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Time optimum switching zones in multiple-valued control systems

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TIME OPTIMUM SWITCHING ZONES IN MULTIPLE-VALUED

CONTROL SYSTEMS

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A

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ABSTRACT

Present control system theory provides the criteria necessary to bring a second-order single-valued system to rest in a minimum time after a step input. The bang-bang type controller has been developed to obtain this optimum response.

However, existing theory fails to provide the necessary information to obtain the optimum response from a second-order rotational system when multiple values of angular position are allowed. This thesis extends the control theory to include these multiple-valued systems by showing the existence of what are called optimum switching zones. These zones predict which of the infinite number of switching curves in a multiple-valued system is the optimum switching curve. The switching zones are developed for three second-order systems.

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LIST OF SYMBOLS

Symbol	Analytical or Physical Meaning of Symbol
θ	Displacement
$\dot{\theta}$	Time rate of change of displacement
θ_o, θ_f	Initial and final displacement
$\dot{\theta}_o$, $\dot{\theta}_f$	Initial and final velocity
t_{ABCD}	Time required to traverse path ABCD
T	Driving torque ($2 x 10^5$ ft-lb)
A	Moment of Inertia ($2 x 10^7$ slug ft^2)
B	Viscous Damping Coefficient
D	Coulomb Friction Coefficient

I. INTRODUCTION

A. Statement of the Problem

The early work of McDonald[4] and Hopkin[3] treat the problem of returning a system to rest in the shortest possible time following a step input. The results of this study for a second-order system are very well stated by Hopkin:

> In order to make a servomechanism output change from one position to another in a minimum time, the control means must be able to bring about the following operation:
>
> 1. The maximum safe forward value of manipulated variable must be applied from the instant of the step input until a specific later time. In response to this maximum safe value of manipulated variable, the servomechanism will be accelerated at a maximum rate for each value of output velocity.
>
> 2. At a specific time the manipulated variable must be reversed to a maximum safe reverse value so that the system will be decelerated at a maximum rate for each value of output velocity. The instant of reversal of the manipulated variable must be so selected that the system is decelerated to a stop precisely at the desired value of servomechanism output.

It was shown that for optimum response the instant of reversal is that instant when the state of the system is coincident with a point in the phase plane on the deceleration trajectory which passes through the desired value of the output. Fig. (1) shows this deceleration trajectory, EF, and therefore the switching curve for a second-order system.

It can be seen that for states of the system described by any points to the right of the switching curve maximum negative effort should be applied until the state of the system is described by any point on the switching curve. At the instant of coincidence

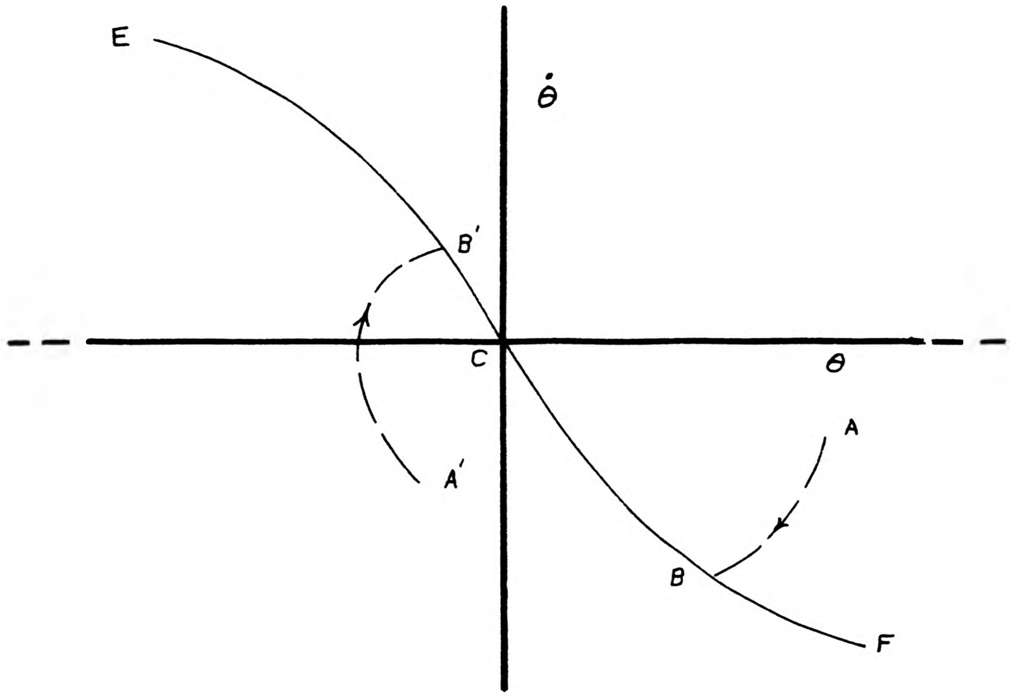

Fig. (1) Optimum Switching Curve for Second-Order System

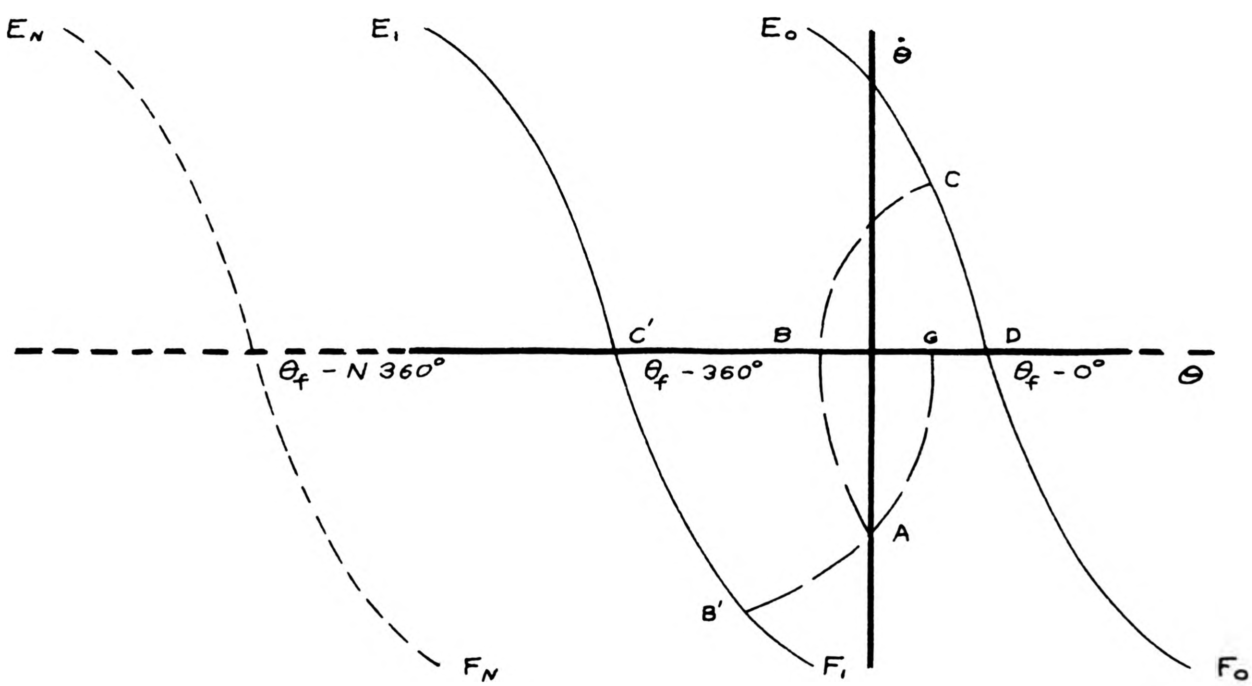

Fig. (2) Optimum Switching Curves for Second-Order Multiple-Valued System.

the driving force should be reversed, bringing the system to rest at the desired position along the deceleration curve. If the initial state of the system is to the left of curve EF, maximum positive effort should be initially applied. Upon intersecting the switching curve, maximum negative effort should be applied. The system will follow the deceleration curve to the desired final position. Paths ABC and A'B'C' of Fig. (1) illustrate the two possibilities.

For multiple-valued rotational systems Fig. (2) is a more complete description. Fig. (2) represents a system where any designated final angular position, θ_f, is equivalent to any of the other angular positions given by $\theta_f \pm N360^\circ$, therefore an infinite number of switching trajectories passing through all of the angular positions, $\theta_f \pm N360^\circ$ ($N = 0, 1, 2, \cdots$), are possible. The representation of Fig. (1) is applicable to rotating systems if multiple values of angular displacement are not allowed. For example, if 360° is not considered equal to 720°.

With the introduction of the multiple values of final displacement and thereby the multiple switching curves, Fig. (2) does not provide enough information to say which curve is optimum. Therefore, some criteria must be established to determine which of the switching curves is the optimum switching curve for the state of the system at the instant being considered. (Optimum in this thesis implies time optimum.) For example, consider a system in a state described by point A, Fig. (2). If curve E_oF_o is the dominant switching curve for this state, then path ABCD is the optimum path. On the other hand, if curve E_1F_1 is the optimum switching curve for state A, then path AB'C' should be followed.

For a physical interpretation of the problem consider a system rotating in a clock-wise direction at the time a command is given to come to rest at a position, θ_f, in a minimum time.

The question which must be answered immediately is: by what series of inputs will the system reach the final state in the least time? The only possible answers are -

a. a counter-clock-wise torque is applied and the system is "backed-up" to θ_f.

b. a clock-wise torque is applied, accelerating the system in a clock-wise direction, until, at a state designated by the switching curve passing through $\theta_f \pm N360^o$, a counter-clock-wise torque is applied bringing the system to rest at $\theta_f \pm N360^o$.

c. either (a) or (b) is applicable, i.e. both paths require the same time.

If the points satisfying (c), for a particular N, form a locus of points, this locus will describe boundaries between a zone where the optimum switching curve passes through $\theta_f + N360^o$; and a zone where the optimum switching curve passes through $\theta_f + (N + 1)360^o$, where N can be zero or any positive or negative integer.

It is the purpose of this thesis to investigate several multiple-valued second-order systems seeking to answer the above question, that is to determine the existence and boundaries of optimum switching zones and to interpret their physical meaning.

It can be seen that, in order for the switching zones to be meaningful, the desired final position must be multiple-valued. This implies a rotational system with no spring like forces.

As an example of a possible use of the optimum switching zones consider a satellite. A bang-bang type control for a satellite is described by Lieberman[1] and is also considered as system (A) below. It is required that the satellite maintain a prescribed attitude with respect to some reference and it is desired to bring the satellite to this attitude in the shortest possible time after tumbling into orbit and/or after any severe disturbance. The reference position may be approached from either a clock-wise or counter-clock-wise direction. It is obvious that a controller that takes into account the optimum switching zones is required and that any other controller would be less than optimum.

As a second example consider a weapon used as a defense against fast moving aircraft or missiles. A tracking radar controls the direction of fire; a search radar and a computer provides target priority information. It is required that the weapon change targets upon command from the computer, in the shortest possible time. (Perhaps a target is destroyed or passes out of range and another target is rapidly approaching.) The weapon has a large inertia and is free to rotate through 360°. A bang-bang type controller is used to provide the quickest possible response. Again the controller must take into account the optimum switching zones to be able to predict the optimum path and therefore to provide the optimum response.

B. Systems to be Considered

The differential equation of a general second-order system is given by

$$A\ddot{\theta} + B\dot{\theta} + C\theta + D = T$$

Consideration in this thesis will be limited to those systems where C is zero. Thus, the three systems to be studied are described by the following differential equations:

A) Inertia only........................... $A\ddot{\theta} = T$

B) Inertia plus Coulomb Friction.......... $A\ddot{\theta} + D = T$

C) Inertia plus Viscous Damping........... $A\ddot{\theta} + B\dot{\theta} = T$

II. REVIEW OF THE LITERATURE

McDonald[4] and Hopkin[3] were the first to show that the relay type servo provides the optimum step function response. The phase plane method of analysis for this type of servomechanism was introduced earlier by MacColl[5] and Weiss[6].

Bushaw[7] is credited with the first treatment of the time optimal bang-bang control. Later LaSalle[8] showed that the best bang-bang system was also the best of all available systems for achieving time optimum control. In 1956 Bellman, Glicksberg, and Gross[9] presented the first general approach to these systems using linear programming techniques. Recently Pontrjagin, Gamkrelidze, and Boltjanski[10] studied the generalized problem from a more sophisticated mathematical viewpoint using the calculus of variations.

Curtin[11] has recently investigated the response of the bang-bang controller for step, ramp and sinusoidal reference by an extensive analog computer study.

III. THEORETICAL ANALYSIS

A. System with Inertia Only

The differential equation describing this system is

$$A\ddot{\theta} = T$$

where here, and throughout this thesis, T is the applied torque, θ is the angular displacement and A represents the moment of inertia. This reduces to the form

$$\ddot{\theta} = KT \qquad (1)$$

where K = 1/A. Integrating Eq. (1):

$$\dot{\theta} = KTt + \dot{\theta}_0 \qquad (2)$$

Integrating again:

$$\theta = \frac{KT}{2}t^2 + \dot{\theta}_0 t + \theta_0 \qquad (3)$$

Eliminating time between Eq. (2) and Eq. (3) yields

$$\theta - \theta_0 = \frac{\dot{\theta}^2 - \dot{\theta}_0^2}{2KT} \qquad (4)$$

For a rotating system there are an infinite number of paths in the phase plane which may be traversed from any one initial point to any final point. Physically this means that the system may rotate any number of times before stopping at any designated final value, Fig. (2) shows two of these paths, AB'C' and ABCD. The following calculations will be for all of the possible paths and will determine which of the paths will bring the system to rest at the designated final value in the least time. This path will be designated the optimum path, where, here and throughout this thesis, optimum implies the time optimum.

Fig. (2) shows the switching curves EF, which are the optimum switching curves for a single valued system repeated every 360° and identifies points to be used in the following calculations.

<u>Calculation of time for path AB.</u> The equation of path AB is given by Eq. (2) with T positive, $\dot{\theta} = 0$ and $\dot{\theta}_o = \dot{\theta}_o$

$$0 = kTt + \dot{\theta}_o$$

Solving for time yields

$$t_{AB} = \frac{-\dot{\theta}_o}{kT} \tag{5}$$

<u>Calculation of time for path BC and path CD.</u> The equation relating to path BC is given by Eq. (3) with T positive, t_{BC} equal to t_{CD} (due to the symmetry of the acceleration and deceleration trajectories in a pure inertial system[1]), $\dot{\theta}_o = 0$, $\theta_o = \theta_B$ and

$$\theta = \theta_B + \frac{[-\theta_B + (\theta_f - N360)]}{2}$$

where N = 0, 1, 2, ··· . Substituting these values into Eq. (3) yields

$$\theta_B + \frac{[-\theta_B + (\theta_f - N360)]}{2} = \frac{kT}{2}t^2 + \theta_B$$

Solving for time yields

$$t_{BC} = \left[\frac{-\theta_B + \theta_f - N360}{KT}\right]^{\frac{1}{2}} \tag{6}$$

But t_{BC} equals t_{CD} , therefore

$$t_{BCD} = 2\,t_{BC} = 2\left[\frac{-\theta_B + \theta_f - N360}{KT}\right]^{\frac{1}{2}} \tag{7}$$

<u>Calculation of total time for path ABCD.</u> The total time is the sum of the times required to traverse each segment of the path, therefore

$$t_{ABCD} = t_{AB} + t_{BC} + t_{CD} = -\frac{\dot{\theta}_o}{KT} + 2\left[\frac{-\theta_B + \theta_f - N360}{KT}\right]^{\frac{1}{2}} \tag{8}$$

<u>Calculation of total time for path AB'C'.</u> The equation of the path from G to point B' is given by Eq. (3) with T negative, $\dot{\theta} = 0$, $\theta_o = -\theta_B = \theta_G$ (due to symmetry) and $\theta = -\theta_B - \left[\frac{-\theta_B - [\theta_f - (N+1)360]}{2}\right]$, therefore

$$-\theta_B - \left[-\theta_B - \frac{[\theta_f - (N+1)360]}{2}\right] = -\frac{KT}{2}t^2$$

Solving for $t_{GB'}$ yields

$$t_{GB'} = \left[\frac{(N+1)360 - \theta_f - \theta_B}{KT}\right]$$

But due to symmetry, $t_{B'C'}$ is equal to $t_{GB'}$ therefore

$$t_{GC'} = 2\left[\frac{(N+1)360 - \theta_f - \theta_B}{KT}\right]^{\frac{1}{2}}$$

Also due to symmetry,

$$t_{GA} = t_{AB} = \frac{-\dot{\theta}_o}{KT}$$

Therefore the total time to traverse path AB'C' is

$$t_{AB'C'} = t_{GC'} - t_{GA}$$

$$= \frac{\dot{\theta}_o}{KT} + 2\left[\frac{(N+1)360 - \theta_f - \theta_B}{KT}\right]^{\frac{1}{2}} \tag{9}$$

<u>Determination of switching zone boundaries.</u> To determine the boundaries where the time to traverse path AB'C' is equal to the time to traverse path ABCD, $t_{AB'C'}$ is set equal to t_{ABCD} and the relationship between θ_f and $\dot{\theta}_o$ is obtained. Equating Eq. (8) and Eq. (9) yields

$$-\frac{\dot{\theta}_o}{KT} + 2\left[\frac{-\theta_B + \theta_f - N360}{KT}\right]^{\frac{1}{2}} = \frac{\dot{\theta}_o}{KT} + 2\left[\frac{(N+1)360 - \theta_f - \theta_B}{KT}\right]^{\frac{1}{2}}$$

Squaring both sides and reducing yields

$$\frac{-\theta_B + \theta_f - N360}{KT} = \frac{\dot{\theta}_o^2}{(KT)^2} + \frac{2\dot{\theta}_o}{KT}\left[\frac{(N+1)360 - \theta_f - \theta_B}{KT}\right]^{\frac{1}{2}} + \frac{(N+1)360 - \theta_f - \theta_B}{KT} \tag{10}$$

If in Eq. (4) T is positive, $\theta = \theta_B$, $\theta_o = 0$, $\dot{\theta} = 0$ and $\dot{\theta} = \dot{\theta}_o$, then,

$$\dot{\theta}_o^2 = -2KT\,\theta_B \tag{11}$$

Substitution of Eq. (11) into Eq. (10) gives

$$\theta_f - 180(2N+1) + \theta_B = \dot{\theta}_o \left[\frac{(N+1)360 - \theta_f - \theta_B}{KT}\right]^{\frac{1}{2}}$$

Squaring both sides and reducing yields

$$\begin{aligned}\left[\theta_f - 180(2N+1)\right]^2 + 2\,\theta_B\left[\theta_f - 180(2N+1)\right] + \theta_B \\ = \frac{\dot{\theta}_o}{KT}\left[(N+1)360 - \theta_f\right] - \frac{\dot{\theta}_o^2}{KT}\theta_B\end{aligned} \tag{12}$$

Substituting Eq. (11) into Eq. (12) and reducing gives

$$\theta_B^2 - 360\,\theta_B - \left[\theta_f - 180(2N+1)\right]^2 = 0$$

Solving the quadratic and choosing the negative radical yields

$$\theta_B = \frac{360 - \left\{360^2 + 4\left[\theta_f - 180(2N+1)\right]^2\right\}^{\frac{1}{2}}}{2} \tag{13}$$

Substituting Eq. (11) into Eq. (13) gives

$$\dot{\theta}_o^2 = KT\left\{-360 + \sqrt{360^2 + 4\left[\theta_f - (2N+1)180\right]^2}\right\} \tag{14}$$

The curves EF of Fig. (3) are the optimum switching curves for this system. The heavy curves are zone boundaries and are plots of Eq. (14) for N equal to 0, 1, and N. See Lieberman[1] for an application of the system and the development of the optimum switching curves.

<u>Interpretation of Figure (3).</u> If, at the instant when a command is given to the system to proceed to an angular position θ_f, the state of the system can be described by any point in zone-I, then the

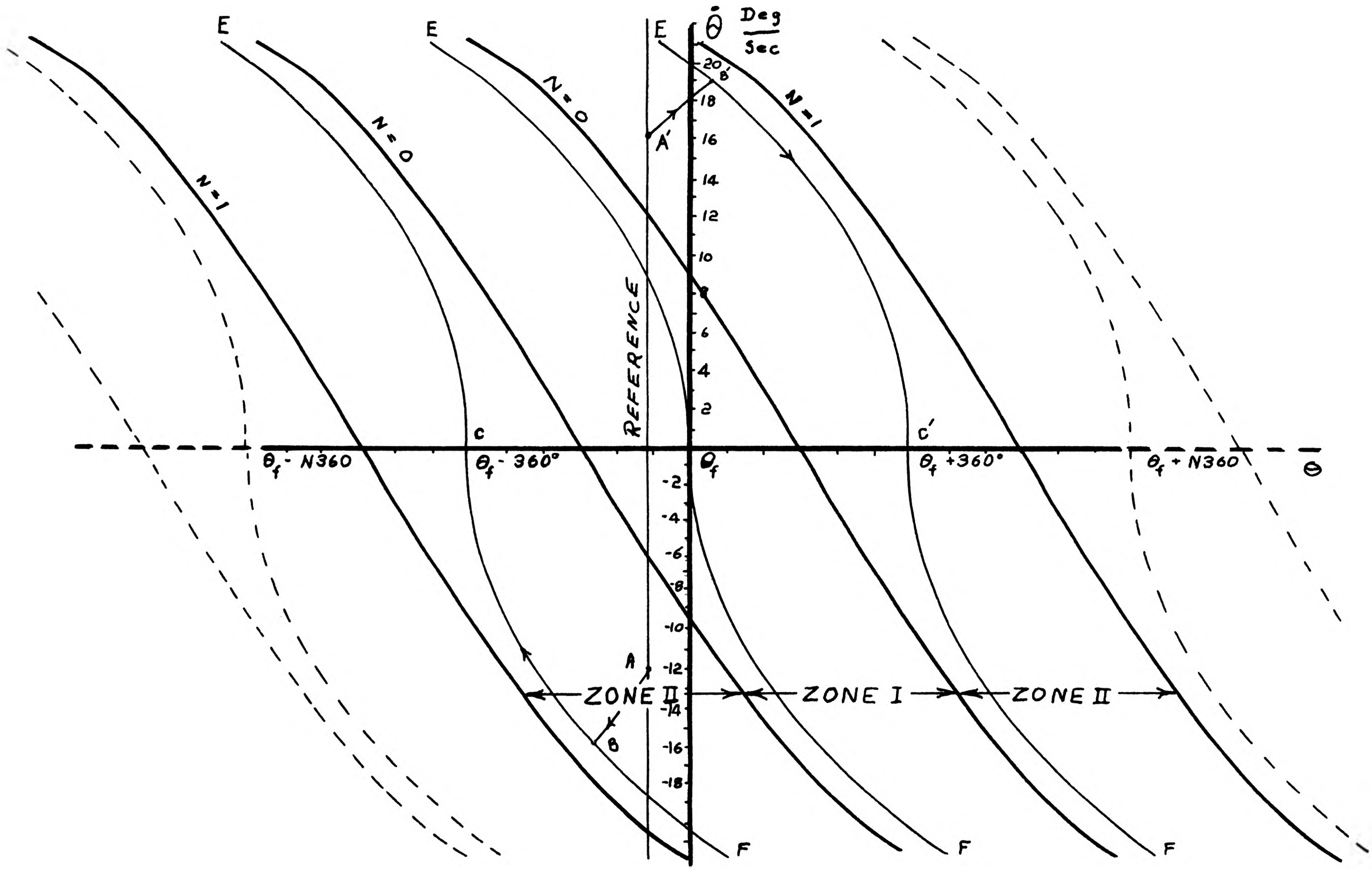

Fig. (3) Optimum Switching Zones for System with Inertia Only.

optimum path is that path which is dictated by the optimum switching curve passing through θ_f. If at the instant of command the state of the system is described by any point in zone-II, then the time optimum path is that path which is dictated by the switching curve which passes through $\theta_f \pm 360^\circ$. (The sign will be that of the velocity at the instant of command.) Physically this means that less time will be required to reach θ_f, if the system is first accelerated in a forward direction and then decelerated to $\theta_f \pm 360^\circ$ rather than decelerated to zero velocity and then accelerated back to θ_f. (The forward direction is the direction of rotation at the instant of command.)

In general, if the state of the system is described by a point in zone-N, the optimum path is dictated by the switching curve passing through $\theta_f \pm (N - 1)360^\circ$. If the point is to the right of the designated switching curve then a negative initial torque is required. If the point is to the left of the designated switching curve then the initial torque would be positive. In either case the torque is reversed upon intersection of the switching curve.

To illustrate the significance of the zones consider path ABC, Fig. (3). The initial state of the system is given by $\theta_o = 0$ and $\dot{\theta}_o = -12$ deg/sec. The negative initial angular velocity in zone-II indicates that the switching curve passing through $\theta_f - 360^\circ$ dictates the time optimum path. (The negative sign is due to the negative initial velocity.) Since the point is to the right of the designated switching curve, the initial command would be for a negative torque to be applied to the system until the phase plane trajectory intersects the switching curve, at which time a positive torque would be applied, bringing the system to rest at $\theta_f - 360^\circ$.

As a second example consider path A'B'C', Fig. (3). The initial state of the system is given by $\theta_o = 0$ and $\dot{\theta}_o = 16$ deg/sec. The positive initial velocity in zone-II indicates that the switching curve passing through $\theta_f + 360^\circ$ dictates the optimum path. Since the point is to the left of the designated switching curve, the initial command would be for a positive torque to be applied to the system until the phase plane trajectory intersects the switching curve, at which time a negative torque would be applied, bringing the system to rest at $\theta_f + 360^\circ$.

B. System with Inertia and Coulomb Friction

The differential equation describing this system is

$$A\ddot{\theta} \pm D = T$$

where D represents the coulomb torque and will have the sign of the velocity. This reduces to the form

$$\ddot{\theta} \pm K_1 = KT$$

where $K = 1/A$ and $K_1 = D/A$. Taking the Laplace transform for a step forcing function yields

$$\theta(s) = \frac{\theta_o}{s} + \frac{\dot{\theta}_o}{s^2} + \frac{KT \pm K_1}{s^3}$$

Taking the inverse Laplace transform yields

$$\theta(t) = \dot{\theta}_o t + [KT \pm K_1] \frac{t^2}{2} + \theta_o \qquad (15)$$

Taking the time derivative yields

$$\dot{\theta}(t) = \dot{\theta}_o + [KT \pm K_1]t \tag{16}$$

Eliminating time between Eq. (15) and (16) leads to

$$\theta - \theta_o = \frac{\dot{\theta}^2 - \dot{\theta}_o^2}{2(KT \pm K_1)} \tag{17}$$

Fig. (4) shows the well known switching curves for this system[2] and identifies points to be used in the following calculations.

In section A zones were developed which predicted the optimum path to any designated final angular position for any initial condition. These zones in effect answer the question: which of the infinite number of possible paths is the optimum path? In this section the time to traverse any of the paths AB'C' is compared to the time to traverse the one path ABCD, which must terminate at θ_f. Therefore, in addition to answering the above question, zones will be developed which answer the question: will it require less time to go back to θ_f or will it take less time to go on to $\theta_f \pm N360^o$ where N is any integer? This provides a means of predicting the number of revolutions which can be traversed in a forward direction in the same time as it would take to bring the system back to θ_f. The curves might be used in a system where it is desired to get the maximum possible number of revolutions while moving the system to θ_f in a fixed time. Possibly they are of academic value only, but the author feels that it is important to point out the existence of these zones.

First consider path ABCD. The system is initially accelerated to

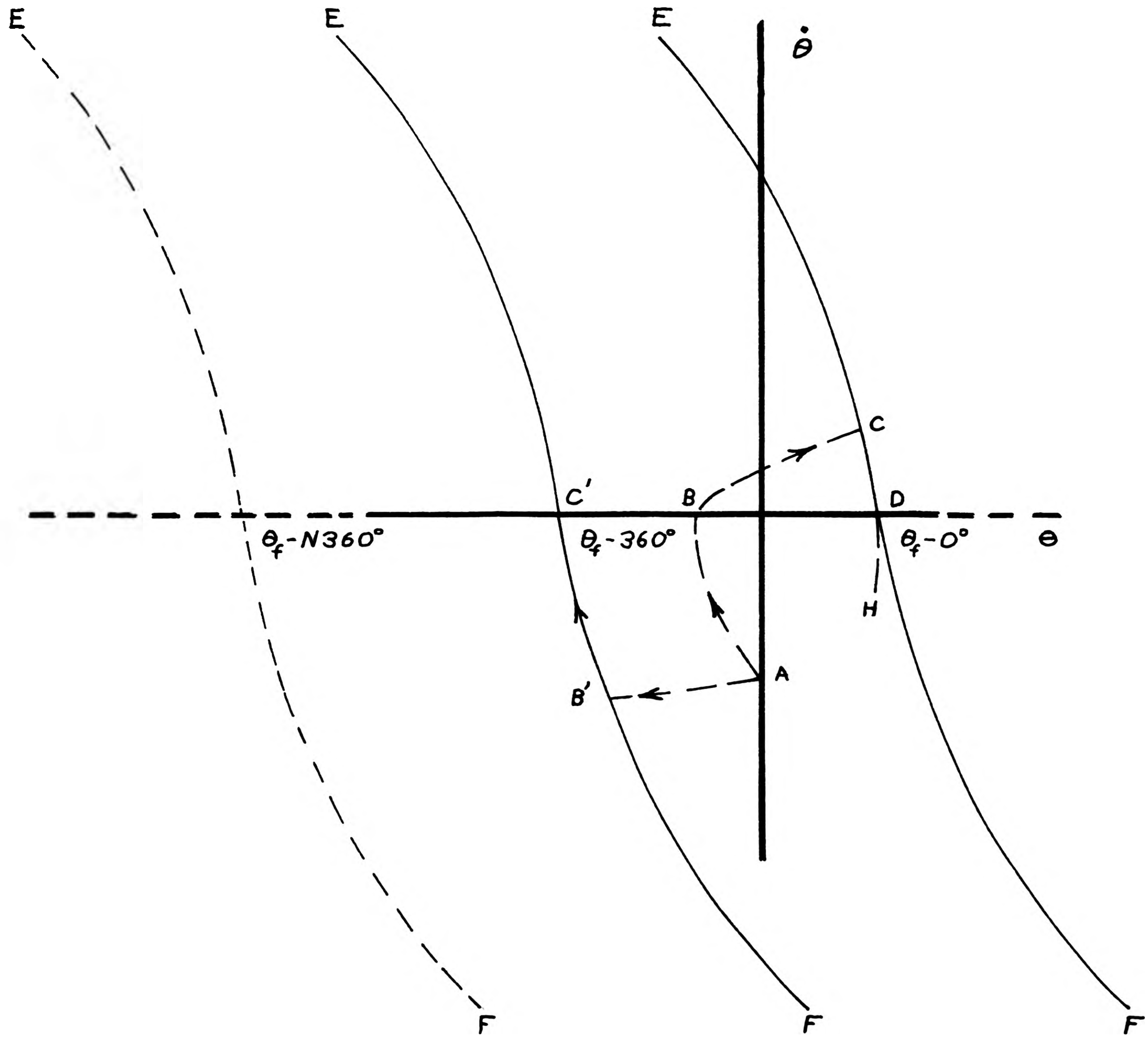

Fig. (4) Optimum Switching Curve for System with Inertia and Coulomb Friction.

a positive velocity $\dot{\theta}_C$ and then decelerated to zero velocity at the designated θ_f.

Calculation of time for path AB. In Eq. (16) let K_1 be negative, T be positive, $\dot{\theta}_o = \dot{\theta}_o$ and $\dot{\theta} = 0$

$$0 = \dot{\theta}_o + (KT + K_1)t$$

Solving for time yields

$$t_{AB} = \frac{-\dot{\theta}_o}{(KT+K_1)} \tag{18}$$

In Eq. (15) let $\theta_o = 0$, $\theta = \theta_B$, K_1 be negative and T be positive

$$\theta_B = \dot{\theta}_o t + (KT+K_1)\frac{t^2}{2}$$

Using time as given by Eq. (18) yields

$$\theta_B = \frac{-\dot{\theta}_o^2}{(KT+K_1)} + \frac{\dot{\theta}_o^2}{2(KT+K_1)} = \frac{-\dot{\theta}^2}{2(KT+K_1)} \tag{19}$$

Calculation of intersection C, Fig. (3). The equation of path BC is given by Eq. (17) with both T and K_1 positive, $\theta_o = \theta_B$ and $\dot{\theta}_o = 0$, therefore

$$\theta - \theta_B = \frac{\dot{\theta}^2}{2(KT-K_1)}$$

and

$$\dot{\theta}^2 = 2(\theta - \theta_B)(KT-K_1) \tag{20}$$

The equation of path CD is given by Eq. (17) with T negative, K_1 positive, $\dot{\theta} = 0$, $\dot{\theta}_o = \dot{\theta}_C = \dot{\theta}$, $\theta_o = \theta_C = \theta$ and $\theta = \theta_f$, therefore

$$\theta_f - \theta = \frac{-\dot{\theta}^2}{2(KT - K_1)}$$

and

$$\dot{\theta}^2 = 2(\theta_f - \theta)(KT - K_1) \qquad (21)$$

Equating Eq. (20) and Eq. (21) and solving for $\theta = \theta_C$, where θ_C is the angular displacement at the intersection point C, yields

$$\theta_c = \frac{\theta_B(KT - K_1) + \theta_f(KT + K_1)}{2KT} \qquad (22)$$

Substituting Eq. (19) in Eq. (22) and reducing leads to

$$\theta_c = \frac{-\dot{\theta}_o^2 (KT - K_1)}{4KT(KT + K_1)} + \frac{\theta_f(KT + K_1)}{2KT} \qquad (23)$$

Note that in calculating θ_C an N term was not included as was done in section A. This does not allow path ABCD to terminate at any point other than θ_f.

<u>Calculation of time for path BC.</u> For path BC the angular displacement as a function of time is given by Eq. (15) with both T and K_1 positive, $\theta_o = \theta_B$, $\dot{\theta}_o = 0$ and $\theta = \theta_C$, therefore

$$\theta_c - \theta_B = (KT - K_1)\frac{t^2}{2}$$

and

$$t_{BC} = \left[\frac{2(\theta_c - \theta_B)}{KT - K_1}\right]^{\frac{1}{2}} \qquad (24)$$

Substituting Eq. (19) and Eq. (23) into Eq. (24) and simplifying yields

$$t_{BC} = \frac{1}{kT-K_1}\left[\frac{\dot{\theta}_o^2(kT-K_1)}{2kT} + \frac{\theta_f(kT+K_1)(kT-K_1)}{kT}\right]^{\frac{1}{2}} \tag{25}$$

Calculation of time for path CD. Time t_{DH} is equal to time t_{CD} and is easily calculated, see Fig. (4). In Eq. (15) T is negative, K_1 is positive, $\theta_o = \theta_f$, $\dot{\theta}_o = 0$ and $\theta = \theta_C$, therefore

$$\theta_c - \theta_f = -(kT+K_1)\,\frac{t^2}{2}$$

and

$$t_{CD} = t_{DH} = \left[\frac{2(\theta_f - \theta_c)}{kT+K_1}\right]^{\frac{1}{2}} \tag{26}$$

Substituting Eq. (23) in Eq. (26) yields

$$t_{CD} = \frac{1}{(kT+K_1)}\left[\frac{\dot{\theta}_o^2(kT-K_1)}{2kT} + \frac{\theta_f(kT+K_1)(kT-K_1)}{kT}\right]^{\frac{1}{2}} \tag{27}$$

Calculation of total time for path ABCD. The time $t_{ABCD} = (t_{AB} + t_{BC} + t_{CD})$, therefore

$$t_{ABCD} = \frac{-\dot{\theta}_o}{kT+K_1} + \frac{2kT}{(kT)^2-K_1^2}\left[\frac{\dot{\theta}_o^2(kT-K_1)}{2kT} + \frac{\theta_f(kT+K_1)(kT-K_1)}{kT}\right]^{\frac{1}{2}} \tag{28}$$

Next consider path AB'C', Fig. (4). The system is initially accelerated in a forward direction to a state described by point B' then is decelerated to zero velocity at the designated position $\theta_f - 360°$.

Calculation of intersection B'. The equation of path AB' is given by Eq. (17) with both T and K_1 negative, $\theta_o = 0$, $\theta = \theta_{B'}$, $\dot{\theta} = \dot{\theta}_{B'}$ and $\dot{\theta}_o = \dot{\theta}_o$, therefore

$$\theta_{B'} = \frac{\dot{\theta}_o^2 - \dot{\theta}_{B'}^2}{2(kT - k_1)}$$

and

$$\dot{\theta}_{B'}^2 = \dot{\theta}_o^2 - 2\,\theta_{B'}(kT - k_1) \qquad (29)$$

The equation of path B'C' is given by Eq. (17) with T positive, K_1 negative, $\theta_o = \theta_{B'}$, $\theta = \theta_f - N360°$, $\dot{\theta}_o = \dot{\theta}_{B'}$ and $\dot{\theta} = 0$, therefore

$$\theta_f - N360 - \theta_{B'} = \frac{-\dot{\theta}_{B'}^2}{2(kT + k_1)}$$

and

$$\dot{\theta}_{B'}^2 = 2(kT + k_1)(\theta_{B'} + N360 - \theta_f) \qquad (30)$$

Equating Eq. (29) and Eq. (30) and solving for $\theta_{B'}$ yields

$$\theta_{B'} = \frac{\dot{\theta}_o^2}{4kT} - \frac{2(kT + k_1)(N360 - \theta_f)}{4kT} \qquad (31)$$

Calculation of time for path AB'. The equation of path AB' is given by Eq. (16) with both T and K_1 negative, $\dot{\theta} = \dot{\theta}_{B'}$ and $\theta_o = \theta_o$, therefore

$$\dot{\theta}_{B'} - \dot{\theta}_o = -(kT - k_1)\, t_{AB'}$$

and

$$t_{AB'} = \frac{\dot{\theta}_o - \dot{\theta}_{B'}}{KT - K_1} \tag{32}$$

Substituting Eq. (31) into Eq. (30) and reducing yields

$$\dot{\theta}_{B'}^2 = \frac{\dot{\theta}_o^2 (KT + K_1)}{2KT} + \frac{(KT + K_1)(KT - K_1)(N360 - \theta_f)}{KT} \tag{33}$$

Substituting Eq. (33) into Eq. (32) and choosing the negative sign for the radical yields

$$t_{AB'} = \frac{\dot{\theta}_o}{KT - K_1} + \frac{1}{KT - K_1}\left[\frac{\dot{\theta}_o^2 (KT + K_1)}{2KT} + \frac{(KT + K_1)(KT - K_1)(N360 - \theta_f)}{KT}\right]^{\frac{1}{2}} \tag{34}$$

<u>Calculation of time for path B'C'.</u> The equation of path B'C' is given by Eq. (16) with T positive, K_1 negative, $\dot{\theta} = 0$ and $\dot{\theta}_o = \dot{\theta}_{B'}$, therefore

$$-\dot{\theta}_{B'} = (KT + K_1)\, t$$

and

$$t_{B'C'} = \frac{-\dot{\theta}_{B'}}{KT + K_1} \tag{35}$$

Substituting Eq. (33) into Eq. (35) and choosing the negative sign for the radical yields

$$t_{B'C'} = \frac{1}{KT + K_1}\left[\frac{\dot{\theta}_o^2 (KT + K_1)}{2KT} + \frac{(KT + K_1)(KT - K_1)(N360 - \theta_f)}{KT}\right]^{\frac{1}{2}} \tag{36}$$

Calculation of total time for path AB'C'. The time required to traverse path AB'C' is the sum of $t_{AB'C'}$ and $t_{B'C'}$, therefore

$$t_{AB'C'} = \frac{\dot{\theta}_o}{kT - K_1} + \frac{2kT}{(kT)^2 - K_1^2}\left[\frac{\dot{\theta}_o^2 (kT+K_1)}{2kT} + \frac{(kT+K_1)(kT-K_1)(N360 - \theta_f)}{kT}\right]^{\frac{1}{2}} \qquad (37)$$

Determination of switching zone boundaries. To determine the boundaries where the time to traverse any of the possible paths AB'C' is equal to the time to traverse path ABCD, $t_{AB'C'}$ is set equal to t_{ABCD} and the relationship between $\dot{\theta}_o$ and θ_f is obtained. Equating Eq. (28) and Eq. (37) yields

$$\frac{\dot{\theta}_o}{kT-K_1} + \frac{2kT}{(kT)^2-K_1^2}\left[\frac{\dot{\theta}_o^2(kT-K_1)}{2kT} + \frac{(kT+K_1)(kT-K_1)(N360-\theta_f)}{kT}\right]^{\frac{1}{2}}$$

$$= \frac{-\dot{\theta}_o}{kT+K_1} + \frac{2kT}{(kT)^2-K_1^2}\left[\frac{\dot{\theta}_o^2(kT-K_1)}{2kT} + \frac{\theta_f(kT+K_1)(kT-K_1)}{kT}\right]^{\frac{1}{2}}$$

which reduces to

$$\dot{\theta}_o + \left[\frac{\dot{\theta}_o^2(kT+K_1)}{2kT} + \frac{(kT+K_1)(kT-K_1)(N360-\theta_f)}{kT}\right]^{\frac{1}{2}}$$

$$= \left[\frac{\dot{\theta}_o^2(kT-K_1)}{2kT} + \frac{\theta_f(kT+K_1)(kT-K_1)}{kT}\right]^{\frac{1}{2}}$$

Squaring both sides and reducing leads to

$$\frac{kT+K_1}{2kT}\left[\dot{\theta}_o^2 + (kT-K_1)(N360-2\theta_f)\right]$$

$$= \dot{\theta}_o\left[\frac{\dot{\theta}_o^2(kT+K_1)}{2kT} + \frac{(kT+K_1)(kT-K_1)(N360-\theta_f)}{kT}\right]^{\frac{1}{2}}$$

Squaring both sides and reducing yields

$$\dot{\theta}_0^{\,2} = -2K_1\theta_f - N360(KT-K_1) + \left\{\left[2\theta_f K_1 + N360(KT-K_1)\right]^2 + \left[(KT)^2 - K_1^2\right](N360-2\theta_f)^2\right\}^{\frac{1}{2}} \tag{38}$$

Fig. (5) shows the optimum switching curves, EF, and the zones bounded by Eq. (38) for N = 1, 2, and 3. See Kaplan[2] for a discussion of the system and development of the switching curves and switching criteria.

Interpretation of Fig. (5). The zones in Fig. (5) compare the time to traverse path ABCD, where for this case path ABCD must terminate at θ_f, with the time to traverse paths terminating at angular positions $\theta_f \pm 360^\circ$, $\theta_f \pm 720^\circ$, $\theta_f \pm 1080^\circ$ etc. In zone-II the system can be brought to rest at $\theta_f \pm 360^\circ$ in less time than it can be brought to rest at θ_f. (The sign will be that of the initial velocity.) In zone-III the system can be brought to rest at $\theta_f \pm 720^\circ$ in less time than it can be brought to rest at θ_f. In zone-IV the system can be brought to rest at $\theta_f \pm 1080^\circ$ in less time than it can be brought to rest at θ_f, and so on and so forth for an infinite number of zones.

In each case the zones extend an infinite distance to the left for negative initial velocities and an infinite distance to the right for positive initial velocities; therefore, zone-III, zone-IV and any additional zones will be in zone-II; zone-IV and any additional zones will be in zone-III, etc.

Although only three boundaries are shown to avoid cluttering the figure, there are an infinite number of these boundaries and this

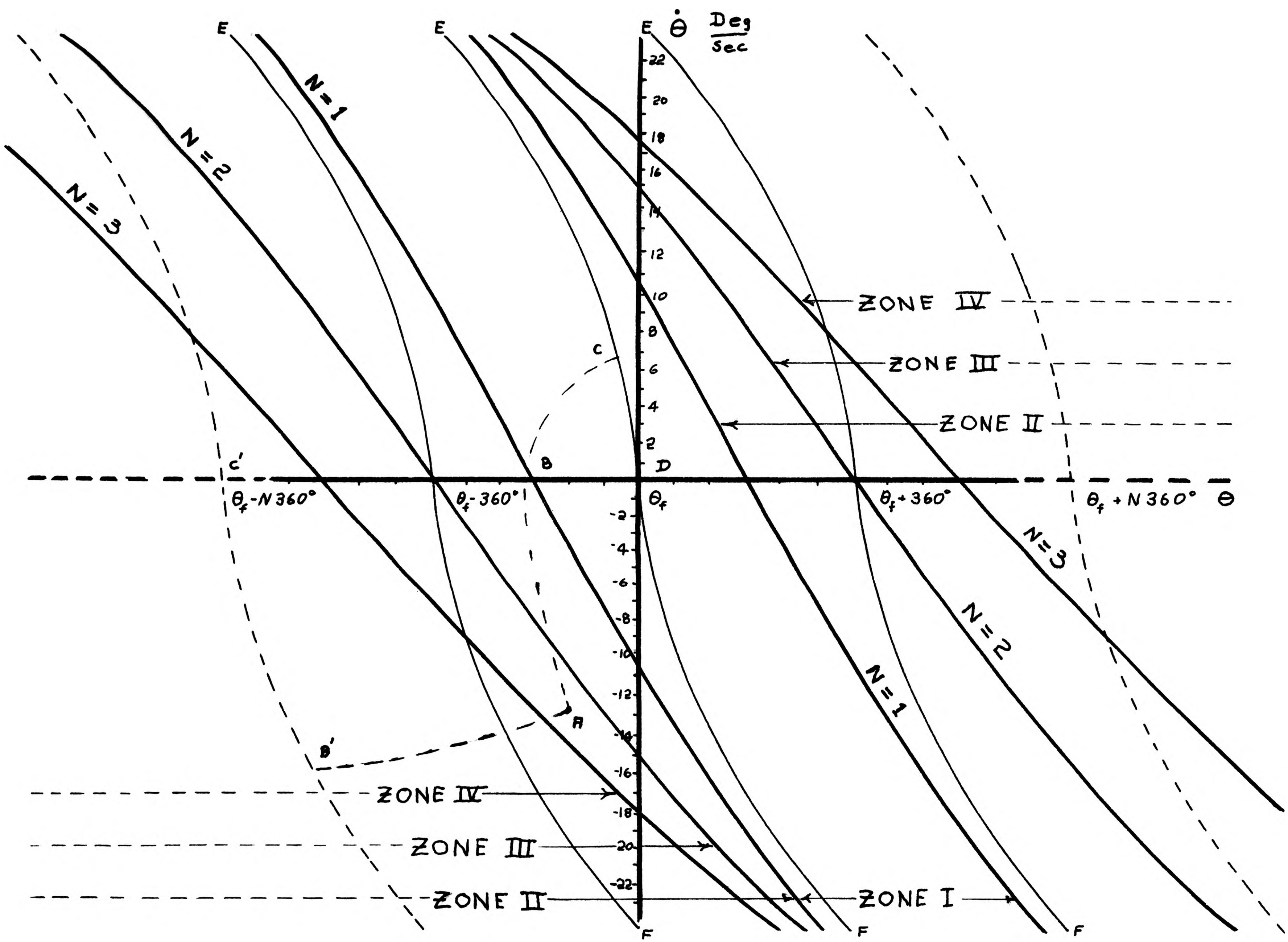

Fig. (5) Zones Which Predict the Number of Revolutions Which Can Be Traversed In A Forward Direction infinite number of boundaries is repeated every 360°.

Note that these zones are in no way optimum zones, they predict only the number of revolutions that may be traversed in a forward direction in the time it would take to stop and back-up to θ_f. To illustrate the meaning of the zones, consider a system in a state described by point A, Fig. (5), that is , a system that has an initial velocity of -13 deg/sec, when a command is given to come to rest at θ_f. The negative velocity in zone-III indicates that the system can continue in the negative direction past $\theta_f -360°$ and make one more complete revolution to $\theta_f -720°$ (path AB'C') in less time than would be required to stop and back-up to θ_f (path ABCD). In other words, approximately one and one-half revolutions can be completed in the direction of the initial velocity in the time it would take to stop and back-up to the desired final position. These zones would be useful in a system for which it is required that a final position be obtained in a fixed time and in which it is desirable to obtain the maximum number of revolutions in the direction of the initial revolution.

In order to establish the optimum switching zones consider the before neglected zone-I. It can be seen that any point in zone-I requires less time to reach θ_f than any other angular position $\theta_f \pm N360°$. Also, for any initial state outside of zone-I there is a path to the desired final angular position which requires less time than the path which terminates at θ_f. For example, every point not in zone-I is in zone-II and points in zone-II require less time to reach $\theta_f \pm 360°$ than to reach θ_f. Therefore it can be concluded that zone-I is the optimum switching zone for the switching curve passing through θ_f. As zone-I is repeated every 360° the optimum switching zones are established for the entire phase plane. Note that the zone boundaries

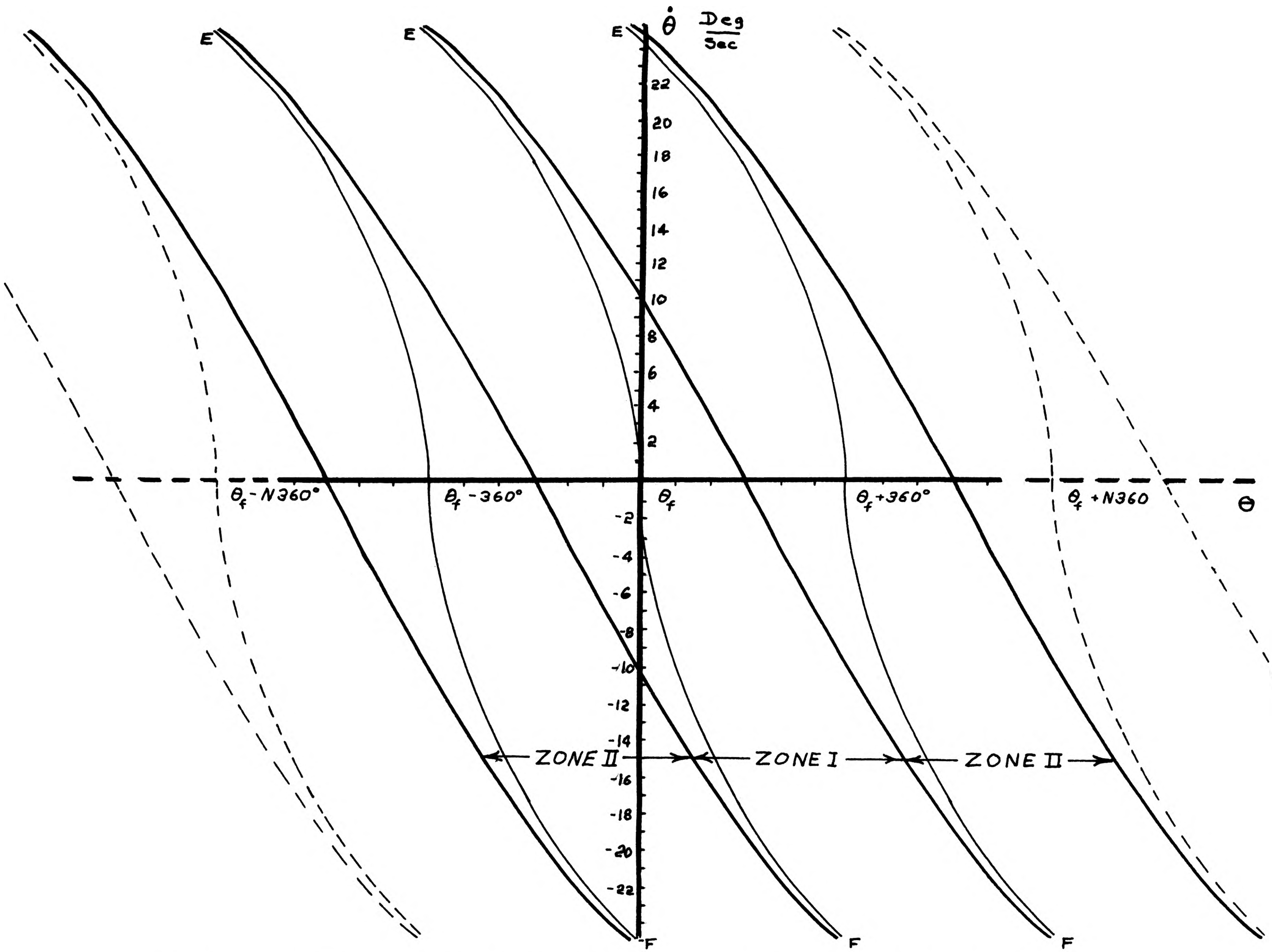

Fig. (6) Optimum Switching Zones for System with Inertia Plus Coulomb Friction.

will be the boundary for N = 1 repeated every 360°. Fig. (6) shows the optimum switching zones. The significance of these zones is exactly the same as that given to the optimum switching zones developed in section A.

C. System with Inertia and Viscous Damping

The differential equation describing this system is

$$A\ddot{\theta} + B\dot{\theta} = T$$

where B represents the viscous-damping coefficient. This reduces to the form

$$\ddot{\theta} + K_1\dot{\theta} = KT$$

where $K = 1/A$ and $K_1 = B/A$.

Applying the Laplace transform for a unit step forcing function and reducing leads to

$$s^2\theta(s) - \theta_0 s - \dot{\theta}_0 + K_1[s\,\theta(s) - \theta_0] = \frac{KT}{s}$$

and

$$\theta(s) = \frac{\frac{\dot{\theta}_0}{K_1} + \theta_0 - \frac{KT}{K_1^2}}{s} + \frac{\frac{KT}{K_1}}{s^2} + \frac{\frac{KT}{K_1^2} - \frac{\dot{\theta}_0}{K_1}}{s + K_1} \qquad (39)$$

Taking the inverse Laplace transform:

$$\theta = \frac{\dot{\theta}_0}{K_1} + \theta_0 - \frac{KT}{K_1^2} + \frac{KT}{K_1}t + \left[\frac{KT}{K_1^2} - \frac{\dot{\theta}_0}{K_1}\right]e^{-K_1 t} \qquad (40)$$

Taking the time derivative yields

$$\dot{\theta} = \frac{KT}{K_1} + \left[\dot{\theta}_o - \frac{KT}{K_1}\right] e^{-K_1 t} \tag{41}$$

Solving for time from Eq. (41) leads to

$$t = \frac{1}{K_1} \ln \frac{\left[\dot{\theta}_o - \frac{KT}{K_1}\right]}{\left[\dot{\theta} - \frac{KT}{K_1}\right]} \tag{42}$$

Replacing time in Eq. (40) and reducing yields

$$\theta - \theta_o = \frac{\dot{\theta}_o}{K_1} \frac{(\dot{\theta}_o - \dot{\theta})}{(\dot{\theta}_o - \frac{KT}{K_1})} + \frac{KT}{K_1^2} \frac{(\dot{\theta} - \dot{\theta}_o)}{(\dot{\theta}_o - \frac{KT}{K_1})} + \frac{KT}{K_1^2} \ln \frac{(\dot{\theta}_o - \frac{KT}{K_1})}{(\dot{\theta} - \frac{KT}{K_1})} \tag{43}$$

At this point in the development of sections A and B expressions were derived for the intersection points C and B', Fig. (2) and Fig. (4), in terms of $\dot{\theta}$ and θ. In this case the author knows of no way of solving for $\dot{\theta}$ explicitly in terms of θ. Therefore the method of setting the velocities equal, as was done in the previous cases, is not applicable. Attempts to equate angular displacements were unsuccessful. Therefore it was concluded that an analytical expression for the switching boundaries was not obtainable.

This left only a trial and error solution. This could have been accomplished using the digital computer facilities of the school, but a graphical method was decided upon. A graphical method, although laborious and less accurate, tends to develop a feel for the system which is useful.

The details of this method are given in Appendix A, but basically the method consists of repeatedly superimposing the various trajectories, reading off the points of intersection, substituting in the appropriate equation, calculating and comparing t_{ABCD} and $t_{AB'C'}$ until the difference between t_{ABCD} and $t_{AB'C'}$ is acceptably small.

This results in a locus of points in the phase plane which represents states of the system for which equal time is required to come to rest at θ_f - N360° or at θ_f - (N + 1)360°, therefore this locus is obviously one of the optimum zone boundaries. The resulting points are tabulated in Table I. It is evident from the symmetry of the switching curves that this boundary is repeated every 360°. Fig. (7) shows the optimum switching curves and the optimum switching zones formed from the calculated boundary repeated every 360°. The significance of these zones is exactly the same as that given to the optimum switching zones developed in section A.

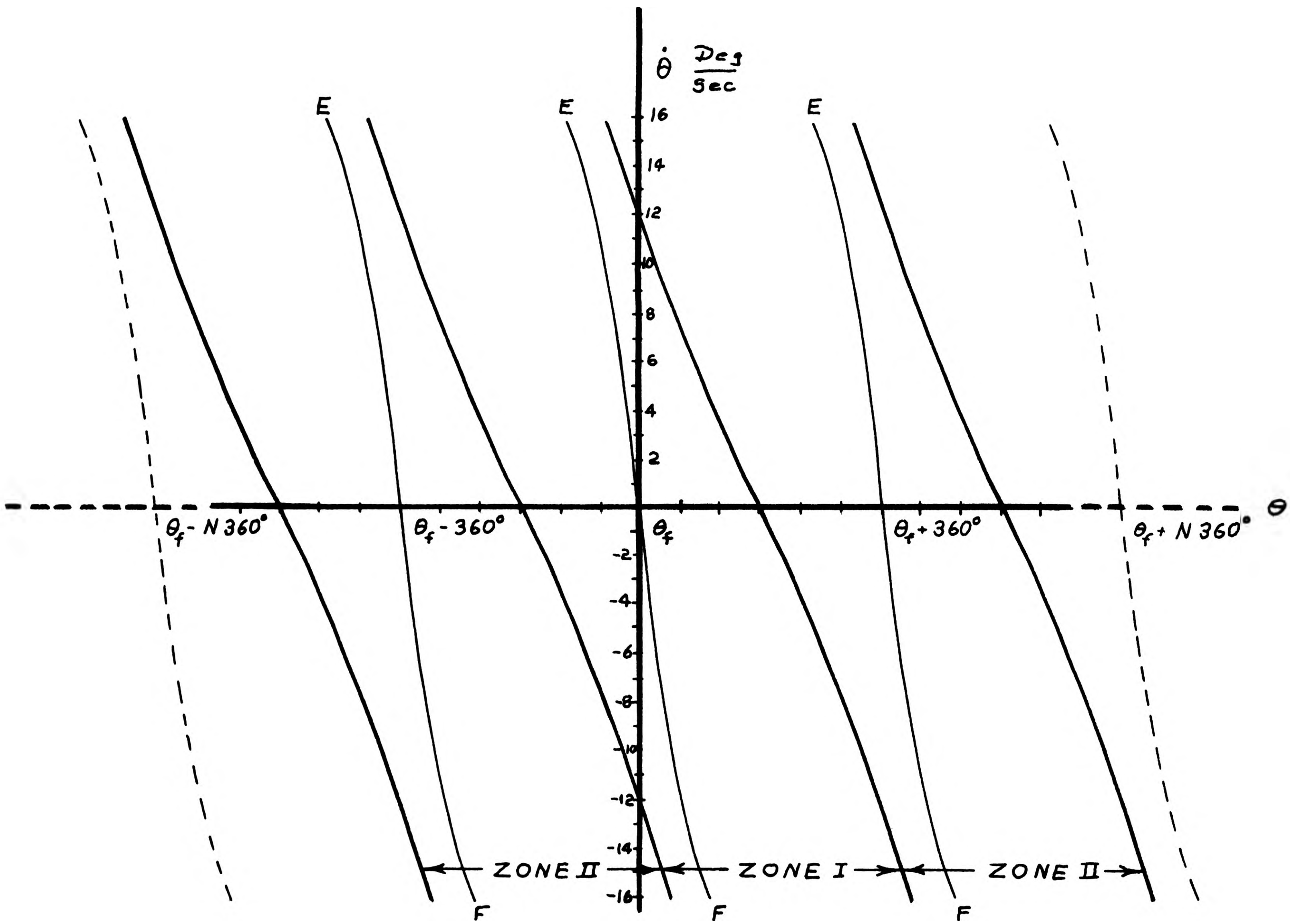

Fig. (7) Optimum Switching Zones for System with Inertia Plus Viscous Damping.

IV. CONCLUSIONS

This thesis points out that present optimum control theory does not provide enough information to obtain the time-optimum step-input response for a second-order multiple-valued control system. This is because it is necessary to determine which of the infinite number of paths that are possible in a multiple-valued system is the optimum path.

Zones were shown to exist which determine the time optimum path for rotational multiple-valued systems. Figs. (3), (6) and (7) show the resulting optimum switching zones for the three systems considered.

It can be concluded that these zones also exist for any system which meets the second-order multiple-valued constraints applied in this thesis (a rotational system with no spring like forces and θ_f equivalent to $\theta_f \pm N360^{\circ}$ where N is any integer). This is true because the single-valued switching curves are simply the deceleration curves of the system which, for the constraints applied, are not dependent on displacement. They can thus be repeated every 360° without crossing. Since the switching curves extend from infinity in one direction to infinity in the other direction without crossing, every point in the phase plane is surrounded by only two switching curves, one of which must be crossed to reach any switching curve not adjacent to the point. Therefore, for any particular point in the phase plane there are only two paths which have the possiblity of being the optimum path to a particular final position. This is because any path which crossed a switching curve to reach the final position would be less than an optimum path, due to the nature of the optimum switching curves. This indicates that the method of equating the time required to traverse two general paths (ABCD and AB'C' of Figs. (2), (4) and (8)) as was done in this thesis will locate optimum switching zones for any second-order system subject to the constraints mentioned above.

Also, zones were developed which predict the number of revolutions which can be traversed in a forward direction in the same time as it would take to bring the system back to some designated final angular position.

APPENDIX A

Graphical Solution for System with Inertia Plus Viscous Damping

It was pointed out in Chapter III that it was not possible to obtain an analytical expression for the optimum switching zone boundaries for a system with inertia plus viscous damping. This was because of the difficulty in expressing the points of intersections C and B', Fig. (8), analytically. A graphical method eliminates this difficulty.

The equations of motion, which were developed in Chapter III, were used to make overlays of the phase plane acceleration and deceleration trajectories of the system. These overlays can be superimposed on a scaled reference coordinate system and the critical intersection values read off. The graphically determined values of displacement and velocity are used in the equations of Chapter III to determine time. Since the optimum switching zone boundaries are desired, a trial and error method is used to locate points where the time to come to rest at an angular position $\theta_f \pm N360^\circ$ is equal to the time to bring the system to rest at $\theta_f \pm (N + 1)360^\circ$ (N is any integer). Fig. (8) shows the switching curves and identifies points to be used in the calculations.

Assume the following constants. KT = 0.573 and $K_1 = 0.0573$. Therefore, from Eq. (42)

$$t = 17.45 \left[\ln (10 - \dot{\theta}_o) - \ln (10 - \dot{\theta}) \right] \qquad (44)$$

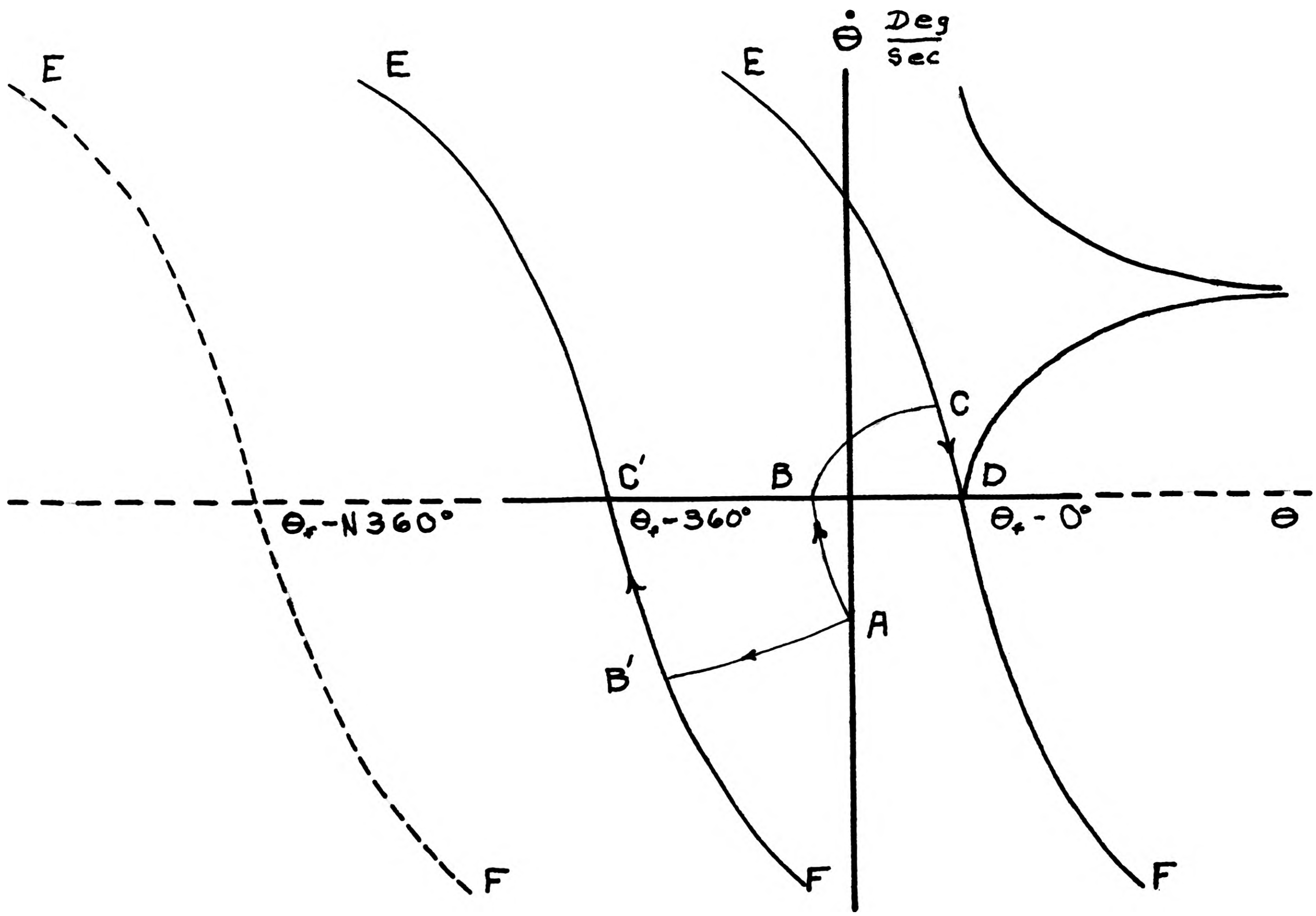

Fig. (8) Optimum Switching Curves for System with Inertia Plus Viscous Damping.

The points which must be determined graphically are initial and final velocity. These points are easily determined on portions of the curves that have a large slope but where the trajectories approach the horizontal a small error in velocity will cause a large error in the time calculated by Eq. (44). For this latter case an average velocity is approximated and the change in angular position is read from the graph. Then time is given by

$$t = \frac{\Delta\theta}{\dot{\theta}_{Average}} \qquad (45)$$

Examples will illustrate both cases.

Overlays were made of Fig. (10) (Overlay-I) and Fig. (11) (Overlay-II). Fig. (10) is a plot of the deceleration curves of the system given by Eq. (39) with the assumed constants and the proper initial conditions. (Note that the deceleration curves are also the switching curves[1].) Fig. (11) shows both the acceleration and deceleration portion of the phase plane trajectory. The shape of the phase plane trajectories are not dependent on displacement therefore Overlay-I and Overlay-II are valid representations of the trajectory throughout the phase plane as long as the zero velocity reference is maintained. That is the overlays can be manipulated right or left but not up or down. By correctly manipulating the overlays all of the trajectories needed in these calculation can be represented.

As a first example of this trial and error method assume $\theta_f = 108^\circ$ and first try initial velocity $\theta_o = -5$ deg/sec.

Step 1. Place Overlay-I over Fig. (9), the reference coordinates, so that the θ_f axis passes through 108° on Fig. (9). This should place the optimum switching curves of Overlay-I at 108° and at 108° - 360° = -252°.

Step 2. Place Overlay-II over Overlay-I so that the axes line up and the curve opens to the right and approaches a positive velocity of 10 deg/sec. Adjust until the curve passes through $\theta = 0°$ and $\dot{\theta}_o = -5$ deg/sec. This provides a picture of path ABCD, Fig. (8). $\dot{\theta}_C$ can be read from the graph as approximately 7.05 deg/sec.

Step 3. Manipulate Overlay-II so that the curve opens left and approaches a negative velocity of 10 deg/sec. Line up the axes and again adjust the curve to pass through $\theta = 0°$ and $\dot{\theta}_o = -5$ deg/sec. This provides a picture of path AB'C'. $\dot{\theta}_{B'}$ can be read as approximately 9.0 deg/sec.

Step 4. Using these values in Eq. (44) yields

$$t_{AC} = 17.45[\ln(10-5) - \ln(10-7.05)] = 17.45[1.625]$$

$$t_{CD} = 17.45[\ln(10+7.05) - \ln(10)] = \underline{17.45[0.533]}$$

$$17.45[2.159] = 37.6 \text{ sec} = t_{ABCD}$$

$$t_{AB'} = 17.45[\ln(10-5) - \ln(1)] = 17.45[1.6094]$$

$$t_{B'C'} = 17.45[\ln(10+9) - \ln(10)] = \underline{17.45[0.6418]}$$

$$17.45[2.2512] = 39.3 \text{ sec} = t_{AB'C'}$$

Step 5. Compare t_{ABCD} and $t_{AB'C'}$. If t_{ABCD} is larger choose a more positive initial velocity. If $t_{AB'C'}$ is larger choose a more negative initial velocity. Repeat steps 1 through 5 until the difference between t_{ABCD} and $t_{AB'C'}$ becomes as small as desired. In this example $t_{AB'C'}$ is larger therefore a more negative initial velocity should be chosen.

Choosing an initial velocity of -6 deg/sec the following parameters were found: $\dot{\theta}_C$ = 7.2 deg/sec and $\dot{\theta}_{B'}$ =-9.1 deg/sec. Using these values in Eq. (44) yields

$$t_{AC} = 17.45[\ln(10+6) - \ln(10-7.2)] = 17.45[1.7430]$$

$$t_{CD} = 17.45[\ln(10+7.2) - \ln(10)] = \underline{17.45[0.5426]}$$

$$17.45[2.2856] = 39.9 \text{ sec} = t_{ABCD}$$

$$t_{AB'} = 17.45[\ln(10-6) - \ln(10)] = 17.45[1.4916]$$

$$t_{B'C'} = 17.45[\ln(10+9.1) - \ln(10)] = \underline{17.45[0.6470]}$$

$$17.45[2.1386] = 37.0 \text{ sec} = t_{AB'C'}$$

Since t_{ABCD} is larger, a point on the zone boundary has been isolated between -5 deg/sec and -6 deg/sec.

As a second example let $\theta_f = 36^\circ$ and try an initial velocity of $\dot{\theta}_o$ = -10 deg/sec.

Step 1. Place Overlay-I over Fig. (9) so that the θ_f axis passes through 36° and 36° - 360° = -324°.

Step 2. Place Overlay-II over Overlay-I so that the axes line up and the curve opens to the right. The curve should approach a velocity of positive 10 deg/sec. θ_C is read as approximately 6.5 deg/sec.

Step 3. Manipulate Overlay-II so that the curve opens to the left and is flat on the lower side. Line up the axes and again adjust the curve to pass through - 10 deg/sec. Obviously Eq. (44)

can not be used as it indicates that an infinite time is required to reach point B'. Therefore an average velocity of -10 deg/sec is assumed. $\theta_{B'}$ is read from the graph as approximately -270°.

Step 4. Calculating time using Eq. (44) and Eq. (45) yields

$$t_{AC} = 17.45[\ln(10+10) - \ln(10-6.5)] = 17.45[1.8806]$$

$$t_{CD} = 17.45[\ln(10+6.5) - \ln(10)] = \frac{17.45[0.5008]}{17.45[2.3814]} = 41.6 \text{ sec} = t_{ABCD}$$

$$t_{AB'} = 270 \text{ deg} / 10 \frac{\text{deg}}{\text{sec}} = 27.0 \text{sec}$$

$$t_{B'C'} = 17.45[\ln(10+10) - \ln(10)] = 17.45[0.6931] = \frac{12.1 \text{sec}}{39.1 \text{sec}} = t_{AB'C'}$$

Step 5. Comparing times shows that t_{ABCD} is larger than $t_{AB'C'}$, therefore the boundary passes at a more positive velocity.

After several trial solutions were made the zone boundary was found to pass through the point $\dot{\theta} = -9.8$ deg/sec.

The points obtained are tabulated in Table-I and are plotted in Fig. (7).

If this graphical method was to be used extensively it would be convenient to calibrate the curves in terms of time so that a change in time from one point on the curve to another point on the curve could be read directly.

TABLE I.

Tabulated Results of Graphical Calculations

Zone Boundary for Inertia Plus Viscous System

Angular Position (Deg.)	Angular Velocity (Deg/Sec)
180	0
144	-2.1
108	-5.3
72	-6.8
36	-9.8
0	-12.0
-18	-14.0
-36	-16.5
-54	-18.5

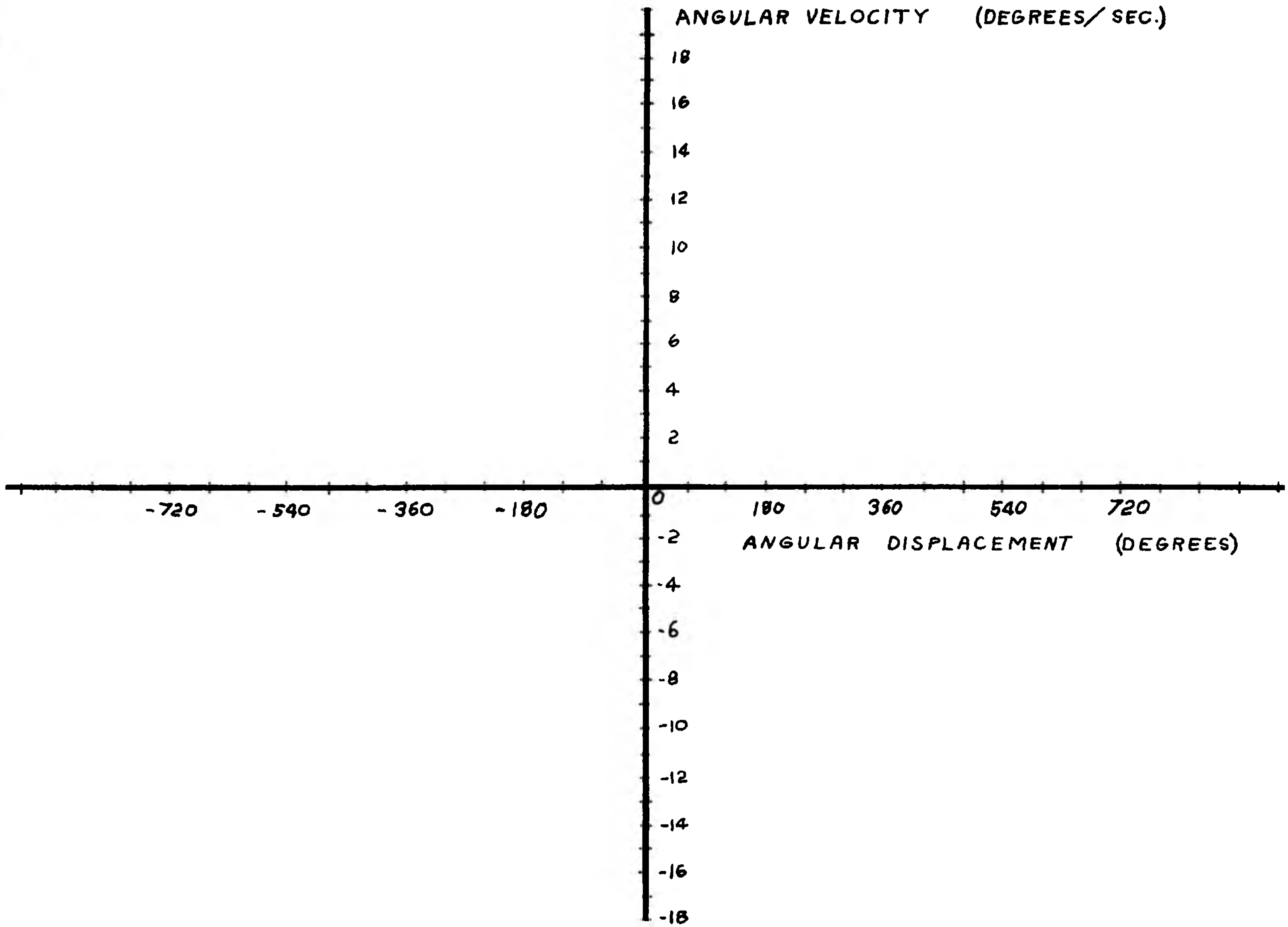

Fig. (9) Reference Coordinates.

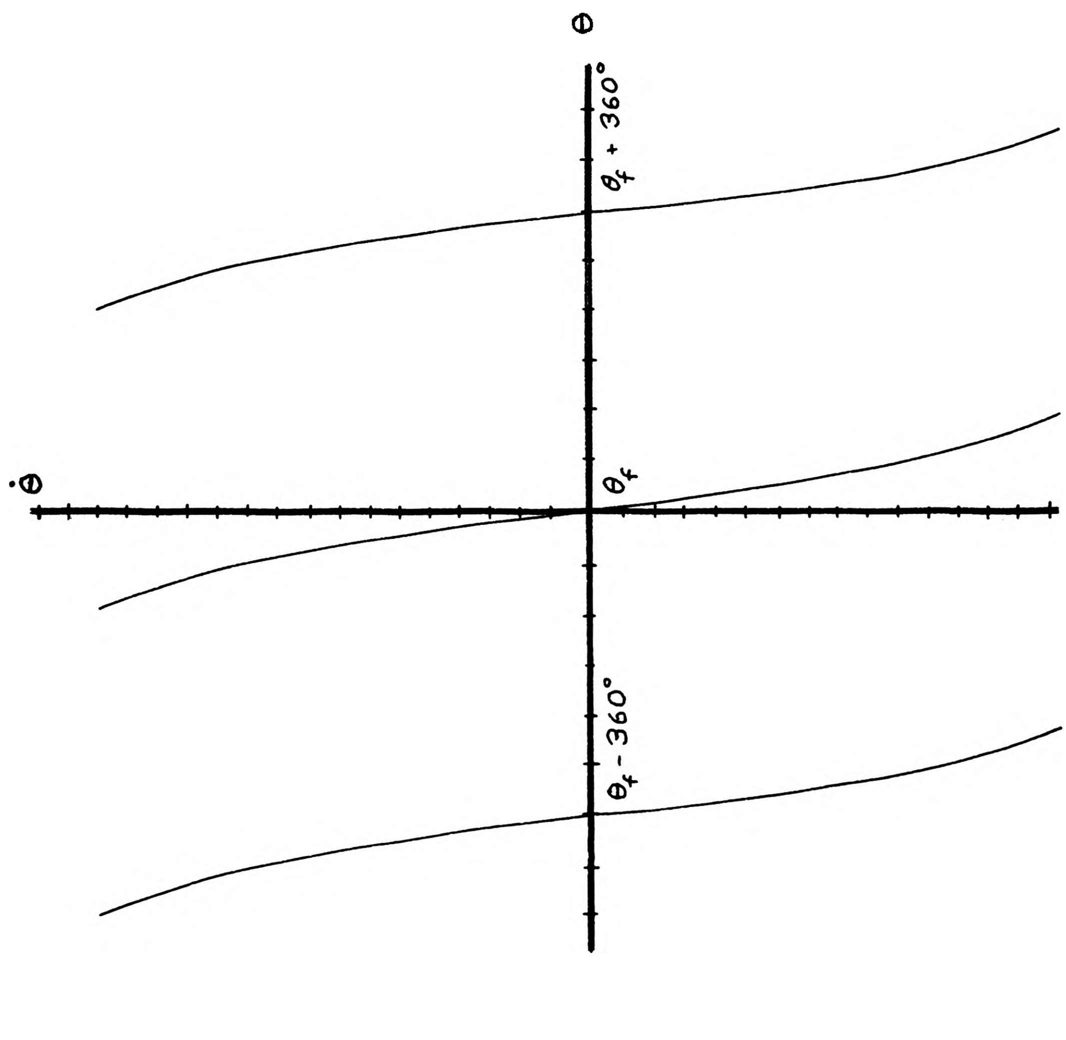

Fig. (10) Overlay-I

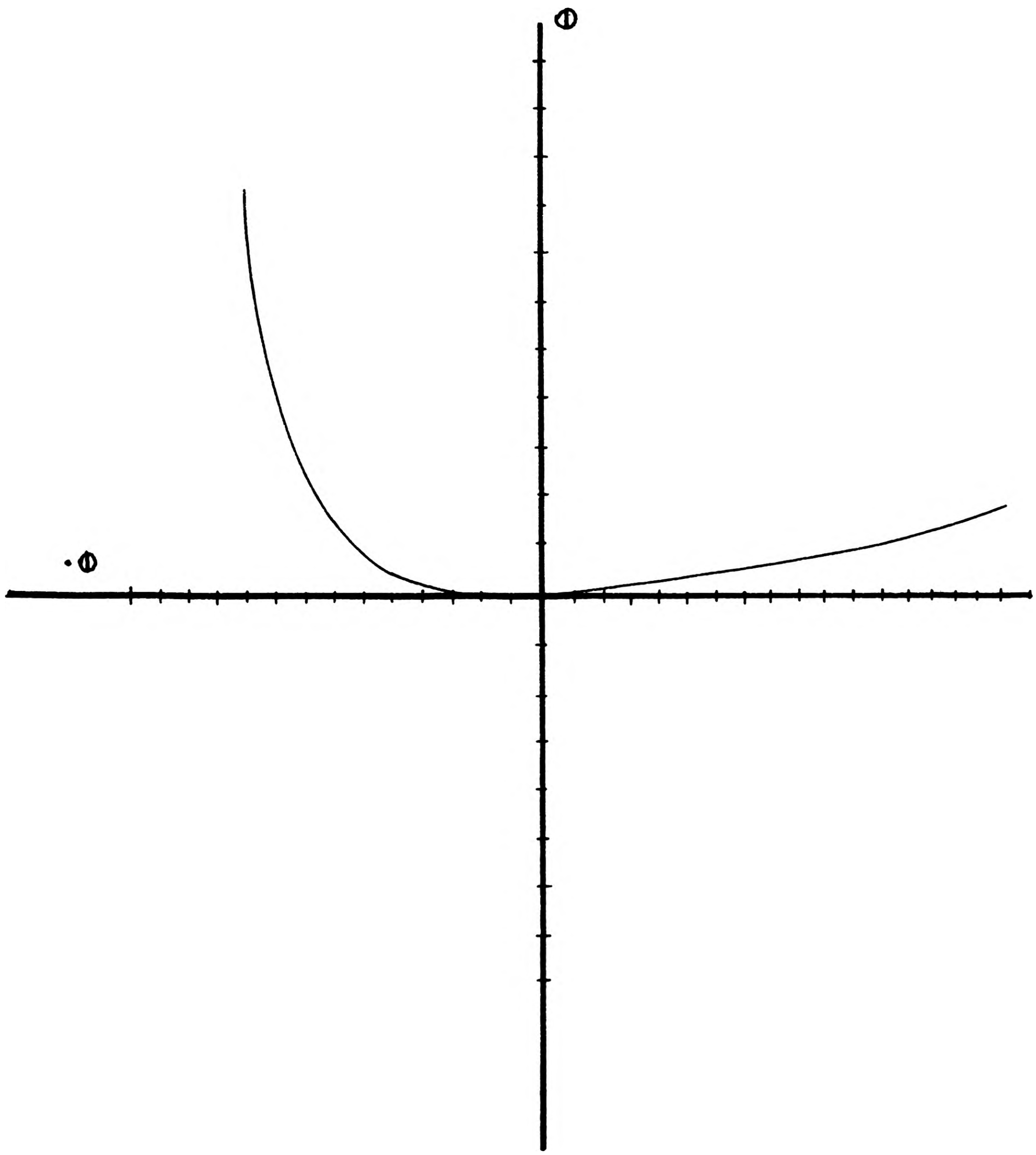

Fig. (11) Overlay-II.

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VITA

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