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The importance of radiation as a means of controlling surface and internal temperatures of a semi-infinite solid

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THE IMPORTANCE OF RADIATION AS A MEANS OF CONTROLLING SURFACE AND INTERNAL TEMPERATURES OF A SEMI-INFINITE SOLID

BY

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A

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ABSTRACT

This problem is concerned with the temperature history of a semi-infinite solid bounded by one flat surface.

Heat was applied by convection from a hot fluid to the flat surface of the solid which was originally at a uniform temperature. Heat was radiated from the flat surface to black space and conducted into the solid according to Fourier's law. The temperature at the surface, and at several points beneath the surface, was determined as a function of time.

A method for solution of such a problem is demonstrated here by the use of numerical analysis and the Royal McBee LGP 30 Digital Computer. The solution is applicable to all similar problems.

The result demonstrates that radiation from the flat surface of the solid is very important because it results in depressing the surface temperature far below that of the hot fluid in contact with its one surface.

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TABLE OF NOTATIONS

A = Area, Ft^2

c = Specific heat, BTU/ Lb ^{o}F

δ = Incremental measurement (Fig. 2)

ϵ = Emissivity

h = Surface conductance, BTU /Ft^2 ^{o}F Hr.

k = Conductivity, BTU/Ft ^{o}F Hr.

l = Length, Ft.

θ = Time, Hr.

Θ = Fourier Modulus

Nu = Nusselt's Number

R = $\epsilon \sigma A$

σ = Stefan-Boltzmann Constant

t = Temperature, ^{o}F

V = Volume, $Ft.^3$

w = Weight Density, $Lb/Ft.^3$

I. INTRODUCTION

The heating of a semi-infinite solid may be accomplished in a number of ways. For example, the solid may be heated by radiation, by convection, or by conduction through contact with another solid. If the application of heat is uniform on all the surface and if the solid is homogeneous, the heat flow within the solid will be one dimensional and normal to the surface of the solid.

In the case of most heat conduction problems, the temperature rather than the heat flux specifies the boundary conditions for the differential equations dictated by Fourier's law for the conduction of heat in solids.

It has been demonstrated by other graduate students of the Mechanical Engineering Department of this school that with proper technique, numerical methods and the digital computer are a powerful combination for solving such problems. An excellent example of this combination is shown in a thesis by B. W. Marshall (1). In his paper the numerical solution was obtained by using the digital computer in conjunction with a method of numerical analysis. That author's problem was similar to this one, with the main difference being that he considered a plate of finite width, whereas the plate under consideration here is of infinite width.

The types of heat flux considered here were radiation, convection and conduction. Figure 1 shows the direction of flow assumed for each mode of heat transfer. The solid was considered to be heated by convection, while the surface was cooled by radiation. Internal heat transfer was accomplished by conduction.

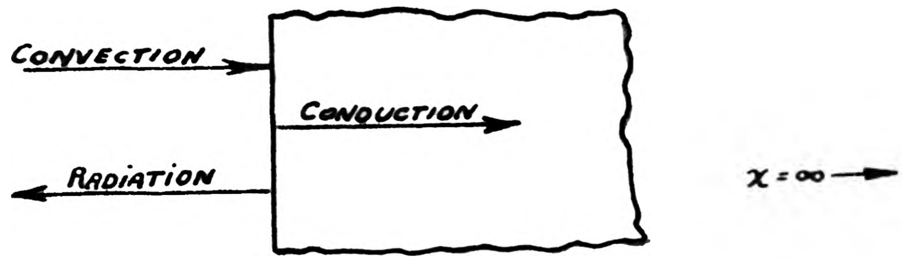


Figure 1. Types of heat flow, and their direction.

The problem to be investigated in this thesis was to determine temperature distribution within the solid as a function of both time, and distance from the surface. This temperature distribution will show the effect of radiation as a means of controlling the surface temperature and the temperature of various internal points.

II. REVIEW OF LITERATURE

The analytical approach to temperature variation in a semi-infinite solid as a function of time may be found in most advanced heat transfer texts. Schneider (2), Kern (3), and Jakob (4) all have a sampling of this type of problem. However, none of the above mentioned authors have considered radiation in these problems.

Some papers written by graduate students of the Mechanical Engineering Department of this school have investigated radiation, but have done so with a finite solid. Notable among these is a paper done by B. W. Marshall (1).

The lack of study concerning radiation in high temperature cooling is also apparent in the space technology field. An article by R. Hawkes (8) in a journal from that field indicates that after 12 years of rocket engine research, Rocketdyne* is now testing an engine with the combustion chamber and expansion cone cooled only by radiation. Rocketdyne engineers now state the radiation cooling would be more effective in space than in the laboratory. It is true that there may be as much as 3 percent heat rejection by conduction and convection to the atmosphere in ground tests, but the effectiveness of the radiation process is hindered by the high energy " back pressure" of the atmosphere. The back pressure referred to here is the re-radiation of heat back to the engine from the atmosphere surrounding it. It was with this in mind that the author of this thesis chose to radiate heat from the surface of the steel plate

* Rocketdyne, division of North American Aviation, Inc.

to so-called black space.

A treatment is presented by Jakob (4) concerning radiation and absorption by gases. Since Jakob's article is on a simplified level, the solution and explanation of the phenomena of radiation and absorption by gases is still largely a matter of conjecture.

III. DISCUSSION

The determination of the transient temperature within a semi-infinite solid of uniform properties and initially at a uniform temperature, while recieving and radiating heat at its one surface, requires the solution of the well Fourier's general equation

$$\frac{\partial t}{\partial \theta} = \left(\frac{k}{c\rho}\right)\frac{\partial^2 t}{\partial x^2} \qquad (1)$$

where t is temperature, θ is time, x is distance, and c, k, and ρ are the specific heat, thermal conductivity, and density respectively. There are solutions to this equation which meet certain boundary conditions. However, there are no exact solutions known to the author which meet the boundary conditions peculiar to this problem.

The conditions involving an infinite length, x, and an infinite time, θ, are the contributing factors to the author's inability to arrive at an exact solution. There are, of course, solutions to differential equations involving an infinite distance or infinite time. However, in combination with the conditions of this problem, the most generally used and taught method (Separation of Variables) will fail to yield an exact solution, as is demonstrated herein.

The conditions for this problem will allow two boundary conditions to become immediately apparent. The initial temperature throughout the plate was 100°F, defining the first boundary condition as

$$T_{(x,\theta)} = K \,, \qquad \text{where } K = 100°F, \ \theta = 0, \ 0 \leqq x \leqq \infty$$

or

$$T(x,0) = 100 \,, \qquad 0 \leqq x \leqq \infty .$$

The fact that the temperature of the plate will ultimately become stable and uniform throughout will cause the second condition to be

$$\frac{\partial T(x,\theta)}{\partial x} = 0, \qquad \text{where } \theta = \infty, \ 0 \leqq x \leqq \infty$$

or, at the surface where x = 0,

$$\frac{\partial T(0,\infty)}{\partial x} = 0 \ . \tag{2}$$

It should be noted that from this second equation, (2), the ultimate temperature can be predicted. Since there is no net heat exchange at the surface of the plate when the condition of stability is finally reached, there exists a heat balance between the convective flow into the face and the radiative flow from the face. This balance is

$$h\,(t_g - t_w) = \sigma \epsilon T_w^4 . \tag{3}$$

Since h, σ, ϵ, and t_g are known, t_w may be found algebraically as 1540° F.

In applying the Separation of Variables Method of solution, the main assumption is that the solution of equation (1) can be written in a "product form", i.e. a function of x multiplied by a function of θ, or

$$T_{(x,\theta)} = X_{(x)} \cdot \theta_{(\theta)} .$$

This assumption is justified by the fact that it will yield solutions to a wide variety of engineering problems.

Substituting this form of the function T into equation (1) yields

$$X\theta' = \left(\frac{k}{c\rho}\right)\theta X'' \quad \text{or} \quad \frac{\theta'}{K\theta} = \frac{X''}{X} \;, \quad K = \left(\frac{k}{c\rho}\right) ,$$

where the prime notation indicates differentiation of that particular function with respect to the only variable present. Since each side of the last expression is independent of the other variable, their common value cannot be a function of either x or θ, and must therefore be a constant, say λ, such that

$$\frac{\theta'}{K\theta} = \lambda \;, \quad \frac{X''}{X} = \lambda$$

or

$$\theta' - \lambda K\theta = 0 \;, \quad X'' - \lambda X = 0 \;.$$

The constant λ may have only three possible ranges or values: greater than 0, exactly equal to 0, or less than 0.

If the value of the constant is to be greater than 0, the solutions of $X'' - \lambda x = 0$ are exponential in the variable x, implying that the temperature will rise exponentially as the distance x increases, which is physically unreasonable , considering the conditions of this problem.

If the value of the constant is less than 0, the solutions of $\Theta' - \lambda K \Theta = 0$ will yield

$$\Theta(\theta) = C e^{-\beta^2 K \theta} , \qquad \text{where } \lambda = -\beta^2 .$$

This implies that as time θ becomes infinite, the function $\Theta(\theta)$ approaches 0, causing the product solution to approach 0. Since it is apparent that the temperature will not approach 0 after considerable time, this solution must also be rejected.

The remaining case is that of $\lambda = 0$. Using this value, we will find the solutions

$$\Theta(\theta) = A \qquad , \qquad X(x) = Bx + C \quad .$$

Again, as the distance x becomes infinite, the product of the linear function and the constant must also approach infinity. This is again clearly not in agreement with the conditions of the problem.

Thus it is shown that the Method of Separation of Variables has failed to yield an exact solution due to the conditions of this problem. A numerical solution method of Dusinberre (5) and others, is found to be satisfactory, which the author now shows.

Conduction through the solid is defined by the Fourier conduction equation

$$q = -kA \frac{dt}{dx} \quad , \qquad (4)$$

where q is the amount of heat flowing per unit time, A is the area normal to the direction of the flow of the heat, and k is the proportionality factor called the thermal conductivity. As applied to this problem, equation (4) takes the form

$$q = -k \frac{A}{L} (t_2 - t_1) \quad , \qquad (5)$$

where L is the length of the flow path from point 2 to point 1, and t_2 and t_1 are the temperatures of points 2 and 1 respectively.

In order to determine an expression for the convective heat flow from a gas to a solid in contact with the gas at the solid's surface, the author applied the general linear solution of the Fourier expression (4). In this process of heat transfer, the resistance to heat flow is generally confined to a thin layer immediately adjacent to the wall surface, and as a result, the temperature gradient is usually limited to this same layer called the boundary layer. This thin layer conducts heat from the wall to the main body of the gas. The thickness of this layer shall be designated as δ_g so that it replaces its equivalent L in the application of the equation. The conductivity of the gas now becomes k_g, and the temperatures t_2 and t_1 now become t_w and t_g, respectively.

Thus we have

$$q = \frac{k_g}{\delta_g} A \, (t_g - t_w) \quad ,$$

where A represents the cross-sectional area of the flow path.

Owing to the difficulty of interpreting and measuring an effective film thickness, it is the usual practice to combine the quotient $\frac{k_g}{\delta_g}$ into a single property h, and therewith express the general boundary heat exchange as simply

$$q = hA \, (t_g - t_w) \, . \tag{6}$$

Radiation from the solid is governed by the Stefan-Boltzmann radiation equation

$$q = (\epsilon \sigma A) t_1^4 = R t_1^4 \quad , \tag{7}$$

in which ϵ is the emmissivity of the surface, and σ is the Stefan-Boltzmann constant.

Equations (5), (6), and (7), when summed, represent the net heat flow responsible for a change in the internal energy at any point after time θ , according to the law of conservation of energy. This change of internal energy is defined as:

$$\frac{cwV}{\Delta\theta}(t_1' - t_1) \quad , \tag{8}$$

where c is the specific heat, w is the weight density, v is the volume of the node, $\Delta\theta$ is the time increment, and t_1' represents the temperature of point 1 after the time increment has elapsed.

Therefore, the heat balance equation is

$$hA(t_g - t_1) - Rt_1^4 + K\frac{A}{L}(t_2 - t_1) = \frac{cwV}{\Delta\theta}(t_1' - t_1) \ . \tag{9}$$

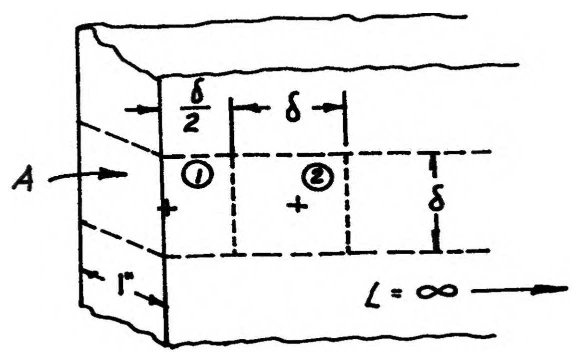


Figure 2. Division of Plate into nodes.

As it is seen from Figure 2, the surface area, A, normal to the path of flow is $\delta \times 1 = \delta$ and the length of the flow path from point 1 to point 2 is $\frac{\delta}{2} + \frac{\delta}{2} = \delta$.
Substituting these into equation (9), we have

$$h\delta(t_g - t_1) - Rt_1^4 + k\frac{\delta}{\delta}(t_2 - t_1) = \frac{cwV}{\Delta\theta}(t_1' - t_1) \ ,$$

and simplifying

$$\frac{h\delta}{k}(t_g - t_1) - \frac{R}{k}t_1^4 + (t_2 - t_1) = \frac{cwV}{\Delta\theta k}(t_1' - t_1) \; . \qquad (10)$$

By defining Nusselts Number as $\frac{h\delta}{k}$ and the quantity called thermal diffusiveity as $\alpha = \frac{k}{cw}$, the substitution of these is now made into equation (10) so that

$$Nu\,(t_g - t_1) - \frac{R}{k}t_1^4 + (t_2 - t_1) = \frac{V}{\alpha\Delta\theta}(t_1' - t_1) \, . \qquad (11)$$

Again, from Figure 2, it is seen that the volume of the node is

$$V = \delta \times \frac{\delta}{2} \times 1 = \frac{\delta^2}{2} \; .$$

Substituting this into equation (11), we have

$$Nu\,(t_g - t_1) - \frac{R}{k}t_1^4 + (t_2 - t_1) = \frac{\delta^2}{2\alpha\Delta\theta}(t_1' - t_1) . \qquad (12)$$

Defining further the Fourier Modulus as

$$\Theta = \frac{\alpha\Delta\theta}{\delta^2} \quad ,$$

and applying this to (12)

$$Nu\,(t_g - t_1) - \frac{R}{k}t_1^4 + (t_2 - t_1) = \frac{1}{2\Theta}(t_1' - t_1) \, ,$$

$$2\Theta Nu\,(t_g - t_1) - \frac{2\Theta R}{k}t_1^4 + 2\Theta(t_2 - t_1) = t_1' - t_1 \qquad (13)$$

Solving (13) for t_1'

$$2\Theta Nu\,(t_g - t_1) - \frac{2\Theta R}{K} t_1^4 + 2\Theta(t_2 - t_1) + t_1 = t_1'$$

$$t_1' = 2\Theta\left[Nu\,t_g + t_2 + t_1\left(\frac{1}{2\Theta} - 1 - \frac{R}{K}t_1^3 - Nu\right)\right] . \tag{14}$$

If the First Law of Thermodynamics is not to be violated, then the coefficient of t_1 has to be greater than, or at least equal to, zero. In equation form, this is

$$\left(\frac{1}{2\Theta} - 1 - \frac{R}{K}t_1^3 - Nu\right) \geqq 0 \quad . \tag{15}$$

Solving equation (15) for the Fourier Modulus Θ ,

$$\frac{1}{2\Theta} \geqq Nu + \frac{R}{K}t_1^3 + 1$$

$$\Theta \leqq \frac{1}{2(Nu + \frac{R}{K}t_1^3 + 1)} = \frac{K}{2(KNu + Rt_1^3 + K)} \quad . \tag{16}$$

This equation represents a limiting value of the Fourier Modulus in order to stay within the requirements of the First Law of Thermodynamics.

The values of the constants used were now selected in order to solve (16) for its limiting value. The selected constants, and their corresponding references, are as follows:

$K = 12.4$ BTU/Hr Ft °F (6)

$W = 488.0$ Lb/Ft^3 (6)

$h = 50.0$ BTU/Hr Ft^2 °F (7)

$\epsilon = 0.75$ (7)

$c = 0.11$ BTU/Lb °F (7)

The initial uniform temperature of the solid was $100^{\circ}F$ and the temperature of the gas at the surface was $2000^{\circ}F$. Having selected the distance between nodal points as $\delta = 0.5$ inches (Figure 2), the limiting value of the Fourier Modulus was now calculated.

Applying the values of the constants to (16), we have

$$\Theta \leqq \frac{12.4}{2\left(\frac{.75 \times .1713 \times 10^{-8} \times 560^3}{24}\right) + 2\left(\frac{50}{24}\right) + 2(12.4)} \leqq 0.426 .$$

Therefore, the maximum value of Θ is 0.426 . The nearest fraction to this which will give a simple solution to the internal equations without exceeding 0.426, is 1/3. This was chosen so that the internal conduction equations would have as simple a numerical form as possible, as is shown later. Having established this value, the author now solved for the time increment $\Delta\theta$ according to the definition

$$\Theta = \frac{\alpha \Delta\theta}{\delta^2} , \quad \therefore \quad \Delta\theta = \frac{\Theta \delta^2 c w}{k} . \qquad (17)$$

Solving equation (17) yields

$$\Delta\theta = \frac{(\frac{1}{3})(\frac{1}{24})^2(488)(.11)}{12.4} = .025 \ hr = 1.5 min .$$

Therefore the time increment for successive calculations is 1.5 minutes.

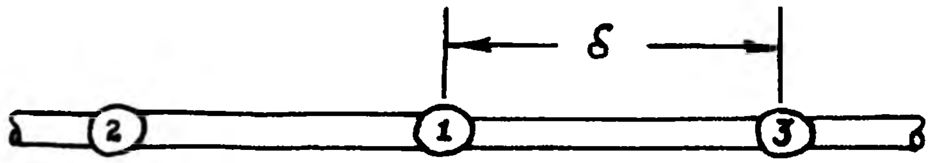


Figure 3. Structure of nodes

Having determined the equation for the surface temperatures and the limiting value of the Fourier Modulus, θ, the author began the final step, that of determining a numerical relationship between successive internal temperatures, governed by the value of the Fourier Modulus. If it is assumed, as in Figure 3, that the solid is composed of nodes joined by conducting rods, each with a uniform conductivity of "k", then it can be stated that the heat flow between nodes is

$$q = -k \; dt \quad . \tag{18}$$

As before, if δ is chosen sufficiently small, then the heat flow can be expressed as the finite difference $-k\Delta t$ where Δt is the temperature difference between adjacent nodal points. Therefore, the total heat conducted in the time increment $\Delta\theta$ is

$$Q = -k \, \Delta t \, \Delta\theta \quad . \tag{19}$$

As described in equation (8), the change in internal conditions is given by

$$(cwV)\, dt \quad ,$$

or expressed as a finite difference,

$$(cwV)\, \Delta t' \quad , \tag{20}$$

where $\Delta t'$ refers to $(t_1' - t_1)$, t' being the temperature after the time increment has elapsed. Then in terms of finite differences, we have the heat balance

$$-\sum k \Delta t \Delta \theta = cwV \Delta t' \, . \tag{21}$$

If the nodes in Figure 3 are now considered, equation (21) will now take the form

$$K_{21}(t_2 - t_1) + K_{31}(t_3 - t_1) = \frac{cwV}{\Delta\theta}(t_1' - t_1) \, . \tag{22}$$

Since this is a one dimensional network based on the conducting area $A = \delta \times L$, then

$$K = k\frac{A}{L} = k\frac{\delta}{\delta} = k \quad .$$

In the surface equation derivation, it was shown that $\alpha = \frac{k}{cw}$, and $\Theta_1 = \frac{\alpha \Delta\theta}{\delta^2}$. These substitutions along with the coefficient k can now be made into equation (22) to give

$$(t_2 - t_1) + (t_3 - t_1) = \frac{1}{\Theta_1}(t_1' - t_1)$$

$$t_1' = \Theta_1\left[t_2 + t_3 + t_1\left(\frac{1}{\Theta_1} - 2\right)\right] \quad . \tag{23}$$

Having defined the Fourier Modulus for this problem as 1/3, equation (23) will now yield

$$t_1' = \frac{1}{3}\left[t_2 + t_3 + (3-2)t_1\right]$$

$$t_1' = \frac{t_2 + t_3 + t_1}{3} \quad . \qquad (24)$$

Having found the equations for the surface temperature, and for the internal points for successive time intervals, the actual computations began. The Flow Chart for the computation of the problem is found on page 26, and the program for the Royal McBee LGP 30 Computer is found on page 25.

It is well known that numerical solutions of partial differential equations are subject to several different types of error. The first of these is truncation error, which is due to the use of finite subdivisions. Naturally, as these subdivisions become smaller and smaller, the numerical results approach the corresponding exact values more and more closely. This is refered to as the convergence of the numerical system. A second kind of error, known as the numerical error, is often thought of as consisting mainly of so-called "round-off" errors. Thus if it were possible to carry along an infinite number of decimal places in the calculations, there would be no numerical error. The way in which numerical errors grow or decay with time is though of as the stability of the difference equation. With regard to convergence, it is shown in Schneider (2) that the largest value of the internal modulus, θ , for which convergence will be guaranteed is $\frac{1}{2}$. This same maximum value also can be used as the criterion for guaranteeing stability. The value chosen in this thesis for θ , is 1/3, which is safely within the limits, so that convergence and stability are both guaranteed.

TABLE 1

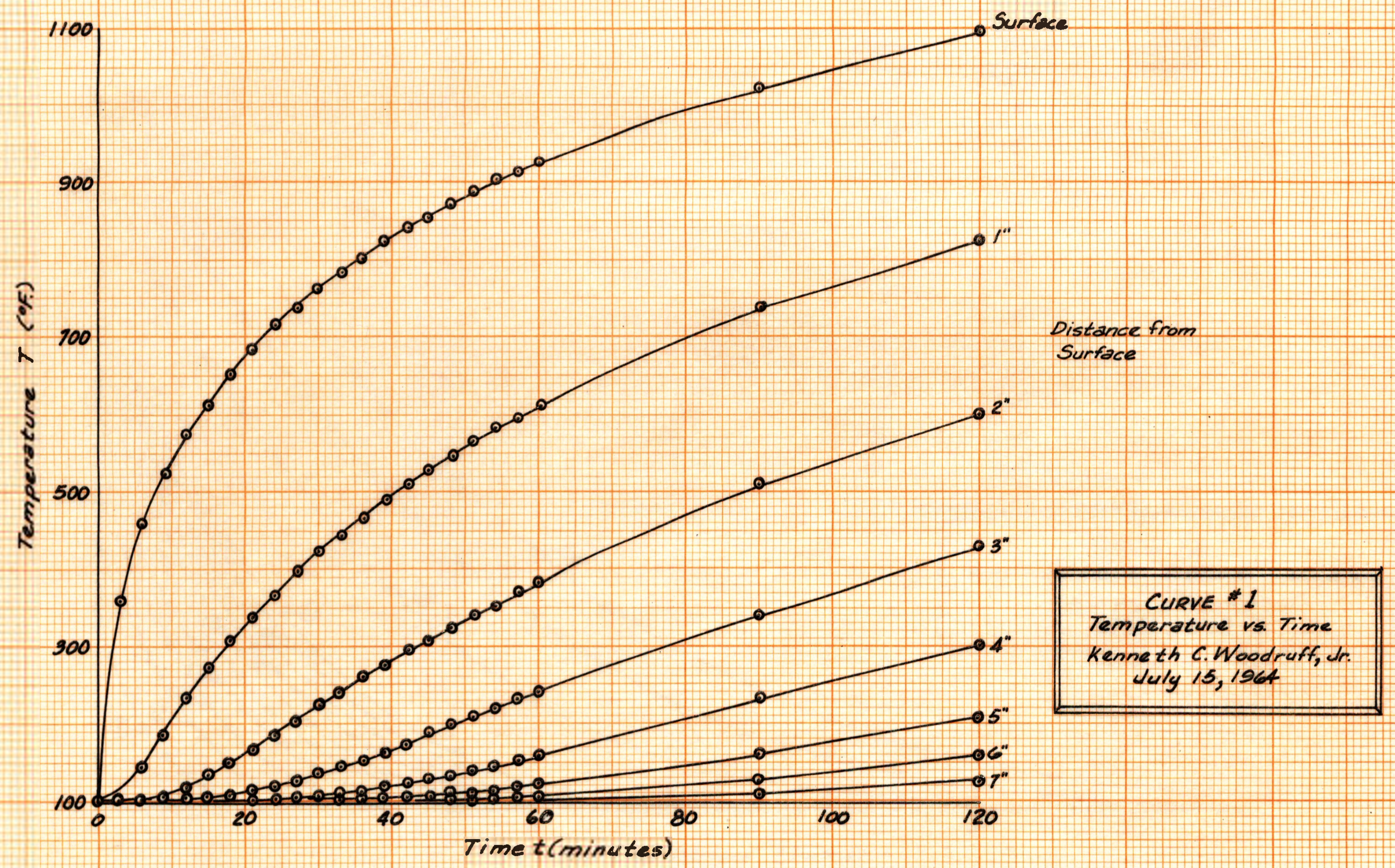
Temperature, °F, at distance from surface

TIME. MIN.	0"	1"	2"	3"
0.0	100.00	100.00	100.00	100.00
3.0	359.88	100.00	100.00	100.00
6.0	456.52	144.64	100.00	100.00
9.0	521.98	189.69	106.71	100.00
12.0	573.29	231.69	118.90	100.94
15.0	615.98	270.25	134.40	103.55
18.0	652.70	305.61	151.74	107.91
21.0	684.99	338.14	170.00	113.82
24.0	713.83	368.21	188.64	121.01
27.0	739.91	396.14	207.31	129.23
30.0	763.72	422.18	225.80	138.23
33.0	785.61	446.57	243.98	147.84
36.0	805.87	469.48	261.77	157.90
39.0	824.73	491.09	279.14	168.28
42.0	842.36	511.51	296.04	178.89
45.0	858.90	530.87	312.49	189.65
48.0	874.48	549.26	328.47	200.48
51.0	889.20	566.77	454.99	311.45
54.0	903.14	583.47	359.07	222.21
57.0	916.38	599.44	373.71	233.03
60.0	928.97	614.72	387.94	243.78
90.0	1029.12	739.12	510.56	344.77
120.0	1099.57	829.34	605.45	431.68

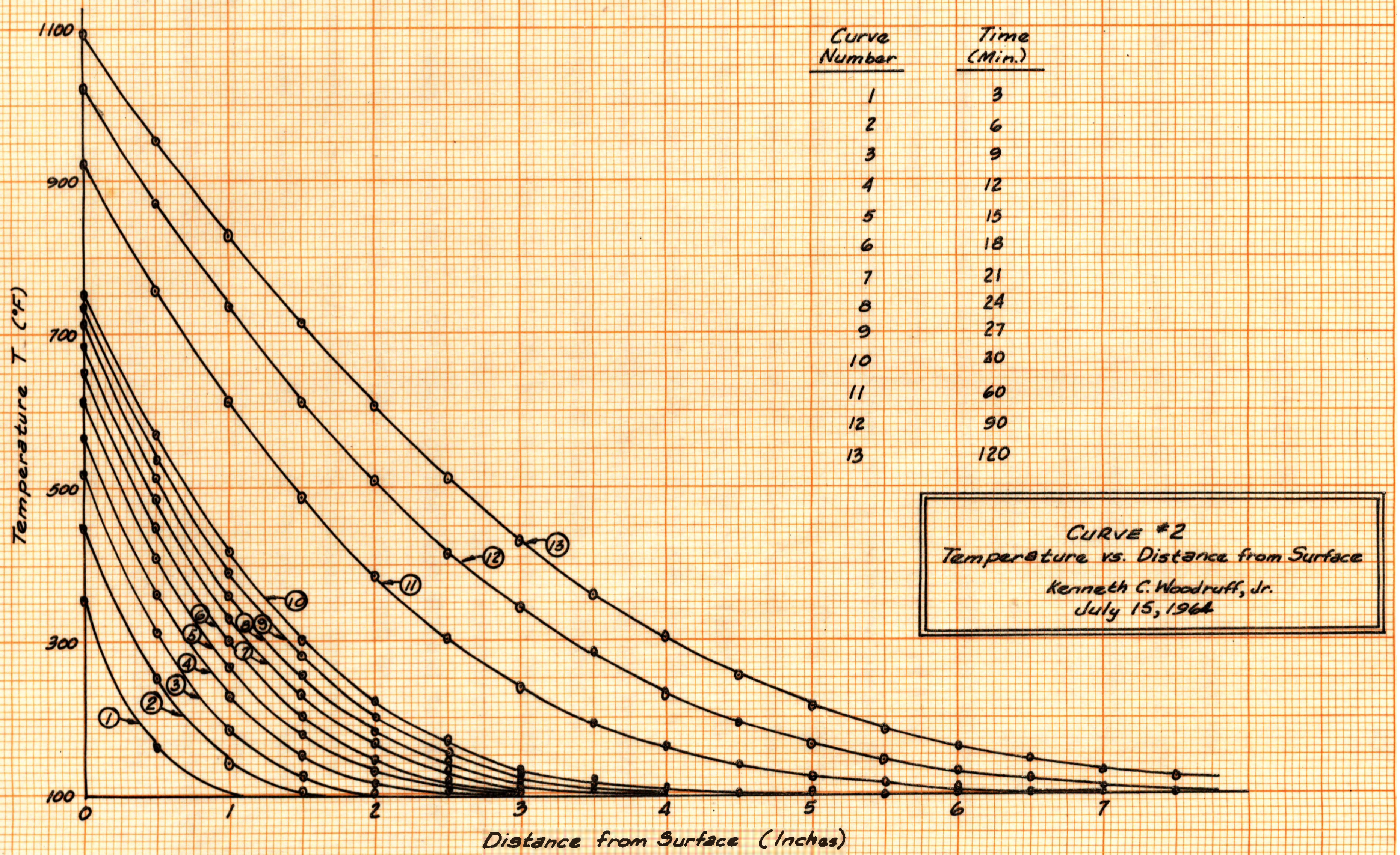
TABLE 1 (Cont'd.)

Temperature, °F, at distance from surface

TIME. MIN.	4"	5"	6"	7"
0.0	100.00	100.00	100.00	100.00
15.0	100.12	100.00	100.00	100.00
18.0	100.61	100.01	100.00	100.00
21.0	101.65	100.10	100.00	100.00
24.0	103.34	100.32	100.01	100.00
27.0	105.70	100.74	100.05	100.00
30.0	108.71	101.42	100.15	100.01
33.0	112.34	102.38	100.33	100.03
36.0	116.53	103.66	100.61	100.07
39.0	121.22	105.26	101.01	100.14
42.0	126.35	107.17	101.55	100.26
45.0	131.87	109.39	102.25	100.45
48.0	137.71	111.91	103.12	100.66
51.0	143.84	114.17	104.15	100.97
54.0	150.21	117.78	105.36	101.36
57.0	156.74	121.08	106.74	101.84
60.0	163.52	124.62	108.29	102.40
90.0	235.02	168.60	131.98	113.62
120.0	305.51	219.92	165.75	133.79



Surface
1"
Distance from Surface
2"
3"
4"
5"
6"
7"
CURVE #1
Temperature vs. Time
Kenneth C. Woodruff, Jr.
July 15, 1964
Temperature T (°F.)
1100
900
700
500
300
100
0
20
40
60
80
100
120
Time t (minutes)


Curve Number
Time (Min.)
1 3
2 6
3 9
4 12
5 15
6 18
7 21
8 24
9 27
10 30
11 60
12 90
13 120
CURVE #2
Temperature vs. Distance from Surface
Kenneth C. Woodruff, Jr.
July 15, 1964
Temperature T (°F)
1100
900
700
500
300
100
0
1
2
3
4
5
6
7
Distance from Surface (Inches)

IV. CONCLUSIONS

Curve No. 1 shows one very important fact, which is that the surface temperature is only going to reach 70 to 75 percent of the temperature of the hot gas to which it is exposed. Under similar circumstances, a problem like this one, but without radiation accounted for, would show a surface temperature very nearly as great as the gas temperature. Even in this problem the coefficient of surface conductance was chosen to be high so that a great amount of heat could be put into the surface. Yet, as the surface temperature passed 500°F it may be seen that the radiation factor became significantly large, and began to approach in value, that of the heat input. The radiation outflow of heat became an insulator which, after the first hour, allowed only a small net amount of heat to flow into the plate.

As it was stated in the introduction, equation (3) may be solved for the ultimate surface temperature of the solid. Inserting in equation (3) the values chosen for this problem yields a temperature of 1540°F at a time of infinity. This would be the temperature computed by this numerical solution if the computer had been allowed to run for an infinite time. The surface temperature curve on Curve No. 1 would, if extrapolated, approach asymptotically this ultimate temperature.

This small amount of net heat flow into the plate is the reason that points more distant from the surface do not have a larger temperature gain in the time intervals beyond 60 minutes. The conductivity used for this steel is not especially high, which also contributes to holding down the internal (beyond 3 in.) temperatures.

As it was pointed out previously, the insulator effect of the radiation from the surface is a very important factor in space problems, particularly in problems of ultra-high speed flight in the near-space region of the atmosphere. In both of these cases the radiation into a void from the body is an important factor in maintaining the proper temperatures, both from a strength standpoint as well as from the standpoint of the operating temperatures for the many components involved. In the field of manned space vehicles, it is obvious how great the need is for a controlled environment.

It can safely be said that the radiation factor is of great importance to this problem, as was borne out by the temperature curves.

V. APPENDIX A.

PROGRAM FOR LGP 30 COMPUTER

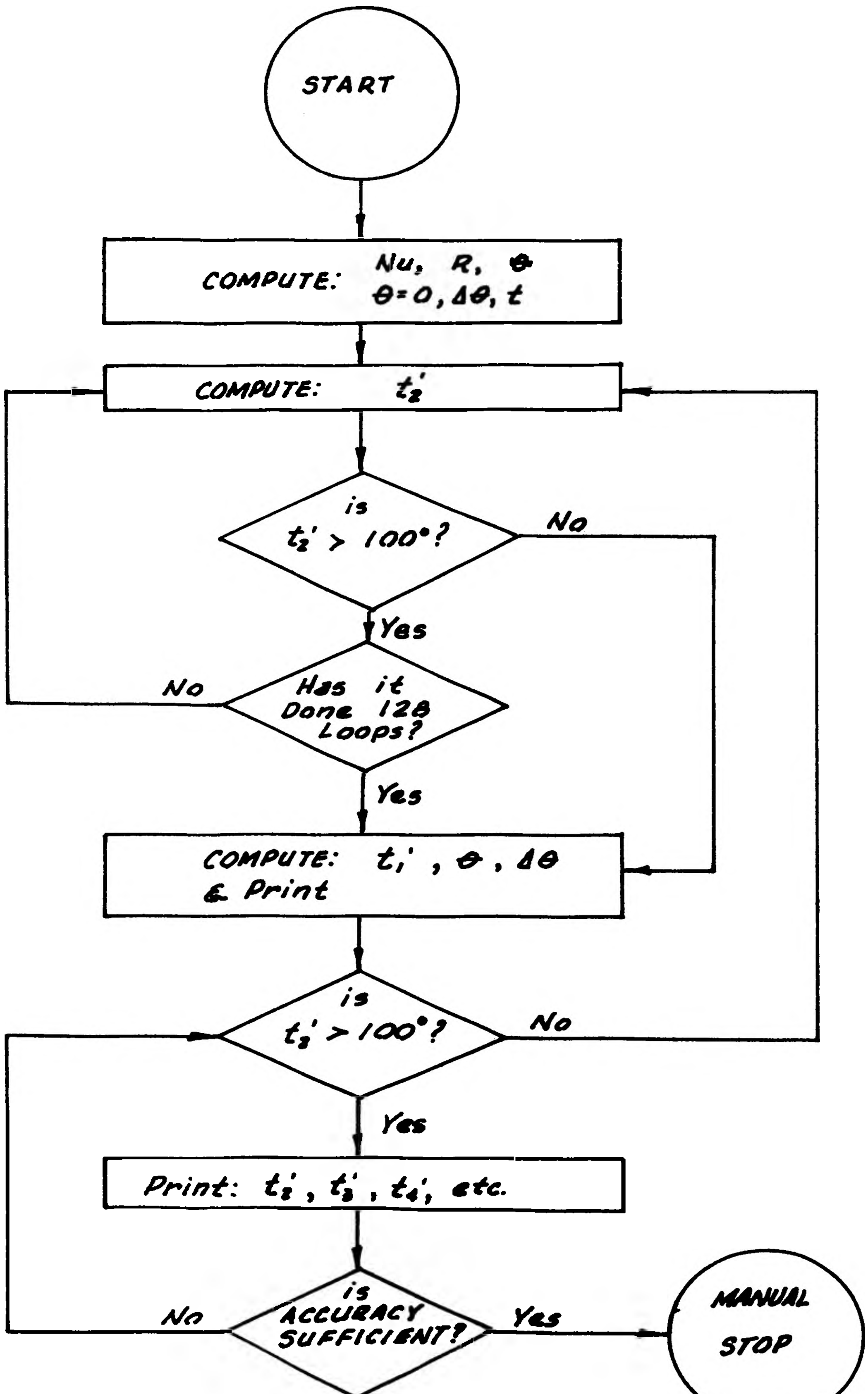
```
;0006000'/0000000'
r6300'u0400'h4000'1c0128'1i0002'1e0000'b4012'1h4400'
1z6006'b4016'm4010'd4014'h4034'b4022'd4014'm4026'
m4010'd4008'h4028'b4002'a4010'h4046'b4012'a4010'h4048'
b4012'a4000'h4052'2c0128'2i0002'2e0000'2b4400'2a4402'2a4404'
d4006'2h5000's4052't6058'2x6039'b4006'a4004'm4008'
a4012'a4012'a4102'h4056'b4400'a4056'h4060'b4028'd4014'
m4060'm4060'h4036'b4002'd4032's4036's4034's4002'm4060'
a4402'a4056'h4038'b4034'm4024'a4038'm4032'h4050'b4044'a4046'
h4044'm0000'b4404'z4010'd0000's4056'z0002'd0000'
1c0128' 1i0002'1e0000'3c0004'3i0002'3e0000'1b5000'1h4402'
s4052'r6144'1b5000'z0002'd0000'1z6138'3z6130'm0000'd0000'
d0000'u6128'b4050's4056'h4400'm0000'u6036'

.00060000'

2'+05-'1''2''3''12''5'+01-'100''124'+01-'50''
11'+02-'460''750''+03-'2460''173'+11-'f'
```

APPENDIX B.

FLOW CHART



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VII. VITA

The author was born on the 4th of May, 1932, in Memphis, Tennessee.

His early education was obtained in the primary school systems of Clearwater, Florida and Union City, Tennessee. After his family moved to St. Louis, Missouri in 1944, he completed his high school education at Western Military Academy, Alton, Illinois, obtaining his diploma in June, 1950.

In the fall of 1950, he first entered the University of Missouri School of Mines and Metallurgy. After completion of four semesters, he withdrew from school to serve in the Armed Forces. He re-entered the University of Missouri School of Mines and Metallurgy upon his release from military service and obtained his Bachelor of Science Degree from that institution.

After a few years spent in business, he entered the University of Missouri School of Mines and Metallurgy as a candidate for the degree of Master of Science, Mechanical Engineering.