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A study of shear failures in footing

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A STUDY OF SHEAR FAILURES IN FOOTINGS

BY

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A

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ABSTRACT

The purpose of this investigation was to conduct a model study of the basic mechanism of failure in shear of reinforced concrete footings through the use of plaster models.

Sixty-nine models were constructed and loaded to failure on a foundation of sand using various combinations of size and depth of model and column size. Load - deformation data and failure loads were recorded.

It was determined that model studies are a feasible procedure to analyse the mechanism of failure in footings. The perimeter of the loaded area was found to be the critical section in determining the ultimate shearing strength of footings rather than some arbitary distance out from the face of the column. A dimensionless plot was developed to correlate all the data and an equation is proposed to predict ultimate loads.

ACKNOWLEDGMENT

The author wishes to express his sincere gratitude to Mr. James E. Spooner and Dr. Thomas S. Fry of the Civil Engineering Department of the University of Missouri at Rolla. Without their dedication to the advancement of knowledge concerning basic structures and the field of foundation engineering, the author would not have been able to thoroughly appreciate all aspects involved in this investigation.

My deepest feeling of gratitude is towards my beloved wife, Jane Williams Cole, for her support and encouragement which has been ever present.

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I. INTRODUCTION

For many years, the methods used in the design of reinforced concrete footings were based largely on the studies made on this subject by Prof. A. N. Talbot (1) and published in 1913. Since the time of this original study, many changes have taken place that have necessitated a continual re-evaluation of accepted procedures to keep footing design abreast of technical progress. There has been a steady evolution in design procedures and improvement in materials. The normal design practice, through many years of practical application, has yielded safe and reliable structures but the ultimate strength design methods for reinforced concrete have been receiving steadily increasing interest during the past ten to fifteen years.

In spite of the efforts of numerous renowned researchers, no general theory is presently available by which footings failing in shear can be thoroughly described. A great deal has been accomplished through these many investigations, and it is indeed fortunate that safe design equations apparently can be developed without a full understanding of the fundamental laws governing the phenomenon under consideration.

The major efforts in the field of study of shear strength in slabs has been devoted to an analysis of the problems encountered in flat plate floor systems. It can be readily seen that the critical area of such a structure is the connection between the slab and column. Maximum bending moments and large shearing forces are concentrated here, and the integrity of the entire structure is to a large extent governed by the degree to which the ultimate strength of this area can be predicted and utilized.

The subject of this investigation, the problem of shear strength in footings subjected to concentrated loading, has many similarities with the modern flat plate floor systems since the footing in many respects acts as a portion of an inverted flat slab. The relationship however is complicated by the fact that the footing rests on soil. These complications become evident when some of the basic assumptions of soil-footing interaction are compared to the actual stress distributions. One such assumption is that the pressure from a concentrated load is assumed to be distributed uniformly over the soil in contact with the bottom of the footing if the load is applied at the center of gravity of the footing's bearing area. In actuality these stresses may vary a great deal and the shape of the distribution is dependent upon the soil character. Thus the methods used in the design of footings are still largely empirical and have been derived from data obtained from research on flat plate floor systems or, as in the case of the work of F. E. Richart, on supporting material other than soil.

For a rational design, more information is needed concerning the behavior of footings on real foundation materials. In an attempt to establish the more significant variables of interest, this investigation studies the behavior of small rectangular plaster footings, resting on sand. The variables investigated include column size, footing depth, and the geometry of the footing.

II. REVIEW OF LITERATURE

Prior to the year 1900 there were two schools of thought regarding the method of failures in shear in reinforced concrete members. One group looked on horizontal shear as the basic cause of shear failures. The shearing stresses were computed using the equation:

$$v = \frac{VQ}{It} \qquad 1$$

where v = unit horizontal shear stress at a distance y from the neutral axis,*

V = total vertical shear at the section,

I = moment of interia of the cross-sectional area with respect to the neutral axis,

Q = first moment of the part of the cross-sectional area cut off at a distance y from the neutral axis,

t = width of the cross-section at a distance y from the neutral axis.

The second school of thought, and the most readily acceptable approach today, considers diagonal tension the basic cause of shear failures. In the early 1900's E. Morsch of Germany proposed the nominal shearing stress equation:

$$v = \frac{V}{bjd} \qquad 2$$

where jd = internal moment arm,

b = perimeter of critical peripheral section.

*Symbols once presented in an equation and explained will not be explained again unless a distinct meaning is intended. A listing of all symbols used is contained in Appendix A.

An evaluation of the principal stress theory reveals the fact that equation 2 is not a valid measure of the diagonal tension above and below the neutral axis. However since the diagonal tensile stress at the neutral axis does equal the vertical shear, the shear stress determined by this approach is considered as an overall measure of the diagonal tension.

The results of the first extensive study of the shear strength of slabs and footings, conducted in this country, were published by A. N. Talbot (1) in 1913. Basically he utilized equation 2, calculating the shear stress at a distance d from the face of the column. The proposed formula was:

$$v = \frac{V}{4(r+2d)\,jd} \qquad 3$$

where r = side dimension of square column,

d = effective depth of slab.

A significant result of Talbot's test was that relatively high values of shear strength were obtained when high percentages of tensile reinforcement were used in the slabs. This publication and later works by Talbot were used extensively throughout the world in design practices.

In 1915, O. Graf and C. Bach of Germany completed a set of experiments mainly designed to investigate flexural strength. A few of the slabs, loaded at the center only, failed in shear rather than in flexure. A portion of the slab, within the area of failure, was pushed out underneath the load and had the form of a truncated cone.

O. Graf completed, and reported on, a series of shear tests on slabs loaded by concentrated loads near supports. The results indicated that the shear capacity decreased as the load was moved away from the supports. He proposed the following shear stress equation:

$$v = \frac{V}{4rt} \qquad 4$$

where t = total depth of slab.

Graf suggested that flexural cracking may have some influence on shear strength as his research had indicated that shear strength increased with concrete strength, but at a lower rate than compressive and tensile strength.

In 1939, F. E. Richart and R. W. Kluge (2) conducted and reported on an investigation of reinforced concrete slabs simply supported on two edges only. The slabs had an effective depth of 5.5 in. and were 5 ft. square. The loaded areas were circular, 6 and 2 in. in diameter. The series of this investigation that developed shear failures was so designed to provide information on the effect of the size and shape of the load-bearing area.

In this latter investigation, the ultimate shear stresses for rectangular slabs, as computed by equation 3, were approximately 0.08 times the cylinder compressive strength f'_c. However in actuality only about 60 percent of this calculated stress was obtained prior to failure of square slabs. One of the conclusions arrived at by the authors was that the stresses obtained by equation 3 are only nominal and arbitrarily chosen, and that an increase in flexural strength of the square slabs would have increased the shear strength.

From 1946 through 1953 an extensive program of investigations on highway bridges was conducted by the University of Illinois and reported on by N. W. Newmark, C. P. Siess, et al. (3) (4) (5). Their first report included tests of 15 models of simple spans, right angle

I-beam bridges. All slabs failed in shear at loads considerably higher than those causing first yielding of the tensile reinforcement. The average value of the ratio of the final shearing load to that causing yielding, was found to be as high as 1.8, thus indicating that the ultimate flexural capacities of the slabs probably were almost exhausted at the time of shear failure. It was a conclusion of the authors that loads at shear failure, to some degree, were dependent on the same factors as the loads at first yielding.

F. E. Richart (6), in 1948, reported on the results of an extensive investigation of reinforced concrete footings. In all, 24 wall footings and 140 column footings were tested to failure. The column footings were composed of a series of 128 seven foot square footings, and the remaining 12 being divided among the rectangular shapes of 6 x 9 ft. and 6 x 10 ft. Variables investigated included: amount, strength, bond characteristics and end anchorage of tensile reinforcement. Other variables evaluated were concrete strength and effective depth of the footings. From the report of the data it is apparent that 106 of these column footings failed in shear.

The most important conclusions of Richart's report were that shear stress rather than bond stress may frequently be a critical feature in the design of a footing. The shear stress at failure, calculated by equation 3, at a distance d from the faces of the columns generally varied from less than 0.05 f'_c to 0.09 f'_c. In addition he found that the value of v/f'_c increased consistently as the effective depth of the footing decreased. In his experiments most of the footings which failed initially due to tension, or excessive bond slip, collapsed finally by a diagonal tension failure with a pyramidal plug of concrete punching through the slab beneath the column.

The effect of extensive cracking in the footing due to high tensile stress in the flexural reinforcement and probably due to slip of bars in the interior of the footing, evidently caused early failure by a diagonal tension collapse.

This investigation by Richart covered a range of materials and designs for which experimental verification was, at that time, largely lacking. Some of the results obtained were very much unexpected. For the first time someone proposed that shearing stress rather than bond stress might frequently be the critical feature in the design of a footing.

In 1953, E. Hognestad (7) published the results of a re-evaluation of the shear failures of footings which were reported by Richart. In this study Hognestad recognized the interaction of flexure and shear. Evaluating the effect of superimposed flexure on ultimate shear strength, he introduced the ratio $\emptyset = V_{test}/V_{flex}$ as one of the parameters in his statistical study of Richart's test results. In this ratio, V_{test} is the observed shear force at shear failure, V_{flex} is the shear force at ultimate flexural strength as computed by the yield-line theory. Hognestad suggested that shear stress should be computed at zero distance around the loaded area because this seemed to provide the best measure of shear strength.

The following ultimate shear strength equation was found to apply within the range of variables covered by Richart's tests.

$$v = \frac{V}{7/8bd} = \left(0.035 + \frac{0.07}{\emptyset_o}\right) f'_c + 130 \text{ psi} \qquad 5$$

Equation 5 was considered by its developer to be valid for values of r/d (the ratio of the column width to the effective depth of the footing) between the limits 0.88 and 2.63 but to be unsafe for values of f'_c below 1800 PSI.

In 1956, E. Hognestad in conjunction with R. C. Elstner (8), reported on the tests results of thirty-eight 6 ft. square slabs which were loaded in the center, and, in the majority of cases, supported along all four edges. The major variables were: concrete strength, percentage of flexural tension and compression reinforcement, percentage of shear reinforcement, and the size of the column. The specific effect of the concentration of the flexural reinforcement was also explored. No effects on the ultimate shear strength were found due to the variation in concentration of the tension reinforcement under the column or the amount of compression reinforcement. The tests indicated that the ultimate shear strength computed from equation 5 was unsafe for high concrete strengths (4500-7300 PSI). Hognestad completed a statistical analysis of this and other series of tests and found that agreement with the actual tests data could be obtained through the use of the equation:

$$v = \frac{V}{7/8bd} = 333 \text{ PSI} + 0.046\ f'_c/\phi_o \qquad 6$$

For slabs with special shear reinforcement, the following equation was suggested:

$$v = 333 \text{ PSI} + 0.046\ \frac{f'_c}{\phi_o} + (gu - 0.050)f'_c \qquad 7$$

where $gu = \dfrac{A_v\ f_y\ \sin\alpha}{7/8\ bd\ f'_c}$ 8 where A_v = area of shear reinforcement.

f_y = yield point of shear reinforcement,

α = inclination of shear reinforcement with base of slab.

Inspection of equation 7 indicates that the shear reinforcement is not fully effective.

An ultimate strength theory for shear which is radically different from earlier approaches to the problem was proposed by C. S. Whitney (9) in a 1957 report. He based his study on previously reported test results of Richart, Elstner and Hognestad on slabs and footings but excluded a number of tests which he considered involved bond failure. These excluded slabs which had a large amount of tension reinforcement consisting of closely spaced bars. For the remaining slabs, Whitney assumed that the shear strength is primarily a function of the ultimate resisting moment "m" of the slab per unit width inside the "pyramid of rupture," i.e., the frustum of a cone or pyramid with surfaces sloping out in all directions from the column at an angle of 45 degrees.

Whitney proposed the following ultimate shear strength equation:

$$v = 100 \text{ psi} + 0.75 \left(\frac{m}{d^2}\right) \left(\frac{d}{\ell_s}\right) \qquad 9$$

where v = shear stress computed at a distance of d/2 from surfaces of loaded areas,

ℓ_s = "shear span." For a slab supported along the edges it is taken as the distance between the support and the nearest edge of the loaded area. For a footing with uniform distribution of the reaction, ℓ_s is taken as half of the distance between the edge of the footing and the face of the column.

Since the test results of specimens with relatively high flexural strengths were omitted by Whitney in developing equation 9, it can only be applied in cases when ϕ_o is close to unity.

An analysis of equation 9 by the author led to the conclusion that the shear strength of a slab can be effectively increased by adding to the amount of flexural reinforcement inside the pyramid of rupture. The shifting of tensile reinforcement from outside to inside the "pyramid of rupture" should also effectively increase the shear strength.

To compute the ultimate shear force V in footings, Whitney subtracted the support reaction inside a distance of d/2 from the faces of the column. The majority of other investigators have subtracted the total support reaction on the base of the "pyramid of rupture," or at a distance d out from the face of the column.

The most thorough, complete and up to date study of the shear strengths of slabs based on practically all of the available data was published by Johannes Moe (10) in 1961. Moe reported on tests of forty-three 6 ft. square slabs which were similar to the test specimens of Elstner and Hognestad. The principal variables were: effect of openings near the face of the column, effect of concentration of tensile reinforcement in narrow bands across the column, effect of column size, effect of eccentricity in applied load, and the effectiveness of special types of shear reinforcement. He also presented a statistical analysis of 260 slabs and footings tested by earlier researchers.

Some of the more important conclusions arrived at by Moe, and of particular interest in this research, are:

1. The critical section governing the ultimate shear strength of slabs and footings should be measured along the perimeter of the loaded area.

2. The shear strengths of slabs and footings are affected by flexural strength.

3. The triaxial state of stress in the compression zone at the critical section influences the shear strength of that section considerably.

4. The shear strength of the concrete is highest when the column size is small compared to the slab thickness.

5. The ultimate shear strength of slabs and footings is predicted with good accuracy by the formula:

$$v_u = \frac{V_u}{bd} = \left[15\ \left(1-0.075\ \frac{r}{d}\right) - 5.25\ \phi_o\right] \sqrt{f'_c} \qquad 10$$

where b = perimeter of the loaded area,

d = effective depth of slab,

r = side length of square loaded area,

V_u = ultimate shear force,

V_{flex} = ultimate shear force if flexural failure had occurred,

$\phi_o = V_u/V_{flex}$

for footings:

$$V_u = \left[1 - \left(\frac{r+d}{a}\right)^2\right] P_u \qquad 11$$

where P_u = total load on footing,

a = side length of square footing slab.

6. Inclined cracks develop in the slabs at loads as low as 50 percent of the ultimate.

7. Loads 50 percent above the inclined cracking load, sustained for three months, did not affect the ultimate shear strength.

8. The effect of openings adjacent to the column may be accounted for by introducing the net value for the perimeter b into equation 10.

9. Concentration of flexural reinforcement in narrow bands across the column did not increase the shear strength. However, such concentration did increase the flexural rigidity of the test slabs, and also increased the load at which yielding began in the tension reinforcement.

10. Some increase in shear strength can be obtained by shear reinforcement. However the anchorage of such reinforcement in the compression zone seems to be problematical, therefore the use of shear reinforcement in thin slabs was not recommended.

11. In cases of moment transfer between square columns and slabs, test results indicate it is safe to assume that one-third of the moment is transferred through vertical shear stresses at the perimeter of the loaded area distributed in proportion to the distance from the centroidal axis of the loaded area. Maximum shear stress due to the combined action of vertical load and moment should not exceed the value expressed by the equation 10.

The limited nature of knowledge previously available regarding the mechanism of failure in shear of slabs and footings under concentrated loads is reflected in the standard specifications of other countries and in the steadily changing approach here in the United States. Various rules, which differ to a considerable degree, are used to determine what is considered to be the critical shear or inclined tensile stresses. The allowable stresses also have a wide variation.

In this country the first standard specifications, published in 1913, stipulated an allowable shear stress in pure shear equal to $0.06\ f'_c$. This shear stress was to be computed by the equation $v = V/bt$, where the critical section was to be taken along the perimeter b of the loaded area, and the t was the total slab thickness. The revised version of 1917 required that the diagonal tension requirements be met but provided no rules by which to determine this diagonal tension stress.

The ACI standard of 1916 allowed a stress in pure shear equal to $0.075\ f'_c$ and required it be computed along the periphery of the loaded area. It wasn't until 1920 that a distinction was made between the two types of possible shear failure. The ACI standard of 1920 stated:

(a) A pure shear failure controlled by the allowable shear stress computed at zero distance from the periphery, and stipulated at 0.10 f'_c.

(b) A so-called "diagonal tension failure" controlled by shear stress computed by the formula $v = V/bjd$ at a distance of d/2 from the periphery, and limited to $0.035\ f'_c$.

The Joint Committee of 1924 specified that the shear stress should be computed at (t - 1½ in.) from the periphery of the loaded area, and the allowable shear stress was computed by:

$$v = 0.02\ f'_c\ (1 + n) \leq 0.03\ f'_c \qquad 12$$

where n = ratio of the area of the reinforcing steel crossing directly through the loaded area to the total area of tensile reinforcement.

The American Concrete Institute adopted the Joint Committee report of 1924 as a standard and only minor changes have been made with respect to shear and diagonal tension in slabs and footings.

The 1963 ACI Building Code (318-63) allowed the following shear stresses, in working stress design, computed on a critical section at a distance d/2 beyond the face of the column.

(a) $2 \sqrt{f'_c}$ unless shear reinforcement of a specified nature is provided.

(b) $3 \sqrt{f'_c}$ with specified shear reinforcement. Allowable stress in shear reinforcement limited to 50 percent of that prescribed under normal reinforcement circumstances of the working stress design.

This nominal shear stress will be computed by:

$$v = \frac{V}{b_o d} \qquad 13$$

where b_o = periphery of critical section.

Under the provisions of the Ultimate Strength Design, the nominal ultimate shear stress v_u will be computed again at a distance d/2 and will be computed by:

$$v_u = \frac{V_u}{b_o d} \qquad 14$$

where V_u = total ultimate shear.

This ultimate shear stress, so computed, shall not exceed $v_c = 4\,\theta \sqrt{f'_c}$, unless specified shear reinforcement is provided, in which case v_u shall not exceed $6\,\theta \sqrt{f'_c}$. When v_c does exceed $4\,\theta \sqrt{f'_c}$, shear reinforcement yield strength shall be 50 percent of that prescribed for normal circumstances under the Ultimate Shear Stress design, where θ = capacity reduction factor of 0.85.

In Germany a completely different approach to the design problem of shear in slabs is practiced. In determining shear as well as flexural stresses, slab strips of prescribed widths are assumed. The widths given for shear computations are different from those in moment, and the widths also vary with the position of the load on the slab. The Germany Specification DIN 1045 of 1943 gives the following formulas for the effective slab strip width in shear:

$$b_1 = r + 2s \text{ and } b_2 = \frac{1}{3}\left(\ell + \frac{r + 2s}{2}\right) \qquad 15$$

where s = thickness of a load-distributing layer on the top of the slab,

ℓ = length of the span of the slab.

The larger of the values b_1 and b_2 can be used. In the case of a load close to one of the supported edges, b shall be taken as r + 5t.

In other countries a combination of the American and German practice is being used. The Norwegian Standard Specifications of 1939 assumed the shearing stresses to be evenly distributed around the loaded area at a distance of 2d/3 from the periphery. It is also necessary to consider a strip of the slab of a certain specified width as a beam and to check the shearing stresses in this beam strip. If a load is placed close to one of the supported edges of a slab, this last check frequently provides the largest shearing stresses.

III. MATERIALS, FABRICATION, AND TEST PROCEDURE

A. MATERIALS

1. The conditions of similitude require similarity between the model physical properties and the prototype structure. When initially considering a material to simulate reinforced-concrete, the first choice was logically a scaled down concrete mix. The properties of such a model material should be identical to those of the prototype material and as such could be used to provide behavior up to failure including the formation of cracks and "elastic" deformations. There are significant disadvantages to a scaled down concrete mix. Two of these are the relatively long time which is required to cure the model, and the alterations in shape that occur due to shrinkage.

These disadvantages lead to the consideration of plaster as a suitable material. The initial setting time can be adjusted from as fast as 10 minutes up to any desirable period by changing the water/plaster ratio. Gain of strength after initial set is rapid. In addition, it was found that plaster can be worked for a period of time after gaining initial set.

The material used for the models of this study was a high strength gypsum plaster, commercially sold as Hydrastone, a product of the U. S. Gypsum Company. An evaluation of well-cured plaster with regard to Young's modulus reveals that its behavior, almost up to failure, is practically linear. The modulus decreases in a non-linear manner with increasing water/plaster ratio and increasing with time. Poisson's ratio is found to have an approximate value of about 0.16. In addition green plaster is found to have a non-linear stress-strain relationship similar to that of concrete. Therefore, plaster, as it affords the structural characteristics of concrete, was chosen as the basic material for use in this model study.

The plaster mixture used had a water/plaster ratio of 0.7. It was not designed to meet any particular compressive strength requirement but this relatively weak strength mix, 1400 to 1500 psi, was found to provide sufficient strength to, in general, insure failure in diagonal tension. Richer design mixtures were tried but difficulty was encountered in having the models fail in diagonal tension without overloading test equipment, removing the air entrained during mixing, and in casting operations.

2. The wire reinforcement used was a uniform ¼ x ¼ inch mesh steel fabric of refined copper alloy steel, 23 gage, plated with 8% electrolytic zinc. This is commonly available as "hardware cloth."

B. FABRICATION OF MODELS

1. While it is best to obtain a geometric similarity between the model and prototype, it is sometimes necessary to depart from this approach. The models of this study are not necessarily related, through dimensional analysis, to a full scale structure or prototype of a footing. This departure from the laws of similitude has been necessary in order to study the range and type of response obtained when varying specific parameters. However, the dimensions or geometry of the models was chosen in such a manner as to duplicate the ratios of column size to depth, column size to width, and length to width, that would be encountered in actual practice. The size of the models used in this investigation was 15 in. x 15 in.,

12.5 in. x 18 in., and 10 in. x 22.5 in., thus all areas were equal at 225 square inches. The effective depth and column size were varied, in combination, from 1, $1\frac{1}{2}$ and 2 inches. See Tables 1, 2 and 3.

2. The molds for the plaster models were constructed with plexiglass. Molds were reinforced where necessary and were found to be quite adequate in producing uniform models. Small plexiglass molds were used for production of the columns and the 1 inch plaster cubes produced were also used in the test for compressive strengths.

Holes were drilled in the bottom of the plexiglass mold plate in order that the reinforcing wire could be tied down to insure uniform effective depth of model. This also insured that the mesh was at the bottom of the footing with only a minimum of cover. No problems were encountered in either maintaining the uniform effective depth or in preventing seeping of the liquid plaster through these holes prior to setting.

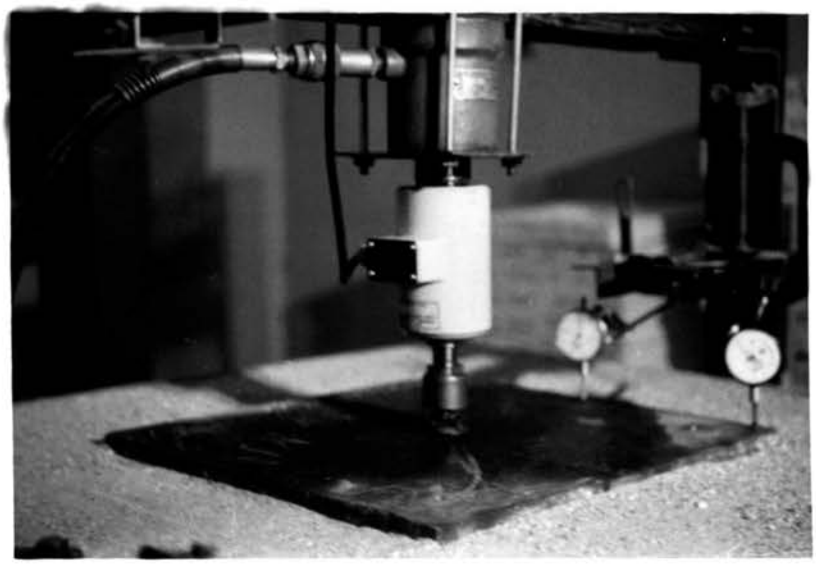
C. TEST PROCEDURE

The models were tested by loading them to failure in the apparatus shown in Figures 1 and 2. The box containing the sand had internal dimensions of 34 inches x 34 inches in plan by 22 inches deep. This container was designed in accordance with the Boussinesq theory to limit the stress on the sides and bottom to 20% of the soil pressure present under the base of the model. A load of 10,000 lbs. or 44.44 psi was assumed for design purposes.

Prior to the actual loading of models the sand was loaded to 10,000 lbs. through an 18 inch square steel plate $\frac{1}{2}$ inch thick. The deformations of the plate were recorded at the center of one side, ½ inch in from the edge, and at a corner, ½ inch from each edge. Pictures of this procedure are shown in Figure 1. After each day's series of model testing was completed, another load deformation test was conducted. The results of one such series of load-deformation testing are shown on Figure 7.

For the actual model loading operations, the model and column being evaluated were leveled and centered under the loading apparatus as shown in Figure 2. The load was applied through the use of a simplex hydraulic pump and load cell. Applied loads and the action of model and column during loading to failure were noted and recorded through the use of a Standard Universal Load Cell with 10 Kip capacity and a Budd/Strainsert Model HW-1 portable strain indicator.

The compressive strength of each model was measured by tests on 1 inch plaster cubes, which were cast at the same time as the model.



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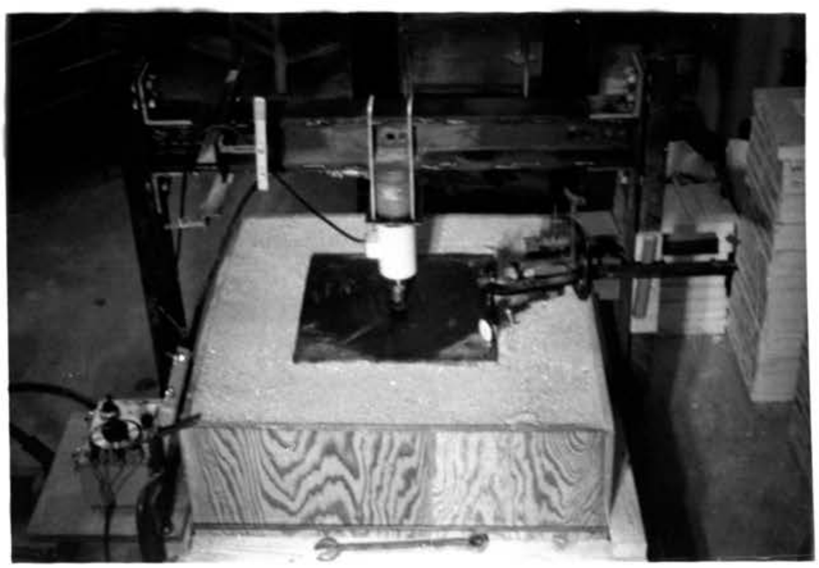

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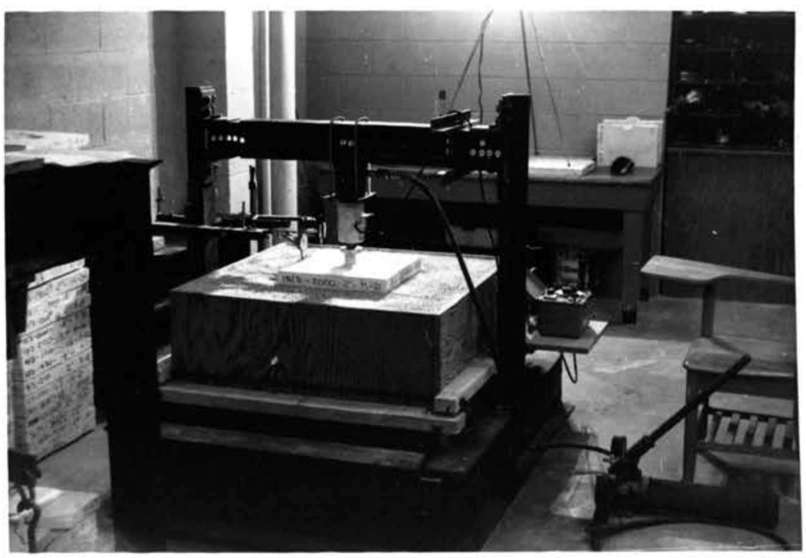

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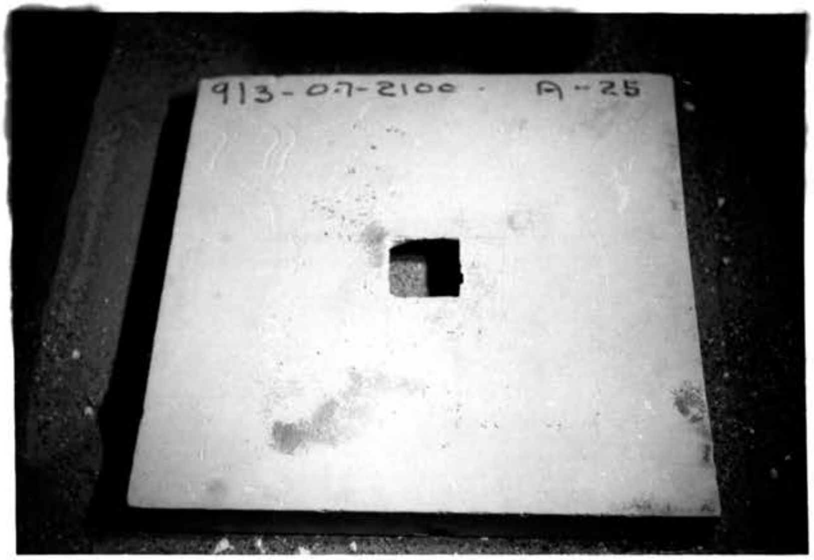
FIGURE 2. MODEL UNDER LOAD



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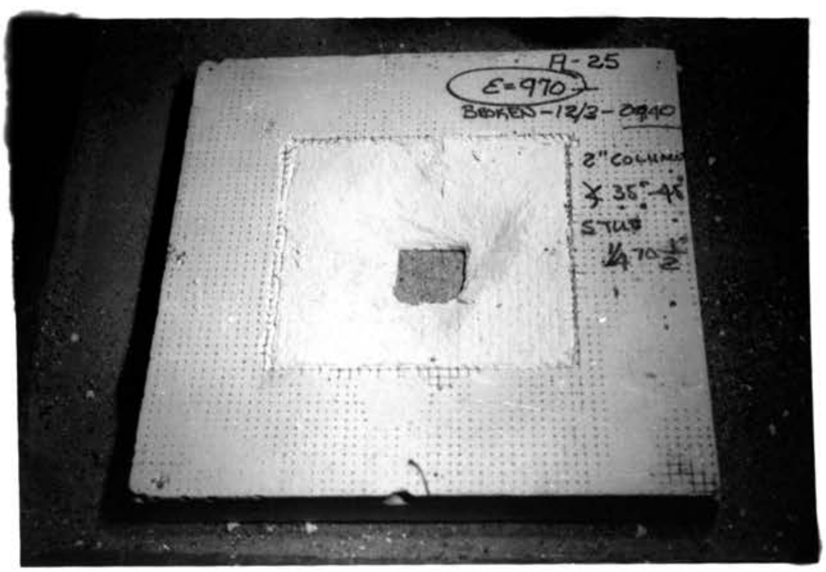

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FIGURE 3. MODEL AFTER FAILURE IN DIAGONAL TENSION

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FIGURE 4. EXAMPLE OF "PYRAMID OF RUPTURE"

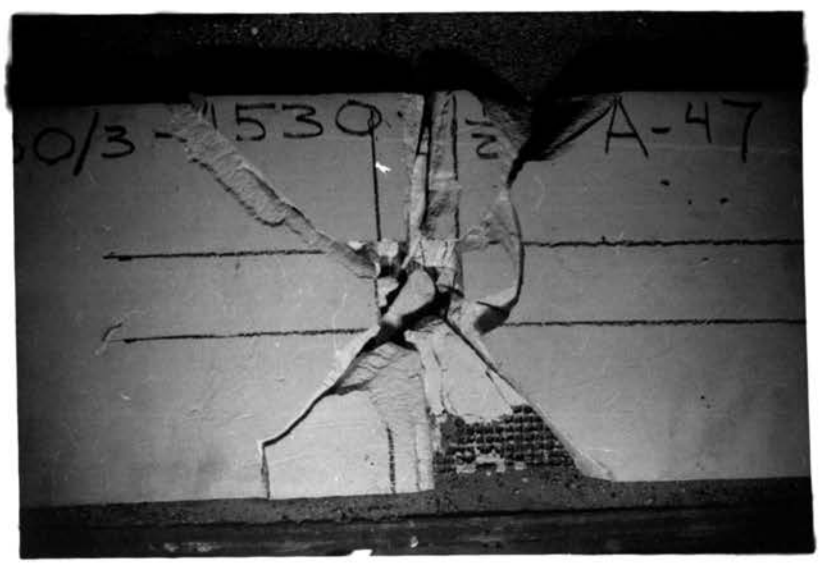

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FIGURE 5. EXAMPLE OF MOMENT FAILURE, 10 in. x 22.5 in. MODEL,

1.5 in. DEEP BROKEN USING A 1.5 in. COLUMN

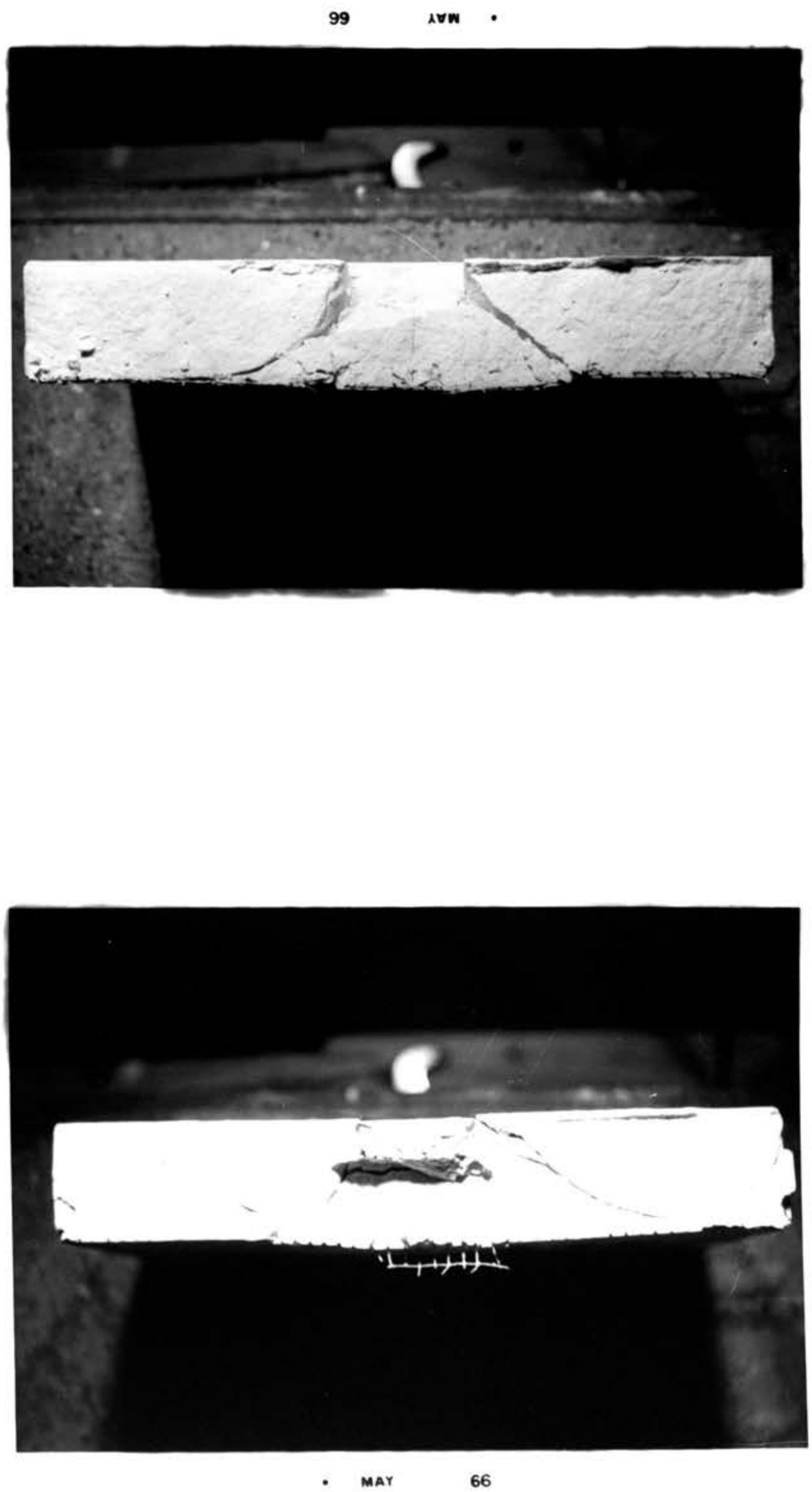


FIGURE 6. EXAMPLE OF FAILURE PATTERNS NOTED IN MODELS

IV. EXPERIMENTAL RESULTS

The results of the laboratory testing have been compiled for analysis and are shown graphically in Figures 7 through 14. The data, from which these graphs are derived, are presented in Tables 1 through 8 in Appendix B.

Figures 8 through 10 are graphical representations of all the data measured for the load versus footing depth for the models. Figure 11 is a composite of these three graphs with only average values being plotted. Average values are plotted on the remaining graphs.

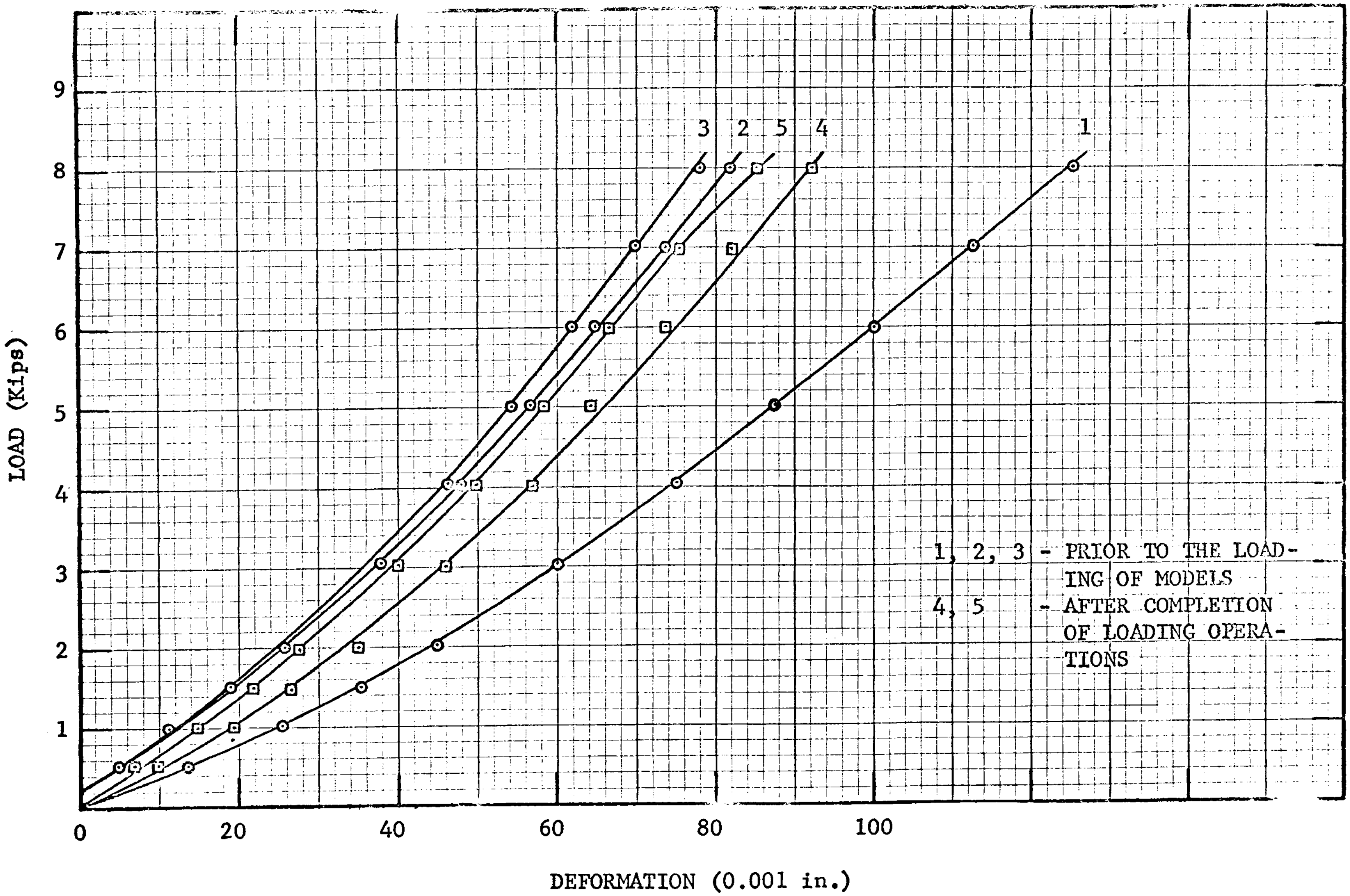


FIGURE 7. LOAD - DEFORMATION RELATIONSHIP

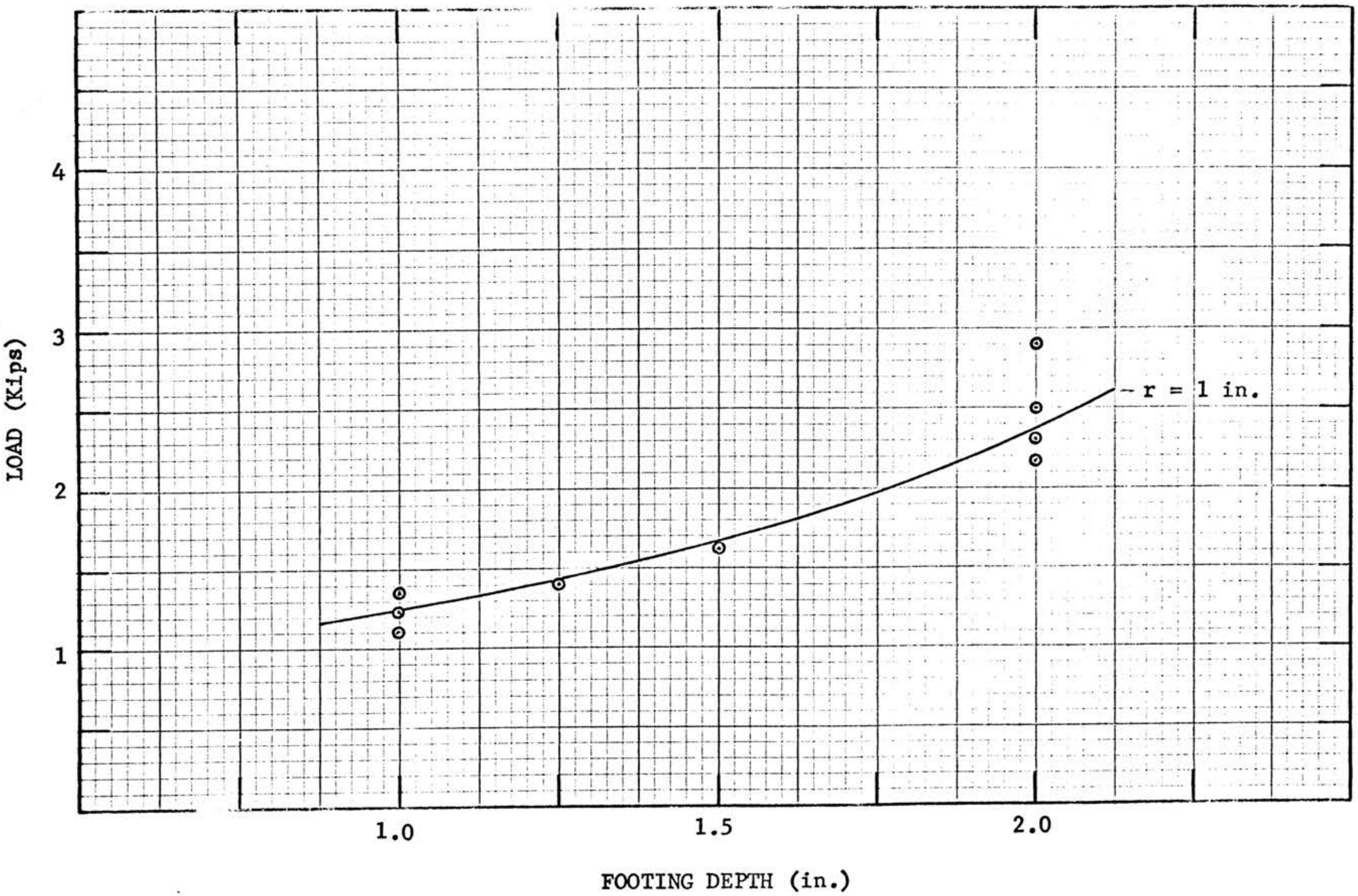


FIGURE 8. LOAD - FOOTING DEPTH, 15 in. x 15 in. MODEL

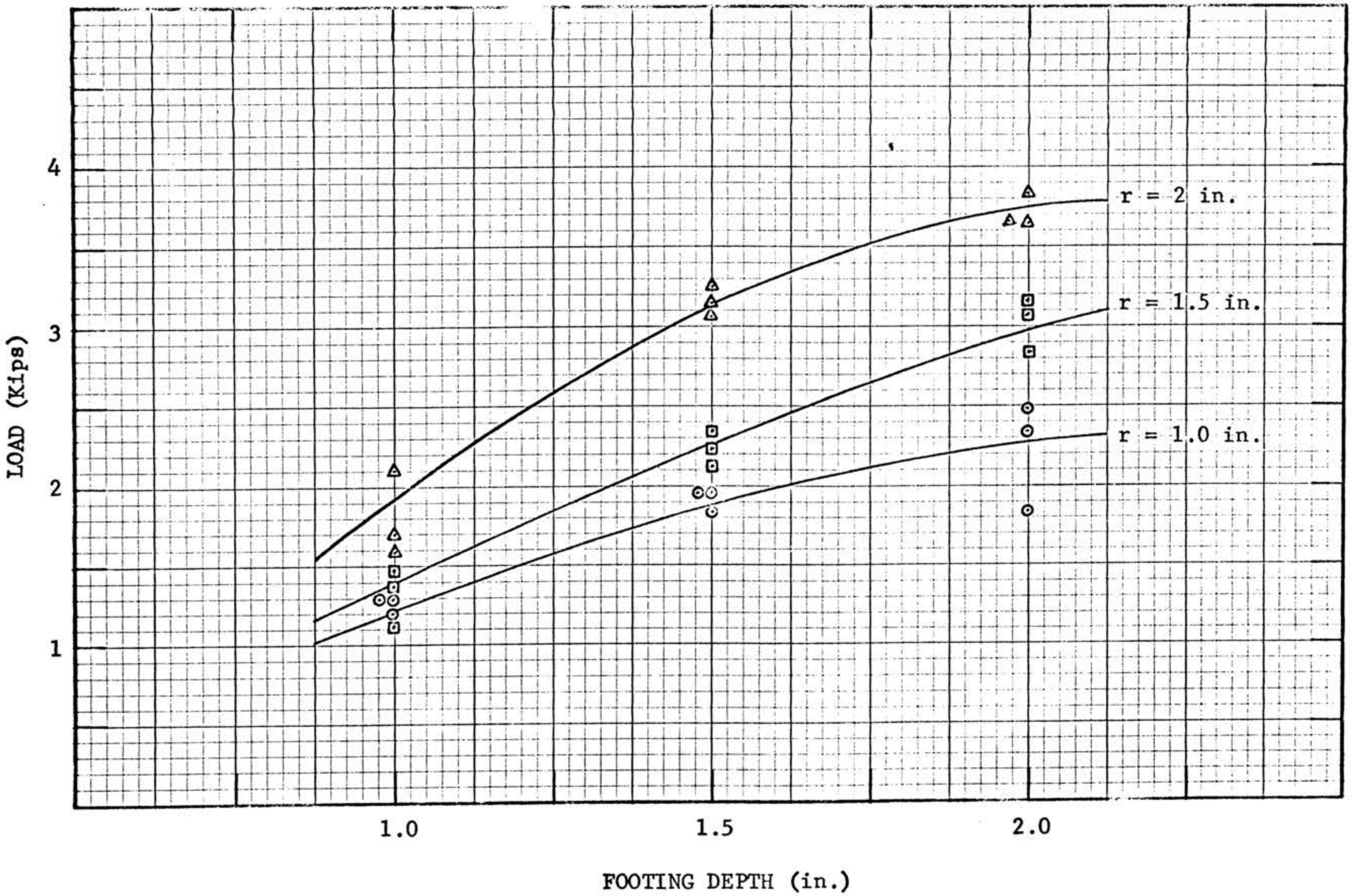


FIGURE 9. LOAD - FOOTING DEPTH. 12.5 in. x 18 in. MODEL

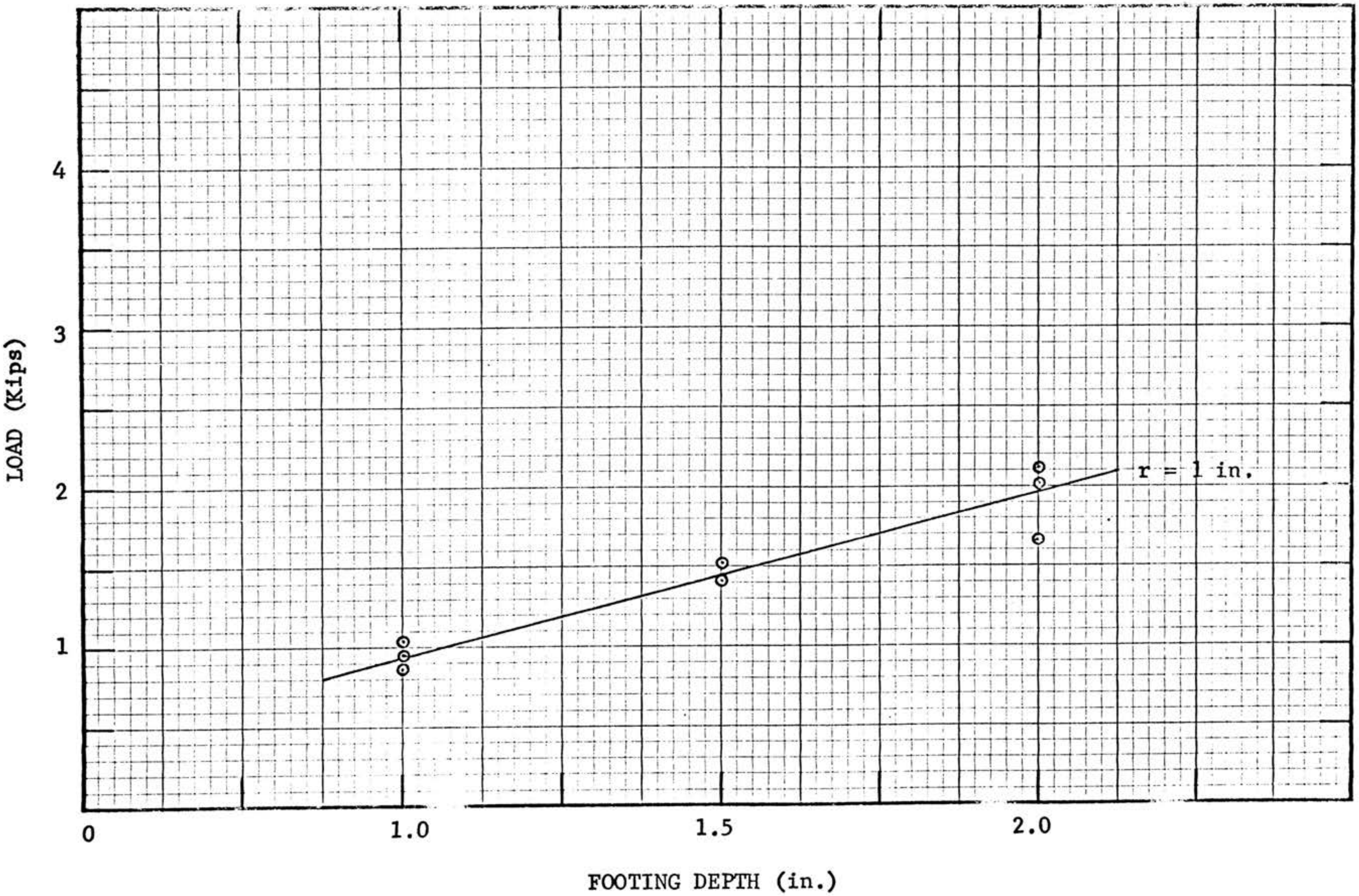


FIGURE 10. LOAD - FOOTING DEPTH, 10 in. x 22.5 in. MODEL

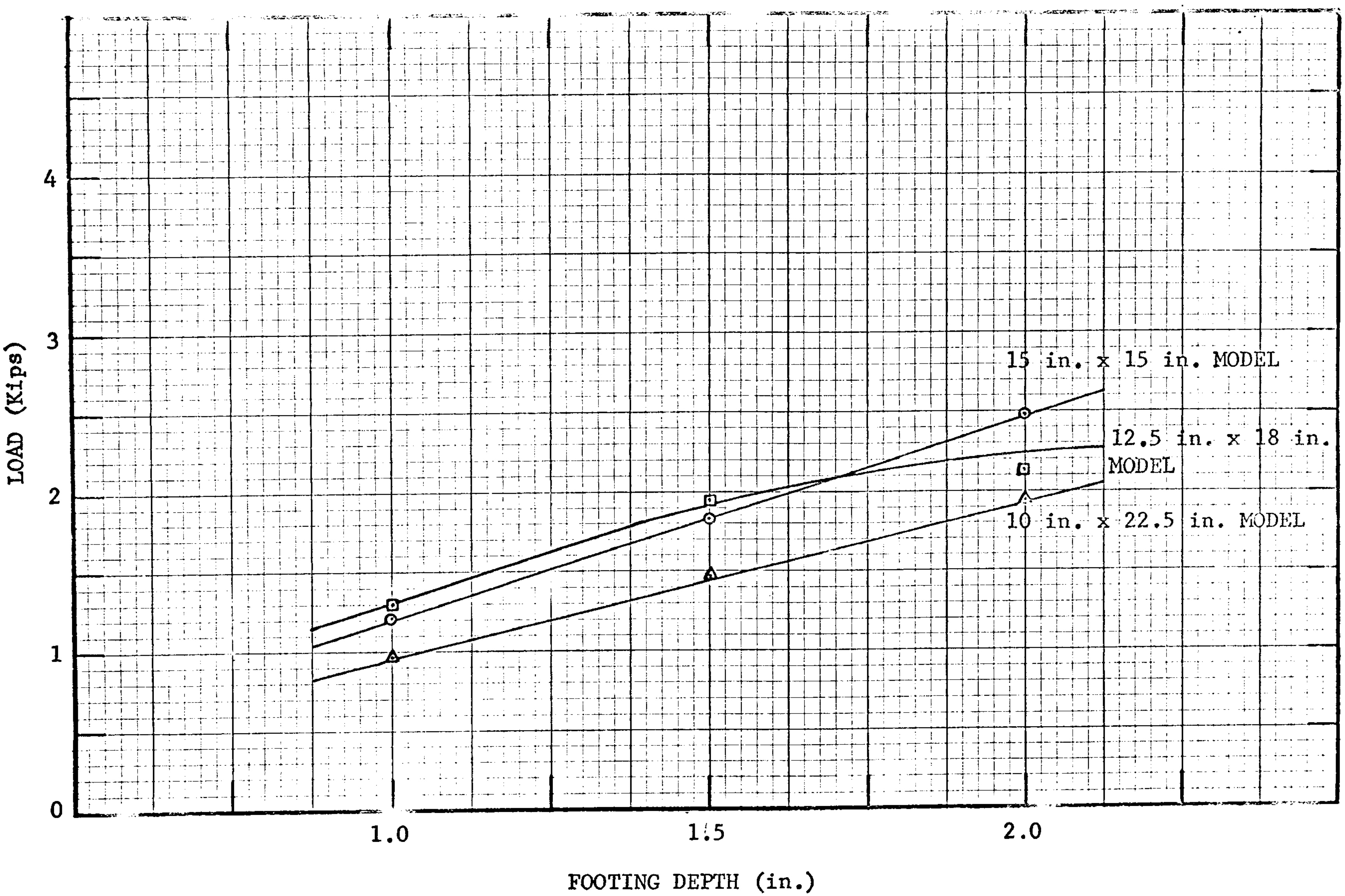


FIGURE 11. LOAD - FOOTING DEPTH (r = 1 in.)

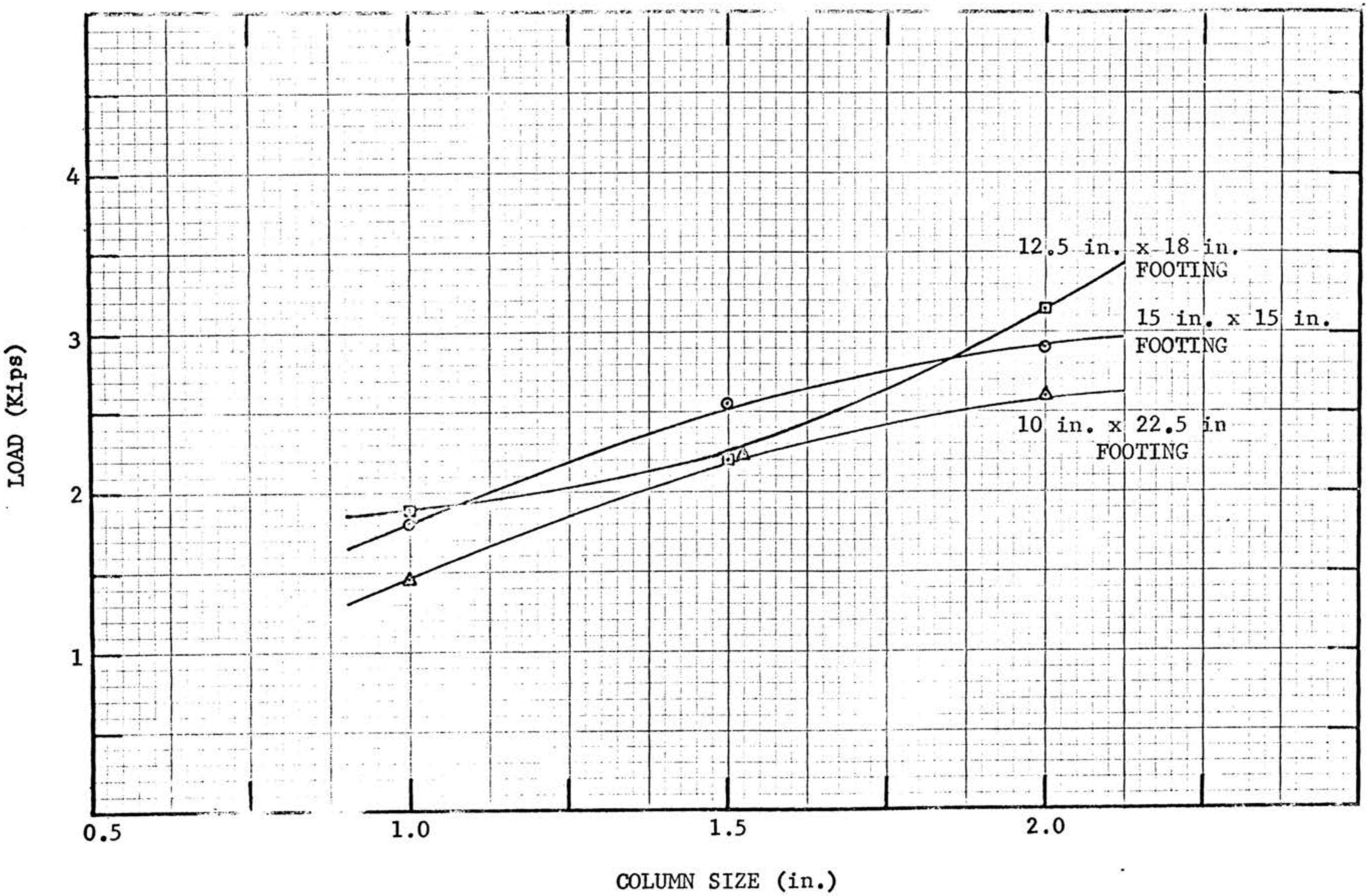


FIGURE 12. LOAD - COLUMN SIZE (d = 1.5 in.)

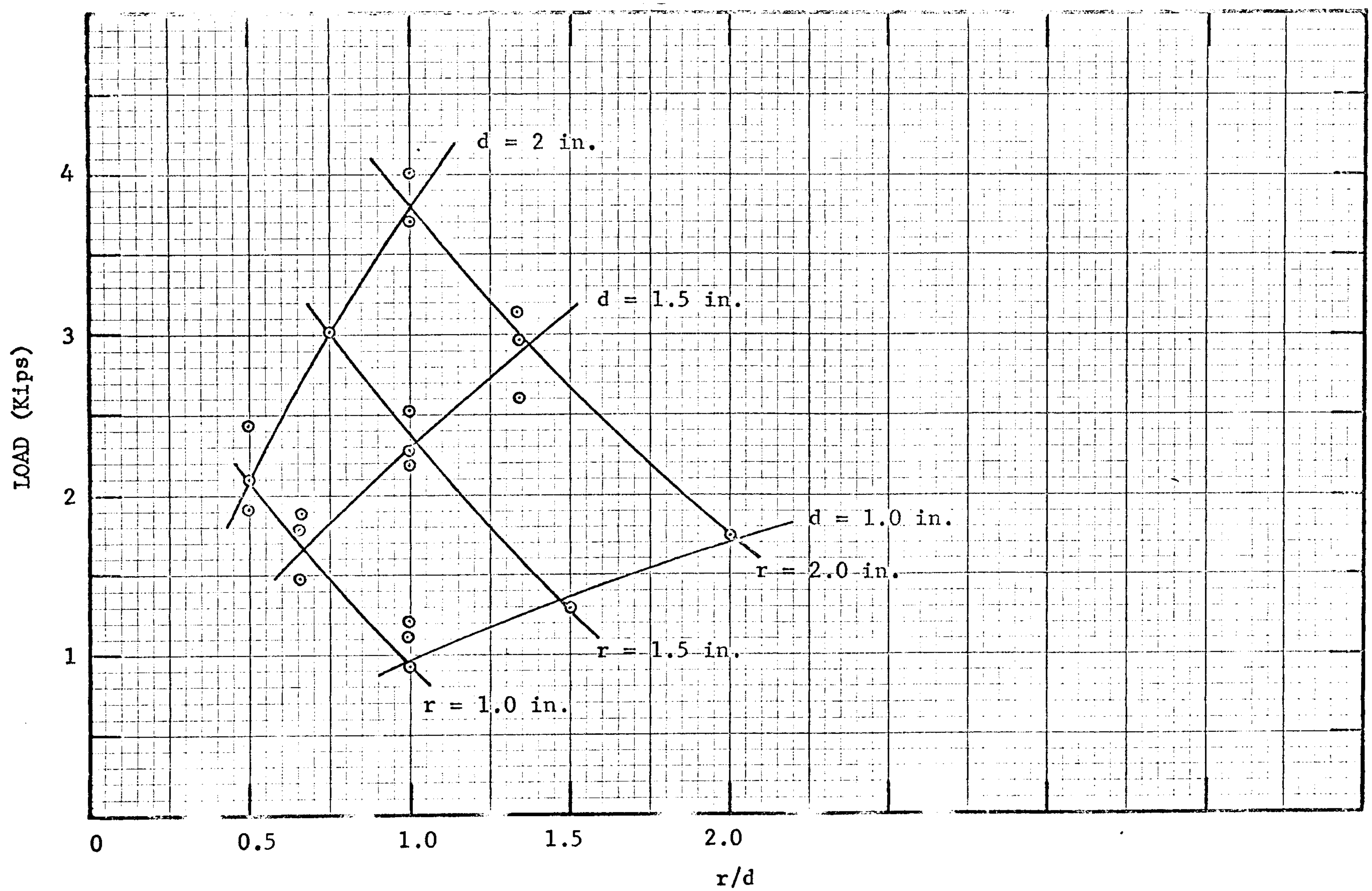


FIGURE 13. LOAD - r/d. COMPOSITE OF ALL FOOTINGS

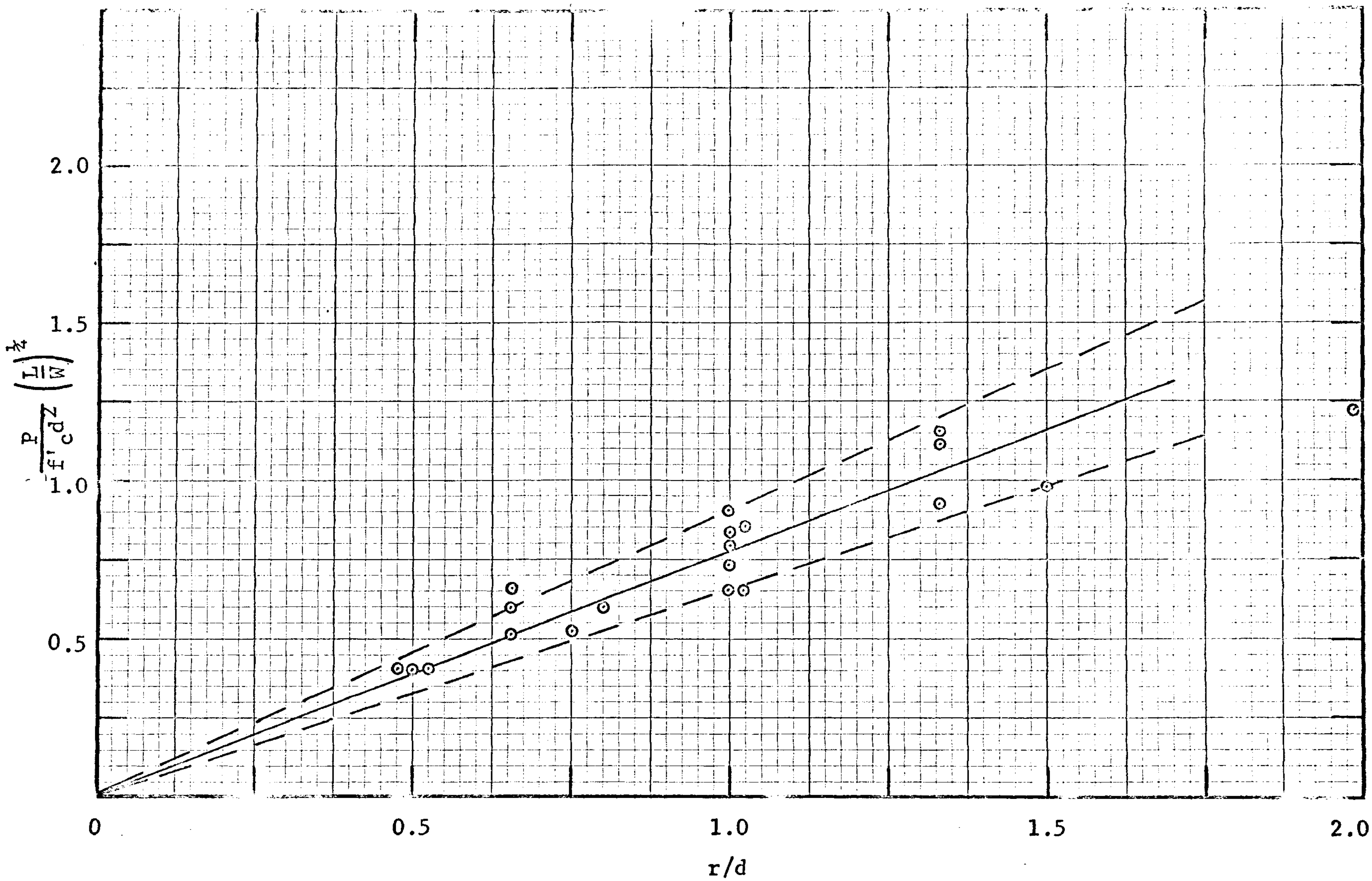


FIGURE 14. DIMENSIONLESS PLOT OF AN AVERAGE OF ALL DATA

V. DISCUSSION OF RESULTS

A. DESCRIPTION OF SERIES

This group of tests was planned to furnish data on footings expected to fail by diagonal tension. Sixty-nine models were made in all, with three basic sizes 15 in. x 15 in. square, 12.5 in. x 18 in., and 10 in. x 22.5 in. The effective depth and column size of each model was varied from 1 to 2 inches. Each model was loaded to failure at each effective depth using a 1 inch square column. Each model was also loaded to failure using an effective depth of 1.5 inch and each of the three column sizes. The 12.5 in. x 18 in. model was loaded to failure using all possible combinations of the effective depths and column sizes. Three samples were run on each set of parameters. The effect of eccentricity in the column load on the shear strength of the square model was investigated through the use of three models and three different offset distances.

In all of these procedures the models were placed directly in contact with the sand. Considerable attention was given to the insuring of a complete contact between the base of the model and the sand. The model was leveled and column and model combination centered directly beneath the load cell to insure center loading. In instances where care was not taken in test preparations it was found that shear failures would occur in the column, or sometimes non-typical shear or moment type failures would occur directly under the column in the model.

A short series of tests were run on the square model with the model being supported along all four edges. This was done in an attempt to evaluate the actual load causing failure in diagonal tension. This was compared to the load causing the same type failure when the model was fully supported and an upward soil pressure was present. The difference between the two was taken as the upward force due to soil pressure.

B. STRESSES IN REINFORCEMENT

Prior to discussing the shearing stresses developed, a discussion of the tensile and bond stresses present in the reinforcing wire would be appropriate. Although strain measurements were not taken to substantiate this assumption, it is assumed that tensile stresses in the reinforcing wire were well below the yield stress of the material. In the thicker and longer models, 10 in. x 22.5 in. x 1.5 in. using the 1.5 in. column, the stress evidently approached the yield stress and probably exceeded it. See Figure 5. The cracking of the plaster induced under circumstances such as this may have a definite influence on diagonal tension failures. When extensive cracking occurs after either a tension or bond failure, the section resisting diagonal tension is undoubtedly decreased and the shearing stresses can be expected to be lower than when a primary shearing failure occurs. The controlling modes of failure are noted on the data sheets in Appendix A.

The reinforcing wire was placed at the bottom of the model with minimum cover to obtain maximum effective depth and uniformity in obtaining this depth in all models and to limit the dowel effect in the area of diagonal tension failure. Each model was closely examined after failure. In no instance was it readily evident that failure occurred primarily in bond.

C. TYPES OF FAILURE

The majority of the models reached the maximum load when a pyramid of plaster beneath the column punched through suddenly and violently. Of the 69 models tested, 58 were considered to have failed in diagonal tension, 10 in a combination of diagonal tension and moment and one failed in moment. It is interesting to note that even though moment cracks frequently occurred in the model there was no appreciable effect on the load carrying capacity of the model.

D. LOADING OPERATIONS

The loading operations were conducted on a medium sand, as classified by the M.I.T. soil classification system. The box was loaded in 10 in. lifts and stabilized through the use of a concrete vibrator. The in-place density was 107.8 lbs/cu ft while the relative density of the material was 0.975.

The container was designed in accordance with the Boussinesq equations in order that the sides and bottom would not play a significant role in determining the response of the models to loading. In other words, an attempt was made to create soil conditions which could be logically anticipated in the field.

For a check of the theories and reasoning applied to this portion of the research, load-deformation tests were conducted throughout the course of the work to determine the loading characteristics of the sand. The curves shown in Figure 7 are the result of one such series of tests. It will be noted that curve No. 1 shows approximately fifty percent greater total deformation than curve No. 2. This is a result of disturbing the top one inch of soil during model leveling procedures of previous loading operations. It should be noted that curves 2 through

4 do have the same basic shape and are within 0.014 in. of each other. In general, all of the before and after load-deformation tests resulted in curves of the same basic shape and magnitude as curves 2 through 4 of Figure 7.

E. MECHANISM OF FAILURE

During the loading operations, carefuly inspection of the models would at times reveal the formation of cracks at the edge of the long side. The cracks, which frequently developed at as low as 60 to 65 percent of the ultimate load, extended rapidly up to the proximity of the neutral axis. After reaching this location there would be very little additional development up to the point of failure in diagonal tension. At the point of failure a relatively narrow depth of the model comprised the compression zone as exhibited in Figure 6. This moment cracking would occur under one edge of the column and infrequently there would be a crack developed under both edges of the column. This is the location of maximum moment substantiated by J. Moe (10). When models did fail at ultimate load, the "pyramid of rupture" was normally as shown in Figures 3 and 4, taking the shape of the frustum of a cone or pyramid with surfaces sloping out in all directions from the column at an angle of approximately 45 degrees.

A close examination of Figure 6 reveals a vertical failure surface in the plaster at the perimeter of the column. This failure surface existed on all models, and ranged from as deep as 1 in. on a two inch deep model using a 1 in. square column, to 1/8 in. on a one inch deep model using a 2 in. square column. Failures occurring in this upper region are considered to be a result of shear, compression and diagonal tension. This type of failure is referred to in the literature as shear-compression failure. The volume shown under this shear-compression failure took the form of a frustum of a cone and is classed as a "pyramid of rupture." This is an example of what is considered failure in diagonal tension by the author.

Another effect which can be noted on Figure 6 is the change in appearance of the plaster in the area associated with compressive stresses due to moment. This change was detectable when the model was split apart and was of maximum depth at the column and went to a minimum at the edge of the model.

Tables 6, 7 and 8 are a study of the nominal shearing stresses at ultimate load on different positions of the critical section. The section at d=0, at the face of the column, seems to give the best agreement between the individual values. It would thus seem that this would tend to support assumptions that the section around the periphery of the loaded area is a critical one. The value $\sqrt{f'_c}$ provided better correlation than f'_c which also agrees with previous work done in reinforced concrete. The improvement noted by the use of $\sqrt{f'_c}$ rather than f'_c was not significant enough to warrant its use in other areas of the thesis.

F. SUMMARY OF RESULTS

The relationship between the load carrying capacity and column size is shown on Figure 12. With increasing size of the column there is an increase in the carrying capacity of the model. With an increase in the effective depth of a model there is a corresponding increase in the load carrying capacity of the model, see Figures 8 through 11. Both of these relationships are to be anticipated.

An interesting relationship was noted when plotting the load versus the r/d ratio, Figure 13. It was found that there is not a straight line relationship between an increase in depth and load carrying capacity. The 2 in. model showed a greater increase in capacity with increasing r/d ratio than the 1 and 1.5 in. deep models.

Through numerous inspections and evaluations of the data with various combinations of the parameters being attempted, a relationship was determined to exist between the dimensionless quantities,

$\frac{P}{f'_c d^2} \left(\frac{L}{W}\right)^{\frac{1}{4}}$ and r/d.

where L = length of model

W = width of model

The power for the second term of the first expression was determined by test to provide the best correlation. The results of this approach are shown on Figure 14. The dotted lines show the limits for the majority of the data with the solid line being an approximate average of all of the data. It should be noted that there definitely is convergence in the lower regions of the r/d ratio and that the lines can be extended to pass through the origin. The results of this inspection would tend to support the suggestions of various researchers of late that the shearing strength should be calculated at the face of the column rather than at d/2.

The three 15 in. x 15 in. models tested with eccentricity of 1, 2 and 3 inches showed little decrease in load carrying capacity from the axially loaded models. See Table 5.

G. COMPARISON WITH OTHER PROCEDURES

The Report of ACI-ASCE Committee 326 on Shear and Diagonal Tension (11) proposes that the ultimate shear capacity for footings with square columns be determined in accordance with the equation:

$$V_u = 16\, d^2 \left(\frac{r}{d} + 1\right) \sqrt{f'_c} \qquad 16$$

where V_u = ultimate shear capacity.

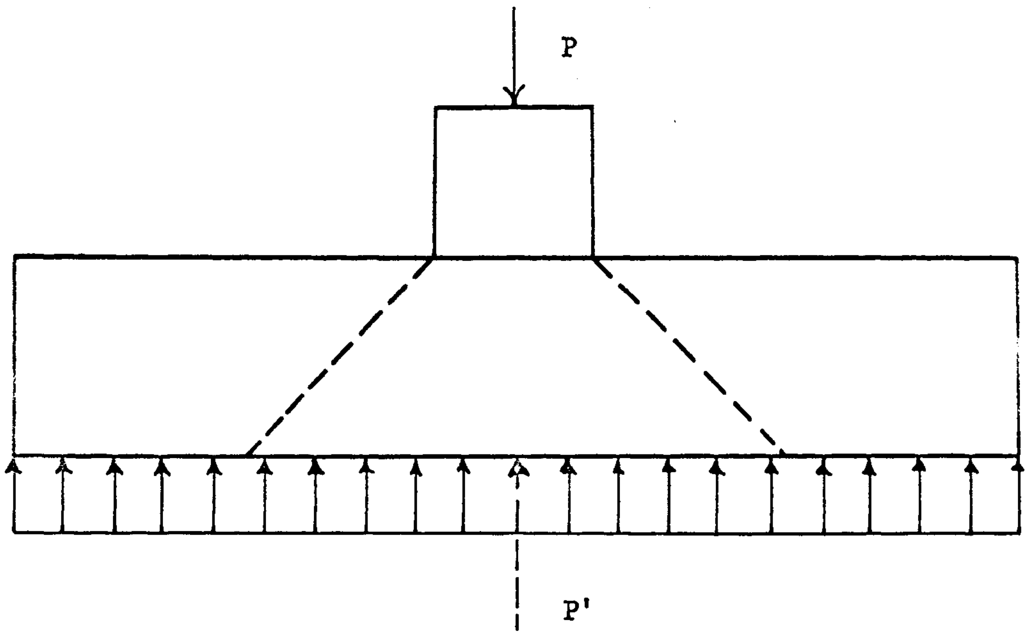
This equation is based on the concept that the shear area is the vertical section which follows the periphery at the edge of the loaded area, and that the ultimate shear stress is a function of $\sqrt{f'_c}$ and r/d. However an analysis of the proposed equation indicates a similarity between it and the ACI Code, 318-63 (12), which is based on the assumption that the shear area is the vertical section which follows the periphery located a distance d/2 beyond the edge of the loaded area, and that the allowable shear stress is proportional to $\sqrt{f'_c}$. Thus, while the joint committee proposes the use of a shear calculated at the face of the column, their equation takes the same form as the ACI-Building code which is calculated at d/2. An analysis of equation 16 indicates that as r/d approaches zero the ultimate shear capacity, V_u, approaches $\sqrt{f'_c}$. The creditability of this assumption is questioned when examining the premis of the concept. If the critical section is taken at the face of the column, the shear area would approach zero as the r/d ratio approaches zero. Thus there seems to be an inconsistency between the assumption and the form of the equation.

An examination of the relationship between $\frac{P}{f'_c\, d^2}\left(\frac{L}{W}\right)^{\frac{1}{4}}$ and r/d, shown on Figure 14, indicates that it will pass through the origin if so extended. Thus as the r/d ratio goes to zero the load carrying capacity of the footing goes to zero. The equation suggested by an analysis of Figure 14 is:

$$P = 0.8 \frac{r}{d} f'_c d^2 \left(\frac{v}{L}\right)^{\frac{1}{4}} \qquad 17$$

If the + 1 value is deleted from the right side of equation 16 it will also pass through the origin and take the same shape as the proposed relationship. This is considered as further evidence that equation 16 contains some inconsistencies.

The three models tested with edge support failed at an average load of 1400 lbs., see Table 4, while those fully supported failed at an average load of approximately 2500 lbs. The increase in load capacity with the footing fully supported is to be anticipated and represents the upward force on the bottom of the "pyramid of rupture." The total load P on the loaded area was used in development of the data for this thesis rather than V, the total shear force. The area of the base of the "pyramid of rupture" varied from 7% of the total model base for the 1 in. deep model using the 1 in. square column to 16% for the 2 in. deep model using the 2 in. square column. The following calculations indicate that there is only a 6% error in using the P quantity and it is assumed adequate from an analysis viewpoint.



PYRAMID OF RUPTURE

$V = P - P'$

$P' = qnA_c = \frac{P}{A_g} \times A_c$

Then:

$V = P\left(1 - \frac{A_c}{A_g}\right)$

From previous discussion:

$0.07 \leq \frac{A_c}{A_g} \leq 0.16$

Or,

$0.84 \leq 1 - \frac{A_c}{A_g} \leq 0.93$

and an average of 0.88

Thus:

$V = P\,(0.88 \pm 0.05)$

Error = 5.7%

Where V = total shear force

P = total load on loaded area

P' = load resisted by area of the "pyramid of rupture"

q_n = net soil pressure for structural design of footing (total load divided by area of footing)

A_g = area of footing

A_c = area of "pyramid of rupture"

VI. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

1. Current design practices are deficient in the area of developed methods and procedures for designing reinforced concrete footings to resist shear and diagonal tension. The necessity for additional investigations in this problem area is apparent when the provisions of the relatively new ultimate strength design methods are evaluated.

2. The load carrying capacity of a model increased with: increasing model effective depth when column size remained constant and with increasing column size when the effective depth was held constant. There was an apparent relationship between the L/W ratio and load carrying capacity. With an increase in the L/W ratio there was a decrease in the load carrying capacity.

3. The use of plaster models to simulate reinforced concrete footings is a valid procedure to evaluate the shear mechanisms of failure. In addition it seems feasible to test these models on various soil conditions attempting to simulate actual field soil conditions.

4. The shear area, or critical section, governing the ultimate shearing strength of footings should be the vertical section which follows the periphery of the loaded area.

5. This research indicates that the ultimate shear stress is a function of f'_c, r/d, d, and the L/W ratio. The relation suggested by this thesis is:

$$P = 0.8 \left(\frac{r}{d}\right) f'_c \, d^2 \left(\frac{W}{L}\right)^{\frac{1}{4}}$$

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APPENDIX A

LIST OF SYMBOLS

LIST OF SYMBOLS

A_c = area of "pyramid of rupture"

A_g = area of model

A_v = area of shear reinforcement

a = side length of square footing

b = width of cross section; also perimeter of critical peripheral section

b_1 = effective slab strip width

DT = diagonal tension

d = effective depth

e = eccentricity of axial load

f'_c = compressive strength of 1 x 1 in. cubes

f_y = yield point of steel

I = moment of inertia

jd = internal moment arm

L = length of model or footing

ℓ = length of the span of the slab

ℓ_s = shear span in slabs

M = bending moment

m = ultimate resisting moment per unit width of slab

P = total load on loaded area

Q = first moment of part of a cross section

q_n = net soil pressure

r = width of column

s = thickness of a load-distributing layer on the top of the slab; also spacing of web reinforcement along longitudinal axis of member

t = width of the cross-section at a distance y from the neutral axis; and total depth of section

V = total vertical shear at the section

V_{flex} = ultimate shear force for flexural failure

V_u = ultimate shear capacity

v = shear stress

v_u = ultimate shear stress

α = inclination of web reinforcement to longitudinal axis of member

θ = capacity reduction factor of 0.85

ϕ = V_{test}/V_{flex}

ϕ_o = V_u/V_{flex}

APPENDIX B

TABULATION OF TEST DATA

TABLE 1. PRINCIPAL RESULTS OF 15 in. x 15 in. MODEL TESTS

MODEL NO.	r in.	d in.	$\frac{r}{d}$	f'_c PSI	P lbs.	TYPE OF FAILURE
A-1	1.00	1.00	1.00	1540	1080	DT
A-6	1.00	1.00	1.00	1500	1300	DT
A-9	1.00	1.00	1.00	965	1200	DT
A-2	1.00	1.25	0.80	1510	1400	DT
A-3	1.00	1.50	0.66	1570	1900	DT
A-5	1.00	1.50	0.66	1560	1750	DT
A-11	1.00	1.50	0.66	1560	1750	DT
A-4	1.00	2.00	0.050	1640	2490	DT
A-13	1.00	2.00	0.50	1875	2900	DT
A-14	1.00	2.00	0.50	1060	2275	DT
A-31	1.00	2.00	0.50	1260	2150	DT
A-17	1.50	1.50	1.00	1206	2450	DT/M
A-19	1.50	1.50	1.00	1390	2650	DT/M
A-22	1.50	1.50	1.00	1720	2600	DT
A-24	2.00	1.50	1.33	1485	3250	DT
A-25*	2.00	1.50	1.33	817	2450	DT
A-27	2.00	1.50	1.33	1050	3300	DT
A-32	2.00	1.50	1.33	1350	2650	DT
A-41	2.00	1.50	1.33	1000	2900	DT

*Not included on graphs where use of compression strength is applicable

DT refers to diagonal tension, M to moment

TABLE 2. PRINCIPAL RESULTS OF 12.5 in. x 18 in. MODEL TESTS

MODEL NO.	r in.	d in.	$\frac{r}{d}$	f'_c PSI	P lbs.	TYPE OF FAILURE
A-7	1.00	1.00	1.00	1500	1200	DT
A-8	1.00	1.00	1.00	1590	1250	DT
A-10*	1.00	1.00	1.00	965	1250	DT
A-52	1.50	1.00	1.50	1525	1425	DT
A-53	1.50	1.00	1.50	1350	1300	DT
A-54	1.50	1.00	1.50	1480	1150	DT
A-55	2.00	1.00	2.00	1495	1550	DT
A-56	2.00	1.00	2.00	1687	1600	DT
A-57	2.00	1.00	2.00	1836	2050	DT
A-12	1.00	1.50	0.66	1560	1800	DT
A-18	1.00	1.50	0.66	1206	1900	DT/M
A-20	1.00	1.50	0.66	1370	1900	DT/M
A-21	1.50	1.50	1.00	1720	2300	DT
A-23	1.50	1.50	1.00	1665	2200	DT
A-26*	1.50	1.50	1.00	567	2150	DT
A-28*	2.00	1.50	1.33	787	3100	DT/M
A-33	2.00	1.50	1.33	1427	3150	DT
A-35	2.00	1.50	1.33	1266	3175	DT
A-15	1.00	2.00	0.50	1440	2450	DT
A-16	1.00	2.00	0.50	1635	2300	DT
A-29*	1.00	2.00	0.50	700	1800	DT

Continued

TABLE 2 (Concluded)

MODEL NO.	r in.	d in.	$\frac{r}{d}$	f'_c PSI	P lbs.	TYPE OF FAILURE
A-34	1.00	2.00	0.50	1167	1800	DT
A-58	1.50	2.00	0.75	1525	3075	DT
A-59	1.50	2.00	0.75	1699	2800	DT
A-60	1.50	2.00	0.75	1570	3175	DT
A-61	2.00	2.00	1.00	1307	3650	DT
A-62	2.00	2.00	1.00	1442	3850	DT
A-63	2.00	2.00	1.00	1537	3650	DT/M

*Not included on graphs where use of compression strength is applicable

DT refers to diagonal tension, M to moment

TABLE 3. PRINCIPAL RESULTS OF 10 in. x 22.5 in. MODEL TESTS

MODEL NO.	r in.	d in.	$\frac{r}{d}$	f'_c PSI	P lbs.	TYPE OF FAILURE
A-36	1.00	1.00	1.00	1260	1000	DT
A-37	1.00	1.00	1.00	1427	950	DT
A-38	1.00	1.00	1.00	1167	900	DT
A-40	1.00	1.50	0.66	1325	1500	DT
A-42	1.00	1.50	0.66	1020	1500	DT
A-43	1.00	1.50	0.66	1710	1400	DT
A-39	1.00	2.00	0.50	1345	1650	DT/M
A-50	1.00	2.00	0.50	1495	2000	DT/M
A-51	1.00	2.00	0.50	1687	2100	DT/M
A-44	1.50	1.50	1.00	1770	2250	DT
A-45	1.50	1.50	1.00	1488	2200	DT
A-46	1.50	1.50	1.00	1670	2200	DT
A-47*	2.00	1.50	1.33	1350	2700	M
A-48	2.00	1.50	1.33	1525	2600	DT
A-49	2.00	1.50	1.33	1732	2500	DT/M

*Not included on graphs where use of compression strength is applicable

DT refers to diagonal tension, M to moment

TABLE 4. PRINCIPAL RESULTS OF 15 in. x 15 in. MODEL TESTS SUPPORTED ON EACH EDGE

MODEL NO.	r in.	d in.	$\frac{r}{d}$	f'_c PSI	P lbs.	TYPE OF FAILURE
A-64	1.50	1.50	1.00	1836	1350	DT
A-65	1.50	1.50	1.00	1525	1400	DT
A-66	1.50	1.50	1.00	1699	1450	DT

TABLE 5. PRINCIPAL RESULTS OF 15 in. x 15 in. MODEL TESTS WITH ECCENTRICITY IN COLUMN LOAD

MODEL NO.	r in.	d in.	$\frac{r}{d}$	e in.	f'_c PSI	P lbs.	TYPE OF FAILURE
A-17, 19, 22	1.50	1.50	1.00	0	1436	2533	DT
A-69	1.50	1.50	1.00	1.00	1455	2300	DT
A-67	1.50	1.50	1.00	2.00	1570	2250	DT
A-68	1.50	1.50	1.00	3.00	1307	2400	DT

TABLE 6. SHEAR STRESS AT VARYING LOCATIONS OF CRITICAL SECTION.
15 in. x 15 in. MODEL

MODEL NO.	SECTION 0 b	SECTION 0 $v/\sqrt{f'_c}$	SECTION d/2 b	SECTION d/2 $v/\sqrt{f'_c}$	SECTION d b	SECTION d $v/\sqrt{f'_c}$
A-1	1.00	6.88	2.00	3.43	3.00	2.50
A-6	1.00	8.40	2.00	4.18	3.00	2.80
A-9	1.00	9.68	2.00	4.85	3.00	3.23
A-2	1.00	7.20	2.25	3.98	3.50	2.06
A-3	1.00	7.25	2.50	2.91	4.00	1.81
A-5	1.00	7.40	2.50	2.96	4.00	1.85
A-11	1.00	7.40	2.50	2.96	4.00	1.85
A-4	1.00	7.68	3.00	2.57	5.00	1.54
A-13	1.00	8.11	3.00	2.79	5.00	1.67
A-14	1.00	8.74	3.00	2.92	5.00	1.75
A-31	1.00	7.30	3.00	2.54	5.00	1.52
A-17	1.50	7.85	3.00	3.92	4.50	2.62
A-19	1.50	7.92	3.00	3.94	4.50	2.63
A-22	1.50	6.96	3.00	3.47	4.50	2.32
A-24	2.00	7.05	3.50	4.02	5.00	2.82
A-25	2.00	7.15	3.50	4.09	5.00	2.86
A-27	2.00	7.22	3.50	5.60	5.00	3.91
A-32	2.00	6.14	3.50	3.40	5.00	1.85
A-41	2.00	7.60	3.50	4.32	5.00	3.05

Continued

TABLE 6 (Concluded)

MODEL NO.	SECTION 0 b	SECTION 0 $v/\sqrt{f'_c}$	SECTION d/2 b	SECTION d/2 $v/\sqrt{f'_c}$	SECTION d b	SECTION d $v/\sqrt{f'_c}$
Average		7.57		3.62		2.20
Standard Deviation		0.568		0.513		0.499
Coefficient of Variance		7.50%		14.20%		22.60%

TABLE 7. SHEAR STRESSES AT VARYING LOCATIONS OF CRITICAL SECTION. 12.5 in. x 18 in. MODEL

MODEL NO.	SECTION 0 b	SECTION 0 $v/\sqrt{f'_c}$	SECTION d/2 b	SECTION d/2 $v/\sqrt{f'_c}$	SECTION d b	SECTION d $v/\sqrt{f'_c}$
A-7	1.00	7.56	2.00	3.82	3.00	2.56
A-8	1.00	7.80	2.00	3.80	3.00	2.61
A-10	1.00	0.01	2.00	5.02	3.00	3.35
A-52	1.50	6.10	2.50	3.64	3.50	2.62
A-53	1.50	5.88	2.50	3.66	3.50	2.52
A-54	1.50	4.96	2.50	3.00	3.50	2.14
A-55	2.00	5.01	3.00	3.34	4.00	2.51
A-56	2.00	4.86	3.00	3.24	4.00	2.44
A-57	2.00	5.97	3.00	3.74	4.00	2.98
A-12	1.00	7.61	2.50	3.04	4.00	1.89
A-18	1.00	9.10	2.50	3.66	4.00	2.28
A-20	1.00	8.54	2.50	3.44	4.00	2.14
A-21	1.50	6.15	3.00	3.09	4.50	2.04
A-23	1.50	5.98	3.00	2.99	4.50	2.00
A-26	1.50	10.01	3.00	5.00	4.50	3.35
A-28	2.00	9.35	3.50	5.26	5.00	3.69
A-33	2.00	6.84	3.50	3.98	5.00	2.78
A-35	2.00	7.40	3.50	4.25	5.00	2.97
A-15	2.00	8.08	3.00	2.70	5.00	1.61
A-16	1.00	7.10	3.00	2.38	5.00	1.42

Continued

TABLE 7 (Concluded)

MODEL NO.	SECTION 0 b	SECTION 0 $v/\sqrt{f'_c}$	SECTION d/2 b	SECTION d/2 $v/\sqrt{f'_c}$	SECTION d b	SECTION d $v/\sqrt{f'_c}$
A-29	1.00	8.50	3.00	2.82	5.00	1.70
A-34	1.00	6.60	3.00	2.20	5.00	1.32
A-58	1.50	6.55	3.50	2.82	5.50	1.78
A-59	1.50	5.68	3.50	2.42	5.50	1.55
A-60	1.50	6.67	3.50	2.86	5.50	1.83
A-61	2.00	6.32	4.00	3.16	6.00	2.10
A-62	2.00	6.32	4.00	3.16	6.00	2.15
A-63	2.00	5.83	4.00	2.92	6.00	1.94
Average		6.69		3.21		2.15
Standard Deviation		1.11		0.523		0.452
Coefficient of Variance		16.6%		16.3%		21.1%

TABLE 8. SHEAR STRESSES AT VARYING LOCATION OF CRITICAL SECTION. 10 in. x 22.5 in. MODEL

MODEL NO.	SECTION 0		SECTION d/2		SECTION d	
	b	$v/\sqrt{f'_c}$	b	$v/\sqrt{f'_c}$	b	$v/\sqrt{f'_c}$
A-36	1.00	7.05	2.00	3.52	3.00	2.35
A-37	1.00	6.28	2.00	3.16	3.00	2.10
A-38	1.00	6.58	2.00	3.38	3.00	2.19
A-40	1.00	6.86	2.50	2.55	4.00	1.72
A-42	1.00	7.83	2.50	3.13	4.00	2.00
A-43	1.00	5.64	2.50	2.28	4.00	1.42
A-39	1.00	5.61	3.00	1.88	5.00	1.12
A-50	1.00	6.46	3.00	2.14	5.00	1.29
A-51	1.00	6.12	3.00	2.11	5.00	1.27
A-44	1.50	5.92	3.00	2.96	4.50	1.98
A-45	1.50	6.34	3.00	3.17	4.50	2.12
A-46	1.50	5.97	3.00	2.99	4.50	1.99
A-47	2.00	6.12	3.50	3.48	5.00	2.52
A-48	2.00	5.54	3.50	3.18	5.00	2.22
A-49	2.00	5.00	3.50	2.86	5.00	2.00
Average		6.22		2.90		1.88
Standard Deviation		0.542		0.464		0.398
Coefficient of Variance		8.70%		16.0%		21.8%

VITA

Major Fred V. Cole was born April 18, 1931, in Schenectady, New York. He received his elementary education and started his high school education in Altamont, New York. He completed his high school education in Cristobal, Panama Canal Zone.

He entered the military service as an enlisted man in March of 1952. He received a commission as an officer in the U. S. Army Corps of Engineers in June of 1953 and has served in the United States and overseas in West Germany and Korea. He received a Bachelor of Science Degree in Military Science from the overseas branch of the University of Maryland in June 1958. He received a Bachelor of Science Degree in Civil Engineering from the Missouri School of Mines and Metallurgy in June 1962.

He is a graduate of the Engineer Officers Candidate School and the Engineer Officers Advance Course, both conducted at Fort Belvoir, Virginia. He married the former Jane L. Williams of Washington, D.C. in June 1955 and they have three daughters, Linda Joan, Jean Lee and Cynthia Ann.

He is an associate member of the American Society of Civil Engineers, a member of the National Society of Professional Engineers, and of Chi Epsilon and Tau Beta Pi, National Scholastic Honorary Fraternities. He is a Registered Professional Engineer in the State of Missouri.

In August of 1963 he was sent to the Missouri School of Mines and Metallurgy by the U. S. Army ~~as an~~ Assistant Professor of Military

Science. He has been enrolled in the Graduate School of the University of Missouri at Rolla since January 1964.