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Equal or Not? An Exploration of Eighth-Grade Students' Experience of Algebra

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(Under the Direction of Yasar Bodur)

ABSTRACT

Over the past two decades, a considerable amount of algebra instruction nationwide has shifted from high school to middle school. In Georgia, all eighth-grade students have been required to take a course that is equivalent to about 80 percent of a traditional Algebra 1 course. The purpose of this qualitative study was to explore how a selected group of eighth-grade students in a suburban Georgia middle school experience algebra within the eighth grade mathematics curriculum. A qualitative research design was used to investigate students' perceptions of algebra, the strategies employed by teachers to teach algebra, students' difficulties with algebra, and students' prior experiences with mathematics. Constructivism provided the theoretical framework for the study. As a theory of active knowing and learning, constructivism is a primary theoretical perspective on learning mathematics (Ernest, 1997).

Purposeful sampling was used to select six eighth-grade participants for the study. Specifically, intensity sampling was used to identify students who had difficulty with algebra to a high, but not extreme, extent (Gall, Gall, & Borg, 2007). Data collection methods included student profiles, individual and focus group interviews, think-aloud interviews, and document analysis. Data were analyzed through the constant comparative method.

Findings from this study indicate that the participants perceive algebra as being too difficult for eighth grade, especially in terms of the pace of instruction. While the participants indicated that it was important for all students to learn algebra, they noted that differentiated instructional strategies are necessary. Data from this study, however, reveal that teachers continue to rely on traditional teaching methods such as lecture and note taking. Participants further noted a benefit from cooperative learning strategies, as well as support and encouragement from teachers. It was also evident from the data that students are relying on memorization of rules or steps to solve algebra problems, rather than developing an understanding of the concepts. In addition, the participants reported feeling unprepared for the algebra they experienced in eighth grade, based on their previous math classes. These findings indicate opportunities for improving students' experience of algebra in two major areas: curriculum and pedagogy.

INDEX WORDS: Mathematics, Curriculum, Algebra, Middle School, Adolescents, Constructivism

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CHAPTER 1

INTRODUCTION

"Mathematics is not only an impenetrable mystery to many, but has also, more than any other subject, been cast in the role as an 'objective' judge, in order to decide who in the society 'can' and who 'cannot'" (Skovsmose, 2005, p. 38). In the name of "equality," standardization of the curriculum requires that all students have access to the "privileged" knowledge of advanced mathematics. All students, regardless of prior academic achievement, are expected to master rigorous mathematics standards set forth by the state. In eighth grade, many of these standards include algebra concepts that were previously taught in high school mathematics courses. As a result, some students are having great difficulty understanding the mathematics they are expected to learn. Many of these struggling students are students of color and students from economically disadvantaged families.

On the 2011 Criterion Referenced Competency Test (CRCT), 14 percent of all eighth grade students in Georgia did not meet standards in mathematics (Georgia Department of Education, 2011c). This percentage equates to approximately 16,650 eighth grade students not meeting standards. Of greater concern is the fact that "meeting standards" corresponded to answering only 31 out of 60 questions, or 51.6 percent, correctly. It is unknown how many eighth grade students "met standards" by answering only 52 to 69 percent of the questions correctly, scores that would typically be considered failing. Further, 2011 CRCT results reveal that 20 percent of Black students in eighth grade, or about 8,990 students, did not meet standards, accounting for over half of the total students statewide who did not meet standards. Similarly, 24 percent of economically disadvantaged eighth grade students did not meet standards compared to only eight percent of students who were not economically disadvantaged.

Through this study, I have sought to understand how students experience algebra in eighth grade mathematics. To understand how students experience algebra, I investigated “their perceptions, purposes, premises, and ways of working things out” (Noddings, 1990, p. 14). I listened closely to students' thoughts and concerns about algebra, and mathematics in general. Listening to student voice can be powerful, as “their perspectives often capture the realities of classroom and school life in vivid detail” (Yonezawa & Jones, 2006, p. 21).

Constructivism provides a theoretical framework for the study. As a theory of active knowing and learning, the roots of constructivism can be located in the traditions of problem solving, misconceptions, and theories of cognitive development (Confrey & Kazak, 2006). In the tradition of problem solving, Polya’s (1945) four stages (understanding, devising a plan, carrying out the plan, and looking back) emphasized that mathematics was more than just memorizing a series of steps. Polya’s first stage, “understand the problem” requires the student to construct an adequate initial representation of the problem (Schoenfeld, 1987). In the tradition of misconceptions, practitioners assumed that student errors could be corrected by simply making the student aware of the error and then providing the student with the correct procedure. Misconceptions, on the other hand, were more difficult to overcome as they seemed to “pop back up like weeds” (Confrey & Kazak, 2006, p. 307). The study of misconceptions established that learning mathematics often required “revisiting and revising ideas” as well as “careful attention to learners’ thoughts and perceptions” (p. 308).

Piaget’s work on theories of cognitive development was highly influential on the development of constructivism. Confrey and Kazak (2006) recognize seven major contributions of Piaget’s work to the development of constructivism:

1) A child’s view is different qualitatively from an adult’s,

2) General stages of development viewed as likely intellectual resources for building ideas occur sequentially and provide important background information for studying children,

3) The development of an idea determines its meaning, rather than a simple statement of a formal definition and set of relationships, known as *genetic epistemology*,

4) Because of the first three premises, one can witness two major kinds of encounters to responses to new ideas and information which are *assimilation* and *accommodation*,

5) The process of moving from action, to operation, to mathematical object, required a level of consciousness that he labeled *reflective abstraction*,

6) The patterns of thought available for reuse and modification were cast as schemes, and

7) Describing foundational ideas often involved a search for *conservation* and invariance. (pp. 308-309)

These contributions encouraged the field to acknowledge that children could be valuable sources of information. Many constructivists trace their lineage to Piaget's work.

In addition, Vygotsky (1978) stressed the importance of social interaction in student learning. Vygotsky's theory suggests that we learn first through interactions with others and then individually "through an internalization process that leads to deep understanding" (Fogarty, 1999, p. 78). Vygotsky's primary topic was the Zone of Proximal Development (ZPD), which describes how interaction with a more capable person can move development forward. Vygotsky's theory of the ZPD promotes the need for active teaching (Blake & Pope, 2008) as students in the ZPD are able to imitate teachers and connect prior understanding with new concepts.

Similarly, Bruner (1960) viewed the task of teaching as one of translation, of representing the subject in a way that students can understand. He argued that any concept can be represented in a concrete way that can be understood by children. The concept can later be expanded upon when the appropriate stage of development has been reached. While Bruner noted the importance of guiding students to abstract thought processes, he also stressed the ineffectiveness of doing so in a manner that is outside the child’s way of thinking. If formal explanations of abstract concepts such as algebra are presented to students too soon, the result can be a lack of conceptual understanding by the students. The student may be able to follow a memorized set of rules or steps, but without understanding their meaning.

Statement of the Problem

There are differing views of when students should receive instruction in algebra. Not that long ago, algebra was commonly considered a ninth grade course and the middle school mathematics curriculum focused on building the conceptual foundation needed for the later study of algebra. Recently, however, algebra instruction has shifted to middle school, with topics such as linear equations being introduced to all students as early as seventh grade (Star & Rittle-Johnson, 2009). In 1990, only about 16 percent of eighth grade students nationwide were enrolled in algebra courses, but several events over the past two decades have led to a substantial nationwide push for more students to take algebra in eighth grade (Loveless, 2008). Reports that middle school students around the world were learning algebra led to a national goal of enrolling students in an algebra course by eighth grade (Loveless, 2008). Further, a study of the Trends in International Mathematics and Science Study (TIMSS) concluded that the eighth grade mathematics curriculum in the United States is equivalent to the average seventh grade curriculum for other participating countries (Greene, Herman, & Haury, 2000). As a result of this push for early algebra, the percentage of eighth graders enrolled in algebra has steadily increased to 24 percent in 2000 and 31 percent in 2007 and currently, nearly half of eighth grade students nationwide are enrolled in an algebra course (Loveless, 2008).

In addition to concerns over global competitiveness, research has shown that grouping students based on ability does not improve academic achievement or promote more positive behaviors or attitudes (Oakes, 2005) and these findings have led to a focus on equality of access to higher level coursework for all students, especially in mathematics. In response, many states, including Georgia, have eliminated or reduced multiple tracks for mathematics classes in middle school and high school. All students, regardless of prior achievement, are expected to master rigorous grade-level content standards. As a result, many students are struggling in their eighth grade math classes because they are not prepared to learn the mathematics they are expected to learn (Loveless, 2008).

Foster (2007) describes three "habits of mind" that students need in order to learn algebra. First, students must be able to reverse mathematical operations. That is, they must be able "to undo mathematical processes as well as do them" (p. 164). Students, therefore, must understand the mathematical processes well enough that they can work backward from the solution to the starting point. Second, algebra students must be able to build rules to represent functions. Building rules involves recognizing patterns in data and then organizing the data into input-output situations. Finally, students learning algebra need the ability to abstract from computation, meaning that they need to be able to think about computations independently of the numbers that are used.

Most eighth grade students are thirteen or fourteen years old and, therefore, are classified as “early adolescents.” According to the National Middle School Association (2003), schools serving early adolescents must be “developmentally responsive” to the unique characteristics and needs of these students. During early adolescence, however, students demonstrate a wide range of individual intellectual development and many may not have developed the habits of mind needed to learn algebra successfully. Some students in eighth grade have developed the capacity for abstract thought processes, such as those required in algebra, while others have not yet transitioned to this higher level of cognitive function (Caskey & Anfara, 2007). These differences can lead to a disparity between the needs of developing adolescents and the experiences afforded them by their social environments, especially schools (Eccles et al., 1993). Further, students’ perceptions of academic competence and self-concept often decline during adolescence (Shapka & Keating, 2005) and young adolescents may also be highly susceptible to influences on their academic self-concept, both positive and negative (Parker, 2010). Thus, it is crucial that mathematics educators consider the unique needs of adolescent students and the potential impact of a rigorous mathematics curriculum on the self-concepts of struggling students.

Purpose of the Study

The purpose of this study was to investigate how a selected group of students in a suburban Georgia middle school experience algebra within the eighth grade mathematics curriculum. The existing literature on teaching algebra in eighth grade presents primarily quantitative studies which describe the number of math courses that students subsequently take in high school and student achievement in those courses. In contrast, this study utilized qualitative methods to gain an in-depth understanding of students' experiences with the curriculum.

Qualitative methods provide the data necessary to develop an understanding of these students' experiences. Specifically, the methods of "asking, watching, and reviewing" (Merriam, 2009, p. 85) were used to collect data and develop an understanding of students' experiences. Individual and focus group interviews were conducted to ask students about their experiences with algebra. Think-aloud interviews also allowed me to watch as students experienced algebraic tasks. In addition, a review of documents such as student work samples and test scores provided supplementary information about the students' experiences. According to Patton (2002), "Understanding comes from trying to put oneself in the other person's shoes, from trying to discern how others think, act, and feel" (p. 49). Qualitative methods, therefore, allowed me to attempt to put myself in the students' shoes.

Research Questions

This study was an exploration of how students experience algebra in the current era of standards based reform. The main research question, therefore, was "What are eighth grade students' experiences with algebra?" More specific research questions are as follows:

1. How do eighth grade students perceive algebra and the eighth grade mathematics curriculum?
2. How do students describe the strategies that teachers employ to teach algebra?
3. What difficulties, if any, do students encounter in learning the algebra concepts required by the eighth grade mathematics curriculum?
4. How do students describe their previous middle school experiences with mathematics?

Overview of Method

This study addressed the research questions through use of a qualitative research design. Six eighth-grade participants were selected using purposeful sampling (Gall, et al., 2007). Specifically, intensity sampling was used to select participants who had experienced a high degree of difficulty with algebra, but not an extreme degree of difficulty.

Data sources included participant profiles, individual participant interviews, think-aloud interviews, a focus group interview, and document analysis. Data were analyzed through the constant comparative method. Emergent themes for each participant were compared and synthesized across all participants. Credibility for the study was sought by using multiple data collection methods, collection of data on multiple occasions over time, rich thick description, researcher reflexivity, and peer examination.

Significance of the Study

Through this study, I created opportunities to hear from students by listening to their thoughts and perceptions about mathematics education. “Young people have unique perspectives on learning, teaching, and schooling” and “their insights warrant not only the attention but also the responses of adults” (Cook-Sather, 2006, p. 359). This study has helped me, as a middle school mathematics teacher, to better understand the experiences of my students from their perspective. With this knowledge, I hope to improve the experiences of future students by improving my own practice and also by working with other teachers to improve their practice.

This study could also be significant for many mathematics educators by making them more aware of the feelings of struggling students. Too often, the blame for poor achievement in mathematics is placed upon students, especially students who are perceived as lazy or unmotivated, without seeking to understand the underlying issues. Listening to students’ perspectives on their experiences can help educators gain a better understanding of these underlying issues and may possibly result in changes in how these educators interact with struggling students.

The perspectives provided by this research could also impact education on a school, district, or state level by offering insight into the mathematical experiences of struggling eighth grade students from a successful suburban school. These struggling students are supposedly receiving the same high quality education, from highly qualified teachers, as their more successful classmates. Their perspectives on the nature of the standards, the quality and "equality" of their education, and the factors that influence their achievement, could lead to changes in curriculum and/or teaching practices that would create more equity in mathematics education.

While I recognize that the rigor of the mathematics curriculum is unlikely to change, my hope is that developing an understanding of the experiences of struggling students will shed some light on what needs to be done to support these students and, as a result, make eighth-grade mathematics a more successful experience for them.

CHAPTER 2

REVIEW OF THE LITERATURE

The Algebra Problem

Algebra in eighth grade was once reserved for a small percentage of mathematically talented students. In 1990, about 16 percent of eighth grade students nationwide were enrolled in algebra courses (Loveless, 2008). Over the next decade, several events led to a substantial push for more students to take algebra in eighth grade. In 1994, the National Center of Education Statistics reported that effective middle schools offered algebra to eighth grade students (Spielhagen, 2006a) and subsequent research concluded that "early access to algebra has a sustained positive effect on students, leading to more exposure to advanced mathematics curriculum and, in turn, higher mathematics performance by the end of high school" (J. B. Smith, 1996, p. 149). Concern began to grow that middle school students around the world were learning algebra and, in 1998, the Clinton administration made enrolling students in an algebra course by eighth grade a national goal (Loveless, 2008). Further, a study of the Trends in International Mathematics and Science Study (TIMSS) concluded that the United States' mathematics curriculum for eighth grade was comparable to the average seventh-grade curriculum for other participating countries, putting eighth grade students in the United States a year behind their global counterparts (Greene, et al., 2000). Since then, the percentage of eighth graders enrolled in algebra has steadily increased to the point that nearly half of eighth grade students nationwide are enrolled in an algebra course (Loveless, 2008). In Georgia, all eighth grade students are required to take a course that is equivalent to about 80 percent of the content of a traditional Algebra 1 course.

Research on 8th Grade Algebra. The study of algebra is often considered a gateway for further study in mathematics and science (Smith, 1996). The point at which students gain access to algebra is also important as taking algebra in eighth grade positions students to enroll in calculus by twelfth grade. The argument for requiring algebra for all eighth grade students rests on the assumption that the same benefits noted in much of the research on eighth grade algebra will be achieved under a universal algebra policy (Stein, Kaufman, Milan, & Hillen, 2011). This research, however, has been based primarily on cases where access to algebra was restricted to only those students deemed to be the most prepared to study algebra.

A standard practice in U.S. middle schools has been to group students in mathematics classes based on their prior mathematics achievement, but research has shown that students assigned to low-level courses often fall further behind. Tracking, therefore, "contributes to low math performance rather than addressing it" (Burris, Heubert, & Levin, 2004, p. 2). Further, research on accelerating instruction implies that an accelerated curriculum is more effective than a remedial curriculum in enhancing the performance of low achievers (Burris, et al., 2004). Spielhagen (2006b) argues that algebra in eighth grade provides "both rigor and opportunity" (p. 38) while also improving mathematics literacy across the student population. Spielhagen recognizes, however, that there will always be students who are not ready for algebra in eighth grade.

Most studies conducted on eighth grade algebra are quantitative in nature and focus on students' subsequent achievement in math and high school course taking patterns. Darling (2010), for example, found that exposure to algebra in eighth grade may lead to completion of higher level math courses in high school and higher ACT achievement scores. Similarly, a study by Diette (2005) revealed that students who took algebra in eighth grade experienced a greater increase in end-of-grade achievement test scores, regardless of their past test scores. Calzada (2002) also investigated the effect of eighth grade algebra on student achievement, finding that students who participated in eighth grade algebra experienced higher passing rates on a number of standardized assessments in grades eight and ten compared to students who did not participate in eighth grade algebra. The results of these studies were limited, however, by the fact that not all eighth grade students were enrolled in the algebra courses. The eighth-graders studied were those who were most likely already considered to be above average in mathematics achievement.

Some argue that policies mandating algebra for all students will not necessarily result in students learning more math (Chazan, 1996; Loveless, 2008). Casey (2005) found that in ten rural North Carolina schools testing all students in algebra, only 8 percent of students scored in the advanced and proficient bands. Students are required to repeat the course in high school at rates of up to 92 percent. High school science teachers also reported that students were unable to apply previously studied math concepts in their classes.

Capraro and Joffrion (2006) investigated the ability of middle school students to translate English language into mathematical symbols or vice versa. The ability to use symbolic algebra to represent and solve linear equations is fundamental for student success with advanced algebra concepts. Results of the study indicated that students “were not procedurally or conceptually ready even at the seventh or eighth grade level to translate from the written word to mathematical equations” (p. 147). These findings suggest that many eighth grade students are not ready for the study of algebra.

In a review of 44 studies of eighth grade algebra (Stein, et al., 2011), universal algebra policies were found to increase access to algebra for all students, but also resulted in more underprepared students enrolled in algebra classes. According to Loveless (2008), nearly one out of every thirteen eighth graders in advanced math classes “knows very little mathematics” (p. 6). The result of these “misplaced” students is an undue burden on the classroom teacher who, according to Loveless, is held responsible for fixing the problem. Several studies (Liang, Heckman, & Abedi, 2012; Stein, et al., 2011) indicate that simply taking algebra will not necessarily result in higher achievement levels or access to more advanced mathematics courses. While universal algebra policies increase the number of students taking and passing algebra, they also increase the number of students failing algebra. In studies where achievement gains were noted, the universal algebra policies were accompanied by strong supports for struggling students. One of the most important supports provided was additional time for algebra instruction.

Chazan (1996) urges those who make decisions about algebra policies to consider the students and teachers who are affected by such decisions. Further, Loveless (2008) argues that the goal should not be for students to take algebra in eighth grade. Instead, the goal should be for more students to learn algebra. Similarly, Stein, et. al. (2011) assert that it may not be necessary for all students to learn the same algebra content at the same time, regardless of their level of preparedness. Instead, algebra policies must “ensure that students receive instruction that both is geared to their needs and moves them toward commonly accepted standards for what it means to be competent in algebra” (p. 485).

Conceptual Difficulties in Algebra. The research literature addresses several factors that contribute to students’ difficulty in learning algebra. Kieran (1992) explored how the content of algebra, the way algebra is taught, and the ways by which students approach algebra can be involved in students’ lack of understanding of algebra. Algebra is considered as the branch of mathematics dealing with “symbolizing general numerical relationships and mathematical structures” and with performing operations on those structures (p. 391). Thus, algebra is initially introduced to students as a generalization of the operations conducted in arithmetic. For example, students are taught to substitute numerical values into algebraic expressions to find the value of the expression. This example represents a *procedural* perspective of algebra because, ultimately, arithmetic operations are carried out on numbers to produce numbers. A *structural* perspective, on the other hand, refers to operations that are carried out on algebraic expressions, rather than numbers. An example of a structural perspective would be simplifying an algebraic expression by combining like terms, as the result is still an algebraic expression. The transition from a procedural perspective to a structural perspective is one that is "accomplished neither quickly nor without great difficulty" (p. 392).

Sfard (1991) describes a three-phase model of conceptual development in mathematics. The first phase, *interiorization*, consists of performing some operation on mathematical objects that are already familiar to the student. In the second phase, *condensation*, the operation is simplified into more manageable parts. The condensation phase lasts as long as an operation is perceived of only from a procedural perspective. The third phase, *reification*, occurs when a student is suddenly able to see a mathematical object in a different way, as separate from the process that created it.

Further, Kieran (1991) explains the learning of algebra as a sequence of procedural-structural development. Students may begin their study of algebra from a procedural perspective, using strategies such as substitution, but they must quickly learn to view algebraic expressions and equations "as objects in their own right" (p. 393) upon which higher level operations can be performed. If a student is unable to conceive of algebraic expressions and equations in this manner, algebraic manipulation will be a source of great conflict. Additionally, students must make adjustments involving the symbolic representation of numerical relationships, such as would be required when translating a problem situation into an equation. Students can experience difficulties related to the use of the equal sign as well as the nature of the operations that are represented in the problem. Additionally, the use of letters (variables) to represent values in algebra, and the multiple ways that variables can be used, can also be the cause of many difficulties for students (L. R. Booth, 1999).

Expressions and Equations in Algebra. In algebra, students must learn to operate on expressions and equations, rather than numbers. According to Schoenfeld (1994), “much of the power of algebra…is that of abstraction” (p. 24); that is, the ability to represent a situation symbolicly and perform operations on those symbols. The operations performed in algebra involve simplifying, solving, factoring, and so on, rather than the basic operations of addition, subtraction, multiplication, and division that students experienced in arithmetic. Being able to perform these algebraic operations, however, does have some intuitive basis in arithmetic. Yet, evidence suggests that students may not be aware of the properties and underlying structure of arithmetic operations and, therefore, cannot generalize these properties and structures to algebraic expressions and equations (Kieran, 1992). For example, many students are unable to judge the equivalence of two numerical expressions without computing. When faced with algebraic expressions, in which computing is not an option, students often have great difficulty.

An algebraic expression is a description of an operation involving one or more variables, such as $x + 2$, $3n$, or $a – b$ (Wagner & Parker, 1999). Students often learn how to simplify expressions by using a set of procedures, rather than learning the underlying structure of the operations being performed. This procedural learning of algebra makes it difficult for students to generalize and apply what they have learned to different types of expressions. Frequently, students who have learned to successfully simplify one type of expression are unable to transfer what they have learned to a different expression (Kieran, 1992).

Many students view algebraic expressions as being incomplete because they cannot find an "answer." When asked to simplify an expression, many students will attempt to solve for the variable by adding "= 0" to the expression to make it part of an equation (Wagner, Rachlin, & Jensen, 1984). This need to transform expressions into equations causes students to be unable to interpret the meaning of variables in expressions because they believe that an expression needs to equal something; that is, they need to be able to carry out the operations and get an "answer."

Carry, Lewis, and Bernard (1980) found that one of the most common errors in simplifying expressions involves simplifying an expression such as $20x - 5$ to $15x$. They referred to this error as the "deletion error" and argued that it may be caused by students over-generalizing certain arithmetic operations. In arithmetic, students learn to perform operations on numbers until they get a single number as the answer. If they generalize this process to algebra, students may feel uncomfortable leaving an operation sign in an expression as the final answer, so they perform any operations they can on the remaining numbers to obtain a single term (Wagner & Parker, 1999). Many students also continue to view variables as labels for concrete objects, rather than a representation of an unknown value, and simply perform arithmetic calculations and attach the variable at the end.

Booth (2008) asserted that conceptual knowledge of the different features of an equation is essential to learning how to solve algebraic equations. These features include the equals sign, variables, like terms, and negative signs, among others. Conceptual knowledge of these features is defined as "understanding the function of the feature in the equation and how changing the location of the feature would affect the overall problem" (p. 571). Student misconceptions of these features can lead to incorrect procedures for solving equations. The use of these incorrect procedures may persist because they lead to a correct solution for some equations, but not for others. In a study of how these misconceptions affect students' ability to solve algebraic equations correctly, Booth (2008) found that students who hold these misconceptions prior to instruction tend to learn less from instruction on how to solve equations.

Use of Variables in Algebra. Understanding the concept of variable is crucial in transitioning from arithmetic to algebra. It is also a concept that is difficult to describe and even more difficult to learn (Schoenfeld & Arcavi, 1999). In arithmetic, letters are often used to represent units, such as meters or minutes. In algebra, the same letters may be used in an expression or equation to represent the *number of* meters or the *number of* minutes. Students can often be confused by these different uses of letters in mathematics, resulting in reading variables as labels, rather than as unknown values. For example, a student may interpret the expression "3a" as "3 apples," rather than as "3 *times* the *number* of apples" (L. R. Booth, 1999, p. 303). When students do interpret variables as representing an unknown value, there is a strong inclination for students to think of the letters as representing specific unique values, rather than as numbers in general. This interpretation can lead to the misconception that different letters must always represent different values (L. R. Booth, 1999).

Students are often introduced to variables in the context of solving an equation, such as $2x + 5 = 19$. When solving such an equation, students are asked to find the value of the variable or the 'unknown' in the equation. In this case, there is only one value for x that makes the equation true. So, while x is called the variable in the equation, its value does not vary, which may be confusing to students. It is important, however, for students to understand a variable as something that can vary (Foster, 2007).

MacGregor and Stacey (1997) studied several hundred students in grades seven through ten in 22 schools and found that the majority of students up to age 15 seem unable to construe variables as generalized numbers or as unknown values. Instead, students ignored the variables, regarded them as labels, or replaced them with numerical values. Evidence indicates that students' difficulties in using variables can be associated with several different misconceptions. Often, students will use intuition and guessing or use analogies to other symbol systems with which they are familiar to interpret the meaning of variables. Interference from new learning can also lead to more opportunities for older students to make mistakes with variables. Teachers may wrongly assume that students have a good understanding of basic concepts when more advanced knowledge is introduced when, in fact, students need much more experience using variables in simpler situations to communicate their understanding. Teaching styles, teaching materials, and approaches to teaching algebra also have a significant effect on students' understanding of variables. Particular teaching approaches, such as using variables as abbreviated words or labels, can lead to students' misunderstanding of variables.

Use of the Equal Sign in Algebra. As suggested by the RAND Mathematics Study Panel (2003), "the notion of 'equal' is complex and difficult for students to comprehend, and it is also a central mathematical idea within algebra" (p. 53). In arithmetic, the equal sign is most often used as an operational symbol to announce an "answer" or as a signal to perform an arithmetic operation (Siegler, 2003). In algebra, however, students must recognize the equal sign as a reflection of a symmetric and transitive relationship between the expressions on either side. The goal in algebra is no longer to simply find the answer, as it was in arithmetic, and students can no longer rely solely on the strategies they used in arithmetic. In algebra, the equal sign must be interpreted as a symbol of equivalence between the left and right sides of an equation.

Kieran (1981) interviewed students about their understanding of the equal sign. Most students described the equal sign in terms of indicating the answer and limited examples to those involving an operation on the left side and the result on the right side. This view of the equal sign may be sufficient for interpreting equations such as $2x + 3 = 15$, but will fall short when students are faced with equations containing variables on both sides of the equal sign. Kieran noted that cognitive strain could be eased considerably if the concept of the equal sign were extended first using arithmetic equalities and then transferred to algebraic equations.

In a study of 375 middle school students, Knuth et al. (2008) asked participants to write a definition of the equal sign. In sixth and seventh grades, operational definitions such as “A sign connecting the answer to the problem” (p. 515) outnumbered relational definitions such as “It means that what is to the left and right of the sign mean the same thing” (p. 515). Eighth grade students showed an improvement in the percentage of relational definitions, but 45 percent of students still defined the equal sign in operational terms. A subsequent finding of this study was that students who held a relational view of the equal sign were more likely than students who held an operational view to solve algebraic equations correctly.

McNeil and Alibali (2005) showed that middle school students’ interpretation of the equal sign as a relational symbol of equivalence depends upon the context of the equation. Students asked to view the equal sign alone or in the context of an “operations equals answer” equation seldom exhibited a relational understanding of the equal sign. In contrast, students presented with equations including operations on both sides of the equal sign were far more likely to demonstrate a relational understanding. Results of a further study (McNeil, et al., 2006) suggest that a greater number of middle school students may understand the equal sign in a relational way, but they are unable to demonstrate that understanding because the equations they see in school, especially in textbooks, frequently appeal to an operational interpretation. Based on these results, middle school students may benefit from seeing more equations with operations on both sides of the equal sign to help enforce their understanding of the equal sign as a symbol of equivalence.

The Mathematics Curriculum

Curriculum Organization. Debates regarding how to organize the school curriculum have been ongoing for over a century. Organization is “an important problem in curriculum development because it greatly influences the efficiency of instruction and the degree to which major educational changes are brought about in the learners” (Tyler, 1949, p. 83). There are several characteristics of curriculum organization that must be considered, including scope, sequence, continuity, relevance, integration, articulation and balance (Oliva, 2001; Ornstein & Hunkins, 2009). Research has shown that the strength of these organizational characteristics can have a significant impact on student achievement (Squires, 2009). Tyler (1949) also noted the importance of evaluation, or assessment, in curriculum development.

Scope. Scope can be defined as the breadth of the curriculum, or “the what” of curriculum organization (Oliva, 2001, p. 458). The content of a course, including the topics, learning experiences, and activities, make up the scope of the curriculum for that course. Saylor and Alexander (1954) defined scope as “the breadth, variety, and types of educational experiences that are to be provided pupils as they progress through the school program” (p. 284). Tyler (1949) recommended identifying elements of the curriculum to serve as “organizing threads” (p. 86). Taba (1945) argued that scope “transcends the coverage of subject matter” and “involves, also, planning a sufficient variety of learning experiences and reactions, academic as well as nonacademic, in as well as out of school” (p. 86). Regardless of how scope is defined, the content of the curriculum is “crucial to academic performance” (Gay, 2000, p. 112).

One of the ongoing debates in determining the scope of the curriculum is that of breadth versus depth. Gardner, Torff, and Hatch (1996) argue that it is more important for students to develop a deep understanding of "*some* key topics or themes" (p. 57) than to require that much specific material be covered. Further, if understanding is the goal of education, we must accept that it is not possible to cover everything. "Indeed, the greatest enemy of understanding is 'coverage'" (p. 57). Developing students' understanding requires that time be devoted to in-depth exploration of a few key topics, allowing students to "engage in continued practice and refinement of performance…on a piece of work that is complex enough to represent the many kinds of knowledge and skill that must come together to produce accomplished performance" (Darling-Hammond, 1997, p. 57). The scope of the curriculum, therefore, should be limited to the "most fundamental understanding" (Bruner, 1960, p. 31) of the key topics in a subject area. The broader the scope of the curriculum, the harder it will be for students to develop an understanding of key topics.

In the current era of standards-based school reform, decisions regarding the scope of the curriculum are primarily made at the state level. Academic standards describe what students should know and be able to do and are mandated by the state as the basis of the curriculum in each subject area. Standardized tests hold teachers and schools accountable for teaching the standards that have been imposed by the state. Often, the sheer number of standards makes it nearly impossible for a teacher to effectively address every one of them. Jackson and Davis (2000) recommend identifying the most "essential concepts and ideas embedded in the lengthy standards and developing a curriculum that reflects these essential concepts" (p. 38). Teachers may also find it necessary to prioritize the standards and focus more on the knowledge and skills that every student should learn.

Sequence. Sequence is defined as the order in which the topics or organizing elements of a course are arranged. It is “the when” of curriculum organization (Oliva, 2001, p. 458). According to Saylor and Alexander (1954), “determination of the sequence of educational experiences is a decision as to the most propitious time in which to develop those educational experiences suggested by the scope” (p. 249). Sequencing requires consideration of several issues, including the maturity, interests, and readiness of the learners, the difficulty of the topic, and the prerequisite skills needed (Oliva, 2001).

Tyler (1949) referred to sequence as “the increasing breadth and depth of the learner’s development” (p. 96) and emphasized the relationship between sequence and continuity in the curriculum. Continuity refers to the vertical repetition of curriculum elements, ensuring that learners revisit important concepts and skills. According to Tyler, sequence refers not only to the repetition of concepts and skills, but rather to “higher levels of treatment with each successive learning experience” (p. 85). This vertical repetition of ideas throughout the curriculum is often referred to as the spiral curriculum.

John Dewey (1938) was the first to use the metaphor of the spiral in curriculum organization, stating that “the new facts and new ideas” obtained by learners “become the ground for further experiences in which new problems are presented. The process is a continuous spiral” (p. 79). Bruner (1960) later popularized the idea of the spiral curriculum, arguing that “any subject can be taught effectively in some intellectually honest form to any child at any stage of development” (p. 33). For example, Bruner argues that “fifth-grade children can play games with rules modeled on highly advanced mathematics; indeed, they can arrive at these rules inductively and learn how to work with them” (p. 38). This does not, however, mean that young children will be able to grasp formal mathematical processes. Schools, therefore, should introduce children to the basic ideas of a subject, those that can be understood intuitively and concretely, at an early age and "revisit these basic ideas repeatedly, building upon them until the student has grasped the full formal apparatus that goes with them" (p. 13).

There are two primary ideas of how the school curriculum should be sequenced. The first focuses on organizing the curriculum to coincide with the different stages of a learner's development while the second supports ordering the content of subject matter according to the prerequisite skills required for each unit of study (Oliva, 2001; Ornstein & Hunkins, 2009). The first approach requires sequencing the curriculum so that it is "developmentally appropriate," while the second approach places subject matter "at the grade level at which it is assumed learners will be able to master it" (Oliva, 2001, p. 461). According to Smith, Stanley, and Shores (1957), each approach taken separately is unrealistic. The first approach is unrealistic because it considers the curriculum to always be flexible enough to be adjusted to each child's needs. The second approach, however, is too inflexible, requiring that lower grade level experiences must prepare students for the content of the higher grade levels. While the second approach has historically been used in organizing the curriculum of American schools, some combination of these two approaches is likely to be the most realistic approach to curriculum sequencing.

If the purpose of learning a subject is to be able to "do" the subject, then the sequence of learning must be built around the key performance tasks that allow students to apply knowledge in a practical manner. As noted by Whitehead (1929), "Let the main ideas which are introduced into a child's education be few and important, and let them be thrown into every combination possible. The child should make them his own, and should understand their application here and now" (p. 2).

According to Reigeluth (2007), the impact of curriculum sequencing on student learning depends upon the relationship between the topics in the curriculum and the scope of topics included in the curriculum. When there is a strong relationship between the concepts that make up the curriculum, the order in which the concepts are taught “will influence how well both the relationship and the concept are learned” (p. 23). As the amount of content in a curriculum increases, the sequence of the concepts becomes increasingly important because students may “have a difficult time organizing so much content logically and meaningfully” if is it not well sequenced (p. 23).

Evaluation and Assessment. Tyler (1949) describes evaluation as a process for determining the extent to which the learning experiences developed within the curriculum “are actually producing the desired results” (p. 105). Tyler notes that evaluation of the curriculum involves assessing “the behavior of students since it is change in these behaviors which is sought in education” (p. 106). He also notes that these assessments must be taken at an early point and again at several later points to identify changes that may have taken place. “Without knowing where the students were at the beginning, it is not possible to tell how far changes have taken place” (p. 106). While paper and pencil tests can measure changes in some student behaviors, there are many kinds of other behaviors that are not easily assessed in this manner.

Tyler (1949) asserts that observations, interviews, and questionnaires are also useful methods of evaluation and assessment. Observations can be useful in assessing “habits and certain kinds of operational skills” (p. 108). Interviews may be helpful in evaluating changes “in attitudes, in interests, in appreciations, and the like” (p. 108). Questionnaires can provide “evidence about interests, about attitudes, and about other types of behavior” (p. 108). Collection of student work samples can also be used to evaluate changes in student behaviors.

The results obtained from any and all of these evaluation methods must be analyzed to identify various strengths and weaknesses and to suggest possible explanations for these strengths and weaknesses. Tyler (1949) argues that the analysis of assessment results should lead to a continuous process of curriculum development. Based on the assessments results, “there is replanning, redevelopment and then reappraisal; and in this kind of continuing cycle, it is possible for the curriculum and instructional program to be continuously improved over the years” (p. 123).

Research on Middle School Mathematics Curricula. Research suggests that curriculum has a significant impact on student achievement. A study of data from the Third International Mathematics and Science Study (TIMSS) indicated that the content of a country’s curriculum impacts student achievement in that country. In examining middle school mathematics data, researchers found statistically significant relationships between various curriculum aspects and learning, as measured by estimated achievement gains from seventh to eighth grade. Results showed that increased curriculum coverage of a topic area is related to larger gains in that same topic area (Schmidt et al., 2001). The study also found that spending more time on topics that required more demanding performance expectations resulted in above average achievement gains in those topic areas. These results support covering fewer topics in math (narrowing the scope) while devoting more time to each topic. Allowing students more time to learn important concepts would help them develop a better understanding of mathematics so they could apply the concepts learned to different situations. Additional time would also allow teachers to differentiate instruction more effectively by providing enrichment for those students who grasped the concept more quickly and providing remediation for those students who need more time.

A more concentrated look into TIMSS achievement data for only the United States revealed that curriculum was significantly related to achievement in U.S. eighth-grade math classrooms (Schmidt, et al., 2001), even after controlling for other differences such as socioeconomic status and prior learning. Overall, the study indicated that increased time spent on a topic resulted in greater achievement scores for that topic. Even small amounts of additional instructional time predicted significant increases in learning.

A previous study of the TIMSS data revealed that math textbooks in the United States include far more topics than German and Japanese textbooks, yet students in both countries significantly outperform U.S. students in mathematics. Mathematics textbooks in the U.S. cover 175 more topics than German textbooks and 350 more topics than Japanese textbooks (Schmidt, McKnight, & Raizen, 1997). These results again point to the importance of depth of coverage over breadth. Attempting to cover more concepts in a shorter amount of time does not lead to increased student achievement in mathematics. Instead, it leads to limited understanding of the mathematical concepts studied and an inability to apply the concepts in problem situations.

National Council of Teachers of Mathematics Standards. According to the National Council of Teachers of Mathematics (NCTM) (2000), “the need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase” (p.1). NCTM asserts that standards play an integral role in the improvement of mathematics education and, therefore, published the Principles & Standards for School Mathematics (2000), which calls for a common foundation of mathematics to be learned by all students. The standards recognize that students “exhibit different talents, abilities, achievements, needs, and interests in mathematics” but emphasize that all students must have access to the highest-quality mathematics education.

According to the NCTM Standards (2000), students in the middle grades should learn algebra both in terms of representing quantitative relationships and as a way of formalizing patterns, functions, and generalizations. The expectations of the algebra standards require that middle grades students work frequently with algebraic symbols to become comfortable in relating algebraic expressions to different representations of quantitative relationships. The standards call for developing an understanding of the different meanings and uses of variables through representing problem situations as algebraic expressions and equations. According to the standards, students in middle school should also learn to recognize and produce equivalent expressions, solve equations, and use formulas. The NCTM algebra standards for grades 6-8 are divided into the following domains: Understand patterns, relations, and functions; Represent and analyze mathematical situations and structures using algebraic symbols; Use mathematical models to represent and understand quantitative relationships; and Analyze change in various contexts.

Georgia Performance Standards. Over the past several years, Georgia schools have implemented a new curriculum, the Georgia Performance Standards (GPS). In middle school mathematics, the scope of each grade level curriculum includes standards in the domains of Number and Operations, Measurement, Geometry, Data Analysis and Probability, and Algebra. Overall, the scope of the middle school mathematics curriculum is very broad and includes a considerable amount of content that was previously taught in high school courses. According to the Georgia Department of Education (2007), “by the time a student has finished 8th grade, they should have learned 80% of the concepts and skills previously taught in Algebra 1, 50% of the content traditionally taught in high school geometry, and a significant amount of statistics and probability previously taught in high school courses" (Georgia Department of Education, 2007, p. 2).

Most of the algebra content previously taught in Algebra 1 is now included in the eighth grade mathematics curriculum, with over half of the grade level standards focused in the algebra domain. Some of the concepts addressed in the eighth grade curriculum include solving equations, including equations with variables on both sides, equations with multiple variables, and absolute value equations, graphing linear equations and inequalities, solving systems of linear equations (both graphically and algebraically), graphing the solution set of a system of inequalities, and recognizing functions.

Results from the 2011 Criterion Referenced Competency Test (CRCT) indicate that fourteen percent of all eighth grade students in Georgia did not meet standards in mathematics (Georgia Department of Education, 2011c). For this particular test, "meeting standards" corresponded to answering at least 31 out of 60 questions, or 51.6 percent, correctly. Twenty-six percent of eighth grade students "Exceeded standards," which corresponded to answering 48 out of 60 questions, or 80 percent, correctly.

A further look at subgroups of students by race reveals that 20 percent of Black students in Georgia and sixteen percent of Hispanic students did not meet standards, compared to nine percent of White students. Similarly, 21 percent of Hispanic students and 14 percent of Black students exceeded standards, compared to 35 percent of White students. Results by socioeconomic status reveal that 24 percent of Economically Disadvantaged students did not meet standards compared to only eight percent of students who were not Economically Disadvantaged. Results for the researcher's school indicate a similar gap between White students and other groups in eighth grade mathematics, with sixteen percent of Black students and 21 percent of Hispanic students not meeting standards compared to eleven percent of White students. Similarly, 31 percent of economically disadvantaged students did not meet standards compared to six percent of students who were not economically disadvantaged (Georgia Department of Education, 2011c).

These data indicate that many students in the researcher's school, and across the state of Georgia, are not mastering the rigorous standards as required by the eighth grade mathematics curriculum. It is also evident that a gap continues to exist in the mathematics achievement of students of color and economically disadvantaged students compared to White students, despite the fact that all students are provided access to the same curriculum. A greater concern may be the number of students considered to be "meeting standards" when they have actually answered little more than half of the test items correctly. Many students, therefore, may be transitioning to high school without the understanding of algebra that is necessary for success in more advanced mathematics courses.

The Equity Problem

Standards-Based Reform. Since the 1980s, educational reform in the United States has been predominantly focused on standards as a means to ensure equity and excellence for all students. The No Child Left Behind Act (NCLB) of 2001 required that all students meet equal academic standards and tied federal funding to schools' performance on standardized tests that measured mastery of these standards. Gutierrez (2007), however, argues that the standards movement is confusing the meaning of *equity* with *equality*. Equity is defined as "justice" or "fairness," while equality means "sameness."

> For example, equality in a mathematics education setting might mean that all students are given the *same* access to powerful mathematics, the *same* quality of

> teachers, the *same* curricular materials, the *same* forms of teaching and the *same* supports for learning. This sounds good, if learning is universal and occurs in a social, political, and historical vacuum. However, in order to redress past injustices and account for different home resources, student identities, and other contextual factors, students need different (not the same) resources and treatment in order to achieve 'fairness.' Beyond holding that school approaches be the same, equality might also mean that student outcomes…are the same – that students all end up in the same place. Yet having all students reach the same goals does not represent 'justice' for students' own desires or identities. (pp. 40-41)

Sleeter (2005) has also noted that high standards are not the same as standardization. Standards are necessary to define "how well students are expected to master a given body of knowledge or skills" (p. 3), but standardization leads to bureaucratization when the curriculum is defined at the state or national level and success is measured in terms of test scores. Research has shown that grouping students based on ability does not improve academic achievement or promote more positive behaviors or attitudes (Oakes, 2005) and these findings, along with the continued achievement gaps between various groups of students, have led to a push for equality of access to higher level coursework, especially in mathematics. Equality, however, does not mean treating everyone the same. As Crenshaw (1997, p. 285) points out "it is fairly obvious that treating different things the same can generate as much inequality as treating the same things differently" (p. 23). Curriculum standardization as a way of eliminating inequity has, instead, resulted in "homogenizing the curriculum, even while classrooms in the United States have become more diverse" (Sleeter, 2005, p. 6).

Some theorists argue that the mandated standardization of the curriculum actually reproduces the socioeconomic status quo because the curriculum is "based on a Eurocentric and patriarchal ideology that presents knowledge as constructed by white males of European ancestry and ignores the experiences of marginalized groups such as African, Latino, and Native Americans and women" (Gutek, 2009, p. 331). While some theorists believe that all students should have access to the "privileged" knowledge of a college-preparatory curriculum and that providing students with this knowledge can help collapse the walls of race and class (Noddings, 1998), others argue that the standard liberal arts curriculum is simply the manifestation of privileged knowledge. A standardized curriculum treats students as a homogenous group who should all know and be able to do the same things, but "over the long term, standardization creates inequities, widening the gap between the quality of education for poor and minority youth and that of more privileged students" (McNeil, 2000, p. 6).

Providing access to algebra and other advanced mathematics courses for all students is an important goal for mathematics education. Students should not be excluded from taking advanced courses, but rather they should be encouraged to do so. Providing access, however, is very different from *requiring* all students to take such courses at a specific time. Noddings (1998) notes that

> Forcing all children to take algebra, physics, and foreign language will not in itself give them a share of privileged knowledge. Indeed, such a move may very well extend the hegemony of the dominant class. Not only will students be deprived of the choices Dewey thought so important in participation in democratic processes, but they may come to believe that there is only one ideal or model of educated persons. In a society that needs a vast array of excellences, this

could be debilitating. For children whose talents are ignored or undervalued, it could be tragic. (p. 70)

Sleeter (2005) also notes the importance of "diverse funds of knowledge" in a democratic society and argues that "it is to our benefit that we do *not* all learn the same thing, beyond the basic skills" (p. 7). There should be many definitions of a well-educated person rather than a single definition based solely on the academic disciplines. The current push for algebra in eighth grade implies that, to be well-educated, one must master these abstract concepts at that time. Students who do not master these concepts are labeled as "Does Not Meet Standards." But, as Pinar (2004) has argued, "There is no *educational* reason why everyone must take advanced algebra or chemistry or study Shakespeare" (p. 226).

NCTM's Equity Principle. Educational equity is a vital component of the vision of the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (2000). The Principles and Standards maintain the importance of all students having the opportunity, as well as the support necessary, to learn mathematics. "Equity does not mean that every student should receive identical instruction; instead it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (p.11).

A primary requirement for equity in mathematics education, according to NCTM's Equity Principle (2000), is high expectations for all students. But, high expectations are not enough. To accomplish the goal of equity, support must be provided as needed that is responsive to students' prior knowledge, strengths, and interests. Additionally, achieving equity requires resources and support for all classrooms and all students. Besides instructional resources, support must include professional development for teachers on understanding the strengths and needs of the diverse students they teach. Making equity a reality for all students is “both an essential goal and a significant challenge” (p. 11).

The “Achievement Gap.” For over two decades, research has indicated the existence of inequities in students’ mathematics achievement between students of color and Whites and between students of different socioeconomic classes. The term “achievement gap” has evolved to describe “circumstances in which some students, primarily those from racially, culturally, and linguistically marginalized and poor families, achieve less than other students” (Nieto & Bode, 2008, p. 12). While most research focuses on Black and White students, the “achievement gap” is also evident among students of other ethnic and racial backgrounds. Scores on the National Assessment of Educational Progress (NAEP) show an overall improvement in student achievement; however, the gap between African American and Hispanic students compared to White students remains very large. For example, in eighth grade mathematics in 2008, the average scale score for White students was 290, compared to 262 for Black students. While these scores represent an improvement for both groups, the gap of 28 points between the average scores is actually higher than it was in 1996 when the gap was 24 points. Since 1996, the gap between White students and Black students in eighth grade mathematics has ranged from 27 to 30 points on each test administration (Barton & Coley, 2008).

Nieto and Bode (2008) assert that the term “achievement gap” is somewhat of a misnomer as it suggests that students alone are responsible for their learning when student achievement is, in fact, influenced by many factors and is directly related to the conditions and contexts in which students learn. According to Joseph D’Amico (2001), the two major causes of the “achievement gap” are sociocultural and school-related factors. Sociocultural factors include ethnicity, poverty, and level of parent education, among others. One school-related factor is the expectations that teachers and schools have of students, which can unfortunately be associated with students' racial and social class backgrounds. Other factors, noted in a Policy Information Report produced by Educational Testing Services (Barton & Coley, 2008), include teacher preparation, teacher experience and attendance, class size, rigor of curriculum, technology-assisted instruction, and school safety. Nieto and Bode (2008) agree, stating that schools that have narrowed the "achievement gap" are characterized by "well-trained and motivated teachers who are teaching in their subject area; a curriculum that is culturally sensitive, challenging, and consistent with high academic standards; and a school culture that reflects a focus on high academic achievement among all students" (p. 13).

Differences in students' mathematics achievement have remained relatively constant despite the focus of public debate on the issue. The majority of research in mathematics education during this time has focused on achievement and cognition, with little attention to social or cultural issues (Lubienski, 2007). Ladson-Billings (1997) noted that such research merely summarizes achievement gaps and proposes solutions, without addressing the underlying problems and issues.

Some explanations for the differences in achievement have included variation in resources, insufficient opportunity to learn, inequalities of educational experiences in urban schools, and a lack of connection between the curriculum and students' cultures and experiences. While most of these explanations are to some extent agreed upon within the mathematics education community, other beliefs about the causes of inequity are more controversial. These views point to "inequitable and unjust relations of power and analyze the interconnections of oppressive societal structures" (Gutstein, 2007, p. 52).

Martin (2009) argues that just as race is socially constructed, achievement differences and gaps are also socially constructed; that is they do not tell us anything “factual, objective, or indisputable” about African-American, Hispanic, Native American, Asian-American, or White children. “What these so-called gaps do highlight are the adverse conditions under which some children are often forced to learn, the privileged conditions afforded to others, and how forces like racism are used to position students in a racial hierarchy” (p. 300). The adverse conditions to which Martin refers are the traditional teaching methods still practiced in many classrooms. As the number of culturally diverse students continues to increase, it becomes more compelling that all educators become more attentive to ways to adapt their practice to meet all students’ needs (Irvine & Armento, 2001).

Apple (2006) argues that schools do not only mirror societal inequalities based on social class and race, but they also produce these inequalities. He contends that policies intended to raise standards and make schools more competitive are damaging to those students who were already the least advantaged. Research indicates that reforms adopted with the goal of raising achievement may often be experienced by the students as disempowering and demotivating (Gillborn & Youdell, 2000). These experiences could lead to feelings of being treated unfairly, sending the message to students that not only is the world unfair, but that schools also privilege those who are already privileged in terms of class and race. This message is certainly not what we want to teach our students, but it may very well be what they learn when schools are led by the assumption that rigorous standards and high-stakes testing will somehow magically resolve deep-seated educational and social issues (Apple, 2006).

Adolescent Development

Adolescence is typically defined as the second decade of life, beginning around age 10 and ending in the early 20s. Due to the significant amount of psychological and social growth that occurs during this time period, the adolescent years are also viewed as a series of phases, beginning with early adolescence (about ages 10-13), middle adolescence (about 14-17 years old) and late adolescence (about ages 18-21). These phases correspond roughly to the way adolescents are grouped in schools, with early adolescents typically attending middle school or junior high school, middle adolescents attending high school, and late adolescents attending college (Steinberg, 2008). According to the National Middle School Association, adolescents undergo “more rapid and profound personal changes between the ages 10 and 15 than at any other time in their lives” (2003, p. 3). Many of these changes occur during adolescence due to cognitive development and psychosocial development and may influence students’ performance in and understanding of algebra.

Cognitive Development. Adolescent learners experience considerable change in their patterns of thinking as they grow toward more mature and abstract ways of thinking. This cognitive growth, however, occurs gradually and irregularly, so most early adolescents require “ongoing, concrete, experiential learning in order to develop intellectually” (National Middle School Association, 2003, p. 3). Although young adolescents begin to develop the capacity for abstract thought processes, this transition “varies significantly across individuals as well as across and within content areas” (Caskey & Anfara, 2007, p. 2).

As they develop, adolescents become less restricted to concrete ways of thinking and are able to reason about what is possible. They are increasingly able to reason deductively as well as inductively and to think in hypothetical terms. Adolescents also develop greater capacity with abstract concepts and begin to think about thinking itself (metacognition), allowing them to manage their own thinking and explain to others the processes they are using (Steinberg, 2008). Not all adolescents, however, develop these ways of thinking at the same age, nor do they employ them regularly in a variety of situations.

Studies of logical reasoning abilities for different age groups indicate that during early adolescence, in particular, individuals show noticeable development in reasoning, information processing, and expertise. Improvements in these abilities result in an increased capability of "abstract, multidimensional, planned and hypothetical thinking" during this period (Steinberg, 2005, p. 70). To a significant degree, however, these developments during adolescence are experience-driven. That is, the activities that adolescents engage in affect which connections in the brain will be strengthened and which will wither. For this reason, there is a high degree of variability in cognitive development between individuals during early adolescence. As individuals enter adolescence, development does not continue along the same pathways or to the same levels for everyone. Specifically within certain content areas, the range and depth of knowledge acquired by individuals becomes much more varied than in childhood. These differences are due, in part, to the experiences of each individual, as pruning of unused connections in the brain is guided by the activities in which an adolescent engages. Experience, therefore, is extremely influential during the adolescent years, making it is especially important for educators to consider the experiences that the developing adolescent brain should have (Kuhn, 2006).

Psychosocial Development. Some of the major psychosocial developments of adolescence include identity, autonomy, and achievement. While these issues are not unique to adolescence, they do play an important role in the overall development of adolescents and can contribute to the problems that some adolescents experience (Steinberg, 2008).

During adolescence, individuals experience changes in the ideas they have of themselves, how positively or negatively they feel about themselves, and the sense of who they are. Research indicates that early adolescents initially experience positive changes in self-concept after transitioning to middle school, but later experience declines to ratings at or below their elementary school values. Academic self-concept, in particular, is susceptible to significant declines during early adolescence, possibly due to the increased emphasis on grades rather than learning that is often associated with the transition to middle school (Parker, 2010).

Early adolescence is a time when students are more and more self-conscious, have a need for positive support from teachers and other adults, and desire increased autonomy and involvement in decision making. Often, there is a mismatch between these needs and the opportunities given to adolescents by their environments, especially schools. Referred to as stage-environment fit, this fit between the developmental needs of early adolescents and their educational environments is critically important (Eccles, et al., 1993).

Additional research suggests that the early adolescent years may begin a decline in school performance for some students that could ultimately lead to academic failure or school dropout. Declines have also been documented in intrinsic motivation, interest in school, self-concept, and confidence in one's intellectual abilities (Eccles, et al., 1993). Kuhn (2006) argues that disposition, more than competence, needs to be a focus of educators and others who are concerned with the development of early adolescents. Simply through the activities they choose, adolescents become better at what they are already good at. At the same time, however, they are developing a sense of personal identity, especially a sense of what they are good at and, conversely, what they are not good at.

Adolescents place great meaning and value on what they do. They draw on these meanings, both positive and negative, in defining who they are. Activities that are valued positively lead to a greater investment of time and effort, which leads to greater proficiency and thus greater valuing (Kuhn, 2006). Consequently, activities that are negatively valued may be avoided, leading to less expertise and less valuing. When these negatively valued activities include mandatory school subjects, such as algebra, attention to the disposition construct is essential in helping students experience success.

Theories of Cognitive Development

Piaget's Cognitive-Developmental Theory. Jean Piaget's theory regarding the developmental stages of children's cognition is one of the most well known developmental theories in the field of education. Piaget's work has provided mathematics educators with important insights into how children learn mathematical concepts and ideas (Ojose, 2008). Piaget (1969) asserted that the cognitive development of a child occurs through a continuous succession of stages. Each stage is an extension of the previous one and the experiences in each stage also build the foundation for movement to the next stage. While the order of succession of the stages remains constant, the ages at which they occur may vary with the individual child. These differences may depend on a variety of factors, including maturity, experience, culture, and ability (Ojose, 2008). Piaget identified four primary stages of cognitive development: sensorimotor, preoperational, concrete operational, and formal operational.

Sensorimotor intelligence. This stage generally occurs in children from birth to two years of age and is characterized by the progressive acquisition of object permanence, or the understanding that objects outside the child's field of vision still exist (Crain, 2011; Ojose, 2008). Mathematically, evidence suggests that children in this stage have some understanding of numbers and counting and can link numbers to objects, such as one dog or two cats (Ojose, 2008). Children in this stage should be provided a variety of daily activities that involve counting.

Preoperational thought. In this stage, which typically occurs in children from two to seven years of age, children begin learning to think by using symbols and internal images, but their thinking is not systematic or logical (Crain, 2011). This stage is also characterized by an increase in language ability and an egocentric perspective. Children in this stage may link together unrelated events, do not understand point-of-view, and cannot reverse mathematical operations (Ojose, 2008). For example, a child who understands addition such as 4 + 3 = 7 cannot perform the reverse operation of subtracting three from seven. Students in this stage should be provided concrete materials, such as blocks, to assist with problem solving tasks.

Concrete Operations. Generally occurring in children between the ages of seven and eleven (Crain, 2011), this stage is characterized by remarkable cognitive growth. Language development and acquisition of basic skills increase dramatically during this stage. Children at this stage develop the ability to think systematically, but require concrete experiences in order to do so. Children in this stage of development benefit from hands-on activities and the use of manipulatives in mathematics instruction (Ojose, 2008).

Formal Operations. This stage typically occurs anywhere between the age of 11 and adulthood, but may not occur in all individuals, and is characterized by the ability to think on abstract and hypothetical levels (Crain, 2011). A child at this stage begins to reason using pure symbols (Ojose, 2008). The dependence on the use of symbols in algebra requires that students have reached this stage prior to studying some of the abstract concepts included in the algebra curriculum.

Piaget recognized that children move through these stages at varying rates, but maintained that children do move through the stages in the same order. To Piaget, the stages were not genetically predetermined but simply represented increasingly complex ways of thinking (Crain, 2011). These ways of thinking are constructed by each individual child, spurred by the innate tendencies of human beings to organize our thinking and to adapt to our environment.

Cognitive organization is the tendency for thought to consist of systems whose parts are incorporated to form a whole. These systems work together and there are interrelationships between cognitive activities. As children develop through the cognitive stages, changes occur in the nature of cognitive organization and thoughts become organized into schemes, regulations, functions, concrete operations, or formal operations (Miller, 2011).

Cognitive adaptation refers to the interaction between an individual and the environment. Adaptation involves two processes: assimilation and accommodation. Assimilation is the merging of new experiences into one's existing cognitive organization. Accommodation is the development of new cognitive structures by modifying and adapting existing structures in response to the environment. Accommodation, however, can only occur in small steps and is related to the present cognitive level of the child. If an experience is too different from a child's current level of understanding, accommodation is not possible. In algebra, for example, if a student does not understand the meaning of a variable and the basic concepts of solving an equation, the experience of solving a system of equations would be too large a step to attempt and accommodation would not be possible. The processes of assimilation and accommodation are closely intertwined in all cognitive activity and Piaget often defined adaptation as an equilibrium between assimilation and accommodation, where neither process is dominant (Miller, 2011).

Vygotsky's Sociocultural Theory of Cognitive Development. In Lev Vygotsky's view, cognitive development cannot be understood without considering the sociocultural context of the child. This context defines and shapes children and their experiences which, in turn, shape the future of the culture (Miller, 2011). Vygotsky (1978) acknowledged that the "natural line" of development, coming from the child alone, is important and may dominate cognitive development up to about two years of age, but after this, development of thought is increasingly influenced by the "cultural line" of development.

Vygotsky (1934/1986) also asserted that the highest levels of thinking, those involving abstract or theoretical reasoning, require instruction in concepts such as writing and math. While children may develop some concepts on their own, the development of purely abstract thinking requires instruction in abstract sign systems. It is necessary, therefore, for students to be instructed in algebra. They will not develop an understanding of algebraic concepts on their own without guidance. Vygotsky (1978) defined the zone of proximal development as the distance between a child's "actual developmental level as determined by independent problem solving" and the higher level of "potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). The actual developmental level is the level of development that is established based on what a child can do on his/her own. Vygotsky further described the zone of proximal development as functions that are in the process of maturation, functions that are "the 'buds' or 'flowers' of development rather than the 'fruits' of development (p. 86).

Vygotsky's zone of proximal development challenged the prevailing notion that the mental development of children could only be measured by their independent activity, rather than their imitative activity. Vygotsky argued that a person can only imitate concepts and skills that are within their development level. For example, if a child is struggling with an arithmetic problem and the teacher models how to solve the problem, the child may then understand the solution. On the other hand, if the teacher were to solve a problem in higher mathematics, outside of the child's developmental level, the child would be unable to understand the solution regardless of how many times s/he imitated it (Vygotsky, 1978).

Children, however, are able to imitate actions that exceed the limits of their independent capabilities. Using imitation, children are able to accomplish much more in collaboration with peers or under the guidance of adults. Therefore, learning which focuses on previously reached development levels is ineffective as it does not aim for a higher level of the developmental process. Vygotsky's theory of the zone of proximal development suggests that the only "good learning" is that which occurs in advance of development (Vygotsky, 1978, p. 89).

Vygotsky's theory, therefore, supports the notion that students should be encouraged to learn algebra, perhaps even in eighth grade. It should be noted, however, that each child's zone of proximal development is different and should be considered before introducing a child to a concept that is outside his or her developmental level.

Bruner's Theory of Readiness for Learning. Bruner (1960) asserted that "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (p. 33). Bruner supports his statements by examining the process of children's intellectual development, the act of learning, and the notion of the spiral curriculum.

Intellectual Development. Bruner's (1960) ideas on intellectual development are based in the stages previously identified by Piaget. To Bruner, the task of teaching was one of translation, of representing the subject "in terms of the child's way of viewing things" (p. 33). He argued that any idea can be represented in a way that can be understood by school-age children and "can later be made more powerful and precise" (p. 33). For example, children in the stage of concrete operations are capable of understanding intuitively and concretely many of the basic ideas of mathematics, but only in terms of concrete operations. Children in this stage can play mathematical games with rules that are based on higher mathematics. They will struggle, however, if given a formal mathematical description of the rules. After practice in concrete operations and when the appropriate stage of development has been reached, children can be introduced to the formal mathematics.

Bruner (1960) noted the importance of helping children progress from concrete thinking to the use of more abstract modes of thought, but stressed the futility of attempting to do this by presenting formal explanations that are outside the child's way of thinking. When this happens, the child learns only to apply certain rules, such as mathematical formulas, without understanding their significance or connectedness because the rules have not been translated into his/her way of thinking. Instruction should, however, provide challenging opportunities for the child to move ahead in his/her development.

The act of learning. According to Bruner (1960), learning takes place in a series of episodes. Good learning episodes reflect what has been previously learned and allow for generalization beyond it. These episodes involve three processes that occur almost simultaneously. First, acquisition of new information occurs. This information often contradicts or replaces what a person has previously known. At a minimum, this process is a refinement of knowledge. The second process described by Bruner is transformation, the process of manipulating acquired knowledge to make it fit new tasks. Bruner describes transformation as comprising “the ways we deal with information in order to go beyond it” (p. 48). The third process of learning is evaluation in which a person judges whether the way they have dealt with information is appropriate to the task.

The spiral curriculum. As previously discussed, Bruner (1960) popularized the idea of the spiral curriculum. He asserted that there are basic ideas at the core of all mathematics. For students to gain command of these basic ideas and use them effectively, “requires a continual deepening of one’s understanding of them that comes from learning to use them in progressively more complex forms” (p. 13). If these basic ideas are put in formalized terms as equations, they are out of reach of the young child. The child must first be allowed to understand the concepts intuitively. Instruction in these basic ideas should begin “as intellectually honestly and as early as possible” (p. 54). The curriculum should then “revisit these basic ideas repeatedly, building upon them until the student has grasped the full formal apparatus that goes with them” (p. 13).

CHAPTER 3

METHOD

In this study, I utilized a qualitative research design to examine students' experiences with middle school mathematics, particularly the algebra concepts required by the eighth grade curriculum. Qualitative research draws from the philosophies of constructionism, phenomenology, and symbolic interactionism and, therefore, a primary purpose of qualitative research is to understand "how people interpret their experiences, how they construct their worlds, and what meaning they attribute to their experiences" (Merriam, 2009, p. 14). Qualitative research allows for the study of a phenomenon in depth and detail. The wealth of detailed information obtained in a qualitative study "increases the depth of understanding of the cases and situations studied" (Patton, 2002, p. 14). To develop this in-depth understanding, "the raw data of personal experiences are vital" (Ornstein & Hunkins, 2009, p. 140).

An important aspect of qualitative research is a focus on meaning and understanding. A key concern is developing an understanding of the phenomenon of interest from the perspective of the participant, not the researcher. This perspective is often referred to as the emic perspective, or insider's perspective, as opposed to the etic, or outsider's, perspective (Gall, et al., 2007). Merriam (2009) argues that "research focused on discovery, insight, and understanding from the perspectives of those being studied offers the greatest promise of making a difference in people's lives" (p. 1). The purpose of this study was to understand the personal experiences of eighth grade students, from their own perspectives, as they encounter the algebra concepts required by the eighth grade mathematics curriculum.

Theoretical Framework

This study draws upon the constructivist view of how students learn and understand mathematics. Constructivism is a theory of active knowing and learning, the roots of which can be traced back to at least the eighteenth century and the work of Giambattista Vico. Vico asserted that humans can know nothing other than the cognitive structures they have themselves put together (von Glasersfeld, 1996). Von Glasersfeld (1996) hails Jean Piaget as the "most prolific constructivist in our century" (p. 6) and most constructivists in education trace their roots to Piaget (Noddings, 1998). Piaget's influence has established constructivism as a primary theoretical perspective on learning mathematics (Ernest, 1997). For Piaget, knowledge is a collection of the conceptual structures that have been adapted within a subject's range of experience, which will always include social interactions with other subjects. Von Glasersfeld (1996) summarizes Piaget's theory of cognition as consisting of two basic concepts, assimilation and accommodation. In Piaget's theory of learning, learning takes place when a scheme leads to perturbation, rather than producing the expected result. This perturbation then leads to accommodation which establishes new equilibrium (von Glasersfeld, 1996). Learning, therefore, is a process of self-organization in which the learner attempts to eliminate perturbations by reorganizing his or her activity.

A primary claim of the constructivist approach is that knowledge cannot be transmitted; it must be constructed by the learner (Noddings, 1998; Skovsmose, 2005). Constructivists following Piaget's theories previously distinguished between "developmental learning" and "rote learning" (Noddings, 1998, p. 115). These constructivists believed that developmental learning requires active participation by the learner and makes a "lasting difference in how students approach problems and new situations" while rote learning is "passive, temporary, and useless for further learning" (p. 115). It has been noted, however, that if all cognition is active, then rote learning must be active as well. Constructivists now distinguish between strong and weak constructions, with active learning being considered stronger than passive learning. Many constructivist teachers have thus moved away from lecturing and believe that they need to know "what and how students are thinking" in order to aid their learning (p. 116).

Piaget's constructivism has had a considerable impact on research in mathematics education and constitutes a key theoretical perspective on learning mathematics. Piaget's methodology focuses on the use of the clinical interview, in which participants perform specific tasks in the presence of an interviewer. As the participant completes the task, the interviewer may use prompting or questioning to gather additional information. This method is used to study the participant's understanding of mathematical topics and can provide in-depth information about an individual's thinking and cognitive processing (Ernest, 1997).

School Profile

A school profile was developed to provide information about the history and the setting of the school. This profile helps to provide a context of the students in their learning environment. Demographic information about the students and staff, academic performance data, and other pieces of information are included in the profile.

Leader Middle School (LMS) is located in Coweta County, about 40 miles south of Atlanta, Georgia. Coweta County has experienced tremendous growth over the past ten years and in some areas has transitioned from a rural community into a suburban community. LMS is the newest of six middle schools in the district, serving approximately 1,000 students in grades six through eight. The school opened in the fall of 2006 with about 750 students. While enrollment at other middle schools in the district has declined during this time, LMS is positioned in an area of the county that continues to experience growth.

The administration and faculty at Leader Middle School has remained stable over the past five years. The principal and many other faculty members have been at LMS since the school opened. Currently, there are 65 faculty members at LMS, 80% of whom are female. Over half of the faculty (58%) has advanced degrees.

Each grade level at LMS is housed on a separate hallway, along with special education resource rooms for the grade level. Connections classes are mostly located on a fourth hallway, although some connections classes are held on the grade-level hallways due to limited classroom space in the building. Each student attends four seventy-minute academic classes (Literature/Language Arts, Math, Science, and Social Studies) and two forty-five minute Connections classes each day. Connections classes change at the end of each nine-week period, with the exception of Band and Chorus which are year-long classes.

Leader Middle School currently serves approximately 1,000 students. Of these students, 70% are White, 18% are Black, 7% are Hispanic, and 3% are Multiracial. The students at LMS come from a wide range of economic backgrounds, with 28% of the students qualifying for free or reduced lunch. Ten percent of the students at LMS have been identified as having some type of disability (Georgia Department of Education, 2011b). Most of these students with disabilities are served in the general education classroom through a co-teaching model. LMS does have, however, three self-contained special education classes for students with mild to moderate intellectual disabilities, emotional/behavioral disorders, and autism.

Students at LMS perform very well academically and the school has made Adequate Yearly Progress (AYP) every year since it opened. On the 2011 Criterion-Referenced Competency Test (CRCT), 88.8% of students at LMS met or exceeded standards on the Math portion of the test and 96.7% of students met or exceeded standards on the Reading portion of the test (Georgia Department of Education, 2011a). Additionally, 90% of 8^{th} grade students met or exceeded standards on the Georgia Middle Grades Writing Assessment (Georgia Department of Education, 2011b). Slogans such as “The School of Champions” describe the culture that is present at Leader Middle School. The principal, staff, and students have a competitive spirit and hold themselves to high expectations. Accomplishments are celebrated frequently in academics, athletics, and extracurricular areas such as band, chorus, and art.

Participants

The participants in this study were students at Leader Middle School who have experienced the eighth grade mathematics curriculum, specifically students who have had difficulty with the algebra domain of the curriculum. Students were selected using a purposeful sampling procedure. The goal of purposeful sampling is to select cases that are “likely to be ‘information-rich’ with respect to the purposes of the study” (Gall, et al., 2007, p. 178). Specifically, intensity sampling was used to select students who have had difficulty with the algebra concepts taught in the eighth grade math curriculum. Gall, Gall, and Borg (2007) define intensity sampling as a strategy to select cases that exhibit a key characteristic “to a high or low, but not extreme, extent” (p. 181). Participants selected for this study, therefore, were students who had experienced a high degree of difficulty with algebra, but not an extreme degree of difficulty. In other words, the participants were not failing their math class, but had experienced greater difficulty in eighth grade mathematics compared to previous years.

Eighth grade math teachers were asked to recommend students to participate in the study, based on students’ classroom performance with algebra concepts, including scores on benchmark assessments administered during the school year. Teachers were instructed to recommend students who were motivated to learn and had put forth good effort, yet had still struggled. I was not looking for students who were struggling due to a lack of effort or students who had a history of difficulty in mathematics. I was interested in speaking to students who had performed relatively well in mathematics prior to reaching eighth grade, but had struggled with the algebra concepts required by the eighth grade curriculum.

Ten students were recommended by the teachers and each recommended student was preliminarily interviewed by the researcher to determine his/her willingness to participate in the study. During this interview, I explained the purpose and methodology of the study and answered any questions the students had about the study. Students were informed that all information shared during the study would remain confidential. To ensure confidentiality of all participants' responses, each participant selected a pseudonym to be used throughout the study. Parents of the recommended students were initially contacted by phone to discuss the purposes of the study and parental informed consent letters were sent home to be signed (see Appendix C). Students were also provided a minor's assent letter to be signed prior to participating in the study (see Appendix D).

Six of the ten recommended students returned their parental consent documents in time to participate in the study. Of these six students, three were male and three were female. The male students consisted of two Black students and one White student. The female students consisted of two White students and one Black student. I selected all six of these students to participate in the study as they provided a good representation of race and gender and had each demonstrated an interest in the study during the initial interview. Students' socioeconomic status could not be disclosed and, therefore, is not considered in this study.

Data Collection

Merriam (2009) describes qualitative data collection as "asking, watching, and reviewing" (p. 85). The data collected for this study incorporated all three of these methods in a variety of ways. The primary source of data involved interviewing, or "asking", students about their experiences with algebra concepts. Some of these interviews also involved “watching” the students as they worked through an algebra task. Additionally, data collection included “reviewing” school and student profiles and documents such as student work samples.

Student Participant Profiles. Student participant profiles were created to help develop a sense of background and context for each participant. To develop the profiles, I reviewed each participant’s permanent record, specifically looking at the mathematical history of each student, including grades and standardized test scores. Students were also asked several background questions during the first individual participant interview to provide further information.

Individual Participant Interviews. The primary source of data for this study was a series of individual participant interviews. According to Yonezawa and Jones (2006), student voice can be a “powerful tool to shaping educational reform and policies” as “their perspectives often capture the realities of classroom and school life in vivid detail” (p. 21). To collect these vivid details, participants were interviewed using a semi-structured interview protocol. All interviews were conducted during the students’ connections class period (with permission of the teacher) or after school to ensure that academic class time was not interrupted. The interviews were audio-recorded and transcripts were produced. Participants were asked to review the transcripts to ensure the accuracy of the interview.

Individual participant interviews were conducted in several sessions (see Appendix A for interview questions). The purpose of the first session was to develop rapport and obtain background information on each participant. Information gathered at these sessions was used in developing the student profiles. A second individual session was conducted to gather information on participants’ perceptions of algebra, difficulties with algebra, strategies employed by their teachers, and previous experiences with mathematics. A third individual interview was then conducted using the think-aloud protocol approach.

Think-Aloud Interviews. The think-aloud protocol is based in the clinical interview method used by Piaget. Ernest (1997) considers this method a "seminal contribution to qualitative research methodology in mathematics education" (p. 29). The goal of using this protocol was to draw out "the inner thoughts or cognitive processes that illuminate what's going on in a person's head during the performance of a task" (Patton, 2002, p. 385). During these think-aloud interviews, participants were asked to work several algebra tasks (see Appendix B for tasks) while thinking aloud about their understanding of the problem. The participant's written work was completed using a Livescribe Echo smartpen, which captured the student's written work in conjunction with the student's voice. As the participant was working through each task, I asked questions to encourage the participant to talk about what she or he was thinking. Task selection for the think-aloud interviews was based on concepts identified in the literature as being problematic for many students (expressions and equations, use of the equal sign, and use of variables), as well as related concepts observed by the researcher as being problematic for students in her school (linear equations and systems of linear equations).

These individual interviews were followed by a focus group interview with all participants. This series of interviews was important to capture and describe how the students experience algebra – "how they perceive it, describe it, feel about it, judge it, remember it, make sense of it, and talk about it with others" (Patton, 2002, p. 104).

Focus Group Interview. A focus group interview was conducted after all individual and think-aloud interviews had been completed. As with the individual interviews, the focus group interview was held during the participants' connections classes. This interview was also audio-recorded and transcribed and participants were asked to review the interview transcripts for accuracy. For the focus group interview, the research questions for the study were presented directly to the group for the discussion. Specific follow-up topics related to each question were also derived from participants' responses to questions during the individual interviews (see Appendix C).

A benefit of focus group interviews is that participants are able to hear each other's responses which may lead them to make additional comments as they hear what other participants have to say (Patton, 2002). This interaction among the participants can enhance the quality of the data (Krueger & Casey, 2000) while also helping the participants to feel more comfortable. Participants who may have been uncomfortable sharing their stories in an individual interview may draw "confidence and a sense of safety" (Patton, 2002, p. 389) from being part of an interview group.

The focus group interview was moderated by the researcher. As Merriam (2009) points out, it is important that the moderator be familiar with group processes. The role of the moderator is not only to pose questions to the focus group members, but also to guide discussion *between* the group members (Patton, 2002). The researcher has studied the use of focus groups in her doctoral coursework and has also participated in focus groups, both as a member and as part of an interview team for focus groups of teachers, students, and parents. Therefore, the researcher has the necessary knowledge and experience to moderate a focus group session.

Documents. According to Merriam (2009), many types of documents can be used to "uncover meaning, develop understanding, and discover insights" relevant to a research problem (p. 163). Similarly, Patton (2002) describes documents as "a particularly rich source of information" (p. 293). Document analysis can provide a researcher with information about things that cannot be observed or reveal things that occurred before the research began. Documents may also stimulate additional inquiry that can be pursued by observations and interviews (Patton, 2002).

The documents gathered for this study included student work samples and prior test scores, including benchmark tests and standardized tests. Teachers were asked to provide copies of all student work during the data collection period. These work samples could include class work, homework, quizzes, tests, projects or other papers. The collected work sample documents provided additional information about the participants' understanding of algebra. For the purpose of data analysis, all documents were photocopied and any identifiable information erased.

Data Analysis

Merriam (2009) describes data analysis as "the process of making sense out of the data" (p. 175). Patton (2002) also argues that "making sense out of massive amounts of data" is a challenge in qualitative inquiry (p. 432). Data collected for this study were analyzed using the constant-comparative method of data analysis, which involves "comparing one segment of data with another to determine similarities and differences" (Merriam, 2009, p. 30). To begin the analysis, all data were compiled into a computer database and then broken into segments. These segments were based on the research questions to be answered and, therefore, were named as follows: Student Perceptions of Algebra, Teacher Strategies, Difficulties with Algebra, and Previous Experiences with Mathematics. Data were reviewed multiple times by the researcher and relevant pieces of data coded. Each interview transcript was coded separately and the codes were then compared across transcripts to identify categories or themes "that capture some recurring pattern that cuts across" the data (p. 181). According to Gall, Gall, and Borg (2007), these categories must "adequately encompass and summarize the data" (p. 467).

Findings for each participant were synthesized to develop a textural description of each participant's experience of algebra. A textural description is "an account of an individual's intuitive, prereflective perceptions of a phenomenon from every angle" (Gall, et al., 2007, p. 496). Additionally, the themes for each participant were compared and synthesized across all participants as a basis for a structural description. A structural description is an "account of the regularities of thought, judgment, imagination, and recollection that underlie the experience of a phenomenon and give meaning to it" (p. 496).

Validation Strategies

Qualitative research does not seek to capture an objective truth or reality; nevertheless, it is important to ensure the credibility of the findings. One of the best known methods for increasing internal validity of a qualitative study is triangulation. Various methods are described in the research literature for ensuring the quality and rigor of qualitative research (Gall, et al., 2007). Credibility for this study was sought by using multiple data collection methods, collection of data on multiple occasions over time, rich thick description, researcher reflexivity, and peer examination.

Data were triangulated among the various forms of data that were collected in the study, including participant profiles, individual interviews, focus group interviews, and documents. Interviews were conducted on multiple occasions and follow-up questions were asked as needed. Rich, thick description is achieved by presenting the participants' voices as direct quotations and by providing a detailed description of each participant. I also kept a record of my thoughts and reactions in a reflective journal during the course of the study in order to be aware of my personal biases related to the study. Finally, I enlisted the assistance of a colleague who is familiar with qualitative data analysis to discuss the findings as they emerged and to review a draft of the report.

Summary

This study utilized a variety of data collection methods in order to understand the experiences of eighth grade mathematics students with algebra concepts. These methods included participant profiles, individual participant interviews, focus group interviews, and document analysis. Data were analyzed to identify themes for each participant as well as themes that occur across participants. Collection of data from multiple sources and on multiple occasions, along with rich thick description, researcher reflection, and peer examination provide validity for the study.

CHAPTER 4

RESULTS

In this chapter I present the results obtained from data analysis. The purpose of this study was to explore students' experience with the eighth grade mathematics curriculum, particularly the algebra concepts required by the curriculum. The primary source of data for this study was student interviews. Six students were interviewed individually in two sessions (see Appendix A for interview questions). The interview questions were organized into segments to address each of the research questions. Following the individual student interviews, each student participated in a think-aloud interview during which he or she solved a series of algebra problems while discussing the problem aloud. Finally, a focus group interview was conducted with all students to provide for additional discussion. Prior to the discussion of findings for each group of interviews, a short vignette is presented. These vignettes provide a context for the participants' words which are used to present the findings of the study.

Data Analysis

The data collected for this study were analyzed using the constant-comparative method of data analysis. Data analysis began following the first individual student interview as I listened to the recorded interviews and produced transcripts. After the transcripts were produced I listened to the recorded interviews again while comparing them to the transcripts. After all individual student interviews were conducted and transcribed, the data were compiled into a computer database and divided into segments based on the research questions to be explored. The segments, therefore, were named as follows: Student Perceptions of Algebra, Teacher Strategies, Difficulties with Algebra, and Previous Experiences with Mathematics.

After compiling the individual student interview data, I reviewed each student's interview transcripts multiple times and coded relevant pieces of data. I coded the data for each student individually and then compared the codes across students to look for patterns. Codes were then grouped into categories. The codes used are listed below.

For the segment of the data related to Student Perceptions of Algebra, the following codes emerged:

P-c Perceptions: Confidence in algebra ability

P-d Perceptions: Difficulty of algebra

P-dl Perceptions: Dislike algebra

P-fp Perceptions: Fast pace of instruction

P-i Perceptions: Importance of algebra in real life

P-te Perceptions: Teacher expectations

P-u Perceptions: Understanding of algebra

Next, I analyzed the segment of data related to Teacher Strategies and identified the following codes:

TS-c Teacher Strategies: Cooperative learning

TS-d Teacher Strategies: Direct instruction

TS-eh Teacher Strategies: Extra help

TS-es Teacher Strategies: Encouragement/support

TS-he Teacher Strategies: High Expectations

TS-ii Teacher Strategies: Individualized instruction

TS-ip Teacher Strategies: Independent practice

TS-m Teacher Strategies: Mastery learning

TS-sc Teacher Strategies: Structured classroom

TS-st Teacher Strategies: Students teach

TS-tp Teacher Strategies: Test prep

As I analyzed the segment of data related to Difficulties with Algebra, the following codes emerged:

D-e Difficulty: Easier concepts

D-fp Difficulty: Insufficient time to learn

D-fs Difficulty: Formulas and steps

D-i Difficulty: Integers

D-lc Difficulty: Lack of connection to real world

D-le Difficulty: Linear equations

D-lu Difficulty: Lack of understanding

D-r Difficulty: Radicals

D-v Difficulty: Variables

D-wp Difficulty: Word problems

Finally, for the segment of data related to Previous Experience with Mathematics, the following codes were identified:

PE-p Previous Experiences: Preparation for eighth grade math

PE-tn Previous Experiences: Teacher experience, negative

PE-tp Previous Experiences: Teacher experience, positive

Data from the student think-aloud interviews were used to supplement the interview data related to students' difficulties with algebra. During the think-aloud interviews, students used a Livescribe smartpen to work the algebra problems. This pen recorded a video of each student's writing as well as his or her voice. The audio portion of each think-aloud interview was transcribed and then comments were inserted based on the recorded written work. These comments helped to identify what the student was doing or writing at the time. After compiling the transcripts and reviewing them for accuracy, relevant pieces of data were coded. Several of the same codes emerged that were identified in the individual student interviews, including:

P-c Perceptions: Confidence in algebra ability

D-fs Difficulty: Formulas and steps

D-i Difficulty: Integers

D-lc Difficuluty: Lack of connection to real world

D-le Difficulty: Linear equations

D-lu Difficulty: Lack of understanding

D-wp Difficulty: Word problems

Additional codes that emerged from the think-aloud interviews included:

D-eq Difficulty: Equations

D-fr Difficulty: Fractions

D-io Difficulty: Input-output table

D-m Difficulty: Misconception

After reviewing the individual student interview transcripts and think-aloud interview transcripts, and coding the data as described above, I compared the codes across students to look for patterns. I noted the codes that occurred most frequently across participants and, from these patterns, I identified areas for further discussion during the focus group interview. Following the focus group interview, I generated a transcript of the interview data and reviewed it several times. As the focus group interview questions were related to the individual interview questions, the focus group interview data were coded using many of the same codes from the individual student interviews. Additional codes that emerged from the focus group interview data included:

TS-g Teacher Strategy: Grading practices

TS-hw Teacher Strategy: Homework

TS-r Teacher Strategy: Resources

Validation Strategies

Credibility for this study was sought by using multiple data collection methods, collection of data on multiple occasions over time, rich thick description, researcher reflexivity, and peer examination. Data were triangulated with the various forms of data that were collected in the study, including participant profiles, individual interviews, focus group interviews, and documents.

Student interviews were conducted on multiple occasions over the course of a one month period. Each subsequent interview was held approximately one week after the previous interview, allowing time for transcription and researcher reflection between interviews. Participants' own words were used to present the findings of the study, providing rich thick description of their experience with algebra.

During the course of the study, I also kept a reflective journal to record my thoughts and reactions. I made notes in this journal after each student interview and at the end of each interview day. According to Patton (2002), the period after an interview "is critical to the rigor and validity" of a qualitative study, as it is a time for "reflection and elaboration" (p. 383). During this time, "insights can emerge that might otherwise have been lost" (p. 384). I also added to these notes as I was transcribing the interviews. This reflective journal represented the beginning of my data analysis as it contained my initial thoughts about student responses. Based on my experience as an eighth grade math teacher, some of the students' responses surprised me, while others did not. I used the journal to document any ideas or interpretations that arose during this reflective time. Questions or comments on information shared by the participants were noted for follow-up in the next interview. For example, during the individual interviews most students responded that learning algebra is important, but they did not believe they would use it in their lives outside of school. This discrepancy was noted in my reflective journal for follow-up during the focus group interview. This process of researcher reflection aids in ensuring the quality and rigor of the research (Gall, et al., 2007).

During data analysis, I also enlisted the assistance of a colleague who is familiar with qualitative data analysis. My colleague reviewed the coded interview transcripts with me and helped me determine the final list of codes to be used for analysis. During our discussion, we added some codes and combined some related codes. For example, in the segment of data related to students' perceptions of algebra, I began with several specific codes regarding students' confidence in algebra, such as test anxiety, lack of confidence, etc. These codes were later combined to form a more general code: confidence in algebra ability. In the segment of data related to teacher strategies, our discussion led to the addition of codes for mastery learning and structured classroom. In the segment of data related to students' difficulties with algebra, I began with concept-specific codes for various concepts that participants reports as being easier than other concepts, but later combined these into one code: easier concepts.

My colleague and I also discussed the findings for each research question as they emerged. Before I began writing my report, my colleague and I discussed the structure of how the findings should be reported. We decided to organize the findings by research question. As I was writing my report, my colleague reviewed several drafts of the report and offered feedback to improve the clarity of the report. For example, I had originally tried to include the findings from the think-aloud interviews in the same section with the findings from the other interviews. After a discussion with my colleague, I decided it would be best to report the findings in two separate sections. I kept the findings from the individual student interviews and the focus group interview together as the questions were connected. The think-aloud interviews, however, were quite different and, therefore, were best reported in a separate section.

Participant Profiles

In this section, I present a short profile of each student participant. These profiles were compiled using data collected during the initial student interview as well as documents such as attendance records and test scores. For the CRCT, a score below 800 is considered “Does Not Meet Standards,” a score of 800 to 849 is considered “Meets Standards” and a score of 850 or above is considered “Exceeds Standards.”

Allen. Allen is a fifteen year old African American male. He is originally from Tampa, Florida and moved to his current home when he was in 3rd grade. He attended elementary school in the district and has attended LMS for 6th, 7th, and 8th grade. Allen's parents are divorced and he lives with his mother and their dog, Sally, a German Shepherd/Lab mix. Allen's father lives in Florida. He also has two older sisters who are both college softball players in New York. He is able to visit them occasionally, and also attends some of their softball games when they travel. "I just got back from like California and Florida." Allen is a member of the LMS track team and also plays football. He's been asked about playing basketball, but says "it's not my thing."

Allen's favorite subject is social studies because "it comes easy for me. I like to learn about like the Indians and stuff like that." His least favorite subjects are math and science. In math, he says "the class went faster this year" and it "gets confusing sometimes." Outside of school and sports, Allen enjoys just hanging out with his friends.

Overall, Allen has been an average math student in middle school. In 6th grade, he had a yearly average of 87 and in 7th grade his yearly average was 74. In 8th grade, his report card scores were up and down, with a 72 for the first nine-week grading period, an 80 for the second nine-week period, a 65 for the third nine-week period, and an 84 for the fourth nine-week period. Allen had ten absences from math class during the year, most of which occurred during the third and fourth nine-week periods. Allen's math CRCT scores have been relatively consistent throughout middle school. He scored 811 on the 6th grade CRCT, 823 in 7th grade, and 810 in 8th grade. A breakdown of Allen's CRCT scores by domain is shown in Table 1.

Table 1

Allen's CRCT scores by domain

Domain	6th Grade % correct	7th Grade % correct	8th Grade % correct
Algebra	61%	58%	53%
Data Analysis & Probability	100%	89%	90%
Geometry	42%	60%	43%
Number Operations	44%	67%	62%
Measurement	42%	Na	na

Allen reported that he usually has time to complete his math homework in class. But, if he does not finish it in class, he will usually do it first when he gets home to "get it out of the way." Sometimes he calls his mother to ask for help or he can also call his sisters "'cause they love math." According to Allen, his mother and sisters are both able to help him with the math when needed. He does not think there is any more they could do to help him, other than "just [make him] study more."

Bob. Bob is originally from Charleston, South Carolina, but moved to Georgia when he was about two years old. He lived in a nearby school district for several years and moved to the LMS school district at the beginning of sixth grade. He is a fourteen year old Black male and lives with both parents, his eleven year old sister, and his two cousins who are ten and fifteen years old. He describes his cousins as being "like a brother to me."

Bob is very involved in sports, including football, soccer, and track. Bob says his favorite sport is football because "there's a lot of contact in it and...if I have a bad day at school I like to get all my anger out on the football field." He is a very good football player as well and was selected to play in the Army All-American game in Texas.

Bob reports that math had always been his favorite subject until this year. Overall, he says he still likes math. He enjoys the process of being introduced to something new "and I don't get it at first and then people start to help me along and then I understand it." Bob's least favorite subject is reading. While he does not have any problems with reading, it is just not something that he loves to do: "some people love to read like every day but I'm not one of those people."

Bob has been an average math student throughout middle school, with a 77 yearly average in sixth grade math and a 75 yearly average in 7th grade math. In eighth grade, his report card grades fluctuated from 66 for the first nine-week grading period to 74 for the second nine-week grading period, 60 for the third nine-week grading period, and 78 for the fourth nine-week grading period. On the math CRCT, Bob scored 807 in 6th grade and 816 in 7th grade. On the first administration of the 8th grade CRCT, Bob scored 781, which is considered "Does Not Meet Standards." On the retest, however, he earned a passing score of 820. A breakdown of his scores by domain is shown in Table 2.

Table 2

Bob's CRCT scores by domain

Domain	6th Grade % correct	7th Grade % correct	8th Grade % correct	8th Grade Retest % correct
Algebra	56%	58%	40%	73%
Data Analysis & Probability	56%	78%	40%	40%
Geometry	83%	53%	29%	86%
Number Operations	33%	58%	46%	62%
Measurement	33%	na	na	na

Bob stated that when he has math homework, he will usually "procrastinate or something" and "try to get everything else done first." He says he gets distracted when doing his homework and his mother has to frequently remind him to finish his work. Bob reports that his mother is able to help him with the math homework and his dad tries to help, but "he didn't really like math in school either so my mom mostly does everything." When his mother helps him, Bob reports that she will "make me keep doing it over and over again...but sometimes I just want to take a break" and when his dad helps him Bob feels "like me and him are on the same page."

Callie. Callie is a thirteen year old White female who was born in North Carolina and moved to Georgia in third grade. She has attended LMS since 6th grade. Callie lives with both of her parents and is an only child, although she has "neighborhood friends and stuff." She lives "across from a park so there're always people." She was a member of the LMS pep squad and also took voice lessons for several years. She reports that she is "not very athletic" so she is not involved in any sports. When asked about anything else she does outside of school, Callie replied, laughing, "I'm always here." Callie's mother is a 6th grade teacher at LMS.

Callie's favorite subject is literature as it "just comes more easy...than math and science 'cause those are more logical like having to work things out and I have more of reading and answering questions type skills." Callie's least favorite subject is math. She says in math "it's harder to pick answers 'cause if you mess one little thing up it screws the whole problem up." Callie reports that math has been her least favorite subject since 5th grade when she had "a teacher who kept telling me I was bad at math and I started believing it. And in 5th grade I didn't pass the CRCT and so ever since then I have like really bad anxiety problems."

Callie has mostly been a "B" student in middle school math. In 6th grade, she earned a yearly average of 87 and in 7th grade she earned a yearly average of 84. In eighth grade, her report card grades have been 84 for the first nine-week grading period, 94 for the second nine-week period, 78 for the third nine-week period, and 84 for the fourth nine-week period. After not passing the math CRCT in 5th grade, Callie scored 815 on the 6th grade CRCT and 846 on the 7th grade CRCT. For 8th grade, she scored 823 on the math CRCT. A breakdown of her scores by domain is shown in Table 3.

Table 3

Callie's CRCT scores by domain

Domain	6th Grade % correct	7th Grade % correct	8th Grade % correct
Algebra	67%	67%	63%
Data Analysis & Probability	78%	89%	90%
Geometry	58%	87%	57%
Number Operations	56%	92%	69%
Measurement	42%	na	na

Callie stated that when she has math homework, she will usually work on it first "while I'm fresh." She uses her notes and the textbook to help her, but sometimes she is unable to do the work and she has to "just put some like words down and stuff to make it look like I've done it because, you know, so the teacher will think that I tried, 'cause I did try." When this happens, she will "usually try to get it the next day." Callie said that her mother is not really able to help her with math anymore "'cause it's getting too hard." She does, however, have people at school that she can turn to for help with math.

Hope. Hope is a fourteen year old White female who is originally from Atlanta, Georgia. She plays softball and volleyball and is a member of the Builder's Club at LMS. She has been a student at LMS since 6th grade and also attended elementary school in the district. She has a 17 year old brother and they live with both parents. Hope enjoys writing and her favorite subject is language because "it feels like I do really good in that subject." Hope's least favorite subject is math because "it's just really hard for me to understand."

Hope has been an "A/B" student in math throughout middle school. Her yearly average for 6th grade math was 91 and for seventh grade math was 88. For eighth grade, despite her difficulties, she has also maintained all A's and B's. For the first nine-week grading period, Hope earned an 86. She then earned an 82 for the second nine week period, a 90 for the third nine week period, and an 87 for the fourth nine week period. During the year, she was absent from math class only four times. Hope's CRCT math scores have also been relatively good during middle school. She scored 813 on the 6th grade math CRCT, 825 on the 7th grade math CRCT, and 823 on the 8th grade math CRCT. A breakdown of her scores by domain is shown in Table 4.

Table 4

Hope's CRCT scores by domain

Domain	6th Grade % correct	7th Grade % correct	8th Grade % correct
Algebra	56%	63%	63%
Data Analysis & Probability	78%	89%	80%
Geometry	58%	60%	43%
Number Operations	56%	67%	85%
Measurement	50%	na	na

Hope reports having difficulty with math homework, saying "usually I can understand a little more in class but when I get home I kind of forget it all." When this happens, she usually asks her parents or brother for help. She says "they do a lot to help me."

Jake. Jake is a fourteen year old White male who was born in Alabama and moved to his current home when he was about three years old. He lives with both parents and has three siblings. His older sister and brother, 27 and 25 years old, have moved out on their own and Jake lives with his parents and his younger sister, who is 11 years old. He has attended LMS for 6th, 7th, and 8th grades.

Jake is involved in several sports, including football, baseball, basketball, soccer, and track. He states that football and baseball are his two favorite sports. He was selected to play on the high school baseball team as an eighth grader and plans to also play high school football. He hopes to be able to go to college for one of the two sports. Jake stated that his favorite subject is social studies because it "comes more easier" to him. He likes that "it shows like how the world has changed throughout the years and how people used to do things then and how people do things now." Jake's least favorite subject is math because "I do the worst in that. It's hard. It's not like it's the funnest thing to do." Jake reported that math has always been hard for him.

Jake was a "B" student in math prior to 8th grade, earning a yearly average of 88 in 6th grade math and a yearly average of 82 in 7th grade math. In 8th grade, Jake began the year with a 70 for the first nine-week report card period and then improved to a 76 for the second nine-week period. He earned an 85 for the third nine-week period and an 81 for the fourth nine-week period. He was absent from math class on seven occasions during the school year.

On the math CRCT, Jake earned a score of 815 in 6th grade and a score of 843 in 7th grade. On the 8th grade math CRCT, Jake earned a score of 803, which is barely above the score of 800 needed to "Meet Standards". A breakdown of his scores by domain is shown in Table 5.

Table 5

Jake's CRCT scores by domain

Domain	6th Grade % correct	7th Grade % correct	8th Grade % correct
Algebra	78%	75%	50%
Data Analysis & Probability	78%	78%	80%
Geometry	67%	80%	57%
Number Operations	22%	83%	46%
Measurement	42%	na	na

When working on his math homework, Jake stated that he will usually start with the problems that look easier. If he cannot figure out how to do the problems, he will just wait until the next day in class to ask questions. Jake said his parents try to help him, but "they haven't done their math in too long either" so it is difficult for them. They try to get on the internet to find help for him and have also tried a tutor in the past.

Sky. Sky is a fourteen year old African American female. She has attended Leader Middle School for 6th, 7th, and 8th grades. She also attended elementary school in the district. Sky lives with both of her parents and has one dog, a Rottweiler who she describes as "really nice." She also has one sister who is 21 years old and no longer lives at home. The family will be moving soon, so she will be going to a different high school than most of her friends, but still in the same district. She said, "It's scary 'cause I don't know many people going to that school but it's okay because my best friend goes there." Sky enjoys playing basketball, which she has done since she was seven years old, and has recently taken up boxing. She explained that boxing is "harder work and you have to have a lot of discipline." Sky's favorite subject is language because she enjoys writing, including writing in her journal. Her least favorite subject is math. While Sky "used to be really good at math," it has never been her favorite subject. She reported that it has become much more "difficult to understand this year."

Sky has generally been a "B" student in math during middle school, with an 83 yearly average in 6th grade and an 82 yearly average in 7th grade. In 8th grade, she began the year with an 85 average for the first nine-week grading period, but dropped to 78 for the second nine-week period and 70 for the third nine-week period. She improved her math grade to 82 for the fourth nine-week period which, according to the students, is mostly review. Sky also had very good attendance during the year, missing her math class only two times.

In 6th grade, Sky scored 789 on the CRCT, which falls below the score of 800 needed to "Meet Standards." In 7th grade, she improved her CRCT score to 835 and for 8th grade she scored 815. A breakdown of her scores by domain is shown in Table 6.

Table 6

Sky's CRCT scores by domain

Domain	6th Grade % correct	7th Grade % correct	8th Grade % correct
Algebra	50%	79%	50%
Data Analysis & Probability	56%	89%	80%
Geometry	25%	60%	86%
Number Operations	33%	67%	69%
Measurement	33%	na	na

Sky reports that when she gets home from school she tries to complete her math homework first "'cause math usually takes the longest, but when I get home I feel like during the day everything went so fast I get lost really quickly during homework in math. So I just try to do the things I know how to do first." Fortunately, Sky's neighbor is an algebra teacher and can help Sky occasionally when she is really struggling. She can also call her older sister for help. Sky's parents both work full time and are often "just getting home or not home yet" when she is doing her math homework. She wishes they could help her more, but understands that they are very "busy, 'cause my dad has his own business and my mom is a nurse."

Findings from Individual and Focus Group Interviews

Sky enters the conference room and sits next to me at the table. Today's interview will focus on her perceptions of algebra, difficulties with algebra, strategies used by her teachers, and her previous experiences with mathematics. I ask Sky what comes to her mind when she thinks about algebra. "Equations," she says. "I get nervous when I think about algebra." She tells me that she doesn't like anything about algebra and that she dislikes it because "it's sometimes hard to understand." I ask Sky about the appropriateness of the algebra she is learning for eighth grade students. She tells me how a lot of math this year she "didn't know the basics of how to do it, so it was harder…if it could have been explained longer, if we had more time to understand it, that would have been better." She continued, saying, "The pace was really fast. You learn something different every day." She didn't think the math was too hard for eighth grade, but "it's just too much." Sky described a typical day in her math class as completing warm-up problems, taking notes, and then working on class work or homework "over everything you talked about that day."

In the following sections, I discuss the findings from the individual and focus group interviews, organized by each of the research questions.

Students' perceptions of algebra. Four primary themes emerged from the data collected about students' perceptions of algebra as part of the 8th grade mathematics curriculum: (1) eighth grade is too early to study algebra, (2) algebra is important to learn, (3) the pace of instruction is too fast, and (4) instruction should be differentiated for students who are less prepared to learn algebra.

Eighth grade is too early to study algebra. All six students overwhelmingly responded that studying algebra in eighth grade is difficult. Callie described eighth grade math as a "huge jump" from what she had learned in seventh grade and Sky expressed that eighth grade required a lot of math "that we didn't know the basics of how to do." Three students mentioned getting nervous when they see algebra or an automatic feeling that they would not do well. These feelings were evident during the think-aloud interviews as several students questioned their responses, even if the responses were correct. The majority of students also reported that there is nothing they like about algebra. Callie stated “I really don’t like anything about it. Just if it’s easier sometimes, I’ll get it, but I still won’t like it. The solving for *x* and all that kind of stuff, I get that…but I still don’t like it.”

The students indicated the belief that some of the material currently taught in 8th grade math would be more appropriate in high school courses. Bob stated that “we think it’s like 10^{th} grade or 9^{th} grade work and it shouldn’t be even be taught to 8^{th} graders.” Jake described how the math that he was required to learn in 8th grade is similar to what most adults learned in high school and argued that the content of 8th grade math may not be appropriate for all students:

> [In high school] Your brain’s more developed and you can kind of more put this in your mind and you’re more like mature about it. And you can really get focused on it. But something like with us, I mean, not everybody is mature enough, they can’t really focus and take it all in. It’s just not, I don’t think it’s appropriate.

Algebra is important to learn. During the individual interviews, five of the six students stated that algebra is important to learn. Jake, however, did not believe algebra was important to learn “unless you’re gonna go in a career that you have to use stuff like that…I guess construction, people who do banking, I guess that could be useful. But… if you do certain careers I don’t think you really need it.” The other students had indicated that algebra would be necessary in high school as well as in many jobs. During further conversation, however, they seemed unable to connect the algebra they were doing in the classroom to its usefulness in the real world.

During the focus group interview, Bob changed his opinion, stating “most jobs don't involve algebra. Like a little bit of them. Banks and stuff like that might, but jobs that I guess most people have don't.” Sky then agreed with Bob, but Callie noted that “I think that we should learn algebra, like a little bit, but not a lot, so we can still know like bank accounts and stuff.” She noted that people in some careers may need to know more, “but that should be your choice.”

During the think-aloud interviews it became apparent that students did not have a clear understanding of how the algebra problems they worked in the classroom could be useful in the real world. After graphing a linear equation and solving a system of linear equations, each student was asked how those concepts could be used in the real world. None of the six students were able to give an example of how the problem could be applied in the real world. Allen stated, “I’m not sure if you use it in the real world” and most of the students had similar comments.

The pace of instruction is too fast. Throughout the interview process, all six of the students mentioned the amount of material they had to learn in 8th grade math and/or the fast pace at which the material was taught. They recognized that the amount of material dictated the pace, as the teachers were required to cover the entire curriculum during the year. Most of the students, however, felt that the fast pace of instruction prevented them from developing an understanding of the concepts. Callie expressed this very well when she said,

> the pace was ridiculous. We moved way too fast...I felt like if we would have just slowed down and let us get a hang of everything, we would actually be better prepared for high school than if we went fast. Because a lot of kids would just be getting it and then we'd move on.

Several students mentioned that they would learn something one day and then move on to something new the next day. Hope stated that "if you don't get it that first day then you pretty much don't get it at all." The students all felt that if they had been given more time to learn new concepts, they would have understood the concepts much better. Hope stated, "it takes me a while to understand how to do it. And we have to go at a fast pace to be able to cover everything. So it's really hard for me to understand it at that pace.”

Instruction should be differentiated. The students were somewhat divided on their thoughts on all students being required to take the same courses. The three girls thought it was important for all students to learn the same math, but they also recognized that there might need to be some differences in how it is taught. Sky pointed out that "you'll have to take the same test, like the CRCT" so that is a reason that all students should learn the same math. Callie agreed, but thought that some students may need to "have extra help and have it taught in a different way." The boys all stated that students should not all learn the same math. Allen pointed out that "some students may be slower and some may be faster" so perhaps these students should not be in the same class. Jake felt that students like himself, that might not understand math as well, should be in a class "where they could get more one on one instruction." Other students, he said, "just get it" and they should be in a more advanced class. Bob described being the last student in the class to finish a problem:

> sometimes it gets really frustrating. I just want to stop and throw my paper away and just walk off...I'll be over there just working at a slow pace and I'll feel like everyone else is done around me. And I feel like everyone is waiting on me. So I try to do it really fast. Then I get mad 'cause I don't do something right and I have to start all over.

Strategies that teachers employ to teach algebra. The following three themes emerged from the data collected about the strategies used by teachers to teach algebra: (1) traditional direct instruction methods are the primary means of instruction, (2) students benefit from collaboration with peers, and (3) teacher encouragement and support is valued by students.

Traditional direct instruction. When students were asked to describe a typical day in their math class, all students described a very traditional classroom with warm-up exercises, direct instruction through lecture and note-taking, practice problems worked independently and then reviewed by the teacher, and the assignment of homework. As Bob said, "we'd take notes, like a whole bunch of notes" and Jake said "we take notes pretty much every day." Sky also pointed out that the teacher

> takes notes really fast so you have to keep up and write really fast. And sometimes you may not listen 'cause you're trying to copy it all down. So when you get home it's just like 'what do I do now?' You don't really know how to study 'cause you were trying to keep up.

While most students reported that they are allowed to use their notes when taking a quiz, they did not always find the notes to be helpful.

All students reported having time to begin their homework in class because, as Allen said, "we went so fast, we had plenty of time to do it". Callie reported that this strategy works well for her because if she has questions about the homework, she is able to ask the teacher or another student because "if I don't get it when I get home I won't be able to do it." Hope and Jake also reported doing a lot of worksheets in math class.

Collaboration with peers. Five students reported that they are occasionally allowed to work in groups. These students indicated a benefit from working in groups or observing another student work out a problem on the board. Bob explained that working in groups helped him "because I get to hear it from the other students. They come up with ways to work the problems, too, instead of just hearing the ways [the teacher] makes them up." In the focus group interview, however, Allen pointed out that working in groups is not always helpful because “sometimes you've got people who don't know and you're just as confused as they are, too. So you're like 'Wow, that's not really helping.'” Sky agreed with him and added “and if they don't get it, they'll start talking about the game last night…I don’t like group work.”

Callie stated that she likes when other students go to the board and work problems because it allows her to see what these students "are messing up on" and helps her not "feel quite as bad." Sky also liked going to the board to work problems because “when you go up to the board and work it out, she'll help you while you're doing it and you'll get to see where everyone else is messing up too. You get to learn from your classmates.” Jake also indicated a benefit of working with peers, stating that "one person may tell you it and then another person will tell you it and it may click better in your head, better than how that one person told you." From these comments, it seems that observing other student work problems in a whole group setting with participation by the teacher may be more beneficial to these students than working in a collaborative group.

Teacher encouragement and support. Four students described how their math teachers, either current or previous, encouraged them and provided extra support to help them be successful. Jake explained how a previous teacher provided "a lot of one on one, like after school" when he needed help and Sky also mentioned the benefit of after-school tutoring. The students also mentioned how teachers demonstrated their belief in the students' ability to be successful. Bob described how his teacher pulled him aside after he received a low grade on a quiz: "she pulled me aside and said, 'I know you studied for this quiz...cause you can tell...I know you got a low grade, but I'm still proud of you for working hard.'" These words of encouragement helped Bob to feel more confident about his work. His teacher allows students to make corrections on quizzes for an improved grade, so instead of being upset about his grade, Bob was motivated to complete the corrections and bring up his grade. As Allen said, making corrections "shows us the mistakes we made" and "helped a lot". Sky commented that her teacher would "tell you that you can do better and she'll try to explain it to you more so you can do better next time." Encouragement and support from teachers appeared to have a big impact on the students' attitudes towards mathematics.

The students also discussed the impact of teachers who are not encouraging or do not have high expectations for students. Hope described how previous teachers "believed in me and thought I could do it" but this year she did not get that feeling from her teacher. The negative feelings she has about her teacher have caused her to have increased anxiety about doing well in math class. Callie also reported having teachers in the past that did not have high expectations of her and it was "disappointing, I guess, like that they don't believe that much in us."

Students' difficulties with algebra. Aside from the fast pace of instruction, which was mentioned repeatedly by all students throughout the interview process, the following themes emerged from the data related to students' difficulties with algebra: (1) Content-related difficulties, and (2) Process-related difficulties. Overall, students noted more process-related difficulties than content-related difficulties.

Content related difficulties. Three students mentioned integers as a source of difficulty for them in algebra. Jake commented: "I always get confused with like the adding, subtracting negative and positive. I figure it out then I'm like do I put a negative sign in front of it or just leave it there?" Integers are introduced to students in seventh grade and the concept is used significantly throughout seventh grade. Many students, however, have difficulty recalling the rules related to same signs or different signs or they will confuse the rules for multiplication and division with the rules for addition. Callie described how she had trouble in 7th grade when integers were introduced and said "I'm still struggling with that...I just hate negative numbers." Sky also mentioned having difficulty with integers.

Variables were mentioned by two students as another content-related difficulty. Hope stated that "it's usually the letters that confuse me. I don't know, I just get really confused… and how you have to figure out like what they are and what they mean." When asked about this difficulty Jake replied, "All the *x*'s and *y*'s everywhere and all the numbers and letters mixed in. It's like 'oh, man.'"

Process-related difficulties. Process-related difficulties mentioned by students included the number of steps involved in solving some algebra problems and the formulas that had to be memorized. When there are several steps to solving a problem, "if you mess up one little thing, it like messes up the whole problem," according to Callie. Even when she realizes that something is wrong, Callie said it is "hard to see where I went wrong, because I get confused with some of the steps sometimes". Often, students think they have found the answer and then realize there are more steps to completing the problem. I could sense Jake's frustration as he told me:

> once you get the answer and then you got a do a whole 'nother step. Like you gotta plug it in to some other problem and do it. That kind of gets confusing. 'Cause you're like 'where does this go?' you know 'where does this go?' And it's all a big jumble.

Bob described how he had difficulty remembering the different formulas used in eighth grade math, such as Pythagorean Theorem, slope-intercept form, point-slope form, the formula for slope, and others. He described studying the night before the CRCT: "I had to stay up almost the whole night remembering all these equations and how to put the numbers in the right order and stuff like that." Sky also noted that

> You have to remember all these steps and all these formulas and stuff. And it's hard to remember. We get like 50 formulas this year that you have to remember. And it's hard to remember everything. So yeah it's a lot of steps. And when you're going really fast it's hard to understand.

Another process-related difficulty, word problems, was mentioned by every student as an area of concern. Callie described word problems as being "really, really hard" because "when you have a word problem, you have to kind of figure out how to set it up. And I just can't figure it out." Sky expressed similar concerns:

I don't know, when I see a word problem it makes me nervous cause they're not easy…There is always something to distract you in the word problem and I always get distracted…You have to go and find the information and set up the problem. And if you set up the problem wrong and you answer it then you answer it incorrectly and the whole problem is incorrect.

Bob and Allen both described having difficulty interpreting word problems. Bob explained how word problems "have a whole bunch of numbers in there" and he has to choose which numbers to use. He claims he is "not really a good decision maker" and second guesses himself a lot, so he will often go back to a word problem and rework it a different way. Allen described how many students' initial reaction to a word problem can be problematic: "when it's short it's like 'ok, I got it' but when it's long it's like 'oh, I don't want to do this'.

Students' previous experience with mathematics. The primary theme that emerged related to students' previous experience with mathematics was that the prior years' math classes had not adequately prepared the students for the algebra they would experience in eighth grade. According to Jake, "sixth and seventh grade, it was hard, but it was definitely more like wide scaled. And once you get in eighth grade, you're there. It's hard, hard, hard, hard."

Two students indicated that seventh grade math had been very similar to sixth grade math and, therefore, had not helped as much to prepare students for eighth grade math. Bob said,

In sixth grade we learned to add and multiply fractions, and I'd already learned that from elementary school and then in seventh grade we learned the same thing. So I was like, 'didn't we already learn this?' So it felt like I was still in sixth grade doing the same work. If we would have done more work like we were gonna do this year, it kinda would have helped me out just a little more in eighth grade.

Sky also mentioned that the students “didn't have a lot of word problems last year, like with algebra in them.” Now, in eighth grade, “there's word problems on every single test or quiz” and it is very difficult when “you're not used to doing them in sixth and seventh grade.” Allen pointed out that “there was some stuff we did in seventh grade that I’ve never seen any other time” and he wished they could have spent more time studying concepts that would have prepared him for eighth grade. Callie had a similar concern, stating “I wish we would have touched on a little more eighth grade stuff towards the end of seventh grade, just kind of show us, so it’s not just like ‘whoah’ when you get there.”

Findings from Think-Aloud Interviews

> *Allen copies the equation on his paper, 5n+20n=(n+20). “Since n+20 is in parentheses, you do that one first. So you times the 5 by n and 20.” Allen rewrites the equation as 5n+20n=5n+100. “You would add, no you would subtract 5n from both sides.” He writes -5n on each side of equation and rewrites the equation as n+20n=100 and then 21n=100. “Yeah, I think I did this wrong.” I ask Allen to look back over his work and see what he thinks is wrong. He says, “I think you would add 5n.” Allen changes his previous work to +5n on each side of the equation and rewrites the equation as 10n+20n=100 and then 30n=100. “Well 30 can’t go into 100.” Allen uses a calculator to divide 100 by 30. “I’m thinking you do 5n+100 which would be 105n, then 20n+5n.” He rewrites the equation as 105n=25n. “Yeah I messed up big time.”*

I have Allen start over, beginning at the second step after he had used the distributive property correctly. He recopies the equation, 5n+20n=5n+100, and I ask him what he is trying to do when solving the equation. "You need to get the n by itself, so you have to add or subtract." He decides to subtract 5n on each side of the equation. He rewrites the equation as 20n= and then pauses. "Would that be zero?" I answer his question with my own question about what he subtracted. "Oh, 5n, which would be zero. That would be 20n=100." Allen divides 100 by 20 and says, "Yeah, it's 5." I ask Allen about what confused him on this equation and he replied, "I got confused when it says 5n+20n=5n. I didn't think it would be that many of the n's there. Uh, yeah, I got confused at that point."

In this section, I present the findings from the student think-aloud interviews. During the think-aloud interviews, participants were asked to work several algebra tasks (see Appendix B for tasks) while thinking aloud about their understanding of the problem. Selection of tasks for the think-aloud interviews was based on concepts identified in the literature as being problematic for many students (expressions and equations, use of the equal sign, and use of variables), as well as related concepts I had observed as being problematic for students in my school (linear equations and systems of linear equations). The findings from the think-aloud interviews are organized by these conceptual areas and are relevant to the exploration of the third research question: What difficulties, if any, do students encounter in learning the algebra concepts required by the eighth grade mathematics curriculum?

Use of the equal sign. Research indicates (Kieran, 1981) that many students perceive the equal sign in terms of indicating the answer, such as $3 + 2 = 5$. This view of the equal sign can be problematic for students faced with equations containing variables on both sides. As such, participants in this study were asked to solve two equations with variables on both sides. The first equation, $6x - 11 = -2x + 5$ was also designed to assess students' understanding of equivalence by including negative terms on both sides of the equation. Callie, Hope, and Sky each solved this equation correctly, showing and explaining all steps. Allen solved it correctly after identifying a minor arithmetic error. Bob and Jake, however, had difficulty solving the equation.

Bob demonstrated a good understanding of the steps needed to solve the equation. He understood that he need to "get *x* by itself" but he got confused when trying to combine the like terms that were on opposite sides of the equation: "I have to combine like terms so I bring the negative $2x$ over here to this positive $6x$." In doing this, however, Bob subtracted $2x$ on both sides of the equation, rather than adding $2x$. Callie had explained this step when she correctly solved the equation: "you do the opposite and this is a negative so you're going to add." Bob made the same error when attempting to combine 11 and 5 in the equation.

Of all the students, Jake had the most difficulty with the first equation. After copying the equation on his paper, he said "This confuses me with this equal sign here. And there's two *x*'s in this problem, so that's even more confusing." Jake made a number of errors in attempting to solve the equation. First, he added $6x$ on both sides of the equation, rather than subtracting it. At the same time, when he wrote $+ 6x$ on the right side of the equation he wrote it underneath the 5 and added $6x$ plus 5 to get $11x$. He thought this might be incorrect and said "maybe you should do the 11 and the 5…'cause they don't have the *x* on it, 'cause those two are like, what's that word called? Well the *x*'s go together and the numbers go together." At this point it seemed he might be on the right track, but then after subtracting 11 from both sides of the equation (although he should have added) he rewrote the equation incorrectly as $6x = -2x + 6$ and said "I'm thinking that you do the 6x with the 6, but I don't know."

I attempted to guide Jake by asking him what the goal is when solving an equation. I was looking for the response "to get the variable by itself" or something similar. Four of the other students had mentioned getting the variable by itself when they were solving the equation. Jake's response, however, was "to get the right answer." After a few additional attempts at guiding questions, Jake said

> That equal sign is messing me up…It makes me feel like it's, it's weird. I don't know. It kinda looks like they're supposed to go together and they're supposed to go together [referring to the opposite sides of the equation] and it just confuses me.

At this point I questioned Jake about the meaning of the equal sign and he stated "I don't know in this problem. It's just over here like saying all kinds of stuff. 6x-11=-2x+5. I have no clue how either of those equal each other." I realized that Jake was thinking of the equal sign as giving the answer, that the right side of the equation was "the answer" to the left side of the equation. I confirmed this by asking Jake about the meaning of the equal sign in general, rather than in terms of this specific equation and his response was that the equal sign means "that it equals something. Like 4 plus 4 equals 8."

On the second equation, $5n + 20n = 5(n + 20)$, Callie, Hope, and Sky again solved the equation correctly, showing and explaining all of the steps. Allen solved the equation correctly after identifying an error in combining like terms. He had correctly used the distributive property on the right side of the equation, but when subtracting $5n$ on both sides of the equation, he left $n + 20n$ on the left side. It seemed that he was uncomfortable subtracting $5n$ minus $5n$ and leaving nothing. When I asked Allen to describe what had caused him difficulty with that equation he said, “it just confused me that there were so many *n*’s.”

Again, Bob and Jake had difficulty with this equation. Bob correctly used the distributive property on the right side of the equation and combined like terms on the left side. He then added $5n$ on both sides rather than subtracting, resulting in an incorrect response. After Bob had completed all of the think-aloud tasks, I went back to the first two equations with him and explained where he had made his mistakes. After going back through the equations, he seemed to have a good understanding of how to solve them and he was very appreciative that I had explained it to him.

Jake, on the other hand, had difficulty beginning work on the equation: “I think you start off somewhere in here, with the parentheses. But I don’t know exactly what you’re supposed to do when you start off in here.” I attempted to guide Jake in the right direction without telling him exactly what to do. He eventually rewrote the equation correctly as 5n+20n=5n+100, but then said “I have a feeling there’s gonna be like division or something in this one. I think this is gonna correspond somehow right here [referring to the 5n on each side]. I want to add that to that [the 5n on the left and 5n on the right], but then I don’t want to add that to that. At this point I decided it would be best to bring a different set of equations for Jake to work on the next time we met. I would need to go back to much simpler equations and then work up to equations with variables on both sides. Unfortunately, Jake was absent on the next day of interviews, which was the final day, and I was unable to schedule another time to meet with him.

Use of variables. To assess the participants’ understanding of the use of variables, students were asked to complete a three-part word problem in which they must represent a problem situation using an expression, make an input-output table for three different values of the variable, and write and solve an equation to find a specific value of the variable.

On this task, four of the six students easily wrote the correct expression $8.95y + 145$. The other two students, Callie and Sky, lacked confidence and asked questions such as “do I just set it up like an equation?” but ultimately wrote the correct expression. Jake explained the expression he had written by saying, “It’d be like you don’t know how many certain amount of yards yet, so you’d multiply that. But then it’s automatically $145 to install the carpet.”

For the input-output table, Jake was the only student who completed the table with no guidance. Allen and Bob did well, but forgot to add the $145 for installation to each of the outputs. Callie, Hope, and Sky had a difficult time getting started and required a number of guiding questions to get them on the right track. The biggest source of discomfort seemed to be in selecting the values to use for inputs. The students did not seem accustomed to selecting their own values for inputs. When selecting her numbers, Hope asked if she needed both positive and negative numbers for inputs. I responded with a question about what the inputs represented and she decided that negatives were not needed “because if I used negatives it wouldn’t really be accurate ‘cause you wouldn’t have a negative amount of carpet.” After completing the input-output table, students were asked to choose one input-output pair and explain its meaning in terms of the problem. All students were able to complete this part of the task successfully.

The final part of this task was to determine how many yards of carpet could be installed for $324. To find the solution, students could take the expression they written previously and make it an equation, $8.95y + 145 = 324$, and solve the equation. Of the six participants, Sky was the only one who recognized quickly that an equation could be used to find the solution. Callie and Bob wanted to use a guess and check method by substituting different values for y in the expression until they reached an answer of \$324. Allen wanted to simply divide \$324 by \$8.95, forgetting about the cost of installation. Hope recognized that the expression could probably be used somehow, but was confused about where to put the \$324. Through a number of guiding questions, all students eventually arrived at the correct solution.

Linear equations. For this task, participants were asked to graph the following linear equation: $-5x + 3y = 18$. Allen and Sky were able to complete the entire task successfully. Hope correctly rearranged the equation into slope-intercept form to make it easier to graph, but used the origin as the *y*-intercept of the graph instead of the point (0, 6). Bob and Callie made minor errors in rearranging the equation, but graphed the incorrect equation correctly, and Jake was unable to rearrange the equation at all.

Sky was the only participant to use mathematical language, such as slope and *y*-intercept, in her explanation of the steps. She was also the only student who could explain the meaning of the resulting graph: "It means that what you're graphing, the answers would be on this line" although she did not sound confident in her response. The other students could not explain what the line represented. Callie stated, "It's just something we have to do…Like I know it graphs the *y*-intercept, but I don't know. I don't really know what else. We were just told to do it."

Systems of linear equations. For this task, participants were asked to solve the following system of equations: $x – 3y = -2$ and $4x + 7y = 11$. I did not ask Jake to complete this task because he had experienced such difficulty in working with the equations in the previous task. Of the five participants who completed this task, all demonstrated a good understanding of the steps required to find the solution. Allen, Bob, Callie, and Sky all used the substitution method and solved one of the equations for either x or y and then substituted the value found for that variable in the second equation. Allen and Bob each made minor errors in solving for *x* or *y*, both related to negatives, and arrived at an incorrect solution. Hope had some difficulty getting started with the task because she was accustomed to using the elimination method of solving a system of equations, which was possibly not the easiest method to use for this particular system. With a bit of guidance to rearrange the equations to a form that she was more comfortable with, she was able to find the correct solution to the system.

None of the students, however, were able to explain what the solution, (1, 1), represented. As Bob said, “I don't know. I've never, no one ever taught how, what the lines and everything represent. They just told me do the problem and move on to the next one.” Similarly, none of the students could think of a real world situation in which solving a system of equations could be useful. Allen stated, “I’m not sure if you use it in the real world.”

Summary

Analysis of the data from this study revealed that students perceive algebra as being too difficult for eighth grade. Students acknowledged that algebra is important for some jobs, but for the most part did not believe that they would personally use algebra in their jobs or in other ways in the real world outside of school. The students overwhelmingly believed that the pace of instruction in their eighth grade mathematics class was unreasonable. All of the students who participated in the study had difficulty with algebra during the year and frequently cited the fast pace of instruction as the source of their difficulties with algebra. Most students asserted that some form of curriculum differentiation should be available for students such as themselves. This differentiation might be in the form of a different a class or simply different teaching methods.

The data also revealed that the participants’ teachers often rely on traditional teaching methods such as lecture, note taking, and independent practice. The teachers do occasionally allow for cooperative group work, but the students do not always find group work to be helpful. The students do, however, report that they benefit from observing peers during group work or when asked to work problems on the board. It was also evident from the data that the students have a strong need for encouragement and support from their math teacher. Several of the students demonstrated a lack of confidence in their algebraic abilities, perhaps due to their perception that their teacher has little confidence in their ability. The students who reported having an encouraging teacher seemed to have more confidence in their ability to be successful in algebra.

While the fast pace of instruction seemed to be the primary source of difficulty for many students, the students also reported difficulties with the concept of integers, the number of formulas and steps needed in algebra, and word problems. It seems that the students are relying on a set of memorized rules or steps to solve algebraic problems, rather than developing an understanding of the concepts. Therefore, when the student cannot recall the rules or steps, he or she is unable to arrive at a correct answer for a problem. These findings were supported by the students' performances during the think-aloud interviews.

When completing the tasks assigned for the think-aloud interviews, two students had difficulty solving equations with variables on both sides. All students were able to write an expression to represent a word problem, but several had difficulty with other parts of the task, including setting up an equation. In terms of linear equations and systems of linear equations, most of the students appeared to have an understanding of the steps involved in finding a solution, but they had little to no understanding of what that solution represented or how the concepts could be applied in the real word.

Overall, the students reported feeling unprepared for eighth grade mathematics, based on their previous math classes. It was noted that the seventh grade math class had repeated many of the concepts of the sixth grade math class, while also including a number of concepts that did not spiral up to eighth grade mathematics. The students felt that the seventh grade math class could have included more work on some of the concepts they would encounter in eighth grade in order to better prepare them for the rigor of eighth grade mathematics.

CHAPTER 5

DISCUSSION: CONCLUSIONS AND IMPLICATIONS

The data collected during this study have provided valuable information about students' experiences with algebra in eighth grade. The study participants offered great insight into the difficulties with algebra that are experienced by many eighth grade students. Their difficulties, however, were not related to specific algebraic concepts as I had anticipated. The findings from the study revealed to me several opportunities for improving students' experience of algebra in eighth grade. These opportunities are related to two major areas: curriculum and pedagogy.

Curriculum

Curriculum organization has been noted as an important factor in student achievement in mathematics (Squires, 2009). There are several characteristics of curriculum organization that influence instruction and student learning, including scope, sequence, continuity, relevance, integration, articulation, and balance (Oliva, 2001; Ornstein & Hunkins, 2009). The findings from this study reflect primarily on the scope, sequence, continuity, and articulation of the mathematics curriculum as well as the developmental appropriateness of the curriculum.

Scope. The lack of time allowed for students to learn new algebraic concepts was the primary concern of the students who participated in this study. The pace of instruction in the eighth grade mathematics curriculum is directly related to the scope of the curriculum, which was mentioned several times by all students during the interviews. Most students described a learning environment where they were introduced to a new concept at least every two to three days. During the individual interview, Jake described his frustration by saying, “You might not even know what happened last week and you’ve already gotta learn something else here and then next thing you know CRCT is here and you don't know half the stuff you did." Other students expressed similar concerns.

Bruner (1960) argued that the scope of the curriculum should be limited to the "most fundamental understanding" (p. 31) of the key topics in a subject area. The eighth grade mathematics curriculum, however, consists of fifty-five different elements across four different domains. The scope of the curriculum also assumes that students have mastered the concepts and skills taught in prior grade levels, and allows little, if any, time for review and remediation. The result is that many students are not allowed time to develop a good understanding of one concept before moving on to another. These students may be able to complete accurate computations related to the concepts, but are unable to later apply these concepts to problem situations. Developing students' understanding of algebra requires time for in-depth exploration of a few key topics, allowing students to engage in continued practice and improvement of performance (Darling-Hammond, 1997).

Research supports the need for additional instructional time in algebra. Spending more time on topics that require more demanding performance expectations has been shown to yield increased achievement in those topic areas (Schmidt, et al., 2001). In addition, studies of universal eighth grade algebra policies have shown that requiring all eighth grade students to take an algebra course can lead to increased achievement "if accompanied by increased time for mathematics instruction and intensive support for lower-achieving students" (Stein, et al., 2011, p. 485).

Participants in this study expressed the need for additional time and support to learn algebra concepts. Several students mentioned the need for after-school tutoring and one student expressed the need for math support classes, which are offered to high school students. Overall, the students did not feel that the concepts they were required to learn were beyond their cognitive level. While they did feel that some concepts might be more appropriate in high school, they seemed confident that they could have understood those concepts if only they were provided more time before moving on.

Sequence, continuity, and articulation. The students who participated in this study did not feel that the previous years' mathematics curricula, particularly the seventh grade curriculum, had helped to prepare them for the algebra they would experience in eighth grade. This finding has implications for the sequencing, continuity, and articulation of the mathematics curriculum. The importance of curriculum sequence and continuity is well documented in the literature (Bruner, 1960; Dewey, 1938; Tyler, 1949). Sequence is defined as the order in which the concepts of a particular course are arranged. Continuity and articulation are both dimensions of sequencing (Olivia, 2001). Continuity is the "vertical reiteration of major curriculum elements" (Tyler, 1949, pp. 84-85). It is not, however, simply repetition of content, but repetition with increasing levels of complexity. Bruner (1960) referred to this as the spiral curriculum, where children are introduced to basic ideas of a subject at an early age and these ideas are then revisited repeatedly, "building upon them until the student has grasped the full formal apparatus that goes with them" (p. 13). Articulation is the planned sequencing of content across grade levels, ensuring that the next grade level picks up where the previous grade level left off (Olivia, 2001).

The idea that important concepts and skills are repeated and built upon throughout the curriculum is essential to effective curriculum sequencing. As noted previously, students often begin to learn algebra as generalized arithmetic. It is important, therefore, that students have a good conceptual knowledge of arithmetic operations and that this prior knowledge is used to help students generalize the operations to algebra.

A review of the middle school mathematics curriculum indicated that, overall, the content of each grade level appears to build on that which came before, but there are several standards in sixth and seventh grades that do not appear in the mathematics curriculum again until high school, particularly standards from the Measurement and Geometry domains. Much of this content is not directly relevant to the eighth grade curriculum, which may contribute to students' feelings that eighth grade is "a huge jump" (Callie, Individual Interview) from what they had learned in prior grades. Several students mentioned probability as being one of the easiest concepts in eighth grade. It is also one of the few concepts that are included in the curricula of all three middle school grade levels. The continuity of this portion of the curriculum allowed students to develop a better understanding of the concepts.

It would seem that much of the preparation for algebra would be included in the seventh grade mathematics curriculum. The scope of the seventh grade curriculum, however, includes a significant number of standards related to the domains of Geometry and Statistics and Probability. Based on the state curriculum map, students begin the year with the study of data analysis, including different types of graphs and measures of central tendency. They move from this unit into working with algebraic expressions and the properties of real numbers and then to operations with rational numbers, including integers, fractions, and decimals. The next two units focus primarily on standards from the Geometry domain and, with the exception of proportionality, do not appear to build upon the previous units. While students are introduced to negative numbers and solving equations in 7th grade, it appears that adequate instructional time is not spent on these concepts and many students have not mastered these skills when they enter eighth grade. Some students may be able to accurately complete computations with integers or solve two-step equations because they have memorized a series of steps or "rules." They have not, however, developed a conceptual understanding of what they are doing and are often unable to apply these skills to more complex problems.

Learning theories support the need for students to develop an understanding of basic concepts before moving on to more complex topics. Bruner (1960) argued that concepts must be initially presented in concrete terms that the students can understand and then expanded upon at a later time. These ideas are important to building a solid foundation for the study of algebra. Instruction in the prior grade levels must present concepts in such a way that students can develop a good conceptual understanding of arithmetic. This understanding should then be generalized to algebra in small steps. Algebraic understanding in eighth grade students could be strengthened if more time were devoted to learning basic algebraic concepts in seventh grade before attempting to apply those concepts to more difficult situations in eighth grade.

Developmental appropriateness. The participants in this study noted that eighth grade may be too early for all students to study algebra. They argued that some of the concepts they were expected to learn might be more appropriate for high school students. Jake made the strongest argument for this point, stating that some eighth graders are not mature enough to focus on algebra.

Research in the area of adolescent development has indicated that the development of abstract thought processes, such as those required for the study of algebra, vary considerably across students (Caskey & Anfara, 2007) and most early adolescents require "ongoing, concrete, experiential learning" (National Middle School Association, 2003, p. 3). The National Middle School Association (2003) has also noted that adolescents from the ages of 10 to 15 undergo “rapid and profound” changes during this time, more so than any other time in their lives (p. 3). These changes can have a significant impact on adolescents’ academic performance, especially in abstract subjects such as algebra. Experience is extremely influential during the adolescent years (Kuhn, 2006) so it is especially important that the experiences provided in learning algebra are developmentally responsive to the needs of different students.

Piaget’s (1969) theory of the developmental stages of children’s cognition also recognizes that the ages at which the stages occur may vary with the individual child. Piaget further argued that learning must occur in small steps and if experiences are too different from a child’s current level of understanding, accommodation is not possible. Accommodation is the development of new cognitive structures by modifying and adapting existing structures. If the concepts of algebra are too different from a student’s current mathematical knowledge, he or she will be unable to develop the new cognitive structures needed to learn algebra.

Similarly, Vygotsky (1978) asserted that learning experiences should remain within a child's zone of proximal development. The zone of proximal development is the difference between what a child can do on his/her own and what the child can do with adult guidance or collaboration with more capable peers. Vygotsky’s theory implies that instruction at a higher level is appropriate, if the necessary supports are provided. Instruction, however, that is too far beyond the child’s current level is outside the zone of proximal development. In this situation, it is unlikely a child would be able to understand the concept being explained, regardless of how many times the teacher modeled it.

Pedagogy

Several findings from this study are related to pedagogy, or the methods used in algebra instruction. The study participants identified the need for differentiated instruction in algebra as well as support from their teacher and peers. I also noted an overall lack of conceptual understanding by the students and an inability to connect algebra to the real world.

Differentiated instruction. Half of the study participants thought that all students should be required to take the same math, while the other half thought that there should be different classes based on students' ability level. All students, however, noted that there should be differences in how algebra is taught to students of different ability levels, even if they are all required to learn the same content. For example, during the individual interview, Callie pointed out that some students may need extra help or to have concepts taught in a different way.

Research has indicated that grouping students based on ability does not improve academic achievement or promote more positive behaviors or attitudes (Oakes, 2005). It has also been noted that offering lower level courses often causes the students in those courses to fall even further behind (Burris, et al., 2004). Despite the beliefs of three of the study participants, it does not appear that they would be better off in a different math class from their more capable peers. Instead, instruction within that class should be differentiated to meet the needs of all learners.

Differentiation is a conceptual approach to teaching and learning that involves careful examination of learning goals, ongoing assessment of student needs, and instructional adaptations in response to data about readiness levels, interests, and preferences in learning, among other factors (Brimijoin, 2005). While it may seem that a standards-based curriculum and differentiated instruction cannot coexist, McTighe and Brown (2005) argue that the two must coexist in order for schools and districts to achieve accountability targets. In a differentiated classroom, learning is active rather than passive, all students are held to high expectations and expected to achieve mastery of the learning goals, and there is a balance between individual and collaborative opportunities for students to learn new information (Moon, 2005).

Teacher and peer support. When asked about teaching strategies that helped them learn algebra, several students mentioned working in groups or observing other students work out problems. During the individual interview, Bob stated that he sometimes understands better when another student explains a concept, rather than just hearing the teacher's explanation. The participants also mentioned the importance of teacher encouragement and support. It was important to the students that their teacher believed in them and their ability to be successful.

Research indicates that adolescents' engagement in learning is likely to improve when they perceive their educational environment as safe, supportive, and one of mutual respect among peers, were cooperative learning and sharing of ideas are encouraged and respected (Ryan & Patrick, 2001; Wentzel & Watkins, 2002). It is also noted that teacher encouragement and support can strongly influence students' work habits (Turner & Patrick, 2004). In a study of 33 high school students (Certo, Cauley, Moxley, & Chafin, 2008), caring teachers were described as those who were encouraging, helpful, and tried to relate to the students. These teachers listened to students, cared about their grades, and offered appropriate support. Findings of this study indicate that students are motivated to learn by teachers who care.

A number of studies have supported the claim that cooperative learning promotes achievement and can also contribute to other positive affective outcomes (Roseth, Johnson, & Johnson, 2008; Slavin, 1981). The study participants indicated that cooperative learning strategies often helped them to learn algebra, but they also noted that working in groups is not helpful when everyone in the group is confused or if some group members are not focused on the task. The biggest benefit seemed to be when they were allowed to observe another student working a problem on the board. Research has indicated that observing someone else's problem solving strategy can increase achievement (Azmitia, 1988). Jake noted that when a different person explains a problem, "it may click better in your head" than when someone else explained it.

The success of cooperative learning also supports Vygotsky's theory of the importance of social interaction in student learning, which suggests that students learn first through interactions with others, both teachers and peers. They then internalize what they have learned to develop understanding (Fogarty, 1999). Vygotsky's theory of the zone of proximal development implies that interaction with a more capable peer can move learning forward. Vygotsky argued that children can use imitation to accomplish tasks that are above their independent capabilities. In collaboration with peers or under the guidance of an adult, children can move forward to higher levels of the developmental process.

Conceptual understanding and real world connections. The findings from this study indicate that many students are not developing a conceptual understanding of algebra; nor are they able to connect what they do in algebra to real world situations. Hiebert and Carpenter (1992) describe understanding as "recognizing relationships between pieces of information" (p. 67). Conceptual knowledge is "rich in relationships" while procedural knowledge is "a sequence of actions" (p. 78). Both types of knowledge, however, are needed to be successful in mathematics. Procedural knowledge allows students to complete mathematical tasks efficiently. It is conceptual knowledge, however, that is essential for student understanding.

Many of the participants in this study demonstrated good procedural knowledge when completing the algebra tasks during the think-aloud interviews. They could correctly complete the steps to solve an equation, to graph a linear equation, and to solve a system of linear equations. They seemed to understand that the solution to an equation was the value that would make the equation true. When asked, however, about the meaning of the graph of a linear equation, only one student understood that the line represented all of the possible values for x and y that would make the equation true. Other students stated that they didn't know what it meant, they were just told how to do it. Similarly, most students were able to arrive at a correct solution to the system of equations presented in the think-aloud interviews. None of the students, however, were able to explain what this solution, (1, 1), represented in terms of the system of equations. Some mentioned that it was a coordinate that could be graphed, but did not recognize that it was the point where the two lines would intersect if both linear equations were graphed on the same coordinate plane. Again, comments such as "They just told me to do the problem and move on to the next one" were common from the students. Research has shown that this issue is common among algebra students (Wagner & Parker, 1993).

There are a number of benefits to improving students' conceptual knowledge, and thereby their understanding, of mathematics. Developing an understanding of mathematics improves students' ability to remember and also reduces the amount that must be remembered as new connections are constructed between new knowledge and existing knowledge. Understanding also enhances a student's ability to transfer existing knowledge to new situations (Hiebert & Carpenter, 1992).

The NCTM's Equity Principle states that the mathematics curriculum should be "interesting for students and help them see the importance and utility of continued mathematical study for their own futures" (p. 13). While the participants in this study agreed that algebra is important, they had no understanding of its utility in their futures. These students have been taught to memorize steps for solving problems; yet, they are unable to describe how they might one day use these skills in the real world.

In middle school, there is often more emphasis on grades than on learning (Parker, 2010). Teachers and students both may find it more efficient to simply focus on procedures for solving algebraic problems. In my experience, some students become frustrated with constructivist teaching approaches and have said to me, “Just tell me how to do it.” Learning algebra, however, is about far more than memorizing the steps to solving a problem. Students need to encounter the algebraic concepts they learn in authentic, real world scenarios to help make the connections needed for developing understanding.

Authentic learning is “higher-order learning that is used to solve problems – problems that are meaningful, challenging, and complex” (Glatthorn, 1999, p. 5). These problems are situated in real life and presented in their context. The theory of situated cognition suggests that students gain a deeper understanding of learning targets when they actively construct knowledge in contexts that they find important and interesting (Brown, Collins, & Duguid, 1989). The use of authentic problems in mathematics instruction has been shown to improve students’ problem solving skills (Bottge, 1999; Bottge, Heinrichs, Chan, & Serlin, 2001).

Typical word problems in mathematics do not foster authentic learning, as they do not represent authentic mathematical practice. During the focus group interview, the students discussed how the word problems they typically see would not be useful in the real world. Sky commented, “You could use it, but you wouldn't really want to. If you're like putting marbles in a jar, you wouldn't say ‘Hmmm, what's the probability if I grab a marble?…some people may think like that, but I don't know, I wouldn't. I would just put the marbles in a jar.” These types of word problems are often written in a language that is common only to other word problems and, therefore, are foreign to authentic mathematical practice. They may also lead students to misconceive how mathematics is actually used in the real world (Brown, et al., 1989). This idea would support why the study participants believed that algebra would only be useful to them in the future if they chose to be "a teacher or work in a bank."

Implications for Curriculum

It is important to consider the scope of the curriculum as a significant factor that limits students' time for learning algebraic concepts. Considering that Georgia has recently adopted the Common Core State Standards (CCSS), there are curriculum changes being implemented beginning in the 2012-2013 school year. Several concepts have been moved from the eighth grade curriculum to other grade levels, including absolute value, inequalities, set theory, and probability. Other concepts, however, have also been moved into the eighth grade curriculum. Most of the content that is new to eighth grade is from the Geometry domain, including transformations, similarity, and volumes of cones, cylinders, and spheres. It remains to be seen if this change in curriculum will provide students the time necessary to develop a conceptual understanding of algebra.

The CCSS for Mathematics are designed to stress "conceptual understanding of key ideas" and also return continually "to organizing principles such as place value or the properties of operations" (Common Core State Standards Initiative, p. 4). Research-based learning progressions were considered in developing the standards in terms of how students' "mathematical knowledge, skill, and understanding develop over time" (p. 4). Development of algebraic understanding begins in first grade with the following standards:

> Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following*

equations are true and which are false? 6 = 6, 7 = 8 – 1,5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 +? = 11, 5 = □ – 3, 6 + 6 = □ (Common Core State Standards Initiative, p. 15)*

Both of these standards represent critical understandings in algebraic thinking. If developed early and supported throughout arithmetic instruction, these concepts will help to build a solid foundation for the study of algebra. The importance of using arithmetic equalities to build understanding of the equal sign was noted by Kieran (1981). In addition, McNeil and Alibali (2005) found that presenting students with equations including operations on both sides of the equal sign, such as 4 + 1 = 5 + 2 in the example above, helped students develop a relational understanding of the equal sign. It will be important for educators to use many of these types of examples and not only equations of the "operations equals answer" context.

In second grade, students will begin representing addition and subtraction word problems, "with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem" (Common Core State Standards Initiative, p. 19). By third grade, students are expected to "solve two-step word problems using the four operations" and "represent these problems using equations with a letter standing for the unknown quantity" (p. 23). This expectation progresses to multi-step word problems, using only whole numbers, in fourth grade and to word problem involving fractions in fifth grade. The full concept of variable is then introduced in sixth grade, when students are expected to "use variables to represent numbers and write expressions when solving a real-world or mathematical problem" and "understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set" (p. 44). These expectations for young students to represent word problems using equations contradicts the findings of Capraro and Joffrion (2006), which indicated that many students, even in middle school, were not ready "to translate from the written word to mathematical equations" (p. 147). The implementation of these rigorous standards in the lower grades will prove challenging for educators but, if successful, can have a significant impact on students' algebraic understanding in middle school.

There are also major changes on the horizon in terms of how students will be assessed and, therefore, how teachers are held accountable. The common assessments that are to be developed to assess students' understanding of the grade-level standards will be very different from the assessments that most students and teachers are accustomed to. Two consortia are developing assessments aligned to the CCSS and both have indicated that students will be expected to respond to open-ended assessment items and performance tasks, in addition to the traditional selected-response items. Georgia is a governing state in the Partnership for Assessment of Readiness for College and Careers (PARCC) and is expected to use the PARCC assessment beginning in the 2014-2015 school year.

The PARCC mathematics assessments in grades three through eight will consist of two summative components. The first component will be a performance-based assessment and "will focus on applying skills, concepts, and understandings to solve multi-step problems requiring abstract reasoning, precision, perseverance, and strategic use of tools." The second component will be an end-of-year assessment and "will call on students to demonstrate further conceptual understanding" of grade-level standards and "demonstrate mathematical fluency" (Partnership for Assessment of Readiness for College and Careers, 2012).

In mathematics, PARCC has identified, at each grade level, clusters of standards that are considered to be major clusters, supporting clusters, and additional clusters. These clusters are related, in part, to the weighting of standards on the PARCC assessment. Standards in the major clusters, therefore, are expected to be assessed more heavily than standards in the supporting and additional clusters. There is concern that teachers may interpret the additional clusters as being "extra" or "optional" standards and either not teach these standards or simply "cover" them briefly. These standards, however, are essential for success in the next grade level and should not be dismissed as unnecessary.

Implications for Pedagogy

As noted by McTighe and Brown (2005) rigorous academic standards and high-stakes accountability measures "are not a passing fad" (p. 243). They do not, however, imply a one-size-fits-all standardization of teaching practices. Teachers will continue to be faced with classrooms filled with learners at varying levels of readiness and they must continually reflect on their teaching practices to ensure that the needs of all learners are met. To do this, teachers need to develop knowledge and understanding of differentiation. Implementing differentiation can be very difficult, especially for secondary teachers who serve over 100 students per day. To help teachers develop the needed skills in differentiation, professional learning is needed that actually models differentiation. This professional learning might be characterized by mentoring, coaching, or study groups. "By differentiating professional learning, teachers can live differentiation as they are learning about it" (Brimijoin, 2005, p. 260).

Flexible grouping is one strategy that teachers may consider for differentiating instruction. Flexible grouping can allow some student groups additional time on a concept while providing time for enrichment with other groups of students. This strategy also provides the benefits of cooperative learning mentioned earlier. Teachers would benefit from professional learning on flexible grouping, as this strategy can be difficult to implement in middle school classrooms. Additionally, mastery learning strategies (Guskey, 1997), such as allowing students to retake tests and quizzes to demonstrate mastery could be beneficial to students. Students often view the test as the end of learning for a particular concept and allowing a retake test will provide time for students to continue working on previous concepts. This strategy also sends a message to the students that the teacher cares about their performance and wants them to improve.

It is also important that students' conceptual understanding of arithmetic be developed prior to instruction in algebra. The primary responsibility to develop this understanding lies with educators prior to eighth grade, such as sixth and seventh grade. Eighth grade teachers, however, should communicate with their colleagues in the lower grades regarding the misconceptions that students have in algebra. By working together, educators can improve their own understanding of these misconceptions, thereby improving their instructional methods and students' understanding. Some teachers in lower grades may not realize that how they teach arithmetic concepts can later impact a student's understanding of algebra.

It is not always easy, however, to convince teachers in other grade levels to change their practice. Eighth grade teachers at LMS have approached seventh grade teachers, asking that they try to focus more on students' understanding of algebraic concepts, rather than on memorization of steps. The response, however, was that their CRCT scores were high and they did not want to do anything to jeopardize those scores in the future. It seems that the focus on accountability, especially the possibility of teacher pay being based on performance, often leads to superficial coverage of topics in order to pass a test and little consideration is given to students' understanding of the concepts and readiness for the next grade level.

With the implementation of the CCSS and the impending changes in assessment methods, teachers in the lower grades will no longer be able to depend on memorization of steps to lead to students' high performance on the summative assessments. Perhaps these changes will lead to more focus on teaching for understanding, rather than teaching to the test. If students understand the concepts, they will perform well on the test. Mathematics educators at all levels need to consider how their teaching strategies impact students' understanding of algebraic concepts. Some teachers, however, may need professional learning opportunities and coaching on ways to teach for student understanding.

Educators should also consider how they can provide additional instructional time for students struggling with algebra. There are many possibilities that could be considered based on the specific situation. Ideally, this additional time should be provided during the school day, as after-school tutoring is often inconvenient for the student, the teacher, and the parents. One option might be a support math class during Connections class time. LMS currently offers a similar class, but it is a very small class and is designed for students who have had low scores on the CRCT in prior years. Many students who struggle with algebra in eighth grade do not qualify for the class. An additional class is needed for students who need additional time to work on grade level mathematics concepts. Funding, however, is not available to all schools for additional courses or teachers. State funding is provided for support math classes in high school and perhaps should be considered for middle school as well. Teachers can also find ways to provide additional time for struggling students by implementing differentiated instructional strategies and cooperative learning strategies as mentioned earlier.

Limitations of the Study

A primary limitation of this study is that data was collected from only a selected group of students from one middle school. Generalizability of the findings, therefore, may be limited, but it is important for students' voices to be heard. As noted by Patton (2002), "While one cannot generalize from single cases or very small samples, one can learn from them – and learn a great deal, often opening up new territory for further research" (p. 46). A second limitation of this study is the possibility that participants may not have been completely honest and forthcoming in their responses due to fear that I may share information with their teachers. I believe the students were very honest, however, as I worked to build relationships with these students that would allow them to feel comfortable talking to me about their experiences and to trust that I would keep their responses confidential. I assured students on several occasions that their responses would remain confidential.

Suggestions for Further Research

With the adoption of the Common Core State Standards (CCSS) by 45 states (Common Core State Standards Initiative, 2012), a higher percentage of U.S. students will be learning algebra in the eighth grade. While the CCSS curriculum for eighth grade mathematics does not include algebra to the same extent as the former GPS curriculum, there remains a significant focus on linear equations in eighth grade. With the majority of the nation's eighth graders studying algebra in eighth grade, there will be a host of opportunities for further research.

A primary limitation of this study was the small sample of participants. With most of the nation's eighth graders studying algebra under the new CCSS curriculum, a larger study of students' experience with algebra is needed. Also, if "all students must have the opportunity to learn and meet the same high standards" (Common Core State Standards Initiative), further research should focus on the support needed by many students to be successful in eighth grade algebra. Simply adopting high standards for all students is not enough. Many students need support in eighth grade algebra that they are not receiving and additional research in this area is critical.

The participants in this study noted that algebra should be differentiated and that working with peers was beneficial to their understanding of algebra. Based on these findings, additional studies of the effectiveness of differentiated instruction and cooperative learning on students' achievement and attitudes in eighth grade algebra could be very beneficial to teachers who are working to implement these strategies in their classrooms. Studies related to professional learning in these areas is also necessary as many teachers realize the benefits of these strategies and want to implement them in their classrooms, but need additional support to implement them effectively.

Study participants also reported that encouragement and support from their teachers was important for their success in algebra. This finding suggests the need for additional research on the impact of teacher-student relationships on student achievement, especially in middle school when students are experiencing a myriad of psychological changes.

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APPENDIX A

Student Interview Questions

Interview 1

1. Tell me a little about yourself, such as where you are from, how old you are, etc.
2. How long have you been at this school?
3. What is your favorite subject? Why is that your favorite subject?
4. What is your least favorite subject? Why?
5. What sports or extra-curricular activities are you involved in?
6. Tell me about your family, such as who lives in the same house as you, their ages, etc.
7. Describe a typical night when you are working on homework or studying for a math test.
8. What kind of assistance do you have at home when you are working on math homework?
9. How do you think your parent or guardian could help you do better in math?
10. Is there anything else you would like to tell me about yourself or your family?

Interview 2

1. When you think about algebra, what is the first thing that comes to your mind? Tell me about that.
2. What do you like about algebra?
3. What do you dislike about algebra?
4. Do you think algebra is important? Tell me more about why you feel that way.
5. Do you believe that the content of the 8th grade math curriculum is appropriate for 8th grade students? Tell me more about why you feel that way.
6. Do you think that all students should be required to learn the same math? Tell me more about why you feel that way.

7. How do you feel about the amount of material in 8th grade math and the pace of instruction?
8. Tell me what a typical day is like for you in math class.
9. Tell me about some features of your math class that help you learn algebra better.
10. Tell me about some features of your math class that make it difficult for you to learn algebra.
11. How have your teachers (current or previous) helped you learn algebra more easily?
12. How have your math teachers demonstrated high expectations for you and others in your class?
13. What challenges do you face with algebra?
14. Which concept has been the easiest for you in math this year? What do you think made it easier?
15. Which concept has been the most difficult for you this year? What do you think made it difficult?
16. How did your previous math classes help prepare you for the algebra required in the 8th grade curriculum?
17. Looking back over your time in middle school, what experiences in your math classes stand out and why?
18. Is there anything else you would like to share with me about your experiences in math?

Research Question	Interview Questions
How do eighth grade students perceive algebra and the eighth grade mathematics curriculum?	1, 2, 3, 4, 5, 6, 7
How do students describe the strategies that	8, 9, 10, 11, 12

teachers employ to teach algebra?	
What difficulties, if any, do students encounter in learning the algebra concepts required by the eighth grade mathematics curriculum?	13, 14, 15
How do students describe their previous middle school experiences in mathematics?	16, 17, 18

APPENDIX B

Tasks for Student Think-Aloud Interviews

Use of the Equal Sign

Solve each of the following equations. Explain your steps as you go.

1. $6x - 11 = -2x + 5$

2. $5n + 20n = 5(n + 20)$

Use of Variables

3. The Home Station Store is having a sale on carpet for stairs. The sale price is \$8.95 per yard plus \$145.00 for installing the carpet.

    a. Make a table that shows five inputs and outputs for getting carpet for your stairs.
    b. Choose one pair of inputs and outputs from the table and explain what it means in this situation.
    c. If you spend \$324.00, how many yards of carpet can you have installed?

(Collins & Dacey, 2011, p. A14)

Linear Equations

Graph the following linear equation. Explain what the graph represents.

4. $-5x + 3y = 18$

Systems of Linear Equations

Solve the system of equations. Explain the meaning of the solution.

5. $x - 3y = -2$
   $4x + 7y = 11$

APPENDIX C

Focus Group Questions

1. What are your perceptions of algebra and the eighth grade mathematics curriculum?
    - Difficulty
    - Amount of material
    - Pace of instruction
    - Real-world use, importance of learning algebra

2. What are some strategies that teachers employ to teach algebra? What do you think about these strategies?

    - Warm-Ups
    - Notes/lecture
    - Homework
    - Group work
    - Students teach the class/work at the board

3. What difficulties, if any, do you encounter in learning algebra concepts?

    - Variables
    - Integers
    - Number of steps
    - Memorizing formulas
    - Word problems

4. What are your previous middle school experiences with mathematics?

    - 6^{th} grade – fun
    - 7^{th} grade – slower pace
    - Teacher expectations

APPENDIX D

Parental Informed Consent

Dear Parent or Guardian:

A study will be conducted at your child's school over the next few weeks. Its purpose is to learn about students' experiences in mathematics. In particular, we will be asking questions on how students feel about mathematics and how they perform in mathematics. Students will also be asked to solve problems and "think aloud" about their understanding of the problem.

If you give permission, your child will be asked to complete a series of face-to-face interviews. Each session of the study will take approximately 30-45 minutes for your child to participate.

Your child's participation in this study is completely voluntary. The risks from participating in this study are no more than would be encountered in everyday life; however, your child will be told that he or she may stop participating at any time without any penalty. Your child may choose to not answer any question(s) he/she does not wish to answer for any reason. Your child may refuse to participate even if you agree to her/his participation.

In order to protect the confidentiality of the child, a number and not the child's name will appear on all of the information recorded during the experiment. Your child will also choose a pseudonym to be used in the final research report. All information pertaining to the study will be kept in a locked filing cabinet in an office at my home. No one at your child's school will see the information recorded about your child.

If you have any questions or concerns regarding this study at any time, please feel free to contact Jan Reyes, Curriculum Studies doctoral candidate, at (678) 416-3683, or Dr. Yasar Bodur, advisor, at (912) 478-7285.

To contact the Office of Research Services and Sponsored Programs for answers to questions about the rights of research participants please email IRB@georgiasouthern.edu or call (912) 478-0843.

If you are giving permission for your child to participate in the study, please sign the form below and return it to your child's teacher as soon as possible. Thank you very much for your time.

Jan Reyes
Curriculum Studies doctoral candidate

Dr. Yasar Bodur
Teaching & Learning, Professor

Investigator's Signature: ____

Child's Name: ____

Parent or Guardian's Signature: ____

Date: ______________________________________

APPENDIX E

Minor's Assent Letter

Dear students,

I am Jan Reyes, a graduate student at Georgia Southern University, and I am conducting a study on Eighth Grade Students' Experiences of Algebra.

You are being asked to participate in a project that will help me learn about your experiences with algebra. If you agree to be part of the project, you will be asked to complete a series of individual interviews and participate in a group interview with other participants.

You do not have to help me with this project. You can stop whenever you want. If you do not want to answer some of the questions during interviews, you do not have to. You can refuse to help me with the project even if your parents have said you can.

None of the teachers or other people at your school will see the answers to the questions that I ask you. All of the answers that you give me will be kept in a locked cabinet in a room at my home, and only I or others who are helping me will see your answers. We are not going to put your name on the answers that you give us, so no one will be able to know which answers were yours.

If you or your parent/guardian has any questions about this form or the project, please call me at (678) 416-3683 or my advisor, Dr. Yasar Bodur, at (912) 478-7285. Thank you!

If you understand the information above and want to do the project, please sign your name on the line below:

Yes, I will participate in this project: ___________________________________

Child's Name: ____

Investigator's Signature: ____

Date: ________________

APPENDIX F

Sample Comments by Code

Students' Perceptions of Algebra

P-c Perceptions: Confidence in algebra ability

"Yeah, I think I did this wrong."

"Wait, can it be decimals or no?"

P-d Perceptions: Difficulty of algebra

"It's gonna be hard."

P-dl Perceptions: Dislike algebra

"I really don't like anything about it. Just if it's easier sometimes, I'll get it, but I still won't like it."

P-fp Perceptions: Fast pace of instruction

"if they'd take it slower it would be better"

"we have to go at a fast pace to be able to cover everything"

P-i Perceptions: Importance of algebra in real life

"I don't think it's like really too much important to like really learn algebra unless you're gonna go like in a career that you have to like use kinda stuff like that."

P-te Perceptions: Teacher expectations

"you may not know what the teachers expect you to know"

P-u Perceptions: Understanding of algebra

"I really just didn't understand it that much."

"I just don't really understand algebra."

Teacher Strategies

TS-c Teacher Strategies: Cooperative learning

"if we got a mistake wrong the class will help you"

TS-d Teacher Strategies: Direct instruction

"She writes, we have vocabulary terms, she'll write that and we have to copy that. And then she'll show us examples and we'll have to follow along."

TS-eh Teacher Strategies: Extra help

"usually helped me by like getting me some tutors or tutoring me after school and like sending me home with extra stuff to help me"

TS-es Teacher Strategies: Encouragement/support

"She said, 'I know you got a low grade, but I'm still proud of you for working hard.'"

TS-he Teacher Strategies: High Expectations

"they like believed in me and thought I could do it"

TS-ii Teacher Strategies: Individualized instruction

"she used to do a lot of one on one"

TS-ip Teacher Strategies: Independent practice

"then she'll give us worksheets"

TS-m Teacher Strategies: Mastery learning

"if we got a bad grade, she would let us do corrections"

TS-sc Teacher Strategies: Structured classroom

"you get used to the class after a while. It's the same thing."

TS-st Teacher Strategies: Students teach

"I like when she lets us go up to the board and like each work one"

TS-tp Teacher Strategies: Test prep

"usually we have a warm up and then we'll do the Coach book"

TS-g Teacher Strategy: Grading practices

"It wouldn't count as a grade cause we just learned it but it was like a lesson quiz to see where we were."

TS-hw Teacher Strategy: Homework

"If you didn't get it in school you have to do homework over it. And especially if you have like 15 problems to do, then you're just confusing yourself even worse trying to work out 15 problems."

TS-r Teacher Strategy: Resources

"Usually I'll go on the internet and see if there's like a video or something"

Students' Difficulties with Algebra

D-e Difficulty: Easier concepts

"You'll have a basic equation and you'll have to subtract on both sides and it'll be a short equation. That was the easiest."

D-eq Difficulty: Equations

"those equations are difficult for me"

"But if you hand me a long equation, I'm not gonna be able to do it"

D-fp Difficulty: Insufficient time to learn

"it's just if we had more time to do all of that it would have been better. Because you learn so much in so little time and you may not know how to begin it"

D-fr Difficulty: Fractions

"Would you get a fraction? Seven doesn't go into 16."

D-fs Difficulty: Formulas and steps

“And since 6 is your whole number, you would find 6 on the coordinate plane.”

“once you get the answer and then you got a do a whole ‘nother step”

D-i Difficulty: Integers

“I get confused with some of the steps sometimes and the integers”

D-io Difficulty: Input-output table

"But I don’t know how to set it up"

"Input and outputs, so [labels first column ‘Input’ and second column ‘Output’]. And you have, hmmm, I’m stuck."

D-lc Difficulty: Lack of connection to real world

“I’m not sure if you use it in the real world”

D-le Difficulty: Linear equations

"I know it graphs the y-intercept, but I don’t know."

D-lu Difficulty: Lack of understanding

“you would probably just plug in some number for *y*”

D-m Difficulty: Misconception

[Copies the first equation. 6x-11=-2x+5]. Oh, I have to combine like terms so I bring the negative 2x over here to this positive 6x. [Subtracts 2x on each side of the equation]. And that’s 4x minus 11 equals positive 5. And then I have to bring the negative 11 over here so I get x by itself. [Subtracts 11 from each side of the equation].

D-r Difficulty: Radicals

“I don’t get it when you have to like multiply square roots and stuff”

D-v Difficulty: Variables

"All that like *x* and *y*'s everywhere and all the like numbers and letters mixed in. It's like oh, man."

"it's usually like the letters that confuses me"

D-wp Difficulty: Word problems

"There is always something to distract you in the word problem and I always get distracted."

Students' Previous Experiences with Algebra

PE-p Previous Experiences: Preparation for eighth grade math

"I don't think it did that much preparing. I mean, 6^{th} and 7^{th} grade, it was hard, but it was definitely more like wide scaled."

"I think that we did a lot of math this that we didn't know the basics of how to do it"

PE-tn Previous Experiences: Teacher experience, negative

"Teachers before if you made a bad grade on a quiz or something, they wouldn't really say anything about it too much."

"I've had teachers in the past who haven't really had high expectations which is kind of like disappointing"

PE-tp Previous Experiences: Teacher experience, positive

"my previous math teachers always like encouraged me and they always had hope in me"