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Modeling Solitary Waves in the Earth's Magnetosphere

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I. Abstract

Solitary waves occur in many mediums, both on Earth and in space. These waves are a single isolated wavelength or half wavelength of a full wave. This research focuses on observing these waves at the Earth's magnetopause, the region around Earth where the Earth's magnetic field controls the motion of particles. This research applies a Gaussian function to model solitary waves in Mathematica, utilizing electric potential data gathered by NASA's Polar spacecraft. The code is able to loop through the data set, finding fit parameters for the function for each wave. It can also loop through the entirety of our data set, 183 separate waves observed by Polar. With more refinement, we hope to obtain confidence levels for the value being given in the fit, which will help us make the model more accurate. This research is ongoing, with the ultimate goal being to fit the large number of events we have and to obtain a distribution of wave sizes and other parameters in order to allow us to infer what geometric shape the solitary wave may have had.

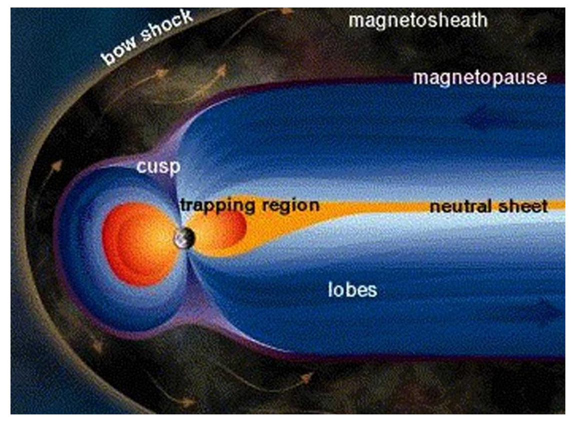
II. Introduction

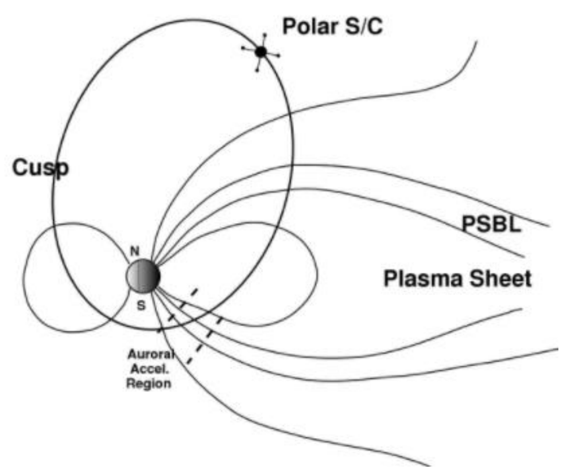
A solitary wave is a type of nonlinear wave. It appears as a single isolated half or full wavelength of a linear wave mode. This type of wave has been observed in many different mediums, including plasma at the magnetopause.

There are many types of nonlinear waves that can be observed at the Earth's magnetopause. This study will focus on soliton data gathered at the magnetopause to characterize solitary waves.

This image shows a solitary wave in water traveling towards the shore.
Image courtesy of newscientist.com

Data was taken by NASA's Polar spacecraft, which orbited the Earth and took data in several regions of the magnetosphere, including the cusp and auroral acceleration region.
The data we looked at was electric potential with respect to time.





Images courtesy of NASA

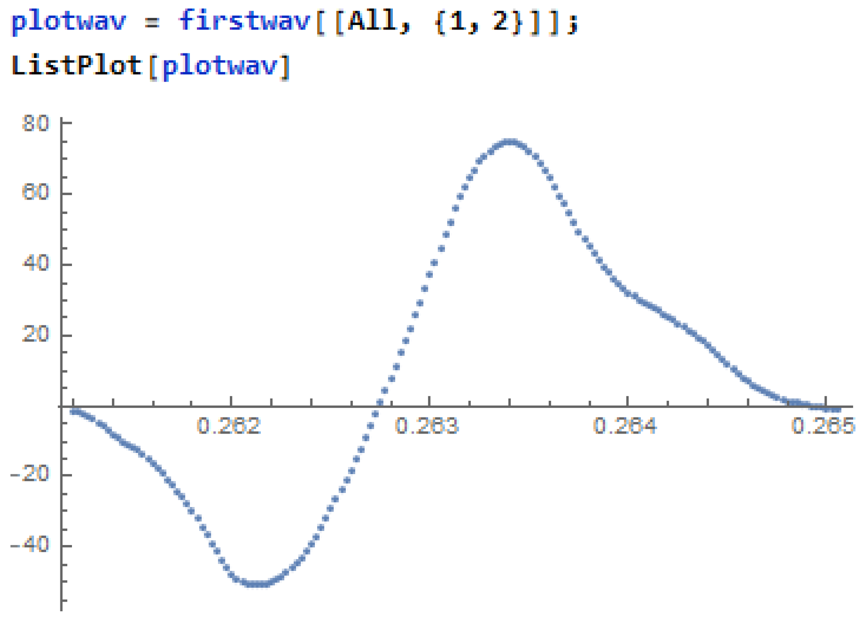
III. Theory

Solitary waves are a type of shock wave which exist in plasma. Even though there is no actual collision, a magnetic shock is generated (the bow shock) as the Earth's dipole magnetic field moves through solar wind plasma.[2] Solitary waves are potential and density disturbances that travel as an isolated nonlinear single pulse.[2]
We would expect the electric field for these waves to follow the shape of a Gaussian function.[1] Since our data was electric potential, we expected the model to follow the general form of the equation:

$$\vec{F} = \vec{\nabla}\varphi$$

where F represents the electric field and φ is electric potential.

The data plotted was parallel electric field, so we used the derivative of the Gaussian (this is the derivative along the parallel electric field direction).



This is a scatter plot of the data. In accordance with our theory, this looks like the fitting function that would work best is the inverse of a Gaussian function.

IV. Model

We modeled this data using the derivative of a Gaussian function, as this was the function that both was expected by our theory and worked the best for our data. Ultimately, the goal was for the program to run autonomously, with no input at all from the user. Mathematica's NonLinearModelFit requires a "guess" for the fit parameters, in order to get a good fit. I was able to code for Mathematica to use information from the data for 2 of the parameters, while the other 2 parameters were consistent enough to not need to change with each trial.
The code loops through the data, finding values for these fit parameters, then using NonLinearModelFit to model the data itself. It then selects the next data set. The "goodfits" variable is a list of the data sets that look like they will fit the function best. These are my "simple cases" – those that should work well so that we can begin to develop the model itself. With future work we hope to expand this list to include all of the data sets.
The code loops through the first 4 data sets right now, to give a small representation of what these fits will look like. These are the 4 plots shown (overlaid over the scatterplots of the data points themselves).

```
goodfits = {1, 4, 6, 7, 8, 11, 12, 13, 17, 18, 19, 20, 21, 25, 27, 28, 29, 32,
   35, 38, 39, 40, 43, 46, 48, 49, 50, 53, 54, 55, 59, 60, 61, 64, 65, 66, 67,
   68, 69, 70, 71, 73, 74, 77, 79, 80, 82, 83, 84, 85, 86, 91, 93, 94, 96, 97,
   100, 101, 102, 103, 105, 106, 108, 112, 113, 114, 115, 116, 117, 118,
   119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 133, 137, 138,
   139, 140, 141, 144, 145, 149, 150, 151, 152, 153, 154, 155, 157, 158,
   159, 161, 164, 165, 170, 174, 175, 178, 179, 180, 181};
n = Length[goodfits]

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For[i = 1, i < (n + 1), i++,
 filename = "/Users/Jess/Downloads/1997_ion_sws/all_ions_only_1997/" <>
   liszt[[goodfits[[{i}]]]];
 firstwav = ReadList[filename, {Number, Number, Number, Number}];
 plotwav = firstwav[[All, {1, 2}]];
 maxampl = plotwav[[1, 2]];
 minampl = plotwav[[1, 2]];
 m = Length[plotwav];
 For[j = 1, j < m, j++, If[maxampl < plotwav[[j, 2]],
   maxampl = plotwav[[j, 2]]]];
 For[j = 1, j < m, j++, If[minampl > plotwav[[j, 2]], minampl = plotwav[[j, 2]]]];
 tinit = plotwav[[1, 1]];
 tfin = plotwav[[m, 1]];
 origin = ((tfin - tinit) / 2) + tinit;
 offset = (maxampl + minampl) / 2;
 ClearAll[x];
 model = (x - a) * b * Exp[-c * (x - a)^2] + d;
 nlm = NonlinearModelFit[plotwav, model,
   {{a, origin}, {b, 170000}, {c, 1500000}, {d, offset}}, x];
 Print@Show[Plot[nlm[x], {x, tinit, tfin}], ListPlot[plotwav],
   PlotRange → {{tinit - 0.0001, tfin + 0.0001}, {maxampl + 2, minampl - 2}},
   AxesOrigin → {tinit, 0}]]
```

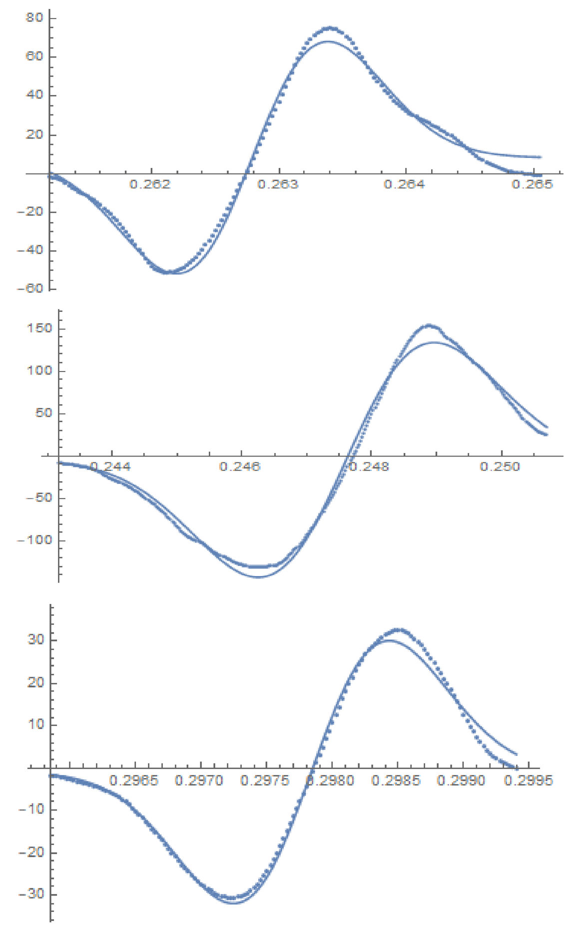

V. Future Investigation

This project is still a work in progress. There are many options for future investigation into solitary waves. Future investigation could take the form of:

- Finding error parameters: how close are our fits to the data themselves, and how can we improve this model to accurately show the data?
- Analyzing all of the data sets that clearly show a solitary waveform, even if it does not look as good or clean and adapting our model to account for these cases
- Statistical analysis of these events, and finding a size distribution of events
- Evaluate the events from a geometric perspective: these waves are 3 dimensional, but the spacecraft does not necessarily cut through the center of the wave. With this analysis we hope to get an idea of what part of the wave the spacecraft may have passed through, as well as what geometric shape would make sense for these events.

VI. Acknowledgements

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I would like to acknowledge Dr. Jim Crumley, for all of his help with this project over the course of the semester.

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