Optimalization by peak-holding

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OPTIMALIZATION BY PEAK-HOLDING

BY

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A

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ABSTRACT

The problem presented is a study of the peak-holding method of optimalization. This method applies to a system that has an extremum.

After describing how the complete system, consisting of the controller and the controlled system, works; variations in the controller are presented. These variations are changes in the final stage of the controller, the servo. A phase-plane analysis shows how the output of the controlled system varies with its rate of change. Limitations are presented which tell why the system cannot be kept at its peak value at all times. Finally an analog simulation is used to study the system and verify the theoretical results.

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CHAPTER I

INTRODUCTION

The peak-holding method of control applies to a system that has an extremum (a maximum or a minimum). In this paper an analysis was made on a system with a positive peak (a maximum); a similar analysis applies to a system with a negative peak (a minimum). The peak-holding method keeps a system with an extremum operating near its peak value.

A peak-holding device remembers the peak value. The peak-holding method works by comparing the present value of the output to the peak value. When the difference between the peak value and the present value has reached an allowed deviation a controller changes the sign of the rate of change of the output of the system. In this manner the system is kept in the vicinity of its peak.

If the controller could make the allowed deviation zero then the system would have its peak value at all times. An investigation was made on the limits of this deviation due to noise and switching. It was shown that the allowed deviation cannot go to zero.

The final stage of the controller is a servo. The purpose of this servo is to transmit the signal from the controller to the controlled system. A problem arises in the selection of a servo. What type of servo will work? If the servo can have a relatively simple transfer function then it should be used. Four different transfer functions, chosen for their simplicity, were considered and it was determined which of these was satisfactory.

A phase portrait, which is useful in visualizing what happens to the system, was constructed by means of isoclines. The jump in the rate of change of the output of the system is distinctly shown in the phase portrait. The phase portrait also shows that as the allowed deviation from the peak state becomes smaller the number of switchings required becomes greater and is thus a limitation on the controller.

If the output of a system can be improved by the peak-holding method, then the work done in this thesis may be useful in designing a controller for this system.

CHAPTER II

REVIEW OF THE LITERATURE

According to Tsien and Serdengecti[15] optimalizing by peak-holding was developed by Li[6] in 1951. In his work Li described the peak-holding method and applied it to an internal combustion engine. In this application two variables controlled the system.

In 1952 Shull[11] applied the technique to an aircraft cruise control. He used the optimalizing control to obtain the optimum performance of an aircraft under extreme changes of environment (such as ice deposits on the wings). In 1957 Genthe[7] applied Shull's work to turbojet aircraft.

Tsien[14] published a book in 1954 containing a chapter on the peak-holding method of control. In 1955 Tsien and Serdengecti[15] published an article nearly identical to the chapter in Tsien's book. Both of these publications gave a complete description of the peak-holding method with main emphasis on the dynamic effects. These dynamic effects considered were the lag in response which occurs in an actual system.

In 1957 White[16] applied peak-holding to a chemical process. Steam flow was controlled to obtain a maximum percentage of product from a catalyst bed. In his paper White gives a description of a commercial model of the controller he developed.

Farber[6] and Cosgriff[4] approached the problem emphasizing the use of logic in the controller. If the input to the system and the output of the system are increasing no change is made; if the input is increasing and the output is decreasing, the input is reversed.

An analog computer circuit was presented by Maybach[9] in 1963. He gave a brief description of the peak-holding method and introduced the peak-holding computer circuit. According to Maybach the peak-holding circuit was developed by G.A. Korn (University of Arizona).

The work done in this thesis is not a special application of peak-holding, but an extension of some of the work done on the general method. A more detailed investigation of noise is made. No previous work had considered the different servos presented. The examination in the phase plane is new. An analog circuit similar to Maybach's was used to simulate the system.

CHAPTER III

DESCRIPTION OF SYSTEM

A. The Controlled System

As an aid in reading this thesis a few terms will now be defined. The controlled system is the system being controlled; its characteristics are given in Figure 1. The complete system is the controlled system and the controller and will be referred to as the system.

The total input to the controlled system is denoted by X(t) and the total output by Y(t). When the input to the controlled system is X_o the peak value of Y_o occurs. Of primary interest in the controlled system are variations about the peak value; for this reason a new set of co-ordinates is chosen with X_o and Y_o as a reference. Variations about this point are given in terms of x(t) and y(t) as shown in Figure 1. This can be expressed mathematically as

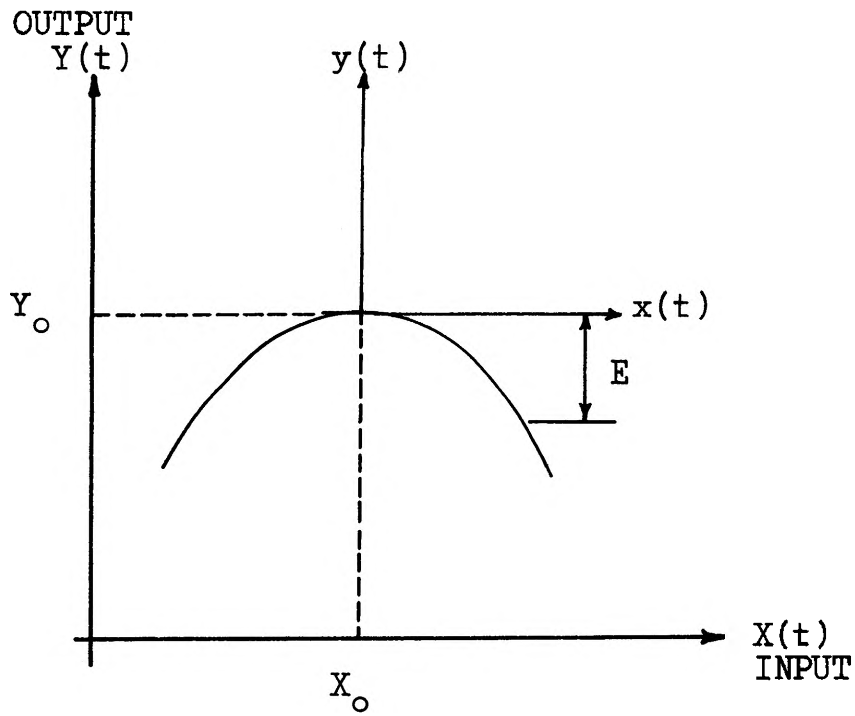
$$x(t) = X(t) - X_o \tag{1}$$

and

$$y(t) = Y(t) - Y_o \tag{2}$$

The exact relationship between Y(t) and X(t) of a controlled system may or may not be known; Maybach[9] and Tsien[14] represented it by letting

$$Y(t) = Y_o - K(X_o - X)^2 \tag{3}$$



THE CONTROLLED SYSTEM

FIGURE 1

This relationship can be expressed as

$$y = -Kx^2 \tag{4}$$

if equations (1) and (2) are subsituted into (3). This equation is an approximation using the first three terms of Taylor's series expanded about the point $x = 0$. The coefficients of the first two terms in this case are zero. The value of K is determined by the characteristics of the controlled system.

E shown in Figure 1 is the maximum allowed deviation from the peak value. Maximum allowed deviation occurs when $y = -E$.

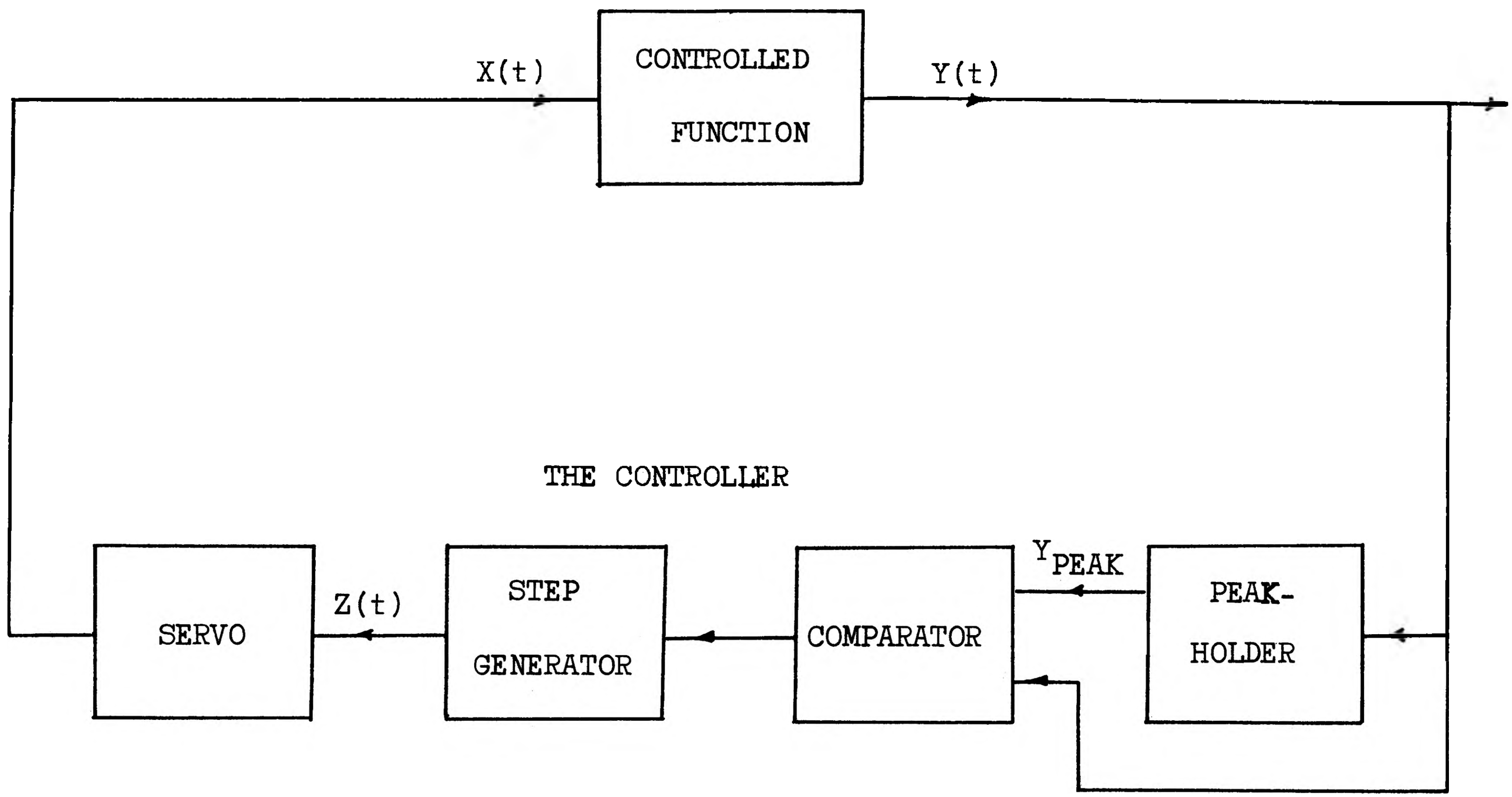
B. The Controller

Bibbero defines a controller as:

> A mechanism which measures the value of a variable quantity of condition and operates to correct or to limit deviation of this measured quantity from a selected reference.

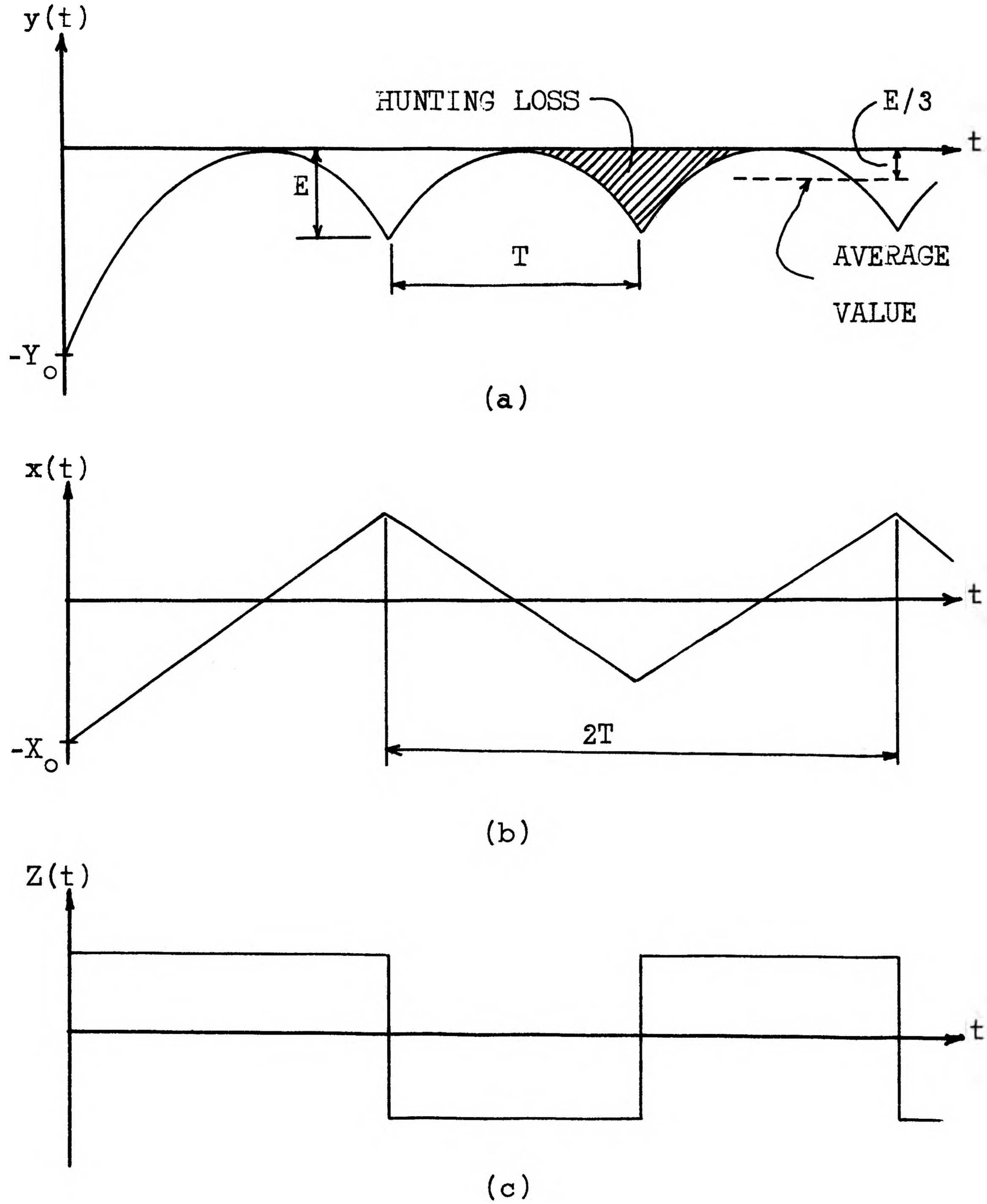
The reference that will be considered in the system is Y_o which is the same as $y = 0$, and the limit of deviation is E. In a design problem E will have a fixed chosen value. Stated in another way, the purpose of the controller is to find the optimum point and keep the controlled system in the neighborhood of this point. The neighborhood in this case has bounds of Y_o and E. A complete block diagram of the system is given in Figure 2. Figure 3 shows the various signals when the servo is an integrator.

To understand how the system works, consider the system



BLOCK DIAGRAM OF SYSTEM

FIGURE 2



SIGNALS IN A TYPICAL SYSTEM

FIGURE 3

when it is at rest; x is equal to $-X_o$. For this analysis x will vary linearly with time. As x increases with time y also increases according to equation (4). When x equals zero the maximum value of y occurs; if x increases further y decreases. In this analysis x continues to increase; therefore y decreases and approaches the maximum allowed deviation, -E, as shown in Figure 3a. When y = -E it is mandatory that switching occur to keep y within its limits. At this time switching does take place causing x to decrease which causes y to increase, reach a peak again, and decrease. As soon as y reaches -E again switching occurs and x increases causing y to increase. This process continues and y becomes a periodic function as shown in Figure 3a and x becomes periodic as shown in Figure 3b.

The switching described in the preceding paragraph takes place in the controller of the system. The controller consists of a peak-holder, a comparator, a step generator, and a servo. Figure 2 shows the relationship of each of these in the controller; they will now be described in the following paragraphs.

The peak-holder is a memory device which records the maximum value of Y. The peak-holder continuously decides whether the present value is larger or smaller than the preceding value. If it is larger the peak-holder continues making these decisions; if it is smaller the peak-holder holds the larger value. The peak-holder does as its name implies; it decides when a peak is reached and holds it.

The output of the peak-holder and Y are continuously fed into the comparator. When their difference is E the comparator sends a signal to the step generator.

When the step generator receives this signal it changes states. This change in states is a change in the sign of its output, Z. If the system is operating properly the output of the step generator will be as shown in Figure 3c.

The output of the step generator is the input to the servo. The servo must transmit this change in Z to the controlled system in a way that will keep $y > -E$. An integrator was used for the response shown in Figure 3. Different servos will be discussed in the next chapter.

In some controlled systems it is desirable to keep E as small as possible, but there are limitations on E which will be discussed in Chapter VI. If E takes on a fixed value the average value of Y will be greater than $Y_o - E$. When the servo is an integrator

$$Y_{average} = Y_o - E/3$$

The quantity E/3 is called the hunting loss. To keep the hunting loss small, E must be small, but E can be made only as small as the limitations of noise and switching will allow. Therefore the hunting loss also has a limit. The hunting loss and average value are shown in Figure 3a.

CHAPTER IV

THE SERVO TRANSFER FUNCTION

The servo, which is fed by the step generator, is the final stage of the controller. When the step generator changes states the servo's output, x, must correct the output of the controlled system.

A general transfer function, G(s) could be written for the system and an analysis made; however the purpose of the servo is to control the controlled system in the simplest possible manner. Chosen for their simplicity, the following transfer functions will be considered.

1. Gain, $G_1(s) = K_1$
2. Integration, $G_2(s) = K_2/s$
3. A simple pole, $G_3(s) = K_3/(s + 1/T_3)$
4. Double Integration, $G_4(s) = K_4/s^2$

The K's in the above transfer functions are the static loop sensitivities.

When the servo transfer function is simply a fixed gain, x will have a discrete value of K_1Z (Z is always a + or - constant value). According to equation (4) the output of the controlled system will also have a discrete value which depends on the value of x. Because the output of the controlled system has only a discrete value the gain transfer function, $G_1(s)$, is unsuitable.

The next transfer function to be considered is K_2/s. Because the output of the controlled system is periodic it makes little difference when the analysis begins. This analysis will begin when $y = -E$. At this time switching occurs and the step generator changes states. The output of the servo, x, immediately changes directions which causes y to increase, reach a peak, and decrease until $y = -E$. The step generator again changes states causing x to change directions and y increases; the process continues in this manner. This case is illustrated in Figure 3.

The third case to be considered takes the form

$$G_3(s) = K_3/(s + 1/T_3)$$

Making the same assumption as in the second case switching occurs when $y = -E$. The input to the servo is a unit step and

$$x(s) = K_3/s(s + 1/T_3)$$

when expressed in Laplace transforms. Solving for x in the time domain

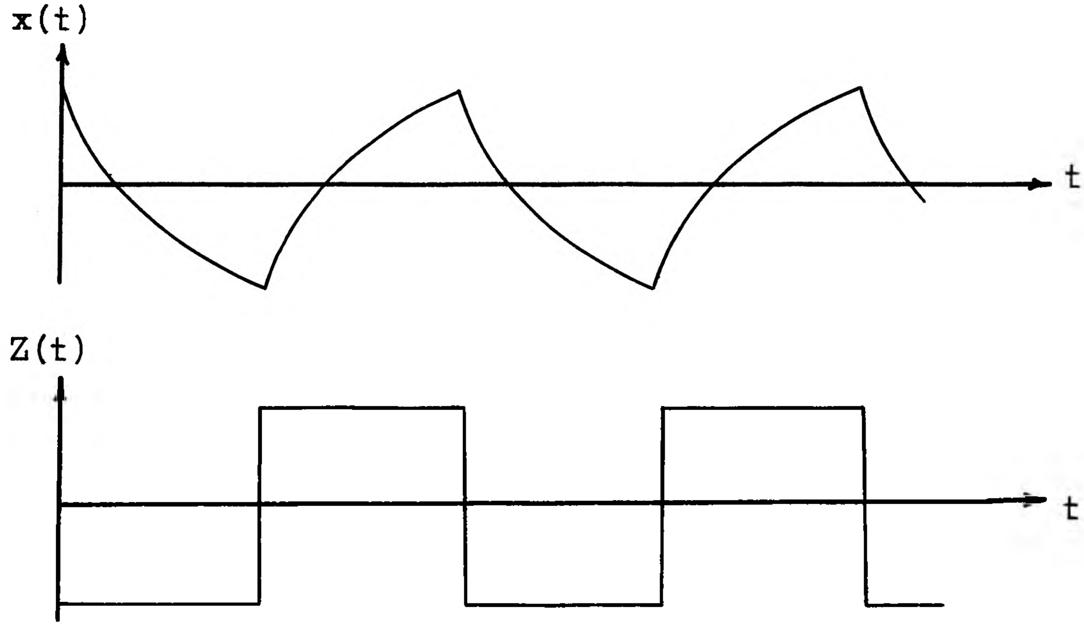
$$x(t) = K_3T_3(1 - e^{-t/T_3})$$

If switching occurs properly, the output of the step generator will be a periodic square wave and the output of the servo will take the form in Figure 4.

Using the series relation

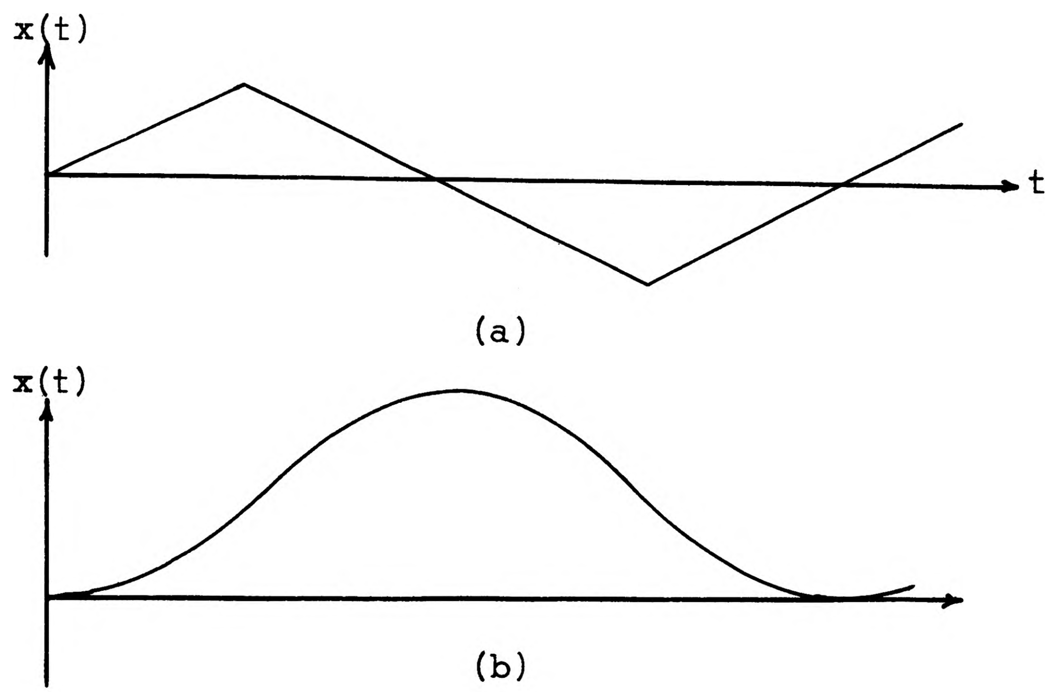
$$e^{-x} = 1 - x + x^2/2! - \ldots,$$

the transfer function, $G_3(s)$, takes the approximate form



RESPONSE FOR SERVO TRANSFER FUNCTION $G_3(s)$

FIGURE 4



RESPONSE FOR SERVO TRANSFER FUNCTION $G_4(s)$

FIGURE 5

$$e^{-x} = 1 - x$$

when x is sufficiently small ($|x|<.1$). In terms of x(t) and t

$$x = K_3T_3(1 - 1 + t/T_3)$$

$$x = K_3t$$

The transfer function has the same form as an integrator as long as $t/T_3<.1$.

One important difference should be noted in the integrator and $G_3(s)$. If a unit step is fed into an integrator the output has no bound, but $G_3(s)$ does. Applying the final value theorem

$$\lim_{s\to 0} sF(s) = \lim_{t\to\infty} f(t)$$

gives no bound on $G_2(s)$, but $G_3(s)$ has the limit K_3T_3.

As a comparison of $G_2(s)$ and $G_3(s)$ let E have a fixed value; the corresponding values of x can be found from the relationship $y = -Kx^2$ and $Y = -E$. Solving for x gives $x = \pm(E/K)^{.5}$. If $K_3T_3<|\pm(E/K)^{.5}|$ then y can never equal -E. This means switching cannot occur. Therefore $G_3(s)$ will not work in the system when $K_3T_3<|\pm(E/K)^{.5}|$. Since $G_2(s)$ has no limit on x then it will work for any value of E.

As a final consideration to the servo transfer function an examination of the double integrator will be made. Assume x is increasing and switching occurs when y = -E. If K_4/s^2 is split into K_4/s and $1/s$ the output, x, can be determined by considering each separately. The output of K_4/s

will be a triangular wave as in $G_2(s)$. This triangular wave is fed into the second integrator which will give a parabolic output. In this case the output of the first integrator will be $\dot{x}$ and the output of the second integrator will be x. If $\dot{x}$ takes the triangular shape in Figure 5a, then x will take the form in Figure 5b. When switching occurs x continues to increase making y become smaller which makes the double integrator unsuitable in the system.

CHAPTER V

PHASE-PLANE ANALYSIS

A study of the output of the controlled system in the phase plane is useful in seeing the relationship between Y and $\dot{Y}$. In the preceding chapter it was found that the servo transfer functions K_2/s and $K_3/(s + 1/T_3)$ were the only useful transfer functions of the four considered. When $t/T_3 < .1$ both of these functions can be represented by letting

$$x = At + C$$

When the system is starting from rest $C = -X_o$. When switching occurs, A and C in the equation for x will change. The linear relation of x to time will be the only one considered in the phase-plane analysis. Taking the first and second derivatives of x with respect to time gives

$$\dot{x} = A$$

and

$$\ddot{x} = 0$$

Consider the output-input relation for the controlled system to be

$$Y = Y_o - K(X_o - X)^2$$

or

$$Y = Y_o - Kx^2$$

Taking the derivative of Y with respect to time gives

$$\frac{dY}{dt} = \dot{Y}(t) = -2Kx\dot{x}$$

Applying the method of isoclines as given by Truxal[13], let

$$U(t) = \dot{Y}(t)$$

and

$$\dot{U}(t) = \ddot{Y}(t) = -2K(x\ddot{x} + \dot{x}\dot{x})$$

Y can now be expressed in terms of $\dot{Y}$ and δ, where δ is a constant $\frac{d\dot{Y}}{dY}$. Continuing with the phase-plane development

$$\frac{\dot{U}}{\dot{Y}} = \frac{d\dot{Y}}{dY} = \delta = \frac{-2K(x\ddot{x} + \dot{x}\dot{x})}{-2K(x\dot{x})}$$

Recall that $\dot{x} = A$ and $\ddot{x} = 0$.

$$\delta = \frac{\dot{x}}{x}, \quad x = \frac{\dot{x}}{\delta} = \frac{A}{\delta}$$

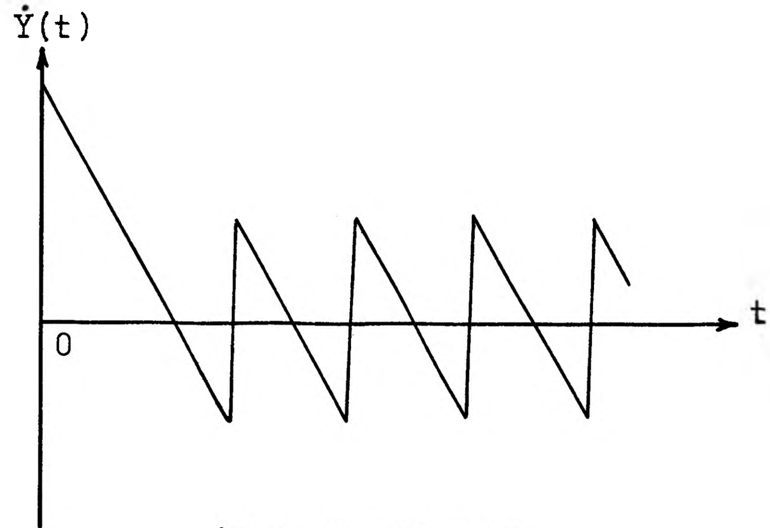
$$\dot{Y} = -2Kx\dot{x} = \frac{2KA^2}{\delta}$$

$$Y = Y_o - K\frac{A^2}{\delta^2} = Y_o + \frac{\dot{Y}}{2\delta}$$

A plot of $\dot{Y}$ versus time is given in Figure 6a. A phase portrait for the system, starting from rest, is given in Figure 6b. Thus

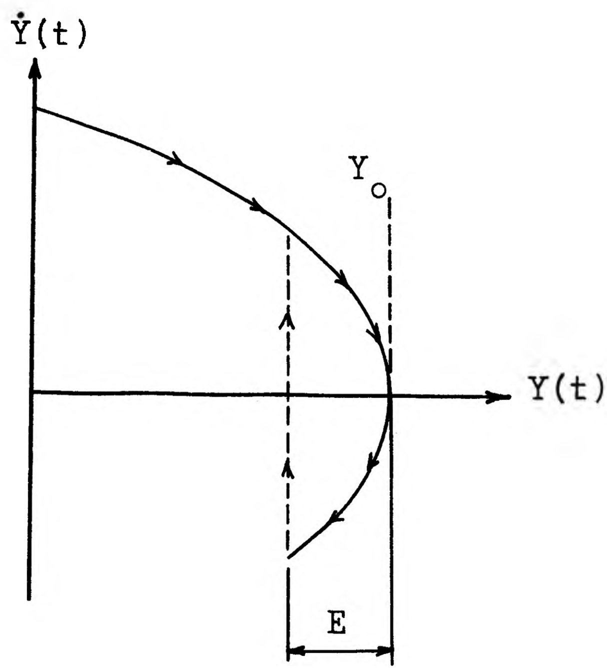
$$\dot{Y}(0^+) = -2K(-X_o)A = 2KX_oA$$

If A is constant and the maximum allowed deviation, E, takes on different values, a family of curves will occur and is shown in Figure 7a. In Figure 7a the time required to go from point 2 to point 3 is constant for both values of E. A certain amount of time is required to go from 1 to 2 and from 3 to 4 in the E_2 case. This illustrates that the larger the value of E the longer the period, T, of



Ẏ(t) VERSUS TIME

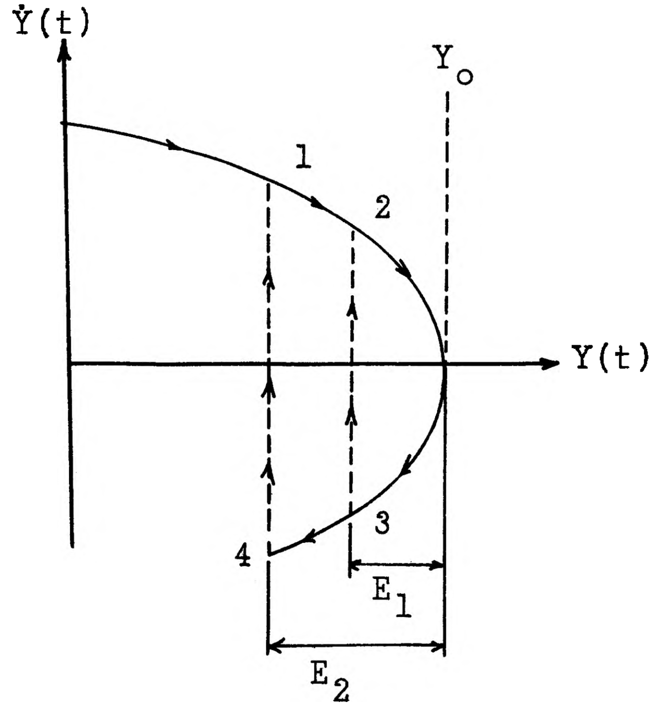
(a)



PHASE PORTRAIT DURING TYPICAL OPERATION

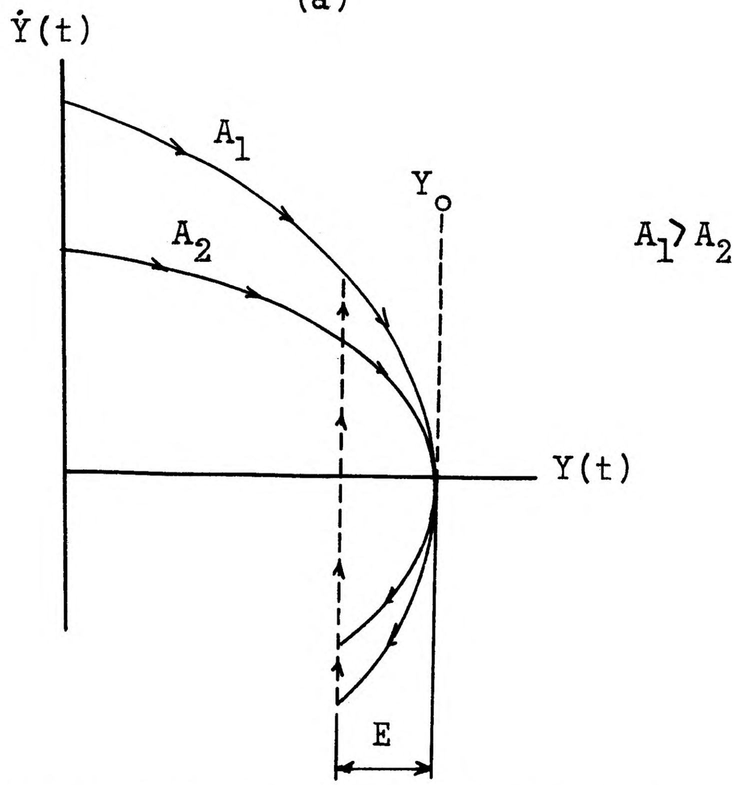
(b)

FIGURE 6



PHASE PLANE WITH DIFFERENT E'S

(a)



PHASE PLANE WITH DIFFERENT GAINS ON SERVO

(b)

FIGURE 7

Y will be. Another way of looking at this is that over a long period of time the system with the greater E will require a smaller number of switchings. This will be considered in the limitations on E in the next chapter.

Next consider a constant value of E and variations in A, which is the gain in the servo. For an illustration let A_1 and A_2 be two A's and $A_1 > A_2$. The starting points in the phase portrait will be

$$\dot{Y}_1(0^+) = 2KX_oA_1$$

and

$$\dot{Y}_2(0^+) = 2KX_oA_2$$

Figure 7b shows the phase portrait with these two different values of A. If E is constant as shown in the figure then the system with A_1 will have a shorter period than A_2. This is true since $|\dot{Y}|_{A = A_o} > |\dot{Y}|_{A = A_2}$ for all time.

By using different values for A and E, a multitude of different phase portraits can be obtained. One thing common to all of these is the jump from a negative $\dot{Y}$ to a positive $\dot{Y}$. If inertial effects of a system are considered[15] this jump cannot occur as shown in the phase portrait.

CHAPTER VI

STABILITY AND LIMITATIONS

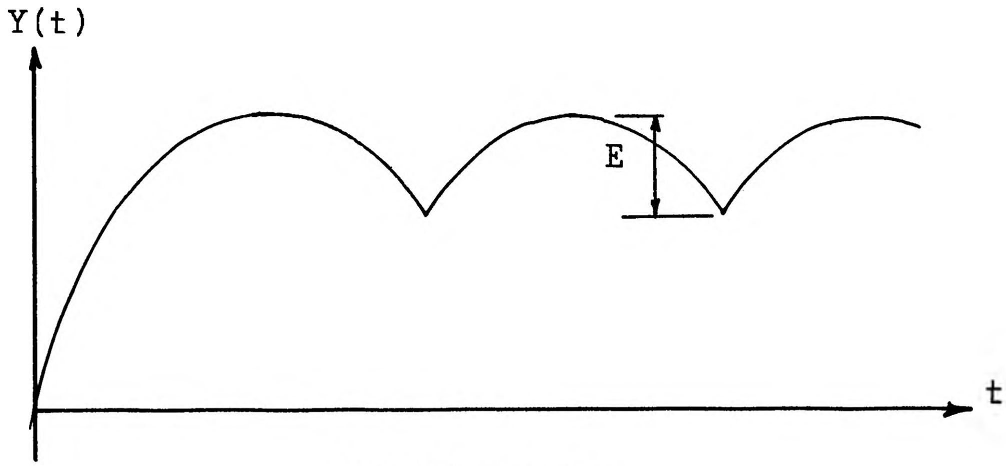
In any control system the stability of the system and limitations on operation should be considered. Instability can occur in this system when noise is introduced. Both noise and switching speed are limitations on E.

Noise can be represented in many different ways, but for the purpose of this stability analysis a sinusoid will be used to represent noise in the system. The noise will be expressed as $N\sin\omega_n t$ where N is the maximum noise level and ω_n is the frequency. The frequency of the noise is assumed to be much greater than the frequency of the controlled system output. $N\sin\omega_n t$ will be introduced to various parts of the system to see how it effects the stability. In the following paragraphs a subscript, n, on any of the symbols means the corresponding normal signal plus noise.

Assume noise is present in the controlled system output, Y. This is expressed as

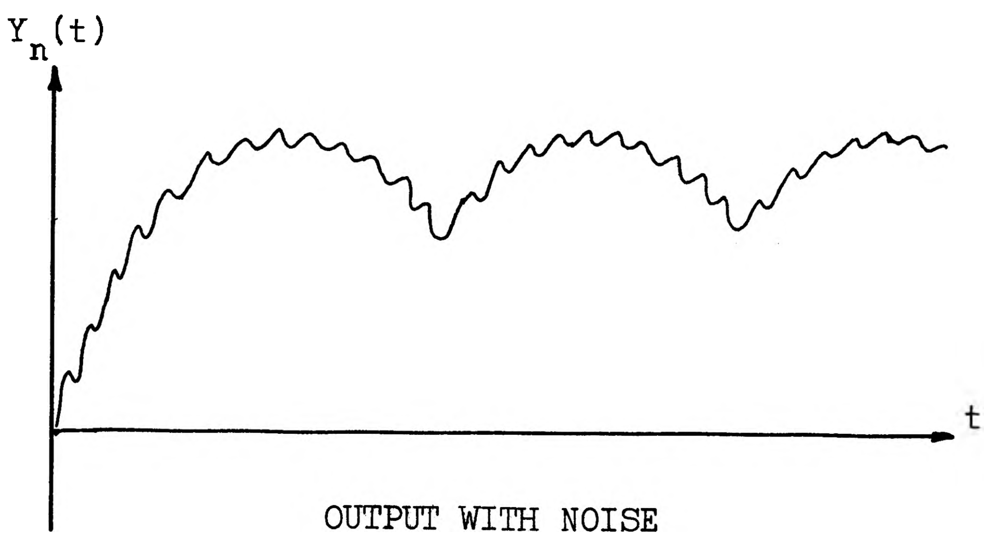
$$Y_n = Y + N\sin\omega_n t$$

A graphical interpretation of this is given in Figure 8b; Figure 8a is Y without noise. This noise in the output of the system will cause the peak-holder to hold a maximum false peak of $Y_o + N$. Also the value of Y_n will sometimes be smaller than Y by this same value of N. Considering $Y_o + N$



NORMAL OUTPUT

(a)



OUTPUT WITH NOISE

(b)

FIGURE 8

peak and Y - N as a new minimum, switching occurs when

$$(Y_o + N) - (Y - N) = E \tag{7}$$

Rearranging equation (7) gives

$$Y_o - Y = E - 2N = E'$$

E′ is the value of deviation at which switching appears to occur. If $E<2N$ the system will be unstable and switching will occur at the frequency of the noise. Even when $E>2N$ switching will occur at a greater than normal rate because of the false peak and false minimum.

Suppose that noise is present in the system at the servo output and is expressed as

$$x_n = x + N\sin\omega_n t$$

Since $y = -Kx^2$

$$y_n = -K(x + N\sin\omega_n t)^2$$
$$= -K(x^2 + 2xN\sin\omega_n t + N^2\sin\omega_n{}^2 t) \tag{8}$$

The second two terms of equation (8) are unwanted terms. Using the analysis of the preceding paragraph instability occurs when $E<(2N^2 + 2x_{max}N)$.

Noise in the output of the step generator will be sent through the servo. Since in the useful case, the servo is an integrator its output will contain a sinusoid of the same frequency but of different magnitude. The analysis is then the same as the case just analyzed.

If noise is introduced at the output of the comparator and is large enough then the step generator will start changing states. This will cause the actual output of the comparator to lose control of the multivibrator.

From the discussion in the preceding paragraphs it is obvious that noise is a limitation on the system. If there is no noise in the system what would be the limitations? As E becomes smaller the period of Y becomes smaller as described in the phase-plane analysis and shown in Figure 7. Switching occurs more frequently as E decreases so that switching time in the comparator and the rise time of the step generator become limitations as E decreases.

Summing up the results of this chapter, noise can cause the system to become unstable and is a limitation on E; if E is not limited by noise it will be limited by the time required for the comparator and step generator to change states.

CHAPTER VII

ANALOG SIMULATION AND RESULTS

A. The Analog Circuit

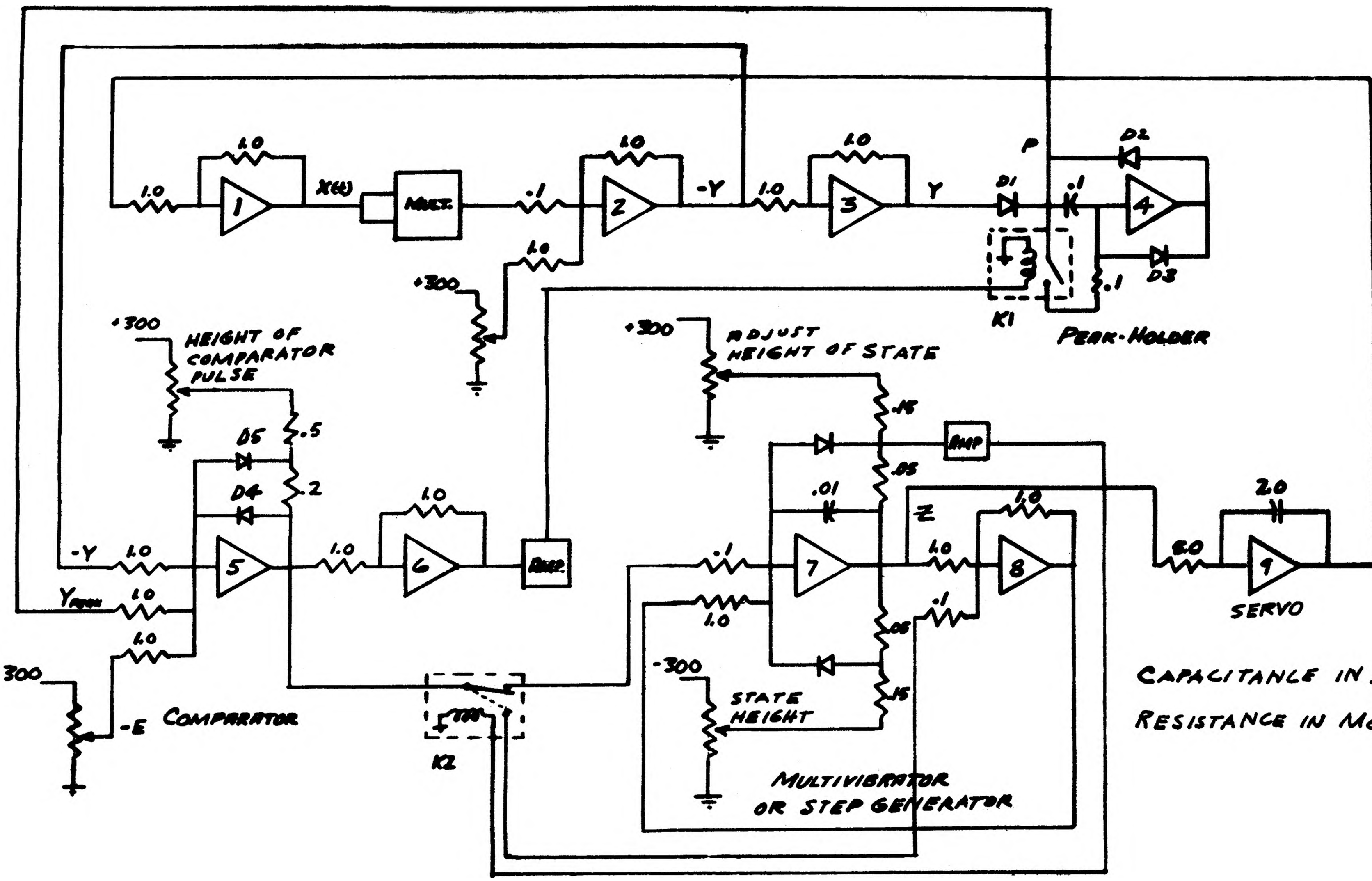
The analog computer provided a model for the system that was studied. The basic circuit used for the simulation is shown in Figure 9. The servo did not always have the transfer function shown in Figure 9.

Except for the peak-holder all of the individual circuits can be found in most books on analog computing. Before discussing the operation of some of these circuits a brief description of how the system works will again be given. This discussion will be given with the idea that the reader will follow the diagram of Figure 9.

This description will start at the output of the step generator or multivibrator which has the output Z. When the computer goes into operation the multivibrator will go either to the positive or negative state. Its output is sent through a servo which has an output of x, and starts at $x = 0$. If the output of the multivibrator is positive or negative the servo will have (with the sign changer considered) respectively a positive increasing or negative increasing output. Recalling that

$$Y = Y_o - Kx^2$$

it does not matter whether x is positive or negative, Y



CAPACITANCE IN µf

RESISTANCE IN Megohms

FIGURE 9 THE ANALOG COMPUTER CIRCUIT

will decrease. When Y has deviated from its peak value by E the comparator sends a signal to the multivibrator causing it to change states. This causes x to change directions and Y increases, reaches a peak, and decreases. When Y has again deviated by E from its peak the multivibrator again changes states and x changes directions. The process then continues in this manner.

1. The Multivibrator. The comparator used in this simulation has only a negative output. The multivibrator used goes to its positive state when it receives a negative signal and its negative state when it receives a positive signal. When the system is operating properly a series of negative pulses is sent to the multivibrator from the comparator. The multivibrator must change states each time one of these pulses is received. To accomplish this a relay, K2, was used; it is energized when a positive voltage is sent to the relay's amplifier. When the multivibrator is in the positive state the relay is energized, and when the multivibrator is in the negative state the relay is de-energized. If the relay is de-energized the negative pulse from the comparator goes to amplifier 7; if the relay is energized the negative pulse goes to amplifier 8. Assuming that the relay is de-energized a negative pulse to amplifier 7 would drive the multivibrator to the positive state. If the relay is energized the negative pulse goes to amplifier 8 which in turn

sends a positive pulse to amplifier 7. This positive pulse to the multivibrator causes it to go to the negative state. Thus the relay causes the multivibrator to change states everytime it receives a negative pulse.

The relay and amplifier used in this circuit were taken from the work done by Curtin[5]. The relay is a small high speed type and is driven by the amplifier shown in Figure 10.

2. <u>The Servo Transfer Function.</u> The output of the multivibrator goes to the servo which in turn sends a signal to the controlled system. All the servo transfer functions considered in Chapter IV except gain were used in the analog simulation. The first servo that was considered was the integrator which is shown in Figure 9.

The imperfect integrator,

$$G_3(s) = \frac{K_3}{s + 1/T_3}$$

was simulated by the circuit in Figure 11. Its transfer function is

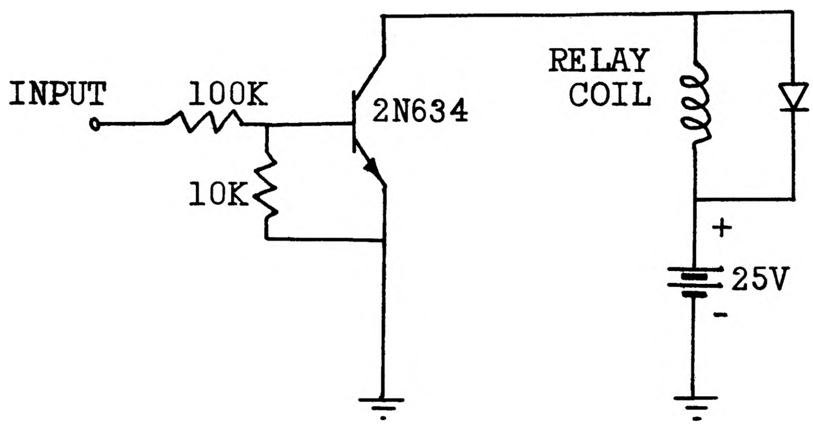
$$G_3(s) = \frac{1}{R_1C} \cdot \frac{1}{s + 1/R_2C}$$

As R_2 becomes larger this transfer function approaches the perfect integrator.

Finally the double integrator,

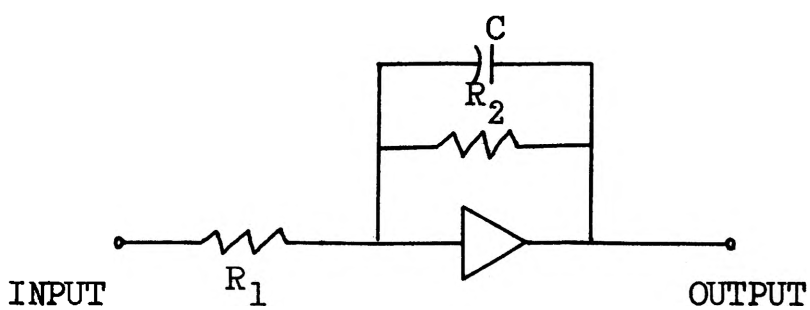
$$G_4(s) = K_4/s^2$$

was considered. To get this transfer function it was only necessary to add another integrator to the one that is



AUXILIARY AMPLIFIER AND RELAY COIL

FIGURE 10



TRANSFER FUNCTION $G_3(s)=\frac{K_3}{s+1/T_3}$

FIGURE 11

already in Figure 9.

3. <u>The Multiplier.</u> The output of the servo goes to the multiplier. The multiplier used operates as follows. If any two functions u and v are fed into the multiplier its output is -.01uv. In the simulation u and v were both x so that the output is $-.01x^2$. This value was sent through amplifier 2 with a gain of 10 and was added to Y_o. This made the output of amplifier 2 $-(Y_o - Kx^2)$, which is -Y. In this particular case K = .1.

4. <u>The Peak-Holder.</u> After -Y is sent through a sign changer it goes to the peak-holder. With ideal diodes the peak-holder characteristics are shown in Figure 12. The dashed line indicates the response when actual diodes are used. Note in the circuit that the peak value is recorded at point P.

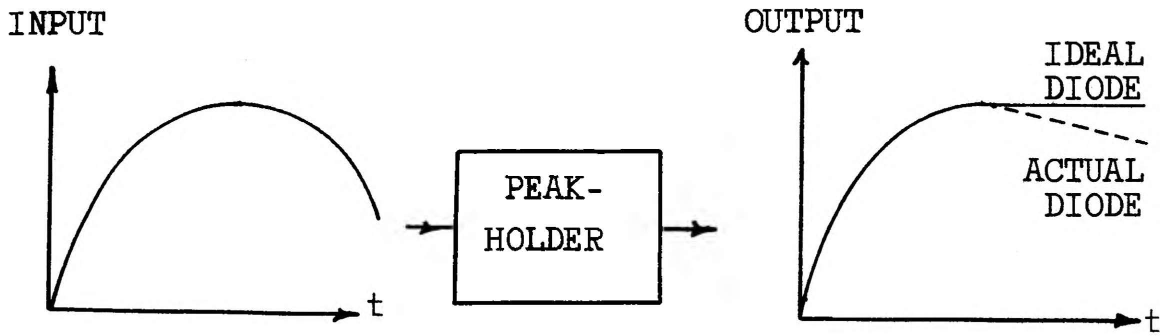
An analysis of the peak-holder showing the above results will now be given. First the circuit is redrawn as shown in Figure 13. The G's represent the conductance in general and the cases of reverse and forward bias will be considered after solving the node voltage equations. Using the notation on Figure 13.

$$e_o = - e_2$$

$$e_y(G_1 + G_2 + G_3) - e_1G_1 - e_2G_3 - e_oG_2 = 0$$

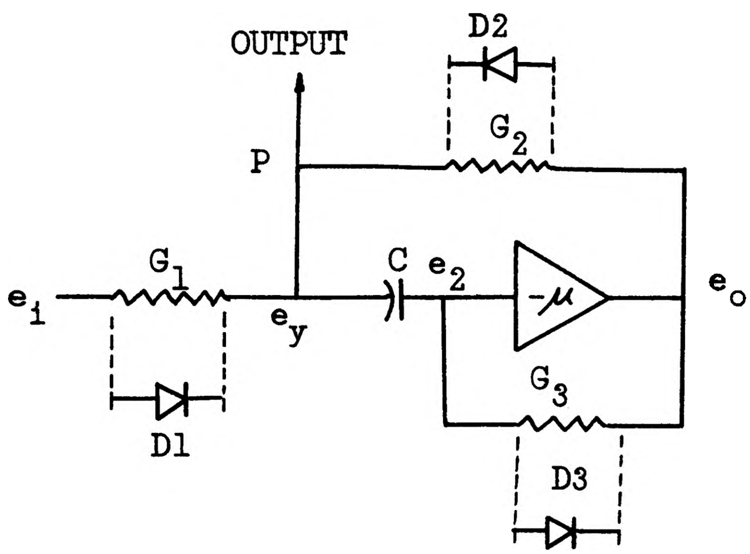
and

$$e_o(G_1 + G_2) - e_2G_1 - e_yG_2 = 0$$



PEAK-HOLDER RESPONSE

FIGURE 12



PEAK-HOLDER REDRAWN WITH CONDUCTANCES

FIGURE 13

Using the fact that μ is very large and solving the equations

$$e_y = e_i \frac{G_3}{C}\left[\frac{1}{s + \frac{1}{C}\left(\frac{G_1G_2 + G_1G_3 + G_2G_3}{G_1 + G_2}\right)}\right] \qquad (9)$$

When e_i is increasing with time D1 is forward biased. Also D3 will be forward biased because D3 and the capacitor act as a differentiator making e_o negative. Since e_y is positive and e_o is negative D2 will be reverse biased. When a diode is forward biased its conductance is large; when reversed biased its conductance is small. This means G_1 and G_3 are large, and G_2 is small. This simplifies equation (9) to

$$e_y = e_i \frac{G_3}{C}\left[\frac{1}{s + 1/(C/G_3)}\right]$$

The time constant, $\frac{C}{G_3}$, is very small since C = .01µf. and G_3 is very large. This small time constant means that e_y will follow e_i very quickly. To show this the following example was developed.

A typical value of conductance was chosen for G_3 (1 mho) and the value of .01µf. was used for the capacitance. When these values are subsituted

$$e_y = e_i \frac{10^4}{s + 10^4}$$

To check the response to an input, e_i was made a unit step. When expanded by partial fractions and expressed in the time domain

$$e_y(t) = 1 - e^{-t/10^{-4}}$$

This means after 4 time constants, .4 milliseconds, the output approximately equals the input. This shows that when e_i is increasing the output follows quickly.

When the input to the peak-holder begins to decrease D1 and D3 are reverse biased and D2 is foward biased. Subsituting the proper values for the conductance in equation (9) gives

$$e_y = e_i \frac{G_3}{C} \left[\frac{1}{s + \frac{1}{C}(G_1 + G_3)} \right]$$

Typical values for G were again used and a large time constant appeared in the transfer function. Using the arguments of the preceding example it was determined that response is slow when the input voltage is decreasing.

The voltages, e_y and e_i, were used for a general analysis of the peak-holder. In the system simulated the input is Y. Because the diodes did not hold the peak exactly the relay K1 was not used. It was originally set up to be used as a reset for the peak-holder.

5. <u>The Comparator.</u> The comparator has three inputs, -Y, Y_{peak}, and -E. As long as the sum of these is negative the output of amplifier 5 will be positive and the diode, D4, connected from the output of the amplifier to the input of the amplifier will clamp the output at approximately zero. When the sum of the inputs becomes positive the

output of the amplifier becomes negative and D4 is reverse biased. This negative value is limited by D5. The comparator is only in this negative state for a short time because when it goes negative the multivibrator changes states. When it changes states Y begins to increase again which makes the sum of the inputs to the comparator negative. When the input is negative the output is essentially zero.

B. The Experimental Results

The results and a discussion of some of the results obtained from the analog simulation are given in this section.

When the analog computer was put into operation it was observed that the peak-holder had jumps in its output as shown in Figure 14b. Figure 14b was recorded with the leads changed from Y to Y_{peak} to show that the peak-holder had a higher output than Y. At this point it was decided that a check of the peak-holder's response to some other signal might prove useful in explaining the jump. A sinusoid was introduced and the results are shown in Figure 14a.

As mentioned earlier in this chapter the peak-holder at times acts like a differentiator. This means that noise in Y will cause the output of the peak-holder to jump. According to the manufacture of the multiplier up to 50 mv. of noise can be expected on the output of the multiplier. Assumthe worst case this noise was sent to amplifier 2 and 3 with a gain of 10 into the peak-holder. This meant the input to the peak-holder was .5 volts of noise. Since the peak-holder sometimes acted as a differentiator with a high gain this noise caused the jump in the peak-holder output.

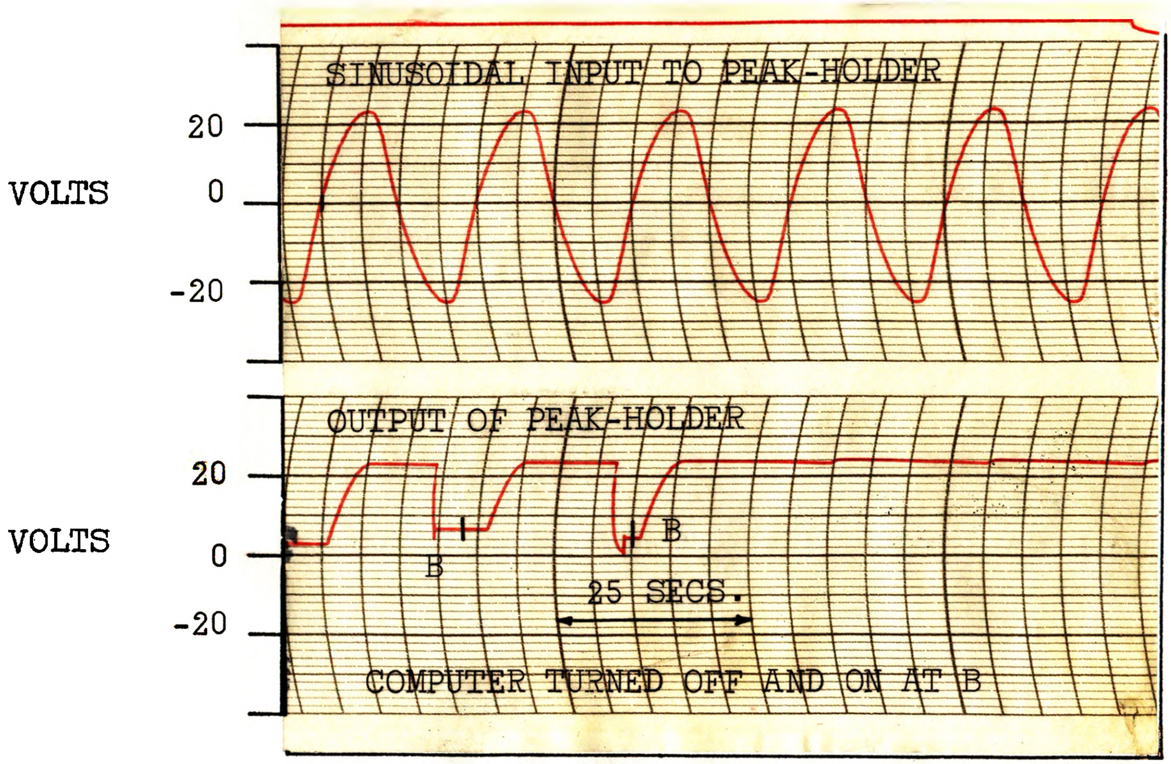
The various servo transfer functions were simulated and recorded in Figures 15, 16, and 17. The transfer functions used are noted on each diagram. When the double integrator was used the computer was turned off and on since after a few seconds the system went unstable.

Noise was introduced in the system through a megohm resistor connected to the input of amplifier 3. When E was 20 volts, the input noise (a sinusoid) amplitude was varied and instability resulted when the peak-to-peak value was 8 volts as shown in Figure 18.

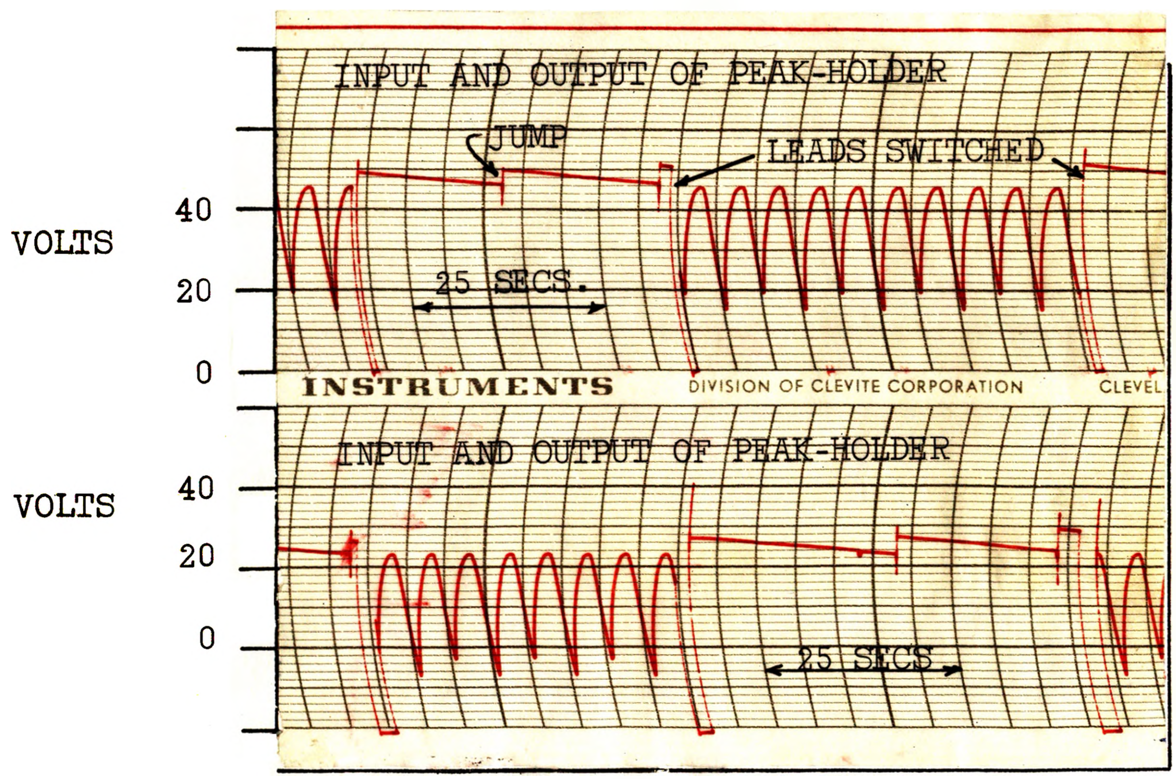
The phase portrait is a plot of Y versus $\dot{Y}$ and should take the form shown in Figures 6, 7, and 8. In the analog simulation a differentiator is seldom used because of noise. As mentioned earlier the multiplier adds noise to Y so it was not possible to get $\dot{Y}$ by using a differentiator. An attempt was made to get the derivative by using a preferred differentiator from Chestnut and Mayer[3], but this proved unsuccessful. It was found that the best way to obtain $\dot{Y}$ was to use the relation in Chapter V

$$\dot{Y} = -2Kx\dot{x}$$

In the useful case the servo was an integrator so $\dot{x} = Z$. Also x was the input to the multiplier. Using these values $\dot{Y}$ was determined. Figure 19a shows Y and $\dot{Y}$ versus time; Figure 19b shows the phase portrait.



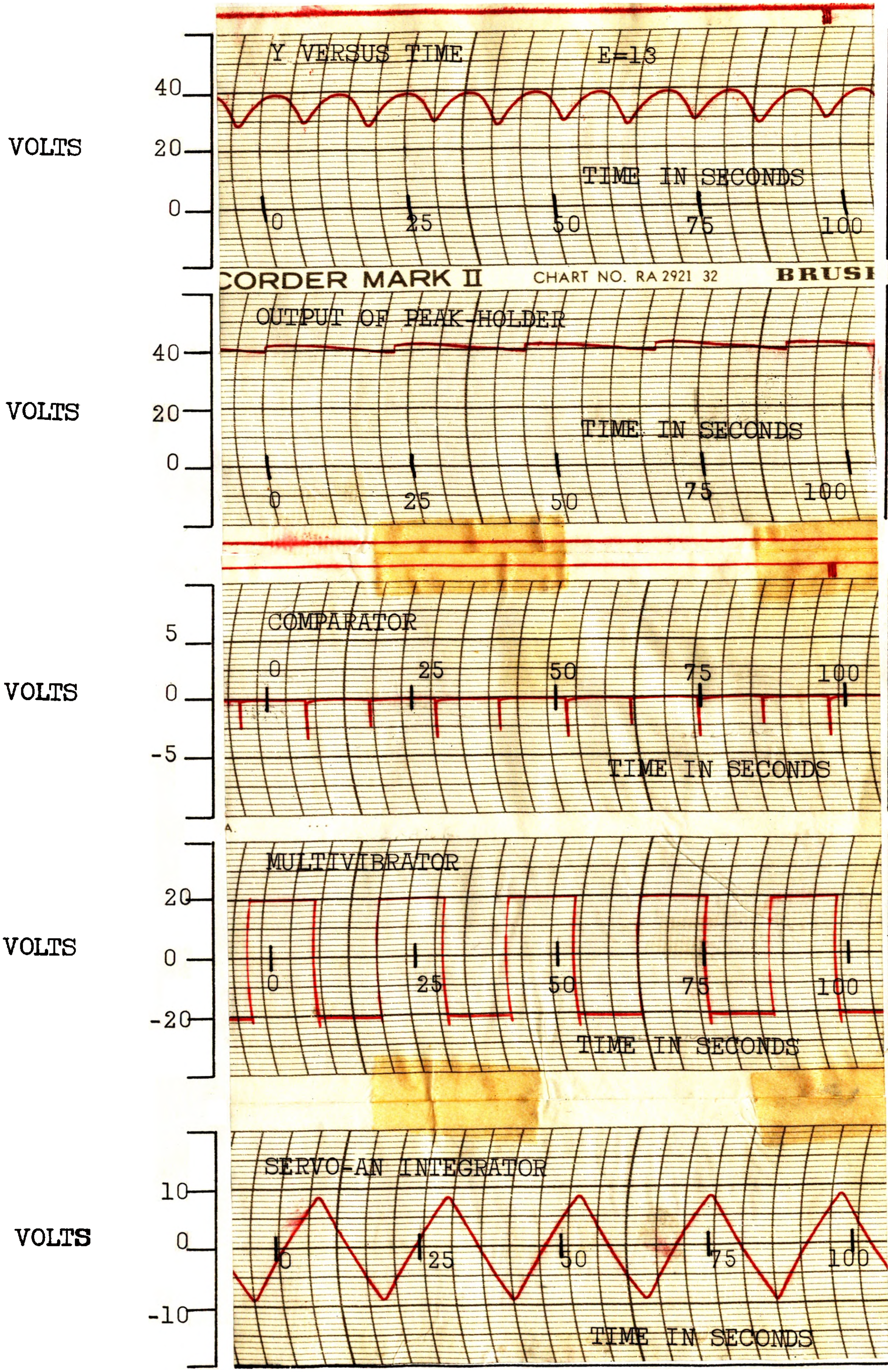
(a)



(b)

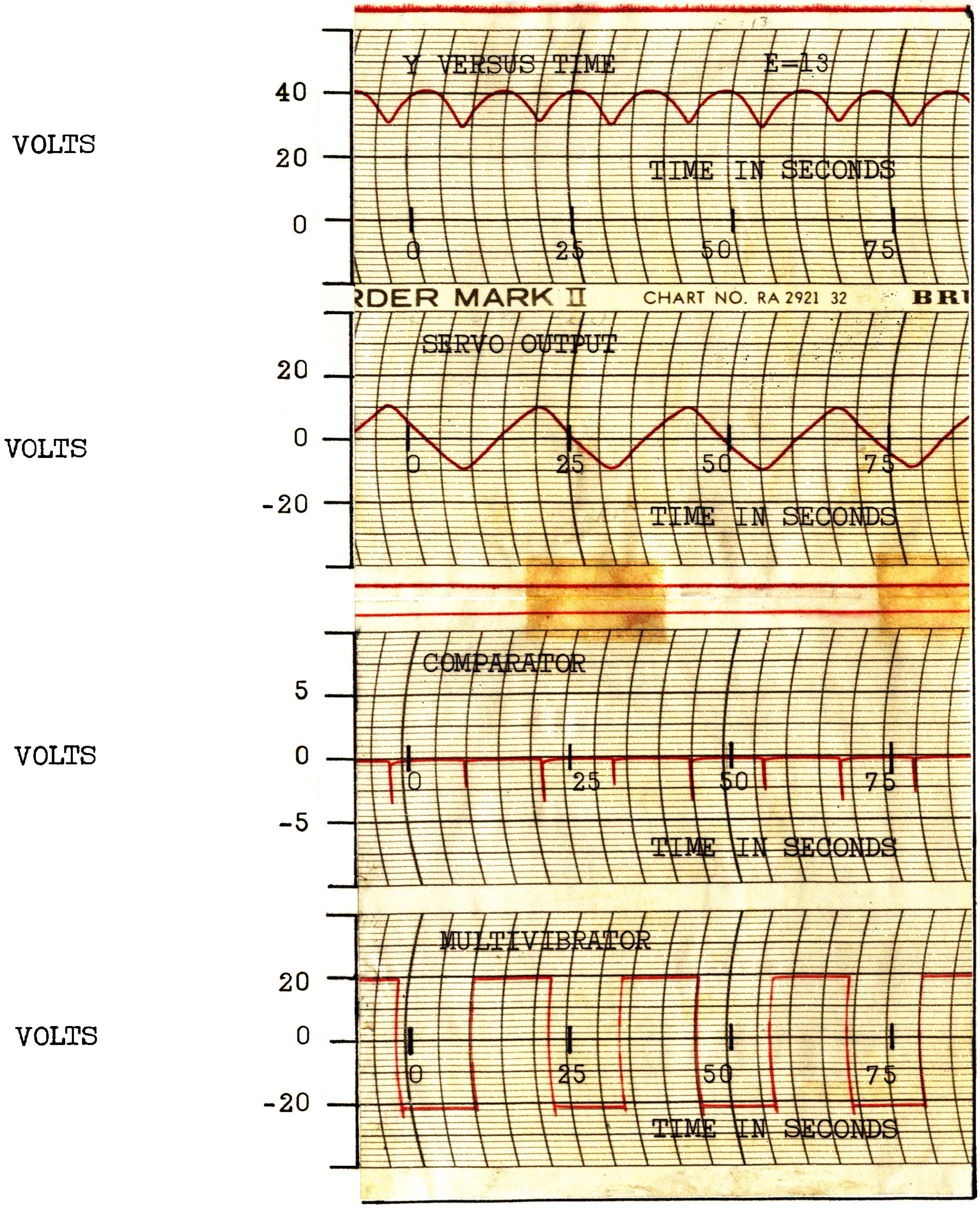
RESPONSE OF PEAK-HOLDER

FIGURE 14



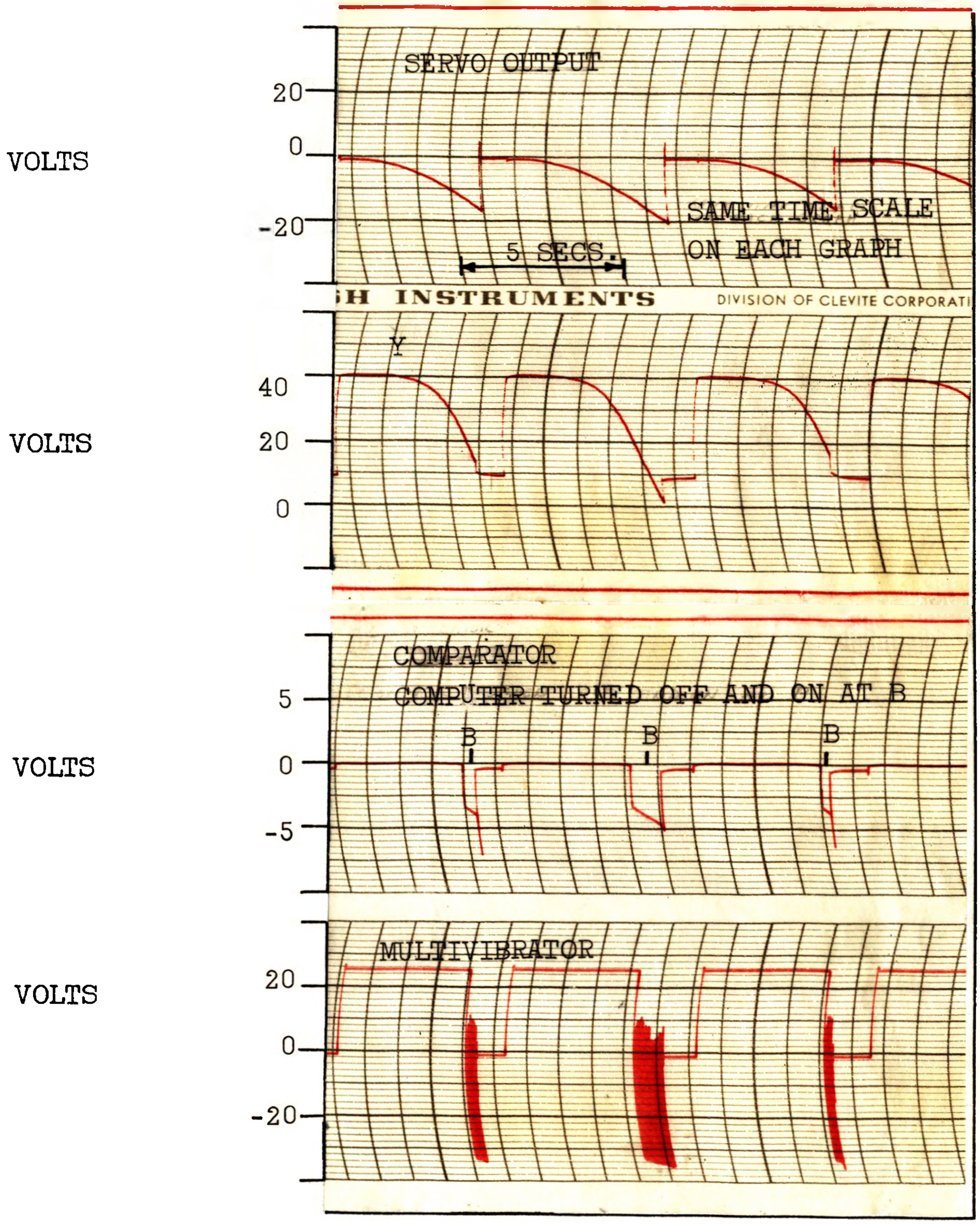
SYSTEM RESPONSE WITH THE SERVO AN INTEGRATOR

FIGURE 15



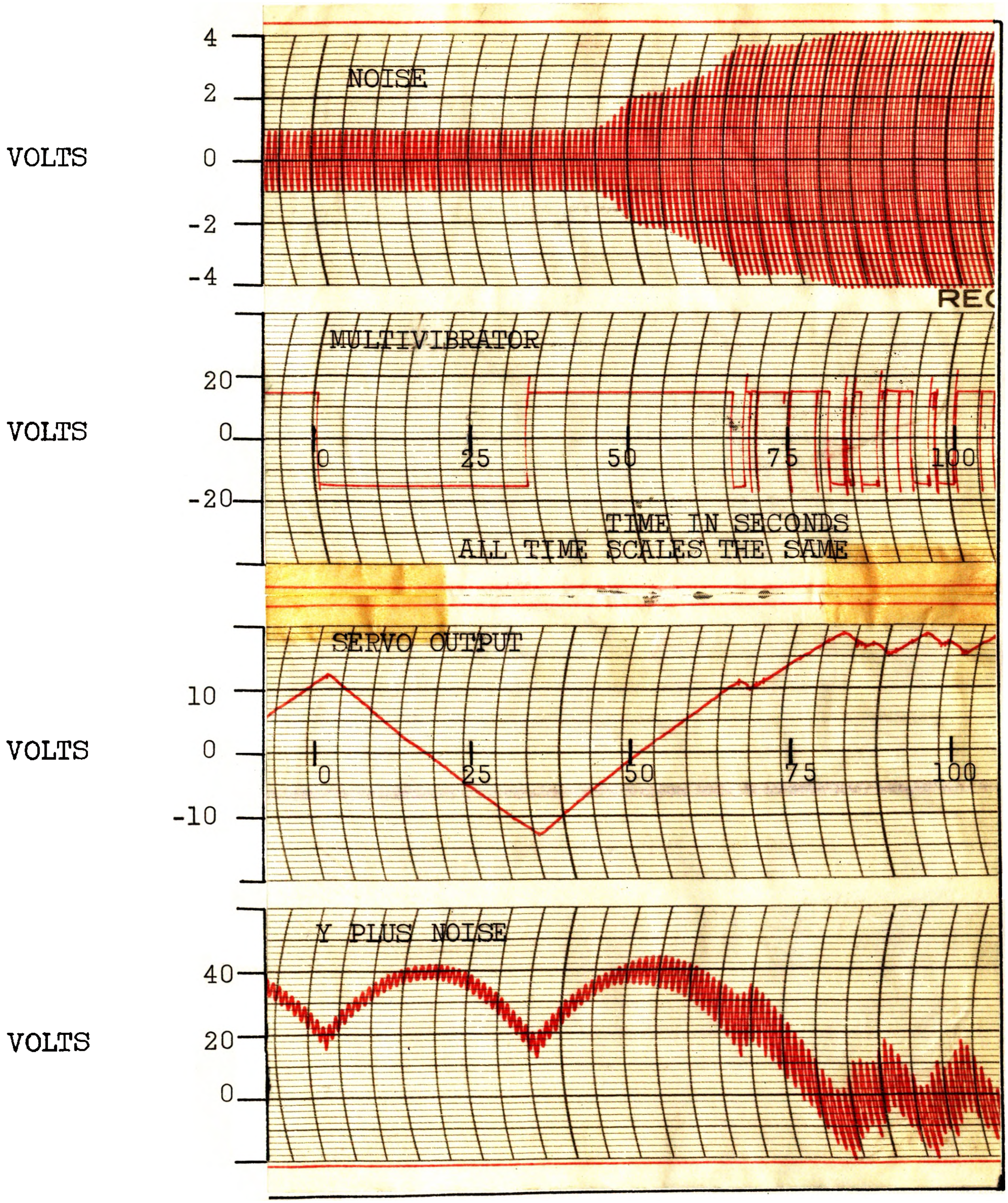
RESPONSE WITH SERVO AN IMPERFECT INTEGRATOR

FIGURE 16



RESPONSE WITH SERVO TRANSFER FUNCTION, $G(s)=K_4/s^2$

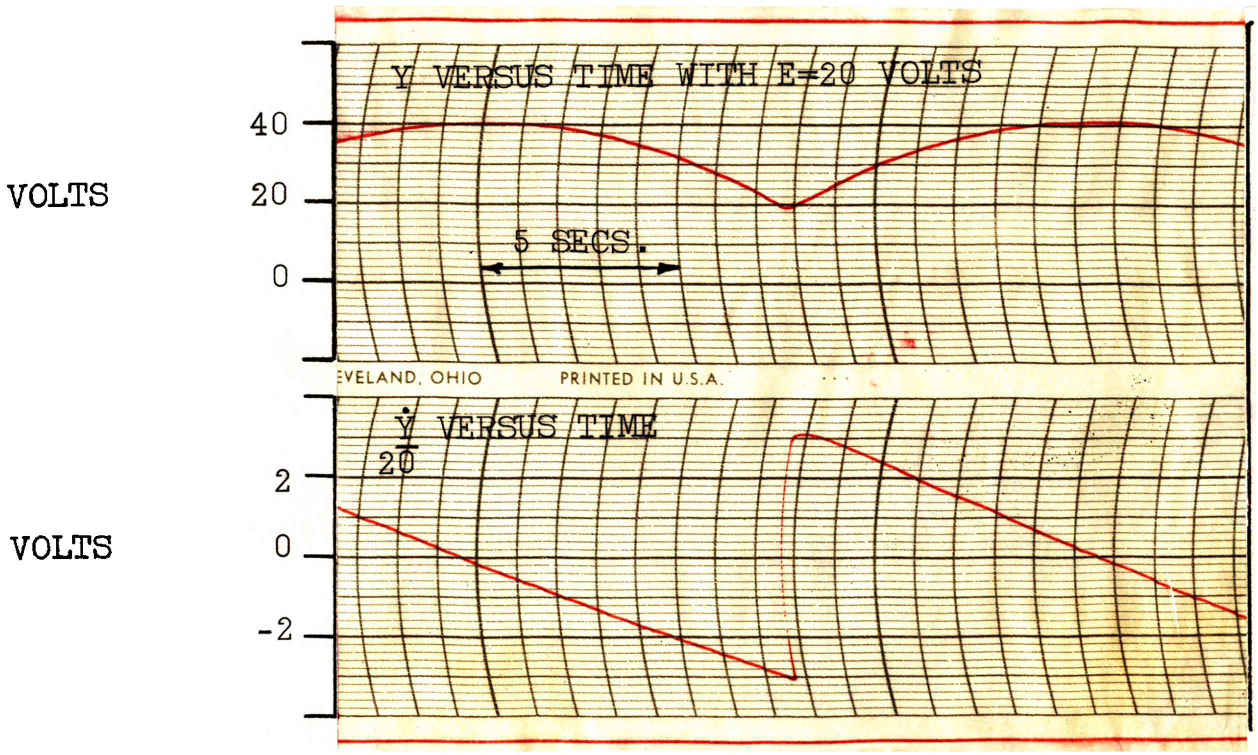
FIGURE 17



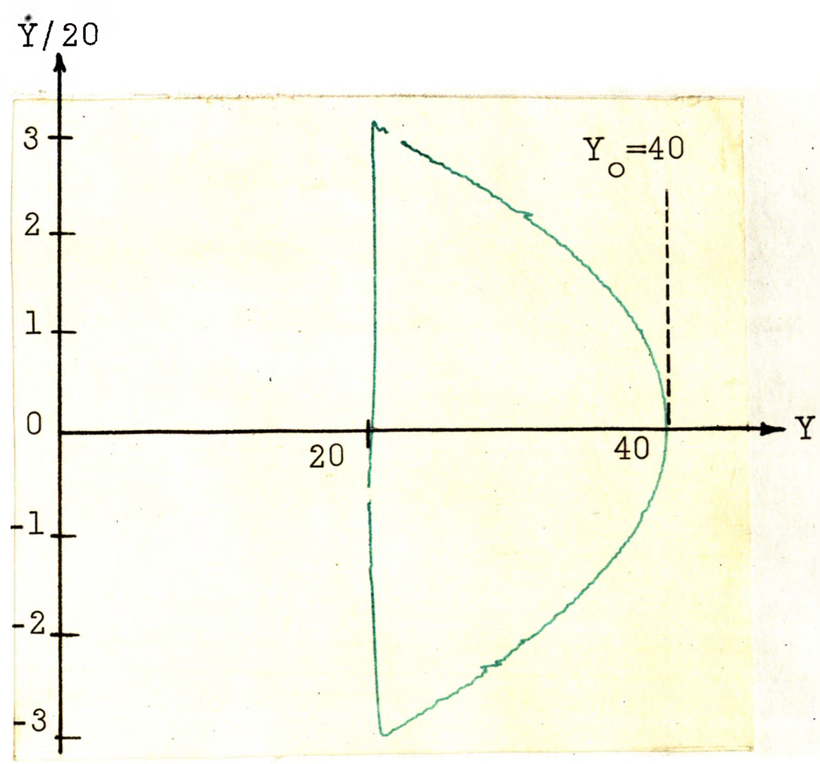
INSTABILITY DUE TO NOISE IN OUTPUT

OF CONTROLLED SYSTEM

FIGURE 18



(a)



(b)

PHASE PORTRAIT

FIGURE 19

CHAPTER VIII

DISCUSSION AND CONCLUSIONS

The peak-holding method is one way to control a system with an extremum[1]. As presented in this thesis a hunting loss of E/3 occurs. A modification of the peak-holding method may make the hunting loss less than E/3. As an example of a modification let the servo be a double integrator and let the multivibrator change states when y = -E/2. The system will respond favorably and the hunting loss will be E/2. However if switching can occur properly (noise and switching are not limitations) at E/2 a single integrator would give a hunting loss of E/6.

The work done in this thesis can be applied to certain systems with multiple inputs. In the system considered by Li[8] two variables controlled the efficiency of an internal combustion engine. In this two variable case each input had an output, and the total output was the product of the individual outputs. These individual outputs were both considered separately and had separate controllers. If a system has many input-output relations of this type then the work done in this thesis can be applied to each part of the system separately.

The sinusoid used to represent noise in this system showed the effects of an unwanted signal. In an actual control system chances are noise other than a sinusoid is in the system. In an actual system an investigation of where and what kind of noise is in the system should be made.

Although a phase portrait is not a necessity in studyin this system it is useful in seeing what happens to Y. When the servo is something other than an integrator the phase portrait may be difficult to plot as shown by equation (7).

The analog simulation presented was useful in the study of the system. A better simulation could have been made if the diodes used in the peak-holder where ideal, and no noise was introduced by the multiplier. The difficulty in getting switching to occur properly in the analog simulation clearly showed that in an actual system switching can be a big problem.

The problem of application to an actual system presents many problems, but once these problems are solved the peak-holding method can present a savings in material, time, and energy.

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