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Integrated scattering of microwaves by small metal spheres

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INTEGRATED SCATTERING OF MICROWAVES BY SMALL METAL SPHERES

BY

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THESIS

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INTRODUCTION

When electromagnetic radiation falls upon a material body of any kind, the associated electric field induces periodic oscillations of the electrons of the material synchronous with the incident radiation. The material serves as a secondary source and emits energy in the form of scattered radiation with a frequency equal to that of the incident electromagnetic wave. The polarization and intensity of the scattered radiation are determined by the size, shape, electric constants, and interactions among the scattering elements.

The scattering of electromagnetic waves is an old problem in physics. Such names as Rayleigh, Debye, and Mie have become associated with scattering, in recognition of their contribution to the subject. Because of the complexity of the general solution, applying to particles of arbitrary size, shape, orientation, and index of refraction, the complete solution probably will never be obtained. In some special cases however, notably the case of plane electromagnetic waves scattered by independent isotropic spherical particles as treated by Mie, a complete and rigorous solution has been obtained. It yields for the scattered intensities a complicated series expression involving the index of refraction of the sphere relative to that of the surrounding medium, the dielectric constant of the surrounding medium, and a parameter $\alpha = 2\pi a/\lambda$, where a is the radius of the sphere and λ is the wavelength of the incident radiation. For very small particles the Mie theory reduces to the celebrated Rayleigh theory. For larger particles little numerical application of the Mie theory has been made because, with presently available mathematical methods, computational difficulties are excessive.

Until quite recently there had been no satisfactory experimental verification (in the microwave region) of the theory, except perhaps in the region of very small α values, where the simpler Rayleigh law is valid. There are two main reasons for this: (1) numerical application of the theory as set forth by Mie is extremely laborious for larger values of α, and (2) serious experimental difficulty has been encountered in producing suitable space distributions of sufficiently uniform scattering particles in the larger diameter range. Quite recently however, Aden and also Seigert have carried out excellent verification of the theory for microwave back-scattering.

In this investigation scattering area coefficients are determined for chrome-plated steel spheres in the diameter range from 5/32 to 14/32 inches, using microwave radiation having a wavelength of a nominal 3.2 centimeters. Spheres with diameters accurate to one ten thousandth of an inch were obtained commercially. Single layers of randomly distributed spheres, having area coverage factors in the approximate range from 5 to 12 per cent were obtained. From transmission data using these single layer samples the scattering area coefficients were computed and plotted as a function of $\alpha = \frac{2\pi a}{\lambda}$. The results were compared with the theory.

REVIEW OF LITERATURE

The foundation for electromagnetic scattering was laid by Lord Rayleigh[1] in 1871 when he applied the electromagnetic theory to the problem of light scattered by the molecules of a gas. According to Rayleigh the oscillating electric field of light incident upon a transparent isotropic particle, whose radius is small compared to the wavelength of the light, induces an oscillating electric moment in the particle which then behaves as a linear electric oscillator.

If the linear dimension of the particles is comparable to the wavelength of the incident radiation, or if the particles are metallic, they cannot be considered optically small and the character of the light scattered by them is more complex than that given by the Rayleigh dipole radiator theory.

The general solution for the case of a plane wave incident upon particles of arbitrary size, shape, and orientation has never been worked out. However, for the special case of a spherical particle the solution has been obtained by Mie[2]. The Stratton[3] treatment of the Mie theory expresses the solution in the form of orthogonal spherical vector wave functions.

Since the paper by Mie in 1908 many workers have produced a considerable number of papers on all the aspects of

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1. Rayleigh, Lord, Phil. Mag. Vol. 41, p. 447 (1871) or Scientific Papers, Vol. I, pp. 87-110 (1871)
2. Mie, G., Ann. Physik, Vol. 25, p. 377 (1908)
3. Stratton, J. A., Electromagnetic Theory, The McGraw-Hill Book Co., Inc., 1944

the scattering problem. Important contributions, theoretical, numerical, or experimental, have come from Rayleigh, Stratton, Aden and others.

The case of back-scattering from water hemispheres has been treated by Aden[4]. One of the difficulties involved in his experiment was in obtaining water spheres that would maintain their size. However, Aden overcame that difficulty by using small containers in the form of thin hemispherical shells made of Styrofoam and filling these with water. The water hemispheres were mounted on aluminum discs, then placed in their proper position on a screen. When exposed to electromagnetic radiation the conditions were the same (by the method of images) as for a complete sphere in free space. The back-scattering area coefficient was plotted as a function of the diameter-to-wavelength ratio, with $\lambda = 16.230$ centimeters. Figure 1 shows his results.

A brief theoretical treatment of the scattering for a plane wave by a metallic sphere is given by Siegert[5]. He investigated the case of the metallic sphere to obtain an approximation to what might be expected from an actual radar target of more general shape. The back-scattering cross section of a radar target may be defined as equal

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4. Aden, A. L., Journal of Applied Physics, Vol. 22, p. 601 (1951)

5. Siegert, A. J. F., Radar System Engineering (MIT Radiation Lab. Series, Vol. #1) McGraw-Hill Book Co., Inc., New York, 1947, pp. 63-65

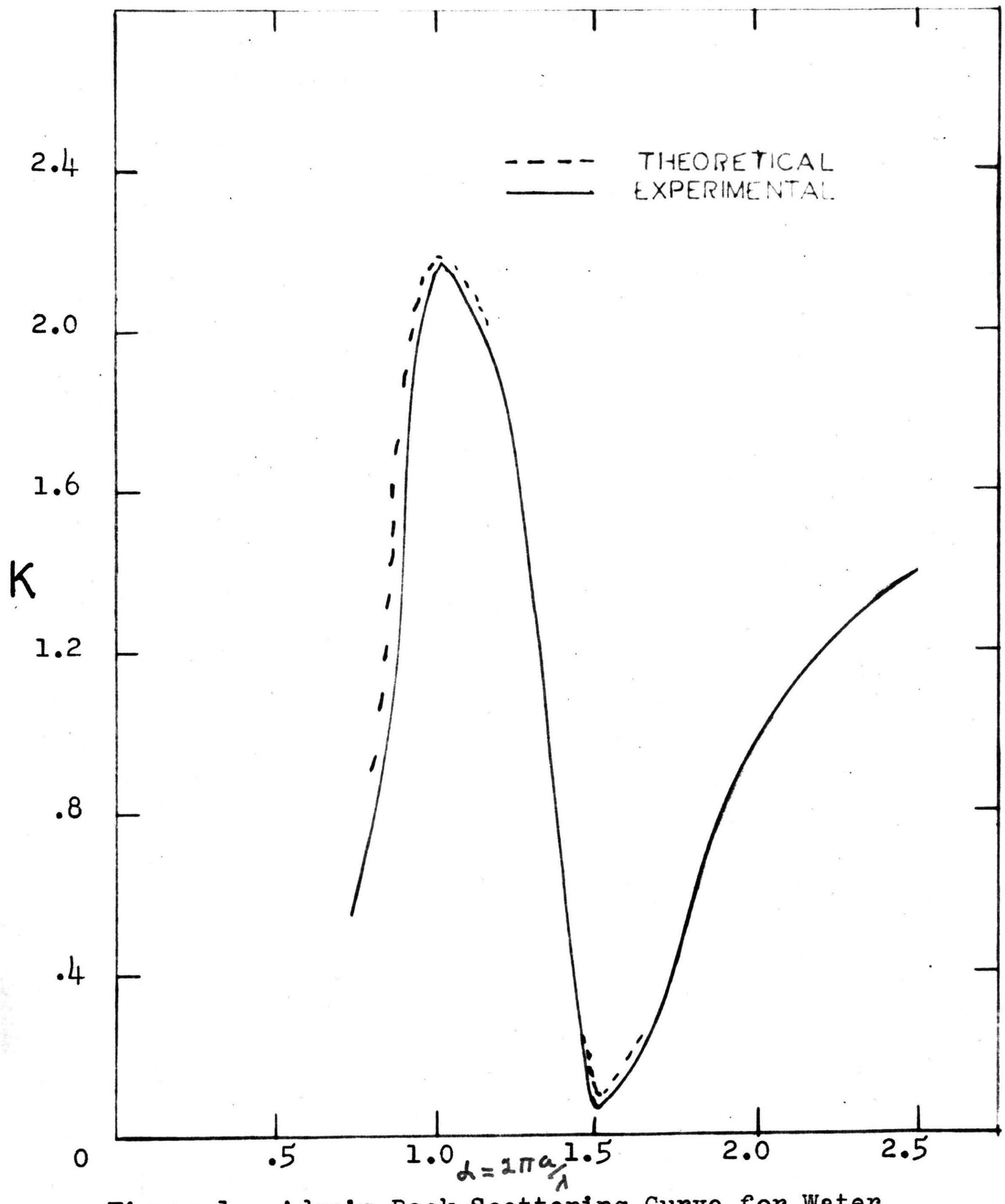

Figure 1. Aden's Back-Scattering Curve for Water Hemispheres.

to 4π times the power per unit solid angle which is scattered back towards the transmitter for unit intensity in the wave incident upon the target. Siegert presented his results in the form of a graph where he plotted the back-scattering area coefficient, k, versus the wavelength-to-diameter ratio.

The determination of the character of the oscillations to be expected from a conducting sphere is not easy. However, the first few modes of oscillation, in the case of a sphere with infinite conductivity, have been determined. According to Stratton[6], to produce resonance of a perfectly conducting sphere at its lowest natural frequency, the wavelength of the incident radiation in the external medium is given by

$$\lambda = \frac{2\pi a}{.86}$$

This suggests that the value of the back-scattering area coefficient corresponding to $\lambda = 0.86$ should be the first maximum of Figure 1.

Stratton, J. A., Electromagnetic Theory, New York, The McGraw-Hill Book Co., Inc., p. 558, 1944

SCATTERING AREA COEFFICIENT

By scattering, as used in this paper, is meant the removal of energy from the incident beam by reflection, diffraction, absorption, or reradiation.

The scattering cross section of a scattering element is defined as the area perpendicular to the beam through which enough energy falls per second from the incident plane wave to equal the total scattered intensity. The scattering cross section is used to express the ability of a particle to scatter energy incident upon it. However, the usual term to express the same thing is the scattering area coefficient, K, which is defined as the ratio of the scattering section to the cross sectional area of the sphere.

It would be expected on a basis of geometrical optics that the intensity scattered by a sample containing m spheres per unit area would be given by

$$\Delta I = I_0 - I_1 = I_0 \pi a^2 m, \qquad (1)$$

where I_0 is the intensity incident upon the sample, I_1 is the intensity which has passed straight through the sample, ΔI is the scattered intensity, and a is the sphere radius. By definition of the scattering area coefficient, K, the actual scattered intensity will be K times as great as that given by Equation (1), or

$$\Delta' I = I_0 - I_1 = I_0 K \pi a^2 m, \qquad (2)$$

This permits one to compute the value of K from the intensities.

In the experiment to follow, it will be galvanometer deflections instead of intensities which will be recorded. These deflections are proportional to the square roots of intensities, so that:

$$K = \frac{\dfrac{d_0^2 - d_1^2}{d_0^2}}{\pi a^2 m} \qquad (2')$$

where d_o is the scale deflection with sample removed, d_1 is the scale deflection for the sample in the beam, and $\pi a^2 m$ is the area coverage factor (referred to as A.C.F. hereafter in this paper). In the experiment Plexiglas sheet was used to support the scattering spheres. Some loss in the transmitted intensity was, of course, caused by the presence of the Plexiglas. To correct for this effect d_o was taken always with a blank sheet of Plexiglas in place of the sample.

Similarly, some scattering is produced by the presence of the holes drilled in the Plexiglas. In order to account for the effects of these holes, the transmission of a drilled Plexiglas sheet may be substituted for the plain Plexiglas. In this case equation (2') becomes

$$K_{corrected} = \frac{1 - \left(\frac{d_1}{D_o}\right)^2}{A.C.F.} \quad . \qquad (3)$$

D_o is the scale deflection when drilled Plexiglas is placed in the microwave beam.

GENERAL DISCUSSION OF THE EXPERIMENTAL APPARATUS

Radiation of nominal 3.2 centimeter wavelength is produced by a klystron generator. This radiation is rendered plane by a parallel-plate microwave lens. It then falls on the scattering sample which consists of a plane distribution of metal spheres supported by Plexiglas. Radiation which is transmitted undeviated (not scattered) by the sample is picked up by a receiver. The system is shown schematically in Figure 2.

The microwave generator consists of a type 723 A/B-2K25 klystron coupled to a short section of wave guide which propagates energy into space through a pyramidal horn antenna. The purpose of the horn is two-fold: (1) to give the greatest possible attenuation to unwanted modes that might be excited in the waveguide and to give the greatest possible gain to the dominant mode, and (2) to produce a directional effect (beam) in the forward direction.

The microwave lens serves to convert an approximately spherical wave into a 12-inch by 12-inch parallel, plane-polarized beam, which was used to illuminate the scattering sample.

The receiver consists in general of (1) a pyramidal horn which serves to match the waveguide to free space with the least possible attenuation (2) a loop which detects the presence of the magnetic field, (3) a crystal rectifier which converts alternating current to direct current, and (4) a sensitive direct current galvanometer with its telescope and scale. The galvanometer was a Leeds and Northrup

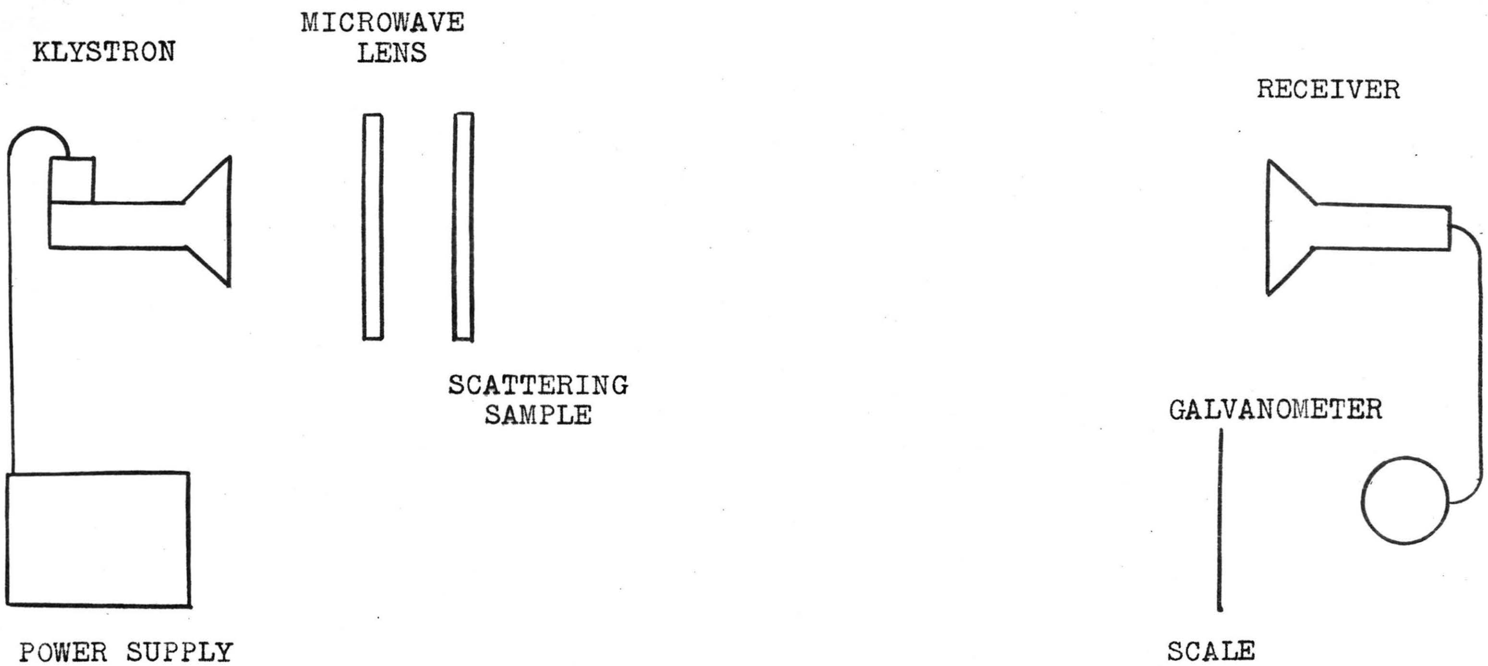

Figure 2. Block Diagram of Experimental Apparatus.

Type R, having a sensitivity of 0.00035 $\frac{\mu A}{mm}$ for the scale one meter from the mirror.

A power supply delivers voltages to the various elements of the microwave generator tube. The frequency at which a klystron generator operates is somewhat dependent upon the acceleration grid and repeller voltages. The voltages applied to the tube did not vary more than one volt in 180 volts.

The wavelength generated is not known but from the literature relating to similar klystrons the probable maximum range over which this klystron can be tuned is of the order of magnitude of 1,000 megacycles/second. In order to get a frequency variation of this magnitude a mechanism must be used for warping the cavity. In this investigation, however, the cavity remained unchanged throughout the entire investigation. The repeller and grid voltages were held sensibly constant so that the actual variation in frequency was of the order of only a few megacycles/second. The nominal frequency was 9,400 mc./sec..

PRODUCTION OF THE SCATTERING SAMPLES

It is desirable to have a scattering sample in which (1) the particles are randomly distributed in a plane, (2) the separation is large enough to permit independent oscillation of the spheres and (3) the supporting medium has the same index of refraction as the surrounding atmosphere.

In choosing spheres for the scattering sample it is desirable to have spheres of uniform size and the largest conductivity possible. Copper or aluminum would be desirable metals due to their high conductivities. Unfortunately, however, spheres made of these metals are not readily available. Spheres made of chrome-plated steel were chosen because of their uniformity in size and availability*. The manufacturer's tolerance was given as the 1/10,000 of an inch for the sizes used.

Materials having an index of refraction very nearly unity can be obtained commercially. One of these is Styrofoam**. However, this desirability is more than offset by the fact that it is difficult to form. Polystyrene and Plexiglas are also commonly used in microwave equipment; both have an index of refraction of 1.6, but the losses are lower for Polystyrene than for Plexiglas. However, Plexiglas was chosen because it was available locally, and because it is less troublesome to drill.

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* Manufactured by R. J. Bearings Co., St. Louis, Missouri

** Manufactured by Dow Chemical Co., Midland, Michigan

One of the major problems involved in producing a scattering sphere sample is to devise a method by which one may place a small number of spheres on a supporting medium in a random fashion. The definition of random is not obvious. However, it is usually interpreted as being the antonym of regular or orderly. In other words, if a group of particles can be shown to have no definite order then it may be stated that they are random.

The air rifle shown in Figure 3 was constructed for the purpose of obtaining a random distribution. In order to study the pattern from the air rifle it was found that a considerable number of impacts were necessary. A carbon paper overlay was placed over sheets of white paper which were in turn backed by an aluminum plate. The air rifle was held in the manner shown and air pressure of 40 p. s. i. was applied to the gun. After each box of approximately 500 shots had been fired from a distance of 100 inches, the impact area was photographed. This procedure was continued until 6 boxes of shot were fired. By comparing photographs (Figures 4 and 5 are typical) it was found that the relative density of the pattern changes with the number of shots fired. In other words, the gun does not have a characteristic barrel pattern. Thus, the pattern produced by this air gun system is, to a high degree, a random one.

For the purpose of locating the spheres on the supporting medium a master pattern, shown in Figure 6, was produced with the air gun system, using approximately 1500

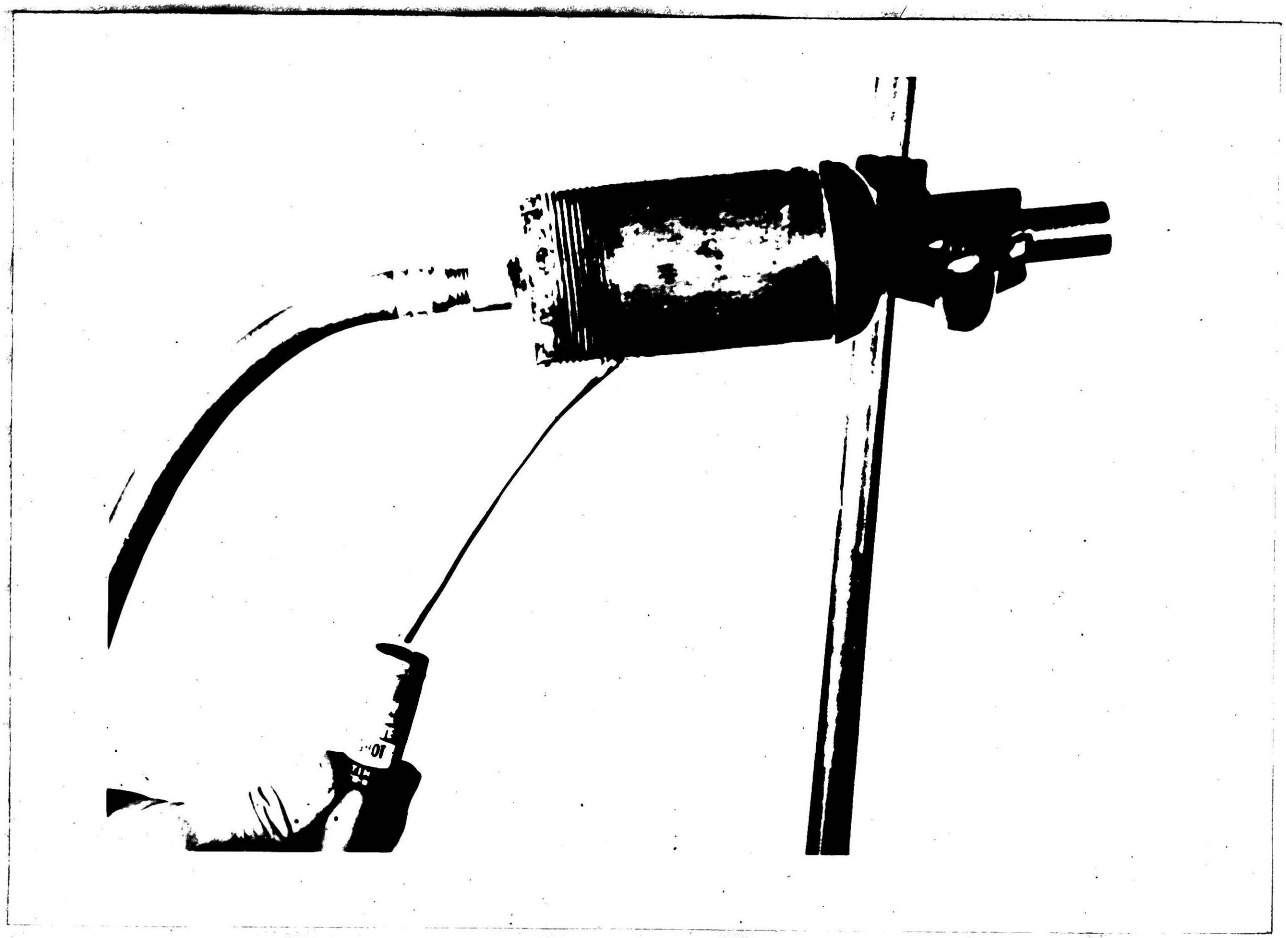

Figure 3. Air Rifle Used in Obtaining Random Distribution.

Figure 4. Pattern Produced by Air Gun After 1,500 Shots.

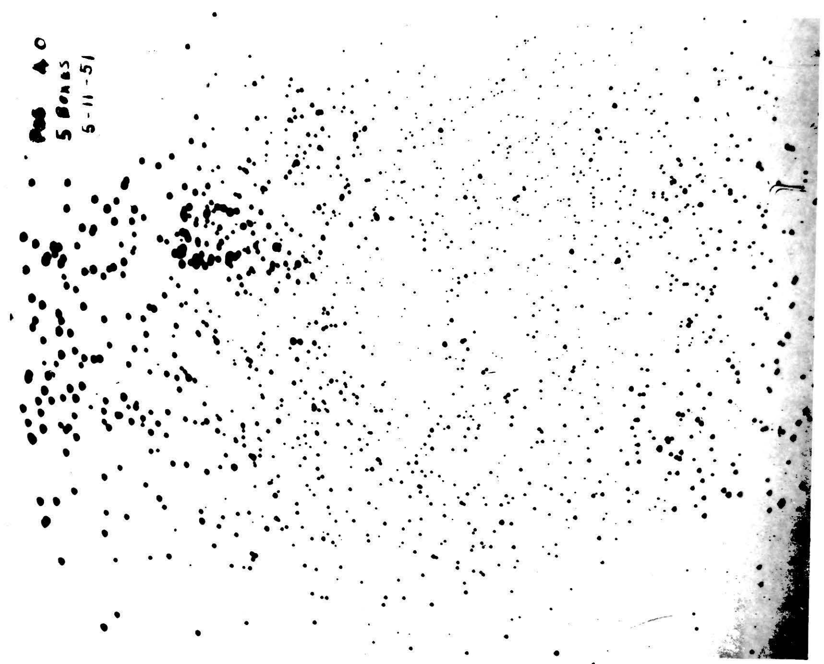

Figure 5. Pattern Produced by Air Gun After 2,500 Shots.

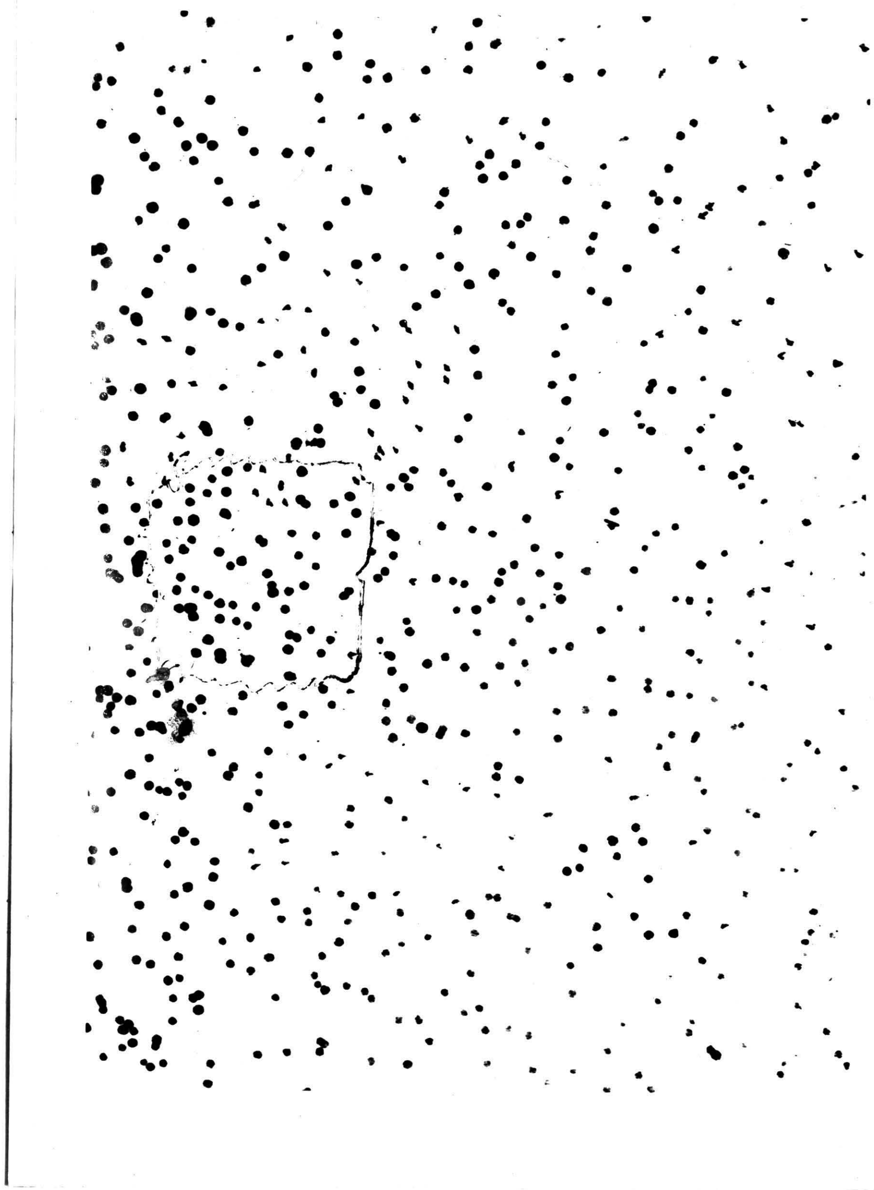

Figure 6. Master Pattern for Preparing Samples. The Portion Used is Indicated.

shots. An opaque projector was used for the purpose of marking the Plexiglas so that it might be drilled to receive the spheres.

The criterion for independent oscillation of the spheres, according to Kock[7], is that the sphere separation be greater than $\lambda/4$. The number of spheres were computed so that this criterion would be obeyed, and the magnification of the pattern was adjusted so that approximately the computed number of holes would be drilled into the Plexiglas. One of the samples appears in Figure 7. The samples were prepared by placing a small quantity of Polystyrene Coil Dope* into the drill holes and putting the spheres into the holes.

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7. Kock, W. E., Bell Laboratories Record, May 1946.

* Manufactured by American Phenolic Corporation, Chicago, Ill.

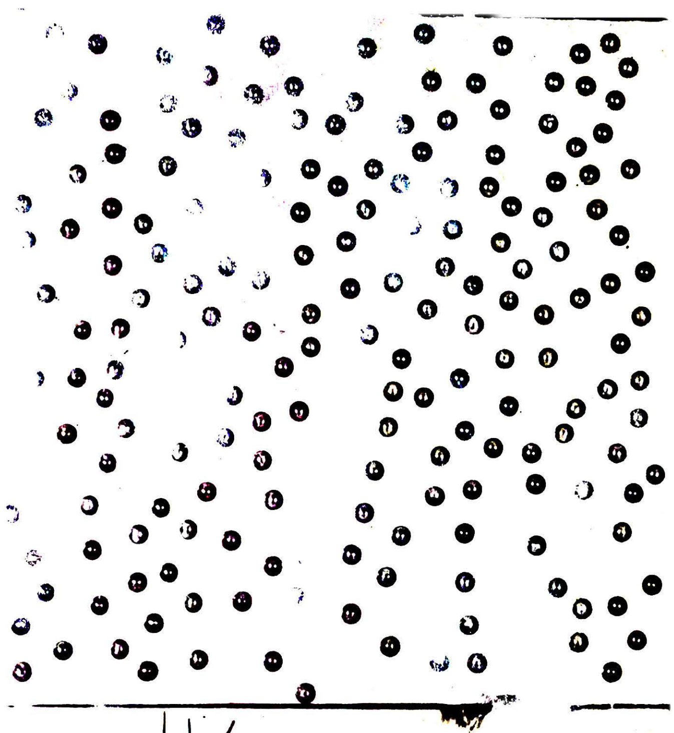

Figure 7. A Typical Sample.

MICROWAVE LENS DESIGN AND CONSTRUCTION

Since the theory applies only to the case of plane waves and the microwave transmitter used propagates into space an approximately spherical wave one must either have a considerable separation between the transmitter and the scattering sample or devise some method of converting the spherical wave into a plane wave. The former method was rejected as impracticable in the present investigation for two reasons: (1) the distance between the transmitter and receiver would be prohibitive under the housing conditions available, and (2) after any considerable distance from the transmitter the power density would be so small that detection of the magnetic field would be difficult.

One familiar with the analogies existing between optics and microwaves would be tempted to consider the construction of a lens similar to those used in the visible region of light. This would perhaps be a suitable design if the lens were made of some soft thermoplastic, such as Polystyrene, instead of glass. The difficulties with a dielectric lens, such as this, would be size, weight, and the shaping operation.

Recently, however, Kock[8] has developed a synthetic dielectric having a variable index of refraction determined by the metallic particles inserted into the dielectric.

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8. Kock, W. E., "Experiments with Metal Plate Lenses for Microwaves," BTL Report MM-44-160-67, 1944; "Metal Plate Lens Design Consideration," MM-44-160-195, 1944

This type of synthetic dielectric is quite light in weight and is highly desirable in that the index of refraction can be controlled. However, because the Styrofoam supporting structure is troublesome to construct, this type of lens was not considered feasible.

For some time parallel-plate microwave lenses have been employed in connection with radar. The design is based upon the fact that the apparent wavelength for an electromagnetic wave guided by plane parallel conduction sheets may be longer than the wavelength in free space. This implies an index of refraction less than unity, for the index of refraction is the ratio of the velocity in free space to the velocity of the same wave in the given medium. The wavelength between two parallel conducting plates is given by:

$$\lambda g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda c}{2b}\right)^2}} \tag{4}$$

where λg is the wavelength in the parallel-plate waveguide, λ is the wavelength in free space, λc is the effective wavelength perpendicular to the plates, and b is the separation of the plates. The index of refraction is given by:

$$n = \frac{V_m}{V_s} = \frac{\lambda}{\lambda g} = \sqrt{1-\left(\frac{\lambda c}{2b}\right)^2} \tag{5}$$

where n is the index of refraction, V_m is the velocity in the medium, and V_s is the velocity in free space.

Then, for a given wavelength, the index of refraction can have values ranging from zero to nearly one. In practice it is customary to select a value for n of 0.5 in order to minimize the thickness of the lens and to avoid as much mismatch (reflection) as possible.

The contour of the lens may be found by the principle of equality of optical paths along rays between pairs of wavefronts. Applying this principle one finds[9] for the contour of the lens in polar cordinates:

$$R = \frac{(1-n)f}{1-n\cos\theta} \tag{6}$$

where n is the index of refraction, R is the radius vector from the origin to the lens surface, θ is the angle between the radius vector and a fixed line, and f is the focal length of the lens. The equation is seen to be an ellipse when n is less than unity.

For the design of the lens the wavelength was assumed to be 3.2 centimeters and the index of refraction to be 0.5. From Equation (4) and these values of λ and n the plate separation b is found to be 1,8475 centimeters.

In order to produce a 12 x 12 inch beam, the chord joining the extremities of the surface is found to be 30 centimeters. Since the angular width of the microwave beam is 20 degrees, the focal length of the lens should be about 85 centimeters. Substituting these values for f and n into Equation (6) and transferring the origin to the center of

9. Southworth, G. C., Principles and Application of Wave-Guide Transmission, D. Van Nostrand Co., Inc., New York, 1950. p. 459.

the ellipse, the equation of the contour becomes:

$$4y^2 + 3x^2 = 9633.4 \text{ cm}^2 \tag{7}$$

The surface will be an ellipse of revolution which is given by:

$$4z^2 + 4y^2 + 3x^2 = 9633.4 \text{ cm}^2, \tag{8}$$

The contour of the plates can then be calculated by substituting for Z the value 0, b, 2b, ..., 8b.

The lens, shown by the photograph in Figure 8, was found to have a focal length of 86.0 centimeters as nearly as could be determined by a 5/2 wavelength dipole antenna. The detectable divergence of the beam was found to be 1 inch in 19 feet or 0.044 degrees.

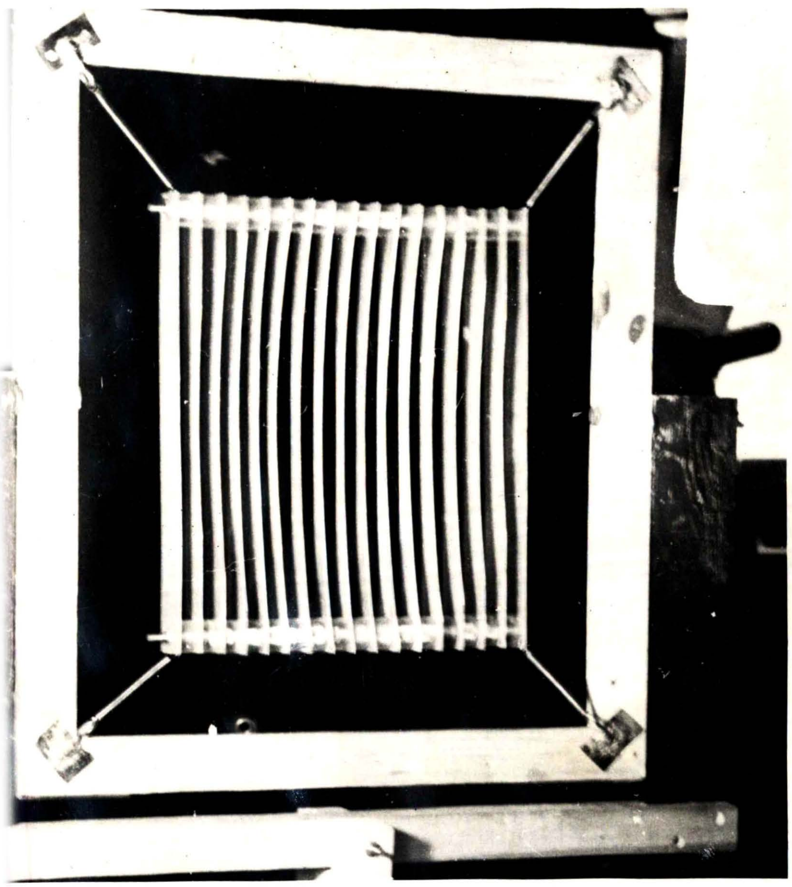


Figure 8. Parallel Plate Microwave Lens, Viewed From Concave Side

MEASUREMENTS

The experiment proper consisted mainly of transmission measurements made on the various samples using a galvanometer, and the system described in a preceding section. The wavelength was held constant by carefully controlling the grid and repeller voltages of the klystron.

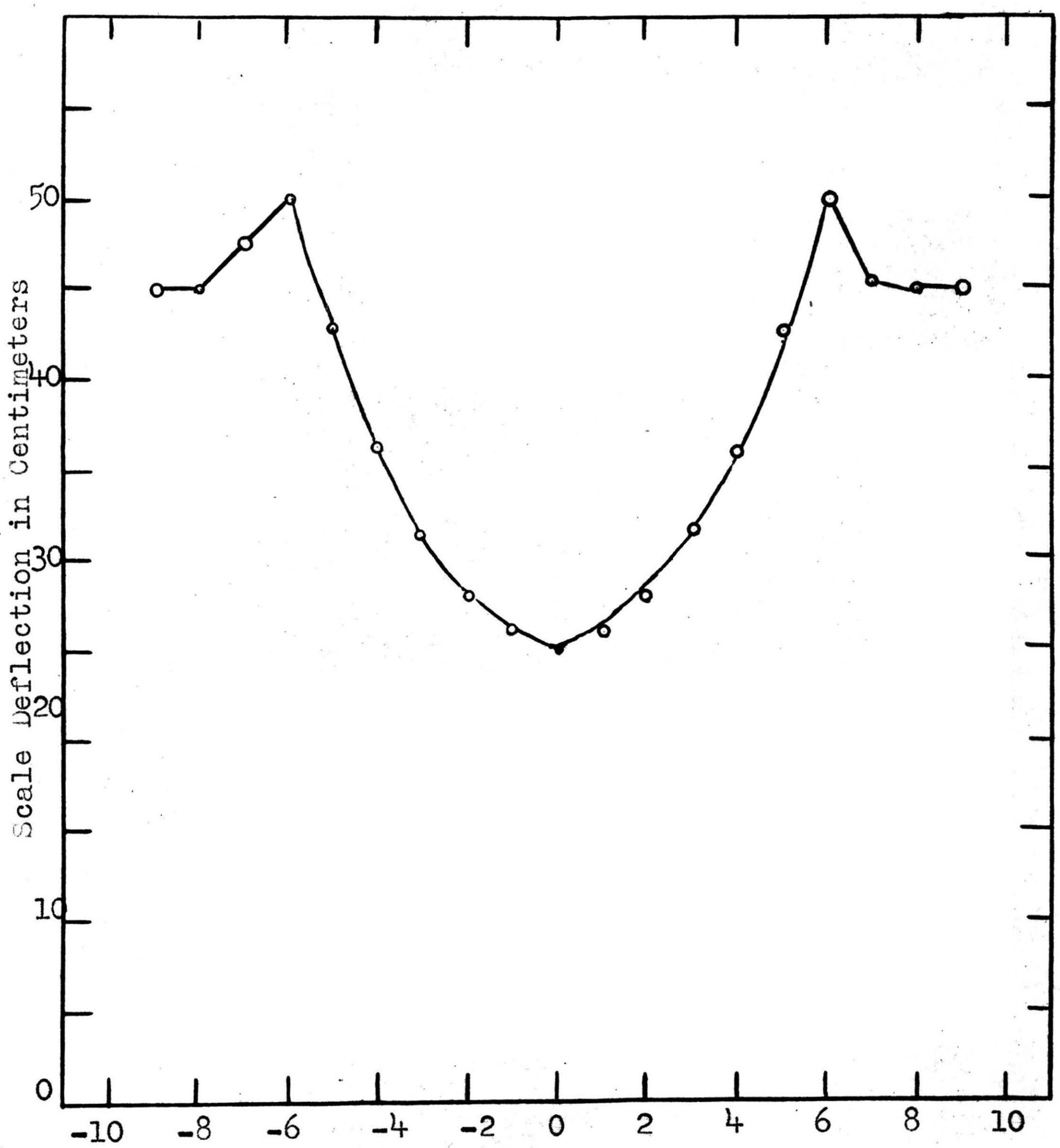
The 12 X 12 inch sheet of Plexiglas was placed between the microwave lens and receiver as shown in Figure 2, and the power was adjusted until full scale deflection was obtained. With the power input held constant the drilled Plexiglas was substituted for the undrilled and the deflection recorded. Next the drilled Plexiglas was replaced with the scattering sample and the scale deflection recorded. The observed deflections along with the computed average scattering area coefficients, both corrected and uncorrected, can be found in Table I-A. By "corrected" is meant "based on the transmission through drilled Plexiglas". Equation (3) was used to compute the value of K in this case.

In order to show that the scattering area coefficient was independent of the A.C.F., two scattering samples were prepared for a sphere diameter of 11/32 inches. The area coverage factors were 6.10 and 12.2 per cent. For these samples the computed value of K can be found in Table I-B.

The Plexiglas supporting the spheres was approximately 13 inches square. However, the square into which the spheres were placed was 12 inches on a side. All 144 square inches were illuminated by the incident radiation, but the opening of the receiver horn is only 5 inches square. Any radiation received from the outer edges of the scattering sample is, of course, undesirable. In order to test the effects of a change in the area of the sample, a scattering sample was prepared covering an area on the Plexiglas of 5 inches square. This was placed in the beam and, with the power constant, its position was changed until the minimum scale deflection was obtained. The observed deflections may also be found in Table I-B.

The galvanometer deflection was observed to change as the scattering sample was moved laterally across the front of the lens. An investigation of these effects was made using the 5 X 5 inch sample described above. The center of the microwave lens was taken as the reference point and the distances plotted represent the distance between this point and a vertical line drawn through the center of the scattering sample. The results are found in Figure 9.

In an effort to avoid the effects of the holes Polystyrene Coil Dope was used to fix the smaller spheres to the Plexiglas. For the 5/32 inch spheres the K value was computed, and compared with the K value obtained with holes. The differences in these K values were found to be negligible.



Distance in Inches from Center of Lens

Figure 9. Curve Showing the Bilateral Effects of Scattering Sample.

RESULTS

The curve in Figure 10 shows the variation of the scattering area coefficient, K, with α in the neighborhood of the fundamental resonance of the sphere. According to Stratton's theory the fundamental resonance of an infinitely conducting sphere occurs at an α value of 0.86. Thus, the experimental results are in agreement with the theory, since the maximum is seen to lie between $\alpha = 0.86$ and 0.935.

The effects of a change in the A.C.F. on the computed value of K was found to be 0.01. The deviation can be seen to be in the third significant figure. Therefore the scattering area coefficient is considered to be independent of the area coverage factor, provided the spheres are sufficiently separated to scatter independently.

It can be seen in Figure 9 that energy is scattered into the receiver by that portion of the sample immediately outside of the effective 5 X 5 inch area. Therefore it would seem reasonable to expect the K values as computed from the 12 X 12 samples to differ from those computed from the 5 X 5 samples. The computed value of K is only 0.03 larger than for 12 X 12 inch sample.

This experiment is subject to two types of errors, indeterminate and determinate. The indeterminate errors in α are due to the manufacturer's tolerance of $\pm$ 1/10,000 inch in sphere diameter and to the instability of the microwave generator. Therefore the variation in α due to the

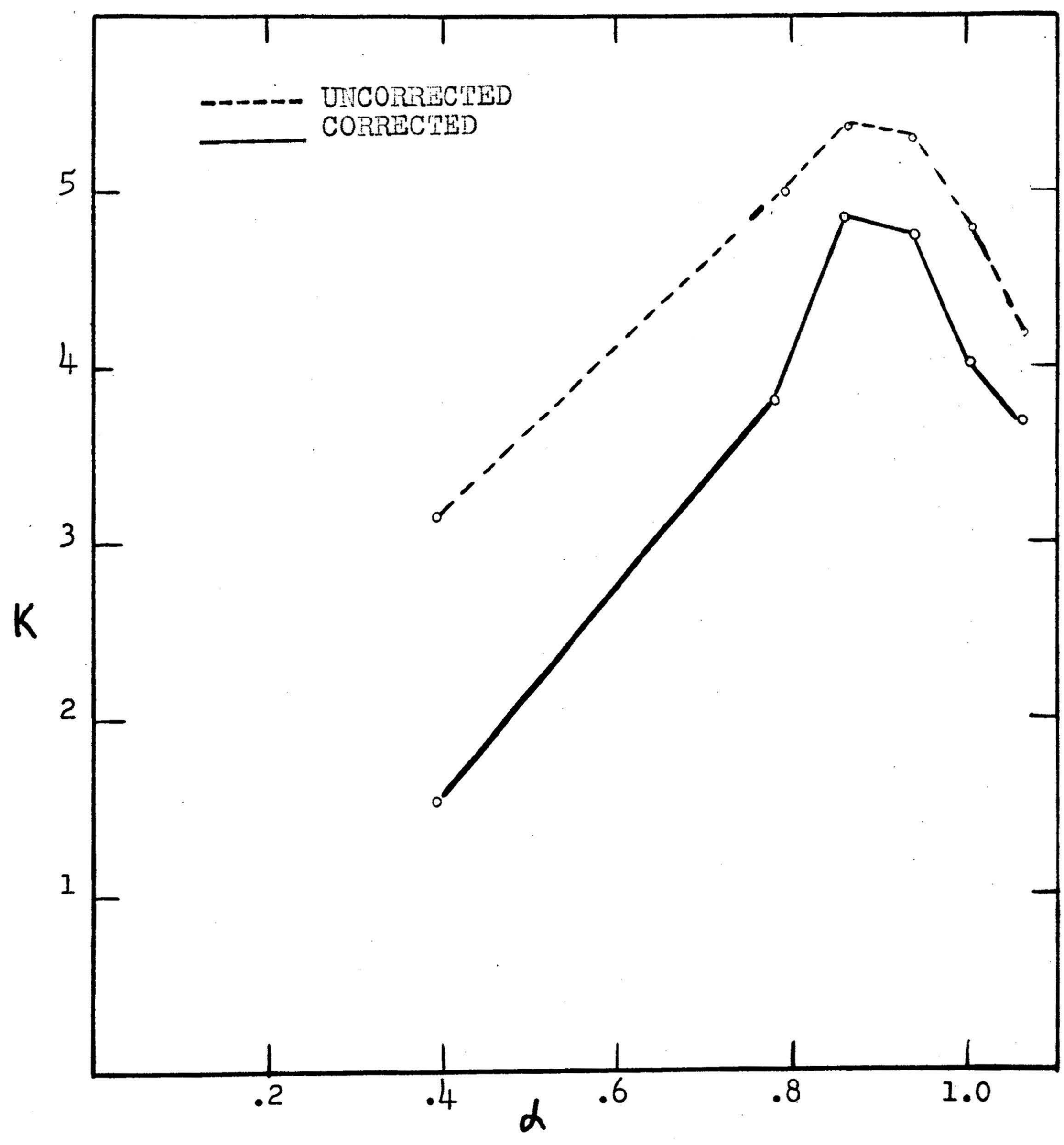


Figure 10. Scattering Area Coefficient as a Function of

α

tolerance of the spheres is found to be only 0.00025. The variation in α is given by

$$\Delta\alpha_D = \frac{\pi}{\lambda}\,\delta_D\,, \tag{9}$$

where $\Delta\alpha_D$ is the variation in α, δ_D is the variation in diameter. Similarly,

$$\Delta\alpha_\lambda = \frac{\pi D}{\lambda^2}\,\delta_\lambda \tag{10}$$

is the variation due to the instability of the klystron. This variation is not computed here. However, it is believed to be small since the frequency drift for similar klystrons is of the order of a few megacycles/second.

The indeterminate error in the computed value of K is due to the error in reading the deflection and to the tolerance of the spheres. An indication of the consistency of the data is given by Figure 11. The uncertainty in the mean deflection as a result of erratic errors in the reading of the scale is found to be $\pm$ .03 centimeters (A.D.) for the seven observations used. For the 11/32 inch spheres, this uncertainty in the scale deflection and the 1/10,000 inch diameter tolerance yield

$$\Delta K_{d_1} = \pm .0066,$$
$$\Delta K_{D_0} = \pm .0040,$$
$$\Delta K_a = \pm .0028,$$
$$K_{total} = \sqrt{\overline{\Delta K_{d_1}}^2 + \overline{\Delta K_{D_0}}^2 + \overline{\Delta K_a}^2} = \pm .0082.$$

Thus, the uncertainty in K is seen to be quite small.

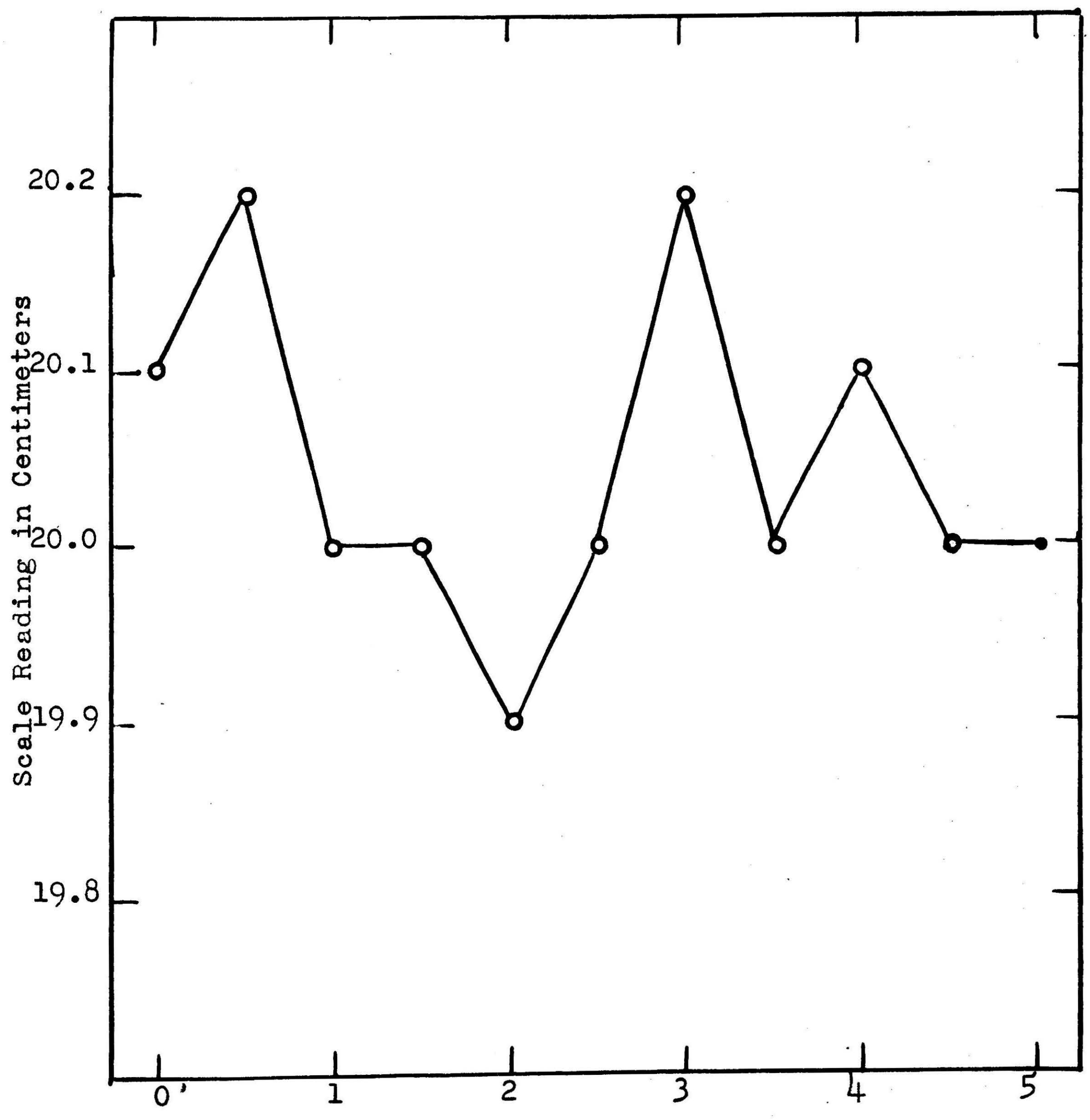


Figure 11. Stability Curve Giving the Scale Deflection as a Function of Time

There is, however, an appreciable determinate error in α due to the fact that the wavelength was not determined. Taking the maximum deviation of the generator to be 1,000 megacycles/second, sphere diameter as 11/32 inch, and λ as 3.2 cm, the variation in α is 0.095. This error in α is seen to be quite large in comparison to the indeterminate error. This error could have been eliminated entirely had equipment for measuring the wavelength been available.

TABLE I

Readings were taken with λ=3.2 cm, repeller voltage 180 volts, grid voltage 116 volts, grid current 19 milliamperes.

TABLE I-A

For 12 X 12 Sample

Sphere Diam.	d_o	A.C.F.	D_o	d_1	K Corr.	K Uncorr.	α
5/32 inch	50	7.2%	46.9	44.2			
			46.9	44.0			
			47.0	44.1			
			46.9	44.1			
			46.9	44.0			
			46.8	44.1			
			46.9	44.2			
	50	7.2%	46.9	44.1	1.53	3.16	0.395
5/16	50	5%	47.5	43.1			
			47.6	43.4			
			47.5	43.3			
			47.3	43.4			
			47.5	43.3			
			47.5	43.4			
			47.4	43.2			
	50	5.0%	47.47	43.3	3.80	5.0	0.780
11/32	50	12.2%	46.8	30.0			
			46.5	29.7			
			46.9	29.9			
			46.9	29.9			
			47.0	30.1			
			46.7	29.8			
			46.8	29.9			
	50	12.2%	46.8	29.9	4.85	5.37	0.86
3/8	50	9.96%	47.3	34.2			
			47.0	34.3			
			47.1	34.2			
			47.5	34.3			
			47.2	34.1			
			47.4	34.1			
			47.6	34.4			
	50	9.96%	47.3	34.33	4.75	5.30	0.935

Table I - continued

Sphere size	d_o	A.C.F.	D_o	d_1	K Corr.	K Uncorr.	α
13/32	50	9.4%	46.9	36.9			
			47.0	37.0			
			47.0	36.9			
			46.7	36.9			
			47.0	36.9			
			46.9	36.8			
			46.8	36.9			
	50	9.4%	46.9	36.9	4.15	4.80	1.005
7/16	50	10.0%	47.7	38.0			
			47.8	37.9			
			47.8	37.8			
			47.9	37.9			
			47.5	37.9			
			48.0	38.0			
			47.9	38.0			
	50	10.0%	47.80	37.93	3.7	4.20	1.060

TABLE I-B

5 X 5 Inch Sample

Sphere size	d_o	A.C.F.	D_o	d_1	K Corr.	K Uncorr.	α
11/32	50	12.2%		30.1			
				29.9			
				29.7			
				29.8			
	50	12.2%		29.8		5.40	0.860
11/32	50	6.1%		41.1			
				41.0			
				40.9			
				41.0			
	50	6.1%		41.0		5.41	0.860

SUMMARY

The system used in this experiment consists of a nominal 3.2 cm. microwave generator, microwave lens, scattering sample, and receiver.

Transmission measurements were made on samples consisting of metal spheres randomly distributed in a plane and irradiated with the plane electromagnetic radiation of nominal 3.2 centimeter wavelength. The spheres were chrome-plated ball bearings in the diameter range 5/32 to 14/32 inches and were supported by Plexiglas.

From these transmission measurements, scattering area coefficients were computed as a function of the parameter $\alpha = 2\pi a/\lambda$. These values of the scattering area coefficients were plotted as a function of α.

The scattering area coefficient was found experimentally to be independent of the area coverage factor, as long as the average sphere separation is greater than $\lambda/4$.

Comparisons were made with theoretical treatment for integrated scattering by Stratton. The results are in satisfactory agreement with the theory.

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