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Parameters in ablation heat sheilds

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PARAMETERS IN ABLATION HEAT SHIELDS

BY

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A

THESIS

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Aaron J. Miller (Advisor) ____________

S J Pagano ____________

I. ABSTRACT

The desired result of this paper is a method of determining temperature distribution and burn off rate in a semi-infinite rod or shield subjected to a large heat flux. The possible methods of determination were investigated, and it was felt that an analytical approach turned out to be the most feasible.

The analytical analysis employed involves the use of the Fourier's general equation for one dimensional heat flow. Transformations from stationary to moving coordinates reduces the problem from unsteady state to steady state boundary conditions. The resulting equations can be applicable to the extent for the heat flow problems involving the change of phase.

II. PREFACE

The stated purpose of this thesis is to investigate the temperature distribution in a semi-infinite rod with one dimensional heat flow and an application of large quantities of heat applied to the end of the rod. In such an environment the end of the rod may be ablated, melted into the liquid phase, forming a liquid film through which the heat must flow. It might also be melted and evaporated or melted and/or evaporated or sublimed through a porous residue from the end. A situation such as this is of engineering interest as a one dimensional heat shield with applications to fire walls, nose cones, etc. In making investigations of this type the problem could be approached or solved by one of four general methods*(1): analytical, graphical, experimental, and numerical. Although each of these methods has its particular applications and advantages over the others, the problem under study is to be treated analytically. The reader who is familiar with the mechanics of heat transfer will recognize that no claim to originality can be made on the general method. It is desired to find the most convenient and best way for the solution of a problem such as this and then to formulate a general procedure to follow in the solution.

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*All typed numbers refer to bibliography.

It should be pointed out that justification of the analytical analysis is not the stated purpose.

The author would like to express his sincere appreciation to Dr. Aaron J. Miles for his suggestions and guidance on this problem, and he wishes also to thank Professor Gordon L. Scofield for his interest and assistance.

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LIST OF SYMBOLS

Q........heat added, Btu/hr-ft^2.

q_r.......heat radiated, Btu/hr-ft^2.

q'''.....internal heat quantity consumed as in latent heat or heat of ablation, Btu/lb$_m$.

q........heat conducted into rod, Btu/hr-ft^2.

k........thermal conductivity, Btu/hr-ft-°F.

ρ........density, lb$_m$/ft^3.

c_p.......specific heat, Btu/lb$_m$-°F.

t........temperature, °F.

t_∞......temperature at $x = \infty$, °F.

x........coordinate normal to ablative surface.

x'.......$x - v\theta$, which has its origin on receding ablative surface.

α....... $\frac{k}{\rho c_p}$ thermal diffusivity, ft^2/hr.

σ.......0.174×10^{-8} Stefan-Boltzmann's constant, Btu/hr-ft^2-°R^4.

ϵ.......emissivity.

t_a.......ablative temperature, °F or °R.

t_m.......melting temperature, °F.

θ........time, hr.

v........rate of burn off, ft/hr.

s........thickness of film, ft or in.

III. INTRODUCTION

The subject of ablative heat shielding is not new. There are numerous references in the literature. NASA's Mercury capsule represents a successful application in bringing back men, monkeys, and instruments from space. However neither NASA nor the maker of the capsule, McDonnell Aircraft Corporation, has made public the details or the performance characteristics of this shield. There certainly must be numerous though less sophisticated applications in the general engineering field involving melting rate, ablation rate, etc.

It is the purpose of this study to determine the parameters associated with this phenomena, and their individual and collective effectiveness. The problem under investigation involves finding a method for determining the temperature distribution, rate of burn off and the effectiveness of material with various physical parameters as a heat shield. The heat is applied at a steady rate to a semi-infinite cylinder or rod which is subjected to a large heat flux. Some of the parameters to be considered are, thermal conductivity, heat of sublimation, ablation or melting, effect of a screening porous residue, etc. A semi-infinite cylinder or rod is one whose one end is bounded by one plane and the other end goes to infinity in the x-direction. Large heat flux means that the heat is transferred to the cylinder is so great that ordinary cooling methods are insufficient to prevent destruction of the surface of application, and may or may not be applied at a steady rate.

IV. REVIEW OF LITERATURE

The study of temperature variation in a semi-infinite cylinder as a function of time has been undertaken by many investigators during the past years. Much of this work is presented in the textbooks on heat transfer. The method of approach differs widely from one investigation to another, however the assumptions made and the results obtained are similar and compatible.

The analytical approach to this type of problem may be found in the more advanced texts, among those of note are the works of H. S. Carslaw (2), H. S. Carslaw and J. C. Jaeger (3) Ingersoll and Zobel (4) have also presented similar material.

A somewhat simplified form but a completely logical and valid methods of treating the problem may be found in the works by Schneider (5), Kern (6) and Jakob (7). The material presented by these authors also belongs to the analytical approach but is less rigorous with many of their results conveyed in the form of charts and graphs.

Dusinberre (8) presents material dealing with the numerical analysis of heat flow involving transient and steady flow. This material involves similarities to the subject of this investigation with finite and definite surface boundary conditions.

In all the literature investigated, the approach to the solution of the problem has been developed in an expression for temperature distribution in a semi-infinite cylinder from the standpoint of a given or assumed surface temperature argument. The author of this thesis could find no work employing the approach of an expression to determine the temperature history in a semi-infinite cylinder as a function of time and a large heat flux. Because of this, the following thesis was undertaken.

V. DISCUSSION

In determining the temperature at any point within a semi-infinite cylinder insulated laterally at any time as a function of the applied heat flux and parameters of the solid cylinder, a knowledge of the temperature behavior of a solid undergoing transient thermal conduction from this applied heat flux is required. It is assumed that the cylinder under discussion in this paper is homogeneous and that at any given time the applied heat flux is distributed equally over the bounding plane. Under these conditions the heat flow will be considered to be in one direction and normal to this bounding plane.

The heat flow in a solid takes place by conduction and is a function of time, temperature gradient, and the physical properties of the solid. A general conduction equation based on these parameters is then necessary in order to determine the effect of the applied heat flux.

Since conduction in this case takes place in one direction only, this must satisfy Fourier's conduction equation (9)

$$\frac{dQ}{d\theta} = -kA\frac{dt}{dx} \qquad (1)$$

dQ is the amount of heat flowing in differential time $d\theta$. A is the area normal to the flow through which this heat is being conducted, k is a proportionality factor called the thermal conductivity, and $\frac{dt}{dx}$ is the temperature gradient.

Temperature decreases with position in the direction of heat flow and therefore, if the heat flow $\frac{dQ}{d\theta}$ is taken as positive the temperature gradient must be negative as shown in Equation (1).

A more general equation is desired, however. Consider the differential volume in Figure 1. Its faces are parallel

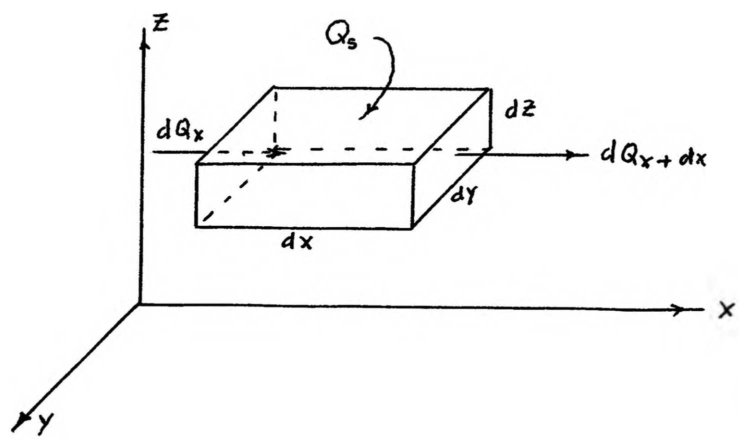

Figure 1. General heat conduction through a differential element

to the x, y, and z coordinate planes. If this differential parallelepiped is from a semi-infinite rod, the bounding face of which is parallel to the y-z plane, then the conduction in either the y- or z-direction is eliminated and hence conduction can take place only in the x-direction.

Let the amount of heat entering the parallelepiped in differential time $d\theta$ be dQ_x and the amount of heat leaving the parallelepiped be dQ_{x+dx}. In order for the total energy to be conserved, the heat entering the parallelepiped must be equal to the heat leaving it plus whatever energy Q_s is stored within it to raise its internal energy level.

Expressed mathematically:

$$dQ_x = dQ_{x+dx} + dQ_s \tag{2}$$

From equation (1)

$$dQ_x = -dy\,dz\,k\left(\frac{dt}{dx}\right)d\theta.$$

The heat leaving the parallelepiped will be

$$dQ_{x+dx} = -dy\,dz\,k\,\frac{\partial}{\partial x}\left(t + \frac{\partial t}{\partial x}dx\right)d\theta$$

or

$$dQ_{x+dx} = -dy\,dz\,k\,\frac{\partial t}{\partial x}\,d\theta - dy\,dz\,k\left(\frac{\partial^2 t}{\partial x^2}\right)dx\,d\theta.$$

The amount of heat stored within the parallelpiped in time $d\theta$ is known to be the product of the specific heat, volume, and density of the particular material and its temperature rise, or

$$dQ_s = \rho\,c_p\,dx\,dy\,dz\,\frac{\partial t}{\partial \theta}\,d\theta. \tag{3}$$

From equation (2)

$$-dy\,dz\,k\left(\frac{\partial t}{\partial x}\right)d\theta = -dy\,dz\,k\,\frac{\partial t}{\partial x}\,d\theta - dy\,dz\,k\left(\frac{\partial^2 t}{\partial x^2}\right)dx\,d\theta + \rho\,c_p\,dx\,dy\,dz\,\frac{\partial t}{\partial \theta}\,d\theta$$

which reduces to

$$\rho\,c_p\,\frac{\partial t}{\partial \theta} = k\,\frac{\partial^2 t}{\partial x^2} \tag{4}$$

For a material with physical properties unaffected by temperature, k, c_p and ρ are constant. When grouped as $\frac{k}{\rho c_p}$ they form a new constant α called the thermal diffusivity. Equation (4) can then be written

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha}\,\frac{\partial t}{\partial \theta} \tag{5}$$

This is Fourier's general conduction heat transfer equation for one-dimensional flow (10), and all one-dimensional conduction problems must satisfy this equation regardless of initial and boundary conditions.

VI. STATEMENT OF THE PROBLEM

Consider a semi-infinite rod of ablative material. That is a rod with one end only and extending to infinity in the x - direction. Take the origin of the x-coordinate at one end and let the temperature at $x = \infty$ be a convenient constant t_∞ . Heat is applied to the end of the rod in any chosen manner such as by radiation, convection, by a steady blast of plasma, or a combination of these. If the surface of the rod is insulated, the applied heat can be radiated from the end of the rod, consumed in ablation, conducted into the rod or transferred by a combination of these.

It is beyond the scope of this thesis to deal with the factors that determine the rate of heat application to the end of the rod or to an ablation surface. This is the problem discussed in most publications. It is the performance of the rod under the application of a known quantity of heat per unit time and per unit area that is of interest here.

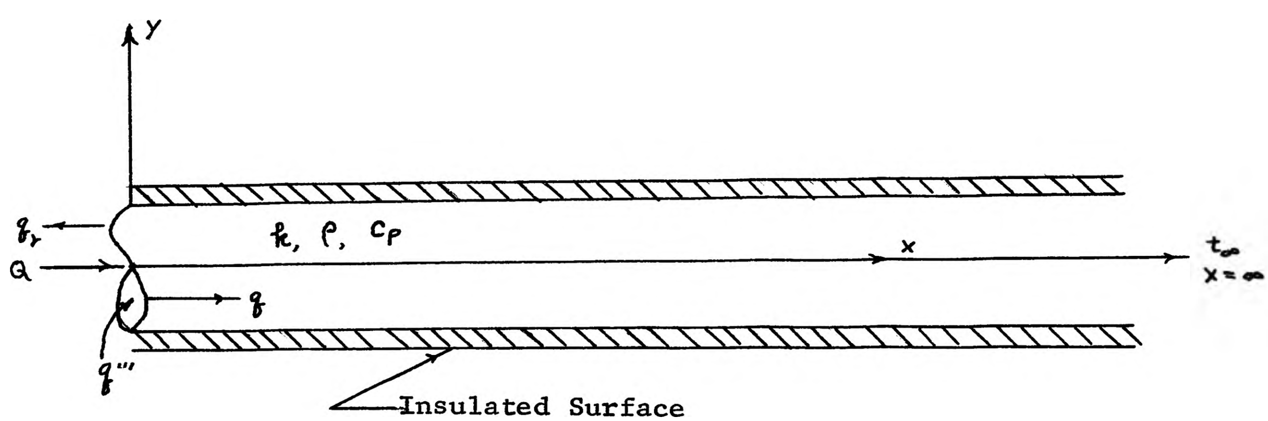

Figure 2. Ablative Rod or Shield

The applied heat Q in Btu/hr-ft^2 flows through the face of the rod at x = 0. The melting and burn off begins when the face x = 0 reaches the ablative temperature t_a. The ablative liquid is removed immediately on formation except for a liquid film with a constant thickness s which is always attached to the face moving with the same rate as the burn off velocity v.

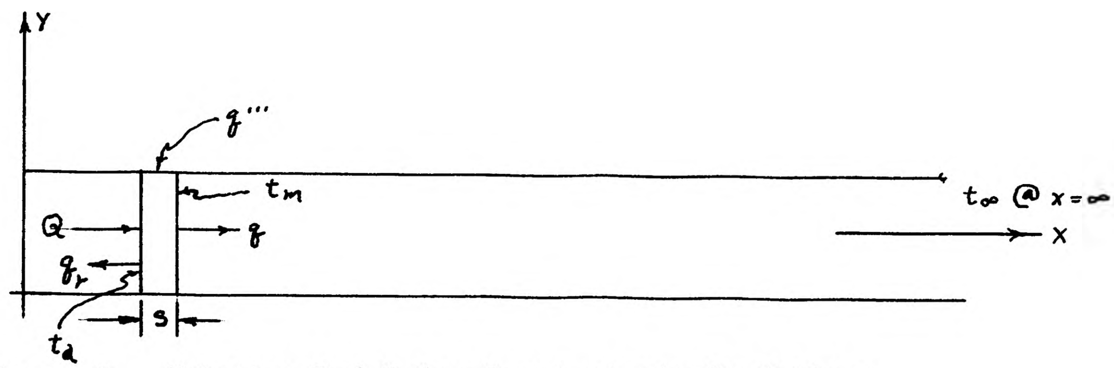

Figure 3. Ablative Rod Under Quasi-steady Conditions

The heat balance after application of heat is

$$Q = q_r + q'''\rho v + q \,, \qquad (6)$$

where q_r represents heat radiated in, q''' is the heat of ablation and q is the heat conducted into the rod.

Since heat is added continuously to the end of the rod along the x - direction and since the liquid film moves at the same rate as the burn off takes place, hence, this phenomena can be considered as a moving heat source (12). That means that the heat source is on the x' -axis of a rectangular coordinate system which is moving with a velocity v with respect to a stationary one. v is directed parallel to x. With this scheme a stationary observer on the x-axis would notice a change in temperature of his surroundings as the source moved along, while if the observer were stationed at a point on the moving x' -axis he would notice no such change in temperature. This condition of "apparent" steady-state temperature has been verified experimentally, and has come to be known as the quasi-steady state. This state is represented mathematically by $\frac{\partial t}{\partial \theta} = 0$ in the moving coordinate system.

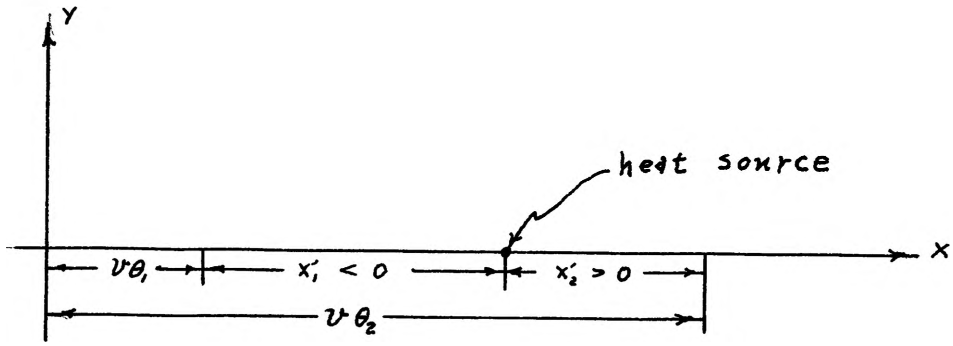

Figure 4. Dual Coordinate System For Moving Heat Sources $(x' = x - v\theta)$

Now consider the suggested transformation from stationary to moving coordinates. In the stationary system the temperature must satisfy the equation (5) $\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha}\frac{\partial t}{\partial \theta}$.
Two new variables are defined (only one being chosen arbitrarily),

$$x' = x - v\theta \quad \text{and} \quad \theta' = \theta .$$

Now $\frac{\partial x'}{\partial x} = 1, \quad \frac{\partial x'}{\partial \theta} = -v, \quad \frac{\partial \theta'}{\partial x} = 0, \quad \text{and} \quad \frac{\partial \theta'}{\partial \theta} = 1 .$

Therefore $\frac{\partial t}{\partial x} = \frac{\partial t}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial t}{\partial \theta'}\frac{\partial \theta'}{\partial x} = \frac{\partial t}{\partial x'} \; ; \quad \frac{\partial^2 t}{\partial x^2} = \frac{\partial^2 t}{\partial x'^2} ,$

and $\frac{\partial t}{\partial \theta} = \frac{\partial t}{\partial x'}\frac{\partial x'}{\partial \theta} + \frac{\partial t}{\partial \theta'}\frac{\partial \theta'}{\partial \theta} = -v\frac{\partial t}{\partial x'} + \frac{\partial t}{\partial \theta'} .$

Substituting these partial derivatives in equation (5)

$$\frac{\partial^2 t}{\partial x'^2} = \frac{1}{\alpha} \left(- v \frac{\partial t}{\partial x'} + \frac{\partial t}{\partial \theta'} \right).$$

Since this is in the moving coordinate system,

$\frac{\partial t}{\partial \theta'} = 0$, and

$$\frac{\partial^2 t}{\partial x'^2} = - \frac{v}{\alpha} \frac{\partial t}{\partial x'} \qquad (7)$$

or $$\frac{d^2 t}{d x'^2} = - \frac{v}{\alpha} \frac{dt}{dx'}$$

The general solution

$$t = C_1 e^{-\left(\frac{v}{\alpha}\right) x'} + C_2 \qquad (8)$$

must, in this case, satisfy the following boundary conditions stated in terms of transformation variable x':

$$\frac{dt}{dx'} = 0 . \qquad \text{as } x' \to \infty , \; t \to t_\infty \quad (a)$$

$$Q = q_r + q'''\rho v - k \frac{dt}{dx'} \qquad \text{as } x' \to 0 \quad (b)$$

Then $$\frac{dt}{dx'} = - C_1 \frac{v}{\alpha} e^{-\left(\frac{v}{\alpha}\right) x'} .$$

To satisfy the first boundary condition

$$t_\infty = C_1 (0) + C_2$$

hence $C_2 = t_\infty$.

For the second boundary condition

$$Q = q_r + q'''\rho v - k \left[- C_1 \frac{v}{\alpha} e^{-\left(\frac{v}{\alpha}\right) x'} \right]_{x'=0} ,$$

whereby

$$C_1 = (Q - q_r - q'''\rho v) \frac{\alpha}{k v} .$$

Substituting C_1 and C_2 back into equation (8)

$$t = [Q - q_r - q'''\rho v] \frac{\alpha}{k v} e^{-(\frac{v}{\alpha})x'} + t_\infty, \qquad (9)$$

In equation (9) the applied heat Q is given as stated in the problem but heat radiated q_r and the velocity of burn off v need to be determined.

First consider q_r, then according to Stefan-Boltzmann's law of total radiation (13) the total energy emitted by a black body is proportional to the fourth power of the absolute temperature of the body and its formula is:

$$q_r = \epsilon \sigma t_a^4. \qquad (10)$$

ϵ is the emissivity of the material, σ is the Stefan-Boltzmann's constant and t_a is the ablative temperature (absolute). The values of ϵ and t_a can be found in most material handbooks.

Since q''' and ρ may be found from the International Critical Tables the only unknown left to be determined is v the velocity of burn off. In reality q''' represents the following: The heat required to change the phase of the material, the heat of fusion, evaporation, sublimation, or oxidation, or perhaps a combination of these. The heat flow rate in the rod can also be represented by the following heat conduction equation: (14)

$$q = -k \frac{dt}{dx'} = v \rho c_p (t_m - t_\infty) \qquad (11)$$

t_m is the melting temperature of the material which can also be found from the International Critical Tables or other sources. In order to obtain $\frac{dt}{dx'}$, differentiate equation (9) with respect to x'

$$\frac{dt}{dx'} = [Q - q_r - q'''\rho v]\frac{\alpha}{kv}\left(-\frac{v}{\alpha}\right)e^{-\left(\frac{v}{\alpha}\right)x'}$$

$$\therefore \quad -k\left.\frac{dt}{dx'}\right]_{x'=0} = Q - q_r - q'''\rho v$$

From equation (11)

$$v = \frac{-k\frac{dt}{dx'}}{\rho c_p (t_m - t_\infty)}$$

$$= \frac{Q - q_r - q'''\rho v}{\rho c_p (t_m - t_\infty)}$$

After transposing and rearranging, this yields

$$v = \frac{Q - q_r}{\rho[c_p (t_m - t_\infty) + q''']} . \qquad (12)$$

Finally with $x' = x - v\theta$ equation (9) becomes

$$t = [Q - q_r - q'''\rho v]\frac{\alpha}{kv}e^{-\left(\frac{v}{\alpha}\right)(x - v\theta)} + t_\infty , \qquad (13)$$

where q_r and v are represented by equations (10) and (12) respectively.

VII. CONCLUSIONS

The temperature distribution and the burn off rate in a semi-infinite cylinder or shield subjected to a large heat flux can be determined by equations (13) and (12) respectively.

The method derived in this thesis is concerned with the expression for heat flux and various physical parameters and is not dependent upon a knowledge of the surface or given temperature behavior as is most of the published material on the subject. The use of coordinate transformations changes the problem to be treated from unsteady state to steady (quasi-steady) state boundary conditions and reduces the non-linear differential equation which was not shown in this thesis to a linear differential equation. This, therefore, gave a much simpler approach to the solutions to the problem.

It is felt that this method of approach could be applied to the solution of other problems involving the change of phase, and that a family of solution equations could be derived to include the most commonly encountered heat flow expressions.

From equations (12) and (13), it is noted that both temperature distribution and burn off rate depend on the heat flux applied and the time, since the rest of the physical parameters are constant.

Equation (13) gives an exact solution to the problem in question, since it involves finding the temperature at any point desired or at any chosen time along the semi-infinite rod.

VIII. SUMMARY

In the preceding work the writer has made use of Fourier's general expression for one dimensional heat flow with transformation of coordinates and applied initial and boundary conditions to arrive at the solution to the problem of temperature distribution and burn off rate in semi-infinite rod as a function of large heat flux and physical parameters of the material.

The resulting equation is relatively simple in application, and it is the hope of the author that this method of approach can be extended to include those various boundary conditions as applied to similar types of problems of a more complex system.

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IX. VITA

The author was born in Foochow, Fookien Province, China on June 24, 1934. He was raised and brought up in the northern part of China, where he completed his elementary school and part of the middle school education.

In 1949 he went to Formosa with his parents. There he graduated from the middle school in June 1952. In September of the same year he entered the Taiwan Provincial Junior College of Commerce and Law where he majored in Public Finance Administration.

In the fall of 1956 he came to the United States and enrolled in Coe College, Cedar Rapids, Iowa and graduated with a B. A. degree in Business Administration the following June. In September of 1957 he enrolled in pre-engineering at the University of Omaha, Omaha, Nebraska and a year later transferred to the Missouri School of Mines and Metallurgy. He received a B. S. degree in Mechanical Engineering in July 1961.

In September of 1961 he registered as an unclassified graduate student and has worked toward an M. S. degree in Mechanical Engineering.