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Unsteady state heat transfer from a hot body to a liquid

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UNSTEADY STATE HEAT TRANSFER
FROM A HOT BODY TO A LIQUID

BY

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A

THESIS

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Approved by

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INTRODUCTION

The problem investigated in this thesis is that of unsteady state heat transfer from a hot body to a liquid, a quenching problem. Unsteady state or transient heat transfer is heat transfer in which the temperature distribution changes with time.

If a hot body was suddenly quenched in a large mass of liquid, a film of vapor would form between the hot body and the liquid. For example, if a hot body was quenched in water a film of steam would form between the hot body and the water. It is the purpose of this thesis to determine the heat transfer from the hot body to the liquid and the temperature distribution in the hot body for such a situation. In this thesis the hot body will be assumed a semi-infinite solid.

A large class of important industrial problems require the prediction of temperatures and heat transfer rates in a solid structure being heated or cooled by immersion in a large mass of fluid. Such problems are relatively common in metallurgical processes where it is necessary to estimate heating or cooling rates of large solid ingots of various shapes. This information is then used to predict the time required for such objects to attain prescribed temperature levels for purposes of melting, hot-working, heat-treatment, and the like. Heating and cooling rates are also of extreme practical interest in the canning industry where perishable canned foods are chilled by immersion; in the paper industry where wood logs are immersed in steam baths preparatory to pulping and veneer cutting; in the manufacture of bricks, glass, and rubber products; in the prediction of allowable combustion times in rocket-engine nozzles; and in the calculation of allowable acceleration rates for airborne vehicles subject to high-speed transient aerodynamic heating.

The results of an analytical approach to heat conduction problems are by no means limited to heat transfer alone. Parts of the theory find application in various static and current electricity problems, gravitational problems, and the methods of development are of general application in mathematical physics.[1]

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(1) L. R. Ingersoll, O. J. Zobel, and A. C. Ingersoll, Heat Conduction With Engineering and Geological Applications, 1st Ed., N. Y., McGraw-Hill, 1948, p. 5.

The author chose this unsteady state heat transfer problem because of an interest in the analytical solutions of such problems, and to his knowledge, this particular problem had not been solved previously.

REVIEW OF LITERATURE

The study of heat transfer is principally concerned with the distribution of temperature and the temperature history within the object under consideration. From the available literature it can be seen that there are essentially four available methods for the evaluation of these temperature fields: (1) analytical, (2) graphical, (3) numerical, and (4) experimental.

The formal analytical approach involves the derivation and solution of mathematical expressions for the temperature as a function of space-time coordinates. The solution must satisfy the differential equation from which it was derived as well as certain initial and boundary conditions imposed by the specific problem itself. The first work done on this subject by mathematical analysis was done by Fourier.[2]

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(2) J. Fourier, The Analytical Theory of Heat, translated with notes by A. Freeman, London, The University Press, 1878, pp. 104-115, pp. 323-332.

The more recent works of Carslaw and Jaeger[3] and Schneider[4]

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(3) H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, 2nd Ed., (London) Oxford at the Claredon Press, 1959, pp. 50-64

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(4) P. J. Schneider, Conduction Heat Transfer, Reading, Mass., Addison-Wesley, 1955, pp. 229-271.

contain the analytical solutions of many of the more difficult problems of unsteady state heat transfer. Some of the previously mentioned material has been presented in a simpler and more readable form by Ingersoll, Zobel and Ingersoll.[5]

(5) L. R. Ingersoll, O. J. Zobel, and A. C. Ingersoll, Heat Conduction with Engineering and Geological Applications, 1st Ed., N. Y., McGraw-Hill, 1948, pp. 78-108.

The graphical method is based on properties of the characteristic field equations and numerical principles. Some graphical methods, as well as analytical solutions, may be found in works by Jakob[6] and Jakob and Hawkins.[7]

(6) M. Jakob, Heat Transfer, N. Y., John Wiley & Sons, 1949, pp. 380-398.

(7) M. Jakob and G. A. Hawkins, Elements of Heat Transfer, N. Y., John Wiley & Sons, 1957, pp. 73-79.

The numerical method is based on finite differences. Some numerical methods may be found in a work by Jakob and

Hawkins.[8]

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(8) Ibid., pp. 71-73.

The experimental method can range anywhere from temperature measurements at points in a model of the prototype structure to the use of analogic experiments which take advantage of the mathematical analogy between heat transfer and other potential-field phenomena. Some methods involving the thermal-electrical analogy are described by Jakob and Hawkins.[9]

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(9) Ibid., pp. 79-97.

There are various papers available on this and closely related subjects; however, the majority of this material is experimental in nature and is in the form of tables and curves.

TABLE OF UNITS

SYMBOL	UNITS	SIGNIFICANCE
θ	hour	time
t	F	temperature, degrees F.
Q	Btu	quantity of heat
x	ft	distance normal to surface
A	ft^2	area
V	ft^3	volume
k	Btu $hr^{-1}ft^{-1}F^{-1}$	thermal conductivity
C	Btu $lb^{-1}F^{-1}$	specific heat
q	Btu hr^{-1}	heat flow per unit time
ρ	lb ft^{-3}	density
α	ft^2 hr^{-1}	diffusivity
C'		constant
C_1		constant
C_2		constant
$\mathcal{L}(u)$		Laplace transformation
$\bar{u}$		Laplace transformation
h_1	Btu $hr^{-1}ft^{-2}F^{-1}$	film coefficient between liquid and vapor
h_v	Btu $hr^{-1}ft^{-2}F^{-1}$	film coefficient for vapor and solid
H	Btu $hr^{-1}ft^{-2}F^{-1}$	Combined film coefficient

DISCUSSION

Since the heat flow in the semi-infinite solid takes place by conduction, it will be found to follow Fourier's law as stated in his conduction equation. This law expressed mathematically is

$$\frac{dQ}{d\theta} = -KA\frac{dt}{dx} \tag{1}$$

where dQ is the amount of heat flowing in differential time $d\theta$, A is the area of the section across which Q is flowing, -dt/dx is the temperature gradient or the rate of change of temperature, t, with respect to the length of path x, and k is the thermal conductivity of the material. The area, A is taken normal to the direction of heat flow.[10]

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(10) W. H. McAdams, Heat Transmission, 2nd Ed., N. Y., McGraw-Hill, 1942, pp. 6-7.

To arrive at a more general equation for heat flow by conduction, the following analytical reasoning will be used.

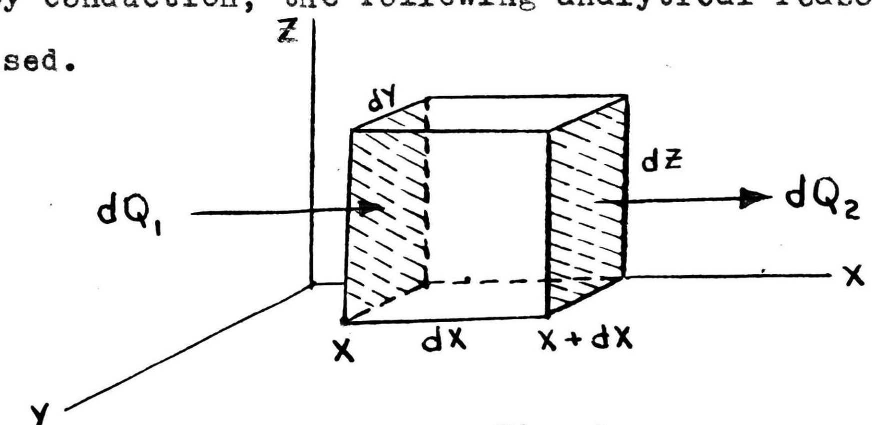

Fig. 1

Referring to figure 1, an elemental cube of volume $dv = dxdydz$ receives a differential quantity of heat dQ_1 Btu through its left yz face in the time interval $d\theta$. Assume all but the left and right yz faces of the elemental cube are insulated. In the same time interval the quantity of heat dQ_2 leaves at the right face. A storage term dQ can be defined as the difference between the heat entering and the heat leaving or

$$dQ = dQ_1 - dQ_2. \tag{2}$$

According to equation (1) the heat entering the left face may be given by

$$\frac{dQ_1}{d\theta} = K\,dY\,dZ\left(-\frac{\partial t}{\partial x}\right) \tag{3}$$

The temperature gradient $-\partial t/\partial x$ may vary with both time and position in the cube. The variation of $-\partial t/\partial x$ as a function of x is $-\partial^2 t/\partial x^2$. Over the distance dx from x to x+ dx the total change in the temperature gradient will be the rate of change of the gradient multiplied by the distance over which the gradient changes or $(-\partial^2 t/\partial x^2)dx$. Then at x the gradient is $-\partial t/\partial x$, and at x+dx the temperature gradient is

$$-\frac{\partial t}{\partial x} - \frac{\partial^2 t}{\partial x^2}\,dx.$$

The heat flowing out of the right face, dQ_2, in the same form as equation (3) is given by

$$\frac{dQ_2}{d\theta} = K\,dY\,dZ\left(-\frac{\partial t}{\partial X} - \frac{\partial^2 t}{\partial X^2}\,dX\right). \tag{4}$$

The net amount of heat stored in the cube in differential time $d\theta$, will then be the difference between the amount of heat flowing in and that flowing out or

$$\frac{dQ}{d\theta} = \frac{dQ_1}{d\theta} - \frac{dQ_2}{d\theta}$$

which can be written as

$$\frac{dQ}{d\theta} = K\,dY\,dZ\left(-\frac{\partial t}{\partial X}\right) - K\,dY\,dZ\left(-\frac{\partial t}{\partial X} - \frac{\partial^2 t}{\partial X^2}\,dX\right)$$

which reduces to

$$\frac{dQ}{d\theta} = K\,dX\,dY\,dZ\left(\frac{\partial^2 t}{\partial X^2}\right). \tag{5}$$

It is known that the amount of heat, dQ, that is stored in a solid body is dependent upon the density of the material ρ, the volume of the body, in this case $V=dxdydz$, its specific heat C, and the instantaneous temperature difference across the body. $d\theta$ represents the increment of time during which the heat is stored. Written mathematically

$$\frac{dQ}{d\theta} = \rho\,C\,dX\,dY\,dZ\left(\frac{\partial t}{\partial \theta}\right). \tag{6}$$

Hence there are two expressions for the amount of heat stored in this elemental cube, namely equations (5) and (6).

Equating these equations

$$\rho C\, dX\, dY\, dZ \left(\frac{\partial t}{\partial \theta}\right) = K\, dX\, dY\, dZ \left(\frac{\partial^2 t}{\partial X^2}\right)$$

which reduces to

$$\frac{\partial t}{\partial \theta} = \frac{K}{\rho C} \frac{\partial^2 t}{\partial X^2}.$$

Letting $k/\rho C = \alpha$, the equation can be written as

$$\frac{\partial t}{\partial \theta} = \alpha \frac{\partial^2 t}{\partial X^2}. \tag{7}$$

This is the general conduction equation when the flow of heat is in the x - direction only. It expresses the conditions that govern the flow of heat in a body, and the solution of any particular problem in heat conduction must satisfy this equation regardless of what the initial and boundary conditions may be.

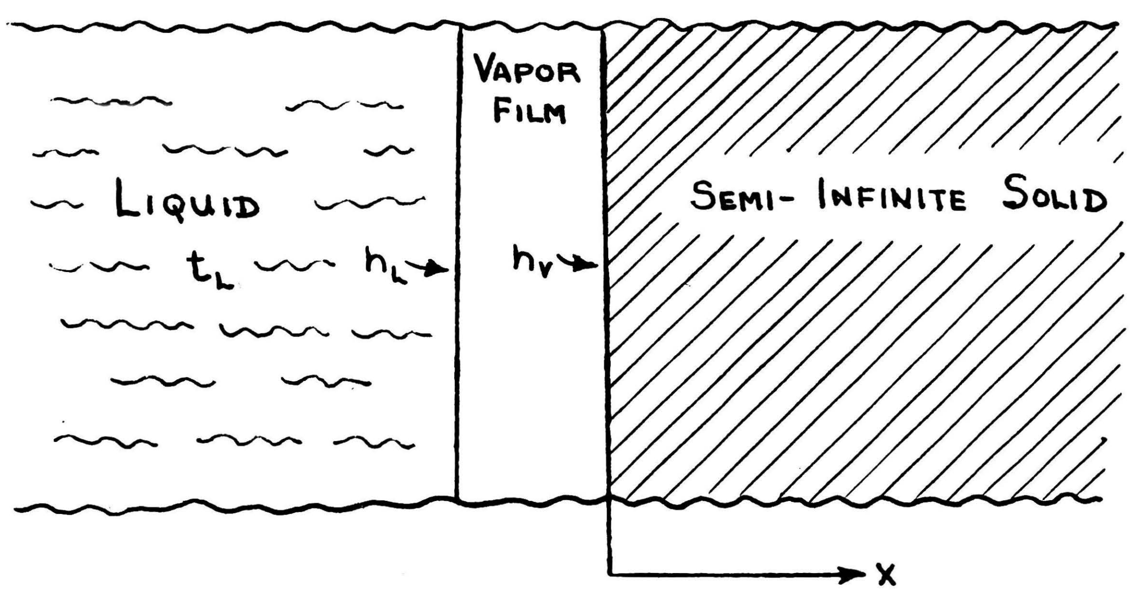

Fig. 2

Figure 2 is a schematic representation of the hot body, the vapor film, and the liquid involved in this investigation. From Figure 2, it should be noted that the x co-ordinate system originates at the surface of the hot body and the positive direction extends to the right into the solid.

One initial and one boundary condition that can be readily determined are imposed on equation (7) by this specific problem. The first is directly concerned with the initial temperature of the hot body, just as it is quenched in the liquid. This initial condition can be stated mathematically as

$$t = t_i \text{ (A CONSTANT)}; \text{ AT } \theta = 0, \; x \geq 0.$$

The boundary condition is a direct result of the equivalence of the heat transferred out of the surface of the hot body by conduction and the heat transferred by convection in the vapor film and the liquid. It can be determined as shown below

$$Q_{\text{CONDUCTION}} = Q_{\text{CONVECTION}} \tag{8}$$

The equivalent film coefficient for convection is given by

$$H = \frac{1}{1/h_L + 1/h_V} \tag{9}$$

The equivalent film coefficient will be assumed independent of the surface temperature (t) of the hot body in this investigation. Therefore Equation (8) can be written as

$$KA \frac{\partial t}{\partial X} = HA(t - t_L). \tag{10}$$

(Contrary to the usual notation, the temperature gradient is positive in this case since the temperature increases as the distance (x) increases.) After cancelling like terms and rearranging equation (10), the boundary condition can be written as

$$\frac{\partial t}{\partial X} = \frac{H}{K}(t - t_L); \quad \text{At } X = 0,\ \theta > 0. \tag{11}$$

Now, let $T = t - t_i$, then equation (7), the initial condition, and the boundary condition can be expressed as functions of T. Equation (7) transforms into

$$\frac{\partial^2 T}{\partial X^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \theta} \tag{12}$$

The initial condition becomes

$$T = t_i - t_i = 0; \quad \text{At } \theta = 0,\ X \geq 0.$$

By the reasoning shown

$$t - t_L = T + t_i - t_L = T - (t_L - t_i) = T - T_L$$

the boundary condition, equation (11), becomes

$$\frac{\partial T}{\partial X} = \frac{H}{K}(T - T_L); \quad \text{At } X = 0,\ \theta > 0. \tag{13}$$

The problem now is to integrate the partial differential equation, equation (12), to get a particular solution that satisfies the given conditions. There are two more or less common methods used to integrate such partial differential equations, the first being the separation of variables method, and the second the Laplace transform method. In this investigation the Laplace transform method will be used to find the solution of this problem as it is not easily determined by the separation of variables method.

The Laplace transformation of a function f(x), symbolized here as $\mathcal{L}(u)$, is defined as the operation of multiplying f(x) by e^{-ux} and integrating over all positive values of the variable x, as

$$\mathcal{L}(u) = \int_0^{\infty} e^{-ux} f(x)\, dx$$

The value of u may be real or imaginary, and in either case its real part must be sufficiently large to ensure the convergence of the given integral. Transforms of various functions f(x) can therefore be obtained by direct integration. The inverse transform is the function itself.[11]

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(11) Schneider, op. cit., p. 113.

However, there are many tables of transform pairs now in use from which transforms and inverse transforms can be directly determined.

Applying the Laplace transformation with respect to θ on both sides of the partial differential equation, equation (12), it becomes

$$\int_0^\infty e^{-u\theta} \frac{\partial^2 T}{\partial X^2}\, d\theta = \frac{1}{\alpha}\int_0^\infty e^{-u\theta} \frac{\partial T}{\partial \theta}\, d\theta. \qquad (14)$$

By using

$$\mathcal{L}\left[\frac{df}{dX}\right] = u\,\mathcal{L}(u) - f(0)$$

and interchanging the order of integration and differentiation on the left side of equation (14), the equation becomes

$$\frac{\partial^2}{\partial X^2}\int_0^\infty e^{-u\theta} T(X,\theta)\, d\theta = \frac{1}{\alpha}\left[u\,\mathcal{L}(u) - T(X,0)\right]. \qquad (15)$$

The left-hand integral transformation is a function of u and the single variable x, therefore the partial derivative goes over to a total derivative.

Next, using the initial condition

$$T = 0; \quad \text{At} \quad \theta = 0, \quad X \geq 0$$

and the abbreviation $\bar{u} = \mathcal{L}(u)$

equation (15) becomes

$$\frac{d^2\bar{u}}{dX^2} = \frac{1}{\alpha}\left[u(\bar{u}) - 0\right] = \frac{u}{\alpha}\,\bar{u}$$

rearranging, it becomes

$$\frac{d^2\bar{u}}{dx^2} - \frac{\mu}{\alpha}\bar{u} = 0 . \tag{16}$$

The general solution of this equation can be obtained by the use of operators, as follows

$$(D^2 - \mu/\alpha)\bar{u} = 0$$

$$(P^2 - \mu/\alpha) = 0$$

$$P = \pm\sqrt{\frac{\mu}{\alpha}} .$$

The general solution of equation (16) can now be written as

$$\bar{u} = C_1 e^{\sqrt{\frac{\mu}{\alpha}}x} + C_2 e^{-\sqrt{\frac{\mu}{\alpha}}x} . \tag{17}$$

However, if T and therefore $\bar{u}$, is to remain finite as $x \rightarrow \infty$ then $C_1 = 0$. The solution can then be written as

$$\bar{u} = C' e^{-\sqrt{\frac{\mu}{\alpha}}x} . \tag{18}$$

Now, the boundary condition

$$\frac{\partial T}{\partial X} = \frac{H}{K}(T - T_L); \quad \text{At } X = 0,\ \theta > 0$$

can be used as a boundary condition for equation (18) by applying the Laplace transformation $\mathcal{L}(u)$ with respect to θ on both sides of the equation. Performing this operation the boundary condition becomes

$$\int_0^\infty e^{-u\theta}\frac{\partial T}{\partial X}\,d\theta = \frac{H}{K}\int_0^\infty e^{-u\theta}T\,d\theta - \frac{H}{K}\int_0^\infty e^{-u\theta}T_L\,d\theta . \tag{19}$$

By using the same reasoning as was used before and

$$\mathcal{L}(\text{CONSTANT}) = \frac{1}{\mu}(\text{CONSTANT})$$

it can be seen that the partial differential equation, equation (19), goes over to a total differential equation which can be written as

$$\frac{d\bar{u}}{dx} = \frac{H}{K}\bar{u} - \frac{HT_L}{K}\frac{1}{\mu}; \quad \text{AT } x = 0, \ \theta > 0. \tag{20}$$

Now, equation (20) can be used as a boundary condition for equation (18) to determine the constant C'. This is done in the following manner. First the derivative of equation (18) with respect to x will be found and evaluated at x = 0. This gives

$$\left.\frac{d\bar{u}}{dx}\right|_{x=0} = C'\left(-\sqrt{\frac{\mu}{\alpha}}\right)\left. e^{-\sqrt{\frac{\mu}{\alpha}}x}\right|_{x=0} = -C'\sqrt{\frac{\mu}{\alpha}}. \tag{21}$$

This equation must be equal to equation (20), setting the two equal

$$-C'\sqrt{\frac{\mu}{\alpha}} = \frac{d\bar{u}}{dx} = \frac{H}{K}\bar{u} - \frac{HT_L}{K}\frac{1}{\mu}; \quad \text{AT } x = 0, \ \theta > 0. \tag{22}$$

Next the substitution $\bar{u}|_{x=0} = C'e^{-\sqrt{\frac{\mu}{\alpha}}0} = C'$ will be made. Equation (22) becomes

$$-C'\sqrt{\frac{\mu}{\alpha}} = \frac{H}{K}C' - \frac{HT_L}{K\mu}$$

solving for C'

$$C' = \frac{H T_L}{K u \left(H/K + \sqrt{u/\alpha} \right)} . \tag{23}$$

Next, replacing C' in equation (18) by equation(23), equation (18) becomes

$$\bar{u} = \left[\frac{H T_L}{K u \left(H/K + \sqrt{u/\alpha} \right)} \right] e^{-\sqrt{\frac{u}{\alpha}}\, x} . \tag{24}$$

The inverse transform of equation (24) can be determined from a table of transform pairs. Carslaw and Jaeger[(12)]

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(12) <u>Op. cit.</u>, p. 380

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list the transform of

$$\frac{e^{-\sqrt{u/\alpha}\, x}}{u \left(H/K + \sqrt{u/\alpha} \right)}$$

as

$$\frac{K}{H} \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha\theta}} \right) - \frac{K}{H} e^{(H/K)x + \frac{H^2}{K^2}\alpha\theta} \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha\theta}} + \frac{H}{K}\sqrt{\alpha\theta} \right)$$

where erfc, which is known as the complementary error function is defined as

$$\operatorname{erfc} X = 1 - \operatorname{erf} X = \frac{2}{\pi} \int_X^{\infty} e^{-\lambda^2} d\lambda$$

The inverse transform of equation (24) is therefore by comparison,

$$\frac{T}{T_L} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha\theta}}\right) - \left\{\exp.\left[\frac{H}{K}x + \frac{H^2}{K^2}\alpha\theta\right]\right\} \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha\theta}} + \frac{H}{K}\sqrt{\alpha\theta}\right) \quad (25)$$

or, by rearranging and expressing in terms of the original temperatures, can be written as

$$\frac{t_i - t}{t_i - t_L} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha\theta}}\right) - \left\{\exp.\left[\frac{H\sqrt{\alpha\theta}}{K}\left(\frac{x}{\sqrt{\alpha\theta}} + \frac{H\sqrt{\alpha\theta}}{K}\right)\right]\right\} \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha\theta}} + \frac{H}{K}\sqrt{\alpha\theta}\right) \quad (26)$$

Equation (26) gives the temperature distribution in the semi-infinite solid as a function of position (x) and time (θ).

By evaluating equation (26) at $x = 0$ and solving for t, the surface temperature of the semi-infinite solid as a function of time (θ) can be found. This expression is

$$t = (t_L - t_i)\left[1 - \left(\exp.\frac{H^2}{K^2}\alpha\theta\right) \operatorname{erfc} \frac{H}{K}\sqrt{\alpha\theta}\right] + t_i \quad (27)$$

The general equation for heat transfer involving film coefficients is

$$Q = H A \Delta t \quad (28)$$

where Q is the amount of heat flowing per unit time, A is the area of the section taken normal to the direction of heat flow, t is the temperature difference across the section under consideration, and H is the equivalent film coefficient.

The heat transfer from the hot body to the liquid can now be written using equations (27) and (28). The temperature difference for this problem is given by

$$\Delta t = t - t_L = (t_L - t_i)\left[1 - \left(\exp. \frac{H^2}{K^2}\alpha\theta\right) \operatorname{erfc} \frac{H}{K}\sqrt{\alpha\theta}\right] + t_i - t_L$$

which reduces to

$$\Delta t = (t_i - t_L)\left(\exp. \frac{H^2}{K^2}\alpha\theta\right) \operatorname{erfc} \frac{H}{K}\sqrt{\alpha\theta}.$$

The equivalent film coefficient is given by equation (9). Taking the area as one square foot, the heat flowing per unit time is given by

$$q = H(t_i - t_L)\left[\exp.\left(\frac{H}{K}\sqrt{\alpha\theta}\right)^2\right] \operatorname{erfc} \frac{H}{K}\sqrt{\alpha\theta}. \qquad (29)$$

Equation (29) gives an equation for determining the unsteady state heat transfer from a hot body to a liquid.

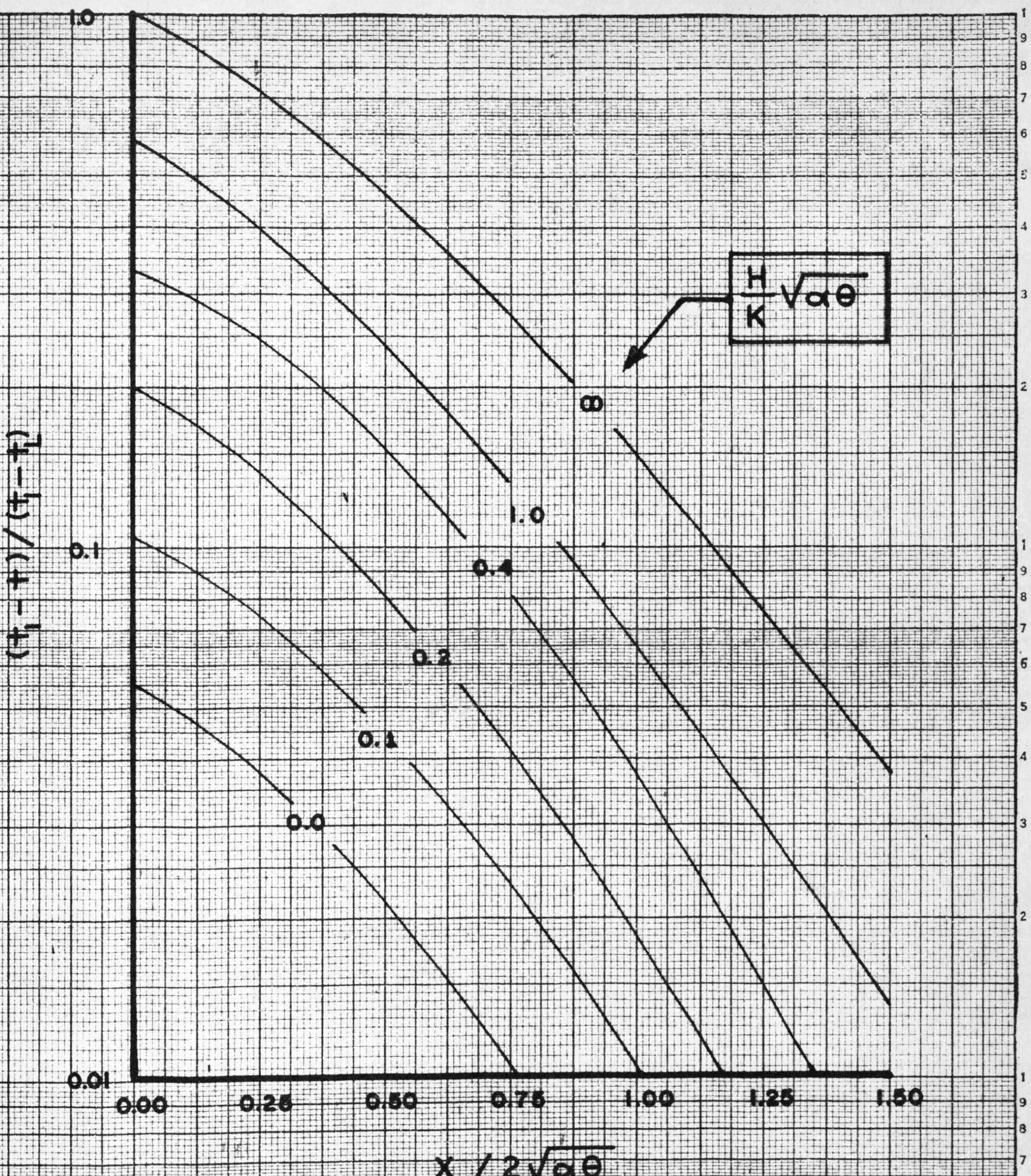
Since the mathematical solutions for the temperature distribution and heat transfer are rather long, two sets of curves have been drawn to expedite the determination of temperatures and heat rates. Plate No. 1, a graph of $(t_i-t)/(t_i-t_L)$ for different values of $x/2\sqrt{\alpha\theta}$ and $H\sqrt{\alpha\theta}/k$ has been plotted to aid in determining the temperature distribution. Plate No. 2, a graph of q for different values of $H(t_i-t_L)$ and $x/2\sqrt{\alpha\theta}$ has been plotted to aid in determining heat rates. The points necessary for plotting these graphs were obtained from equations (26) and (29) respectively.

TABLE I

TEMPERATURE HISTORY IN A SEMI-INFINITE SOLID

From Equation (26)

$\frac{H\sqrt{\alpha\theta}}{k}$		1.0	0.4	0.2	0.1	0.05
$\frac{x}{2\sqrt{\alpha\theta}}$			$(t_i-t)/(t_i-t_L)$			
0.00	1.0000	0.5724	0.3292	0.1983	0.1036	0.0540
0.25	0.7237	0.3782	0.2500	0.1204	0.0748	0.0336
0.50	0.4795	0.2290	0.1239	0.0700	0.0373	0.0193
0.75	0.2886	0.1266	0.0664	0.0270	0.0195	0.0085
1.00	0.1573	0.0629	0.0327	0.0180	0.0095	------
1.25	0.0771	0.0274	0.0146	------	------	------
1.50	0.0339	0.0121	------	------	------	------



TEMPERATURE HISTORY IN A SEMI-INFINITE SOLID

PLATE I

TABLE II

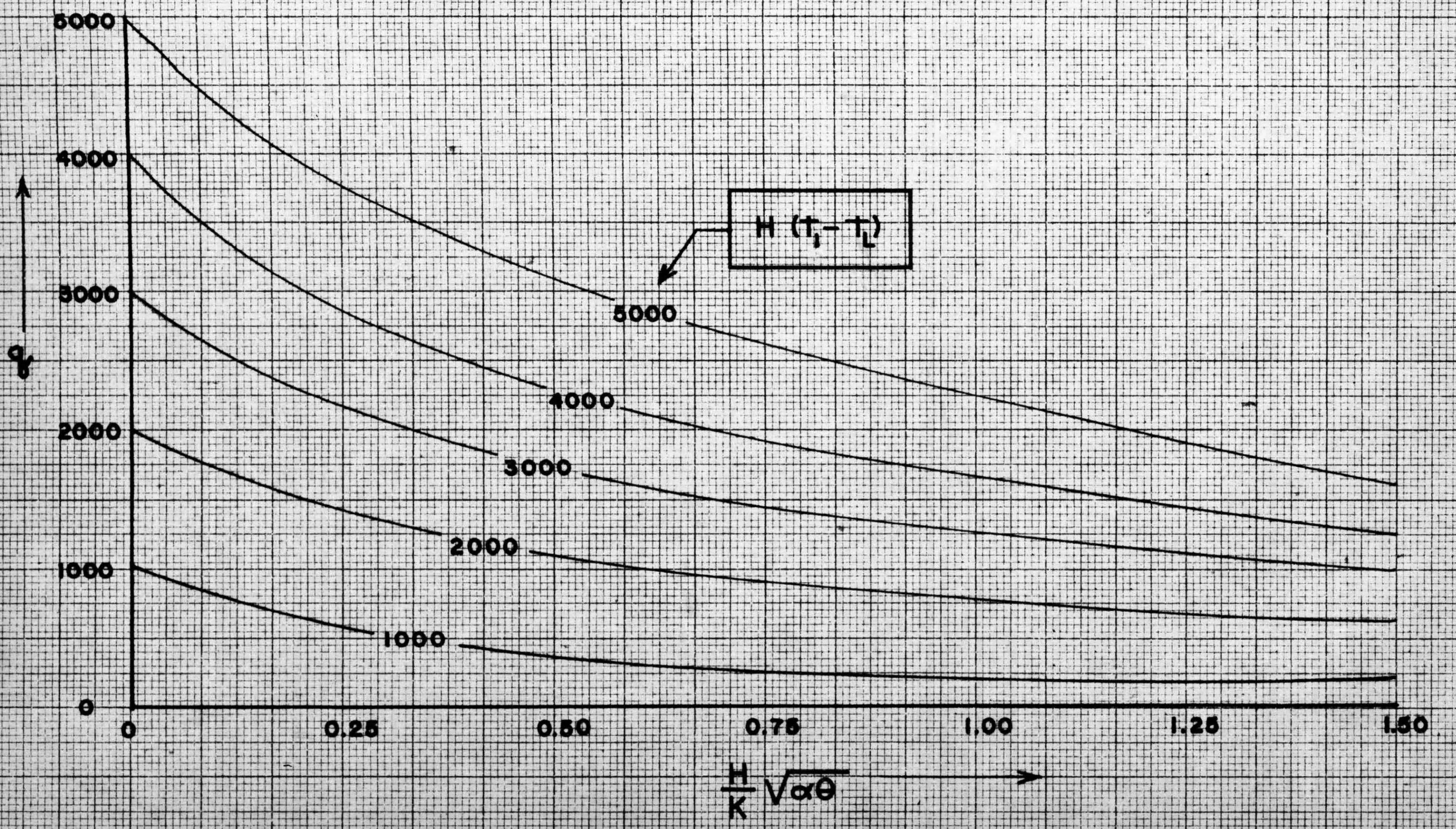
DATA FOR DETERMINING THE HEAT TRANSFER FROM A SEMI-INFINITE SOLID TO A LIQUID

From Equation (29)

$H(t_i - t_L)$	5000	4000	3000	2000	1000
$\frac{H\sqrt{\alpha\theta}}{k}$			q		
0.00	5000	4000	3000	2000	1000
0.25	3852	3082	2311	1541	704
0.50	3078	2463	1847	1231	616
0.75	2531	2024	1518	1012	506
1.00	2138	1710	1283	855	427
1.25	1839	1471	1103	736	368
1.50	1608	1286	965	643	322

PLATE 2

CHART FOR DETERMINING THE HEAT TRANSFER FROM A SEMI-INFINITE SOLID TO A LIQUID



CONCLUSIONS

The analytical reasoning and mathematical relationships, as set fourth in this investigation, determine a relationship for the unsteady state heat transfer from a hot body to a liquid when a vapor film forms between the hot body and the liquid. It also provides a relationship for determining the temperature distribution within the hot body as a function of position and time.

This solution was determined for the case of a semi-infinite solid quenched in a large enough mass of liquid to maintain the liquid temperature essentially constant. Before the solution can be put to practical use, the film coefficients involved will have to be determined experimentally.

When the necessary film coefficients are determined, this solution should furnish a relative quick and simple method for determining the unsteady state heat transfer from a hot body to a liquid and the temperature distribution within the hot body.

SUMMARY

The temperature distribution in a hot body suddenly quenched in a large mass of liquid involving a vapor film between the hot body and the liquid was found by the use of Laplace transforms. From this relationship the temperature of the surface of the hot body at any time was determined. By subtracting the temperature of the liquid from the temperature of the surface of the hot body the temperature difference was determined. By using the general equation for heat transfer involving film coefficients the equation for the heat transfer from the hot body to the liquid was obtained.

Graphs were included to expedite the determination of temperatures and heat rates.

There are many unsteady state heat transfer problems left to be investigated. Other problems are readily suggested by extending this problem for such various shaped hot bodies as parallelpipeds, cubes, and spheres. The author hopes that at some future date the film coefficients involved will be determined experimentally for various materials so that this solution can be put to practical use.

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VITA

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