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Methods for calculating the minimum amortization tonnage and maximum present value for an ore body

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METHODS FOR CALCULATING THE MINIMUM AMORTIZATION TONNAGE AND MAXIMUM PRESENT VALUE FOR AN ORE BODY

BY

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A

THESIS

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ABSTRACT

The discussion is divided into three parts. Part I describes a procedure for determining the mining rate that gives the minimum amortization tonnage required of an ore body, and the amount of that tonnage. The basic idea is that amortization tonnage is equivalent to the estimated capital costs, including interest, divided by the estimated operating profit per ton. Both capital costs and operating profit per ton can be expressed as functions of mining rate, by making use of empirical equations. Therefore, amortization tonnage can be expressed as a function of mining rate. The amortization tonnage is calculated for a series of mining rates, one of which will indicate a minimum tonnage.

Part II describes a procedure for calculating the optimum mining rate to obtain maximum present value of an ore body, when mining rate is the only variable. An expression is derived which gives present value in terms of mining rate, and this expression is solved using various rates. The results are plotted, and the resulting graph will reveal a maximum present value at one mining rate.

Part III shows how optimum operating conditions for maximum present value can be determined when mining method, mining sequence, milling method, cut-off, and mining rate are variable. The procedure consists essentially of calculating the present value for all practical combinations of the above variables, and selecting the combination that gives the highest present value. If desired, cut-off can be eliminated as one of the variables by making it equivalent to the operating and capital cost of mining a ton of ore.

INTRODUCTION

STATEMENT OF THE PROBLEM.

The problem is to outline two procedures. The first is a procedure for calculating the minimum amortization tonnage required of an ore body. An amortization tonnage is the amount of ore of a certain grade that an ore body must contain to repay the capital cost of mining it. This tonnage is not constant for any given ore body but rather is a function of the mining rate. As the mining rate increases, the operating cost usually decreases, owing to improved efficiencies at higher outputs. If the grade remains constant, this results in an increase in operating profits. However, capital costs also increase with mining rate, because the required plant capacity increases, and the combined effect is to give a definite minimum amortization tonnage at a certain mining rate. The procedure outlined allows the determination of this rate and tonnage, under conditions most likely to be found in practice.

The second procedure is one for calculating the maximum present value possible for an ore body. The factors that affect the present value of an ore body are complex, and can be divided into two groups:

1. Those beyond the control of the engineer.
    a. Marketing factors.
    b. Nature of the ore body, and other physical factors (i.e., location) that apply.

2. Those under the control of the engineer.

   a. Mining and milling methods.

   b. Mining rate.

   c. Cut-offs, or blending ratios.

   d. Mining sequence.

To get an idea of the complexity of the subject, consider an assay wall type deposit. The cut-off determines the reserves, the reserves determine the mining rate, and the mining rate determines the cost, which in turn determines the cut-off. When it is also realized that several mining methods may be equally applicable (each with different recoveries and costs) and that the absolute maximum mining rate for each method is not necessarily the best, then the complexity of the problem becomes apparent. This is without even considering the first group of factors.

IMPORTANCE OF THE SUBJECT.

A knowledge of the amortization tonnage, and present value, is essential in the development of mineral properties. Amortization tonnage is most important in the exploratory stage. It gives a goal to be proven or disproven by exploratory development as rapidly and as cheaply as possible, keeps exploration expenditures within reasonable limits, and gives assurance that capital investment is in line with ore reserves. Naturally, the minimum amortization tonnage is of the most interest to the engineer at this stage.

Present value is important when proven ore reserves exist. It is the best measure of the wealth of the reserves, (1), (2), (3), (4) and operating conditions must be chosen to give the maximum possible present value for the ore body.

As these two items are so important, and as the factors that affect them are so complex (especially for present value), it is desirable that they be calculated by systematic and logical procedures, such as outlined here. This will reduce the possibility of errors, and greatly speed the work.

REASONS FOR ITS SELECTION.

The importance of knowing amortization tonnage is well known, and the necessity of planning for maximum present value is often mentioned in the literature, yet the writer knows of no published work that shows how to calculate a minimum amortization tonnage, or gives a system for calculating maximum present value. As the writer's special interest is in mineral property development, it was not long before this lack became apparent, so it was decided to do something about it, the result being this thesis.

The procedure for calculating minimum amortization tonnage makes use of rudimentary empirical equations. While no claim is made that these equations hold true in practice, they serve to illustrate the method. More refined equations could be obtained by means of analytical geometry, (5), and while their use is not absolutely necessary, there is always the interesting possibility that a general expression linking two variables (i.e., mining rate, and operating cost per ton) may be found, under more or less defined conditions. This would be of great assistance in later work.

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REVIEW OF LITERATURE

An article by H. M. Callaway (6) is interesting as it introduces a mathematical expression to show the inverse proportionality between operating cost per ton, and mining rate. This is also done in the procedure outlined here for calculating minimum amortization tonnage, but in this case the expression is different, as is its purpose, and its sphere of application.

Mr. Callaway is concerned with the effect on cut-off grade of reductions in mining rate, which often occur during periods of reduced metal prices when operators try to mine higher grade ore. He shows that a reduction of mining rate on an established property will increase mining costs, and hence will increase cut off grade, due to the element of fixed costs. He gives a method whereby the increase in grade necessary to balance a given reduction in tonnage can be calculated. The expression for cost per ton in terms of mining rate is:

$$C = \frac{a}{m} + b.$$

Where C = cost per ton.
a = fixed costs for the period of one month.
m = tons mined and milled per month.
b = variable cost per ton.

In order to break even, the revenue from a ton of ore must exactly equal the cost of mining and milling that ton, which leads to the expression:

$$R \times SP = \frac{a}{m} + b$$

Where

R = recovery grade.

SP = market price.

From this, Mr. Callaway gets the expression:

$$R = \frac{\frac{a}{m} + b}{SP}$$

which is used to calculate the increase in grade necessary to offset a given decrease in tonnage.

In the part of the present discussion dealing with amortization tonnage, the purpose is to pick the mining and milling rate that gives the least amortization tonnage required of an ore body, as yet undeveloped. In this case, Mr. Callaway's formula for cost per ton in terms of mining rate is not applicable. It employs fixed costs, such as depreciation, which suggests a fixed plant capacity, the variations in mining rate occurring by operating at various levels under full capacity.

What is required in the present case is a formula linking cost to rate on a series of possible plants, each operating at full capacity. The expression used is:

$$\frac{C}{T} = kT^n$$

Where

C = total operating cost, mining and milling, over the period of one year. Includes taxes.

T = tons mined and milled per annum.

k & n = constants.

k & n are evaluated in a particular case by estimating the cost per ton at two different mining rates, substituting in the general expression to get two equations that are solved for k & n.

With regards to blending high and low grade ores, an article by E. T. Wood (7) is interesting. Mr. Wood gives a formula which can be used to calculate the additional dollars to be gained per ton of high grade uranium ore by blending with low grade. The formula is:

$$95A_B\ (1+R) - 95A_H - R(TMC - \frac{C}{T} + 3.00) = \$/Ton_H$$

Where:

A_B = assay of blended mix.

A_H = assay of high grade ore.

R = ratio of low grade tons blended with each ton of high grade.

TMC = total mining cost for high grade tons only.

C = total fixed dollar investment.

T = tons of high grade ore.

This formula only applies to ores sold under the AEC Circular 5, and when A_H & A_B lie between 0.20 and 0.50% U_3O_8. The article states that it can be shown by the equation that with specific grades of ore available for blending, the maximum additional dollars to be gained by blending is realized at a blended grade of 0.20%. Using a blended grade of 0.20% U_3O_8, the formula is equated to zero, and blending curves plotted to show the break even point at different mining costs for various assays of high grade, A_H, in terms of either "R", or the assay of the low grade, "A_L." A_H, A_L, and R are related in a nomograph, so that if the value of any two are known, the value of the third can be found.

Figures 1 and 2 illustrate the blending curves, and Figure 3 relates "R", "A_L" and "A_H". The following example illustrates the use of figures 1 to 3. Assume the available high grade to assay 0.40% U_3O_8, and the mining cost to be $8.00 per ton, including administrative costs. The break even value of "R" is determined to be 2.37 from Figure 1, and consequently "A_L" to be 0.115% U_3O_8 from Figure 3., or using Figure 2., this cut-off value of "A_L" is determined directly to be 0.115% U_3O_8, and the ratio "R", 2.37 is obtained from Figure 3. In other words, if 2.37 tons of 0.115% U_3O_8 were mined and blended with one ton of 0.40% U_3O_8, the same dollar income would be realized as mining only the one ton of 0.40% U_3O_8. If, however, low grade material assaying more than 0.115% U_3O_8 is available, blending will return a greater profit than mining the high grade alone.

Mr. Wood also includes in his article a nomograph showing the additional dollars to be gained by blending one ton of ore of 0.20 to 0.50 percent with ore below 0.20 percent in grade. Blend assay 0.20 percent. The total dollars to be gained by blending any specific ore body is obtained by multiplying the dollars read from the graph by the tons of high grade ore in the ore body.

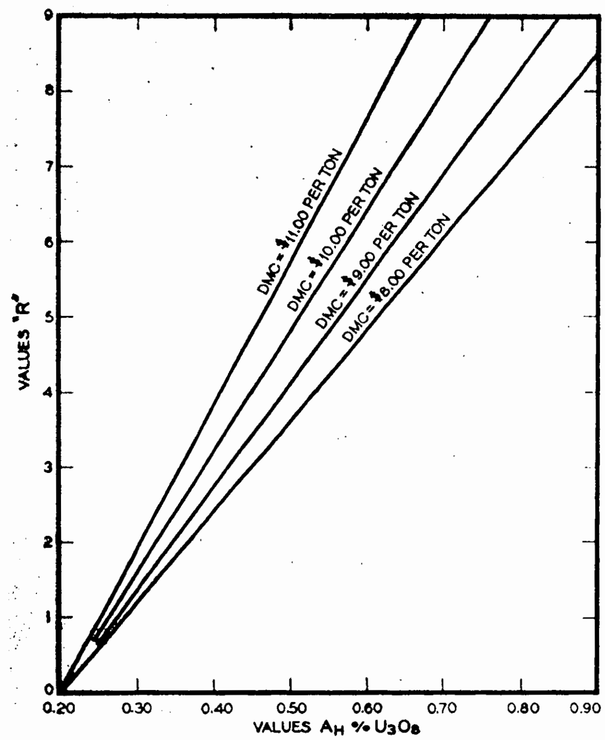


Fig. 1—Blending is profitable only for values of "R" to left of *DMC* or direct mining cost line. Assay of blend is 0.20 pct U_3O_8. Note that equation changes for values of high grade ore above 0.50 pct (see text).

Figure 1-Blending curve, showing break even point in terms of R.

(after Wood)

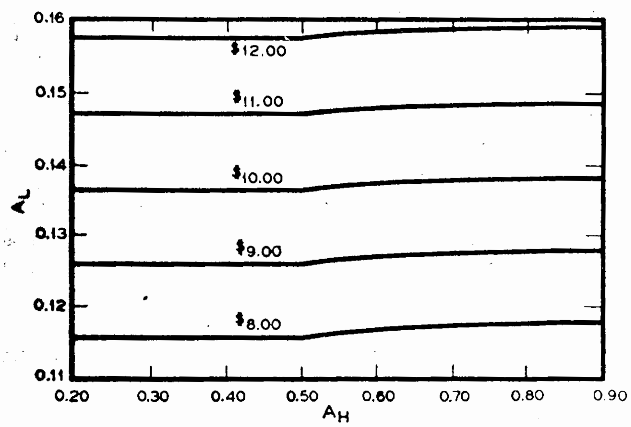


Fig. 2—Minimum values of low grade ore that can be blended to 0.20 pct U_3O_8 at mining costs shown.

Figure 2-Blending curve, showing break even point in terms of A_L.

(after Wood)

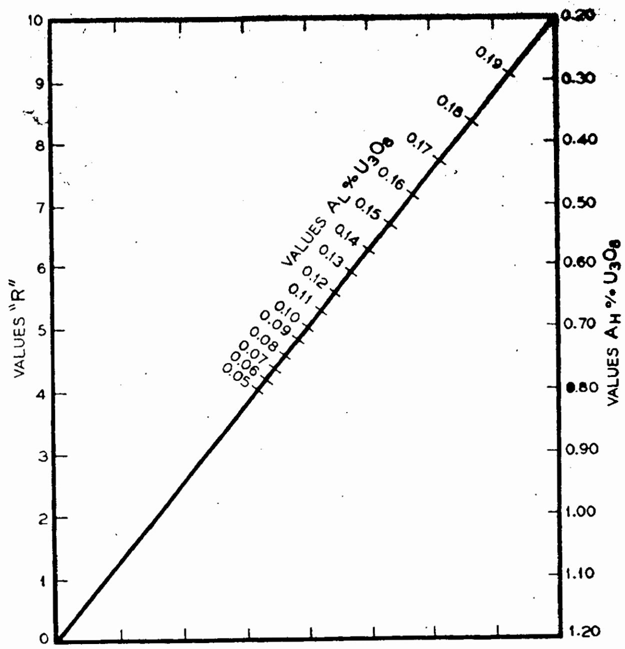


Fig. 3—Graph of $R = \frac{A_H - 0.20}{0.20 - A_L}$

Figure 3-Nomograph relating R, A_H, and A_L.
(after Wood)

As with Mr. Callaway's article, Mr. Wood's article refers to an established mining operation, with a specific total fixed dollar investment. It is this fact that under certain conditions makes blending profitable, the decrease in average grade being offset by decrease in costs at the higher tonnage, due to the element of fixed costs. Mr. Wood's treatment is rather limited when it comes to considering blending in relation to present value, which is the important criteria in these discussions. Being based on total fixed dollar investment as it is, his treatment is effectively limited to one rate of production. Blending results in large increases in ore reserves, which must be accompanied by increases in profit per ton and mining rate, if present value is to be appreciably increased. Using Mr. Wood's method, the engineer is denied the possibility of increasing present value through an increase in mining rate. In the present discussion, this is not the case.

A talk given by J. A. Patterson (8) at the 1958 meeting of the A.I.M.E. in New York, gives an interesting account of the manner in which U_3O_8 cut-off grade is determined for the ore bodies in the Ambrosia Lake, New Mexico uranium district. Statistical methods are used for analyzing the variations in ore reserves, mining costs and profits, with variations in cut-off grade, to enable the mine operator to select the cut-off that will return the maximum profit.

The first step is to construct a tons of ore vs. cut-off grade curve for the ore body under consideration. An example of this curve is shown in Figure 4.

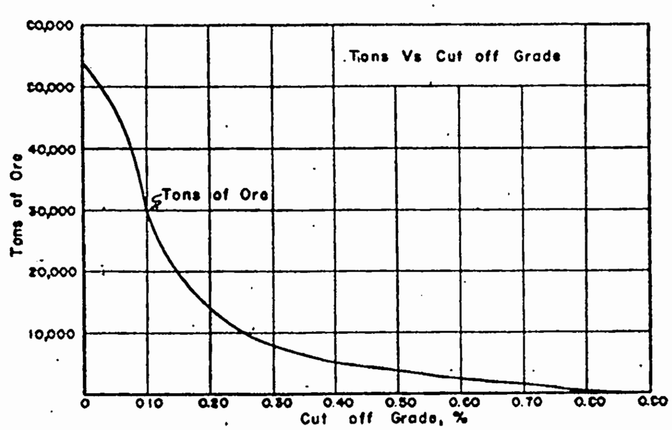


Figure 4-Tons of ore vs. cut-off grade.

(after Patterson)

The nature of the ore bodies, and the sampling methods at Ambrosia Lake allow the use of statistical data handling techniques, based on sample lengths, which greatly simplify the drawing of this curve. The curve shows that ore reserves increase rapidly with decreasing cut-off.

The tons vs. cut-off curve is then used to draw an average grade vs. cut-off curve, as illustrated in Figure 5. This curve shows that average grade decreased with decreasing cut-off.

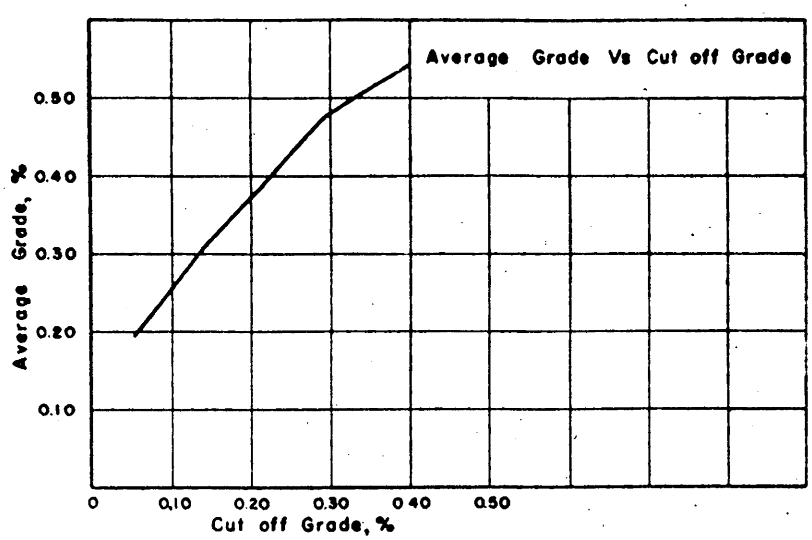


Figure 5-Average grade vs. cut-off grade.

(after Patterson)

A graph is then constructed showing the total cost of mining the ore body at various values of cut-off. The total cost is equivalent to the cost per ton times the tonnage at the appropriate cut-off. The cost per ton is estimated separately for each cut-off, because changing cut-off changes the reserves and with them the scale of operation and the cost.

The gross value of the ore body at different cut-off values is then plotted. The difference between the gross value and the total cost of mining at each cut-off gives the profit at that cut-off. Profits are plotted against cut-off, and the cut-off that gives the maximum profit is accepted as optimum. Figure 6 illustrates the variation of total mining cost, total gross value, and total profit with cut-off.

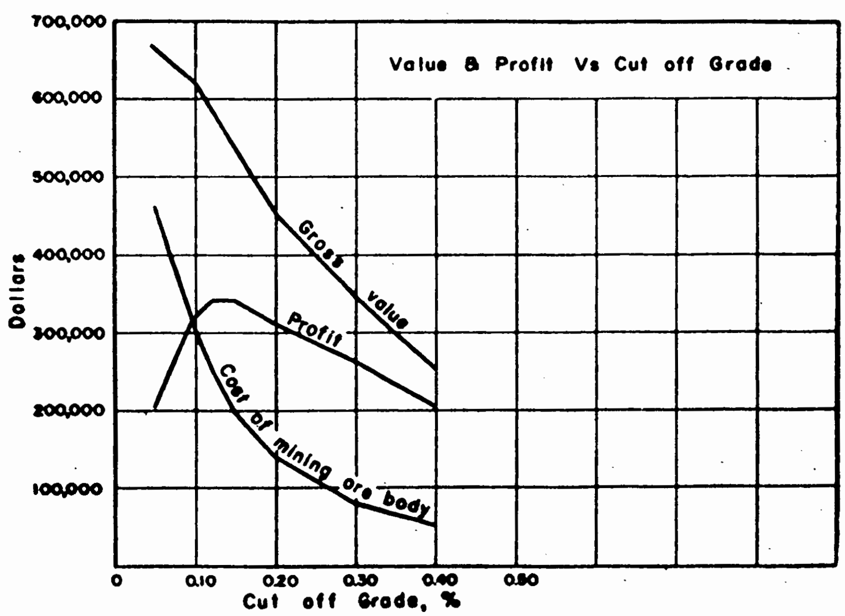


Figure 6–Variation of total mining cost, total gross value, and total profits, with cut-off.

(after Patterson)

The drawback to the method outlined by Mr. Patterson is that it employs total net profits rather than present value, as the basis for fixing optimum cut-off, contrary to the statements of most writers who say that planning for maximum present value is of prime importance.

A talk presented by W. O. Hotchkiss and R. D. Parks at the 1936 meeting of the A.I.M.E. meeting in New York (9) sets forth in a general way some items that can be of use to a mine operator in calculating which of several methods of mining applicable to his own ore body will produce the greatest total present value of profits, as a result of its individual relative cost, recovery, rate of production, and profit. In their talk, they stated that recovery and profits in the mining business do not go hand in hand because usually some part of an ore body can be recovered at a lower cost per ton than a higher proportion of it, and because present value is a better measure of the value of a property than total net profits, especially for purposes of comparison.

They give an example of an ore body, 25% of which is cheaper to mine than the remainder and show that even though mining the easy 25% alone results in lower total net profits, it actually gives a greater present value than mining the two parts of the ore body together. Another more general example brings out this point that the total of future income does not in any way represent present value, and that present value is largely dependent on the time period during which the income is received. Examples are given that show that planning for maximum present value rather than maximum total net profits is also of advantage to fee interests as well as shareholders, even in the case of flat rate royalties.

Other variable factors that must be considered by an operator when planning for production are mentioned in the article. For instance, if a decline in prices over a series of years is expected it will be an incentive to speed production to the extent that there is a demand for the product. On the other hand, the prospect of an increasing price curve may be sufficient to offset the increased present value of a larger immediate production. Of course, the increase in present value brought about by prompt realization is offset by present expenditures for increased plant capacity, etc. Not so obvious is the adverse affect on present value of the longer deferment period required to build the larger plant.

Under conditions of fluctuating demand there is always the possibility that added plant capacity, once provided, may be useful during only part of the operating life. Also, any plant constructed for present industrial or metallurgical processes may be made obsolete by technical advancements. If such advancements can be anticipated, it may well affect the policy of operations. For example, assume a metal mine is operating on a relatively low scale of metallurgical recovery. The operators expect that over a period of years they will be able to devise means of greatly improving the metallurgical work. These probable improvements may well be so important as to offset other inducements for larger present production. The present value of the deferred increase in production at the improved recovery might easily be greater than the additional present worth of prompt realization at low recovery.

Other intangible factors that must be considered when planning for production are community responsibility, possible future need for minerals left in unrecoverable state, and others.

DISCUSSION

DETERMINING MINING RATE THAT GIVES MINIMUM AMORTIZATION TONNAGE, AND THE AMOUNT OF THAT TONNAGE.

USE OF EMPIRICAL EQUATIONS

In the example given in this section use is made of elementary empirical equations to express the relationship between mining rate and various other quantities, such as operating cost per ton, plant and equipment costs, and others. No claim is made that the equations used actually apply in practice, their purpose is merely to illustrate the method of use. On an actual job, equations could be derived that were closer to fact by making estimates, to give an example, of cost per ton at various mining rates, plotting the results and using methods of analytical geometry to get the equations linking them. (5)

Actually, the job could be done without the use of empirical equations, but it is felt that by employing them in a number of cases some general relationship might become apparent which could be very useful and time saving in later work.

<u>Operating cost per ton in terms of mining rate</u>.

Based on the idea that costs are inversely proportional to mining rate, cost per ton is expressed in terms of mining rate by the following equation.

$$\frac{C}{T} = kT^n$$

Where

C = total operating cost, mining and milling, over a period of one year. Includes normal taxes.

T = tons mined and milled per annum.

k and n = constants.

The constants, k and n are evaluated in any particular case by estimating the operating cost per ton $(\frac{C}{T})$ at two different mining rates, and substituting in the general expression to get two equations that are solved for k & n. To show how this is done, consider the following example. The cost of production is estimated at two possible mining rates. They are:

	mining rate	Dollars/Ton
Small production	146,184	9.18
Large production	797,953	7.22

The cost of production includes taxes. To estimate the taxes it is first necessary to estimate the cost exclusive of taxes. Then in conjunction with the value of the ore it is possible to arrive at profits before taxes. Knowing the profits and tax regulations, it is possible to arrive at total taxes, and taxes per ton, which is added to the original operating cost per ton.

Applying these figures to the valuation of k and n in the general expression:

$$\frac{C}{T} = kT^n$$

$$9.18 = k(146{,}184)^n \qquad (1)$$
$$7.22 = k(797{,}953)^n \qquad (2)$$

$$\log 9.18 = \log k + (n \log 146{,}184) \qquad (3)$$
$$\log 7.22 = \log k + (n \log 797{,}953) \qquad (4)$$

Subtracting (4) from (3) gives

$$\log 9.18 - \log 7.22 = n(\log 146{,}184) - n(\log 797{,}953)$$

$$0.96284 - 0.85854 = n(\log 146{,}184 - \log 797{,}953)$$

$$0.10430 = n(5.16495 - 5.90195)$$

$$0.10430 = - n(0.73700)$$

$$n = - \frac{0.10430}{0.73700}$$

$$n = - 0.1415$$

Substitute $n = - 0.1415$ in (1) and solve for k.

$$9.18 = k(146{,}184)^{-0.1415}$$

$$k = 9.18\ (146{,}184)^{0.1415}$$

$$k = 49.39$$

Therefore, the equation relating operating cost per ton to mining rate in this case is

$$\frac{C}{T} = \frac{49.39}{T^{(0.1415)}}$$

The relationship between cost per ton and mining rate is shown diagramaticly in Figure 7.

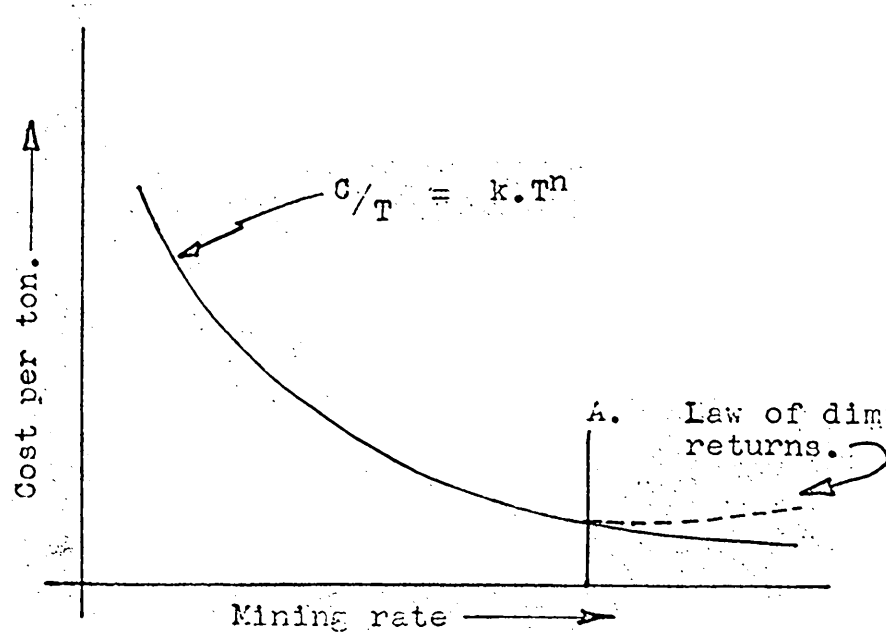


Fig. 7 - OPERATING COST PER TON VS. MINING RATE.

According to the law of diminishing returns, this curve is not strictly correct, but should commence an upward swing at higher production rates, as shown in Figure 7. The commencement of the upward swing in unit costs corresponds to the point where all available working faces are being worked at full efficiency. If extra men are employed in the existing working faces, the rate of production will be increased, but at the cost of efficiency, resulting in higher unit costs.

In the case of manufacturing industries which operate non-wasting assets, a certain amount of inefficiency and higher unit costs can be tolerated if the increased production results in an over all increase in annual profits. In the case of a mine, which operates a wasting asset (i.e., an ore body), this situation would seem undesirable because there is only a limited number of units of ore available, and anything that would tend to reduce the profit per unit would reduce the total return.

However, from a present value point of view, increased unit costs may be desirable, if they are accompanied by increased production which would tend to reduce the time required to exploit the ore body, and hence increase the present value. Therefore, any expression linking unit costs and production rates should reflect this upward swing, although for purposes of illustration, the expression $\frac{C}{T} = kT^n$ will be used in this discussion.

Plant and equipment costs in terms of mining rate.

A very simple relationship is used, expressing plant and equipment costs as so much per ton of daily capacity.

i.e.: $$F_{PE} = \frac{C_{PE} \cdot m}{T}$$

Where

C_{PE} = total plant and equipment costs.

F_{PE} = factor for cost per ton of daily capacity.

T = tons mined and milled per annum.

m = working days, per annum.

Cost per foot of hoisting shaft in terms of mining rate.

Cost per foot is assumed to be directly proportional to the mining rate, because as the mining rate increases, so must the size of the shaft and hence the cost per foot. The general expression used is:

$$c/f = k_1T + k_2$$

Where c/f = cost per foot.

T = tons mined and milled per annum.

k_1 & k_2 = constants, evaluated by estimating the cost per foot at two rates of production, and substituting these values in the general expression to give two equations that are solved for k_1 & k_2.

For example, suppose the cost per foot for a shaft large enough to handle 100,000 tons per annum is 349 dollars. The same shaft, if it was made large enough to handle 200,000 tons per annum, could cost 574 dollars per foot. Then:

$$349 = k_1 . 100,000 + k_2 \quad (1)$$
$$574 = k_1 . 200,000 + k_2 \quad (2)$$

Subtracting (1) from (2) gives

$$225 = k.100,000$$

$$k_1 = 0.00225$$

Substituting $k_1 = 0.00225$ in (1) gives

$$349 = 225 + k_2$$

$$k_2 = 124.$$

<u>Deferment period in terms of mining rate</u>.

Deferment period is assumed to be directly proportional to mining rate, because generally speaking the greater the mining rate, the bigger the plant that must be built, and the longer the period required to build it. The general expression used is:

$$d = k_3.T + k_4$$

Where

d = years of deferment.

T = tons mined and milled per annum.

k_3 & k_4 = constants, evaluated the same way as k_1 and k_2.

CALCULATION OF MINIMUM AMORTIZATION TONNAGE

A knowledge of minimum amortization tonnage is very important when contemplating the development of a mineral property because if exploration development proves this minimum tonnage not to exist, then work on the property should cease.

Amortization tonnage is obtained by dividing the total estimated capital costs (including interest) by the expected operating profits per ton. Both capital costs and operating profits are functions of mining rate, so an expression can be obtained for amortization tonnage in terms of mining rate. Various values of mining rate are then taken, and the corresponding values for amortization tonnage calculated. This procedure will disclose a mining rate that gives an amortization tonnage that is lower than any other, and this is the one required.

Capital requirements.

Exclusive of interest.

Capital will be required for two purposes, plant and equipment, and pre-production development.

Plant and equipment. As mentioned in the section headed "Empirical equations," plant and equipment will be expressed in terms of tons of daily capacity. The general expression used is:

$$C_{PE} = \frac{F_{PE} \times T}{m}$$

the symbols having been defined previously.

Development. Development includes all shaft sinking, cross cutting, drifting, raising and winzing necessary to develop enough working places for full production. For purposes of this discussion, it will be assumed that all pre-production development is completed at the same time as the plant is completed, at the end of the deferment period, so that full production commences more or less immediately at the end of this period.

To estimate the cost of pre-production development, proceed as follows:

1. Estimate the maximum level interval, as determined by physical considerations.

2. Plan on developing one level at a depth which will give backs equivalent to the maximum level interval.

3. Estimate the cost of hoisting and ventilation shafts to this depth. The cost of ventilation shafts (C_{VS}) is assumed to be constant here, but the cost of the hoisting shaft will be a function of the mining rate:

$$C_{HS} = D\ (c/f)$$

$$= D\ (k_1 . T + k_2)$$

Where C_{HS} = total cost of hoisting shaft.

D = depth.

$(k_1 . T + k_2)$ = cost per foot of hoisting shaft in terms of mining rate.

4. Estimate the cost of all other pre-production development work.

   a. Work, the cost of which is <u>not</u> included in the expression for operating cost per ton in terms of mining rate. This includes stations and main haulage cross cuts. (symbol, C_{XC})

   b. Work, the cost of which <u>is</u> included in the expression for operating cost per ton in terms of mining rate (symbol, C_{ND}). Allowance must be made for the fact that this work is both capitalized and charged as an operating expense. Do this by dividing its cost by the amortization tonnage Symbol, A_t and subtracting the result from the expression for cost per ton.

<u>Total</u>. The total capital cost, exclusive of interest, is

$$D(k_1 . T + k_2) + C_{VS} + C_{XC} + C_{ND} + \frac{F_{PE} \times T}{m}$$

<u>Interest on capital</u>.

The interest on the capital invested must be considered as an expense. Suppose for simplicity sake the total capital to be invested in the property to bring it into production is "C." Suppose also that this capital is invested over a deferment period of "d" years, with an equal amount being invested each year. Let the productive life of the mine equal "n" years.

At the end of the deferment period, the capital invested has amounted to

$$\frac{C}{d}(1+r)^d + \frac{C}{d}(1+r)^{d-1} \text{-----------------------------} + \frac{C}{d}(1+r)$$

or:

$$\frac{C}{d}\left[(1+r)^d + (1+r)^{d-1} \text{-----------------------------} + (1+r)\right]$$

The sum of the terms in the square brackets is:

$$\frac{1}{r}\left[(1+r)^{d+1} - (1+r)\right]$$

and the total capital cost at the end of the deferment period is:

$$\frac{C}{d}\left\{\frac{1}{r}\left[(1+r)^{d+1} - (1+r)\right]\right\}$$

This is the amount that must be considered as being invested in the property at the commencement of operations. For convenience, call it C'.

The interest on C' over the productive life of the mine is also an expense, and must be considered as adding to the invested capital.

To return the principle, C', the investor must get back $\frac{C'}{n}$ at the end of the first year, and so on down to the last year. In the meantime he has had:

$\frac{C'}{n}$ invested for 1 year.

$\frac{C'}{n}$ invested for 2 years.

$\frac{C'}{n}$ invested for n years.

Therefore, the value of the investment, with interest is:

$$\frac{C'}{n}\left[(1+r) + (1+r)^2 \text{------------------} (1+r)^n\right]$$

The sum of the terms in the square brackets is:

$$\frac{(1+r)\ .\ \left[(1+r)^{n}-1\right]}{r}$$

and the total value of the investment is:

$$\frac{C'}{n}\left\{\frac{(1+r)\ .\ \left[(1+r)^{n}-1\right]}{r}\right\}$$

or:

$$\frac{C}{d.n}\left\{\frac{1}{r}\quad\left[(1+r)^{d+1} - (1+r)\right]\right\}\left\{\frac{(1+r)\quad\left[(1+r)^{n}-1\right]}{r}\right\}$$

In this expression, $(k_3T + k_4)$ can be substituted for "d" as pointed out previously in the section on empirical equations, and $\frac{At}{T}$ can be substituted for "n" where

n = years of productive life.

A_t = amortization tonnage.

T = tons mined and milled per annum.

<u>Profit per ton</u>.

The profit per ton is equivalent to the value of the ore per ton minus the cost per ton.

$$P = V - \left[kT^n - \frac{C_{ND}}{A_t} \right]$$

Where

P = profit per ton.

V = value of ore per ton.

kT^n = operating cost per ton in terms of mining rate.

$\frac{C_{ND}}{A_t}$ = cost per ton of development work included both as a capital cost, and as an operating cost in the expression $\frac{C}{T} = kT^n$. (See page 31)

<u>Method of calculating minimum amortization tonnage</u>.

The amortization tonnage is equivalent to the estimated capital costs (including interest) divided by the expected profit per ton, i.e.

$$A_t = \frac{\dfrac{\left\{D(k_1T+k_2)+C_{VS}+C_{XC}+C_{ND}+\dfrac{F_{PE}\cdot T}{m}\right\}\left\{\dfrac{T}{A_t} \cdot \dfrac{1}{(k_3T+k_4)}\right\}}{V - \left[kT^n - \dfrac{C_{ND}}{A_t}\right]}}{\left\{\dfrac{1}{r}\left[(1+r)^{k_3T+k_4+1} - (1+r)\right]\right\}\left\{\dfrac{(1+r)}{r}\left[(1+r)^{At/T} - 1\right]\right\}}$$

This expression can be rewritten as

$$\frac{A_t^{\,2}}{T}\left[V - kT^n\right] - \left[At/T \cdot C_{ND}\right] = \frac{\dfrac{\left\{D(k_1 . T+k_2)+C_{VS}+C_{XC}+C_{ND}+\dfrac{F_{PE}.T}{m}\right\}}{(k_3 . T + k_4)}}{\left\{\dfrac{1+r}{r}\left[(1+r)^{At/T} - 1\right]\right\}\left\{\dfrac{1}{r}\left[(1+r)^{k_3T+k_4+1} - (1+r)\right]\right\}}$$

To calculate the minimum amortization tonnage, proceed as follows:

1. Pick a value of T.

2. Substitute in the equation to obtain an expression in the form

$$k' A_t^2 - k'' A_t = k''' \left[(k'''')^{A_t/T} - 1 \right]$$

3. By a process of iteration, arrive at the value of A_t that satisfies this expression. This is the amortization tonnage required at that particular rate of mining.

4. Repeat for several values of T, one of which will indicate a minimum amortization tonnage.

Evaluation of results.

If diamond drilling, and/or exploratory underground development indicate that an amortization tonnage is contained in the ore body above the proposed first level, and if physical conditions are such as to permit mining at the appropriate rate, then it is possible to proceed with development designed to block out reserves.

Diamond drilling, and exploratory underground development may indicate that the minimum amortization tonnage does not occur in the ore body above the proposed first level, or if it does, that conditions are such that prevent mining at the necessary rate. There might also be indications that if deeper levels were opened up, an amortization tonnage would not be contained above them, either. Under these conditions it is clear that it is not possible to develop an amortization tonnage for the level interval and mining and milling method considered. The next step will be to repeat the calculations, using various combinations of mining and milling method and level interval, in the hope of finding one which will permit the development of an amortization tonnage. If no suitable combination is found, then the prospect must be abandoned.

Application.

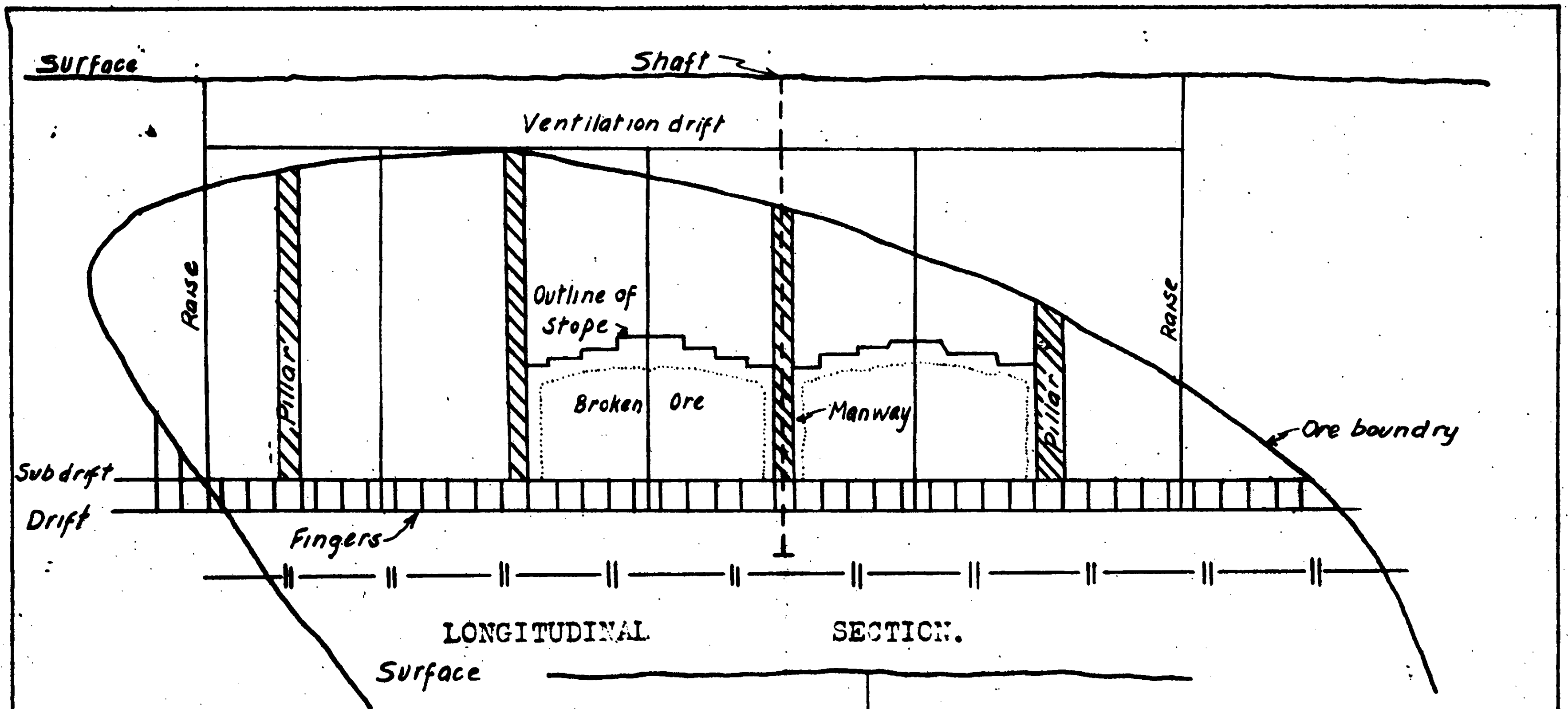
Find the minimum amortization tonnage for the hypothetical ore body illustrated in Figure 8, given the following details:

Value of ore, V (Estimated from earlier work)	=	\$20.00/ton
Cost per ton, kT^n	=	$49.39(T)^{-0.1415}$
Cost of normal development, C_{ND} Made up of:	=	\$141,100.00
Drifting, 940' @ \$60/foot	=	56,600
Sub drifting, 910' @ \$28.30/foot	=	25,750
Fingers, 1040' @ \$12/foot	=	12,500
Raises, 1290' @ \$20/foot	=	25,750
Vent. drifts 730' @ \$28.30/foot	=	20,500
Total		\$141,100
Cost of stations, crosscuts, C_{XC}	=	11,000
Cost of ventilation shaft, C_{VS} (Included in raise cost.)	=	Zero
Depth of hoisting shaft, D	=	330 feet
k_1 & k_2 (From section on empirical equations)	=	0.00225 and 124
Factor for plant and equipment, F_{PE}	=	1500
Interest rate, r	=	0.05
k_3 & k_4 (From section on empirical equations.)	=	0.00001 and 1.50

Table I and II illustrate the calculation of this problem, and Figure 9 illustrates the results graphically. It can be seen that the minimum amortization tonnage is 63,500 tons, and occurs at a mining rate of 17,500 tons per annum. It now remains to check these results against the actual conditions. From Figure 8, the tonnage of ore indicated above the first level is 312,000 tons. At 90 percent recovery, there will be 280,000 tons of minable ore with an indicated average grade of $20.00 per ton. This is greatly in excess of the required 63,500 tons, and as a mining rate of 17,500 tons per annum looks physically feasible, it is alright to proceed with development designed to block out reserves. If the development showed an average grade of less than $20/ton, the figure used in the original calculations, the calculations would have to be repeated to give a new minimum tonnage and rate.

Use of digital computer.

The expression for amortization tonnage shown on page 34 would lend itself very well to solution by a digital computer. A program could be set up which would instruct the machine to determine the amortization tonnage that would equate the left and right hand sides of the expression for any given mining rate. Through the use of a series of mining rates, the machine would supply the amortization tonnage for each rate. The mining rate which gave the minimum amortization tonnage would then be readily apparent.



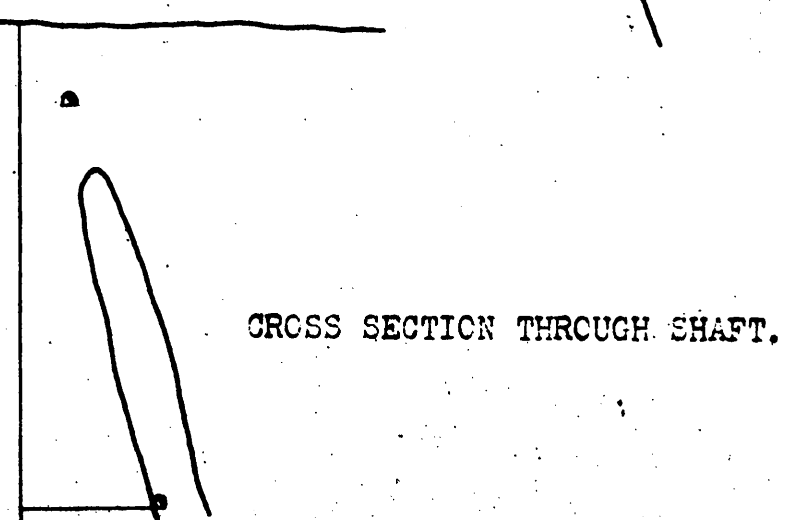


Fig.8- HYPOTHETICAL ORE BODY DEVELOPED TO FIRST LEVEL.

Scale: 1 inch = 125 feet.

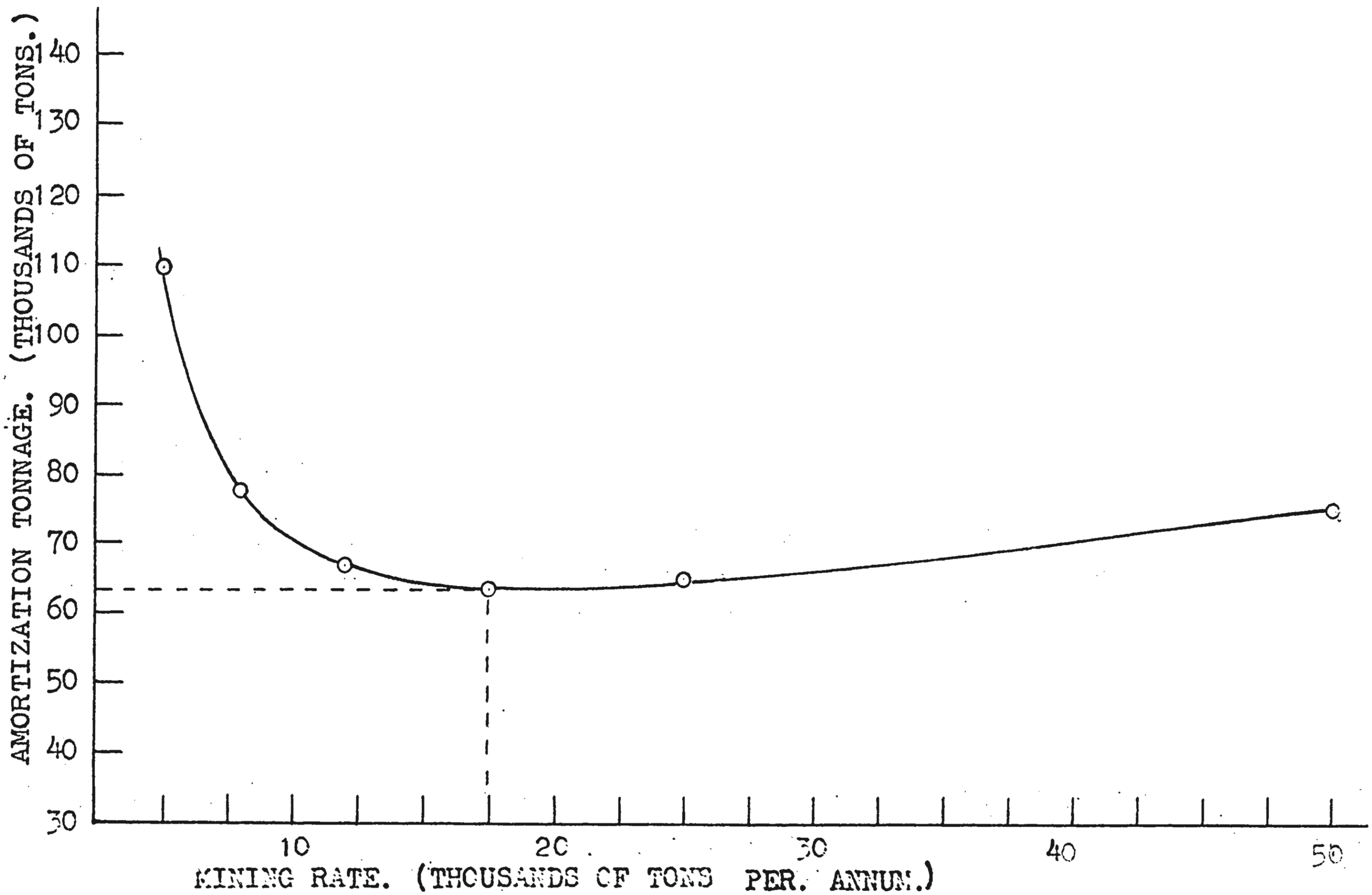


Fig. 9 - AMORTIZATION TONNAGE VS. MINING RATE.

TABLE I. CALCULATION SHEET FOR DETERMINING MINIMUM AMORTIZATION TONNAGE.

$$\overbrace{(A_t)^2\left[\frac{V - kT^{(n-1)}}{T}\right]}^{\text{"a"}} - \overbrace{(A_t)\cdot\frac{C_{ND}}{T}}^{\text{"b"}} = \left\{D(k_1T+k_2) + C_{VS} + C_{XC} + C_{ND} + \frac{F_{PE}\cdot T}{m}\right\}\left\{\frac{1}{r}\left[(1+r)^{k_3\cdot T + k_4 + 1} - (1+r)\right]\right\}\left\{\frac{1}{(k_3\cdot T + k_4)}\right\}\overbrace{\left\{\frac{(1+r)}{r}\left[(1+r)^{A/T} - 1\right]\right\}}^{\text{"c"}}$$

1	2	3	4 "a"	5	6 "b"	7	8	9	10	11	12	13	14	15
T.	V.	kT^{n-}	$\frac{2-3}{1}$	C_{ND}	$\frac{5}{1}$	D	k_1T+k_2	7X8	C_{XC}	C_{VS}	F_{PE}	12X1	m	$\frac{13}{14}$
	$20.00	$\frac{49.39}{T^{(0.1415)}}$		141,100		330	0.00225.T + 124		11,000	0	1500		300	
1,000.	"	18.53	0.00142	"	141.10	"	126	41,500	"	"	"	$1.5X10^6$	"	$0.5X10^4$
3,000.	"	15.91	0.00136	"	47.00	"	131	43,000	"	"	"	$4.5X10^6$	"	$1.5X10^4$
5,000.	"	14.80	0.00104	"	28.4	"	135	44,500	"	"	"	$7.5X10^6$	"	$2.5X10^4$
8,000.	"	13.25	0.000765	"	17.6	"	142	46,800	"	"	"	$12X10^6$	"	$4.0X10^4$
12,000.	"	13.07	0.000579	"	11.75	"	151	49,750	"	"	"	$18X10^6$	"	$6.0X10^4$
17,500.	"	12.40	0.000435	"	8.05	"	163	53,800	"	"	"	$26.3X10^6$	"	$8.79X10^4$
25,000.	"	11.79	0.00033	"	5.64	"	180	59,000	"	"	"	$37.5X10^6$	"	$12.5X10^4$
50,000.	"	10.68	0.000186	"	2.82	"	237	78,500	"	"	"	$75.0X10^6$	"	$25.0X10^4$

1	16	17	18	19	20	21	22	23	24	25 "c"	Multiply "a" in table by "a" in equation, etc.
T.	Add 9,10 11,5,15.	$\frac{1}{r}$	(1+r)	$k_3T + k_4 + 1$	18^{19}	20-18	21X17	$\frac{1}{k_3T+k_4}$	18X17	$\frac{16X23}{22X24}$	
		$\frac{1}{0.05}$	1.05	0.00001T + 1.5 + 1							These equations solved for A_t in table II
1,000.	198,600	"	"	2.51	1.1305	0.0805	1.61	0.660	21	$4.432X10^6$	$A_t^2(0.00142) - A_t(141.10) = 4.432X10^6\left[(1.05)^{A_t/0.1X10^4} - 1\right]$
3,000.	210,100	"	"	2.53	1.1315	0.0815	1.63	0.652	21	$4.689X10^6$	$A_t^2(0.00136) - A_t(47.00) = 4.689X10^6\left[(1.05)^{A_t/0.3X10^4} - 1\right]$
5,000.	218,600	"	"	2.55	1.1325	0.0825	1.65	0.645	21	$4.886X10^6$	$A_t^2(0.00104) - A_t(28.40) = 4.8855X10^6\left[(1.05)^{A_t/0.5X10^4} - 1\right]$
8,000.	238,900	"	"	2.58	1.1340	0.0840	1.68	0.634	21	$5.344X10^6$	$A_t^2(0.000765) - A_t(17.6) = 5.344X10^6\left[(1.05)^{A_t/0.8X10^4} - 1\right]$
12,000.	261,850	"	"	2.62	1.1362	0.0860	1.72	0.619	21	$5.854X10^6$	$A_t^2(0.000579) - A_t(11.75) = 5.854X10^6\left[(1.05)^{A_t/1.2X10^4} - 1\right]$
17,500.	293,000	"	"	2.675	1.1397	0.0897	1.79	0.596	21	$6.582X10^6$	$A_t^2(0.000435) - A_t(8.05) = 6.582X10^6\left[(1.05)^{A_t/1.75X10^4} - 1\right]$
25,000.	336,100	"	"	2.75	1.1440	0.0940	1.88	0.572	21	$7.590X10^6$	$A_t^2(0.00033) - A_t(5.64) = 7.59X10^6\left[(1.05)^{A_t/2.5X10^4} - 1\right]$
50,000.	480,600	"	"	3.00	1.1580	0.1080	2.16	0.500	21	$10.90X10^6$	$A_t^2(0.000186) - A_t(2.82) = 10.9X10^6\left[(1.05)^{A_t/5.0X10^4} - 1\right]$

TABLE 11. CALCULATION SHEET FOR DETERMINING MINIMUM AMORTIZATION TONNAGE.

A_t	Left hand side.			Right hand side.		
	①	②	③	④	⑤	⑥
	$A_t^2(k')$	$A_t(k'')$	①–②	$(k^1V)^{A_t/T}$	④–1	⑤ X k'''
75,000.	$A_t^2(0.000186)$ 1,040,000.	$A_t(2.82)$ $21X10^4$	50000T/A $83X10^4$	$(1.05)^{A_t/5X10^4}$ 1.076	0.076	⑤ $X10.9X10^6$ $83X10^4$
65,000.	$A_t^2(0.000330)$ 1,400,000	$A_t(5.64)$ $37X10^4$	25000T/A $103X10^4$	$(1.05)^{A_t/2.5X10^4}$ 1.1352	0.1352	⑤ $X7.59X10^6$ $102.2X10^4$
63,500.	$A_t^2(0.000435)$ 1,750,000	$A_t(8.05)$ $51X10^4$	17500T/A $124X10^4$	$(1.05)^{A_t/1.75X10^4}$ 1.1936	0.1936	⑤ $X6.582X10^6$ $127X10^4$
67,000.	$A_t^2(0.000579)$ 2,600,000	$A_t(11.75)$ $79X10^4$	12000T/A $181X10^4$	$(1.05)^{A_t/1.2X10^4}$ 1.3143	0.3143	⑤ $X5.854X10^6$ $184X10^4$
78,000.	$A_t^2(0.000765)$ 4,650,000	$A_t(17.6)$ $138X10^4$	8000T/A $327X10^4$	$(1.05)^{A_t/0.8X10^4}$ 1.613	0.613	⑤ $X5.344X10^6$ $328X10^4$
110,000.	$A_t^2(0.00104)$ 12,600,000	$A_t(28.4)$ $311X10^4$	5000T/A $949X10^4$	$(1.05)^{A_t/0.5X10^4}$ 2.927	1.927	⑤ $X4.8855X10^6$ $940X10^4$

CALCULATION OF OPTIMUM MINING RATE FOR MAXIMUM PRESENT VALUE WHEN MINING RATE IS THE ONLY VARIABLE.

This section is included because it follows on very naturally from the work on amortization tonnage, and because it is sometimes the case in practice that ore reserves are constant and of uniform grade, and that only one mining and milling method and sequence of mining is applicable or worth considering. In this case, the procedure outlined here can be used to calculate the mining rate that gives the maximum present value.

In the amortization tonnage example given it was shown that 17,500 tons per annum was the minimum mining rate, and mining should not be carried on at less than this rate. If it is, the amortization tonnage will rise rapidly, as will the operating costs, and profits and present value will be greatly reduced. However, it is not necessarily the upper limit to the mining rate. If the measured ore reserves are larger than the minimum amortization tonnage, then a higher mining rate may be justified, because it results in lower operating costs and greater present value, up to the point where the gain in present value is offset by the increased present cost of the extra plant capacity. On looking at it another way, increasing the mining rate above minimum tends to increase both profit per ton (operating) and the amortization tonnage. At the beginning of the increase, the gain in profit per ton is stronger than the gain in amortization tonnage, with a resultant increase in present value. If mining rate is increased too much, the gain in amortization tonnage will become stronger than the gain in profit per ton, and the result will be a reduction in present value. In between these two extremes, there is a mining rate that gives a maximum present value.

MATHEMATICAL RELATIONSHIPS

The relationship between present value, capital cost, operating profits, mining rate, deferment, life, and interest, are illustrated in the following equation.

$$\left(PV+\frac{C}{d}\right)(1+r)^{d}+\frac{C}{d}(1+r)^{d-1}\text{------}+\frac{C}{d}(1+r)=T\left[V-\left(kT^{n}\frac{C_{ND}}{R}\right)\right]\left[\frac{(1+r)^{\frac{R}{T}}-1}{r(1+r)^{\frac{R}{T}}}\right]$$

Where PV = present value
C = capital invested
$= D(k_1T+k_2)+C_{VS}+C_{XC}+C_{ND}+\frac{F_{PE}\cdot T}{m}$

d = years of deferment.
r = interest rate.
T = tons mined and milled per annum.
V = value of ore, per ton.
kT^n = operating cost per ton.
C_{ND} = cost of normal pre-production development.
R = reserves.

The sum of the terms in the left hand side of the equation, exclusive of the first term is:

$$\frac{C}{d}\left\{\frac{1}{r}\left[(1+r)^{d}-(1+r)\right]\right\}$$

Therefore:

$$\left(PV+\frac{C}{d}\right)(1+r)^{d}+\frac{C}{d}\left\{\frac{1}{r}\left[(1+r)^{d}-(1+r)\right]\right\}=T\left[V-\left(kT^{n}\frac{C_{ND}}{R}\right)\right]\left[\frac{1-(1+r)^{-\frac{R}{T}}}{r}\right]$$

and

$$PV = \frac{T\left[V-\left(kT^{n}\frac{C_{ND}}{R}\right)\right]\left[\frac{1-(1+r)^{-\frac{R}{T}}}{r}\right]-\frac{C}{d}\left[\frac{(1+r)^{d}-(1+r)}{r}\right]}{(1+r)^{d}}-\frac{C}{d}$$

METHOD OF CALCULATING OPTIMUM MINING RATE.

To obtain the mining rate that gives the maximum present value in any particular case, take various values for "T" and calculate the corresponding present values. Plotting will reveal a maximum in the mining rate vs. present value curve, and the optimum mining rate and present value can be read off the curve.

APPLICATION

Suppose in the previous example, development to the depth of the proposed first level was undertaken, and revealed recoverable ore reserves of 300,000 tons, with an average value of $19.00 per ton. These are the true figures that have been arrived at by actually blocking out the reserves. The previous figures of 280,000 tons at $20/ton were merely preliminary estimates based on diamond drilling and/or exploratory underground development. These preliminary estimates had to be made to see if they exceeded the required amortization tonnage. As they did, it was permissible to go ahead with the blocking out of the reserves, which reveals 300,000 tons at $19/ton. Under these conditions find the mining rate that gives the maximum present value.

Table III is the calculation sheet for this problem, and Figure 10 shows the results graphically. It can be seen that the maximum present value is $1,825,000, and occurs at a mining rate of 100,000 tons per annum, giving a productive life of three years for this portion of the ore body. The optimum mining rate can be established when it is physically possible to do so, and when there is an adequate market for the full production. When estimating the optimum mining rate and plant capacity, only the measured ore reserves are considered, as only these have an element of risk sufficiently small to justify the large capital expenditures required. However, it is generally fairly certain that additional ore exists that would warrant a larger plant. Therefore, development should be pushed ahead of production until at the end of a given period of time, the measured ore reserves have increased, and additional plant can be installed, and production increased.

When considering additional plant capacity, the present value of the increased earnings should outweigh the present cost of the additional plant required. The effect on sales volumes and prices should be examined closely when contemplating increased production, especially if the mine is large.

USE OF DIGITAL COMPUTER.

As with the equation for amortization tonnage, the equation for present value shown on page 44 could very easily be programmed for solution by a digital computer. Such a program would enable present values for a large number of mining rates to be calculated rapidly. The optimum mining rate for maximum present value could then be determined easier and faster than by manual solution.

TABLE III. CALCULATION SHEET FOR DETERMINING MAXIMUM PRESENT VALUE.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
T.	V.	kT^n	C_{ND}	R	(4)/(5)	(3) - (6)	(2) - (7)	$\frac{1}{(1+r)^{R/T}}$	1 - (9)	(10) /r	(1) X (8) X (11)	D	$k_1T + k_2$	(13) X (14)
	$19.00	$\frac{49.39}{T^{(0.1415)}}$	$141,100.	300,000.				$\frac{1}{(1.10)^{3X10^5/T}}$		(10) /0.10		330	0.00225T + 124	
50,000	"	10.68	"	"	0.47	10.21	8.79	0.565	0.435	4.35	19.1X10^5	"	237	7.85X10^4
100,000	"	9.69	"	"	"	9.22	9.78	0.750	0.250	2.50	24.5X10^5	"	349	11.5X10^4
106,250	"	9.60	"	"	"	9.13	9.87	0.765	0.235	2.35	24.6X10^5	"	363	12.0X10^4
112,500	"	9.53	"	"	"	9.06	9.94	0.775	0.225	2.25	25.0X10^5	"	378	12.5X10^4
150,000	"	9.15	"	"	"	8.68	10.32	0.825	0.175	1.75	27.0X10^5	"	462	15.3X10^4
200,000	"	8.78	"	"	"	8.31	10.69	0.865	0.135	1.35	28.9X10^5	"	574	18.95X10^4
300,000	"	8.29	"	"	"	7.82	11.18	0.910	0.090	0.90	30.15X10^5	"	799	26.4X10^4

(1)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33) =P.V.
T.	C_{VS}	C_{XC}	F_{PE}	(15) X (1)	n	(19) / (20)	(4) + (15) + (16) + (17) + (21)	$k_3T + k_4$	(22) / (23)	$(1+r)^{(23)}$	1-r	(25)X(26)	1+r	(27)-(28)	(29) /r	(24) X (30)	(12) - (31)	(32) / (25)
	0	11,000	1500		300			0.00001T + 1.5		$(1.10)^{(23)}$	0.9		1.10		(29) /0.10			
50,000	"	"	"	7.5X10^7	"	2.5X10^5	480,660	2.0	2.4X10^5	1.21	"	1.09	"	-0.01	-0.10	-2.403X10^4	1.934X10^6	1.600X10^6
100,000	"	"	"	15X10^7	"	5.0X10^5	767,100	2.5	3.06X10^5	1.269	"	1.145	"	0.045	0.45	13.8X10^4	2.312X10^6	1.825X10^6
106,250	"	"	"	16X10^7	"	5.3X10^5	802,100	2.563	3.13X10^5	1.277	"	1.150	"	0.050	0.50	15.65X10^4	2.304X10^6	1.810X10^6
112,500	"	"	"	16.85X10^7	"	5.6X10^5	837,100	2.63	3.18X10^5	1.285	"	1.158	"	0.058	0.58	18.5X10^4	2.315X10^6	1.800X10^6
150,000	"	"	"	22.5X10^7	"	7.5X10^5	1,055,100	3.0	3.5X10^5	1.33	"	1.200	"	0.100	1.00	35.0X10^4	2.35X10^6	1.770X10^6
200,000	"	"	"	30X10^7	"	10X10^5	1,341,600	3.5	3.84X10^5	1.396	"	1.255	"	0.155	1.55	59.4X10^4	2.296X10^6	1.643X10^6
300,000	"	"	"	45X10^7	"	15X10^5	1,916,100	4.5	4.25X10^5	1.536	"	1.380	"	0.280	2.80	182.5X10^4	1.825X10^6	1.190X10^6

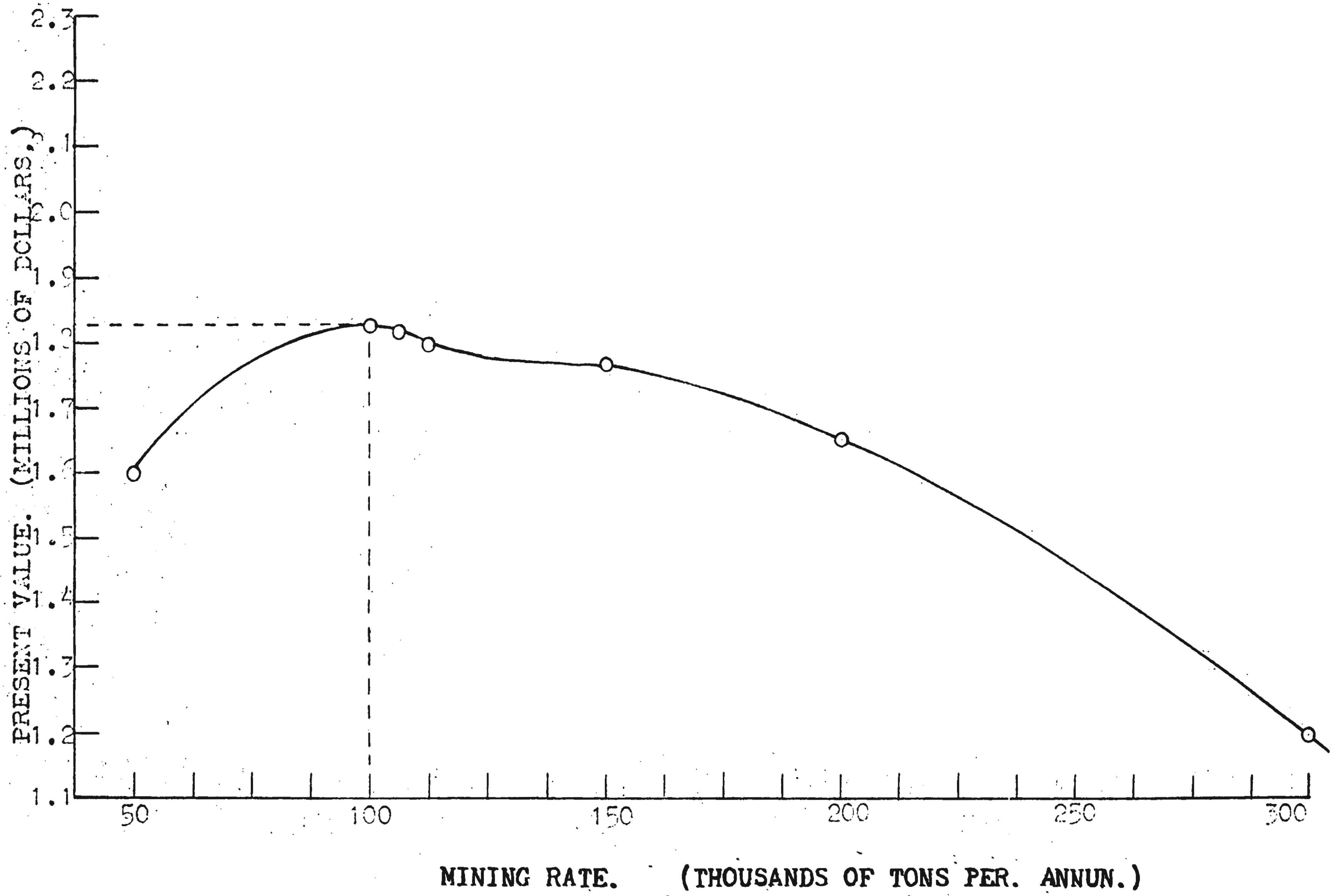


Fig. 10 - Present value vs. annuel production.

USING TOTAL NET PROFITS TO GIVE AN APPROXIMATION TO THE OPTIMUM MINING RATE.

By using an expression for total net profits instead of present value, a rapid approximation to the optimum mining rate can be obtained. However, this figure is only an approximation and must not be accepted as final, as it is based on total net profits, and not present value which is the correct criteria. The mathematical relationship is:

$$TNP = R\left\{V - \left[kT^{n}\ \frac{C_{ND}}{R}\right]\right\} - \left\{D\ (k_1 .T + k_2)\ + C_{VS} + C_{XC} + C_{ND} + \frac{F_{PE} .T}{m}\right\}$$

Where

TNP = total net profits.
V = value of ore per ton.
kT^n = operating cost per ton, mining and milling, in terms of annual production.
C_{ND} = normal development.
R = reserves.

$D(k_1 T + k_2) + C_{VS} + C_{XC} + C_{ND} + \frac{F_{PE} .T}{m}$ = capital cost of plant and capitalized development.

This expression can be simplified to:

$$TNP = R.V - R.kT^{n} - D.k_1 .T - D.k_2 - C_{VS} - C_{XC} - \frac{F_{PE} .T}{m}$$

Differentiating with respect to T,

$$\frac{d\ TNP}{dT} = n\ R.kT^{(n-1)} - Dk_1 - \frac{F_{PE}}{m}$$

To find the value of "T" that gives the maximum total net profit, set the differential equal to zero, and solve the resulting equation for "T":

$$n.R.k.T^{(n-1)} = \quad -Dk_1 \ - \frac{F_{PE}}{m}$$

Solving this expression for T gives:

$$T = (1-n)\sqrt{\frac{n.R.k}{\left[Dk_1 + \frac{F_{PE}}{m}\right]}}$$

Example: Work the previous example using this expression:

$$\begin{aligned} n &= -0.1415 \\ D &= 330 \\ k_1 &= 0.00225 \\ F_{PE} &= 1500 \\ m &= 300 \\ R &= 300{,}000 \\ k &= 49.39 \end{aligned}$$

$$T = 1.1415\sqrt{\frac{0.1415 \times 3.\ 10^5 \times 49.39}{330 \times 0.00225 + \frac{1500}{300}}}$$

$$T = 74{,}300 \text{ tons per annum.}$$

According to these calculations, the mining rate that gives the maximum total net profits is 74,300 tons per annum. Referring to Figure 10, a mining rate of 74,300 tons per annum gives a present value of $1,760,000. Thus, if 74,300 tons per annum were taken as being the mining rate that gave the maximum present value, it would be in error by

$$\left(\frac{1{,}825{,}000 - 1{,}760{,}000}{1{,}825{,}000} \times 100\right) = 3.5\%$$

with regard to present value, and

$$\left(\frac{100{,}000 - 74{,}300}{100{,}000} \times 100\right) = 25\%$$

with regard to mining rate.

DETERMINING MAXIMUM PRESENT VALUE WHEN MINING RATE, MINING AND MILLING METHOD, CUT-OFF, AND MINING SEQUENCE ARE VARIABLE.

The method consists essentially of calculating the present value for all possible combinations of cut-off, mining sequence, etc., that apply in the particular case under consideration, and adopting the combination that gives the greatest present value for that ore body. In this discussion, the various factors will be defined, with the aid of examples where necessary. Then the general method of calculating the present values at the various combinations will be outlined. The combination that gives the greatest present value is readily apparent, and should be adopted.

CUT-OFF.

Cut-off can be defined two ways, as grade (i.e., 2% Cu.) which is the usual way, or as dollars per ton, which is more convenient in certain cases.

Grade.

Cut-off is best expressed as grade when there is only one valuable constituent in the ore. The cut-off grade is illustrated as lines connecting points of equal assay, on sections through the ore zone. For example, in a steeply dipping tabular type of ore zone with considerable width, the sections could be vertical longitudinal sections, say 50 feet apart.

Dollars per ton.

Cut-off is best expressed as dollars per ton when there are several valuable constituents in the ore. As with grade, dollars per ton cut-off is best shown as a series of lines connecting points of equal per ton dollar value, on a series of sections drawn through the ore zone.

To express cut-off as dollars per ton involves a consideration of the proposed method of milling the ore, and to whom the product will be sold, as well as the assays. It is apparent therefore, that if several alternative methods for milling and disposing of the ore are available, then a series of cut-off sections must be drawn for each method. This involves more work than when expressing cut-off as grade, but it is the most convenient approach when the ore contains several valuable constituents. The following example illustrates how dollar per ton cut-off sections can be drawn. Suppose a series of sections through a lead, zinc, copper ore body are available, each section showing the analysis of the ore at points on a regular grid. Suppose also that one possible method of milling and disposal is to produce a bulk concentrate to be sold to a copper smelter. Then, taking each section in turn, and considering the milling method, estimate the analysis of the concentrate that could be produced from the ore at each point on the grid. Also, estimate the ratio of concentration (i.e., tons of concentrate per ton of ore) at each point. Actually, the grade of the concentrate will probably be about the same for all points, depending more on the milling method than on the grade of the ore, but the ratio of concentration will be different at each point, depending largely on the ore grade.

With this information, and knowing the base charges, payments and penalties at the smelter, and using an equitable price for each constituent, it is possible to calculate the dollar value per ton of the ore at each grid point on the section. Iso-value lines can then be drawn on the section, and the same process repeated for the other sections. Figure 11 illustrates this principle diagramatically.

MINING SEQUENCE.

This refers to the sequence in which the various parts of the ore body are mined. Parts of certain ore deposits are of higher grade than other parts, just as some parts are more accessible than others. Generally, the greatest present value will result when the higher grade, more accessible ore is mined first.

In vein mining, there are two basic sequences:

1. Mining down dip, along the full strike length of the ore zone.

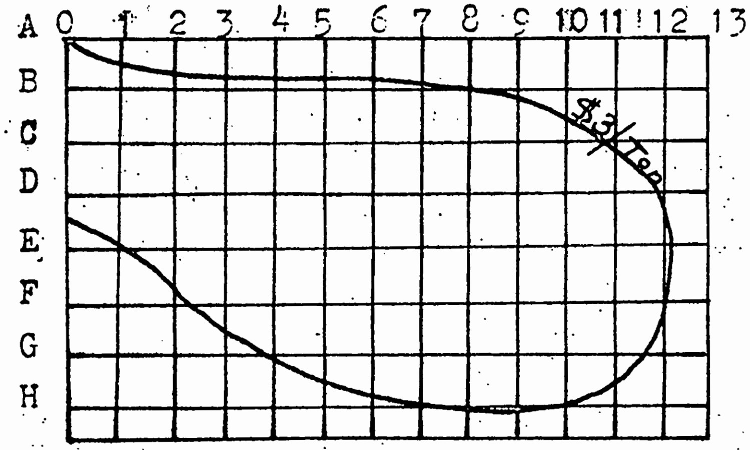
2. Mining along the strike, outward from a shaft which is sunk to approximately the full depth of the ore zone.

All other sequences are combinations of these two. The sequence, along with the cut-off, affects the variation of revenue per ton with time, and operating cost per ton with time, two factors of prime importance when calculating present values.

MINING METHODS.

Mining methods affect operating costs, and recoveries. They also affect capital costs, though not to the same extent as milling methods.

It is quite possible that a mining method with a low recovery may result in a higher over all present value, than one with a higher



11a- Part of vertical section through ore zone.

Constituent	CA	CB	CC	CD etc
Copper	6.0%			
Lead	2.0%			
Zinc	1.5%			
Arsenic	0.5%			
Iron	15.0%			
Lime	5.0%			
Silica	55.0%			
Other	15.0%			

11b- Analysis of ore at point CA

11c- Grade of conc. and ratio of concentration

	Ore. (1 Ton)		Recovery.		Grade of concentrate
	%	lbs.	%	lbs.	
Cu.	6	120	90	108	108/239 X100 = 45.2%
Pb.	2	40	70	28	11.8%
Zn.	1.5	30	15	4.5	1.9%
As.	0.5	10	85	8.5	3.6%
Fe.	15	300	10	30	12.5%
Lime.	5	100	2	2	.8%
Silica	55	1100	5	55	23.0%
Other	15	300	1	3	1.2%
Total	100	2000		239.0	100.0%

RATIO OF CONCENTRATION = 2000/239 = 8.4

VALUE OF CONCENTRATE (FROM SMELTER SCHEDULE)

$25.00 per ton

VALUE OF ORE. $25.00/8.4 = $2.98/ton

Fig. 11- Cut-off expressed as dollars per ton.

recovery, if the cost of the first one is lower. Therefore, all mining methods that appear worthwhile should be investigated.

MINING RATE.

The effect of mining rate on present value was dealt with in a previous part of the discussion.

COMBINING THE VARIABLES.

The procedure to be outlined consists of finding the particular combination of all the above variables that gives the maximum present value for the ore body under consideration. As it is likely that there will be many combinations, it will be convenient to make a sheet similar to that shown in Figure 12 for each combination of cut-off, sequence, and milling and disposal method. The full range of the other variables, mining rate and mining method, can be shown in each sheet. The sheet illustrated in Figure 12 is set up for use with ore bodies containing several valuable constituents. Care must be taken that the right cut-off sections are used with the milling and disposal method being considered. It will be remembered that each method has its own set of sections.

For ore containing only one valuable constituent, the same set of cut-off sections are used for all milling and disposal methods.

In the squares lettered A, B, C, D, etc., in Figure 12, the following information in table form will be placed.

1. In the case of ore containing more than one valuable constituent, the average dollar value per ton or ore mined and milled for each year of life of the mine. In the case of single valuable constituent ore, the average grade of ore mined each year, with the tonnage and grade of concentrate produced each year from this ore.

MILLING AND MARKETING METHOD.	CUT OFF.	SEQUENCE
Bulk concentrate sold to copper smelter.	$20 per ton	Along strike.

MINING METHOD AND RECOVERY.		MINING RATE				
Method	Recovery.	100,000T/A	200,000T/A	300,000T/A	400,000T/A	500,000T/A
CUT AND FILL	98%	A	B	C	D	E
SUB-LEVEL STOPING	60%	F	G	H	I	J
SHRINK STOPING	75%	K	L	M	N	O

Fig.12- COMBINING VARIABLES.

The cut-off sections play a big part in supplying this information, as does the mining sequence and rate, and to a lesser extent, the method.

2. The average operating cost per ton of ore mined and milled for each year of life.

This information comes mainly from the method, rate and sequence of mining.

3. The gross operating profit for each year of life of the mine.

   For the multiple valuable constituent ore, this is obtained by taking the difference between average revenue and cost per ton for each year, and multiplying this difference by the annual production.

   In the other case, gross operating profit per year is arrived at from a knowledge of the tons and grade of concentrate produced each year, and the terms of the schedule under which the product is sold.

4. The estimated tax for each year. This depends on the gross operating profit, and the tax schedule in force in the area.

5. The net operating profit for each year. This is the difference between the gross operating profit, and the tax.

6. The length of the deferment period, and the capital invested during each year of deferment.

METHOD OF CALCULATING PRESENT VALUE.

When the present value is calculated using the values in each square on each sheet, the combination of factors that gives the maximum possible present value will be apparent. An example will best illustrate the method of calculating present value.

The estimated net operating profits from a mine during its productive life of five years are:

1 year	$ 5,000
2 year	$ 7,000
3 year	$10,000
4 year	$ 8,000
5 year	$ 6,000

During the two years required to bring the property into operation, $4,000 capital for plant and equipment will be required each year. What is the present value of the property using a risk rate of 10 percent, and a safe rate of 4 percent? Table IV illustrates the calculation of this problem.

TABLE IV. CALCULATING PRESENT VALUE FOR UNEQUAL ANNUAL RETURNS.

YEAR. (m)	PROFIT.	INTEREST ON CAPITAL (x)	TO SINKING FUND	RATE[1] $(1.04)^{n-m}$	AMOUNT IN (n-m) years
1.	5,000	0.10(x)	5000-0.10x	1.17	5850-0.117x
2.	7,000	"	7000-0.10x	1.125	7860-0.1125x
3.	10,000	"	10000-0.10x	1.109	11090-0.1109x
4.	8,000	"	8000-0.10x	1.040	8325-0.104x
5.	6,000	"	6000-0.10x	1.000	6000-0.100x
				TOTAL.	39125-0.5444x

1. The sum that one dollar will amount to in "n" years if invested at 4 percent compound interest. (n = 5)

The total of the sinking fund installments, with interest, must equal the original capital invested.

Therefore,

$$39,125 - 0.5444\,x = x$$

Solving for x:

$$1.5444\,x = 39125$$

$$x = \$25,300$$

This is the value at the commencement of operations, two years hence. The actual present value is obtained from the expression,

$$(PV + 4000)\,(1.07)^2 + (4000)\,(1.07) = \$25,300$$

$$PV = \frac{25,300 - (4000 \times 1.07)}{(1.07)^2} - 4000$$

$$PV = \$14,400.$$

ELIMINATING CUT-OFF AS AN INDEPENDENT VARIABLE.

In the case of an assay wall type deposit, when the plant is operating at full capacity, it may be possible to eliminate cut-off as an independent variable by making it equivalent to the operating plus capital cost of mining a ton of ore.

i.e. $\text{cut-off } \frac{(\text{dollars})}{\text{ton}} = \frac{\text{operating cost}}{\text{ton}} + \frac{\text{total capital costs and interest}}{\text{Reserves.}}$

However, to determine the reserves, a knowledge of cut-off is required, reserves being inversely proportional to cut-off as illustrated in figure 4. To overcome this difficulty, derive an empirical equation giving reserves in terms of cut-off under the particular conditions that apply. Reserves can then be replaced by a function of cut-off.

For any combination of mining method, milling method, and mining rate, the operating costs and total capital costs are known, so the required cut-off can be found by a process of iteration. It should be noted that each proposed method of milling and marketing will give a different equation relating reserves to cut-off. Also, the mining sequence will cause a variation in operating costs with time, and this will cause corresponding variations in cut-off with time, which however should not be large.

The general procedure for getting optimum operating conditions when cut-off is eliminated as a variable is similar to the procedure previously outlined, except that there are fewer combinations to consider.

DIFFICULTIES.

Of course, the foregoing outline represents ideal conditions, where all the necessary information concerning the ore body, etc., is available before mining commences. In practice, this would seldom be the case, but nevertheless, the same general procedure would have to be attempted with what information was available, unless it was too meager altogether.

Relative changes in costs, product prices, and other factors can occur after the property has been brought into production, and these changes will alter optimum operating conditions. If this occurs, the operating conditions can be reviewed and adjusted to optimum if necessary. Here, as always, the proposed changes should result in a greater present value.

CONCLUSIONS

From the foregoing discussion it can be concluded that amortization tonnage is a function of mining rate and that an ore body has a minimum amortization tonnage which can be found by trial and error computations when amortization tonnage is expressed as a function of mining rate. This process can be greatly simplified by the use of a digital computer.

It is also concluded that present value depends on a number of inter-related factors; cut-off, milling method, mining method and sequence, and mining rate. When mining rate is the only variable, present value can be related to it mathematically, and the optimum mining rate for maximum present value can be found by trial and error computations. Here again, a digital computer will greatly speed the work. Total net profits can be used instead of present value to give a rapid approximation to the optimum mining rate, by the use of calculus.

When several, or all of the factors are variable, the optimum combination for maximum present value can be found only by trial and error computations, that is, calculating the present value for all practical combinations, and choosing that combination which gives the maximum present value. For each combination, the annual profit must be estimated separately for each year of life, and the present value calculated without benefit of formulas. This process appears complicated when outlined in a general manner, but in any particular case, some or all of the variables may be found to be quite limited in range (i.e. only one mining method applicable) and this would greatly simplify the calculations. A further simplification may be effected by expressing cut-off as a function of the operating and capital cost of mining a ton of ore.

This thesis is based on the idea that present value is a better measure of the worth of a mineral property than the total anticipated net profits, and this being the case, operating conditions should be chosen to give maximum present value, rather than maximum total net profits.

In all examples used in this thesis, the plant is considered to be operating at full capacity, the varying mining rates spoken of being considered as the maximum rate possible for each of a series of mills under consideration but not as yet built.

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VITA

Garrett Michael Sainsbury was born August 21st, 1936 at Melbourne, Australia. He received his under-graduate schooling at the Western Australian School of Mines at Kalgoorlie, under a Chamber of Mines scholarship. He graduated in 1956 with the Associate Diploma in Mining Engineering. During the latter two years of the course he was employed full time as a surveyor at the North Kalgurli (1912) Ltd. Gold Mine, as a trainee, as provided by the scholarship. During this time he attended evening classes.

In September, 1957, he enrolled in Graduate School at the University of Missouri, School of Mines and Metallurgy, to study for the degree of Master of Science in Mining Engineering. In February, 1959, having completed the required course work he accepted a position with Climax Molybdenum Co., Climax, Colorado, where he worked until August, 1959. In September, 1959, he accepted a position with Consolidated Denison Mines, Ltd., a uranium producer at Elliot Lake, Ontario.

On completion of his degree, he intends to accept a position with Broken Hill South Ltd., Broken Hill, Australia.