15	5	0.052	0.023	0.021	0.012	0.012	0.012	0.013	0.007	0.011	0.009	0.003	0.004
	10	0.04	0.015	0.015	0.012	0.008	0.009	0.007	0.002	0.006	0.002	0.004	0.001
20	5	0.035	0.03	0.018	0.018	0.016	0.008	0.018	0.005	0.01	0.008	0.006	0.004
	10	0.038	0.019	0.013	0.004	0.005	0.011	0.009	0.005	0.007	0.004	0.002	0.003
25	5	0.047	0.028	0.021	0.018	0.008	0.017	0.011	0.009	0.009	0.008	0.009	0.005
	10	0.038	0.015	0.013	0.009	0.002	0.008	0.004	0.002	0.003	0.003	0.004	0.001

N=500:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.003	0	0	0.001	0	0	0	0	0	0	0	0
	10	0.005	0	0	0	0	0	0	0	0	0	0	0
15	5	0.003	0.001	0.001	0	0	0	0	0	0	0	0	0
	10	0.001	0	0	0	0	0	0	0	0	0	0	0
20	5	0.005	0.003	0	0.001	0	0	0.001	0	0	0.001	0.001	0
	10	0.003	0	0.001	0	0	0	0	0	0	0	0	0
25	5	0.004	0	0	0	0	0	0	0.002	0.001	0	0	0
	10	0.001	0	0	0.001	0	0	0	0	0	0	0	0

N=1000:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.001	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
15	5	0.001	0.001	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
20	5	0.001	0	0	0	0	0	0	0	0	0	0	0
	10	0.001	0	0	0	0	0	0	0	0	0	0	0
25	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0

N=5000:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
15	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
20	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
25	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0

NON-MONOTONE MISSING DATA PATTERN

APPENDIX E t- DISTRIBUTION WITH DIFFERENT DEGREES OF FREEDOM

Table 1: t-distribution (df = 2) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00388	-0.0124 (-1.7968)	0.00463 (0.9412)	0.8993	0.8814 (0.4632)	0.9175 (78.1811)	0.143	0.103	0.078
100		10	-0.00747	-0.0155 (-1.7832)	0.000607 (0.8201)	0.9046	0.8871 (0.5183)	0.9225 (81.7771)	0.126	0.1	0.081
100	15	5	-0.00227	-0.0121 (-1.8270)	0.00753 (0.8660)	0.8431	0.8224 (0.3349)	0.8644 (86.9871)	0.135	0.11	0.097
100		10	-0.00625	-0.0156 (-1.8458)	0.00309 (0.7982)	0.8567	0.8366 (0.3746)	0.8773 (77.7316)	0.138	0.114	0.102
100	20	5	-0.00379	-0.0152 (-1.7996)	0.00763 (0.8104)	0.8033	0.7820 (0.3379)	0.8252 (66.9741)	0.157	0.136	0.115
100		10	-0.00489	-0.0158 (-1.8779)	0.00599 (0.8432)	0.8138	0.7929 (0.4115)	0.8353 (68.5799)	0.153	0.136	0.118
100	25	5	-0.00250	-0.0186 (-1.8256)	0.0136 (2.8872)	0.7565	0.7324 (0.2546)	0.7814 (54.3405)	0.151	0.137	0.123
100		10	-0.00547	-0.0204 (-1.7907)	0.00947 (2.9841)	0.7656	0.7421 (0.3111)	0.7897 (60.7928)	0.145	0.126	0.115
500	10	5	-0.00107	-0.00467 (-0.4636)	0.00253 (0.5231)	0.8957	0.8833 (0.4882)	0.9082 (17.7312)	0.121	0.079	0.059
500		10	-0.00193	-0.00526 (-0.3868)	0.00141 (0.5271)	0.9005	0.8884 (0.4994)	0.9129 (22.9373)	0.111	0.088	0.064
500	15	5	-0.00313	-0.00794 (-1.2621)	0.00169 (0.5087)	0.8458	0.8298 (0.3551)	0.8621 (21.9373)	0.131	0.112	0.081
500		10	-0.00538	-0.00985 (-1.0668)	-0.00090 (0.4832)	0.8553	0.8401 (0.3566)	0.8707 (20.5714)	0.126	0.093	0.079
500	20	5	-0.00518	-0.0121 (-2.4005)	0.00178 (0.6468)	0.8108	0.7897 (0.3426)	0.8324 (265.7)	0.122	0.097	0.09
500		10	-0.00350	-0.0104 (-2.4150)	0.00336 (0.6089)	0.8246	0.8039 (0.3778)	0.8457 (266.7)	0.131	0.111	0.092
500	25	5	-0.00685	-0.0144 (-2.4462)	0.000752 (0.6009)	0.7669	0.7446 (0.2841)	0.7899 (177.6)	0.143	0.121	0.105
500		10	-0.00454	-0.0121 (-2.4073)	0.00297 (0.6388)	0.7856	0.7636 (0.3470)	0.8082 (256.2)	0.130	0.116	0.103

Table 1 continues: t-distribution (df = 2) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00112	-0.00351 (-0.2200)	0.00126 (0.2536)	0.8885	0.8773 (0.5218)	0.8997 (8.4456)	0.098	0.058	0.045
1000		10	-0.00124	-0.00348 (-0.2244)	0.000986 (0.2415)	0.8993	0.8884 (0.6181)	0.9104 (10.1480)	0.012	0.068	0.049
1000	15	5	-0.00253	-0.00570 (-0.6103)	0.000630 (0.2500)	0.8465	0.8326 (0.4390)	0.8605 (12.8859)	0.115	0.073	0.057
1000		10	-0.00281	-0.00587 (-0.6165)	0.000239 (0.2155)	0.8510	0.8383 (0.4603)	0.8639 (10.9615)	0.091	0.072	0.059
1000	20	5	-0.00283	-0.00707 (-1.2158)	0.00140 (0.3479)	0.8117	0.7937 (0.3785)	0.8301 (114.3)	0.115	0.098	0.08
1000		10	-0.00312	-0.00737 (-1.2231)	0.00112 (0.3248)	0.8127	0.7956 (0.4296)	0.8302 (102.3)	0.103	0.091	0.074
1000	25	5	-0.00538	-0.0130 (-2.4249)	0.00222 (0.6530)	0.7674	0.7451 (0.3193)	0.7904 (173.4)	0.129	0.111	0.097
1000		10	-0.00401	-0.00853 (-1.2093)	0.000518 (0.3372)	0.7753	0.7569 (0.3745)	0.7940 (90.8757)	0.114	0.095	0.083
5000	10	5	0.000759	-0.00046 (-0.2378)	0.00198 (0.1395)	0.8973	0.8862 (0.4740)	0.9084 (19.5703)	0.096	0.061	0.05
5000		10	0.000467	-0.00072 (-0.2387)	0.00165 (0.1316)	0.8999	0.8896 (0.5060)	0.9104 (19.0600)	0.086	0.055	0.043
5000	15	5	0.000681	-0.00084 (-0.2155)	0.00220 (0.1273)	0.8410	0.8291 (0.4721)	0.8529 (14.3503)	0.1	0.075	0.059
5000		10	0.000242	-0.00122 (-0.2353)	0.00171 (0.1319)	0.8536	0.8420 (0.5068)	0.8653 (19.0251)	0.09	0.068	0.06
5000	20	5	-0.00007	-0.00199 (-0.3206)	0.00186 (0.1377)	0.8061	0.7902 (0.3758)	0.8223 (39.1734)	0.111	0.085	0.067
5000		10	0.000139	-0.00171 (-0.3120)	0.00199 (0.1467)	0.8161	0.8007 (0.4477)	0.8318 (40.1355)	0.095	0.083	0.073
5000	25	5	0.000360	-0.00183 (-0.3229)	0.00255 (0.1402)	0.7538	0.7375 (0.3404)	0.7704 (40.9543)	0.114	0.088	0.073
5000		10	-0.00018	-0.00228 (-0.3186)	0.00192 (0.1338)	0.7668	0.7506 (0.3608)	0.7833 (38.4704)	0.095	0.074	0.059

Table 2: t-distribution (df = 4) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.00142	-0.00177 (-0.2592)	0.00462 (0.1608)	0.8779	0.8704 (0.5377)	0.8854 (1.9151)	0.099	0.06	0.038
100		10	0.00174	-0.00130 (-0.2200)	0.00477 (0.1484)	0.8878	0.8812 (0.5642)	0.8943 (1.8768)	0.101	0.062	0.039
100	15	5	0.00512	0.000902 (-0.3407)	0.00934 (0.2336)	0.8223	0.8133 (0.4596)	0.8313 (1.8801)	0.101	0.066	0.041
100		10	0.00523	0.00125 (-0.2973)	0.00922 (0.2339)	0.8254	0.8175 (0.5023)	0.8334 (1.9427)	0.086	0.055	0.036
100	20	5	0.00735	0.00235 (-0.2975)	0.0123 (0.2778)	0.7674	0.7568 (0.2924)	0.7782 (1.9293)	0.097	0.062	0.041
100		10	0.00640	0.00170 (-0.3322)	0.0111 (0.2306)	0.7785	0.7699 (0.4482)	0.7873 (1.8023)	0.07	0.052	0.034
100	25	5	0.00461	-0.00116 (-0.3961)	0.0104 (0.3740)	0.7278	0.7167 (0.2609)	0.7391 (2.0607)	0.08	0.053	0.038
100		10	0.00652	0.00101 (-0.3139)	0.0120 (0.3498)	0.7262	0.7169 (0.3781)	0.7356 (1.9113)	0.059	0.039	0.028
500	10	5	0.000421	-0.00103 (-0.0831)	0.00187 (0.0772)	0.8882	0.8828 (0.6229)	0.8936 (2.2678)	0.064	0.022	0.015
500		10	0.000676	-0.00075 (-0.0746)	0.00210 (0.0680)	0.8921	0.8877 (0.6889)	0.8964 (2.3809)	0.044	0.017	0.009
500	15	5	-0.00048	-0.00233 (-0.1019)	0.00137 (0.0923)	0.8316	0.8244 (0.3973)	0.8389 (1.9475)	0.058	0.025	0.012
500		10	0.000324	-0.00148 (-0.0876)	0.00213 (0.0997)	0.8436	0.8383 (0.6074)	0.8490 (1.5502)	0.032	0.02	0.013
500	20	5	0.000223	-0.00204 (-0.1409)	0.00249 (0.1135)	0.7770	0.7687 (0.4118)	0.7854 (2.0534)	0.042	0.022	0.012
500		10	0.000971	-0.00118 (-0.1115)	0.00312 (0.0998)	0.7866	0.7803 (0.4975)	0.7929 (2.1212)	0.029	0.014	0.009
500	25	5	0.00159	-0.00104 (-0.1625)	0.00422 (0.1524)	0.7309	0.7218 (0.3188)	0.7401 (1.5273)	0.031	0.014	0.008
500		10	0.00101	-0.00150 (-0.1639)	0.00353 (0.1329)	0.7405	0.7334 (0.4025)	0.7476 (2.1263)	0.002	0.01	0.005

Table 2 continues: t-distribution (df = 4) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000153	-0.00086 (-0.0501)	0.00116 (0.0672)	0.8838	0.8789 (0.6104)	0.8887 (1.5450)	0.043	0.012	0.004
1000		10	0.000298	-0.00067 (-0.0466)	0.00127 (0.0556)	0.8929	0.8892 (0.6780)	0.8965 (1.4530)	0.027	0.007	0.004
1000	15	5	0.000180	-0.00110 (-0.0602)	0.00146 (0.0708)	0.8307	0.8241 (0.4732)	0.8375 (1.6237)	0.029	0.01	0.005
1000		10	0.000444	-0.00077 (-0.0570)	0.00165 (0.0675)	0.8377	0.8331 (0.6282)	0.8423 (1.4031)	0.017	0.008	0.005
1000	20	5	0.000343	-0.00115 (-0.0664)	0.00184 (0.0749)	0.7812	0.7733 (0.4114)	0.7891 (1.3477)	0.032	0.012	0.006
1000		10	0.00130	-0.00016 (-0.0867)	0.00276 (0.0660)	0.7913	0.7856 (0.5430)	0.7971 (1.3670)	0.017	0.009	0.004
1000	25	5	0.000801	-0.00097 (-0.1307)	0.00257 (0.0931)	0.7291	0.7203 (0.3299)	0.7380 (1.5738)	0.021	0.009	0.005
1000		10	0.00143	-0.00024 (-0.0802)	0.00309 (0.0885)	0.7381	0.7317 (0.4238)	0.7446 (1.2212)	0.012	0.008	0.006
5000	10	5	-0.00064	-0.00109 (-0.0214)	-0.00019 (0.0270)	0.8830	0.8784 (0.6032)	0.8875 (1.0649)	0.01	0.001	0
5000		10	-0.00070	-0.00113 (-0.0202)	-0.00027 (0.0235)	0.8921	0.8892 (0.7110)	0.8950 (1.0263)	0.004	0	0
5000	15	5	-0.00074	-0.00131 (-0.0260)	-0.00017 (0.0294)	0.8314	0.8251 (0.4421)	0.8377 (1.0648)	0.008	0.002	0
5000		10	-0.00073	-0.00126 (-0.0235)	-0.00020 (0.0268)	0.8370	0.8330 (0.6295)	0.8410 (1.0271)	0.003	0	0
5000	20	5	-0.00047	-0.00115 (-0.0333)	0.000213 (0.0298)	0.7858	0.7783 (0.4064)	0.7933 (1.1572)	0.01	0.003	0.001
5000		10	-0.00055	-0.00118 (-0.0337)	0.000079 (0.0336)	0.7908	0.7858 (0.5161)	0.7959 (1.0216)	0.004	0	0
5000	25	5	0.000031	-0.00073 (-0.0351)	0.000792 (0.0417)	0.7258	0.7178 (0.3594)	0.7339 (1.0731)	0.007	0.001	0
5000		10	-0.00044	-0.00117 (-0.0351)	0.000296 (0.0422)	0.7372	0.7317 (0.4972)	0.7429 (1.0110)	0.001	0	0

Table 3: t-distribution (df = 6) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00087	-0.00365 (-0.1477)	0.00192 (0.1698)	0.8799	0.8734 (0.5207)	0.8864 (2.2020)	0.088	0.036	0.016
100		10	-0.00097	-0.00369 (-0.1412)	0.00175 (0.1666)	0.8843	0.8786 (0.5754)	0.8900 (2.2394)	0.08	0.033	0.019
100	15	5	0.000810	-0.00257 (-0.1747)	0.00419 (0.2217)	0.8144	0.8064 (0.4046)	0.8226 (2.1500)	0.064	0.041	0.018
100		10	0.000011	-0.00331 (-0.1985)	0.00333 (0.2178)	0.8254	0.8186 (0.5286)	0.8322 (2.2195)	0.056	0.026	0.012
100	20	5	-0.00414	-0.00832 (-0.2535)	0.000046 (0.2023)	0.7594	0.7498 (0.3179)	0.7692 (2.1673)	0.062	0.038	0.019
100		10	-0.00145	-0.00536 (-0.2624)	0.00246 (0.2169)	0.7748	0.7671 (0.3838)	0.7826 (2.2304)	0.047	0.022	0.012
100	25	5	-0.00046	-0.00520 (-0.2491)	0.00428 (0.2699)	0.7198	0.7092 (0.2967)	0.7305 (1.5217)	0.053	0.035	0.022
100		10	-0.00074	-0.00522 (-0.2387)	0.00373 (0.2478)	0.7188	0.7100 (0.3283)	0.7276 (1.8370)	0.04	0.022	0.01
500	10	5	-0.00038	-0.00163 (-0.0666)	0.000881 (0.0747)	0.8840	0.8790 (0.5063)	0.8890 (1.1093)	0.039	0.01	0.001
500		10	-0.00076	-0.00195 (-0.0713)	0.000442 (0.0789)	0.8918	0.8884 (0.6587)	0.8953 (1.1329)	0.018	0.001	0.001
500	15	5	-0.00088	-0.00246 (-0.0819)	0.000706 (0.0786)	0.8311	0.8245 (0.4510)	0.8378 (1.3188)	0.023	0.01	0.004
500		10	-0.00098	-0.00247 (-0.0774)	0.000511 (0.0835)	0.8390	0.8344 (0.5888)	0.8436 (1.2785)	0.013	0.003	0.001
500	20	5	-0.00111	-0.00302 (-0.0889)	0.000797 (0.1006)	0.7817	0.7742 (0.4091)	0.7893 (1.1425)	0.03	0.006	0.001
500		10	-0.00114	-0.00296 (-0.0846)	0.000674 (0.0883)	0.7866	0.7810 (0.5116)	0.7923 (1.1917)	0.01	0.002	0.001
500	25	5	-0.00102	-0.00321 (-0.0925)	0.00117 (0.1293)	0.7261	0.7173 (0.3397)	0.7350 (1.1570)	0.016	0.006	0.003
500		10	-0.00132	-0.00340 (-0.0911)	0.000747 (0.1050)	0.7382	0.7319 (0.4439)	0.7447 (1.2052)	0.007	0.004	0.002

Table 3 continues: t-distribution (df = 6) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00088	-0.00223 (-0.0742)	0.000469 (0.0603)	0.7788	0.7711 (0.3117)	0.7866 (1.0262)	0.012	0	0
1000		10	-0.00131	-0.00261 (-0.0696)	-0.00001 (0.0686)	0.7864	0.7813 (0.5477)	0.7916 (1.0397)	0.006	0	0
1000	15	5	-0.00034	-0.00146 (-0.0547)	0.000773 (0.0611)	0.8291	0.8228 (0.5098)	0.8354 (1.1284)	0.016	0.001	0.001
1000		10	-0.00048	-0.00155 (-0.0510)	0.000588 (0.0484)	0.8405	0.8363 (0.6335)	0.8447 (1.0364)	0.004	0	0
1000	20	5	-0.00088	-0.00223 (-0.0742)	0.000469 (0.0603)	0.7788	0.7711 (0.3117)	0.7866 (1.0262)	0.012	0	0
1000		10	-0.00131	-0.00261 (-0.0696)	-0.00001 (0.0686)	0.7864	0.7813 (0.5477)	0.7916 (1.0397)	0.006	0	0
1000	25	5	-0.00047	-0.00201 (-0.0789)	0.00108 (0.0839)	0.7297	0.7214 (0.3730)	0.7381 (1.0682)	0.01	0.001	0
1000		10	-0.00046	-0.00195 (-0.0844)	0.00103 (0.0772)	0.7392	0.7335 (0.4085)	0.7448 (1.0373)	0.001	0	0
5000	10	5	0.000483	0.000075 (-0.0239)	0.000890 (0.0228)	0.8833	0.8788 (0.6411)	0.8877 (1.0293)	0.005	0	0
5000		10	0.000439	0.000055 (-0.0208)	0.000823 (0.0199)	0.8915	0.8886 (0.7559)	0.8944 (1.0071)	0.001	0	0
5000	15	5	0.000694	0.000179 (-0.0307)	0.00121 (0.0262)	0.8282	0.8221 (0.5184)	0.8343 (1.0367)	0.008	0	0
5000		10	0.000605	0.000121 (-0.0267)	0.00109 (0.0287)	0.8381	0.8342 (0.6247)	0.8421 (0.9879)	0	0	0
5000	20	5	0.000425	-0.00017 (-0.0377)	0.00102 (0.0301)	0.7766	0.7693 (0.4090)	0.7840 (1.0308)	0.005	0	0
5000		10	0.000464	-0.00011 (-0.0369)	0.00104 (0.0273)	0.7910	0.7862 (0.5267)	0.7959 (0.9773)	0	0	0
5000	25	5	0.000610	-0.00007 (-0.0365)	0.00129 (0.0356)	0.7287	0.7201 (0.3295)	0.7374 (1.0146)	0.003	0	0
5000		10	0.000623	-0.00005 (-0.0337)	0.00129 (0.0304)	0.7394	0.7342 (0.4704)	0.7447 (0.9636)	0	0	0

Table 4: t-distribution (df = 8) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00010	-0.00286 (-0.1414)	0.00265 (0.1750)	0.8772	0.8704 (0.5060)	0.8840 (1.5243)	0.097	0.04	0.015
100		10	-0.00029	-0.00287 (-0.1566)	0.00228 (0.1289)	0.8899	0.8847 (0.6039)	0.8950 (1.5383)	0.08	0.031	0.014
100	15	5	-0.00180	-0.00525 (-0.1982)	0.00165 (0.1981)	0.8249	0.8170 (0.3774)	0.8330 (1.4921)	0.077	0.041	0.019
100		10	-0.00195	-0.00523 (-0.1774)	0.00133 (0.1784)	0.8344	0.8280 (0.5186)	0.8408 (1.2747)	0.056	0.022	0.013
100	20	5	-0.00127	-0.00549 (-0.2059)	0.00296 (0.2352)	0.7694	0.7603 (0.3379)	0.7787 (1.4426)	0.054	0.028	0.013
100		10	-0.00196	-0.00601 (-0.2387)	0.00209 (0.2446)	0.7802	0.7729 (0.4150)	0.7875 (1.2751)	0.046	0.018	0.009
100	25	5	-0.00052	-0.00538 (-0.2481)	0.00435 (0.2519)	0.7202	0.7098 (0.2491)	0.7308 (1.4397)	0.052	0.024	0.016
100		10	-0.00149	-0.00610 (-0.2272)	0.00312 (0.2621)	0.7218	0.7137 (0.3218)	0.7301 (1.6691)	0.027	0.012	0.006
500	10	5	0.00122	0.000074 (-0.0699)	0.00237 (0.0666)	0.8820	0.8771 (0.6109)	0.6109 (1.0943)	0.038	0.003	0
500		10	0.00111	2.797E-6 (-0.0581)	0.00221 (0.0609)	0.8915	0.8882 (0.6698)	0.8948 (1.0514)	0.011	0.001	0
500	15	5	0.000414	-0.00106 (-0.0918)	0.00189 (0.0713)	0.8293	0.8232 (0.4741)	0.8356 (1.0846)	0.026	0.003	0
500		10	0.00111	-0.00031 (-0.0936)	0.00254 (0.0661)	0.8364	0.8320 (0.5858)	0.8407 (1.0448)	0.003	0	0
500	20	5	0.000594	-0.00121 (-0.0976)	0.00240 (0.0873)	0.7770	0.7696 (0.4046)	0.7846 (1.1150)	0.012	0.001	0.001
500		10	0.000999	-0.00071 (-0.0926)	0.00270 (0.0819)	0.7876	0.7822 (0.4985)	0.7930 (1.0799)	0.004	0.002	0
500	25	5	0.000866	-0.00118 (-0.1169)	0.00291 (0.1087)	0.7211	0.7124 (0.2928)	0.7299 (1.0851)	0.014	0.003	0
500		10	0.000407	-0.00153 (-0.0968)	0.00234 (0.1038)	0.7319	0.7258 (0.4436)	0.7381 (1.0483)	0.002	0	0

Table 4 continues: t-distribution (df = 8) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000163	-0.00066 (-0.0544)	0.000984 (0.0417)	0.8874	0.8829 (0.5955)	0.8919 (1.0278)	0.023	0	0
1000		10	0.000479	-0.00032 (-0.0452)	0.00128 (0.0490)	0.8926	0.8894 (0.7061)	0.8958 (1.0202)	0.004	0	0
1000	15	5	0.00105	0.000015 (-0.0568)	0.00209 (0.0556)	0.8306	0.8244 (0.5182)	0.8369 (1.0491)	0.019	0	0
1000		10	0.000771	-0.00023 (-0.0547)	0.00177 (0.0555)	0.8411	0.8370 (0.5995)	0.8452 (1.0137)	0.001	0	0
1000	20	5	0.00177	0.000494 (-0.0738)	0.00304 (0.0750)	0.7831	0.7756 (0.4448)	0.7906 (1.0279)	0.011	0	0
1000		10	0.000858	-0.00034 (-0.0642)	0.00206 (0.0739)	0.7868	0.7818 (0.5321)	0.7919 (1.0071)	0.002	0	0
1000	25	5	0.00114	-0.00029 (-0.0633)	0.00256 (0.0755)	0.7256	0.7174 (0.3393)	0.7339 (1.0311)	0.003	0	0
1000		10	0.00100	-0.00036 (-0.0735)	0.00236 (0.0688)	0.7362	0.7304 (0.4376)	0.7420 (0.9904)	0	0	0
5000	10	5	0.000066	-0.00030 (-0.0156)	0.000436 (0.0172)	0.8854	0.8809 (0.5538)	0.8899 (1.0303)	0.005	0	0
5000		10	-0.00003	-0.00037 (-0.0157)	0.000323 (0.0172)	0.8938	0.8910 (0.7085)	0.8967 (0.9866)	0	0	0
5000	15	5	0.000071	-0.00041 (-0.0237)	0.000549 (0.0245)	0.8324	0.8267 (0.5175)	0.8381 (1.0034)	0.001	0	0
5000		10	0.000054	-0.00040 (-0.0218)	0.000510 (0.0251)	0.8382	0.8342 (0.6017)	0.8421 (0.9755)	0	0	0
5000	20	5	-6.53E-6	-0.00056 (-0.0281)	0.000543 (0.0307)	0.7805	0.7730 (0.3965)	0.7880 (1.0119)	0.001	0	0
5000		10	0.000032	-0.00051 (-0.0243)	0.000577 (0.0291)	0.7929	0.7880 (0.4366)	0.7979 (0.9912)	0	0	0
5000	25	5	-0.00044	-0.00109 (-0.0428)	0.000206 (0.0297)	0.7298	0.7214 (0.3287)	0.7382 (0.9955)	0	0	0
5000		10	-0.00025	-0.00087 (-0.0289)	0.000364 (0.0293)	0.7390	0.7334 (0.4273)	0.7446 (0.9667)	0	0	0

Table 5: t-distribution (df = 10) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.000987	-0.00154 (-0.1524)	0.00352 (0.1321)	0.8831	0.8770 (0.5141)	0.8892 (1.4943)	0.092	0.031	0.011
100		10	0.00199	-0.00045 (-0.1279)	0.00443 (0.1374)	0.8886	0.8836 (0.6415)	0.8936 (1.5162)	0.081	0.022	0.006
100	15	5	0.00241	-0.00096 (-0.1809)	0.00579 (0.1946)	0.8240	0.8161 (0.4036)	0.8320 (1.4025)	0.08	0.029	0.014
100		10	0.00260	-0.00067 (-0.1615)	0.00586 (0.1712)	0.8328	0.8265 (0.4962)	0.8391 (1.4768)	0.056	0.023	0.007
100	20	5	0.00208	-0.00185 (-0.2014)	0.00600 (0.2265)	0.7643	0.7554 (0.3117)	0.7732 (1.1938)	0.042	0.019	0.006
100		10	0.00278	-0.00104 (-0.2044)	0.00660 (0.1778)	0.7779	0.7710 (0.3930)	0.7849 (1.3200)	0.029	0.01	0.007
100	25	5	0.00139	-0.00320 (-0.2518)	0.00598 (0.2613)	0.7195	0.7097 (0.2186)	0.7293 (1.2681)	0.04	0.021	0.011
100		10	0.000829	-0.00354 (-0.2467)	0.00520 (0.2676)	0.7250	0.7172 (0.3617)	0.7329 (1.2162)	0.024	0.008	0.003
500	10	5	0.000483	-0.00064 (-0.0746)	0.00160 (0.0592)	0.8825	0.8775 (0.5454)	0.8875 (1.0828)	0.036	0.007	0
500		10	0.000270	-0.00080 (-0.0475)	0.00134 (0.0596)	0.8905	0.8871 (0.7081)	0.8938 (1.0371)	0.011	0	0
500	15	5	0.000076	-0.00139 (-0.0788)	0.00154 (0.0706)	0.8272	0.8207 (0.4747)	0.8338 (1.0613)	0.024	0.002	0
500		10	0.000817	-0.00053 (-0.0707)	0.00217 (0.0560)	0.8345	0.8301 (0.6005)	0.8390 (1.0296)	0.004	0	0
500	20	5	0.000943	-0.00076 (-0.0952)	0.00265 (0.0884)	0.7751	0.7675 (0.3335)	0.7827 (1.0736)	0.016	0.001	0
500		10	0.000734	-0.00090 (-0.0773)	0.00236 (0.0712)	0.7832	0.7778 (0.5329)	0.7887 (1.0074)	0.001	0	0
500	25	5	0.000200	-0.00180 (-0.0963)	0.00220 (0.1035)	0.7260	0.7173 (0.3211)	0.7348 (1.0454)	0.006	0	0
500		10	0.000432	-0.00148 (-0.0893)	0.00235 (0.1093)	0.7329	0.7267 (0.4214)	0.7391 (1.1481)	0.003	0.002	0.002

Table 5 continues: t-distribution (df = 10) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000297	-0.00050 (-0.0418)	0.00110 (0.0418)	0.8848	0.8803 (0.5666)	0.8894 (1.0350)	0.002	0	0
1000		10	0.000496	-0.00029 (-0.0412)	0.00128 (0.0390)	0.8895	0.8864 (0.7253)	0.8926 (1.0146)	0.001	0	0
1000	15	5	0.000419	-0.00061 (-0.0545)	0.00144 (0.0457)	0.8290	0.8228 (0.5081)	0.8352 (1.0530)	0.01	0.001	0
1000		10	0.000409	-0.00059 (-0.0498)	0.00141 (0.0498)	0.8396	0.8353 (0.6155)	0.8439 (1.0148)	0.002	0	0
1000	20	5	0.000791	-0.00040 (-0.0584)	0.00198 (0.0651)	0.7784	0.7711 (0.4096)	0.7859 (1.0749)	0.007	0.001	0
1000		10	0.000424	-0.00073 (-0.0623)	0.00158 (0.0558)	0.7855	0.7805 (0.5179)	0.7905 (1.0089)	0.001	0	0
1000	25	5	0.000504	-0.00088 (-0.0704)	0.00189 (0.0654)	0.7297	0.7215 (0.3806)	0.7379 (1.0904)	0.005	0.001	0
1000		10	0.000541	-0.00080 (-0.0599)	0.00188 (0.0787)	0.7376	0.7319 (0.4504)	0.7434 (0.9767)	0	0	0
5000	10	5	0.000193	-0.00017 (-0.0203)	0.000561 (0.0202)	0.8883	0.8841 (0.5926)	0.8925 (1.0160)	0.004	0	0
5000		10	0.000155	-0.00019 (-0.0194)	0.000504 (0.0194)	0.8936	0.8908 (0.7375)	0.8964 (0.9874)	0	0	0
5000	15	5	0.000174	-0.00028 (-0.0269)	0.000627 (0.0228)	0.8357	0.8299 (0.4930)	0.8415 (1.0120)	0.004	0	0
5000		10	0.000218	-0.00021 (-0.0273)	0.000651 (0.0240)	0.8395	0.8355 (0.6090)	0.8435 (0.9726)	0	0	0
5000	20	5	0.000104	-0.00043 (-0.0294)	0.000637 (0.0272)	0.7814	0.7743 (0.3929)	0.7887 (1.0012)	0.001	0	0
5000		10	0.000129	-0.00038 (-0.0320)	0.000634 (0.0231)	0.7906	0.7859 (0.5503)	0.7953 (0.9745)	0	0	0
5000	25	5	0.000496	-0.00012 (-0.0290)	0.00111 (0.0264)	0.7298	0.7214 (0.3415)	0.7383 (1.0157)	0.001	0	0
5000		10	0.000345	-0.00025 (-0.0280)	0.000935 (0.0299)	0.7398	0.7345 (0.4592)	0.7451 (0.9690)	0	0	0

Table 6: t-distribution (df = 15) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00145	-0.00398 (-0.1209)	0.00108 (0.1598)	0.8782	0.8720 (0.5090)	0.8844 (1.3834)	0.087	0.025	0.014
100		10	-0.00107	-0.00355 (-0.1290)	0.00140 (0.1307)	0.8856	0.8806 (0.6381)	0.8907 (1.3971)	0.068	0.017	0.007
100	15	5	-0.00163	-0.00482 (-0.1785)	0.00155 (0.1738)	0.8218	0.8140 (0.4544)	0.8296 (1.2044)	0.066	0.027	0.012
100		10	-0.00109	-0.00416 (-0.1677)	0.00198 (0.1697)	0.8337	0.8276 (0.5131)	0.8399 (1.4547)	0.049	0.015	0.006
100	20	5	-0.00195	-0.00567 (-0.2290)	0.00177 (0.2190)	0.7680	0.7590 (0.2731)	0.7771 (1.3354)	0.046	0.018	0.007
100		10	-0.00144	-0.00506 (-0.2042)	0.00218 (0.2439)	0.7808	0.7734 (0.4085)	0.7883 (1.2788)	0.047	0.017	0.007
100	25	5	-0.00274	-0.00713 (-0.2567)	0.00165 (0.3432)	0.7155	0.7058 (0.3151)	0.7254 (1.2199)	0.045	0.021	0.013
100		10	-0.00102	-0.00519 (-0.2366)	0.00316 (0.2926)	0.7202	0.7126 (0.3846)	0.7279 (1.4333)	0.019	0.012	0.005
500	10	5	0.000226	-0.00088 (-0.0540)	0.00133 (0.0677)	0.8863	0.8816 (0.5812)	0.8909 (1.0536)	0.025	0.001	0
500		10	0.000191	-0.00086 (-0.0578)	0.00124 (0.0696)	0.8885	0.8851 (0.6503)	0.8919 (1.0224)	0.011	0	0
500	15	5	0.000068	-0.00129 (-0.0733)	0.00143 (0.0780)	0.8276	0.8214 (0.4893)	0.8339 (1.0560)	0.017	0.001	0
500		10	-0.00024	-0.00155 (-0.0812)	0.00107 (0.0782)	0.8417	0.8374 (0.6278)	0.8459 (1.0115)	0.002	0	0
500	20	5	0.000599	-0.00103 (-0.0977)	0.00223 (0.0815)	0.7855	0.7783 (0.4413)	0.7928 (1.0667)	0.01	0.003	0
500		10	0.000208	-0.00139 (-0.0877)	0.00181 (0.0769)	0.7862	0.7809 (0.4928)	0.7914 (1.0176)	0.002	0	0
500	25	5	-0.00057	-0.00251 (-0.0947)	0.00136 (0.1075)	0.7262	0.7176 (0.3473)	0.7348 (1.0493)	0.007	0	0
500		10	-0.00017	-0.00204 (-0.1021)	0.00171 (0.0983)	0.7304	0.7242 (0.4207)	0.7367 (1.0910)	0.002	0.001	0

Table 6 continues: t-distribution (df = 15) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00013	-0.00091 (-0.0394)	0.000640 (0.0550)	0.8863	0.8817 (0.5771)	0.8909 (1.0537)	0.015	0.001	0
1000		10	-0.00038	-0.00111 (-0.0364)	0.000353 (0.0495)	0.8884	0.8853 (0.6857)	0.8915 (1.0083)	0.005	0	0
1000	15	5	0.000515	-0.00047 (-0.0521)	0.00150 (0.0577)	0.8379	0.8319 (0.4568)	0.8439 (1.0373)	0.009	0	0
1000		10	-0.00012	-0.00105 (-0.0519)	0.000796 (0.0540)	0.8361	0.8319 (0.5835)	0.8404 (0.9830)	0	0	0
1000	20	5	-0.00049	-0.00164 (-0.0633)	0.000649 (0.0653)	0.7773	0.7699 (0.4099)	0.7847 (1.0038)	0.004	0	0
1000		10	-4.96E-6	-0.00110 (-0.0520)	0.00109 (0.0589)	0.7864	0.7815 (0.4998)	0.7913 (0.9901)	0	0	0
1000	25	5	0.000278	-0.00107 (-0.0625)	0.00163 (0.0754)	0.7272	0.7187 (0.3377)	0.7357 (1.0220)	0.007	0	0
1000		10	0.000256	-0.00102 (-0.0679)	0.00153 (0.0766)	0.7331	0.7272 (0.4453)	0.7391 (0.9795)	0	0	0
5000	10	5	-0.00003	-0.00036 (-0.0191)	0.000302 (0.0159)	0.8842	0.8798 (0.5049)	0.8886 (1.0120)	0.005	0	0
5000		10	-4.01E-6	-0.00033 (-0.0208)	0.000320 (0.0188)	0.8920	0.8891 (0.7210)	0.8950 (0.9948)	0	0	0
5000	15	5	0.000048	-0.00037 (-0.0205)	0.000462 (0.0196)	0.8304	0.8244 (0.5048)	0.8364 (1.0092)	0.003	0	0
5000		10	0.000014	-0.00038 (-0.0224)	0.000409 (0.0178)	0.8414	0.8376 (0.6157)	0.8453 (0.9868)	0	0	0
5000	20	5	0.000180	-0.00032 (-0.0256)	0.000678 (0.0230)	0.7793	0.7723 (0.4091)	0.7864 (0.9962)	0	0	0
5000		10	-0.00001	-0.00048 (-0.0232)	0.000454 (0.0204)	0.7853	0.7803 (0.4985)	0.7902 (0.9831)	0	0	0
5000	25	5	-0.00012	-0.00067 (-0.0299)	0.000437 (0.0296)	0.7172	0.7089 (0.3579)	0.7255 (1.0056)	0.002	0	0
5000		10	0.000114	-0.00041 (-0.0257)	0.000643 (0.0279)	0.7361	0.7308 (0.4751)	0.7415 (0.9529)	0	0	0

Table 7: t-distribution (df =20) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00197	-0.00434 (-0.1213)	0.000406 (0.1241)	0.8772	0.8710 (0.3867)	0.8835 (1.1504)	0.091	0.024	0.008
100		10	-0.00211	-0.00436 (-0.1209)	0.000134 (0.1170)	0.8857	0.8810 (0.6675)	0.8904 (1.1431)	0.058	0.018	0.002
100	15	5	-0.00234	-0.00537 (-0.1564)	0.000697 (0.1459)	0.8182	0.8104 (0.3542)	0.8261 (1.2137)	0.055	0.016	0.005
100		10	-0.00198	-0.00490 (-0.1584)	0.000943 (0.1938)	0.8274	0.8214 (0.4581)	0.8334 (1.1395)	0.032	0.01	0.003
100	20	5	-0.00034	-0.00410 (-0.1758)	0.00343 (0.2237)	0.7633	0.7547 (0.3437)	0.7721 (1.1885)	0.04	0.018	0.011
100		10	-0.00249	-0.00601 (-0.1823)	0.00104 (0.1589)	0.7760	0.7690 (0.4487)	0.7830 (1.1867)	0.028	0.01	0.005
100	25	5	-0.00087	-0.00515 (-0.2384)	0.00340 (0.2798)	0.7091	0.6992 (0.2077)	0.7192 (1.1608)	0.043	0.016	0.003
100		10	-0.00141	-0.00558 (-0.2439)	0.00276 (0.2350)	0.7244	0.7167 (0.4100)	0.7321 (1.1510)	0.016	0.006	0.001
500	10	5	-0.00130	-0.00239 (-0.0525)	-0.00021 (0.0485)	0.8822	0.8774 (0.5506)	0.8870 (1.0470)	0.023	0	0
500		10	-0.00094	-0.00198 (-0.0492)	0.000108 (0.0474)	0.8911	0.8880 (0.6998)	0.8941 (1.0129)	0.002	0	0
500	15	5	-0.00140	-0.00276 (-0.0651)	-0.00004 (0.0647)	0.8289	0.8227 (0.5087)	0.8351 (1.0500)	0.017	0.001	0
500		10	-0.00139	-0.00269 (-0.0609)	-0.00009 (0.0512)	0.8382	0.8338 (0.5524)	0.8426 (1.0062)	0.002	0	0
500	20	5	-0.00110	-0.00265 (-0.0839)	0.000451 (0.0777)	0.7731	0.7652 (0.3650)	0.7810 (1.0471)	0.013	0	0
500		10	-0.00152	-0.00306 (-0.0726)	0.000015 (0.0775)	0.7868	0.7815 (0.5131)	0.7921 (1.0706)	0.001	0.001	0
500	25	5	-0.00232	-0.00417 (-0.0893)	-0.00047 (0.0975)	0.7260	0.7174 (0.3106)	0.7346 (1.0295)	0.006	0	0
500		10	-0.00223	-0.00399 (-0.0984)	-0.00047 (0.0872)	0.7310	0.7251 (0.4169)	0.7370 (1.0009)	0.001	0	0

Table 7 continues: t-distribution (df =20) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00120	-0.00195 (-0.0395)	-0.00045 (0.0345)	0.8848	0.8804 (0.5730)	0.8893 (1.0379)	0.015	0	0
1000		10	-0.00119	-0.00192 (-0.0416)	-0.00046 (0.0347)	0.8920	0.8891 (0.6961)	0.8950 (1.0198)	0.002	0	0
1000	15	5	-0.00123	-0.00217 (-0.0469)	-0.00029 (0.0497)	0.8292	0.8230 (0.4886)	0.8354 (1.0337)	0.015	0	0
1000		10	-0.00136	-0.00227 (-0.0496)	-0.00046 (0.0411)	0.8396	0.8355 (0.6295)	0.8437 (1.0085)	0.005	0	0
1000	20	5	-0.00128	-0.00242 (-0.0509)	-0.00014 (0.0620)	0.7725	0.7646 (0.3952)	0.7806 (1.0335)	0.008	0	0
1000		10	-0.00147	-0.00255 (-0.0503)	-0.00038 (0.0592)	0.7829	0.7778 (0.5509)	0.7880 (1.0198)	0.003	0	0
1000	25	5	-0.00232	-0.00366 (-0.0627)	-0.00098 (0.0622)	0.7271	0.7189 (0.3446)	0.7355 (1.0435)	0.005	0	0
1000		10	-0.00181	-0.00311 (-0.0687)	-0.00051 (0.0675)	0.7414	0.7357 (0.4469)	0.7471 (0.9984)	0	0	0
5000	10	5	-0.00030	-0.00064 (-0.0181)	0.000034 (0.0185)	0.8819	0.8773 (0.5804)	0.8865 (1.0180)	0.006	0	0
5000		10	-0.00025	-0.00057 (-0.0195)	0.000078 (0.0182)	0.8884	0.8854 (0.6844)	0.8914 (0.9907)	0	0	0
5000	15	5	-0.00045	-0.00087 (-0.0228)	-0.00002 (0.0218)	0.8292	0.8231 (0.4498)	0.8353 (1.0145)	0.005	0	0
5000		10	-0.00039	-0.00079 (-0.0205)	0.000022 (0.0201)	0.8424	0.8386 (0.6485)	0.8463 (0.9788)	0	0	0
5000	20	5	-0.00068	-0.00118 (-0.0276)	-0.00017 (0.0273)	0.7707	0.7635 (0.3807)	0.7779 (1.0084)	0.002	0	0
5000		10	-0.00058	-0.00106 (-0.0257)	-0.00009 (0.0251)	0.7913	0.7865 (0.5630)	0.7961 (0.9699)	0	0	0
5000	25	5	-0.00058	-0.00115 (-0.0281)	-4.07E-6 (0.0331)	0.7234	0.7152 (0.3420)	0.7316 (1.0025)	0.001	0	0
5000		10	-0.00064	-0.00121 (-0.0290)	-0.00008 (0.0318)	0.7420	0.7368 (0.4991)	0.7473 (0.9664)	0	0	0

Table 8: t-distribution (df = 25) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.000824	-0.00157 (-0.1408)	0.00322 (0.1647)	0.8821	0.8761 (0.5144)	0.8881 (1.2698)	0.092	0.022	0.009
100		10	0.00104	-0.00126 (-0.1186)	0.00335 (0.1600)	0.8861	0.8813 (0.6186)	0.8910 (1.2663)	0.067	0.017	0.006
100	15	5	-0.00059	-0.00365 (-0.1430)	0.00247 (0.1959)	0.8240	0.8163 (0.3397)	0.8319 (1.2743)	0.053	0.015	0.007
100		10	0.000164	-0.00273 (-0.1349)	0.00306 (0.1920)	0.8342	0.8283 (0.5790)	0.8402 (1.1833)	0.037	0.018	0.006
100	20	5	0.000673	-0.00288 (-0.1802)	0.00423 (0.2024)	0.7656	0.7568 (0.3359)	0.7745 (1.2557)	0.047	0.026	0.015
100		10	0.000282	-0.00314 (-0.1490)	0.00371 (0.1900)	0.7719	0.7651 (0.4873)	0.7787 (1.1523)	0.02	0.01	0.005
100	25	5	0.00273	-0.00150 (-0.1955)	0.00695 (0.2067)	0.7149	0.7050 (0.3026)	0.7249 (1.2263)	0.034	0.017	0.005
100		10	0.00272	-0.00134 (-0.1719)	0.00678 (0.1958)	0.7205	0.7129 (0.3420)	0.7281 (1.1345)	0.013	0.004	0.002
500	10	5	-0.00049	-0.00155 (-0.0558)	0.000577 (0.0518)	0.8855	0.8810 (0.6072)	0.8901 (1.0771)	0.029	0.001	0
500		10	-0.00035	-0.00138 (-0.0575)	0.000682 (0.0544)	0.8921	0.8888 (0.7059)	0.8954 (1.0196)	0.01	0	0
500	15	5	0.000245	-0.00113 (-0.1010)	0.00162 (0.0709)	0.8297	0.8234 (0.4622)	0.8360 (1.0373)	0.017	0	0
500		10	-0.00008	-0.00141 (-0.0773)	0.00125 (0.0645)	0.8362	0.8318 (0.6093)	0.8406 (1.0525)	0.003	0.001	0
500	20	5	-0.00014	-0.00182 (-0.0852)	0.00153 (0.1023)	0.7782	0.7706 (0.4048)	0.7859 (1.0507)	0.006	0.001	0
500		10	-0.00023	-0.00183 (-0.1002)	0.00137 (0.0860)	0.7847	0.7793 (0.5461)	0.7901 (1.0111)	0.001	0	0
500	25	5	-0.00052	-0.00245 (-0.1053)	0.00141 (0.0974)	0.7256	0.7168 (0.3202)	0.7345 (1.0253)	0.006	0	0
500		10	-0.00004	-0.00191 (-0.1291)	0.00183 (0.1133)	0.7338	0.7278 (0.4162)	0.7398 (1.0107)	0.002	0	0

Table 8 continues: t-distribution (df = 25) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00026	-0.00100 (-0.0372)	0.000483 (0.0351)	0.8829	0.8784 (0.6203)	0.8875 (1.0360)	0.015	0	0
1000		10	-0.00027	-0.00098 (-0.0385)	0.000443 (0.0336)	0.8918	0.8889 (0.7268)	0.8947 (0.9984)	0	0	0
1000	15	5	-0.00044	-0.00139 (-0.0613)	0.000506 (0.0581)	0.8255	0.8191 (0.4933)	0.8319 (1.0409)	0.012	0	0
1000		10	-0.00053	-0.00143 (-0.0503)	0.000361 (0.0420)	0.8366	0.8325 (0.6055)	0.8408 (0.9910)	0	0	0
1000	20	5	-0.00047	-0.00154 (-0.0570)	0.000611 (0.0549)	0.7786	0.7712 (0.3956)	0.7860 (1.0325)	0.007	0	0
1000		10	-0.00076	-0.00183 (-0.0737)	0.000312 (0.0625)	0.7901	0.7851 (0.5082)	0.7951 (1.0122)	0.001	0	0
1000	25	5	-0.00049	-0.00176 (-0.0843)	0.000792 (0.0655)	0.7262	0.7177 (0.3554)	0.7348 (1.0409)	0.006	0	0
1000		10	-0.00021	-0.00144 (-0.0810)	0.00102 (0.0574)	0.7299	0.7241 (0.4827)	0.7358 (0.9668)	0	0	0
5000	10	5	-4.09E-6	-0.00033 (-0.0170)	0.000317 (0.0186)	0.8854	0.8809 (0.5951)	0.8899 (1.0045)	0.003	0	0
5000		10	0.000072	-0.00024 (-0.0179)	0.000384 (0.0170)	0.8921	0.8893 (0.7368)	0.8949 (0.9940)	0	0	0
5000	15	5	-0.00006	-0.00049 (-0.0242)	0.000371 (0.0237)	0.8330	0.8271 (0.5131)	0.8389 (1.0091)	0.003	0	0
5000		10	-0.00020	-0.00061 (-0.0223)	0.000208 (0.0191)	0.8361	0.8321 (0.5662)	0.8402 (0.9935)	0	0	0
5000	20	5	-0.00047	-0.00096 (-0.0256)	0.000022 (0.0220)	0.7761	0.7691 (0.3911)	0.7832 (0.9945)	0	0	0
5000		10	-0.00043	-0.00091 (-0.0222)	0.000045 (0.0237)	0.7888	0.7840 (0.4754)	0.7936 (0.9699)	0	0	0
5000	25	5	-0.00023	-0.00080 (-0.0260)	0.000352 (0.0266)	0.7263	0.7181 (0.3428)	0.7347 (1.0018)	0.001	0	0
5000		10	-0.00034	-0.00089 (-0.0263)	0.000204 (0.0291)	0.7392	0.7339 (0.4748)	0.7447 (0.9562)	0	0	0

APPENDIX F CHI-SQUARE DISTRIBUTION WITH DIFFERENT DEGREES OF FREEDOM

Table 1: Chi-Square distribution (df = 2) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00016	-0.00499 (-0.3486)	0.00466 (0.2750)	0.8830	0.8750 (0.4887)	0.8911 (2.6478)	0.137	0.085	0.057
100		10	0.00124	-0.00323 (-0.2425)	0.00570 (0.3156)	0.8922	0.8853 (0.6523)	0.8991 (2.5580)	0.125	0.072	0.046
100	15	5	0.00307	-0.00303 (-0.3146)	0.00916 (0.4058)	0.8275	0.8180 (0.3932)	0.8372 (2.5079)	0.111	0.063	0.048
100		10	0.00170	-0.00416 (-0.2954)	0.00757 (0.4391)	0.8340	0.8263 (0.4825)	0.8418 (2.3688)	0.083	0.051	0.037
100	20	5	0.000869	-0.00643 (-0.4042)	0.00817 (0.3966)	0.7682	0.7575 (0.3279)	0.7791 (2.4664)	0.096	0.066	0.044
100		10	0.00396	-0.00293 (-0.3756)	0.0109 (0.3962)	0.7832	0.7740 (0.4180)	0.7926 (2.1579)	0.081	0.054	0.04
100	25	5	0.00143	-0.00689 (-0.4661)	0.00974 (0.4176)	0.7185	0.7063 (0.2564)	0.7309 (2.1231)	0.092	0.07	0.044
100		10	0.00379	-0.00430 (-0.4270)	0.0119 (0.4519)	0.7330	0.7232 (0.3446)	0.7429 (2.0445)	0.067	0.052	0.037
500	10	5	-0.00172	-0.00378 (-0.1046)	0.000329 (0.1223)	0.8826	0.8775 (0.6158)	0.8878 (1.1829)	0.066	0.019	0.007
500		10	-0.00052	-0.00247 (-0.0924)	0.00142 (0.1155)	0.8896	0.8857 (0.6646)	0.8934 (1.2359)	0.043	0.011	0.002
500	15	5	0.000393	-0.00214 (-0.1164)	0.00293 (0.1331)	0.8320	0.8254 (0.4923)	0.8386 (1.1665)	0.035	0.013	0.004
500		10	0.000267	-0.00212 (-0.1232)	0.00266 (0.1501)	0.8423	0.8376 (0.6183)	0.8470 (1.1408)	0.022	0.006	0.002
500	20	5	0.000483	-0.00244 (-0.1554)	0.00341 (0.1628)	0.7776	0.7694 (0.3219)	0.7858 (1.2327)	0.038	0.014	0.005
500		10	0.000248	-0.00253 (-0.1335)	0.00303 (0.1629)	0.7873	0.7815 (0.4975)	0.7930 (1.1750)	0.021	0.004	0.003
500	25	5	-0.00029	-0.00370 (-0.1846)	0.00312 (0.2021)	0.7278	0.7186 (0.3294)	0.7372 (1.2085)	0.028	0.013	0.006
500		10	-0.00004	-0.00332 (-0.1778)	0.00323 (0.1989)	0.7397	0.7333 (0.4763)	0.7460 (1.1396)	0.007	0.003	0.002

Table 1 continues: Chi-Square distribution (df = 2) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00032	-0.00179 (-0.0773)	0.00114 (0.0825)	0.8842	0.8794 (0.5459)	0.8890 (1.1075)	0.037	0.008	0.001
1000		10	-0.00040	-0.00180 (-0.0677)	0.00100 (0.0734)	0.8931	0.8896 (0.6890)	0.8965 (1.0912)	0.025	0.003	0
1000	15	5	-0.00006	-0.00187 (-0.0924)	0.00175 (0.1028)	0.8322	0.8260 (0.4904)	0.8384 (1.0801)	0.031	0.001	0
1000		10	-0.00047	-0.00220 (-0.0840)	0.00126 (0.1053)	0.8393	0.8348 (0.5547)	0.8438 (1.0680)	0.011	0.001	0
1000	20	5	0.000742	-0.00140 (-0.0885)	0.00289 (0.1482)	0.7793	0.7716 (0.4503)	0.7871 (1.1451)	0.018	0.007	0.001
1000		10	-0.00021	-0.00219 (-0.1059)	0.00177 (0.1174)	0.7887	0.7835 (0.5224)	0.7940 (1.1295)	0.005	0.003	0.001
1000	25	5	0.000106	-0.00239 (-0.1369)	0.00261 (0.1203)	0.7318	0.7230 (0.2736)	0.7407 (1.1042)	0.016	0.001	0.001
1000		10	-0.00075	-0.00307 (-0.1121)	0.00158 (0.1341)	0.7374	0.7313 (0.4334)	0.7435 (1.0936)	0.002	0.001	0
5000	10	5	-0.00037	-0.00103 (-0.0370)	0.000293 (0.0328)	0.8912	0.8869 (0.6057)	0.8955 (1.0206)	0.01	0	0
5000		10	-0.00008	-0.00071 (-0.0301)	0.000555 (0.0306)	0.8921	0.8891 (0.7322)	0.8950 (1.0148)	0.002	0	0
5000	15	5	0.000162	-0.00067 (-0.0388)	0.000993 (0.0449)	0.8352	0.8291 (0.4832)	0.8414 (1.0148)	0.006	0	0
5000		10	0.000108	-0.00069 (-0.0361)	0.000909 (0.0519)	0.8448	0.8410 (0.6319)	0.8488 (0.9857)	0	0	0
5000	20	5	0.000015	-0.00096 (-0.0396)	0.000992 (0.0502)	0.7751	0.7678 (0.4093)	0.7825 (1.0146)	0.004	0	0
5000		10	0.000201	-0.00073 (-0.0454)	0.00113 (0.0485)	0.7891	0.7843 (0.5033)	0.7939 (0.9776)	0	0	0
5000	25	5	0.000461	-0.00066 (-0.0565)	0.00158 (0.0612)	0.7248	0.7163 (0.3275)	0.7333 (1.0210)	0.001	0	0
5000		10	0.000339	-0.00072 (-0.0475)	0.00140 (0.0618)	0.7364	0.7307 (0.3795)	0.7421 (0.9555)	0	0	0

Table 2: Chi-Square distribution (df = 4) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00456	-0.0112 (-0.3330)	0.00206 (0.3895)	0.8781	0.8714 (0.4800)	0.8849 (1.5081)	0.105	0.055	0.023
100		10	-0.00335	-0.00977 (-0.3572)	0.00308 (0.4610)	0.8823	0.8767 (0.6038)	0.8880 (1.5089)	0.09	0.044	0.025
100	15	5	-0.00292	-0.0116 (-0.5380)	0.00573 (0.5177)	0.8240	0.8158 (0.4386)	0.8324 (1.4135)	0.078	0.048	0.029
100		10	-0.00559	-0.0139 (-0.4665)	0.00275 (0.4554)	0.8337	0.8268 (0.5159)	0.8406 (1.4461)	0.08	0.048	0.032
100	20	5	-0.00381	-0.0138 (-0.4861)	0.00619 (0.6329)	0.7704	0.7606 (0.3391)	0.7803 (1.8758)	0.08	0.047	0.029
100		10	-0.00711	-0.0168 (-0.4789)	0.00260 (0.5901)	0.7773	0.7693 (0.3765)	0.7853 (1.6907)	0.056	0.038	0.028
100	25	5	0.00133	-0.0103 (-0.5119)	0.0129 (0.7313)	0.7160	0.7052 (0.3022)	0.7270 (1.5186)	0.062	0.043	0.028
100		10	-0.00182	-0.0129 (-0.5841)	0.00925 (0.7063)	0.7278	0.7188 (0.3811)	0.7369 (1.6606)	0.056	0.035	0.022
500	10	5	-0.00178	-0.00474 (-0.1628)	0.00119 (0.1757)	0.8845	0.8797 (0.5978)	0.8893 (1.1238)	0.045	0.005	0.002
500		10	-0.00235	-0.00515 (-0.1523)	0.000454 (0.1651)	0.8910	0.8873 (0.6799)	0.8946 (1.0701)	0.029	0.007	0
500	15	5	-0.00541	-0.00912 (-0.1929)	-0.00170 (0.2021)	0.8282	0.8216 (0.4311)	0.8348 (1.1228)	0.037	0.009	0.001
500		10	-0.00454	-0.00813 (-0.2082)	-0.00094 (0.2204)	0.8334	0.8284 (0.5809)	0.8385 (1.0684)	0.013	0.002	0
500	20	5	-0.00562	-0.00995 (-0.2498)	-0.00130 (0.2222)	0.7717	0.7637 (0.3911)	0.7798 (1.1310)	0.018	0.002	0.001
500		10	-0.00598	-0.0101 (-0.2039)	-0.00189 (0.1977)	0.7863	0.7808 (0.4606)	0.7917 (1.0873)	0.005	0.001	0
500	25	5	-0.00444	-0.00942 (-0.2652)	0.000553 (0.2747)	0.7211	0.7123 (0.3131)	0.7301 (1.0857)	0.019	0.004	0
500		10	-0.00319	-0.00796 (-0.2568)	0.00158 (0.2560)	0.7356	0.7293 (0.4214)	0.7420 (1.1184)	0.005	0.002	0.001

Table 2 continues: Chi-Square distribution (df = 4) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00188	-0.00387 (-0.1027)	0.000111 (0.1093)	0.8826	0.8779 (0.6396)	0.8873 (1.0638)	0.022	0.002	0
1000		10	-0.00134	-0.00331 (-0.0937)	0.000627 (0.1224)	0.8888	0.8857 (0.7306)	0.8919 (1.0312)	0.014	0	0
1000	15	5	-0.00328	-0.00588 (-0.1187)	-0.00069 (0.1405)	0.8297	0.8237 (0.5041)	0.8357 (1.0837)	0.015	0.001	0
1000		10	-0.00281	-0.00522 (-0.1145)	-0.00039 (0.1210)	0.8391	0.8351 (0.5929)	0.8432 (1.0563)	0.009	0.001	0
1000	20	5	-0.00353	-0.00656 (-0.1401)	-0.00051 (0.1491)	0.7763	0.7687 (0.4323)	0.7839 (1.0658)	0.007	0.001	0
1000		10	-0.00283	-0.00567 (-0.1487)	0.000019 (0.1216)	0.7882	0.7830 (0.5067)	0.7934 (1.0291)	0.001	0	0
1000	25	5	-0.00195	-0.00546 (-0.2053)	0.00157 (0.1759)	0.7335	0.7255 (0.3800)	0.7416 (1.0321)	0.008	0	0
1000		10	-0.00135	-0.00464 (-0.1469)	0.00194 (0.2111)	0.7375	0.7319 (0.4786)	0.7431 (0.9927)	0	0	0
5000	10	5	-0.00040	-0.00133 (-0.0464)	0.000521 (0.0444)	0.8804	0.8759 (0.6086)	0.8850 (1.0089)	0.003	0	0
5000		10	-0.00076	-0.00163 (-0.0477)	0.000104 (0.0420)	0.8912	0.8883 (0.7202)	0.8942 (0.9885)	0	0	0
5000	15	5	-0.00053	-0.00168 (-0.0463)	0.000624 (0.0560)	0.8262	0.8201 (0.4433)	0.8323 (1.0127)	0.003	0	0
5000		10	-0.00044	-0.00154 (-0.0518)	0.000672 (0.0575)	0.8405	0.8365 (0.5307)	0.8445 (1.0172)	0.002	0	0
5000	20	5	-0.00054	-0.00195 (-0.0627)	0.000862 (0.0757)	0.7760	0.7686 (0.3824)	0.7834 (1.0027)	0.003	0	0
5000		10	-0.00063	-0.00197 (-0.0671)	0.000704 (0.0696)	0.7843	0.7795 (0.5127)	0.7892 (0.9692)	0	0	0
5000	25	5	-0.00121	-0.00284 (-0.0833)	0.000414 (0.0789)	0.7252	0.7171 (0.3792)	0.7335 (1.0137)	0.001	0	0
5000		10	-0.00118	-0.00272 (-0.0829)	0.000361 (0.0834)	0.7351	0.7295 (0.4174)	0.7407 (0.9700)	0	0	0

Table 3: Chi-Square distribution (df = 6) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00456	-0.0126 (-0.4911)	0.00352 (0.4751)	0.8809	0.8745 (0.5359)	0.8872 (1.2680)	0.108	0.055	0.029
100		10	-0.00375	-0.0113 (-0.4049)	0.00382 (0.4931)	0.8861	0.8808 (0.5454)	0.8915 (1.4159)	0.082	0.047	0.021
100	15	5	0.000138	-0.0102 (-0.5234)	0.0104 (0.5887)	0.8216	0.8131 (0.3944)	0.8301 (1.4107)	0.082	0.045	0.019
100		10	0.00311	-0.00664 (-0.5782)	0.0129 (0.6754)	0.8291	0.8226 (0.5267)	0.8356 (1.3846)	0.058	0.034	0.02
100	20	5	-0.00050	-0.0126 (-0.8003)	0.0116 (0.7776)	0.7714	0.7619 (0.3530)	0.7809 (1.5608)	0.073	0.04	0.022
100		10	0.00303	-0.00868 (-0.6075)	0.0147 (0.7385)	0.7764	0.7685 (0.3782)	0.7844 (1.5740)	0.056	0.034	0.023
100	25	5	0.00311	-0.0112 (-0.7304)	0.0174 (0.8980)	0.7252	0.7148 (0.3037)	0.7356 (1.3771)	0.06	0.043	0.027
100		10	-0.00238	-0.0161 (-0.6790)	0.0113 (0.7390)	0.7311	0.7225 (0.3762)	0.7398 (1.5450)	0.055	0.031	0.014
500	10	5	-0.00077	-0.00429 (-0.1684)	0.00274 (0.1839)	0.8877	0.8827 (0.5944)	0.8927 (1.1519)	0.046	0.006	0.003
500		10	-0.00052	-0.00386 (-0.1666)	0.00282 (0.1946)	0.8934	0.8900 (0.6986)	0.8968 (1.1551)	0.015	0.005	0.001
500	15	5	-0.00065	-0.00500 (-0.1852)	0.00369 (0.2135)	0.8243	0.8179 (0.4769)	0.8308 (1.0670)	0.025	0.008	0
500		10	0.000119	-0.00398 (-0.1911)	0.00422 (0.2120)	0.8415	0.8369 (0.5810)	0.8460 (1.1416)	0.012	0.003	0.002
500	20	5	-0.00115	-0.00631 (-0.2289)	0.00402 (0.2333)	0.7756	0.7679 (0.3614)	0.7835 (1.1829)	0.017	0.003	0.002
500		10	-0.00095	-0.00593 (-0.2226)	0.00403 (0.2454)	0.7852	0.7797 (0.5122)	0.7907 (1.0600)	0.007	0.002	0
500	25	5	-0.00051	-0.00649 (-0.3252)	0.00548 (0.3227)	0.7222	0.7135 (0.2931)	0.7311 (1.0852)	0.015	0.002	0
500		10	0.000664	-0.00502 (-0.2727)	0.00635 (0.2676)	0.7350	0.7290 (0.4382)	0.7411 (1.1645)	0.004	0.002	0.002

Table 3 continues: Chi-Square distribution (df = 6) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00186	-0.00442 (-0.1251)	0.000711 (0.1308)	0.8843	0.8798 (0.5894)	0.8890 (1.0373)	0.019	0	0
1000		10	-0.00136	-0.00378 (-0.1155)	0.00105 (0.1080)	0.8912	0.8880 (0.7161)	0.8944 (1.0609)	0.009	0.001	0
1000	15	5	-0.00084	-0.00393 (-0.1535)	0.00225 (0.1893)	0.8313	0.8250 (0.4943)	0.8377 (1.0713)	0.023	0.003	0
1000		10	-0.00119	-0.00420 (-0.1432)	0.00182 (0.1709)	0.8378	0.8336 (0.6151)	0.8420 (1.0417)	0.004	0	0
1000	20	5	-0.00279	-0.00654 (-0.1787)	0.000953 (0.1703)	0.1703	0.7749 (0.3960)	0.7898 (1.0646)	0.02	0.001	0
1000		10	-0.00228	-0.00588 (-0.1785)	0.00132 (0.1642)	0.7880	0.7829 (0.4787)	0.7932 (1.0268)	0.004	0	0
1000	25	5	0.000440	-0.00376 (-0.1819)	0.00464 (0.1915)	0.7290	0.7205 (0.3254)	0.7376 (1.0490)	0.006	0	0
1000		10	-0.00041	-0.00450 (-0.1940)	0.00369 (0.1913)	0.7379	0.7320 (0.4588)	0.7439 (1.0240)	0.002	0	0
5000	10	5	0.000425	-0.00063 (-0.0505)	0.00148 (0.0546)	0.8839	0.8795 (0.5813)	0.8883 (1.0038)	0.005	0	0
5000		10	0.000218	-0.00081 (-0.0555)	0.00125 (0.0493)	0.8927	0.8900 (0.7366)	0.8954 (0.9922)	0	0	0
5000	15	5	-0.00016	-0.00154 (-0.0725)	0.00122 (0.0759)	0.8327	0.8269 (0.5347)	0.8385 (1.0033)	0.002	0	0
5000		10	0.000257	-0.00107 (-0.0615)	0.00159 (0.0684)	0.8395	0.8354 (0.6518)	0.8435 (1.0192)	0.002	0	0
5000	20	5	0.000631	-0.00099 (-0.0801)	0.00225 (0.0825)	0.7867	0.7796 (0.4443)	0.7939 (1.0027)	0.001	0	0
5000		10	0.000649	-0.00089 (-0.0729)	0.00218 (0.0828)	0.7873	0.7824 (0.5001)	0.7923 (0.9816)	0	0	0
5000	25	5	0.00142	-0.00041 (-0.1006)	0.00325 (0.0928)	0.7251	0.7171 (0.3591)	0.7331 (1.0080)	0.001	0	0
5000		10	0.00169	-0.00013 (-0.0911)	0.00351 (0.1097)	0.7322	0.7266 (0.4461)	0.7379 (0.9782)	0	0	0

Table 4: Chi-Square distribution (df = 8) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00373	-0.0128 (-0.4543)	0.00535 (0.5401)	0.8760	0.8698 (0.5089)	0.8823 (1.4558)	0.09	0.039	0.019
100		10	-0.00271	-0.0114 (-0.4355)	0.00601 (0.5447)	0.8841	0.8789 (0.6026)	0.8894 (1.3851)	0.081	0.035	0.018
100	15	5	0.000248	-0.0117 (-0.6152)	0.0122 (0.6896)	0.8221	0.8142 (0.4601)	0.8300 (1.4212)	0.08	0.035	0.02
100		10	-0.00412	-0.0155 (-0.5597)	0.00723 (0.5364)	0.8313	0.8249 (0.4516)	0.8376 (1.2764)	0.052	0.023	0.014
100	20	5	-0.0110	-0.0249 (-0.6973)	0.00296 (0.7849)	0.7698	0.7605 (0.3468)	0.7792 (1.4120)	0.063	0.03	0.017
100		10	-0.00750	-0.0205 (-0.7205)	0.00548 (0.6467)	0.7772	0.7700 (0.4296)	0.7845 (1.3258)	0.041	0.023	0.011
100	25	5	-0.00396	-0.0195 (-0.7999)	0.0116 (0.8275)	0.7181	0.7081 (0.2727)	0.7281 (1.4002)	0.057	0.029	0.02
100		10	-0.00883	-0.0240 (-0.6968)	0.00638 (0.8616)	0.7212	0.7131 (0.3345)	0.7294 (1.2967)	0.026	0.017	0.009
500	10	5	-0.00244	-0.00652 (-0.1916)	0.00164 (0.1976)	0.8823	0.8773 (0.4700)	0.8873 (1.1269)	0.037	0.005	0.001
500		10	-0.00188	-0.00577 (-0.1904)	0.00200 (0.2010)	0.8889	0.8856 (0.7252)	0.8923 (1.0708)	0.012	0.001	0
500	15	5	-0.00505	-0.0102 (-0.2329)	0.000053 (0.2482)	0.8242	0.8179 (0.4677)	0.8306 (1.0698)	0.022	0.003	0
500		10	-0.00639	-0.0113 (-0.2797)	-0.00145 (0.2182)	0.8349	0.8306 (0.6035)	0.8393 (1.0300)	0.003	0	0
500	20	5	-0.00630	-0.0122 (-0.2931)	-0.00038 (0.2695)	0.7738	0.7659 (0.3318)	0.7818 (1.1368)	0.02	0.006	0.001
500		10	-0.00722	-0.0129 (-0.2539)	-0.00153 (0.2824)	0.7816	0.7765 (0.5325)	0.7868 (1.0796)	0.003	0.001	0
500	25	5	-0.0114	-0.0183 (-0.4155)	-0.00451 (0.3756)	0.7174	0.7086 (0.3020)	0.7264 (1.1028)	0.015	0.005	0.001
500		10	-0.0107	-0.0174 (-0.3797)	-0.00404 (0.3026)	0.7329	0.7269 (0.4427)	0.7390 (1.1006)	0.002	0.001	0.001

Table 4 continues: Chi-Square distribution (df = 8) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00338	-0.00628 (-0.1417)	-0.00048 (0.1478)	0.8839	0.8793 (0.5969)	0.8885 (1.0442)	0.02	0	0
1000		10	-0.00263	-0.00538 (-0.1420)	0.000119 (0.1410)	0.8887	0.8856 (0.6946)	0.8918 (1.0129)	0.002	0	0
1000	15	5	-0.00382	-0.00746 (-0.2038)	-0.00019 (0.1862)	0.8264	0.8199 (0.4845)	0.8330 (1.0717)	0.011	0.001	0
1000		10	-0.00287	-0.00639 (-0.1491)	0.000649 (0.1888)	0.8398	0.8357 (0.5849)	0.8440 (1.0184)	0.002	0	0
1000	20	5	-0.00444	-0.00881 (-0.2696)	-0.00008 (0.2102)	0.7708	0.7633 (0.3266)	0.7785 (1.0595)	0.005	0.001	0
1000		10	-0.00222	-0.00633 (-0.2308)	0.00189 (0.2021)	0.7850	0.7799 (0.5228)	0.7900 (0.9961)	0	0	0
1000	25	5	-0.00462	-0.00962 (-0.2807)	0.000383 (0.2262)	0.7210	0.7125 (0.3223)	0.7296 (1.0176)	0.005	0	0
1000		10	-0.00547	-0.0103 (-0.2569)	-0.00067 (0.2550)	0.7318	0.7261 (0.4684)	0.7376 (0.9877)	0	0	0
5000	10	5	-0.00098	-0.00220 (-0.0584)	0.000241 (0.0557)	0.8825	0.8781 (0.6039)	0.8870 (1.0068)	0.006	0	0
5000		10	-0.00039	-0.00157 (-0.0550)	0.000794 (0.0598)	0.8919	0.8890 (0.7202)	0.8947 (0.9987)	0	0	0
5000	15	5	5.847E-6	-0.00155 (-0.0802)	0.00156 (0.0818)	0.8262	0.8200 (0.4613)	0.8325 (1.0022)	0.002	0	0
5000		10	-0.00072	-0.00223 (-0.0749)	0.000787 (0.0753)	0.8396	0.8357 (0.5708)	0.8435 (0.9888)	0	0	0
5000	20	5	-0.00107	-0.00292 (-0.0871)	0.000776 (0.0797)	0.7758	0.7684 (0.3929)	0.7833 (1.0122)	0.003	0	0
5000		10	-0.00156	-0.00333 (-0.1051)	0.000216 (0.1010)	0.7877	0.7829 (0.4964)	0.7924 (0.9840)	0	0	0
5000	25	5	-0.00206	-0.00427 (-0.1036)	0.000153 (0.1417)	0.7293	0.7211 (0.3646)	0.7376 (0.9972)	0	0	0
5000		10	-0.00184	-0.00388 (-0.1104)	0.000204 (0.1060)	0.7358	0.7303 (0.4270)	0.7414 (0.9798)	0	0	0

Table 5: Chi-Square distribution (df = 10) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.00446	-0.00588 (-0.5437)	0.0148 (0.5181)	0.8799	0.8735 (0.5349)	0.8864 (1.3569)	0.095	0.04	0.019
100		10	-0.00003	-0.0100 (-0.6252)	0.00996 (0.4548)	0.8890	0.8836 (0.5568)	0.8943 (1.3334)	0.09	0.042	0.019
100	15	5	0.000752	-0.0123 (-0.6631)	0.0138 (0.6566)	0.8215	0.8132 (0.3617)	0.8300 (1.3039)	0.083	0.039	0.023
100		10	-0.00031	-0.0127 (-0.7517)	0.0120 (0.6042)	0.8319	0.8255 (0.5343)	0.8384 (1.2531)	0.058	0.029	0.013
100	20	5	0.00451	-0.0108 (-0.8228)	0.0199 (0.8446)	0.7735	0.7643 (0.3385)	0.7829 (1.3380)	0.066	0.036	0.013
100		10	0.00838	-0.00626 (-0.7978)	0.0230 (0.7651)	0.7798	0.7724 (0.4125)	0.7873 (1.2968)	0.048	0.028	0.014
100	25	5	0.00869	-0.00876 (-0.9505)	0.0261 (0.9504)	0.7190	0.7086 (0.2951)	0.7295 (1.3630)	0.047	0.029	0.012
100		10	0.00611	-0.0110 (-0.8126)	0.0232 (0.9791)	0.7324	0.7244 (0.4027)	0.7404 (1.2504)	0.034	0.019	0.007
500	10	5	-0.00258	-0.00699 (-0.1933)	0.00183 (0.2138)	0.8843	0.8795 (0.5912)	0.8891 (1.0667)	0.033	0.003	0
500		10	-0.00123	-0.00540 (-0.2098)	0.00294 (0.2410)	0.8918	0.8884 (0.6774)	0.8952 (1.0430)	0.017	0	0
500	15	5	-0.00357	-0.00928 (-0.3098)	0.00213 (0.2686)	0.8337	0.8274 (0.4174)	0.8401 (1.0647)	0.027	0.003	0
500		10	-0.00392	-0.00940 (-0.2925)	0.00156 (0.3391)	0.8370	0.8327 (0.6065)	0.8414 (1.0089)	0.003	0	0
500	20	5	-0.00453	-0.0113 (-0.3297)	0.00223 (0.3197)	0.7831	0.7756 (0.4339)	0.7907 (1.0928)	0.011	0.004	0
500		10	-0.00288	-0.00939 (-0.3289)	0.00363 (0.3572)	0.7891	0.7838 (0.5221)	0.7945 (1.0316)	0.001	0	0
500	25	5	-0.00586	-0.0138 (-0.4864)	0.00209 (0.4398)	0.7296	0.7208 (0.3072)	0.7386 (1.1107)	0.016	0.002	0.001
500		10	-0.00360	-0.0111 (-0.4317)	0.00395 (0.4055)	0.7317	0.7255 (0.4243)	0.7380 (1.0348)	0.005	0	0

Table 5 continues: Chi-Square distribution (df = 10) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00309	-0.00621 (-0.1531)	0.000031 (0.1697)	0.8833	0.8787 (0.6112)	0.8879 (1.0240)	0.02	0	0
1000		10	-0.00411	-0.00716 (-0.00716)	-0.00716 (0.1627)	0.8892	0.8860 (0.7191)	0.8925 (1.0213)	0.007	0	0
1000	15	5	-0.00353	-0.00763 (-0.1777)	0.000578 (0.1713)	0.8256	0.8194 (0.4648)	0.8319 (1.0611)	0.014	0.001	0
1000		10	-0.00452	-0.00839 (-0.2050)	-0.00066 (0.1853)	0.8413	0.8371 (0.6436)	0.8454 (1.0439)	0.002	0	0
1000	20	5	-0.00703	-0.0118 (-0.2574)	-0.00224 (0.2212)	0.7798	0.7724 (0.3483)	0.7873 (1.0467)	0.007	0	0
1000		10	-0.00537	-0.00994 (-0.2593)	-0.00079 (0.2468)	0.7864	0.7813 (0.5182)	0.7916 (1.0139)	0.002	0	0
1000	25	5	-0.00239	-0.00780 (-0.2860)	0.00302 (0.2590)	0.7275	0.7189 (0.3438)	0.7363 (1.0722)	0.009	0.001	0
1000		10	-0.00455	-0.00974 (-0.2521)	0.000635 (0.2470)	0.7374	0.7317 (0.4443)	0.7431 (1.0242)	0.001	0	0
5000	10	5	-0.00037	-0.00178 (-0.0853)	0.00105 (0.0649)	0.8836	0.8792 (0.6381)	0.8880 (1.0080)	0.006	0	0
5000		10	-0.00078	-0.00212 (-0.0907)	0.000551 (0.0678)	0.8919	0.8890 (0.7345)	0.8948 (0.9903)	0	0	0
5000	15	5	-0.00048	-0.00230 (-0.1092)	0.00133 (0.0998)	0.8261	0.8201 (0.5225)	0.8322 (1.0030)	0.002	0	0
5000		10	-0.00076	-0.00251 (-0.0934)	0.000987 (0.0919)	0.8410	0.8371 (0.6252)	0.8449 (0.9788)	0	0	0
5000	20	5	0.000011	-0.00217 (-0.1234)	0.00219 (0.1193)	0.7754	0.7680 (0.4081)	0.7828 (1.0022)	0.001	0	0
5000		10	-0.00079	-0.00284 (-0.1008)	0.00127 (0.0915)	0.7841	0.7792 (0.5274)	0.7891 (0.9508)	0	0	0
5000	25	5	-0.00036	-0.00288 (-0.1592)	0.00216 (0.1424)	0.7297	0.7213 (0.3334)	0.7381 (0.9878)	0	0	0
5000		10	-0.00050	-0.00293 (-0.1678)	0.00192 (0.1141)	0.7402	0.7347 (0.4312)	0.7457 (0.9624)	0	0	0

Table 6: Chi-Square distribution (df = 15) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00373	-0.0163 (-0.6524)	0.00886 (0.7643)	0.8817	0.8755 (0.4950)	0.8879 (1.3013)	0.095	0.036	0.017
100		10	-0.00484	-0.0165 (-0.6387)	0.00683 (0.6914)	0.8877	0.8826 (0.6179)	0.8927 (1.2823)	0.075	0.032	0.013
100	15	5	0.00243	-0.0135 (-0.8426)	0.0184 (1.1649)	0.8270	0.8193 (0.4774)	0.8348 (1.2615)	0.073	0.035	0.013
100		10	0.00347	-0.0117 (-0.9447)	0.0186 (0.9728)	0.8300	0.8236 (0.4578)	0.8364 (1.2570)	0.05	0.016	0.007
100	20	5	-0.00426	-0.0223 (-0.9979)	0.0137 (1.1397)	0.7679	0.7587 (0.2536)	0.7771 (1.3107)	0.057	0.023	0.012
100		10	0.00390	-0.0136 (-0.9191)	0.0214 (0.9620)	0.7803	0.7730 (0.4373)	0.7877 (1.3165)	0.037	0.023	0.007
100	25	5	-0.00534	-0.0265 (-1.1997)	0.0158 (1.0845)	0.7241	0.7140 (0.2257)	0.7343 (1.3023)	0.044	0.021	0.008
100		10	0.00243	-0.0176 (-1.0253)	0.0225 (1.0118)	0.7277	0.7197 (0.3810)	0.7357 (1.1858)	0.031	0.02	0.011
500	10	5	-0.00157	-0.00709 (-0.2288)	0.00394 (0.3638)	0.8843	0.8796 (0.5699)	0.8891 (1.0408)	0.026	0	0
500		10	-0.00238	-0.00771 (-0.2302)	0.00294 (0.3167)	0.8915	0.8882 (0.6884)	0.8949 (1.0553)	0.012	0.001	0
500	15	5	-0.00231	-0.00927 (-0.2782)	0.00464 (0.3846)	0.8234	0.8171 (0.4452)	0.8297 (1.0910)	0.01	0.002	0
500		10	-0.00439	-0.0112 (-0.3102)	0.00243 (0.3817)	0.8343	0.8298 (0.5656)	0.8388 (1.0405)	0.007	0	0
500	20	5	-0.00530	-0.0138 (-0.4115)	0.00315 (0.3610)	0.7746	0.7670 (0.4213)	0.7823 (1.0659)	0.013	0.003	0
500		10	-0.00267	-0.0108 (-0.3831)	0.00543 (0.4263)	0.7836	0.7784 (0.5124)	0.7890 (0.9996)	0	0	0
500	25	5	-0.00304	-0.0127 (-0.4174)	0.00665 (0.5299)	0.7255	0.7167 (0.3218)	0.7344 (1.0238)	0.01	0	0
500		10	-0.00294	-0.0124 (-0.4475)	0.00648 (0.4841)	0.7357	0.7295 (0.4002)	0.7419 (1.0289)	0.003	0	0

Table 6 continues: Chi-Square distribution (df = 15) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00191	-0.00588 (-0.2298)	0.00205 (0.2305)	0.8823	0.8778 (0.6267)	0.8868 (1.0316)	0.008	0	0
1000		10	-0.00240	-0.00622 (-0.2062)	0.00143 (0.2064)	0.8895	0.8864 (0.6952)	0.8927 (1.0408)	0.003	0	0
1000	15	5	-0.00483	-0.00990 (-0.2507)	0.000249 (0.2845)	0.8376	0.8317 (0.5045)	0.8437 (1.0260)	0.013	0	0
1000		10	-0.00376	-0.00868 (-0.2201)	0.00116 (0.3312)	0.8385	0.8344 (0.6029)	0.8425 (1.0277)	0.003	0	0
1000	20	5	-0.00574	-0.0118 (-0.3005)	0.000276 (0.3624)	0.7843	0.7771 (0.4283)	0.7916 (1.0560)	0.006	0.001	0
1000		10	-0.00426	-0.00996 (-0.2753)	0.00143 (0.3472)	0.7831	0.7779 (0.4841)	0.7883 (0.9800)	0	0	0
1000	25	5	-0.00205	-0.00882 (-0.3557)	0.00472 (0.4430)	0.7278	0.7195 (0.3319)	0.7361 (1.0321)	0.005	0	0
1000		10	-0.00324	-0.00987 (-0.3641)	0.00339 (0.3719)	0.7345	0.7287 (0.3748)	0.7403 (0.9937)	0	0	0
5000	10	5	-0.00086	-0.00260 (-0.1020)	0.000883 (0.0891)	0.8865	0.8822 (0.6092)	0.8909 (1.0094)	0.004	0	0
5000		10	-0.00023	-0.00188 (-0.0704)	0.00142 (0.0860)	0.8917	0.8889 (0.7050)	0.8945 (0.9909)	0	0	0
5000	15	5	-0.00057	-0.00277 (-0.1241)	0.00162 (0.1033)	0.8334	0.8276 (0.4967)	0.8393 (1.0044)	0.001	0	0
5000		10	-0.00080	-0.00294 (-0.1232)	0.00135 (0.1056)	0.8385	0.8347 (0.6406)	0.8424 (0.9986)	0	0	0
5000	20	5	-0.00013	-0.00281 (-0.1403)	0.00256 (0.1460)	0.7799	0.7725 (0.3161)	0.7874 (0.9960)	0	0	0
5000		10	-0.00123	-0.00383 (-0.1586)	0.00137 (0.1321)	0.7871	0.7823 (0.5667)	0.7919 (0.9742)	0	0	0
5000	25	5	-0.00073	-0.00389 (-0.1755)	0.00242 (0.1870)	0.7313	0.7230 (0.3682)	0.7396 (1.0018)	0.001	0	0
5000		10	-0.00011	-0.00315 (-0.1645)	0.00292 (0.1564)	0.7347	0.7292 (0.4527)	0.7402 (0.9543)	0	0	0

Table 7: Chi-Square distribution (df = 20) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.00391	-0.0106 (-0.7782)	0.0185 (0.7539)	0.8861	0.8800 (0.5549)	0.8923 (1.2488)	0.111	0.036	0.013
100		10	0.00239	-0.0120 (-0.7513)	0.0168 (0.7476)	0.8922	0.8872 (0.6561)	0.8971 (1.2474)	0.082	0.029	0.01
100	15	5	-0.00064	-0.0191 (-1.1030)	0.0178 (0.9754)	0.8288	0.8209 (0.4147)	0.8368 (1.2684)	0.079	0.037	0.018
100		10	-0.00504	-0.0225 (-0.9602)	0.0124 (0.8423)	0.8337	0.8277 (0.4500)	0.8397 (1.2093)	0.044	0.018	0.008
100	20	5	-0.00118	-0.0232 (-1.4589)	0.0209 (1.0493)	0.7713	0.7621 (0.3307)	0.7805 (1.3450)	0.058	0.03	0.01
100		10	-0.00154	-0.0224 (-1.2794)	0.0193 (0.9420)	0.7773	0.7704 (0.3796)	0.7843 (1.2063)	0.033	0.013	0.006
100	25	5	-0.00624	-0.0315 (-1.3455)	0.0190 (1.2246)	0.7216	0.7118 (0.2718)	0.7315 (1.3037)	0.048	0.024	0.014
100		10	-0.00650	-0.0311 (-1.3475)	0.0181 (1.2049)	0.7237	0.7160 (0.3596)	0.7315 (1.3076)	0.019	0.009	0.004
500	10	5	-0.00174	-0.00826 (-0.3325)	0.00477 (0.4099)	0.8844	0.8796 (0.5481)	0.8892 (1.0360)	0.027	0	0
500		10	-0.00347	-0.00980 (-0.3707)	0.00287 (0.3391)	0.8919	0.8886 (0.6812)	0.8952 (1.0531)	0.013	0.001	0
500	15	5	-0.00116	-0.00919 (-0.4340)	0.00686 (0.4164)	0.8298	0.8235 (0.3918)	0.8362 (1.0620)	0.018	0.001	0
500		10	-0.00026	-0.00797 (-0.4188)	0.00746 (0.4086)	0.8365	0.8321 (0.5096)	0.8410 (1.0038)	0.003	0	0
500	20	5	0.00242	-0.00708 (-0.5219)	0.0119 (0.4785)	0.7752	0.7676 (0.4450)	0.7829 (1.0617)	0.017	0.001	0
500		10	0.00326	-0.00615 (-0.5140)	0.0127 (0.4976)	0.7839	0.7786 (0.5223)	0.7893 (1.0132)	0.001	0	0
500	25	5	0.00177	-0.00915 (-0.5730)	0.0127 (0.5174)	0.7255	0.7167 (0.3020)	0.7345 (1.1184)	0.015	0.002	0.002
500		10	0.00602	-0.00468 (-0.5666)	0.0167 (0.5526)	0.7327	0.7269 (0.4748)	0.7385 (1.0399)	0.001	0	0

Table 7 continues: Chi-Square distribution (df = 20) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.000167	-0.00428 (-0.2459)	0.00461 (0.2413)	0.8821	0.8774 (0.6009)	0.8868 (1.0299)	0.017	0	0
1000		10	-0.00091	-0.00510 (-0.1871)	0.00328 (0.2283)	0.8931	0.8902 (0.7363)	0.8960 (1.0241)	0.006	0	0
1000	15	5	-0.00079	-0.00643 (-0.2439)	0.00485 (0.2726)	0.8349	0.8288 (0.4867)	0.8410 (1.0277)	0.005	0	0
1000		10	0.000175	-0.00522 (-0.2433)	0.00557 (0.2673)	0.8337	0.8294 (0.6069)	0.8380 (1.0155)	0.002	0	0
1000	20	5	-0.00154	-0.00817 (-0.3207)	0.00509 (0.3626)	0.7769	0.7695 (0.4459)	0.7844 (1.0563)	0.006	0.001	0
1000		10	0.000919	-0.00543 (-0.2776)	0.00727 (0.3615)	0.7853	0.7801 (0.5212)	0.7906 (0.9683)	0	0	0
1000	25	5	0.00119	-0.00686 (-0.3972)	0.00923 (0.4161)	0.7304	0.7219 (0.2291)	0.7389 (1.0819)	0.002	0.001	0
1000		10	0.000992	-0.00652 (-0.3459)	0.00850 (0.3726)	0.7357	0.7299 (0.4721)	0.7416 (0.9935)	0	0	0
5000	10	5	-0.00049	-0.00247 (-0.1352)	0.00149 (0.1105)	0.8889	0.8847 (0.5852)	0.8931 (1.0067)	0.003	0	0
5000		10	-0.00113	-0.00305 (-0.0964)	0.000786 (0.1170)	0.8921	0.8893 (0.7330)	0.8949 (0.9927)	0	0	0
5000	15	5	-0.00187	-0.00436 (-0.1524)	0.000629 (0.1130)	0.8336	0.8277 (0.5549)	0.8394 (1.0128)	0.002	0	0
5000		10	-0.00198	-0.00437 (-0.1233)	0.000401 (0.1191)	0.8410	0.8370 (0.6230)	0.8450 (0.9789)	0	0	0
5000	20	5	-0.00036	-0.00331 (-0.1644)	0.00259 (0.1712)	0.7746	0.7674 (0.3644)	0.7819 (0.9933)	0	0	0
5000		10	-0.00187	-0.00474 (-0.1696)	0.000993 (0.1279)	0.7904	0.7854 (0.5093)	0.7953 (0.9699)	0	0	0
5000	25	5	-0.00026	-0.00379 (-0.1755)	0.00328 (0.1943)	0.7213	0.7128 (0.3458)	0.7298 (1.0081)	0.001	0	0
5000		10	-0.00207	-0.00545 (-0.1642)	0.00131 (0.1690)	0.7415	0.7360 (0.4831)	0.7471 (0.9582)	0	0	0

Table 8: Chi-Square distribution (df = 25) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	0.00394	-0.0125 (-1.0046)	0.0204 (1.0448)	0.8834	0.8775 (0.5473)	0.8892 (1.2765)	0.093	0.037	0.012
100		10	0.00439	-0.0115 (-0.9103)	0.0203 (0.9154)	0.8896	0.8848 (0.5876)	0.8945 (1.2412)	0.067	0.021	0.011
100	15	5	0.0109	-0.00955 (-1.0056)	0.0313 (1.2847)	0.8279	0.8200 (0.4024)	0.8358 (1.3059)	0.067	0.032	0.012
100		10	0.0102	-0.00934 (-1.0786)	0.0298 (1.1050)	0.8299	0.8238 (0.5386)	0.8360 (1.2635)	0.033	0.016	0.007
100	20	5	0.00851	-0.0156 (-1.5416)	0.0326 (1.3789)	0.7680	0.7587 (0.3615)	0.7773 (1.5025)	0.05	0.025	0.01
100		10	0.0176	-0.00525 (-1.1188)	0.0404 (1.2079)	0.7831	0.7758 (0.3865)	0.7904 (1.2724)	0.034	0.02	0.009
100	25	5	0.0218	-0.00624 (-1.4778)	0.0499 (1.6063)	0.7238	0.7134 (0.2921)	0.7343 (1.4293)	0.052	0.031	0.015
100		10	0.0159	-0.0109 (-1.3157)	0.0427 (1.8870)	0.7222	0.7140 (0.3241)	0.7304 (1.2375)	0.026	0.018	0.007
500	10	5	0.00121	-0.00614 (-0.3308)	0.00857 (0.3870)	0.8828	0.8779 (0.5440)	0.8877 (1.0620)	0.025	0.002	0
500		10	0.00274	-0.00420 (-0.3334)	0.00968 (0.3502)	0.8901	0.8868 (0.6951)	0.8935 (1.0433)	0.015	0	0
500	15	5	0.00789	-0.00121 (-0.4302)	0.0170 (0.4650)	0.8296	0.8235 (0.5272)	0.8357 (1.1092)	0.012	0.002	0.001
500		10	0.00359	-0.00517 (-0.4192)	0.0124 (0.4600)	0.8344	0.8299 (0.5657)	0.8388 (1.0105)	0.004	0	0
500	20	5	0.00703	-0.00384 (-0.5794)	0.0179 (0.5187)	0.7792	0.7717 (0.2959)	0.7867 (1.0444)	0.01	0	0
500		10	0.00489	-0.00545 (-0.6575)	0.0152 (0.5726)	0.7876	0.7823 (0.5294)	0.7929 (1.0328)	0.004	0	0
500	25	5	0.00619	-0.00612 (-0.6672)	0.0185 (0.5257)	0.7140	0.7050 (0.2832)	0.7231 (1.0774)	0.01	0.003	0
500		10	0.00501	-0.00685 (-0.6956)	0.0169 (0.6072)	0.7369	0.7307 (0.4234)	0.7431 (1.0123)	0.003	0	0

Table 8 continues: Chi-Square distribution (df = 25) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00096	-0.00604 (-0.2713)	0.00412 (0.2751)	0.8889	0.8846 (0.6443)	0.8932 (1.0549)	0.011	0.001	0
1000		10	-0.00186	-0.00675 (-0.2342)	0.00304 (0.2506)	0.8903	0.8873 (0.7179)	0.8934 (1.0078)	0.003	0	0
1000	15	5	0.00341	-0.00292 (-0.2895)	0.00974 (0.3585)	0.8300	0.8239 (0.4863)	0.8362 (1.0340)	0.004	0	0
1000		10	0.00285	-0.00338 (-0.2928)	0.00909 (0.3144)	0.8382	0.8341 (0.6142)	0.8423 (1.0105)	0.001	0	0
1000	20	5	0.00156	-0.00601 (-0.3595)	0.00914 (0.3955)	0.7803	0.7728 (0.3514)	0.7878 (1.0184)	0.005	0	0
1000		10	0.00398	-0.00321 (-0.3415)	0.0112 (0.3746)	0.7866	0.7818 (0.5051)	0.7914 (0.9607)	0	0	0
1000	25	5	0.00457	-0.00398 (-0.4825)	0.0131 (0.5116)	0.7267	0.7183 (0.3199)	0.7352 (1.0463)	0.004	0	0
1000		10	0.00743	-0.00081 (-0.4401)	0.0157 (0.4029)	0.7363	0.7305 (0.4866)	0.7421 (1.0081)	0.001	0	0
5000	10	5	0.000568	-0.00170 (-0.1225)	0.00284 (0.1175)	0.8810	0.8763 (0.5703)	0.8858 (1.0043)	0.004	0	0
5000		10	0.000604	-0.00154 (-0.1175)	0.00275 (0.1029)	0.8892	0.8863 (0.7174)	0.8921 (0.9813)	0	0	0
5000	15	5	0.00110	-0.00181 (-0.1588)	0.00402 (0.1225)	0.8336	0.8278 (0.4935)	0.8395 (1.0005)	0.001	0	0
5000		10	0.000900	-0.00188 (-0.1561)	0.00367 (0.1190)	0.8404	0.8365 (0.5720)	0.8444 (0.9819)	0	0	0
5000	20	5	0.00176	-0.00162 (-0.1647)	0.00515 (0.1822)	0.7765	0.7693 (0.4120)	0.7839 (1.0120)	0.002	0	0
5000		10	0.00128	-0.00194 (-0.1623)	0.00449 (0.1661)	0.7860	0.7810 (0.5108)	0.7910 (0.9679)	0	0	0
5000	25	5	0.00332	-0.00052 (-0.1914)	0.00716 (0.2014)	0.7307	0.7224 (0.3336)	0.7392 (1.0060)	0.002	0	0
5000		10	0.00286	-0.00081 (-0.1740)	0.00654 (0.1748)	0.7383	0.7329 (0.4370)	0.7438 (0.9630)	0	0	0

Table 9: Chi-Square distribution (df = 30) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.00166	-0.0198 (-0.9737)	0.0165 (1.1006)	0.8757	0.8693 (0.3571)	0.8821 (1.2760)	0.099	0.032	0.01
100		10	0.000708	-0.0167 (-0.8405)	0.0182 (0.9387)	0.8862	0.8813 (0.6233)	0.8911 (1.1664)	0.061	0.02	0.007
100	15	5	0.0110	-0.0121 (-1.1451)	0.0341 (1.3847)	0.8211	0.8135 (0.4019)	0.8288 (1.2977)	0.061	0.027	0.012
100		10	0.00178	-0.0206 (-1.3097)	0.0242 (1.3932)	0.8287	0.8226 (0.5232)	0.8348 (1.2678)	0.036	0.017	0.006
100	20	5	0.00647	-0.0212 (-1.3596)	0.0342 (1.5761)	0.7676	0.7587 (0.2828)	0.7767 (1.3365)	0.047	0.022	0.008
100		10	0.00762	-0.0189 (-1.2136)	0.0342 (1.5830)	0.7734	0.7664 (0.3836)	0.7804 (1.2343)	0.028	0.009	0.004
100	25	5	-0.00314	-0.0351 (-1.9478)	0.0288 (1.5233)	0.7168	0.7071 (0.2528)	0.7267 (1.3334)	0.03	0.019	0.008
100		10	-0.00510	-0.0350 (-1.6143)	0.0248 (1.6076)	0.7217	0.7139 (0.3768)	0.7296 (1.1843)	0.023	0.011	0.004
500	10	5	-0.00140	-0.00912 (-0.3525)	0.00633 (0.4066)	0.8876	0.8829 (0.6007)	0.8923 (1.0427)	0.032	0	0
500		10	-0.00050	-0.00791 (-0.3158)	0.00691 (0.3896)	0.8893	0.8860 (0.6941)	0.8927 (1.0181)	0.007	0	0
500	15	5	0.00190	-0.00809 (-0.5005)	0.0119 (0.6119)	0.8281	0.8218 (0.4193)	0.8345 (1.0546)	0.015	0.001	0
500		10	0.00670	-0.00273 (-0.4782)	0.0161 (0.4612)	0.8402	0.8358 (0.5749)	0.8446 (1.0642)	0.005	0.001	0
500	20	5	0.00392	-0.00789 (-0.5520)	0.0157 (0.5965)	0.7846	0.7773 (0.3962)	0.7920 (1.0947)	0.013	0.004	0
500		10	0.00118	-0.00995 (-0.5338)	0.0123 (0.6074)	0.7844	0.7792 (0.4797)	0.7896 (0.9897)	0	0	0
500	25	5	0.000270	-0.0129 (-0.5823)	0.0135 (0.6850)	0.7216	0.7128 (0.3232)	0.7304 (1.0582)	0.009	0.001	0
500		10	0.00273	-0.0100 (-0.5747)	0.0155 (0.7189)	0.7335	0.7277 (0.4127)	0.7394 (0.9862)	0	0	0

Table 9 continues: Chi-Square distribution (df = 30) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	0.00182	-0.00372 (-0.2497)	0.00737 (0.3676)	0.8808	0.8762 (0.6136)	0.8854 (1.0403)	0.016	0	0
1000		10	0.00229	-0.00300 (-0.2353)	0.00758 (0.3486)	0.8923	0.8893 (0.7149)	0.8954 (1.0123)	0.004	0	0
1000	15	5	0.0148	-0.0129 (-1.4165)	0.0424 (1.4551)	0.4217	0.4186 (0.2674)	0.4249 (0.6302)	0	0	0
1000		10	0.00401	-0.00257 (-0.3016)	0.0106 (0.4021)	0.8422	0.8381 (0.5413)	0.8463 (0.9891)	0	0	0
1000	20	5	0.00130	-0.00678 (-0.3787)	0.00938 (0.4326)	0.7777	0.7702 (0.4037)	0.7854 (1.0413)	0.006	0	0
1000		10	0.000740	-0.00710 (-0.3885)	0.00858 (0.3862)	0.7889	0.7839 (0.4995)	0.7940 (0.9842)	0	0	0
1000	25	5	0.000674	-0.00883 (-0.5465)	0.0102 (0.4112)	0.7290	0.7207 (0.3388)	0.7374 (1.0675)	0.005	0.001	0
1000		10	0.00132	-0.00775 (-0.4242)	0.0104 (0.4159)	0.7365	0.7309 (0.4801)	0.7421 (1.0019)	0.001	0	0
5000	10	5	-0.00083	-0.00341 (-0.1463)	0.00176 (0.1134)	0.8865	0.8821 (0.6336)	0.8909 (1.0128)	0.004	0	0
5000		10	-0.00053	-0.00302 (-0.1299)	0.00195 (0.1243)	0.8888	0.8860 (0.7291)	0.8917 (0.9922)	0	0	0
5000	15	5	0.000193	-0.00299 (-0.1386)	0.00338 (0.1351)	0.8329	0.8270 (0.5054)	0.8389 (1.0187)	0.002	0	0
5000		10	0.000237	-0.00285 (-0.1798)	0.00333 (0.1377)	0.8401	0.8362 (0.6065)	0.8441 (0.9817)	0	0	0
5000	20	5	-0.00060	-0.00437 (-0.1801)	0.00316 (0.1902)	0.7674	0.7596 (0.3904)	0.7752 (1.0042)	0.001	0	0
5000		10	-0.00059	-0.00425 (-0.1859)	0.00307 (0.1629)	0.7893	0.7844 (0.4919)	0.7943 (0.9724)	0	0	0
5000	25	5	-0.00136	-0.00571 (-0.2180)	0.00299 (0.1855)	0.7236	0.7155 (0.3589)	0.7318 (1.0049)	0.001	0	0
5000		10	0.000385	-0.00394 (-0.2915)	0.00471 (0.2197)	0.7308	0.7224 (0.3415)	0.7393 (1.0053)	0.001	0	0

Table 10: Chi-Square distribution (df = 40) with Non-Monotonic Missing data Pattern

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
100	10	5	-0.0103	-0.0308 (-1.1079)	0.0102 (1.1663)	0.8826	0.8765 (0.5194)	0.8887 (1.2463)	0.1	0.036	0.004
100		10	-0.00639	-0.0257 (-1.0271)	0.0129 (0.9829)	0.8854	0.8805 (0.5799)	0.8903 (1.1717)	0.06	0.019	0.003
100	15	5	-0.0208	-0.0467 (-1.4390)	0.00503 (1.3401)	0.8181	0.8104 (0.4235)	0.8260 (1.2334)	0.057	0.023	0.008
100		10	-0.0145	-0.0392 (-1.2448)	0.0102 (1.2999)	0.8254	0.8194 (0.4526)	0.8315 (1.1905)	0.047	0.014	0.003
100	20	5	-0.0219	-0.0516 (-1.2495)	0.00770 (1.5148)	0.7630	0.7543 (0.3106)	0.7719 (1.2632)	0.043	0.018	0.006
100		10	-0.0218	-0.0507 (-1.7745)	0.00714 (1.4194)	0.7747	0.7677 (0.4129)	0.7817 (1.2005)	0.017	0.006	0.003
100	25	5	-0.0293	-0.0650 (-1.8656)	0.00632 (2.1202)	0.7084	0.6984 (0.3153)	0.7185 (1.1952)	0.041	0.02	0.008
100		10	-0.0125	-0.0466 (-1.9053)	0.0216 (1.8007)	0.7212	0.7138 (0.3237)	0.7286 (1.0901)	0.014	0.006	0
500	10	5	-0.00328	-0.0121 (-0.3921)	0.00555 (0.4092)	0.8817	0.8769 (0.6244)	0.8866 (1.0510)	0.031	0.001	0
500		10	-0.00133	-0.00979 (-0.4146)	0.00714 (0.5299)	0.8934	0.8900 (0.6551)	0.8968 (1.0396)	0.008	0	0
500	15	5	-0.00297	-0.0145 (-0.5540)	0.00855 (0.5873)	0.8231	0.8167 (0.4666)	0.8296 (1.0651)	0.014	0.001	0
500		10	-0.00594	-0.0168 (-0.5525)	0.00491 (0.6072)	0.8385	0.8341 (0.5904)	0.8429 (1.0043)	0.001	0	0
500	20	5	-0.00919	-0.0228 (-0.8290)	0.00443 (0.6692)	0.7747	0.7669 (0.3891)	0.7826 (1.0480)	0.011	0	0
500		10	-0.00170	-0.0143 (-0.6467)	0.0109 (0.7081)	0.7863	0.7812 (0.4935)	0.7915 (1.0114)	0.001	0	0
500	25	5	-0.00169	-0.0172 (-0.7991)	0.0138 (0.8032)	0.7313	0.7226 (0.3345)	0.7401 (1.0815)	0.009	0.002	0
500		10	-0.00393	-0.0187 (-0.9864)	0.0108 (0.7401)	0.7358	0.7298 (0.4418)	0.7419 (0.9871)	0	0	0

Table 10 continues: Chi-Square distribution (df = 40) with Non-Monotonic Missing data Pattern:

Results of 1000 Simulations from Normal MAR Mechanism											
N	% Miss	Impute	Average Diff. = True-Imputed	Lower Limit (Minimum)	Upper Limit (Maximum)	Geom. Ave. Ratio of Variance = True/Imputed	Lower Limit (Minimum)	Upper Limit (maximum)	% Count (>1.0)	% Count (>1.05)	% Count (>1.1)
1000	10	5	-0.00538	-0.0119 (-0.3191)	0.00110 (0.3070)	0.8842	0.8799 (0.6179)	0.8886 (1.0268)	0.011	0	0
1000		10	-0.00281	-0.00894 (-0.2778)	0.00331 (0.3139)	0.8931	0.8901 (0.7404)	0.8961 (1.0177)	0.003	0	0
1000	15	5	-0.00165	-0.00970 (-0.3512)	0.00641 (0.4017)	0.8329	0.8269 (0.4889)	0.8391 (1.0361)	0.009	0	0
1000		10	-0.00464	-0.0123 (-0.3779)	0.00303 (0.3363)	0.8448	0.8405 (0.5045)	0.8490 (0.9907)	0	0	0
1000	20	5	-0.00660	-0.0163 (-0.4827)	0.00308 (0.4567)	0.7767	0.7694 (0.4530)	0.7841 (1.0503)	0.003	0.001	0
1000		10	-0.00487	-0.0138 (-0.3798)	0.00405 (0.4157)	0.7895	0.7845 (0.5236)	0.7944 (0.9835)	0	0	0
1000	25	5	-0.00305	-0.0138 (-0.5264)	0.00767 (0.5984)	0.7152	0.7067 (0.3427)	0.7238 (1.0117)	0.001	0	0
1000		10	-0.00281	-0.0131 (-0.5163)	0.00753 (0.6099)	0.7401	0.7346 (0.4319)	0.7456 (0.9594)	0	0	0
5000	10	5	-0.00065	-0.00364 (-0.1787)	0.00234 (0.1678)	0.8841	0.8795 (0.6156)	0.8887 (1.0078)	0.003	0	0
5000		10	-0.00140	-0.00422 (-0.1421)	0.00142 (0.1499)	0.8922	0.8894 (0.7473)	0.8950 (0.9948)	0	0	0
5000	15	5	-0.00307	-0.00667 (-0.1603)	0.000532 (0.1925)	0.8295	0.8234 (0.4886)	0.8356 (1.0146)	0.006	0	0
5000		10	-0.00190	-0.00535 (-0.1674)	0.00156 (0.1858)	0.8376	0.8336 (0.5759)	0.8416 (0.9798)	0	0	0
5000	20	5	-0.00427	-0.00850 (-0.2233)	-0.00003 (0.1845)	0.7709	0.7637 (0.4218)	0.7783 (1.0162)	0.003	0	0
5000		10	-0.00324	-0.00732 (-0.2057)	0.000835 (0.2082)	0.7832	0.7782 (0.5217)	0.7883 (0.9698)	0	0	0
5000	25	5	-0.00597	-0.0109 (-0.2739)	-0.00106 (0.2096)	0.7226	0.7143 (0.3589)	0.7311 (0.9914)	0	0	0
5000		10	-0.00446	-0.00915 (-0.2611)	0.000221 (0.1972)	0.7328	0.7272 (0.4530)	0.7386 (0.9599)	0	0	0

APPENDIX G t-DISTRIBUTION AND CHI-SQUARE DISTRIBUTION WITH 10 AND 30 DFs

Table 1: Significance P-values for t-Distribution with 10 df with Non-Monotonic Missing data Pattern:

N	% Miss	t dist with 10 df Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	50	46	77	62	46 (49)
	15	50	46	91	93	50 (48)
	20	50	51	107	109	62 (61)
	25	50	54	111	118	57 (48)
500	10	49	48	71	73	49 (42)
	15	49	50	82	84	50 (51)
	20	49	51	91	113	52 (50)
	25	49	46	107	127	59 (49)
1000	10	39	39	60	64	40 (44)
	15	39	40	73	68	32 (37)
	20	39	47	93	99	51 (46)
	25	39	48	110	112	48 (49)
5000	10	56	51	71	75	45 (50)
	15	56	47	82	84	38 (44)
	20	56	45	101	113	49 (46)
	25	56	41	113	115	46 (40)

Table 2: Significance P-values for t-Distribution with 30 df with Non-Monotonic Missing data Pattern:

N	% Miss	t dist with 30 df Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	49	50	73	77	57 (54)
	15	49	52	86	85	52 (52)
	20	49	60	105	115	59 (58)
	25	49	59	119	135	59 (60)
500	10	48	53	70	77	56 (50)
	15	48	44	84	104	51 (45)
	20	48	49	101	106	63 (61)
	25	48	54	120	133	61 (53)
1000	10	55	50	73	75	51 (48)
	15	55	46	84	85	48 (53)
	20	55	49	89	105	49 (51)
	25	55	45	116	109	55 (46)
5000	10	52	47	71	75	51 (48)
	15	52	51	89	84	50 (50)
	20	52	48	105	105	60 (54)
	25	52	53	103	128	53 (49)

Table 3: Significance P-values for Chi-Square Distribution with 10 df with Non-Monotonic Missing data Pattern:

N	% Miss	Chi-Sqr dist with 10 df Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	49	54	74	71	50 (53)
	15	49	54	87	83	53 (53)
	20	49	54	92	100	56 (53)
	25	49	57	103	122	56 (59)
500	10	46	45	66	58	43 (41)
	15	46	35	85	71	46 (38)
	20	46	39	93	93	36 (36)
	25	46	33	106	110	33 (33)
1000	10	47	44	70	73	48 (50)
	15	47	48	87	99	50 (46)
	20	47	47	95	107	45 (48)
	25	47	38	113	112	50 (42)
5000	10	60	49	75	76	50 (51)
	15	60	53	102	99	49 (50)
	20	60	53	121	122	56 (63)
	25	60	55	125	131	64 (59)

Table 4: Significance P-values for Chi-Square Distribution with 30 df with Non-Monotonic Missing data Pattern:

N	% Miss	Chi-Sqr dist with 30 df Full Data	Available Data	Mean Substitution	Single Regression Imputation	Multiple Imputation nimpute=5 (nimpute=10)
100	10	47	49	72	66	46 (52)
	15	47	45	83	76	45 (60)
	20	47	43	96	87	48 (41)
	25	47	47	106	117	45 (49)
500	10	48	45	67	68	48 (47)
	15	48	43	73	96	48 (45)
	20	48	41	90	92	49 (45)
	25	48	44	105	113	53 (53)
1000	10	45	41	69	69	47 (41)
	15	45	42	87	98	48 (46)
	20	45	48	105	107	48 (40)
	25	45	46	121	117	50 (49)
5000	10	49	52	75	82	85 (48)
	15	49	49	85	92	48 (50)
	20	49	52	103	111	46 (50)
	25	49	54	124	135	57 (54)

Table 5: Sensitivity and Specificity for t-Distribution with 10 df with Non-Monotonic Missing data Pattern:

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	98.32	73.91	99.02	53.25	98.40	56.45	98.43 (98.11)	76.09 (65.31)
	15	98.80	63.04	99.12	46.15	98.02	34.41	97.68 (97.79)	56.00 (60.42)
	20	98.10	62.75	99.55	42.99	97.87	28.44	97.87 (97.87)	48.39 (49.18)
	25	97.99	57.41	99.44	40.54	97.96	27.12	97.88 (97.48)	52.63 (54.17)
500	10	98.42	70.83	99.25	59.15	98.49	47.95	97.90 (97.91)	59.18 (69.05)
	15	97.89	58.00	99.24	51.22	97.71	33.33	98.11 (97.79)	62.00 (54.90)
	20	97.79	54.90	99.01	43.96	98.31	30.09	97.78 (97.68)	53.85 (54.00)
	25	97.48	54.35	99.33	40.19	98.05	25.20	97.98 (97.27)	50.85 (46.94)
1000	10	98.54	64.10	99.36	55.00	99.04	46.88	98.75 (98.54)	67.50 (56.82)
	15	98.33	57.50	99.35	45.21	98.50	36.76	98.35 (98.34)	71.88 (62.16)
	20	98.64	55.32	99.45	36.56	98.89	29.29	98.42 (98.53)	47.06 (54.35)
	25	98.21	45.83	99.33	30.00	98.87	25.89	98.11 (98.21)	43.75 (44.90)
5000	10	98.42	80.39	99.25	69.01	98.59	57.33	97.70 (98.32)	75.56 (80.00)
	15	97.69	72.34	99.35	60.98	98.25	47.62	96.57 (97.28)	60.53 (68.18)
	20	97.59	73.33	99.33	49.50	98.31	36.28	97.48 (97.48)	65.31 (69.57)
	25	96.87	63.41	99.10	42.48	96.95	25.22	96.86 (69.56)	56.52 (57.50)

Table 6: Sensitivity and Specificity for t-Distribution with 30 df with Non-Monotonic Missing data Pattern:

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	98.11	62.00	99.14	56.16	98.27	42.86	98.52 (98.10)	61.40 (57.41)
	15	97.78	53.85	98.80	44.19	97.60	31.76	97.57 (97.68)	50.00 (51.92)
	20	97.55	43.33	98.88	37.14	97.97	26.96	97.34 (97.24)	40.68 (39.66)
	25	97.34	40.68	99.09	34.45	97.92	22.96	97.45 (97.23)	42.37 (38.33)
500	10	98.42	62.26	99.25	58.57	98.81	48.05	98.41 (98.42)	58.93 (66.00)
	15	98.12	68.18	99.34	50.00	98.66	34.62	98.10 (98.22)	58.82 (68.89)
	20	97.69	53.06	99.33	41.58	97.87	27.36	97.76 (98.19)	42.86 (50.82)
	25	97.57	46.30	99.09	33.33	98.15	24.06	97.66 (97.47)	42.62 (45.28)
1000	10	98.32	78.00	99.24	65.75	98.38	53.33	98.00 (98.11)	70.59 (77.08)
	15	97.90	76.09	99.24	57.14	98.36	47.06	97.79 (98.31)	70.83 (73.58)
	20	97.48	63.27	99.23	53.93	98.66	40.95	97.37 (97.58)	61.22 (62.75)
	25	97.49	68.89	99.32	42.24	97.53	30.28	97.25 (97.38)	52.73 (65.22)
5000	10	98.11	72.34	99.25	63.38	98.49	50.67	98.21 (98.11)	68.63 (70.83)
	15	98.10	66.67	99.12	49.44	97.82	38.12	97.79 (97.68)	62.00 (60.00)
	20	97.58	60.42	98.99	40395	97.43	27.62	97.77 (97.57)	51.67 (53.70)
	25	97.68	56.60	98.89	40.78	98.05	27.34	97.25 (97.68)	49.06 (55.10)

Table 7: Sensitivity and Specificity for Chi-square with 10 df with Non-Monotonic Missing data Pattern:

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	98.52	68.63	99.68	62.16	98.49	49.30	98.42 (98.63)	68.00 (67.92)
	15	98.41	62.96	99.56	51.72	98.69	44.58	98.63 (98.63)	67.92 (67.92)
	20	98.31	61.11	99.67	50.00	98.56	36.00	98.31 (98.31)	58.93 (62.26)
	25	98.30	57.89	99.55	43.69	98.75	31.15	97.78 (97.98)	50.00 (50.85)
500	10	98.43	68.89	99.46	62.12	98.41	53.45	98.01 (98.33)	62.79 (73.17)
	15	97.51	62.86	99.34	47.06	97.95	38.03	98.01 (97.71)	58.70 (63.16)
	20	97.50	56.41	99.23	41.94	98.02	30.11	97.41 (97.30)	58.33 (55.56)
	25	97.21	57.58	98.99	34.91	97.30	20.00	96.90 (96.79)	48.48 (45.45)
1000	10	98.01	63.64	98.92	52.86	98.38	43.84	98.11 (98.32)	60.42 (62.00)
	15	98.21	62.50	99.01	43.68	98.34	32.32	98.11 (98.11)	58.00 (63.04)
	20	97.59	51.06	99.01	40.00	97.87	26.17	97.38 (97.69)	48.89 (52.08)
	25	97.19	52.63	98.99	33.63	97.86	25.00	96.97 (97.18)	42.86 (47.62)
5000	10	97.69	77.55	98.81	65.33	97.84	52.63	97.58 (97.89)	74.00 (78.43)
	15	97.04	60.38	99.44	53.92	97.89	41.41	96.95 (96.74)	63.27 (58.00)
	20	96.62	52.83	99.43	45.45	97.84	33.61	96.19 (97.12)	42.86 (52.38)
	25	96.40	47.27	98.86	40.00	97.12	26.72	96.69 (96.71)	45.31 (49.15)

Table 8: Sensitivity and Specificity for Chi-square with 30 df with Non-Monotonic Missing data Pattern:

N	% Miss	Available Data		Mean Substitution		Single Regression		Multiple Imputation nimpute=5 (nimpute=10)	
		Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
100	10	98.53	67.35	99.35	56.94	98.29	46.97	98.53 (98.84)	71.74 (69.23)
	15	98.22	66.67	98.91	44.58	98.38	42.11	98.12 (98.01)	64.44 (60.87)
	20	97.91	62.79	99.12	40.63	98.36	36.78	97.90 (97.60)	56.25 (58.54)
	25	97.69	53.19	99.11	36.79	98.19	26.50	97.70 (97.79)	55.56 (53.06)
500	10	98.43	73.33	99.68	67.16	98.50	50.00	98.42 (98.22)	68.75 (65.96)
	15	98.01	67.44	99.03	53.42	98.45	35.42	97.79 (97.91)	56.25 (62.22)
	20	97.91	68.29	99.23	45.56	98.35	35.87	98.00 (97.80)	59.18 (60.00)
	25	98.01	65.91	99.33	40.00	98.42	30.09	97.57 (97.89)	47.17 (52.83)
1000	10	98.44	73.17	99.46	57.97	99.24	51.35	98.74 (98.12)	70.21 (65.85)
	15	98.64	76.19	99.78	49.43	98.78	34.69	98.63 (98.32)	66.67 (63.04)
	20	98.42	62.50	99.78	40.95	98.77	31.78	98.00 (97.60)	54.17 (55.00)
	25	98.22	60.87	99.77	35.54	98.30	25.64	97.58 (97.79)	44.00 (48.98)
5000	10	98.31	63.46	99.46	58.67	98.37	41.46	98.62 (98.32)	62.07 (68.75)
	15	98.00	61.22	99.34	50.59	98.46	38.04	97.90 (97.89)	60.42 (58.00)
	20	98.00	57.69	99.44	42.72	98.31	30.63	97.59 (97.68)	56.52 (54.00)
	25	97.25	42.59	99.54	36.29	98.27	25.19	97.14 (97.57)	38.60 (48.15)

APPENDIX H PERCENT COUNT COMPARISON BETWEEN DIFFERENT DISTRIBUTIONS

% Count ≥1.0 for Cauchy distribution, t-distribution, and Normal distribution

N=100:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.154	0.143	0.099	0.088	0.097	0.092	0.087	0.091	0.092	0.101	0.066
	10	0.151	0.126	0.101	0.08	0.08	0.081	0.068	0.058	0.067	0.069	0.06
15	5	0.164	0.135	0.101	0.064	0.077	0.08	0.066	0.055	0.053	0.071	0.044
	10	0.171	0.138	0.086	0.056	0.056	0.056	0.049	0.032	0.037	0.044	0.019
20	5	0.176	0.157	0.097	0.062	0.054	0.042	0.046	0.04	0.047	0.049	0.034
	10	0.183	0.153	0.07	0.047	0.046	0.029	0.047	0.028	0.02	0.021	0.013
25	5	0.209	0.151	0.08	0.053	0.052	0.04	0.045	0.043	0.034	0.05	0.023
	10	0.209	0.145	0.059	0.04	0.027	0.024	0.019	0.016	0.013	0.016	0.011

N=500:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.157	0.121	0.064	0.039	0.038	0.036	0.025	0.023	0.029	0.029	0.023
	10	0.154	0.111	0.044	0.018	0.011	0.011	0.011	0.002	0.01	0.011	0.012
15	5	0.184	0.131	0.058	0.023	0.026	0.024	0.017	0.017	0.017	0.012	0.016
	10	0.178	0.126	0.032	0.013	0.003	0.004	0.002	0.002	0.003	0.003	0.002
20	5	0.197	0.122	0.042	0.03	0.012	0.016	0.01	0.013	0.006	0.01	0.013
	10	0.198	0.131	0.029	0.01	0.004	0.001	0.002	0.001	0.001	0.002	0.001
25	5	0.208	0.143	0.031	0.016	0.014	0.006	0.007	0.006	0.006	0.008	0.006
	10	0.213	0.130	0.002	0.007	0.002	0.003	0.002	0.001	0.002	0	0.002

N=1000:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.139	0.098	0.043	0.012	0.023	0.002	0.015	0.015	0.015	0.015	0.013
	10	0.142	0.012	0.027	0.006	0.004	0.001	0.005	0.002	0	0.003	0.002
15	5	0.150	0.115	0.029	0.016	0.019	0.01	0.009	0.015	0.012	0.008	0.009
	10	0.144	0.091	0.017	0.004	0.001	0.002	0	0.005	0	0	0
20	5	0.188	0.115	0.032	0.012	0.011	0.007	0.004	0.008	0.007	0.011	0.005
	10	0.181	0.103	0.017	0.006	0.002	0.001	0	0.003	0.001	0.002	0
25	5	0.208	0.129	0.021	0.01	0.003	0.005	0.007	0.005	0.006	0	0.002
	10	0.202	0.114	0.012	0.001	0	0	0	0	0	0	0.001

N=5000:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.132	0.096	0.01	0.005	0.005	0.004	0.005	0.006	0.003	0.003	0.002
	10	0.134	0.086	0.004	0.001	0	0	0	0	0	0	0
15	5	0.16	0.1	0.008	0.008	0.001	0.004	0.003	0.005	0.003	0.001	0.001
	10	0.162	0.09	0.003	0	0	0	0	0	0	0	0
20	5	0.193	0.111	0.01	0.005	0.001	0.001	0	0.002	0	0.001	0.001
	10	0.193	0.095	0.004	0	0	0	0	0	0	0	0
25	5	0.219	0.114	0.007	0.003	0	0.001	0.002	0.001	0.001	0.002	0.001
	10	0.218	0.095	0.001	0	0	0	0	0	0	0	0

% Count $\geq$1.05 for Cauchy distribution, t-distribution, and Normal distribution

N=100:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.138	0.103	0.06	0.036	0.04	0.031	0.025	0.024	0.022	0.039	0.012
	10	0.137	0.1	0.062	0.033	0.031	0.022	0.017	0.018	0.017	0.022	0.007
15	5	0.157	0.11	0.066	0.041	0.041	0.029	0.027	0.016	0.015	0.024	0.012
	10	0.157	0.114	0.055	0.026	0.022	0.023	0.015	0.01	0.018	0.012	0.003
20	5	0.171	0.136	0.062	0.038	0.028	0.019	0.018	0.018	0.026	0.015	0.014
	10	0.172	0.136	0.052	0.022	0.018	0.01	0.017	0.01	0.01	0.007	0.003
25	5	0.198	0.137	0.053	0.035	0.024	0.021	0.021	0.016	0.017	0.03	0.012
	10	0.198	0.126	0.039	0.022	0.012	0.008	0.012	0.006	0.004	0.005	0.003

N=500:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.137	0.079	0.022	0.01	0.003	0.007	0.001	0	0.001	0	0
	10	0.137	0.088	0.017	0.001	0.001	0	0	0	0	0	0
15	5	0.162	0.112	0.025	0.01	0.003	0.002	0.001	0.001	0	0	0.002
	10	0.163	0.093	0.02	0.003	0	0	0	0	0.001	0	0
20	5	0.185	0.097	0.022	0.006	0.001	0.001	0.003	0	0.001	0.001	0
	10	0.184	0.111	0.014	0.002	0.002	0	0	0.001	0	0	0
25	5	0.201	0.121	0.014	0.006	0.003	0	0	0	0	0	0
	10	0.200	0.116	0.01	0.004	0	0.002	0.001	0	0	0	0

N=1000:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.124	0.058	0.012	0	0	0	0.001	0	0	0	0
	10	0.126	0.068	0.007	0	0	0	0	0	0	0	0
15	5	0.139	0.073	0.01	0.001	0	0.001	0	0	0	0	0
	10	0.136	0.072	0.008	0	0	0	0	0	0	0	0
20	5	0.173	0.098	0.012	0	0	0.001	0	0	0	0	0
	10	0.171	0.091	0.009	0	0	0	0	0	0	0	0
25	5	0.196	0.111	0.009	0.001	0	0.001	0	0	0	0	0
	10	0.187	0.095	0.008	0	0	0	0	0	0	0	0

N=5000:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.113	0.061	0.001	0	0	0	0	0	0	0	0
	10	0.119	0.055	0	0	0	0	0	0	0	0	0
15	5	0.147	0.075	0.002	0	0	0	0	0	0	0	0
	10	0.147	0.068	0	0	0	0	0	0	0	0	0
20	5	0.181	0.085	0.003	0	0	0	0	0	0	0	0
	10	0.176	0.083	0	0	0	0	0	0	0	0	0
25	5	0.207	0.088	0.001	0	0	0	0	0	0	0	0
	10	0.206	0.074	0	0	0	0	0	0	0	0	0

% Count ≥1.1 for Cauchy distribution, t-distribution, and Normal distribution

N=100:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.132	0.078	0.038	0.016	0.015	0.011	0.014	0.008	0.009	0.013	0.002
	10	0.129	0.081	0.039	0.019	0.014	0.006	0.007	0.002	0.006	0.006	0.003
15	5	0.146	0.097	0.041	0.018	0.019	0.014	0.012	0.005	0.007	0.014	0.002
	10	0.148	0.102	0.036	0.012	0.013	0.007	0.006	0.003	0.006	0.001	0
20	5	0.166	0.115	0.041	0.019	0.013	0.006	0.007	0.011	0.015	0.003	0.005
	10	0.166	0.118	0.034	0.012	0.009	0.007	0.007	0.005	0.005	0	0.002
25	5	0.190	0.123	0.038	0.022	0.016	0.011	0.013	0.003	0.005	0.015	0.007
	10	0.191	0.115	0.028	0.01	0.006	0.003	0.005	0.001	0.002	0.001	0.002

N=500:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.126	0.059	0.015	0.001	0	0	0	0	0	0	0
	10	0.125	0.064	0.009	0.001	0	0	0	0	0	0	0
15	5	0.151	0.081	0.012	0.004	0	0	0	0	0	0	0
	10	0.150	0.079	0.013	0.001	0	0	0	0	0	0	0
20	5	0.176	0.09	0.012	0.001	0.001	0	0	0	0	0	0
	10	0.179	0.092	0.009	0.001	0	0	0	0	0	0	0
25	5	0.192	0.105	0.008	0.003	0	0	0	0	0	0	0
	10	0.190	0.103	0.005	0.002	0	0.002	0	0	0	0	0

N=1000:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.114	0.045	0.004	0	0	0	0	0	0	0	0
	10	0.11	0.049	0.004	0	0	0	0	0	0	0	0
15	5	0.131	0.057	0.005	0.001	0	0	0	0	0	0	0
	10	0.126	0.059	0.005	0	0	0	0	0	0	0	0
20	5	0.166	0.08	0.006	0	0	0	0	0	0	0	0
	10	0.162	0.074	0.004	0	0	0	0	0	0	0	0
25	5	0.187	0.097	0.005	0	0	0	0	0	0	0	0
	10	0.175	0.083	0.006	0	0	0	0	0	0	0	0

N=5000:

% Miss	Impute	Cauchy	t_2	t_4	t_6	t_8	t_{10}	t_{15}	t_{20}	t_{25}	t_{30}	Normal
10	5	0.104	0.05	0	0	0	0	0	0	0	0	0
	10	0.107	0.043	0	0	0	0	0	0	0	0	0
15	5	0.137	0.059	0	0	0	0	0	0	0	0	0
	10	0.138	0.06	0	0	0	0	0	0	0	0	0
20	5	0.175	0.067	0.001	0	0	0	0	0	0	0	0
	10	0.166	0.073	0	0	0	0	0	0	0	0	0
25	5	0.2	0.073	0	0	0	0	0	0	0	0	0
	10	0.197	0.059	0	0	0	0	0	0	0	0	0

% Count ≥1.0 for Chi-square distribution and Normal distribution

N=100:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.137	0.105	0.108	0.09	0.095	0.095	0.111	0.093	0.099	0.1	0.091	0.066
	10	0.125	0.09	0.082	0.081	0.09	0.075	0.082	0.067	0.061	0.06	0.06	0.06
15	5	0.111	0.078	0.082	0.08	0.083	0.073	0.079	0.067	0.061	0.057	0.074	0.044
	10	0.083	0.08	0.058	0.052	0.058	0.05	0.044	0.033	0.036	0.047	0.046	0.019
20	5	0.096	0.08	0.073	0.063	0.066	0.057	0.058	0.05	0.047	0.043	0.047	0.034
	10	0.081	0.056	0.056	0.041	0.048	0.037	0.033	0.034	0.028	0.017	0.025	0.013
25	5	0.092	0.062	0.06	0.057	0.047	0.044	0.048	0.052	0.03	0.041	0.038	0.023
	10	0.067	0.056	0.055	0.026	0.034	0.031	0.019	0.026	0.023	0.014	0.02	0.011

N=500:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.066	0.045	0.046	0.037	0.033	0.026	0.027	0.025	0.032	0.031	0.028	0.023
	10	0.043	0.029	0.015	0.012	0.017	0.012	0.013	0.015	0.007	0.008	0.004	0.012
15	5	0.035	0.037	0.025	0.022	0.027	0.01	0.018	0.012	0.015	0.014	0.013	0.016
	10	0.022	0.013	0.012	0.003	0.003	0.007	0.003	0.004	0.005	0.001	0.002	0.002
20	5	0.038	0.018	0.017	0.02	0.011	0.013	0.017	0.01	0.013	0.011	0.013	0.013
	10	0.021	0.005	0.007	0.003	0.001	0	0.001	0.004	0	0.001	0.001	0.001
25	5	0.028	0.019	0.015	0.015	0.016	0.01	0.015	0.01	0.009	0.009	0.011	0.006
	10	0.007	0.005	0.004	0.002	0.005	0.003	0.001	0.003	0	0	0.001	0.002

N=1000:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.037	0.022	0.019	0.02	0.02	0.008	0.017	0.011	0.016	0.011	0.01	0.013
	10	0.025	0.014	0.009	0.002	0.007	0.003	0.006	0.003	0.004	0.003	0	0.002
15	5	0.031	0.015	0.023	0.011	0.014	0.013	0.005	0.004	0	0.009	0.009	0.009
	10	0.011	0.009	0.004	0.002	0.002	0.003	0.002	0.001	0	0	0.002	0
20	5	0.018	0.007	0.02	0.005	0.007	0.006	0.006	0.005	0.006	0.003	0.008	0.005
	10	0.005	0.001	0.004	0	0.002	0	0	0	0	0	0	0
25	5	0.016	0.008	0.006	0.005	0.009	0.005	0.002	0.004	0.005	0.001	0.001	0.002
	10	0.002	0	0.002	0	0.001	0	0	0.001	0.001	0	0	0.001

N=5000:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.01	0.003	0.005	0.006	0.006	0.004	0.003	0.004	0.004	0.003	0.003	0.002
	10	0.002	0	0	0	0	0	0	0	0	0	0	0
15	5	0.006	0.003	0.002	0.002	0.002	0.001	0.002	0.001	0.002	0.006	0.001	0.001
	10	0	0.002	0.002	0	0	0	0	0	0	0	0	0
20	5	0.004	0.003	0.001	0.003	0.001	0	0	0.002	0.001	0.003	0.002	0.001
	10	0	0	0	0	0	0	0	0	0	0	0	0
25	5	0.001	0.001	0.001	0	0	0.001	0.001	0.002	0.001	0	0	0.001
	10	0	0	0	0	0	0	0	0	0.001	0	0	0

% Count ≥1.05 for Chi-square distribution and Normal distribution

N=100:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.085	0.055	0.055	0.039	0.04	0.036	0.036	0.037	0.032	0.036	0.031	0.012
	10	0.072	0.044	0.047	0.035	0.042	0.032	0.029	0.021	0.02	0.019	0.019	0.007
15	5	0.063	0.048	0.045	0.035	0.039	0.035	0.037	0.032	0.027	0.023	0.027	0.012
	10	0.051	0.048	0.034	0.023	0.029	0.016	0.018	0.016	0.017	0.014	0.014	0.003
20	5	0.066	0.047	0.04	0.03	0.036	0.023	0.03	0.025	0.022	0.018	0.027	0.014
	10	0.054	0.038	0.034	0.023	0.028	0.023	0.013	0.02	0.009	0.006	0.013	0.003
25	5	0.07	0.043	0.043	0.029	0.029	0.021	0.024	0.031	0.019	0.02	0.017	0.012
	10	0.052	0.035	0.031	0.017	0.019	0.02	0.009	0.018	0.011	0.006	0.007	0.003

N=500:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.019	0.005	0.006	0.005	0.003	0	0	0.002	0	0.001	0.002	0
	10	0.011	0.007	0.005	0.001	0	0.001	0.001	0	0	0	0	0
15	5	0.013	0.009	0.008	0.003	0.003	0.002	0.001	0.002	0.001	0.001	0.001	0.002
	10	0.006	0.002	0.003	0	0	0	0	0	0.001	0	0	0
20	5	0.014	0.002	0.003	0.006	0.004	0.003	0.001	0	0.004	0	0.001	0
	10	0.004	0.001	0.002	0.001	0	0	0	0	0	0	0	0
25	5	0.013	0.004	0.002	0.005	0.002	0	0.002	0.003	0.001	0.002	0.005	0
	10	0.003	0.002	0.002	0.001	0	0	0	0	0	0	0	0

N=1000:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.008	0.002	0	0	0	0	0	0.001	0	0	0	0
	10	0.003	0	0.001	0	0	0	0	0	0	0	0	0
15	5	0.001	0.001	0.003	0.001	0.001	0	0	0	0	0	0	0
	10	0.001	0.001	0	0	0	0	0	0	0	0	0	0
20	5	0.007	0.001	0.001	0.001	0	0.001	0.001	0	0	0.001	0	0
	10	0.003	0	0	0	0	0	0	0	0	0	0	0
25	5	0.001	0	0	0	0.001	0	0.001	0	0.001	0	0	0
	10	0.001	0	0	0	0	0	0	0	0	0	0	0

N=5000:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
15	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
20	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
25	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0

% Count ≥1.1 for Chi-square distribution and Normal distribution

N=100:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.057	0.023	0.029	0.019	0.019	0.017	0.013	0.012	0.01	0.004	0.009	0.002
	10	0.046	0.025	0.021	0.018	0.019	0.013	0.01	0.011	0.007	0.003	0.004	0.003
15	5	0.048	0.029	0.019	0.02	0.023	0.013	0.018	0.012	0.012	0.008	0.012	0.002
	10	0.037	0.032	0.02	0.014	0.013	0.007	0.008	0.007	0.006	0.003	0.004	0
20	5	0.044	0.029	0.022	0.017	0.013	0.012	0.01	0.01	0.008	0.006	0.008	0.005
	10	0.04	0.028	0.023	0.011	0.014	0.007	0.006	0.009	0.004	0.003	0.004	0.002
25	5	0.044	0.028	0.027	0.02	0.012	0.008	0.014	0.015	0.008	0.008	0.007	0.007
	10	0.037	0.022	0.014	0.009	0.007	0.011	0.004	0.007	0.004	0	0.002	0.002

N=500:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.007	0.002	0.003	0.001	0	0	0	0	0	0	0	0
	10	0.002	0	0.001	0	0	0	0	0	0	0	0	0
15	5	0.004	0.001	0	0	0	0	0	0.001	0	0	0	0
	10	0.002	0	0.002	0	0	0	0	0	0	0	0	0
20	5	0.005	0.001	0.002	0.001	0	0	0	0	0	0	0	0
	10	0.003	0	0	0	0	0	0	0	0	0	0	0
25	5	0.006	0	0	0.001	0.001	0	0.002	0	0	0	0.002	0
	10	0.002	0.001	0.002	0.001	0	0	0	0	0	0	0	0

N=1000:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0.001	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
15	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
20	5	0.001	0	0	0	0	0	0	0	0	0	0	0
	10	0.001	0	0	0	0	0	0	0	0	0	0	0
25	5	0.001	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0

N=5000:

% Miss	Impute	$\chi^2_{(2)}$	$\chi^2_{(4)}$	$\chi^2_{(6)}$	$\chi^2_{(8)}$	$\chi^2_{(10)}$	$\chi^2_{(15)}$	$\chi^2_{(20)}$	$\chi^2_{(25)}$	$\chi^2_{(30)}$	$\chi^2_{(40)}$	$\chi^2_{(50)}$	Normal
10	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
15	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
20	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
25	5	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0