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ANALOG SIMULATION OF THE VOIGT SOLID

BY

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A

120078

THESIS

submitted to the faculty of the

UNIVERSITY OF MISSOURI AT ROLLA

in partial fulfillment of the requirements for the

Degree of

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1966

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ABSTRACT

A method of reducing a mixed partial differential equation with boundary conditions to a form suitable for programming on the analog computer is developed. Presented along with the computer program set up to be used on the EAI TR-48 analog computer is a systematic check procedure to indicate possible computer malfunctions and programming errors. The results of the computer simulation are presented and supported with observed data.

ACKNOWLEDGEMENTS

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The author is also grateful to Dr. George Clark and Dr. Gerald Rupert of the Mining Department for their work that led to this thesis.

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I. INTRODUCTION

The Voigt equation of viscoelasticity^{1,2*} (sometimes know as the Kelvin-Voigt equation) is a mathematical model of a system that has the properties of elasticity and viscous damping, for example, a spring and a dashpot in parallel is the one dimensional system.

The particular problem treated here applies this model, in spherical coordinates, to the shock wave caused by an underground explosion in a homogeneous medium. Figure 1 is a sketch that illustrates the problem and the coordinate system used. The explosion is located at $r = 0$, in the center of a spherical pit of radius a . The pit is completely surrounded by a homogeneous medium characterized by the parameters ω_0 , c , λ and μ .

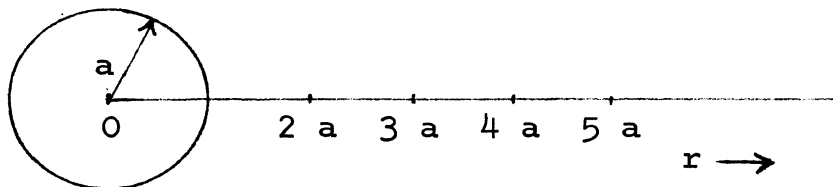


Figure 1. The Coordinate System Used

An explosion of strength B takes place at $t = 0$ and is assumed to have spherical symmetry.

The Voigt equation of viscoelasticity and the boundary conditions for this particular problem were developed by

*Superscripts used in this way are references to the Bibliography

Drs. Clark and Rupert of the Mining Department at the University of Missouri at Rolla and are,

$$\left(1 + \frac{1}{\omega_0} \frac{\partial}{\partial t}\right) \left(\frac{2}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial r^2}\right) = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \quad (1)$$

with,

$$\left(1 + \frac{1}{\omega_0} \frac{\partial}{\partial t}\right) \left[(\lambda + 2\mu) \frac{\partial^2 \phi}{\partial r^2} + \frac{2\lambda}{r} \frac{\partial \phi}{\partial r} \right]_{r=a} = B\delta(t) \quad (2)$$

for $t > 0$

$$\phi = 0 \text{ for } r \gg a, t = 0, \quad (3)$$

$$\frac{\partial \phi}{\partial t} = 0 \text{ for } r > a, t = 0, \text{ and} \quad (4)$$

$$\lim_{r \rightarrow \infty} r\phi = 0. \quad (5)$$

The dependent variable $\phi(r,t)$ is a wave potential function that has little physical significance itself, but the partial derivatives of ϕ can be related to the physical world. Of interest as results of the computation are the displacement, $\frac{\partial \phi}{\partial r}$; strain, $\frac{\partial^2 \phi}{\partial r^2}$; and velocity, $\frac{\partial^2 \phi}{\partial t \partial r}$ of particles located at different radial distances from the explosion site.

Analog computers are well suited to the solution of ordinary differential equations, but partial differential equations require special treatment. The general consensus of opinion ^{3,4,5,6,7,8,9,10} on the subject of solving partial differential equations on the analog computer can best be summarized as a series of points, they are:

1. Several avenues of attack are available for any

problem, and the equipment available is the single most important factor in selecting the one to follow.

2. If the analog computer used has no associated logic and memory capabilities then the most fruitful avenue of attack is to replace derivatives with respect to all but one of the independent variables with finite differences. Usually, time is chosen to be the remaining independent variable.
3. The procedure indicated above tends to push the solution toward instability and so must be used carefully.
4. Each problem in this area is individual, and the best approach to any given problem can only be determined by trial and error.

II. THE DIFFERENTIAL DIFFERENCE EQUATIONS

Since the analog computer has only one independent variable it is necessary to eliminate either r or t from the equation and boundary conditions. This is accomplished by means of differential difference equations, that is, replacing derivatives with respect to one of the independent variables with the ratios of finite differences.

Because the forcing function $\delta(t)$ (from the first boundary condition) is a function of time only it seems wisest to eliminate r and work with continuous equations in t .

There are three principle kinds of difference equations; forward, central and backward. The central difference equations were used because they are easiest to handle and because according to several text books on numerical analysis^{3,11,12} they tend to introduce the least error.

The central difference equations used are:

$$\left(\frac{\partial \phi}{\partial r}\right)_k \approx \frac{1}{2\Delta r} (\phi_{k+1} - \phi_{k-1}) \quad (6)$$

and

$$\left(\frac{\partial^2 \phi}{\partial r^2}\right)_k \approx \frac{1}{(\Delta r)^2} (\phi_{k+1} - 2\phi_k + \phi_{k-1}). \quad (7)$$

k is defined by the relation $r_k = a + k\Delta r$ and takes on the values $-1, 0, 1, 2, \dots$. In order to make the above expression easier to work with let $\Delta r = na$ where a is the radius of the spherical cavity in which the explosion takes place and n is an arbitrary constant. This results in

$$r_k = a + kna = a(1 + kn). \quad (8)$$

Applying relations (6), (7) and (8) to equations (1), (2), (3), (4) and (5) results in the elimination of r as an independent variable. Equation (1),

$$\left(1 + \frac{1}{\omega_0} \frac{\partial}{\partial t}\right) \left(\frac{2}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial r^2}\right) = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \quad \text{becomes,}$$

$$\left(1 + \frac{1}{\omega_0} \frac{d}{dt}\right) \left[\frac{1}{a^2 n(1+kn)} (\phi_{k+1} - \phi_{k-1}) + \frac{1}{n^2 a^2} (\phi_{k+1} - 2\phi_k + \phi_{k-1}) \right] = \frac{1}{c^2} \frac{d^2 \phi_k}{dt^2} .$$

Collecting terms and rearranging yields,

$$\left(1 + \frac{1}{\omega_0} \frac{d}{dt}\right) \left[\phi_{k+1} \left(\frac{1+(k+1)n}{2(1+nk)}\right) - \phi_k + \phi_{k-1} \left(\frac{1-(k-1)n}{2(1+nk)}\right) \right] = \frac{a^2 n^2}{2c^2} \frac{d^2 \phi_k}{dt^2} . \quad (9)$$

Equation (2)

$$\left(1 + \frac{1}{\omega_0} \frac{\partial}{\partial t}\right) \left[(\lambda + 2\mu) \frac{\partial^2 \phi}{\partial r^2} + \frac{2\lambda}{r} \frac{\partial \phi}{\partial r} \right]_{r=a} = B\delta(t) \quad \text{for } t > 0$$

becomes,

$$\left(1 + \frac{1}{\omega_0} \frac{d}{dt}\right) \left[(\lambda + 2\mu) \frac{1}{a^2 n^2} (\phi_1 - 2\phi_0 + \phi_{-1}) + \frac{\lambda}{a^2 n} (\phi_1 - \phi_{-1}) \right] = B\delta(t).$$

The Mining Department has expressed an interest in a solution for a media where $\lambda = \mu$. Making this substitution and simplifying the results gives,

$$\left(1 + \frac{1}{\omega_0} \frac{d}{dt}\right) \left[\left(\frac{3+n}{3-n}\right) \phi_{+1} - \left(\frac{6}{3-n}\right) \phi_0 + \phi_{-1} \right] = \frac{B a^2 n^2}{(3-n)\lambda} \delta(t). \quad (10)$$

Equation (3),

$$\begin{aligned} \emptyset &= 0 \text{ for } r \gg a, t = 0, \text{ becomes,} \\ \emptyset_k &= 0 \text{ for } k \gg 0, t = 0. \end{aligned} \quad (11)$$

Equation (4),

$$\begin{aligned} \frac{\partial \emptyset}{\partial t} &= 0 \text{ for } r > a, t = 0, \text{ becomes,} \\ \frac{d\emptyset_k}{dt} &= 0 \text{ for } k > 0, t = 0. \end{aligned} \quad (12)$$

Equation (5),

$$\begin{aligned} \lim_{r \rightarrow \infty} r\emptyset &= 0, \text{ becomes,} \\ \lim_{k \rightarrow \infty} r_k \emptyset_k &= 0. \end{aligned} \quad (13)$$

Equations (9), (10), (11), (12) and (13) have only one continuous independent variable, therefore, they are in a form suitable for simulation on the analog computer.

III. THE SIMULATION

The boundary value problem to be simulated is

$$\left(1 + \frac{1}{\omega_0} \frac{d}{dt}\right) \left[\phi_{k+1} \left(\frac{1+(k+1)n}{2(1+nk)} \right) - \phi_k + \phi_{k-1} \left(\frac{1+(k-1)n}{2(1+nk)} \right) \right] = \frac{a^2 n^2}{2c^2} \frac{d^2 \phi_k}{dt^2} \quad (9)$$

with,

$$\left(1 + \frac{1}{\omega_0} \frac{d}{dt}\right) \left[\phi_1 \left(\frac{3+n}{3-n} \right) - \phi_0 \left(\frac{6}{3-n} \right) + \phi_{-1} \right] = \frac{B a^2 n^2}{(3-n)\lambda} \delta(t); \quad (10)$$

$$\phi_k = 0 \text{ for } k \geq 0, t = 0; \quad (11)$$

$$\frac{d\phi_k}{dt} = 0 \text{ for } k > 0, t = 0; \quad (12)$$

$$\text{and } \lim_{k \rightarrow \infty} r_k \phi_k = 0. \quad (13)$$

Equation (13) states that at large distances the explosion has no effect and so ϕ_k is zero for large values of k . This allows the use of a finite number of points in space to be used in the simulation. Equations (11) and (12) give the initial condition settings for the integrators whose outputs are ϕ_k and $\frac{d\phi_k}{dt}$, for all $k > 0$, as zero.

Equation (9) and (10) form the heart of the simulation and will be amplitude scaled and time scaled to make them more compatible with the computer used before proceeding further.

Amplitude and time scaling will result in the answers being normalized in such a way as to be independent of the

parameters of the medium used and of the radius of the spherical cavity in which the explosion takes place.

Let P represent normalized ϕ and γ represent normalized time. The scale factors used are

$$\phi_k = \frac{a^2 n^2 \omega_o^2}{2c^2} P_k \quad \text{and} \quad t = \frac{1}{\omega_o} \gamma.$$

The resulting scaled equations are, from equation (9)

$$\left(1 + \frac{d}{d\gamma}\right) \left[P_{k+1} \left(\frac{1+(k+1)n}{2(1+nk)} \right) - P_k + P_{k-1} \left(\frac{1+(k-1)n}{2(1+nk)} \right) \right] = \frac{d^2 P_k}{d^2 \gamma} \quad (14)$$

and from equation (10)

$$\left(1 + \frac{d}{d\gamma}\right) \left[P_1 \left(\frac{3+n}{3-n} \right) - P_o \left(\frac{6}{3-n} \right) + P_{-1} \right] = B' \delta(t) \quad (15)$$

where

$$B' = \frac{2c^2 B}{\omega_o^2 (3-n) \lambda}$$

is the normalized strength of the explosion.

As the first step in the program, equation (15) is solved for $\frac{dP_{-1}}{d\gamma}$ yielding,

$$\frac{dP_{-1}}{d\gamma} = \left(1 + \frac{d}{d\gamma}\right) \left[P_o \left(\frac{6}{3-n} \right) - P_1 \left(\frac{3+n}{3-n} \right) \right] - P_{-1} + B' \delta(t). \quad (16)$$

By implementing this equation in the normal manner on the analog computer $\frac{dP_{-1}}{d\gamma}$ and P_{-1} are generated.

The implementation of the above equation is straight forward except for the implementation of $B' \delta(t)$. $B' \delta(t)$ was simulated by making the initial condition on the integrator

whose output is P_{-1} equal to B' since all of the area under the curve $\delta(t)$ vs t is accounted for by integrating from t equal to zero to t equal to an arbitrary small number.

The rest of the analog computer program is a straightforward simulation of equation (14) generating P_k and $\frac{dP_k}{d\gamma}$ at $k = 0, k = 1, k = 2, \dots$ up to the limits of the computer used. On the EAI TR-48 at the University of Missouri at Rolla the limiting factor is the number of integrators available, this number being 18. Since it takes one integrator for the simulation of equation (16) and two integrators for each simulation of equation (14), the maximum number of points (k) past $r = 0$ that can be simulated is seven. This requires that as an approximation to equation (13) ϕ_k and $\frac{d\phi_k}{d\gamma}$ are considered zero for all k greater than seven. The validity of this approximation is discussed in Chapter V.

The complete analog computer program used is shown in Figure 2 and Table I shows the potentiometer settings for $n = 0.25, 0.5$ and 0.75 . These values of n were chosen because they effectively sample the allowable range of n . n must be greater than zero to have physical significance and must be less than one, for the arrangement of equation used, to avoid changing the sign of the coefficient of P_0 in equation (14) when $k = 0$. If this sign is changed positive feedback is required to simulate the equation and, as a result, the simulation is unstable.

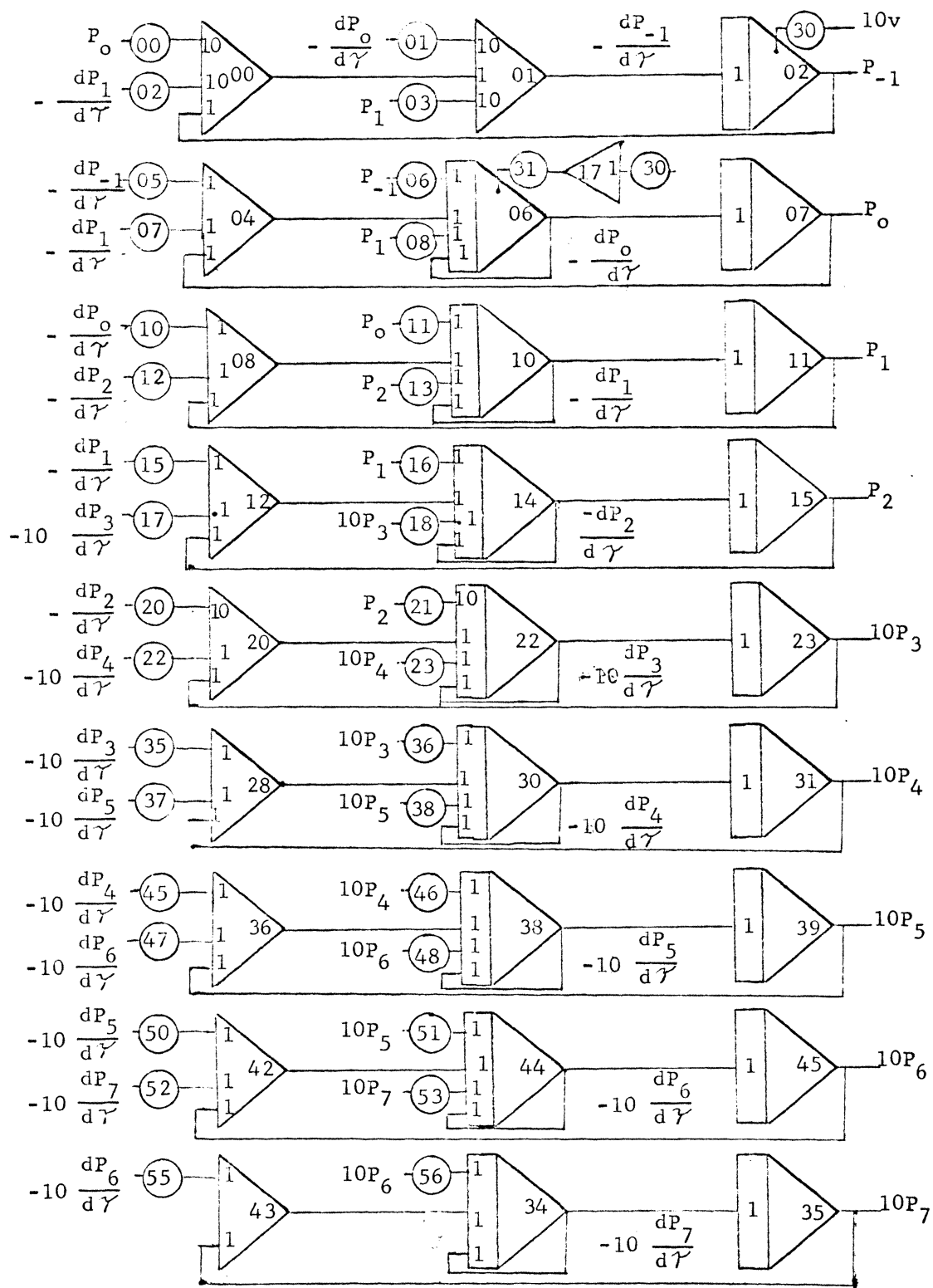


Figure 2. The Analog Computer Circuit

Pot.	n = 0.75	n = 0.5	n = 0.25	Pot.	n = 0.75	n = 0.5	n = 0.25
00	.267	.240	.218	22	.615	.600	.571
01	.267	.240	.218	23	.615	.600	.571
02	.167	.140	.118	30	.500	.500	.500
03	.167	.140	.118	31	.125	.250	.375
05	.125	.250	.375	35	.406	.417	.438
06	.125	.250	.375	36	.406	.417	.438
07	.875	.750	.625	37	.594	.583	.562
08	.875	.750	.625	38	.594	.583	.562
10	.286	.333	.400	45	.421	.429	.444
11	.286	.333	.400	46	.421	.429	.444
12	.714	.667	.600	47	.579	.571	.556
13	.714	.667	.600	48	.579	.571	.556
15	.350	.375	.417	50	.432	.438	.450
16	.350	.375	.417	51	.432	.438	.450
17	.650	.625	.583	52	.568	.562	.550
18	.650	.625	.583	53	.568	.562	.550
20	.385	.400	.429	55	.440	.444	.455
21	.385	.400	.429	56	.440	.444	.455

TABLE I

Table of Potentiometer Settings

IV. CHECKING THE PROGRAM

Due to the complexity of the program and the large number of computer elements used, it became necessary to have a positive check on the wiring of the program and on the operation of the computer elements. The main check used was a static check. The system of equations resulting from assuming a 5 volt input to the simulation at $k = -1$ and allowing all the time derivatives to go to zero was written and solved on the digital computer. This condition was then set up on the computer by removing the initial conditions, adding a constant 5 volt input to amplifier 00, and allowing the solution to reach steady state. Table II is a tabulation of the digital computer results for $n = 0.25, 0.5,$ and 0.75 .

The above procedure, with a check to assure that all of the voltages representing time derivatives were zero, checks on the connection and operation of the amplifiers and those potentiometers that have nonzero voltages as their inputs in the steady-state condition. To obtain a check on the rest of the potentiometers, use was made of the fact that each of the potentiometers that was not checked is next to a potentiometer, with the same setting, that has been checked. Switching the role of the potentiometers within each pair and repeating the static check assures that the rest of the potentiometers are set and connected properly.

Amplifier	Output Variable	n = 0.75	n = 0.5	n = 0.25
02	P ₋₁	3.55	2.94	2.63
07	P ₀	.79	1.30	1.75
11	P ₁	.39	.76	1.23
15	P ₂	.23	.46	.88
23	10P ₃	1.53	3.26	6.30
31	10P ₄	.99	2.17	4.38
39	10P ₅	.62	1.39	2.44
45	10P ₆	.36	.81	1.75
35	10P ₇	.16	.36	.80

TABLE II

Table of Values for Static Check

Input 5 Volts

V. SENSITIVITY

The validity of the results of the program depend to a great extent upon the results not being sensitive to the choice of n and maximum k . Included here are examples of P recorded for the same value of r with different values of maximum k , and P for the same value of r with different values of n .

Figures 3 and 4 contain six recordings of normalized displacement potential at $r = 1.5$ a ($k = 1$) with $\Delta r = 0.5$ a ($n = 0.5$). Each recording was made with successively less computer being used, that is, the first recording was made with the entire simulation ($k_{\max} = 7$), the second was made with the simulation shortened to $k_{\max} = 6$, the next with $k_{\max} = 5$ and so forth to $k_{\max} = 2$. Figure 5 shows three recordings of normalized displacement potential at $r = 3$ a with $k_{\max} = 7, 6$ and 5 . Study of these figures shows that the effect of reducing k_{\max} is to reduce the magnitude of P and to have a slight smoothing effect on the curves. Figure 5 illustrates the reduction in magnitude of P_4 . The shape of all three is, however, the same. This effect is the result of the assumption that all P_k and $\frac{dP_k}{d\gamma}$ are equal to zero for all k greater than k_{\max} .

The smoothing effect is best illustrated by Figures 3 and 4, and practically no magnitude difference is apparent. The smoothing effect is the result of P_1 being affected by a decreasing number of functions, therefore, becoming a simpler function. The lack of magnitude difference can be

explained by the fact that the magnitude at the $r = 1.5 a$ position in the program, being very close to the beginning of the simulation, is more strongly influenced by the strength of the impulse than by the length of the simulation.

Figures 6 and 7 are recordings of P for different values of Δr . Because the normalizing factors for both the strength of the explosion and displacement involve n , the normalized magnitude of P should be larger at the same value of r for smaller values of Δr . Recordings of normalized displacement at $r = a$ for $\Delta r = 0.25 a$, $0.5 a$ and $0.75 a$ in Figure 6 show that the shape is not affected by changing Δr and that the magnitudes are related as expected.

Figure 7, showing normalized displacement at $r = 2.5 a$ for the same values of Δr used above requires more explanation. The shapes of the three recordings are the same but the magnitudes are not related as would be predicted by the scale factors; that is, considering only the scale factor differences, the recording for $\Delta r = 0.25 a$ should be larger than the one for $\Delta r = 0.5 a$. The reason that this did not happen is that the point $r = 2.5 a$ for $\Delta r = 0.25 a$ is very near the end of the simulation and so P is reduced because of the effect demonstrated in Figure 5.

From the above it can be seen that the simulation is almost independent of the choice of Δr and is affected by k_{\max} only for points near k_{\max} .

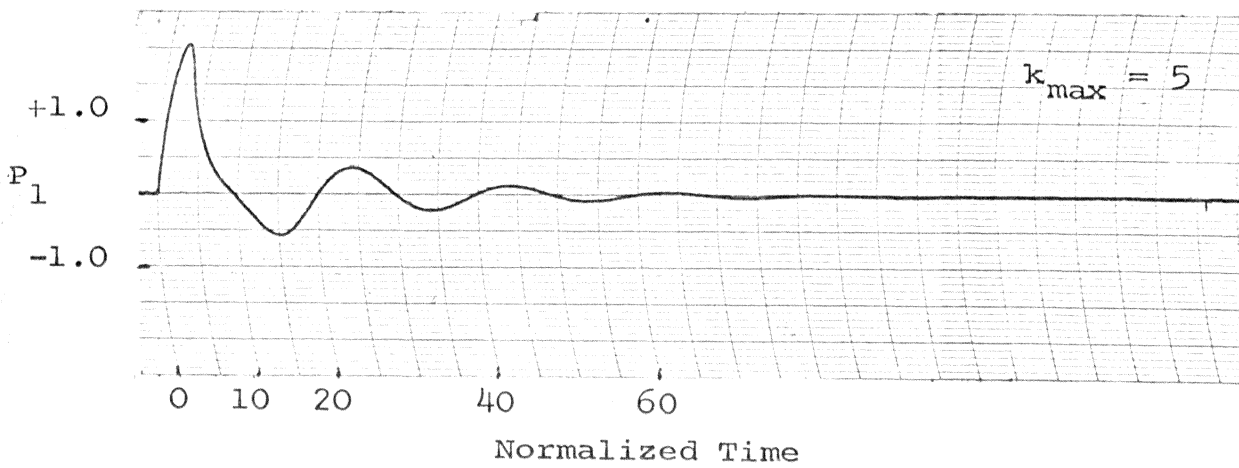
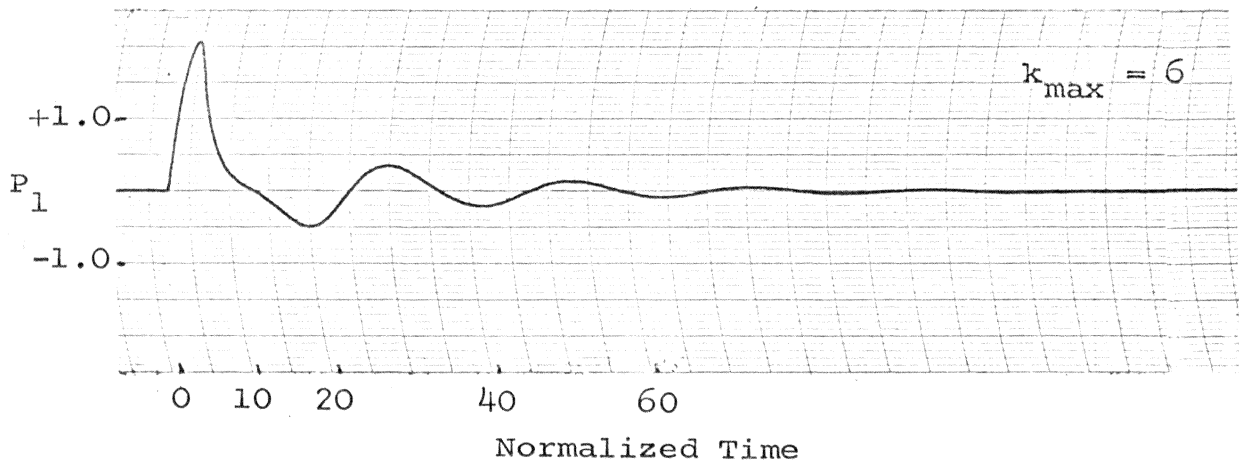
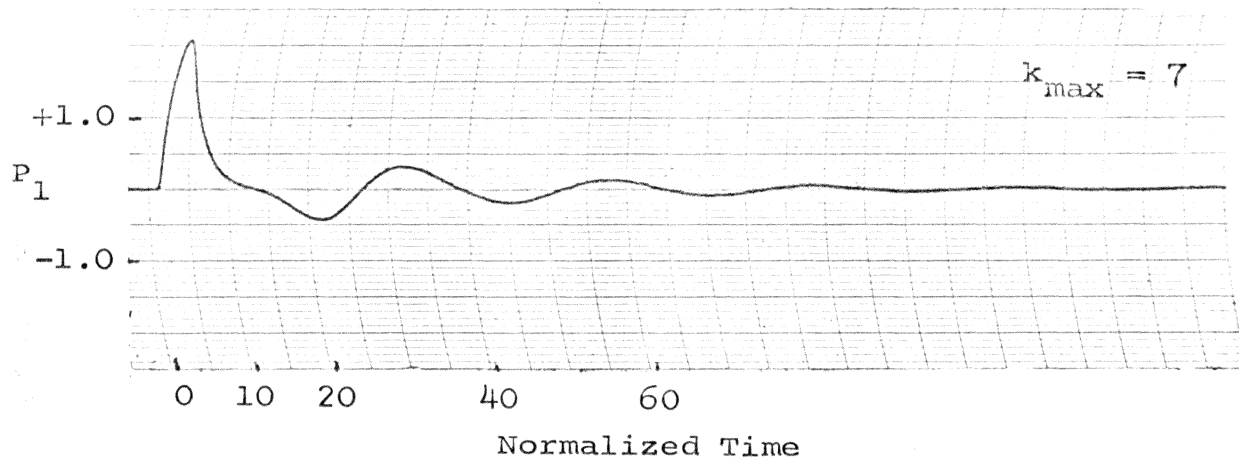


Figure 3. Normalized Displacement Potential at $r = 1.5 a$ with $\Delta r = 0.5 a$

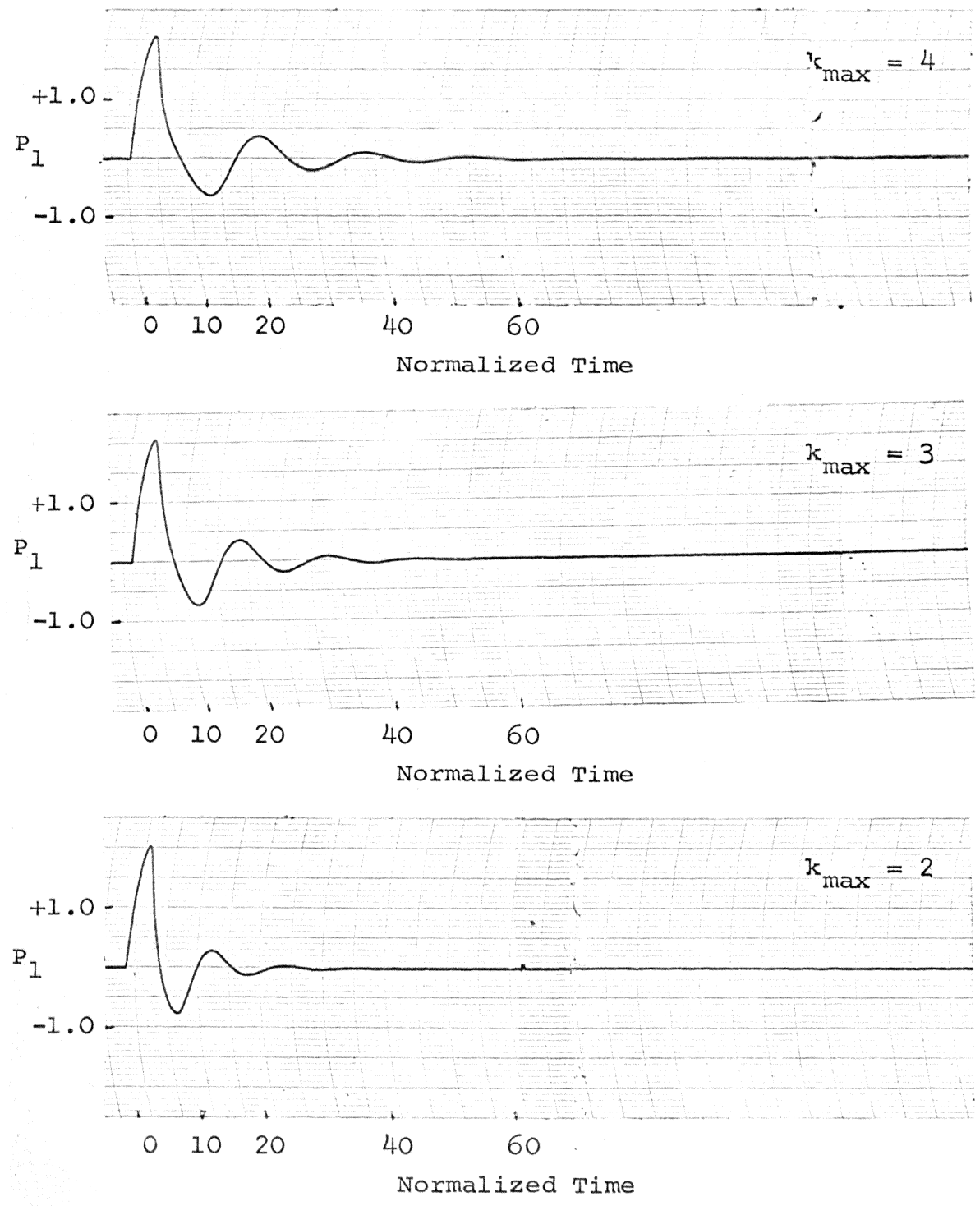


Figure 4. Normalized Displacement Potential at $r = 1.5 a$ with $\Delta r = 0.5 a$

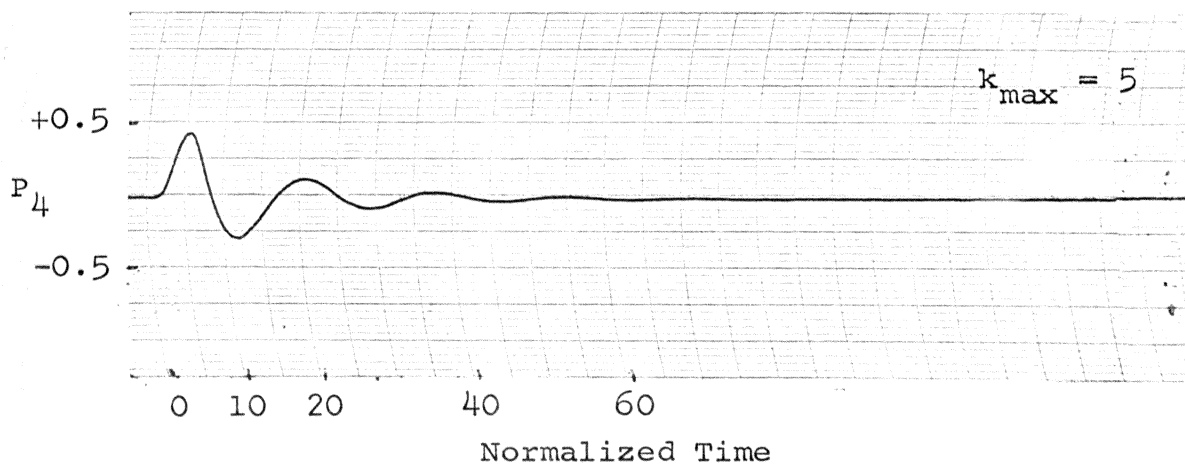
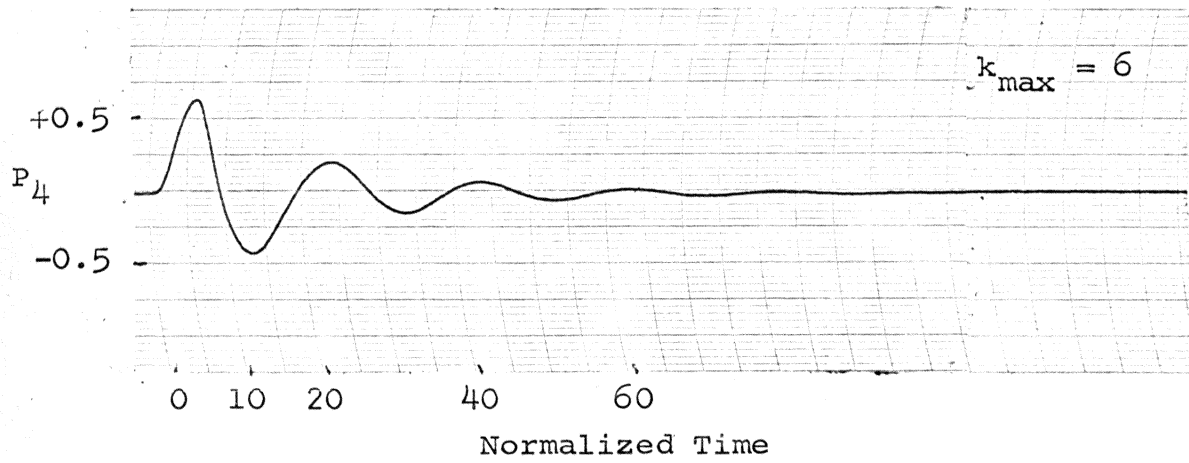
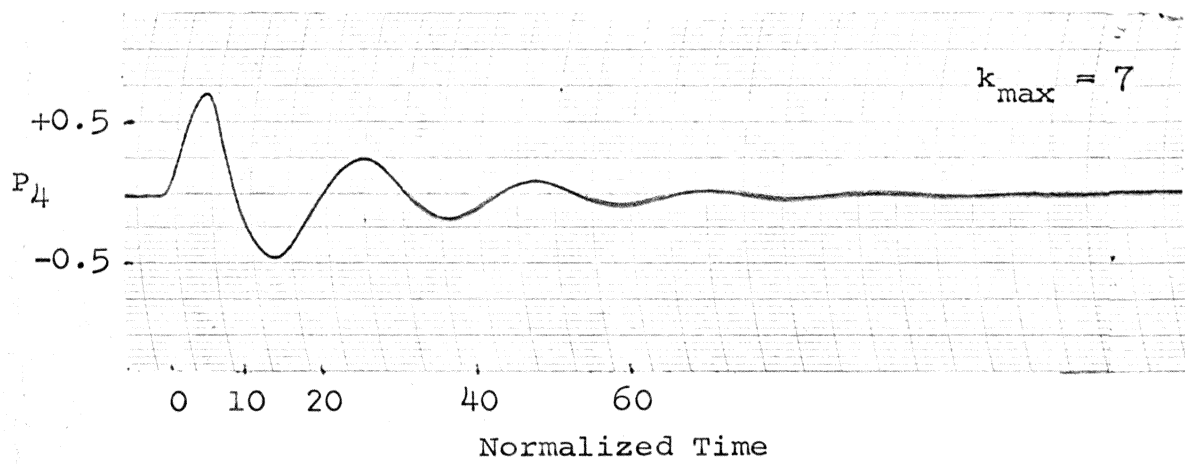


Figure 5. Normalized Displacement Potential at $r = 3 a$ with $\Delta r = 0.5 a$

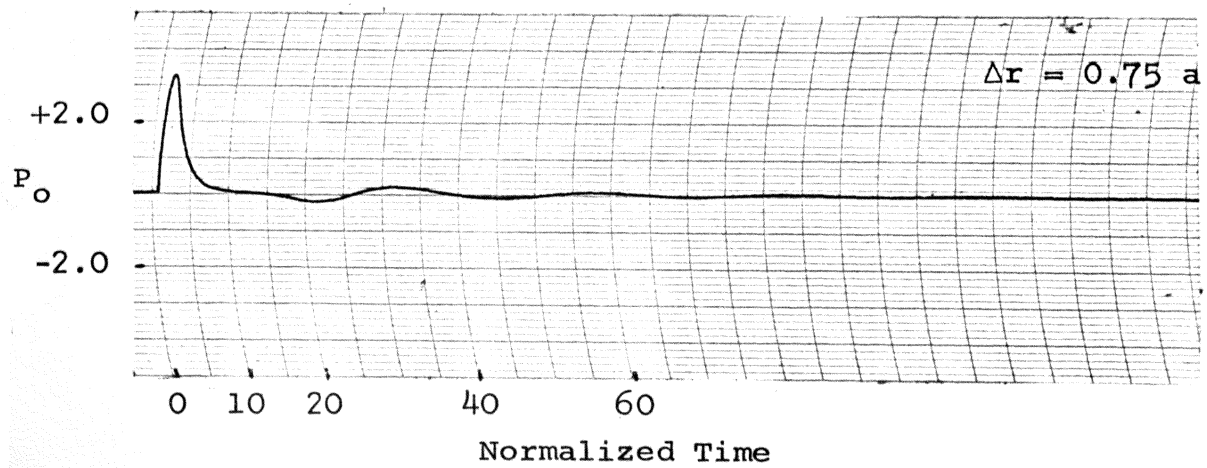
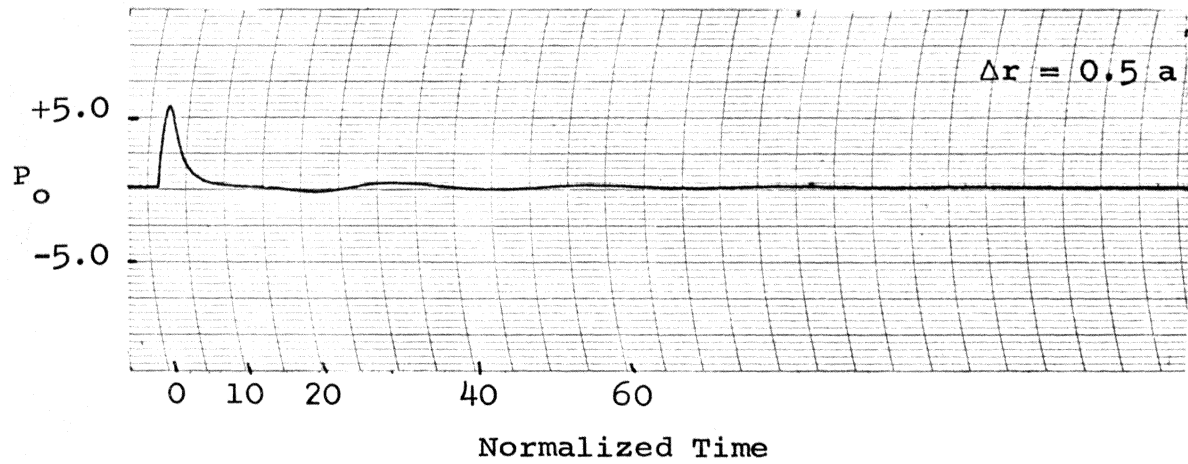
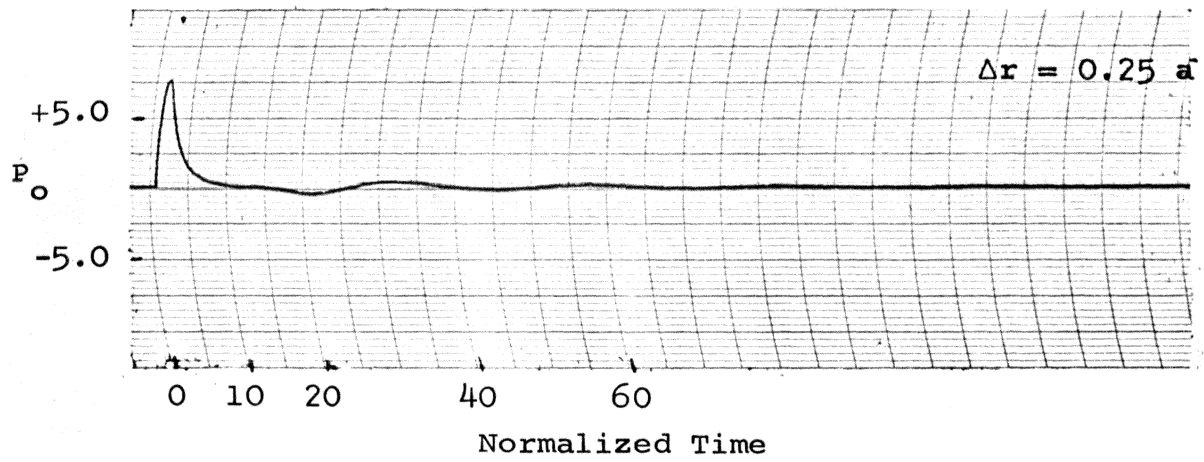


Figure 6. Normalized Displacement Potential at $r = a$

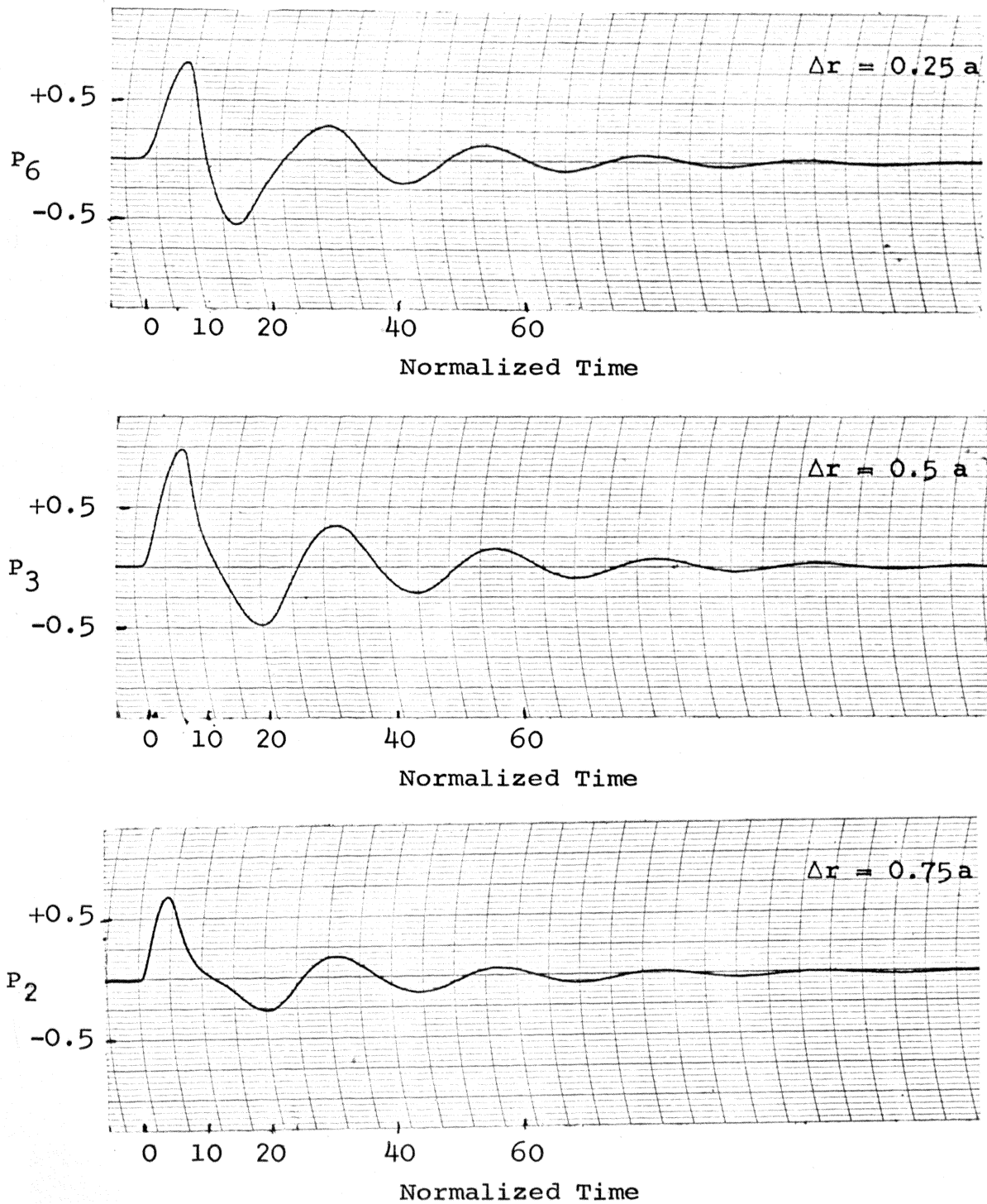


Figure 7. Normalized Displacement at $r = 2.5 a$

VI. THE RESULTS

Of interest as results of the computation are displacement, $\frac{\partial \phi}{\partial r}$; strain, $\frac{\partial^2 \phi}{\partial r^2}$; and velocity, $\frac{\partial^2 \phi}{\partial t \partial r}$. These quantities were obtained in normalized form by the reverse procedure used to eliminate derivatives with respect to r . Recalling the central difference equations from Chapter III, but this time writing them in terms of normalized variables we have

$$\begin{aligned} \left(\frac{\partial P}{\partial r}\right)_k &\cong \frac{1}{2\Delta r} (P_{k+1} - P_{k-1}) \\ \left(\frac{\partial^2 P}{\partial r^2}\right)_k &\cong \frac{1}{\Delta r^2} (P_{k+1} - 2P_k + P_{k-1}) \\ \left(\frac{\partial^2 P}{\partial t \partial r}\right)_k &\cong \frac{d}{dr} \frac{dP_k}{d\gamma} \cong \frac{1}{2\Delta r} \left(\frac{dP_{k+1}}{d\gamma} - \frac{dP_{k-1}}{d\gamma}\right). \end{aligned}$$

Recalling that $\Delta r = na$, $\phi_k = \frac{a^2 n^2 \omega_o^2}{2c^2} P_k$ and $t = \frac{1}{\omega_o} \gamma$, it can be seen that by recording $P_{k+1} - P_{k-1}$ normalized displacement, $\frac{4c^2}{an\omega_o^2} \frac{\partial \phi_k}{\partial r}$, is obtained. By recording $P_{k+1} - 2P_k + P_{k-1}$ normalized strain, $\frac{4c^2}{\omega_o^2} \frac{\partial^2 \phi_k}{\partial r^2}$, is obtained and $\frac{dP_{k+1}}{d\gamma} - \frac{dP_{k-1}}{d\gamma}$ yields normalized wave velocity, $\frac{4c^2}{an\omega_o} \frac{\partial^2 P_k}{\partial t \partial r}$.

On the following pages the results for $n = 0.5$ and maximum $k = 7$ for $k = 1, 2, 3, 4, 5,$ and 6 corresponding to $r = 1.5 a, 2 a, 2.5 a, 3 a, 3.5 a,$ and $4 a$ are presented.

The major characteristics of the computed data, Figures 8 through 13, that can also be observed in experimental data^{13,14} are: the damped oscillations about zero, the increasing time delay as the points get farther from the origin, and the decrease in frequency with time, that is, the second half

cycle takes more time than the first. The output data has been analyzed by Drs. Clark and Rupert and both agree it is consistent with observed data.

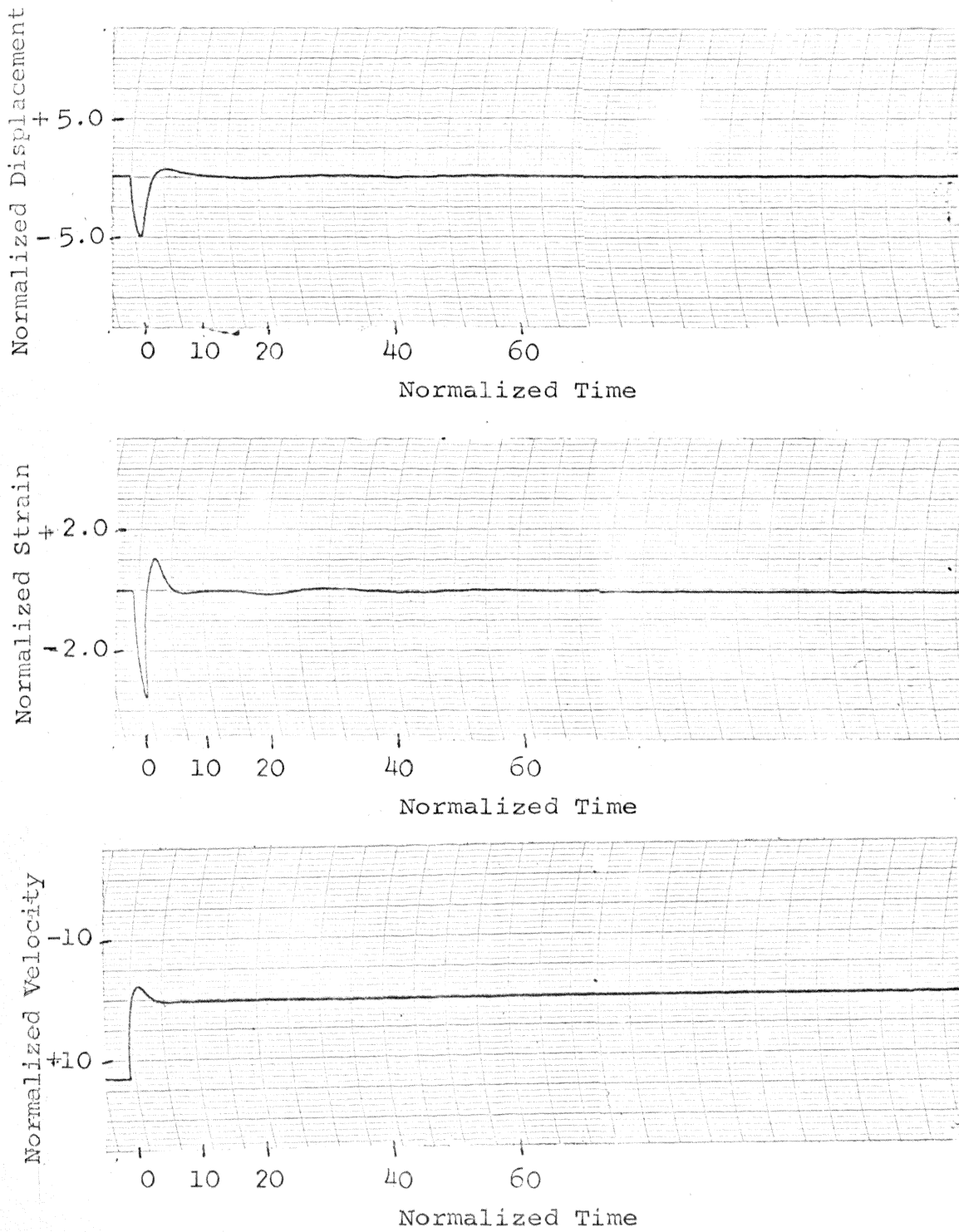


Figure 8. Data for $r = 1.5 a$ $\Delta r = 0.5 a$, $k_{\max} = 7$

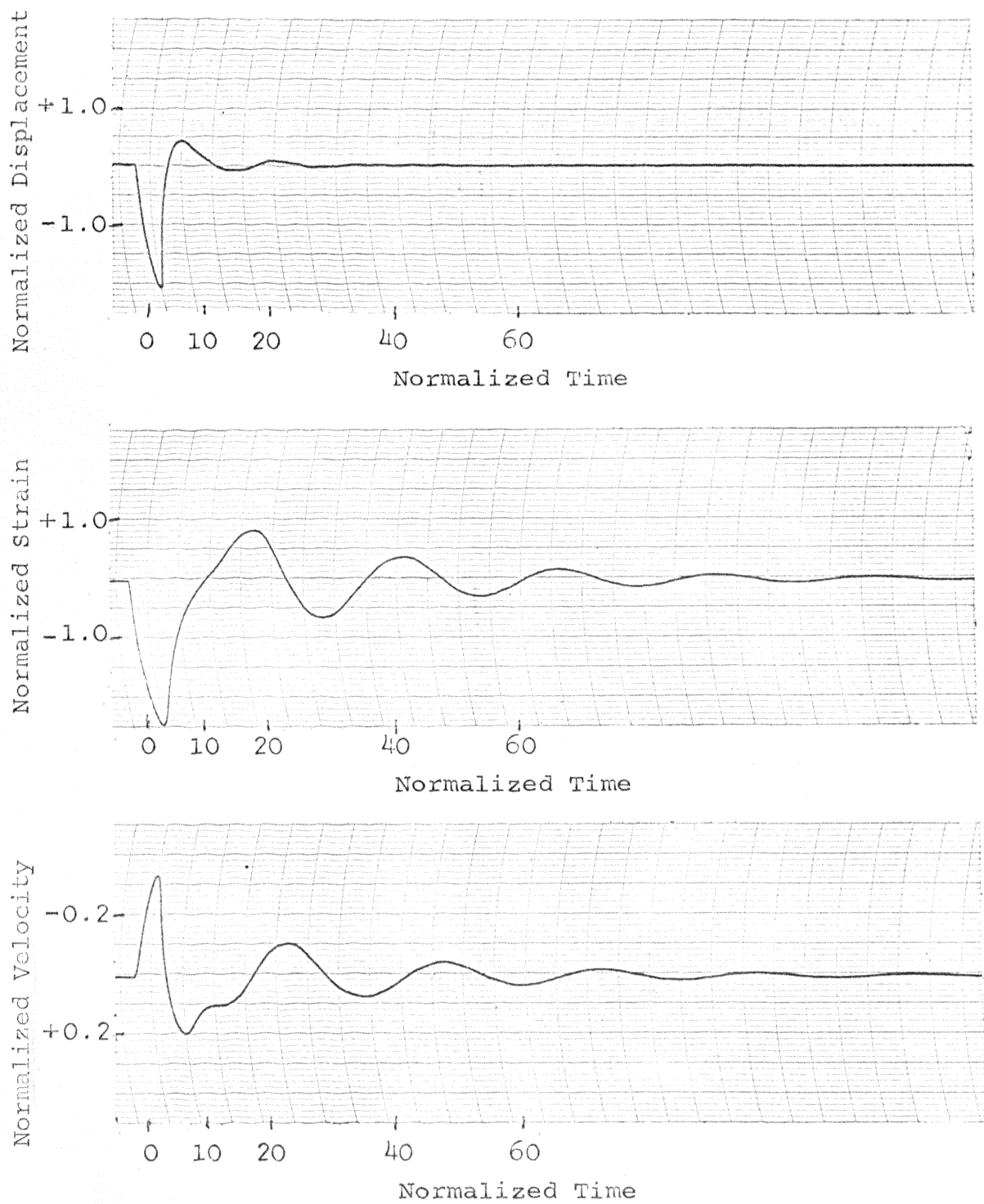


Figure 9. Data for $r = 2.0 a$, $\Delta r = 0.5 a$, $k_{\max} = 7$

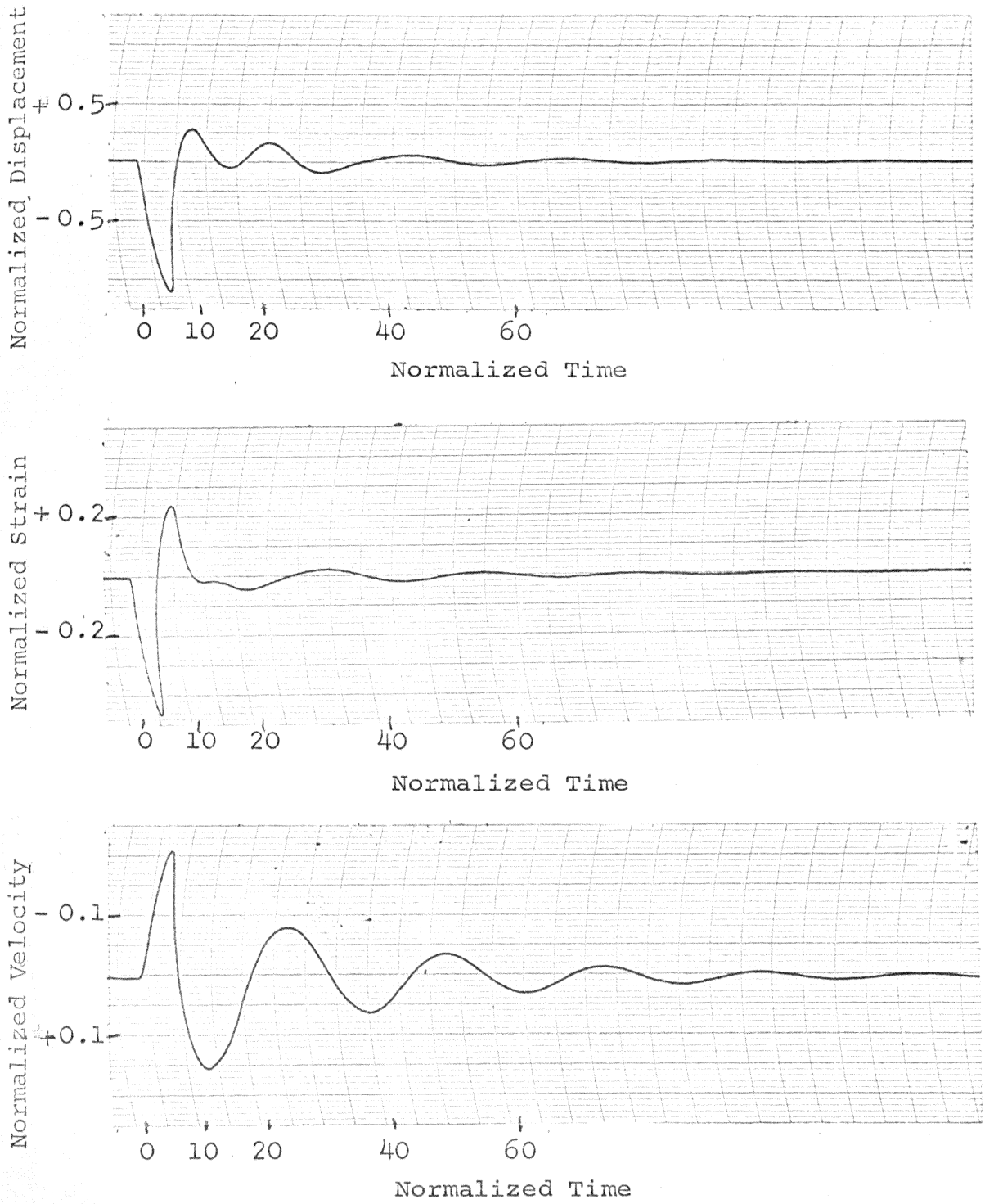


Figure 10. Data for $r = 2.5 a$ $\Delta r = 0.5 a$, $k_{\max} = 7$

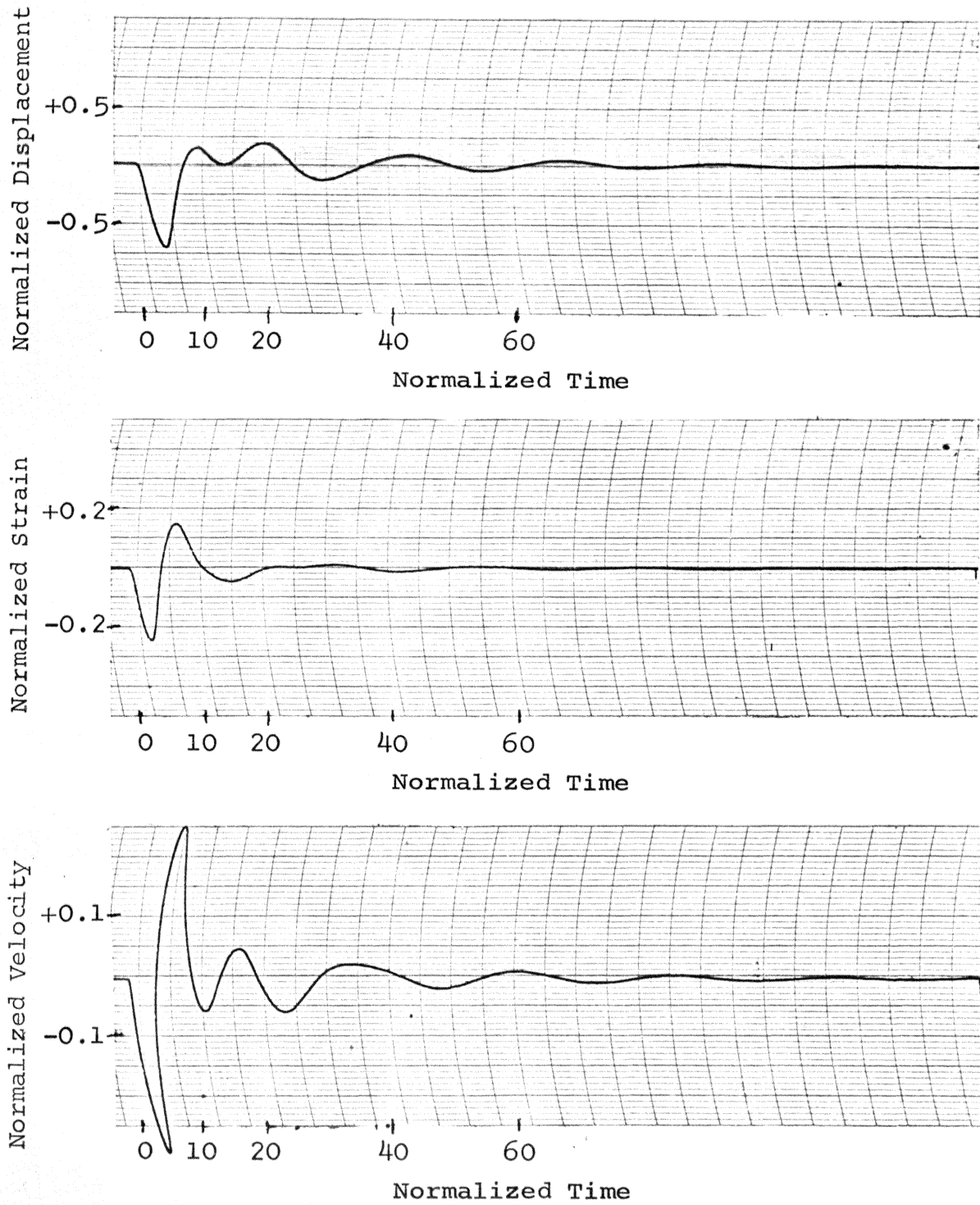


Figure 11. Data for $r = 3.0 a$, $\Delta r = 0.5 a$, $k_{\max} = 7$

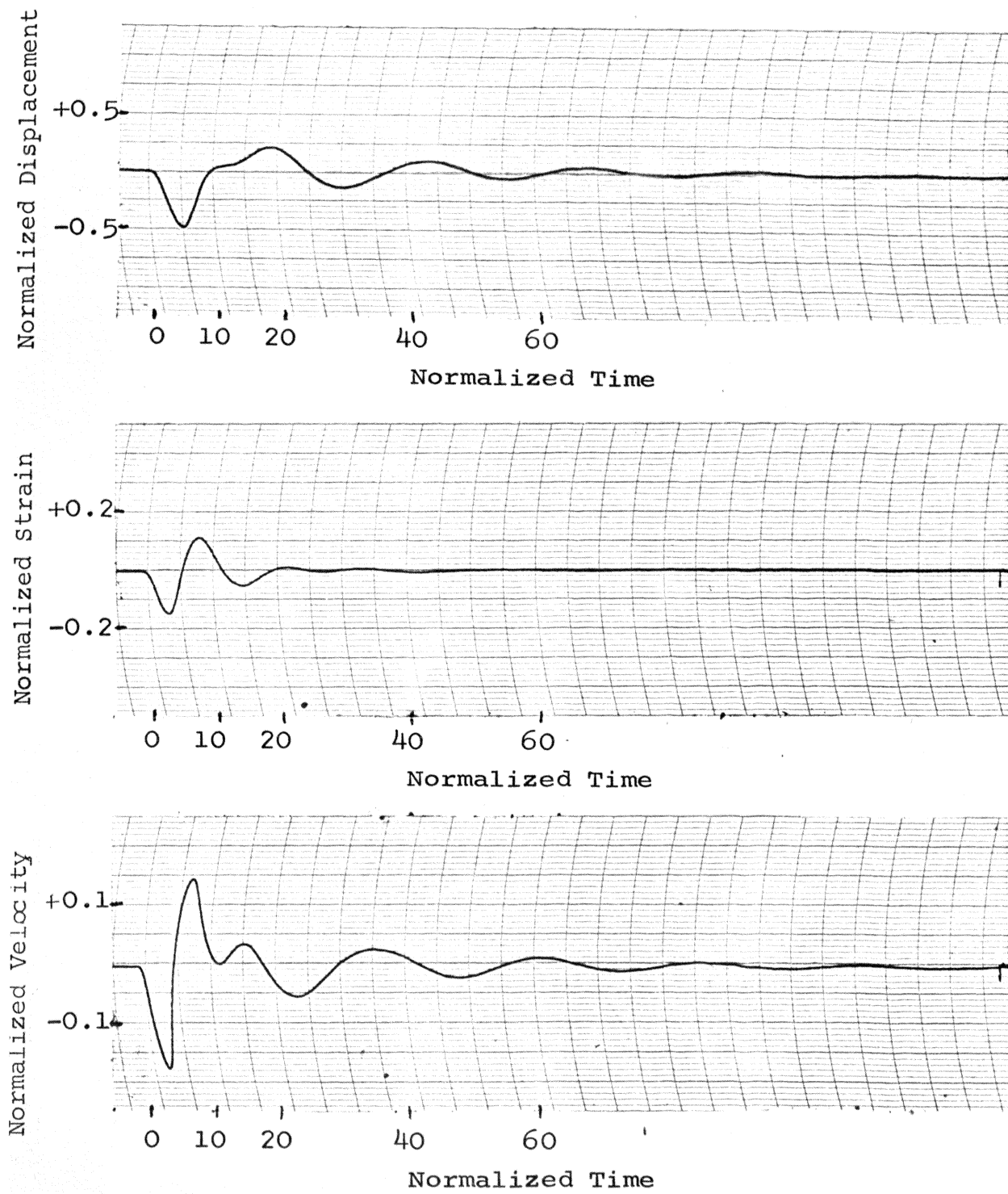


Figure 12. Data for $r = 3.5 a$ $\Delta r = 0.5 a$, $k_{\max} = 7$

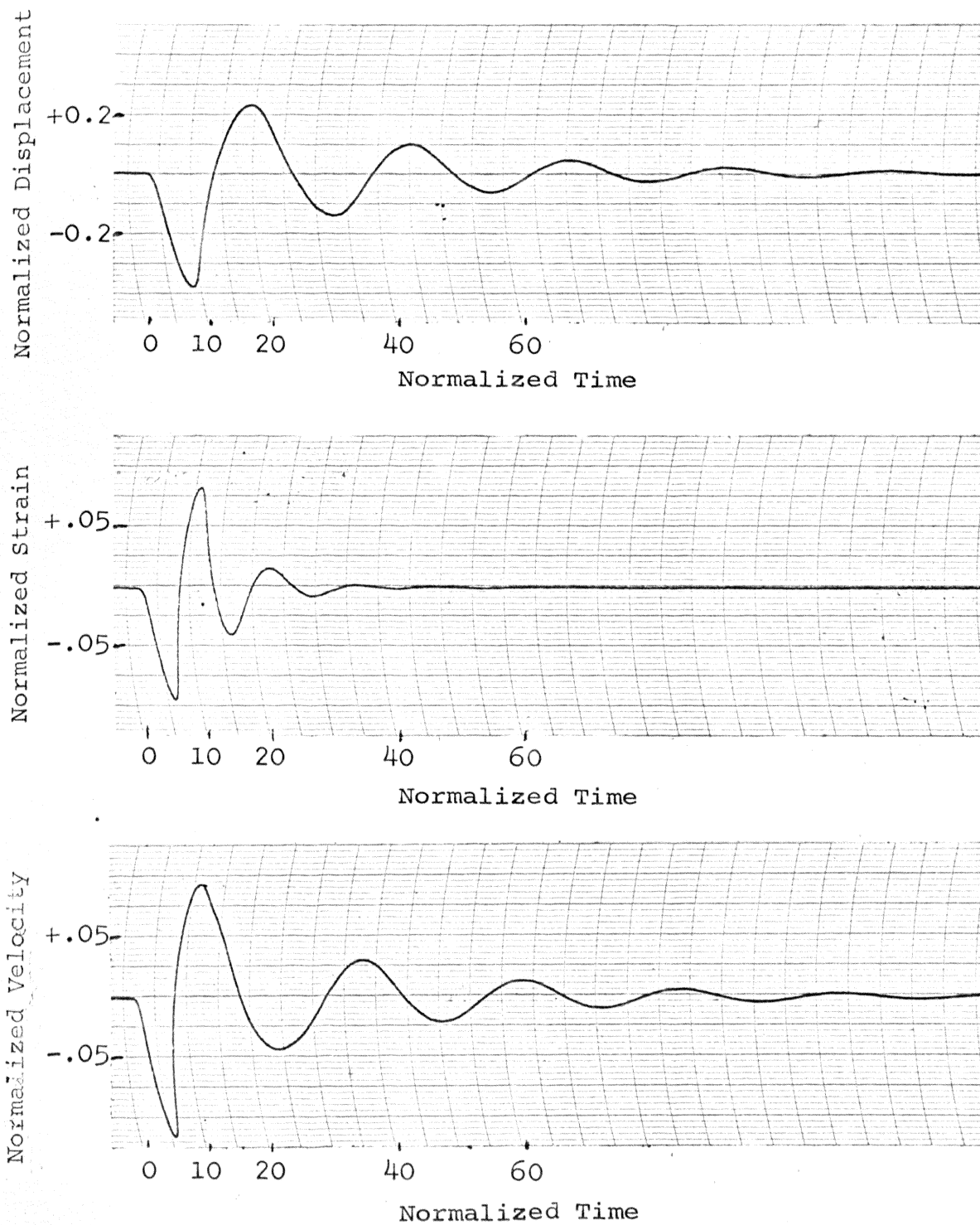


Figure 13. Data for $r = 4.0 a$ $\Delta r = 0.5 a$, $k_{\max} = 7$

VII. CONCLUSIONS

It has been shown that an effective method of solving partial differential equations on the analog computer is to replace derivatives with respect to all but one of the independent variables with differential difference equations. A specific example, the Voigt equation in spherical coordinates was solved and the solution was shown to be relatively insensitive to the mechanics of this type of solution.

The results of the computation have been presented and have been supported by experimental evidence.

The methods used on this problem are sufficiently general to be applied to a large number of other partial differential equations. It should be pointed out however, that there are several types of differential difference equations that can be used. The methods presented here represent the results of a large amount of trial and error investigation of the problem and the solution presented represents the most stable, least sensitive solution to this particular problem. Another similar problem might be better handled with a slightly different approach.

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