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# The analog simulation of heat transfer devices for the smoothing of a fluctuating fluid temperature

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# THE ANALOG SIMULATION OF HEAT TRANSFER DEVICES FOR THE SMOOTHING OF A FLUCTUATING FLUID TEMPERATURE

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## ABSTRACT

Equations are written to describe a heat exchanger with electrical energy input and also a large tank system which can have electrical energy input. The output temperatures were sampled and fedback to define the energy input to the two systems. These systems were simulated on an analog computer and the ability of the two systems to smooth a fluctuating temperature input was tested.

The testing of the systems included trying different types of controllers in the control system as well as different size tanks for the tank system.

A satisfactory smoothing of the output temperature was demonstrated after some modifications.

# ACKNOWLEDGEMENT

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# I. INTRODUCTION

In the design of a constant temperature recirculating bath, it is common practice to use a time proportioning controller to control the energy input or output from the controlled circulating liquid. This bang-bang type control system causes small temperature fluctuations in the output liquid used for circulation. The fluctuations appear as a constant frequency perturbation imposed upon a constant temperature level and the magnitudes of these perturbations depend upon the circulator design.

The problem here undertaken is to design and test by simulation two different methods for removing or reducing these fluctations to a usable level. The two methods to be tried will be a heat exchanger whose output temperature will be controlled by controlling the energy input to the heat exchanger. The other method will be to provide a large capacitance for the fluid to flow through. This smoothing of the temperature fluctuations is required so that this circulating fluid can be used to cool the sink of a cloud-simulation-chamber and keep its temperature at a constant level.

# II. REVIEW OF LITERATURE

Dynamic responses and controller designs for heat exchangers were investigated by several authors such as W. C. Cohen (1), E. F. Johnson (1), H. M. Paynter (2), D. I. Lawson (3), L. Malavard (4), Y. Takahashi (5), ... in the early 1950's.

In January of 1956, R. L. Ford (6) presented his work about "Electrical Analogues for Heat Exchangers" in the Proceedings of IEEE Journal, concerned with the derivation of dynamically accurate electrical analogues for heat exchangers. The author used the concept of the analogies between voltage-temperature and current-heat flow to develop a distributed parameter model of a constant jacket heat exchanger.

The equations were set up as:

$$\frac{\partial \theta}{\partial t} + v \frac{\partial \theta}{\partial x} = \frac{1}{R^* C^*} (T - \theta) \qquad (\underline{1})$$

where

$\theta$ = temperature of fluid in tube

$t$ = time

$v$ = velocity of fluid flow

$x$ = distance along the heat exchanger tube

$T$ = temperature of outer fluid

$R^*$ = thermal resistance per unit length

$C^*$ = thermal capacitance per unit length or by introducing the dimensionless time and distance:

$$\frac{\partial\theta}{\partial t} + \frac{\partial\theta}{\partial\eta} = a\ (T - \theta)$$

where $t = \frac{t}{L}$

$\eta = \frac{x}{I}$

$a = \frac{1}{R^{*}C^{*}v}$

Since the heat exchanger is a distributed parameter system, while the majority of easily adjusted electrical networks are lumped constant systems, it is apparent that approximate methods must be employed. The heat exchanger was divided into a certain number of equal elements and the system was simulated on an electronic analog computer. The steady state response to a step input was considered to be accurate with the errors calculated by comparing the output to theoretical results. The errors found were 7.9, 1.3 and 0.4 per cent for a two, five and ten lumped parameter system respectively.

B. D. Hainsworth and V. V. Tivy (7) wrote about the dynamic analysis of heat exchanger control. The authors ran a series of tests on an analog computer by simulating a commercial water heater of conventional shell and tube design using low pressure steam.

Steam was supplied through a pressure controller. The output temperature of water was compared with the desired temperature by a comparator. The error was sent to a controller, a proportional type, which actuated the diaphragm of a steam control valve.

The mathematical model of the system was then simulated on a Philbrick analog computer and the transient and steady state curves of the system were plotted, for various step function disturbances. The models output compared well with the experimental system.

The authors conclusions about the advantages of the simulation are as follows: it's simple to model; it has rapid output because of time scaling; actual test conditions can be easily duplicated on the computer and different parameters are easy to input in the simulation.

Masami Masubuchi (8) wrote about dynamic response of a multipass heat exchanger.

The same dimensionless equation used by Ford (6) was set up:

$$\frac{\partial\theta}{\partial t} + \frac{\partial\theta}{\partial\eta} = a\ (T - \theta) \qquad (3)$$

a solution was obtained, using a sinusoidal input signal which satisfied the boundary conditions.

The author first investigated a one-two pass heat exchanger and the system was then generalized to a multipass system. All possible parallel and counter flow combinations were discussed in detail. Finally, the results were tested by simulating the most simple case: a one-two pass system with two lumped and then four lumped parameters. Disturbances were also step functions.

The conclusion was that the analog computation was found to be in good agreement with the theoretical results, which were obtained numerically by using a graphical method.

J. R. Schmidt (9), reported work done in simulating a parallel flow heat exchanger on an analog computer. The fluid was water, being heated by hot gas.

The author set up the space lumped equation by a central difference technique, and by taking an energy balance in the fluid, metal and gas, the fundamental equations were written. Substituting all the parameters into the equations, an analog circuit was obtained for this model. A series of tests were run with different input disturbances in forms of step functions and the responses of the system were shown.

## III. DEVELOPMENT OF BASIC EQUATIONS FOR SYSTEM'S DESCRIPTIONS

For the purpose of smoothing the output temperature of a circulating bath, two kinds of systems will be tested and the results will be compared.

### A. Heat Exchanger

The first type will be a heat exchanger with an electrical heating element centered in the shell of the exchanger as shown in Figure 1. The heating element will be assumed to have two parts: the inner part is an electrical resistor, wound around a cylindrical core, the outer part is an electrical insulator. The shell of the exchanger will be consider as a pipe whose outside diameter is completely insulated. Fluid flows in the annular space between the inside of the pipe and the outside of the heating element. This system will be lumped into a certain number of pieces for modeling. A jth increment will be examined first to obtain the basic equations (Figure 2). An energy balance in this jth increment gives:

[energy flow in] - [energy flow out] = [rate of change of lumped system energy]

$$\frac{W}{n} - \frac{T_{1j} - T_{2j}}{Re^*} = m_1^* \, C_{p1} \frac{dT_{1j}}{dt} \qquad (4)$$

where:

n = number of pieces to be lumped,

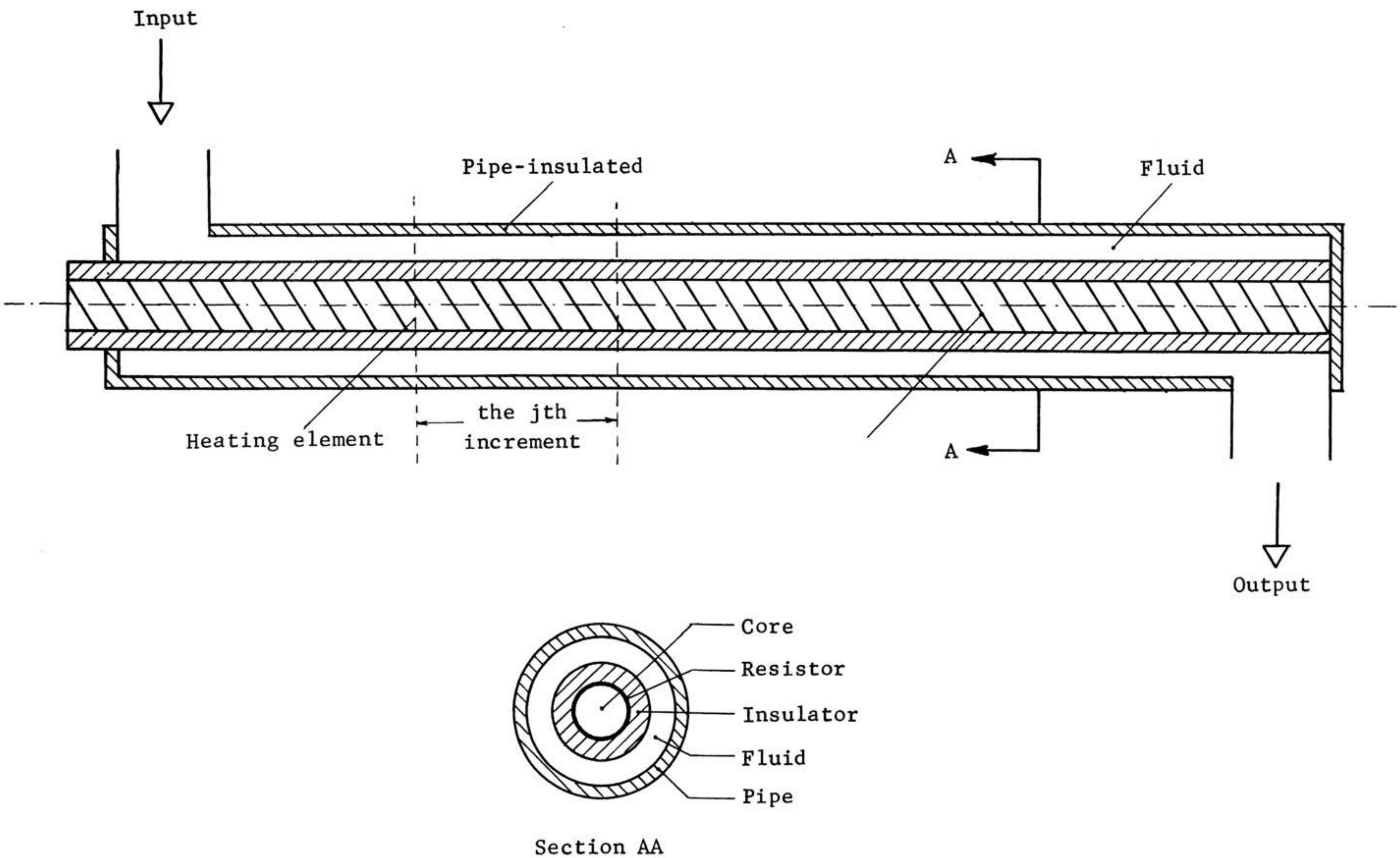


Figure 1. The Heating Element and the Pipe

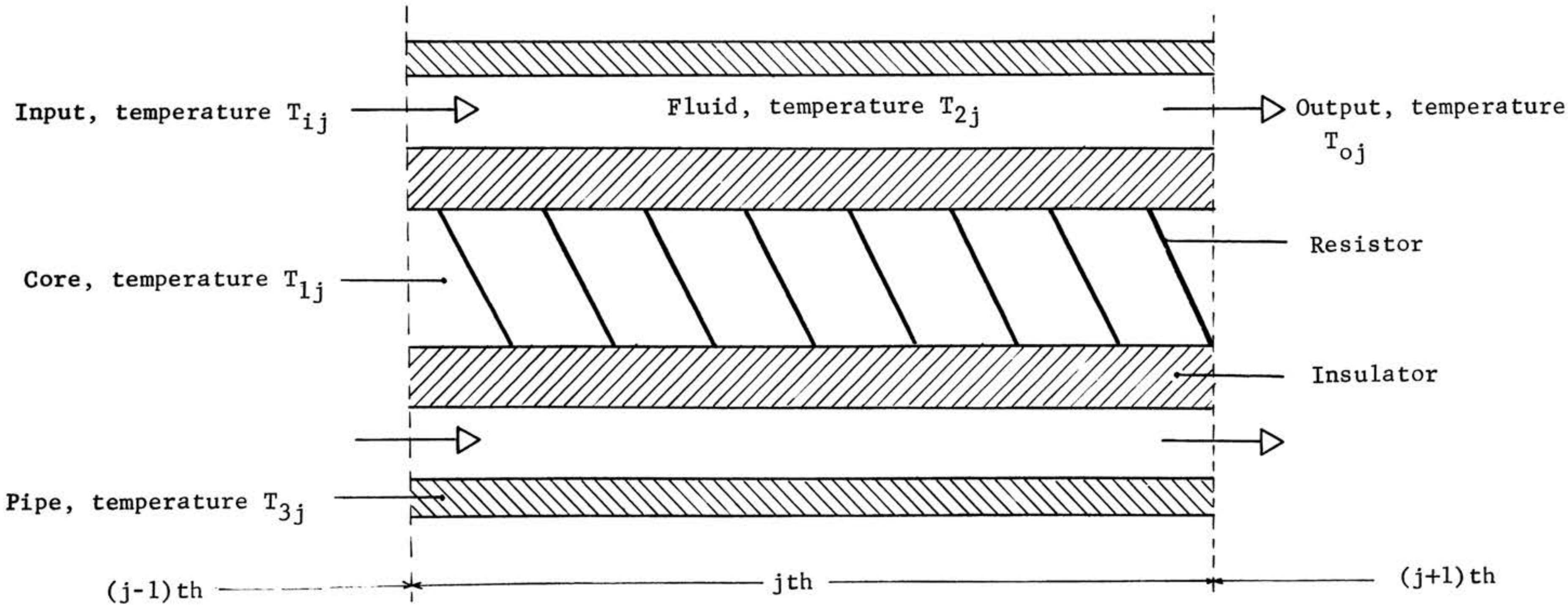


Figure 2. The jth Increment

$\frac{W}{n}$ = electrical energy input for the jth increment, Btu,

$T_{1j}$ = temperature of the electrical resistor and the core, $^oF$,

$T_{2j}$ = temperature of the fluid of the jth increment, $^oF$,

$m_1^* = \frac{m_1}{n}$: total mass of the resistor and the core of the jth increment, $lb_m$,

$C_{p1}$ = specific heat of the core, $Btu/lb_m{}^oF$,

$Re^* = nR_e$: thermal resistance of the insulator of the jth increment, sec $^oF$/Btu,

and $m_1^*C_{p1} = \frac{m_1C_{p1}}{n}$: equivalent capacitance of the core, $Btu/^oF$.

Equation (4) can be rewritten as:

$$T_{1j} = \int_o^t \left(- \frac{T_{1j}}{m_1^* C_{p1}\ Re^*} + \frac{T_{2j}}{m_1^* C_{p1}\ R_e^*} + \frac{W}{n.m_1^* C_{p1}}\right) dt + IC \quad (5)$$

An energy balance in the fluid of the jth increment will give:

[energy flow in] - [energy flow out] = [rate of change of energy],

or:

$$[\overset{o}{m}\ C_{pf}\ T_{ij} + \frac{T_{1j} - T_{2j}}{R_e^*}] - [\overset{o}{m}\ C_{pf}\ T_{oj} + \frac{T_{2j} - T_{3j}}{R_p^*}] = m_f^* C_{pf} \frac{dT_{2j}}{dt} \quad (6)$$

where:

$\overset{o}{m}$ = mass flow rate of fluid, $lb_m$,

$C_{pf}$ = specific heat of fluid, $Btu/lb_m{}^oF$,

$R_e^* = nR_e$: thermal resistance of the insulator for the jth increment, sec $^oF$/Btu,

$R_p^* = nR_p$: thermal resistance of the pipe for the jth increment, sec $^oF$/Btu,

$T_{1j}$ = temperature of the core for the jth increment, $^oF$,

$T_{2j}$ = temperature of the fluid for the jth increment, $^oF$,

$T_{3j}$ = temperature of the pipe for the jth increment, $^oF$,

$T_{ij}$ = input temperature, $^oF$,

$T_{oj}$ = output temperature, $^oF$,

$m_f^* = \frac{m_f}{n}$; mass of the fluid in the controlled volume for jth increment, $lb_m$,

and $m_f^* C_{pf} = \frac{m_f C_{pf}}{n}$ : equivalent heat capacitance of the fluid for the jth increment, Btu/$^oF$.

Equation (6) becomes:

$$T_{2j} = \int_o^t \left(\frac{m}{m_f^*} T_{ij} - \frac{m}{m_f^*} T_{oj} + \frac{T_{1j}}{m_f^* C_{pf} R_e^*} - \frac{T_{2j}}{m_f^* C_{pf} R_e^*} - \frac{T_{2j}}{m_f^* C_{pf} R_p^*} + \frac{T_{3j}}{m_f^* C_{pf} R_p^*}\right) dt + IC \qquad (7)$$

The relation between the output and input temperature of the increment can be obtained by assuming that the mean temperature of the fluid is the average of these two temperatures:

$$\frac{T_{oj} + T_{ij}}{2} = T_{2j}$$

$$T_{oj} = 2\ T_{2j} - T_{ij} \qquad (\underline{8})$$

A logarithmic mean temperature difference could be used here to define the heat transfer from the heat element to the fluid but it is hoped to simulate these equations on an analog computer. The logarithmic simulation would be difficult and the devices used for the simulation tend to be inaccurate, so the average temperature approach was used.

An energy balance for the pipe yields:

[energy flow in] - [energy flow out] = [rate of change of energy],

$$\frac{T_{3j} - T_{2j}}{R_p^*} - 0 = m_3^* C_{p3} \frac{dT_{3j}}{dt} \qquad (\underline{9})$$

where:

$R_p^* = nR_p$ : thermal resistance of the pipe for the jth increment, sec $^oF$/Btu,

$m_3^* = \frac{m_3}{n}$ : mass of the pipe, $lb_m$,

$C_{p3}$ : specific heat of the pipe, Btu/$lb_m$ $^oF$,

and $m_3^* C_{p3} = \frac{m_3 C_{p3}}{n}$ : equivalent heat capacitance for the pipe of the jth increment, Btu/$^oF$.

Equation ($\underline{9}$) becomes:

$$T_{3j} = \int_{o}^{t} \left(\frac{T_{2j}}{m_3{}^{*}C_{p3}\ R_p{}^{*}} - \frac{T_{3j}}{m_3{}^{*}C_{p3}\ R_p{}^{*}}\right)dt + IC \qquad (\underline{10})$$

B. Controller

A comparator controller circuit was constructed to control the output temperature of the heat exchanger to a desired value. The comparator was constructed to compare the sampled output temperature $T_{out}$ from the exchanger to a desired output temperature $T_{set}$ as shown in Figure 3. The difference between $T_o$ and $T_s$ was defined as error e(t) and was sent to the controller. The controller can have any transfer function that its application dictates ( $\frac{m}{e}$ ), for example, it could be a proportional controller where:

$$G(t) = \frac{m(t)}{e(t)} = K,$$

G(t): the time transfer function,

m(t): the output function,

e(t): the error function,

K: proportional gain of the controller.

The output of the controller drives some sort of power supply device which supplies energy as required to the heating element in the heat exchanger.

Combining the heat exchanger and the control system as in Figure 4, a simulation can be prepared, using the developed equations to test

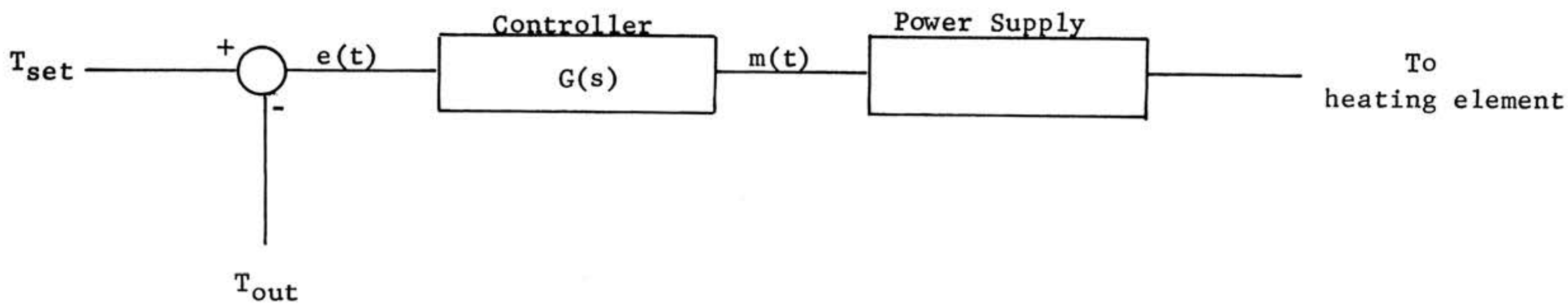


Figure 3. Block Diagram for Comparator Controller

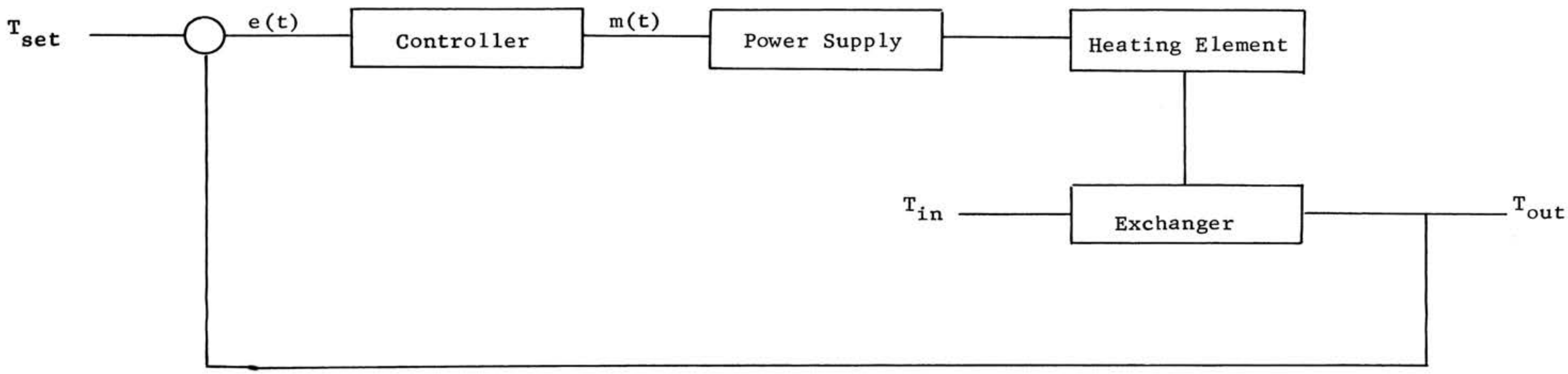


Figure 4. Overall Block Diagram for the Heat Exchanger System

the output response of this type of device for different inputs.

C. Large tank capacitor

The second method for accomplishing the same purpose will be to run the circulating fluid through a big tank, which is stirred continuously to obtain a unique temperature throughout the tank (Figure 5). The tank acts as a single lumped capacitor.

The container is assumed to be made of wood and is completely insulated. An energy balance gives:

[energy flow in] - [energy flow out] = [rate of change of tank energy],

$$\frac{T_f - T_c}{R_c} - 0 = m_c C_{pc} \frac{dT_c}{dt} \qquad (\underline{11})$$

where:

$T_f$: temperature of fluid, $^oF$,

$T_c$: temperature of container, $^oF$

$R_c$: thermal resistance of the container, sec $^oF$/Btu,

$m_c$: mass of the container, $lb_m$,

$C_{pc}$: specific heat of the container, Btu/$lb_m$ $^oF$

and $m_c C_{pc}$: equivalent heat capacitance of the container, Btu/$^oF$.

Equation (11) becomes:

$$T_c = \int_o^t \left(\frac{T_f}{m_c C_{pc} R_c} - \frac{T_c}{m_c C_{pc} R_c}\right) dt + IC \qquad (\underline{12})$$

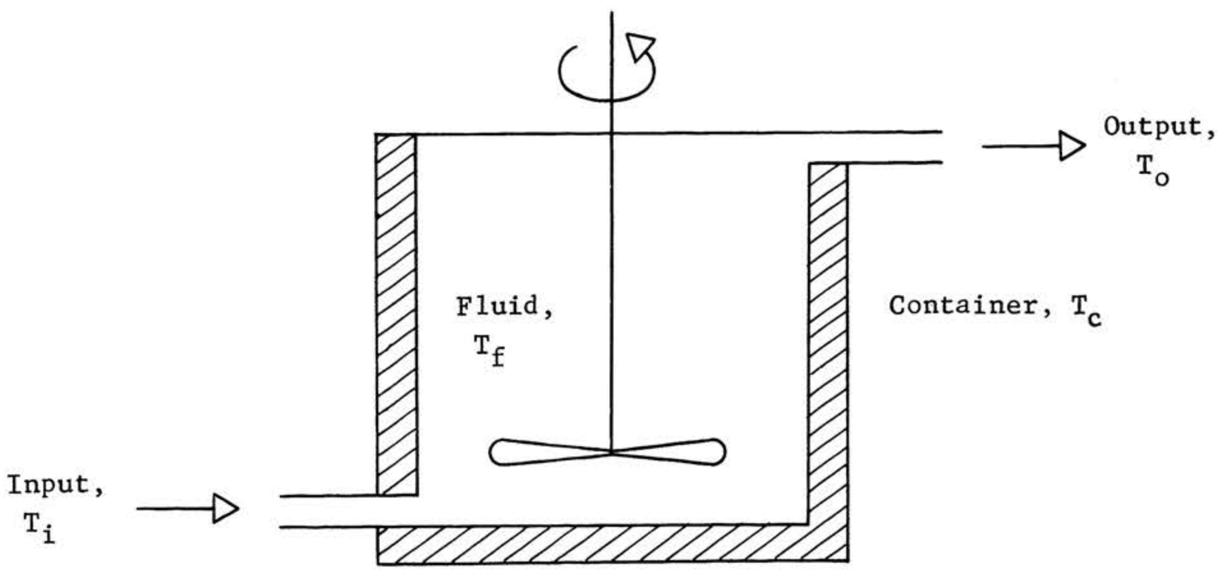


Figure 5. Single Lumped Tank

An energy balance on the fluid in the tank yields:

[energy flow in] - [energy flow out] = [rate of change of fluid energy],

$$[\overset{o}{m}C_{pf}T_i] - [\overset{o}{m}C_{pf}T_o + \frac{T_f - T_c}{R_c}] = m_f C_{pf} \frac{dT_f}{dt} \qquad (\underline{13})$$

where:

$\overset{o}{m}$: mass flow rate of fluid, $lb_m/sec$,

$C_{pf}$: specific heat of fluid, $Btu/lb_m \, {}^oF$,

$m_f$: mass of fluid in the container, $lb_m$,

$T_i$: input temperature, ${}^oF$,

$T_o$: output temperature, ${}^oF$,

$T_f$: temperature of fluid, ${}^oF$,

$T_c$: temperature of container, ${}^oF$,

and $m_f C_{pf}$: equivalent heat capacitance of the tank, $Btu/{}^oF$.

Equation (13) becomes:

$$T_f = \int_o^t \left(\frac{\overset{o}{m}}{m_f} T_i - \frac{\overset{o}{m}}{m_f} T_i + \frac{T_f}{m_f C_{pf} R_c} - \frac{T_c}{m_f C_{pf} R_c}\right) dt + IC \qquad (\underline{14})$$

Again, the mean temperature of the fluid can be considered as the average of the input and the output temperature:

$$T_{out} = 2T_f - T_{in}$$

In the case that electrical energy would be put into the tank by adding a heating element, the energy balance equation (14) would be:

$$[\overset{o}{m}C_{pf}T_i + \frac{T_1 - T_f}{R_e}] - [\overset{o}{m}C_{pf}T_o + \frac{T_f - T_c}{R_c}] = [m_fC_{pf}\frac{dT_f}{dt}] \quad \underline{(15)}$$

where:

$\overset{o}{m}$: mass flow rate of fluid, $lb_m$,

$m_f$: mass of fluid contained in the container, $lb_m$,

$C_{pf}$: specific heat of fluid, Btu/$lb_m$ $^oF$,

$T_1$: temperature of the core of the heating element, $^oF$,

$T_f$: temperature of fluid, $^oF$,

$T_c$: temperature of container, $^oF$,

$T_i$: input temperature, $^oF$,

$T_o$: output temperature, $^oF$,

and $m_fC_{pf}$: equivalent heat capacitance of the tank, Btu/$^oF$.

Equation (15) becomes:

$$T_f = \int_o^t (\frac{\overset{o}{m}}{m_f} T_i - \frac{\overset{o}{m}}{m_f} T_o + \frac{T_1}{m_fC_{pf}R_c} - \frac{T_f}{m_fC_{pf}R_c} + \frac{T_f}{m_fC_{pf}R_c} - \frac{T_c}{m_fC_{pf}R_c})dt + IC \quad \underline{(16)}$$

The equation for the temperature of the heating element will be obtained by an energy balance on that element:

[energy flow in] - [energy flow out] = [rate of change of energy],

$$W - \frac{T_1 - T_f}{R_c} = m_1C_{p1}\frac{dT_1}{dt} \quad \underline{(17)}$$

where:

$W$: electrical energy input, Btu/sec,

$T_1$: temperature of the core, $^oF$,

$T_f$: temperature of the fluid, $^oF$,

$R_e$: thermal resistance of the insulator, sec $^oF$/Btu,

$m_1$: mass of the core and the resistor, $lb_m$,

$C_{p1}$: specific heat of the core, Btu/$lb_m$ $^oF$,

and $m_1C_{p1}$: equivalent thermal capacitance of the core, Btu/$^oF$.

Equation (17) can be rewritten as:

$$T_1 = \int_o^t \left(- \frac{T_1}{m_1C_{p1}R_e} + \frac{T_2}{m_1C_{p1}R_e} + \frac{W}{m_1C_{p1}}\right)dt + IC \qquad (18)$$

If the system were to be controlled by adding energy to the heating element in the tank, the same control system as shown in Figure 4 would be used. Another method for controlling the output temperature could use the bang-bang concept for adding energy to the tank. The control scheme would still be the same as shown in Figure 3, but the controller would be bang-bang.

# IV. PARAMETRIC CALCULATIONS

In the previous chapter, the equations were developed for simulating a heat exchanger and a large tank capacitor. In this chapter, these equations will be applied to a specific heat exchanger and tank system.

The heat exchanger is a small pipe with an electrical heating element centered inside the pipe as shown in Figure 1. The following parameters were measured or calculated for this exchanger:

$m_1 = 0.25\ lb_m$ : total mass of the core and the resistor,

$L_1 = 10$ in : total length of the heating element,

$D_1 = 8/16$ in : diameter of the core, included the resistor,

$C_{p1} = 0.2\ Btu/lb_m\ {}^oF$ : specific heat of the core.

The outer part of this heating element is an insulator, having:

$m_2 = 0.25\ lb_m$ : mass of the insulator,

$D_2 = 10/16$ in : outside diameter of the heating element,

$K_e = 0.5$ Btu/ft. hr. $^oF$ : coefficient of conduction of the insulator,

the heating element provides a surface of heat transfer of

$$S_t = \pi L_1 D_2$$
$$= \pi \times \frac{10}{12} \times \left(\frac{10}{16 \times 12}\right) = 0.136\ ft^2.$$

The pipe is steel, 3/4 in ID, which has:

$m_3$ = 3.8 $lb_m$ : mass of the pipe,

$D_i$ = 26/32 in : inside diameter of the pipe,

$D_o$ = 33/32 in : outside diameter of the pipe,

$L_p$ = 10 in : total length of the pipe,

$C_{p3}$ = 0.113 Btu/$lb_m$ $^oF$ : specific heat of the pipe,

$K_p$ = 30 Btu/ft. hr. $^oF$ : coefficient of conduction of the pipe.

The flow section of fluid is:

$$S_f = \frac{\pi}{4} (D_i^2 - D_2^2)$$

$$= \quad [(\frac{13}{16})^2 - (\frac{10}{16})^2] = 0.212 \text{ in}^2 = 1.47 \times 10^{-3} \text{ ft}^2.$$

The flow rate was chosen to be 1 gal/min = 8.05 $ft^3$/hr, therefore the velocity of the fluid in the heat exchanger is calculated as follows:

$$V_f = \frac{\text{flow rate}}{S_f}$$

$$= \frac{8.05}{1.47 \times 10^{-3}} = 5.480 \text{ ft/hr.}$$

With that flow velocity, the dimensionless Reynolds number is then:

$$N_{R_e} = \frac{D_2 V_f \rho}{\mu}$$

where:

$\rho$ = 62 $lb_m/ft^3$: mass density of fluid.

$\mu = 1.65\ lb_m/ft.\ hr$; absolute viscosity of fluid

$$N_{Re} = \frac{0.052 \times 5.480 \times 62}{1.65} = 10,700$$

A Reynolds number of $N_{Re} = 11,000$ will be used to calculate the convection heat transfer coefficient for both: the heating element wall and the inside pipe wall. It is noticed that the surface coefficient of convection does not have a large effect on the thermal resistance R of the system.

The flow is turbulent and McAdam's (10) equation, as a conservative approximation, was used to obtain the convection heat transfer coefficient for this case:

$$h_c = 0.023 \frac{K}{D} (N_{Re})^{0.8} \left(\frac{C_p\mu}{K}\right)^{0.4}$$

where:

$K = 0.365\ Btu/ft.\ hr.\ ^oF$: thermal conductivity, (11)

$\rho = 62\ lb_m/ft^3$: mass density of fluid, (11)

$C_p = 0.997\ Btu/lb_m\ ^oF$: specific heat of fluid at constant pressure, (11)

$$h_c = 0.023 \times \frac{0.365}{0.052} \times (11,000)^{0.8} \times \left(\frac{0.997 \times 1.65}{0.365}\right)^{0.4}$$

$$= 510\ Btu/hr.\ ft^2\ ^oF.$$

This convection heat transfer coefficient was chosen for both cases, the heating element side and the inner pipe side.

The thermal resistance of the system will be calculated next. The insulator has:

$$R_e = \frac{D_2}{2K_e \cdot \pi D_2 \cdot L_1} \ln \frac{D_2}{D_1} + \frac{1}{h_c \cdot \pi D_2 L_1} \ ,$$

$$D_2 = \frac{10}{12 \times 16} \text{ ft}$$

$$D_1 = \frac{8}{12 \times 16} \text{ ft}$$

$K_e$ = 0.5 Btu/ft. hr. $^{o}$F,

$L_1$ = 10/12 ft.,

$h_c$ = 510 Btu/ft$^2$ hr. $^{o}$F.

$$R_e = \frac{1}{2 \times 0.5 \times \pi \frac{10}{12}} \ln \frac{10}{8} + \frac{1}{510 \times \pi \times \frac{10}{16 \times 12} \times \frac{10}{12}}$$

= 0.21 hr. $^{o}$F/Btu = 755 sec $^{o}$F/Btu.

The thermal resistance of the pipe is:

$$R_p = \frac{D_i}{2\ K_p\ \pi D_i\ L_p} \ln \frac{D_o}{D_i} + \frac{\phantom{1}}{h_c\ \pi D_i\ L_p} \ ,$$

$$D_i = \frac{26}{32 \times 12} \text{ ft.}$$

$$D_o = \frac{33}{32 \times 12} \text{ ft.}$$

$L_p$ = 10/12 ft.,

$K_p$ = 30 Btu/ft. hr. $^{o}$F,

$h_c$ = 510 Btu/ft$^2$ hr. $^{o}$F,

$$R_p = \frac{1}{2\pi \times 30 \times \frac{10}{12}} \ln \frac{33}{26} + \frac{\phantom{1}}{510 \times \pi \times \frac{10}{12} \times \frac{26}{32 \times 12}}$$

$$= 0.004 \text{ hr } {}^{o}F/Btu = 14.4 \text{ sec } {}^{o}F/Btu.$$

The equations for the large tank capacitor will now be applied to a specific system. Three different sizes of tanks will be considered.

The first case will be a 2 ft x 2 ft x 2 ft tank with 500 $lb_m$ of water in it; the following parameters were calculated:

$$\overset{o}{m} = 8.05 \text{ ft}^3/\text{hr} = 0.138 \text{ lb}_m/\text{sec},$$

$$\frac{\overset{o}{m}}{m_f} = \frac{0.138}{500} = 2.8 \times 10^{-4}, \text{ dimensionless}$$

$$R_c = \frac{e}{KA}$$

$e = 1$ in $= 1/12$ ft: thickness of the tank,

$A = 2 \times 2 \times 5 = 20$ ft$^2$: surface area,

$K = 0.07$ Btu/ft hr $^{o}F$: thermal conductivity,

$$R_c = \frac{1}{12} \times \frac{1}{0.07 \times 20} \times 3600 = 215 \text{ sec } {}^{o}F/Btu$$

$$\frac{1}{m_f C_{pf} R_c} = \frac{1}{500 \times 0.997 \times 215} = 0.093 \times 10^{-4} \quad 1/\text{sec}$$

$$\frac{1}{m_c C_{pc} R_c} = \frac{1}{100 \times 0.5 \times 215} = .942 \times 10^{-4} \quad 1/\text{sec}$$

The second tank will be a 2.5 ft x 2.5 ft x 2.5 ft tank with 1000 $lb_m$ of water in it; the following parameters were calculated:

$$\frac{\overset{o}{m}}{m_f} = \frac{0.138}{1000} = 1.38 \times 10^{-4}, \text{ dimensionless}$$

$$R_c = \frac{1}{12} \times \frac{1}{0.07 \times 31} \times 3600 = 144 \text{ sec } {}^{o}F/Btu,$$

$$\frac{1}{m_f C_{pf} R_c} = \frac{1}{1000 \times 0.997 \times 144} = 0.069 \times 10^{-4} \text{ 1/sec},$$

$$\frac{1}{m_c C_{pc} R_c} = \frac{1}{150 \times 0.5 \times 144} = .925 \times 10^{-4} \text{ 1/sec},$$

The last case will be a 3 ft x 3 ft x 3 ft tank with 1500 $lb_m$ of water in it. The parameters would be:

$$\frac{\overset{o}{m}}{m_f} = \frac{0.139}{1500} = .93 \times 10^{-4}, \text{ dimensionless}$$

$$R_c = \frac{1}{12} \times \frac{1}{0.07 \times 45} \times 3600 = 95 \text{ sec } {}^{o}F/Btu$$

$$\frac{1}{m_f C_{pf} R_c} = \frac{1}{1500 \times 0.997 \times 95} = 0.07 \times 10^{-4} \text{ 1/sec},$$

$$\frac{1}{m_c C_{pc} R_c} = \frac{1}{200 \times 0.5 \times 95} = 1.05 \times 10^{-4} \text{ 1/sec}.$$

For a 1000 watt input to the system, the electrical energy W which flows into the system would be:

$$W = 1000 \text{ watt} = 0.95 \text{ Btu/sec}.$$

# V. COMPUTER SIMULATIONS

Now, the equations and parameters were set up on the analog computer in the Mechanical Engineering Laboratory. The analog computer is a Systron Donner SD40 computer. In order to put the problem on the computer, some estimate of maximum values for the variables in each of the systems is required.

The maximum values for the tank are easily estimated but for the heat exchanger, a single lumped system was run with no control and full energy input to define these values.

## A. Single lumped parameter responses

The number of lumped parameters, n, was set equal to 1 in equation (5), which produces the following equation:

$$T_1 = \int_o^t \left(- \frac{T_1}{m_1 C_{p1} R_e} + \frac{T_2}{m_1 C_{p1} R_e} + \frac{W}{m_1 C_{p1}}\right) dt + IC$$

$$m_1 C_{p1} = 0.25 \times 0.2 = 0.05 \text{ Btu/}^oF,$$

$$R_e = 755 \text{ sec } ^oF/\text{Btu},$$

$$W = 0.95 \text{ Btu/sec},$$

$$T_1 = \int_o^t (-0.0265\ T_1 + 0.0265\ T_2 + 20\ EI)\ dt + IC.$$

As a first guess, the maximum temperature for $T_1$ and $T_2$ were:

$T_{1_{max}} = 500\ ^0F$

$T_{2_{max}} = 75\ ^0F$

The scaled equation is:

$$[\frac{T_1}{5}] = \int_o^t \{[-0.0265\ \frac{T_1}{5}] + [0.0054\ T_2] + [4W]\}\ dt + \frac{IC}{5} \qquad (\underline{19})$$

Again, for n = 1, equation (11) yields:

$$T_2 = \int_o^t (\frac{\overset{o}{m}}{m_f} T_i - \frac{\overset{o}{m}}{m_f} T_o + \frac{T_1}{m_f C_{pf} R_e} - \frac{T_2}{m_f C_{pf} R_e} - \frac{T_2}{m_f C_{pf} R_p} + \frac{T_3}{m_f C_{pf} R_p}) dt + IC$$

$m_f = \rho S_f L_p$,

$= 1.47 \times 10^{-3} \times \frac{10}{12} \times 62 = 7.6 \times 10^{-2}\ lb_m$,

$\overset{o}{m} = 8.05 \times 62 = 500\ lb_m/hr = 0.139\ lb_m/sec$,

$C_{pf} = 0.997\ Btu/lb_m\ ^oF$,

$R_e = 755\ sec\ ^oF/Btu$,

$R_p = 14.4\ sec\ ^oF/Btu$,

$m_f C_{pf} = 7.60 \times 10^{-2} \times 0.997 = 7.58 \times 10^{-2}\ Btu/^oF$

$$T_2 = \int_o^t (1.90\ T_i - 1.90\ T_o + 0.018\ T_1 - 0.018\ T_2 - 0.94\ T_2 + 0.94\ T_3)\ dt + IC.$$

The maximum expected value for $T_o$ and $T_3$ will not exceed $80^oF$, therefore:

$$[T_2] = \int_o^t \{[1.90\ T_i] + [-1.90\ T_o] + [0.09\ \frac{T_1}{5}] + [-0.958\ T_2] + [0.94\ T_3]\}\ dt + IC \qquad (\underline{20})$$

The above equation is simulated in Figure 6.

Equation ($\underline{8}$) gives:

$$[T_o] = [2T_2] - [T_i] \qquad (\underline{21})$$

With n = 1, equation ($\underline{9}$) can be rewritten as:

$$T_3 = \int_o^t (\frac{T_2}{m_3C_{p3}R_p} - \frac{T_3}{m_3C_{p3}R_p})\ dt + IC\ ,$$

$m_3 = 3.8\ lb_m$,

$C_{p3} = 0.113\ Btu/lb_m\ ^oF$,

$m_3C_{p3} = 3.8 \times 0.113 = 0.43\ Btu/^oF$,

$R_p = 14.4\ sec\ ^oF/Btu$.

$$T_3 = \int_o^t (0.161\ T_2 - 0.161\ T_3)\ dt + IC.$$

The scaled equation for $T_3$ is:

$$[T_3] = \int_o^t [0.161\ T_2] + [-0.161\ T_3]\ \ dt + IC \qquad (\underline{22})$$

The analog simulation of equations ($\underline{19}$), ($\underline{20}$), ($\underline{21}$) and ($\underline{22}$) are presented in Figure 6. The operational amplifier assignments and

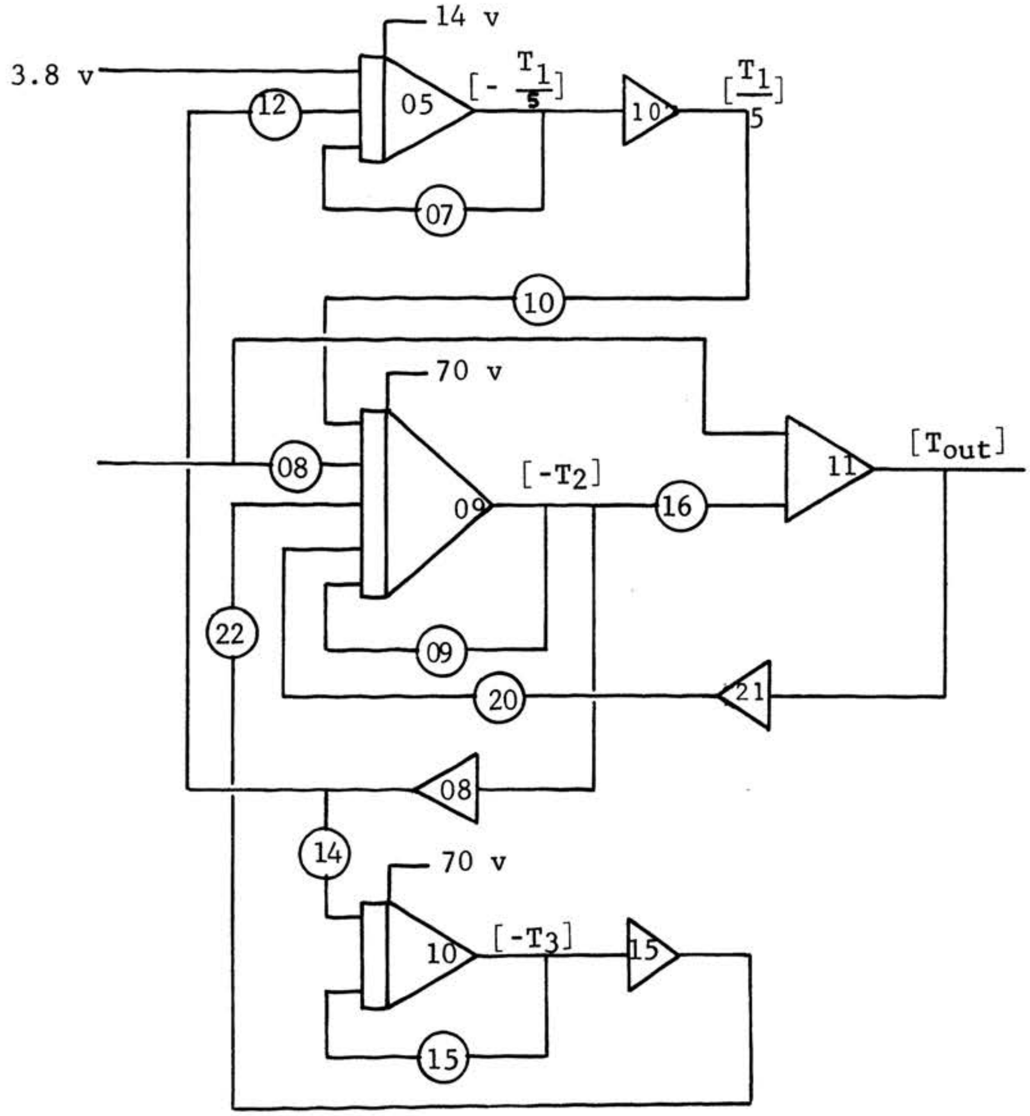


Figure 6. Analog Circuit for a Single Lumped Parameter System

potentiometer settings are listed in Table II and III.

The transient response curves of the system are plotted in Figure 7. These curves show that the maximum value of $T_1$ is about 445 $^oF$ at steady state operating condition, $T_2$ and $T_3$ is not more than 78 $^oF$, therefore, a magnitude scaling factor of 0.2 will be picked up for the output temperature of the heating element and a factor of 1 for $T_2$ and $T_3$.

The time for transient response was found to be less than 150 seconds, so, the analog computer time scale switch was put in the B position which mode the machine time 10 times faster than real time, with this time scaling, a steady state solution is obtained in less than 15 seconds.

B. Three lumped parameter system

The heat exchanger system will be lumped into a certain number of places for improved system description. Masami Masubuchi (8) insulated a parallel counter flow heat exchanger by using a two and four lumped parameter models and compared the results with the distributed parameter model, obtained theoretically. The conclusion was made that a four lumped parameter system produced sufficient accuracy. J. R. Schmidt and D. R. Clark (9), compared the results given by simulating a single three and then seven lumped parameter models. They favored the three lumped parameter system.

The system will be lumped into three equal pieces. A four lumped

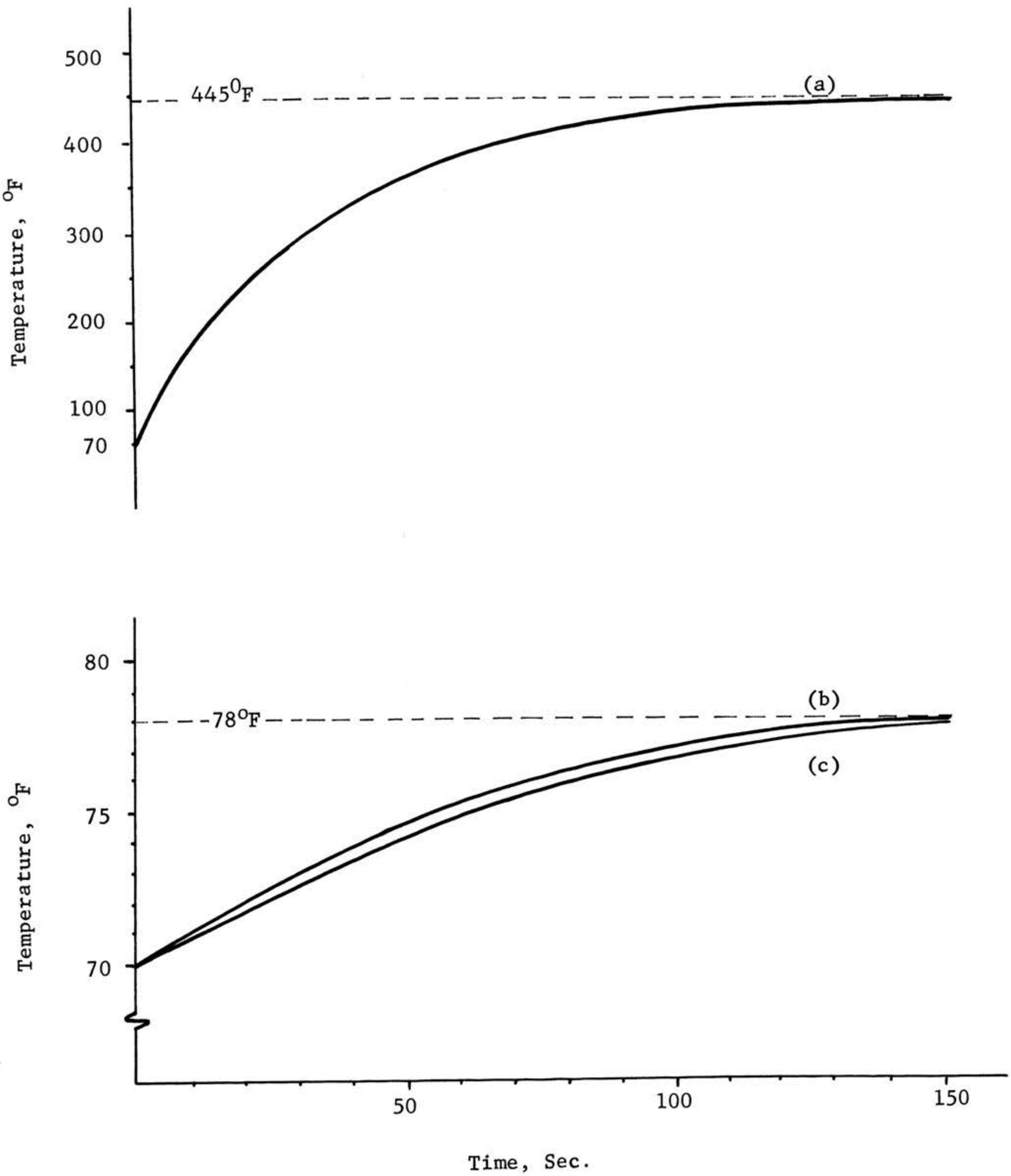


Figure 7. Response of (a) the heating element, (b) the fluid, (c) the pipe, due to 1000 watt input

parameter system would have been closer but the simulation was limited by the number of amplifiers on the analog computer.

With n = 3, equation (5) becomes:

$$T_{1j} = \int_o^t \left(-\frac{T_{1j}}{m_1^* C_{p1} R_e^*} + \frac{T_{2j}}{m_1^* C_{p1} R_e^*} + \frac{W}{2m_1^* C_{p1}}\right) dt + IC,$$

$$m_1^* = \frac{m_1}{3} = 0.083 \ lb_m,$$

$$W = 0.95 \ Btu/sec$$

$$T_{1j} = \int_o^t (-\ 0.0265\ T_{1j} + 0.0265\ T_{2j} + 20\ W)\ dt + IC$$

The scaled equation is:

$$\left[\frac{T_{1j}}{5}\right] = \int_o^t \{[-0.0265\ \frac{T_{1j}}{5}] + [+0.0054\ T_{2j}] + [4W]\}\ dt + IC \quad (\underline{23})$$

Equation (7) can be rewritten:

$$T_{2j} = \int_o^t \left(-\frac{\overset{o}{m}}{m_f^*} T_{ij} - \frac{\overset{o}{m}}{m_f^*} T_{oj} + \frac{T_{1j}}{m_f^* C_{pf} R_e^*} - \frac{T_{2j}}{m_f^* C_{pf} R_e^*}\right.$$

$$\left. - \frac{T_{2j}}{m_f^* C_{pf} R_p^*} + \frac{T_{3j}}{m_p^* C_{pf} R_p^*}\right) dt + IC$$

$$\overset{o}{m} = 0.139 \ lb_m/sec$$

$$m_f^* = \frac{m_f}{3} = \frac{7.60 \times 10^{-2}}{3} \ lb_m,$$

$$C_{pf} = 0.997 \ Btu/lb_m \ {}^oF,$$

$$R_e^* = 3R_e = 3 \times 755 \ sec \ {}^oF/Btu,$$

$$R_p^* = 3R_p = 3 \times 14.4 \text{ sec } ^oF/Btu,$$

$$T_{2j} = \int_o^t (5.7\ T_{ij} - 5.7\ T_{oj} + 0.018\ T_{1j} - 0.018\ T_{2j} - 0.94\ T_{2j} + 0.94\ T_{3j})\ dt + IC.$$

The scaled equation is then:

$$[T_{2j}] = \int_o^t \{[5.7\ T_{ij}] + [-5.7\ T_{oj}] + [0.09\ \frac{T_{1j}}{5}] + [-0.0958\ T_{2j} + [0.94\ T_{3j}]\}\ dt + IC \qquad (\underline{24})$$

$$C_{p1} = 0.2 \text{ Btu/lb}_m\ ^oF,$$

$$R_e^* = 3R_e = 3 \times 755 \text{ sec } ^oF/Btu.$$

Equation (8) gives:

$$[T_{oj}] = [2T_{2j}] + [-\ T_{ij}].$$

From equation (13) :

$$T_{3j} = \int_o^t (\frac{T_{2j}}{m_3^* C_{p3} R_p^*} - \frac{T_{3j}}{m_3^* C_{p3} R_p^*})\ dt + IC,$$

$$m_3^* = \frac{m3}{3} = 1.27 \text{ lb}_m,$$

$$C_{p3} = 0.113 \text{ Btu/lb}_m\ ^oF,$$

$$R_p^* = 3R_p = 3 \times 14.4 \text{ sec } ^oF/Btu,$$

$$T_{3j} = \int_o^t (0.161\ T_{2j} - 0.161\ T_{3j})\ dt + IC$$

The scaled equation is:

$$[T_{3j}] = \int_o^t \{[0.161\ T_{2j}] + [-0.161\ T_{3j}]\}\ dt + IC. \quad (\underline{26})$$

For the continuity of the lumping system, the following conditions must be satisfied:

$$\begin{aligned} T_{o,j-1} &= T_{ij}, \\ T_{oj} &= T_{i,j+1}\ . \end{aligned} \quad (\underline{27})$$

The lumping system must also satisfy the boundary conditions:

$$\begin{aligned} T_{ij}\ \rfloor_{j=1} &= T_{in} \\ T_{oj}\ \rfloor_{j=3} &= T_{out} \end{aligned} \quad (\underline{28})$$

By using all the equations (23) through (28) for j = 1,2,3, all the fundamental equations are obtained as follows:

$$[\frac{T_{11}}{5}] = \int_o^t \{[-0.0265\ \frac{T_{11}}{5}] + [0.0054\ T_{21}] + [4W]\}\ dt + \frac{IC}{5} \quad (\underline{29})$$

$$[T_{21}] = \int_o^t \{[5.7\ T_{in}] + [-5.7\ T_{01}] + [0.09\ \frac{T_{11}}{5}] + [-0.958\ T_{21}] + [0.94\ T_{31}]\}\ dt + IC \quad (\underline{30})$$

$$[T_{01}] = [2\ T_{21}] + [-T_{in}], \quad (\underline{31})$$

$$[T_{31}] = \int_o^t \{[0.161\ T_{21}] + [-0.161\ T_{31}]\}\ dt + IC \qquad (\underline{32})$$

$$[\frac{T_{12}}{5}] = \int_o^t \{[-0.0265\ \frac{T_{12}}{5}] + [0.0054\ T_{22}] + [4W]\}\ dt + \frac{IC}{5} \qquad (\underline{33})$$

$$[T_{22}] = \int_o^t \{[5.7\ T_{01}] + [-5.7\ T_{02}] + [0.09\ \frac{T_{12}}{5}] + [-0.958\ T_{22}] + [0.94\ T_{32}]\}\ dt + IC \qquad (\underline{34})$$

$$[T_{02}] = [2\ T_{22}] + [-\ T_{01}] \qquad (\underline{35})$$

$$[T_{32}] = \int_o^t \{[0.161\ T_{22}] + [-0.161\ T_{32}]\}\ dt + IC \qquad (\underline{36})$$

$$[\frac{T_{13}}{5}] = \int_o^t \{[0.0265\ \frac{T_{13}}{5}] + [-0.0054\ T_{23}] + [4W]\}\ dt + IC \qquad (\underline{37})$$

$$[T_{23}] = \int_o^t \{[5.7\ T_{02}] + [-5.7\ T_{out}] + [0.09\ \frac{T_{13}}{5}] + [-0.958\ T_{23}] + [0.94\ T_{31}]\}\ dt + IC \qquad (\underline{38})$$

$$[T_{out}] = [2T_{23}] - [T_{02}] \qquad (\underline{39})$$

$$[T_{33}] = \int_o^t \{[0.161\ T_{23}] + [-0.161\ T_{33}]\}\ dt + IC \qquad (\underline{40})$$

The equations (29) through (40) are simulated in Figure 8. The assignments for operational amplifiers and potentiometer settings are

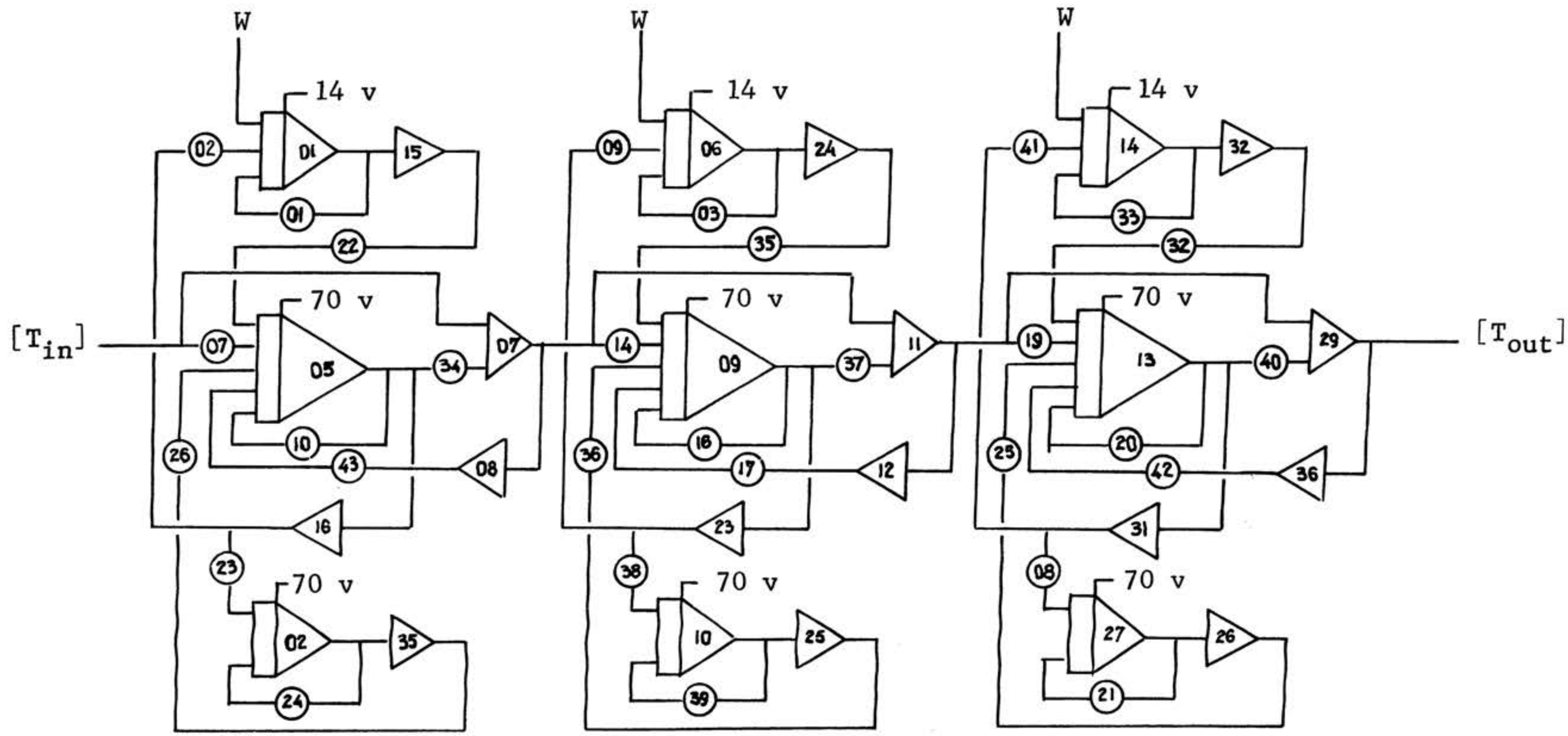


Figure 8. Analog Circuit for the Heat Exchanger

shown in Table IV and V.

The controller can be easily adapted to this model with the output temperature [$T_{out}$] being sampled and fedback to the comparator and compared to [$T_{set}$] (Figure 9). The error -[e] is the output of the comparator. This error signal is sent to the controller. There will be two types of controllers used, a proportional controller and a proportional plus derivative type controller.

The proportional controller is just simply a gain times the error function. The equation for this operation is:

$$G_1(t) = \frac{m(t)}{e(t)} = K \qquad (41)$$

where:

$G_1(t)$ : time transfer function of the controller

$m(t)$ : output function

$e(t)$ : input function or error function

$K$ : gain of the controller

This function can be simply simulated with a potentiometer. The block diagram and analog circuit for proportional controller are presented in Figures 9 and 10. Amplifier assignments and potentiometer settings are found in Table VI and VII.

The proportional plus derivative controller can be approximated by the following transfer function in the Laplace domain:

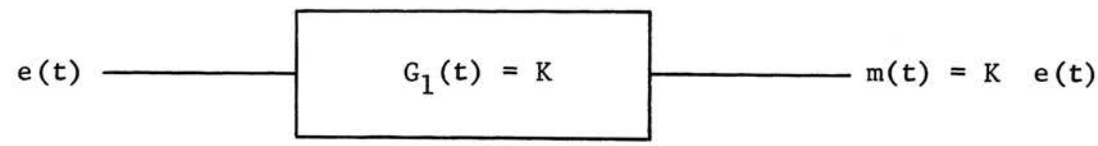


Figure 9. Block Diagram for a Proportional Controller

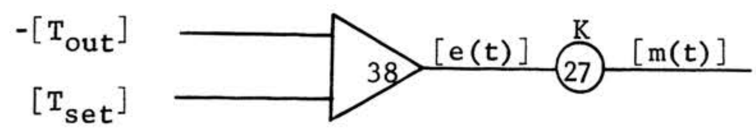


Figure 10. Analog Circuit for Comparator and Proportional Controller

$$G_2(s) = \frac{m(s)}{e(s)} = K\,\frac{1 + \tau_1 s}{1 + \tau_2 s} \qquad (42)$$

with
$$\frac{1}{\tau_2} > \frac{1}{\tau_1}$$

where:

$G_2(s)$ : time transfer function in Laplace domain

$m(s)$ : output function in Laplace domain

$e(s)$ : input function in Laplace domain

$K$ : proportional gain of the controller

$\tau_1$ & $\tau_2$ : the controller time constants.

This function has a block diagram and an analog simulation shown in Figures 11 and 12. Amplifier assignments and potentiometer settings are found in Tables VIII and IX.

The output [m(t)] of the controller drives the power supply which can only react to a positive [m(t)]. So the power supply (Figure 13) is constructed, using diodes to insure no reaction to negative [m(t)] and to size the heating element. For a 1000 watt heating element, the output v(t) is limited to the range:

$$0 \leq v(t) \leq 3.8 \text{ volts.}$$

This output will be the heater input for each of the three lumped systems.

It is desired to see the response of this system with either the

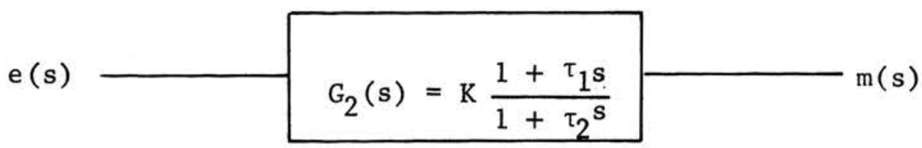


Figure 11. Block Diagram for a Proportional Plus Derivative Controller

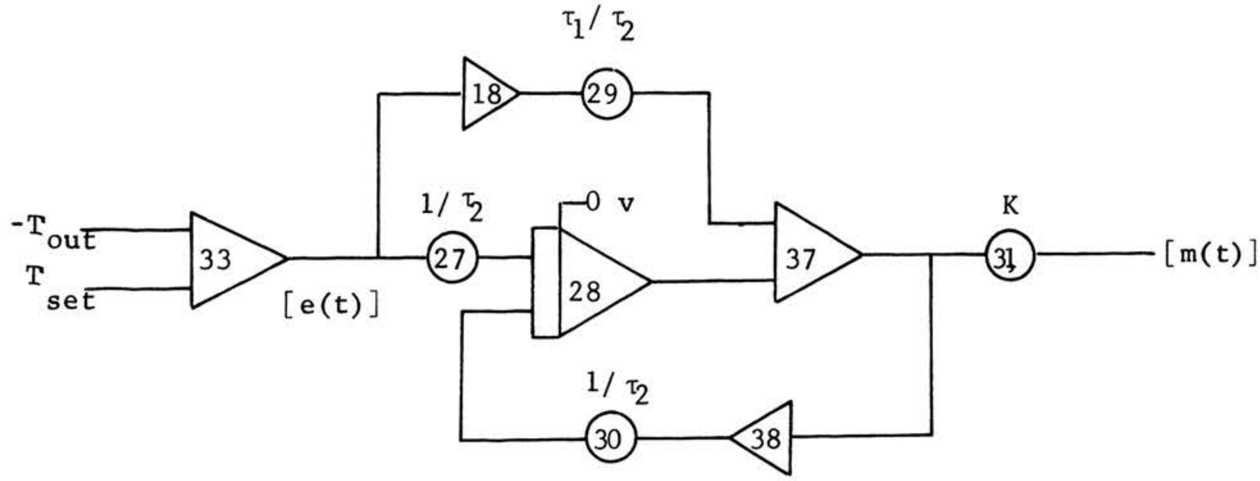


Figure 12. Analog Circuit for Comparator and Proportional Plus Derivative Controller

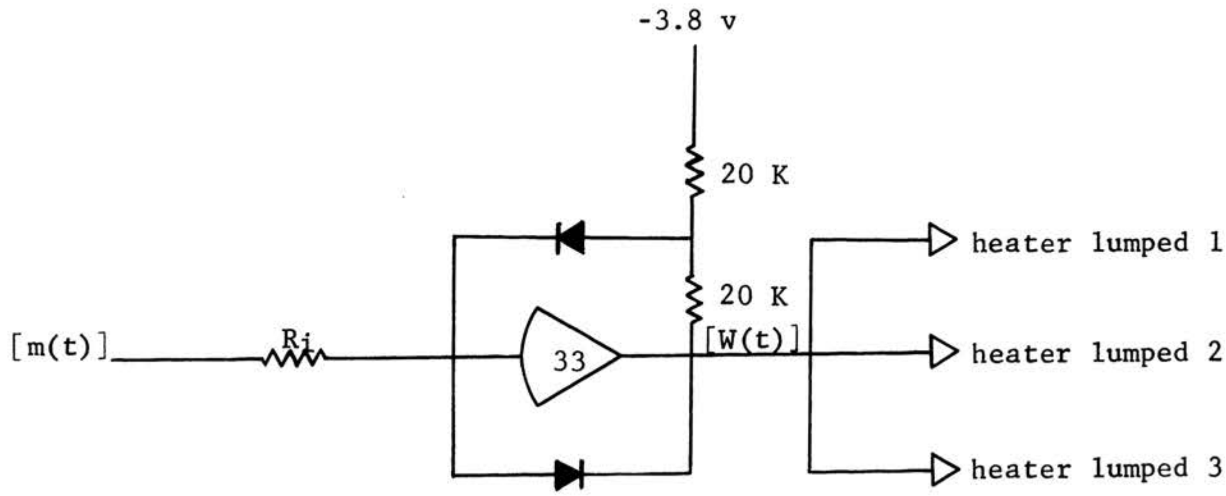


Figure 13. Power Supply Simulation

proportional controller at proportional plus derivative controller with a fluctuating input temperature to the system. This fluctuating input temperature is assumed to be produced by a constant temperature circulating bath. In examining the circulating bath system fabricated by Drs. Sauer and Flanigan for use with the cloud simulation chamber, the output of this system appeared to be a constant level with a 2 cycle per minute imposed fluctuation. The amplitude of this fluctuation was approximately 0.4 degrees Fahrenheit, peat to peak.

This fluctuating signal was assumed to be a sine wave with a frequency of 2 cycles per minute. So the equation describing the entire input signal will be:

$$[T_i] = [T_i]_{\text{constant level}} + [0.2 \sin \omega t]_{\text{fluctuating component}} . \tag{43}$$

where:

$$\omega = 2 \text{ cpm} = 0.21 \text{ rad/sec}$$

Equation (43) can be simulated as shown in Figure 14. Amplifier assignments and potentiometer settings are found in Tables X and XI.

The analog simulation can now be shown in Figure 15 which includes the fluctuating input temperature, the three mass lumped system, the distribution of input energy and the control loop with the controller being represented only by G(s). The system can be run with either of the controllers shown in Figure 10 or 12.

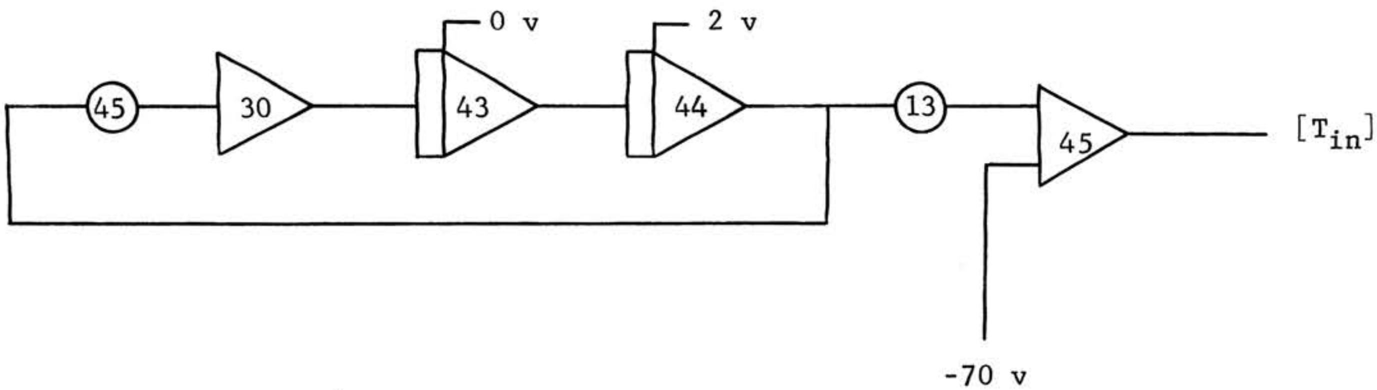


Figure 14. Circuit for Generating [$T_{in}$]

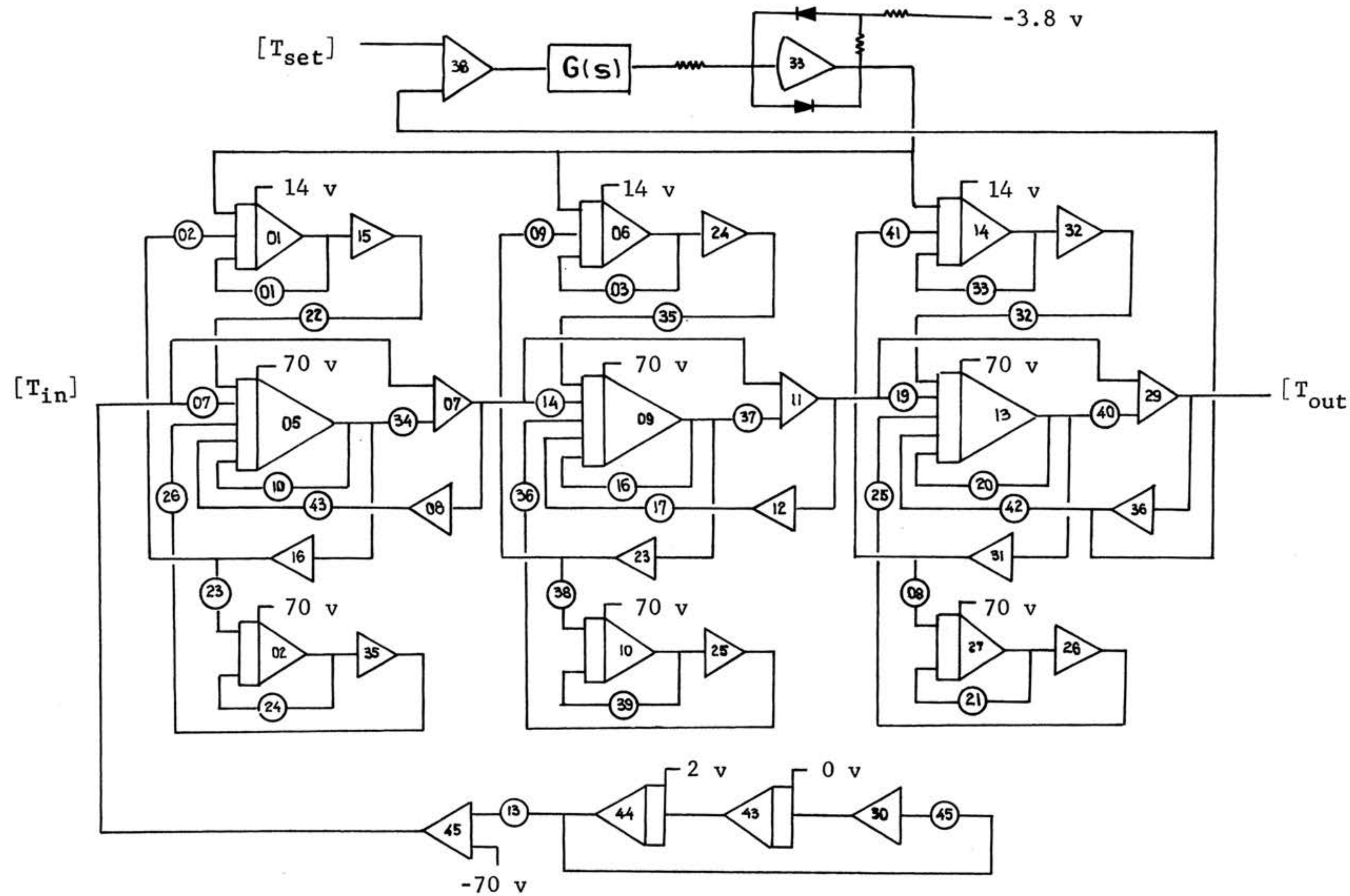


Figure 15. Analog Simulation for the Heat Exchanger, the Fluctuation Input, the Controller and the Power Supply

C. Single lumped tank

Another method is tried here for accomplishing the same purpose as stated previously. The fluid is run through a big tank, which is stirred continuously to obtain a unique temperature throughout the tank.

For the 2 ft x 2 ft x 2 ft tank, the following equations were obtained from equations (12) and (14):

$$[T_c] = \int_o^t \{[0.942 \times 10^{-4}\ T_f] + [-0.942 \times 10^{-4}\ T_c]\}\ dt + IC \tag{44}$$

$$[T_f] = \int_o^t \{[2.8 \times 10^{-4}\ T_i] + [-2.8 \times 10^{-4}\ T_o] + [0.093 \times 10^{-4}\ T_f] + [-0.093 \times 10^{-4}\ T_c]\}\ dt + IC \tag{45}$$

For the 2.5 ft x 2.5 ft x 2.5 ft tank, equations (12) and (14) yield:

$$[T_c] = \int_o^t \{[0.925 \times 10^{-4}\ T_f] + [-0.925 \times 10^{-4}\ T_c]\}\ dt + IC \tag{46}$$

$$[T_f] = \int_o^t \{[1.38 \times 10^{-4}\ T_i] + [-1.38 \times 10^{-4}\ T_o] + [0.069 \times 10^{-4}\ T_f] + [-0.069 \times 10^{-4}\ T_c]\}\ dt + IC \tag{47}$$

For the 3 ft x 3 ft x 3 ft tank, the following scaled equations were formed:

$$[T_c] = \int_o^t \{[1.05 \times 10^{-4}\ T_f] + [-1.05 \times 10^{-4}\ T_c]\}\ dt + IC \tag{48}$$

$$[T_f] = \int_o^t \{[0.93 \times 10^{-4}\, T_i] + [-0.93 \times 10^{-4}\, T_o] + [0.07 \times 10^{-4}\, T_f] + [-0.07 \times 10^{-4}\, T_c]\}\, dt + IC \qquad (\underline{49})$$

All these equations are simulated in Figure 16. Amplifier assignments and potentiometer settings are listed in Tables XII and XIII A time scaled factor of $10^4$ was used.

Energy will be put into the 2 ft x 2 ft x 2 ft tank by a heating element. The input power is controlled by a proportional and then a bang-bang controller. Equation (19) gives:

$$[\frac{T_1}{5}] = \int_o^t \{[-0.0265\, \frac{T_1}{5}] + [0.0054\, T_2] + [4W]\}\, dt + \frac{IC}{5} \qquad (\underline{19})$$

An analog circuit for simulating all the equations from (44) to (49) and equation (19) is shown in Figure 17. Amplifier assignments and potentiometer settings are listed in Tables XII and XIII.

The input power of the heating element is controlled by a proportional controller of gain the same as shown in Figure 10. The tank system is then controlled by a bang-bang controller. A bang-bang controller works in two positions only: it is off completely if it receives an error less than the value of the dead band or it is on with maximum output power when it receives an error greater than the value of the dead band from the comparator. In order to simulate this controller an electro-magnetic switch of type 3324 on the computer is used. The switch is on when a voltage of 28 volts is applied, and off when there

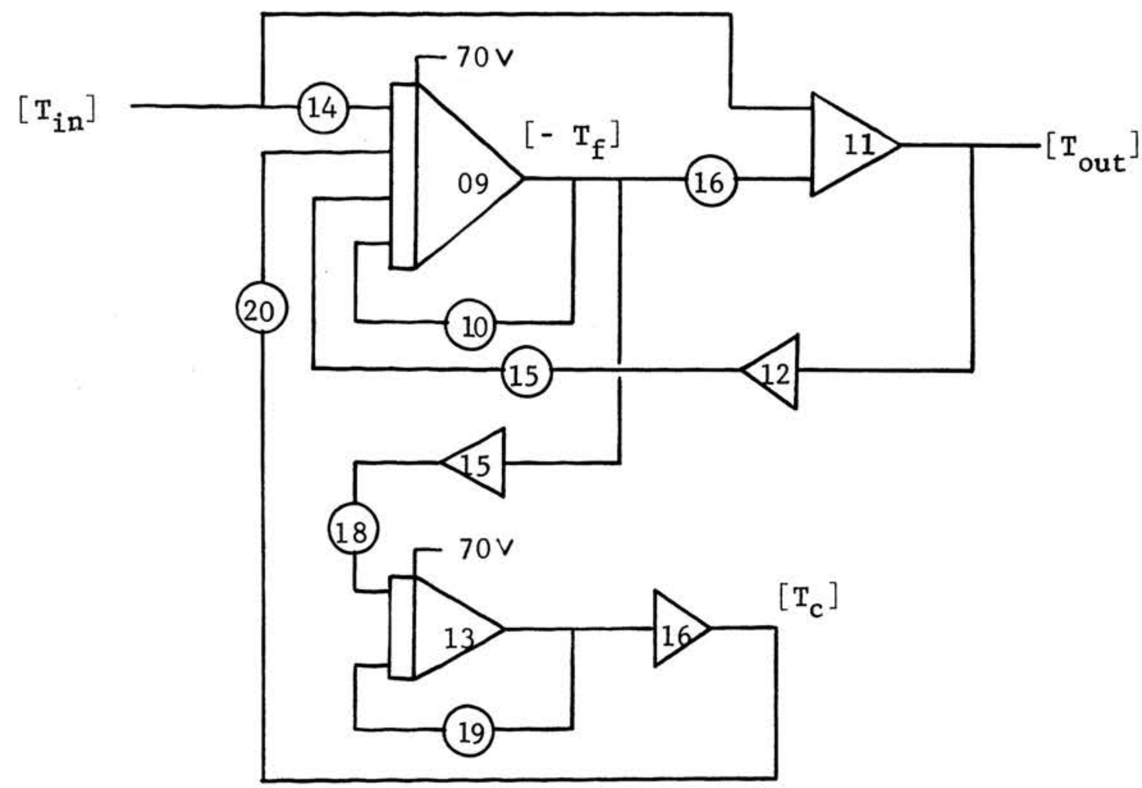


Figure 16. Analog Circuit for the Single Lumped Tank with no Input Heat

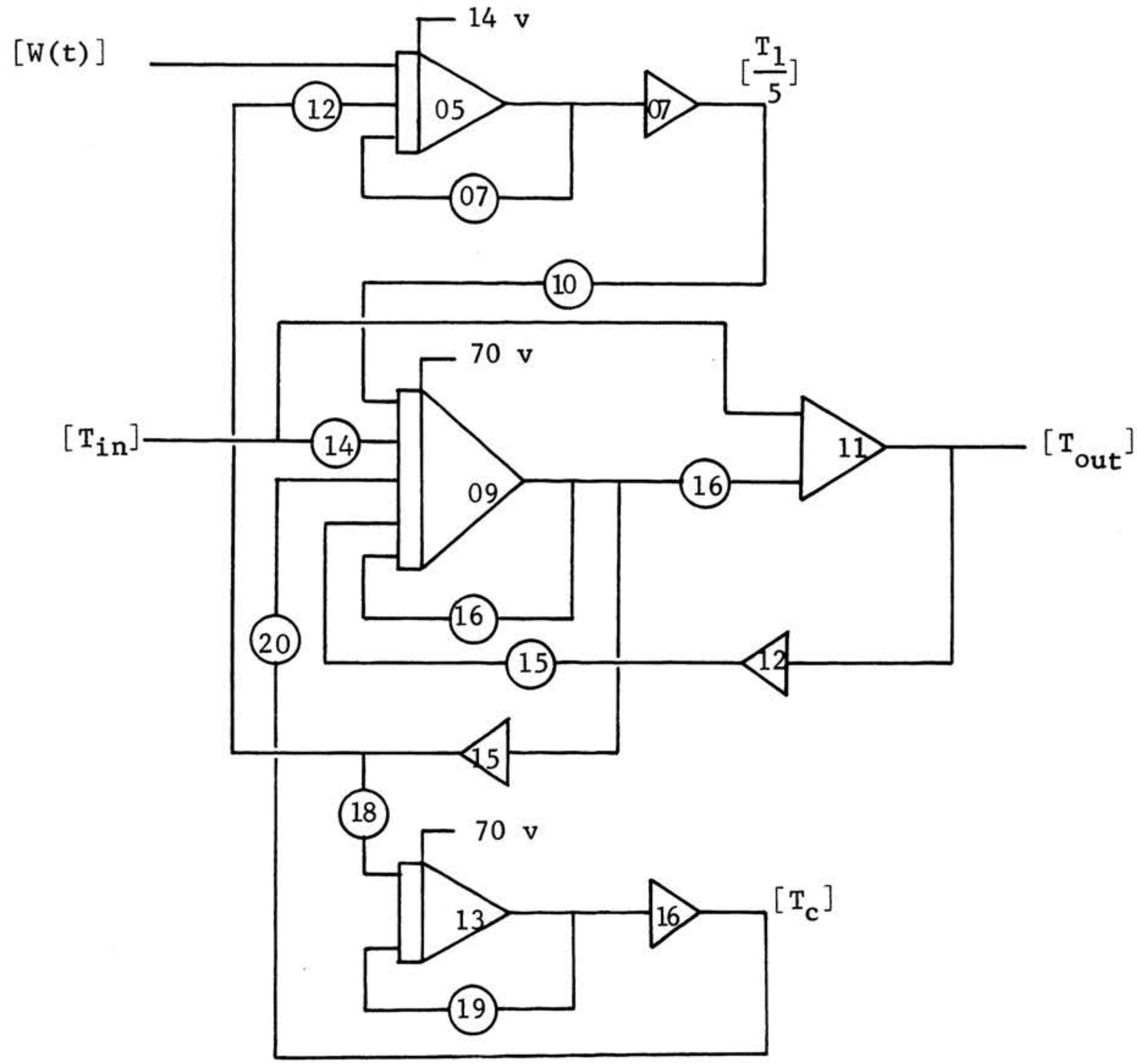


Figure 17. Circuit for the Tank and the Heating Element

is no voltage applied (Figure 18). A circuit is used to provide this logic from the error in order to drive the switch and its simulation is shown in Figure 19.

The analog simulation of the tank system now can be shown in Figure 20, which includes the single lumped tank, the heating element and the controller, being represented by G(s). The system can be run with either of the controllers, shown in Figure 10 or 19. Amplifier assignments and potentiometer settings will be found in Tables XIV and XV.

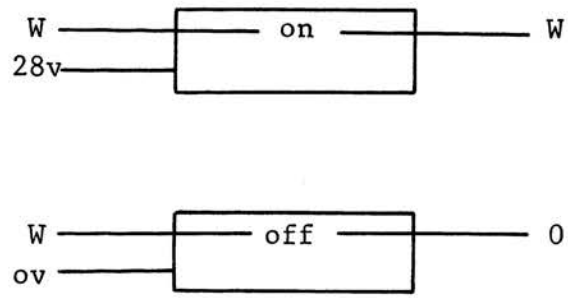


Figure 18. Electro-magnetic Switch

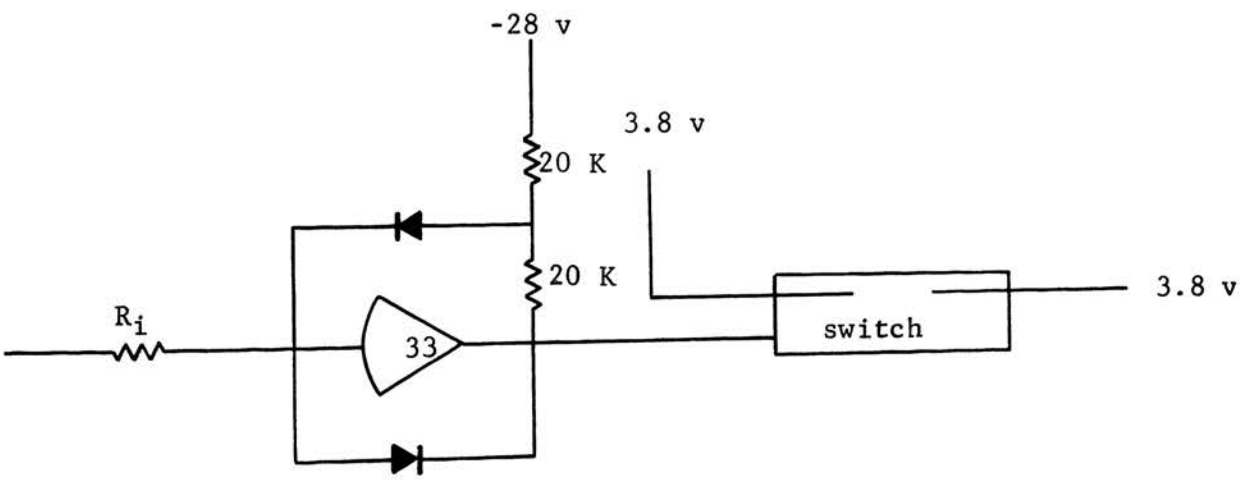


Figure 19. Simulation for a Bang-bang Controller

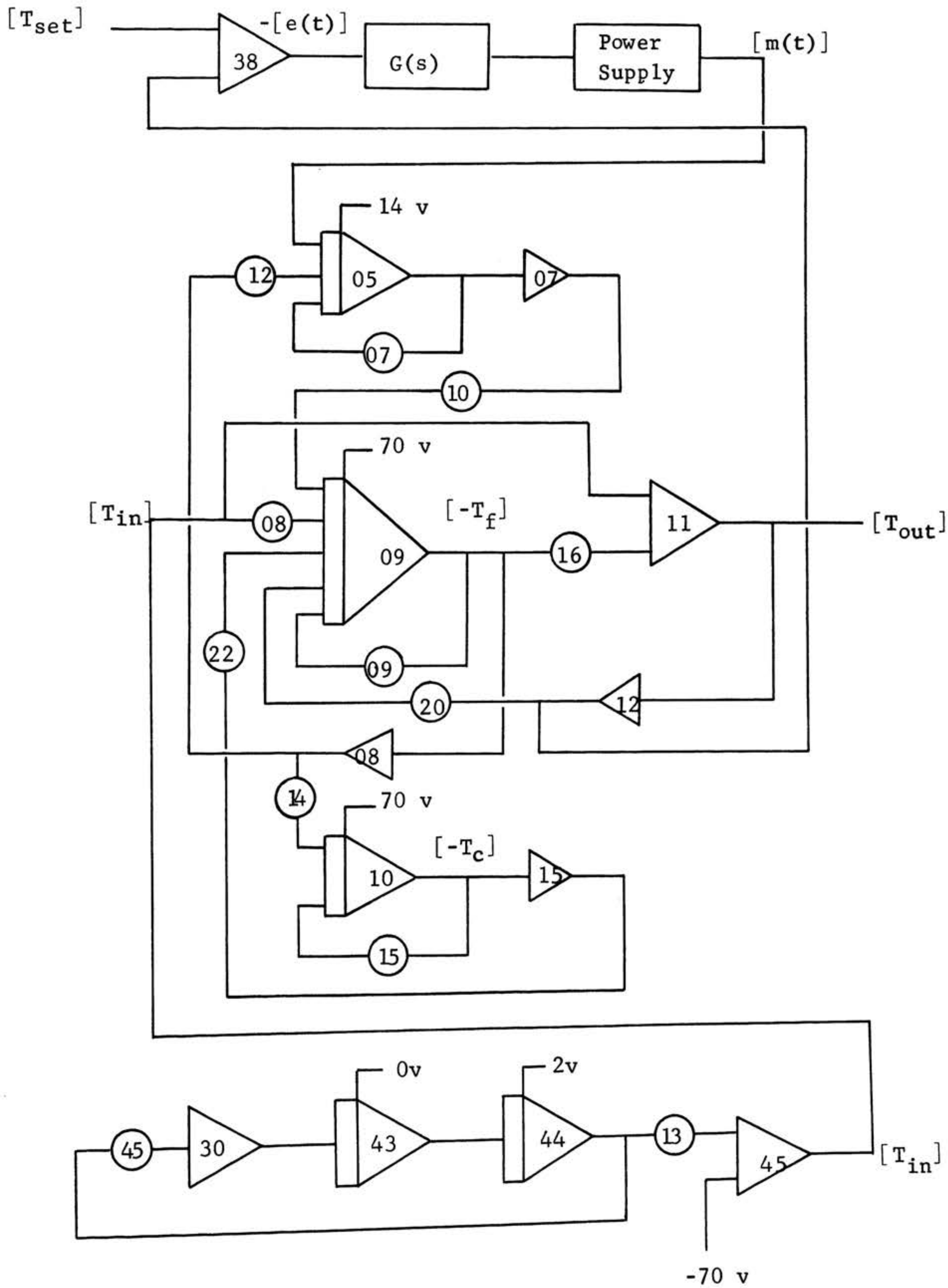


Figure 20. The Tank System with the Heater and Controller

# VI. RESULTS

The first test to be conducted was to run the three lumped mass system with G(s) (Figure 3) set up as a proportional controller (Figure 10) and with gain, k, set to 10. The system was driven with the fluctuating $T_{in}$ (Figure 14) and $T_{set}$ set at 70.5 $^{o}$F. The results of this run are shown in Figure 21. The ratio of output fluctuation to input fluctuation was 0.58 which was considered to be too large. So the next step was to increase the value of proportional gain and the new value tried was 100 with the results shown in Figure 26. The output fluctuation to input fluctuation ratio produced was 0.40 and again, the gain was increased to 500 with the results shown in Figure 23. The output fluctuation to input fluctuation ratio was 0.37.

It can be seen that the output fluctuation to input fluctuation ratio was not being strongly influenced by large changes in proportional gain, so the controller was changed to one with both proportional and derivative control as shown in Figure 12.

This system was hooked up as shown (Figure 15) and tested with various controller parameters not only was the gain adjusted but also the time constants $\tau_1$ and $\tau_2$ providing good anticipation of forthcoming error. The different runs are shown in Figures 24, 25, 26, 27, 28, and 29. In these different runs, gain was varied from 5 to 500 and $\tau_1/\tau_2$ was varied from 1.5 to 10. Using this controller with a gain of

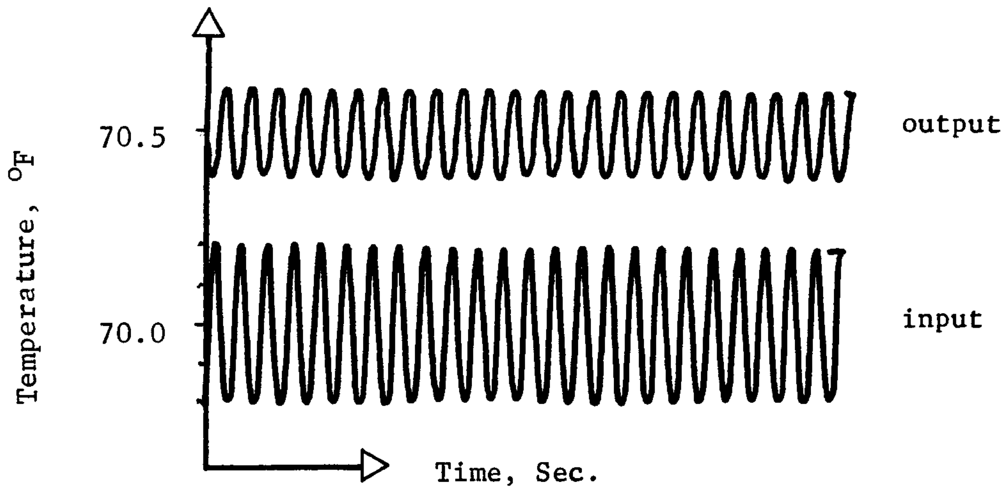


Figure 21. Proportional Controller, K = 10.

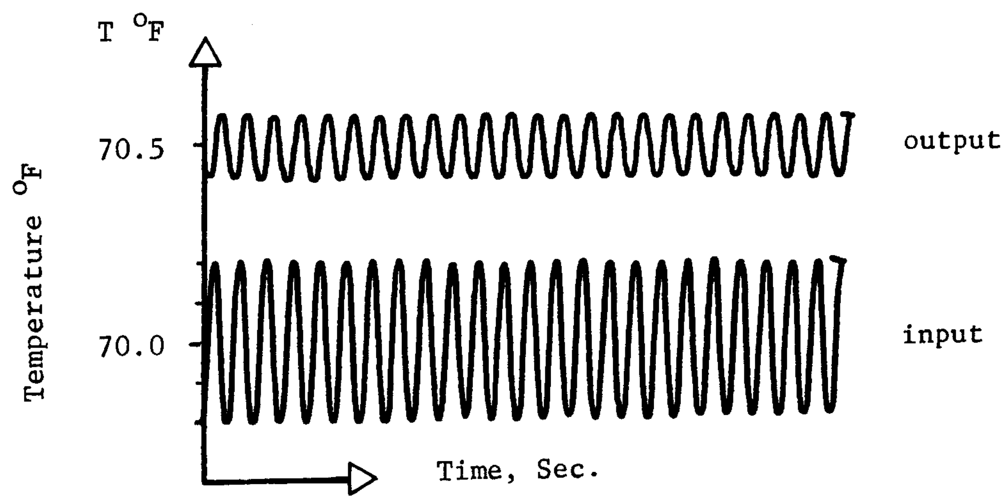


Figure 22. Proportional Controller, K = 100

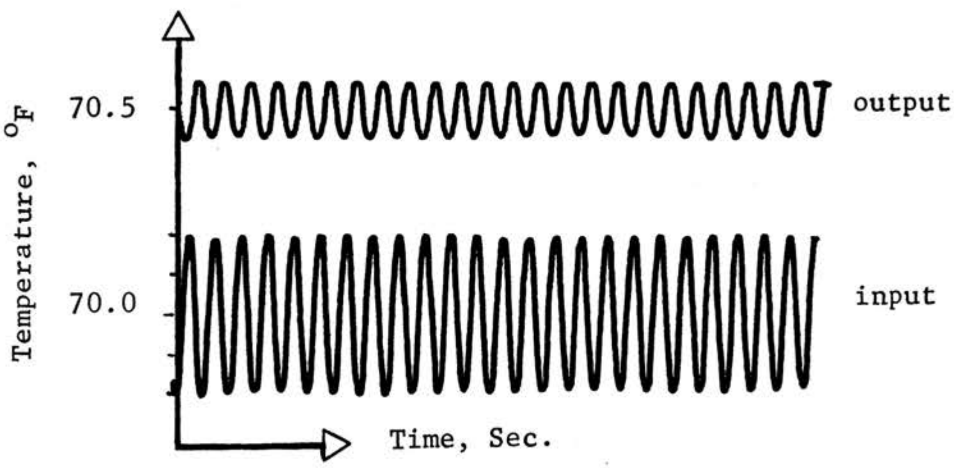


Figure 23. Proportional Controller, K = 500.

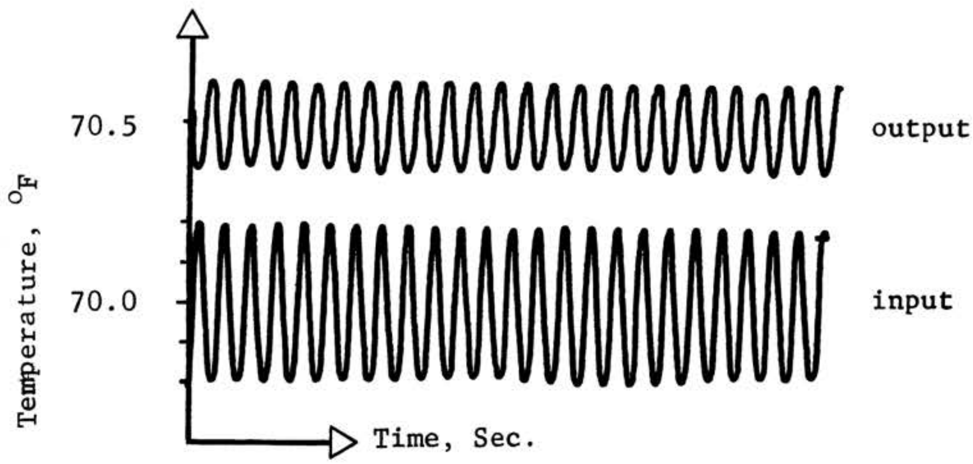


Figure 24. Proportional Plus Derivative Controller, K = 5, $\tau_1/\tau_2 = 1.5$

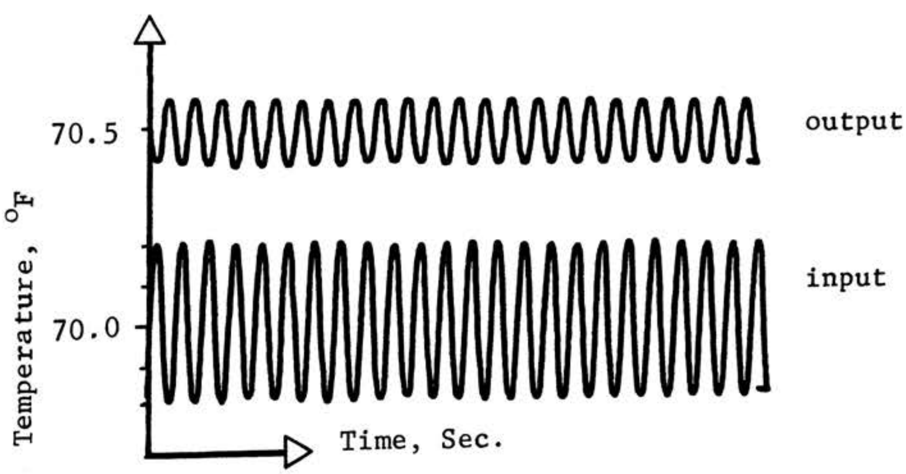


Figure 25. Proportional Plus Derivative Controller, K = 10, $\tau_1/\tau_2 = 1.5$

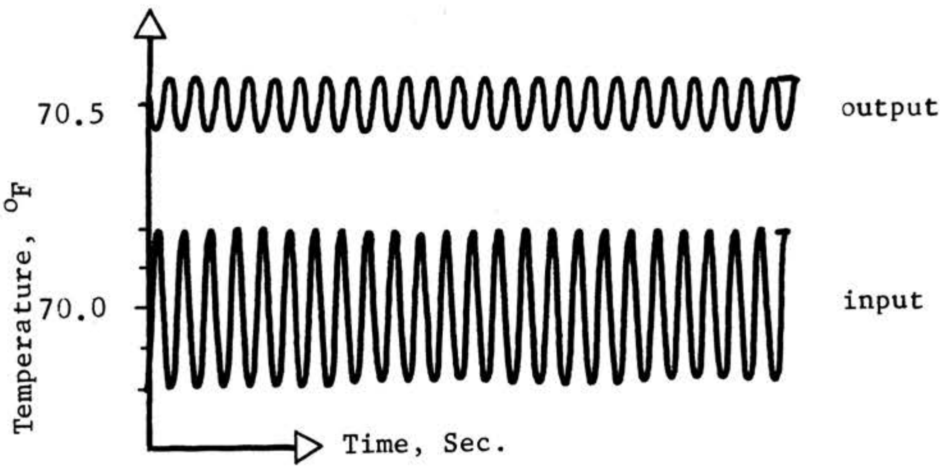


Figure 26. Proportional plus derivative controller, K = 50, $\tau_1/\tau_2 = 5$

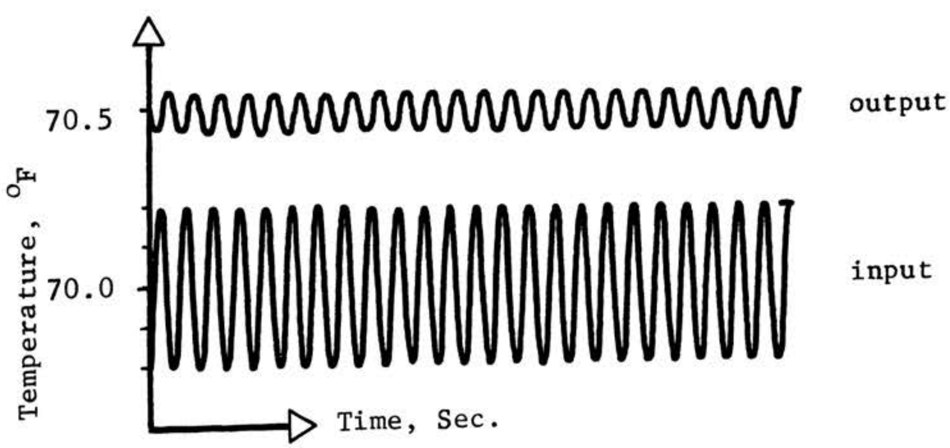


Figure 27. Proportional plus derivative controller, K = 50, $\tau_1/\tau_2 = 10$.

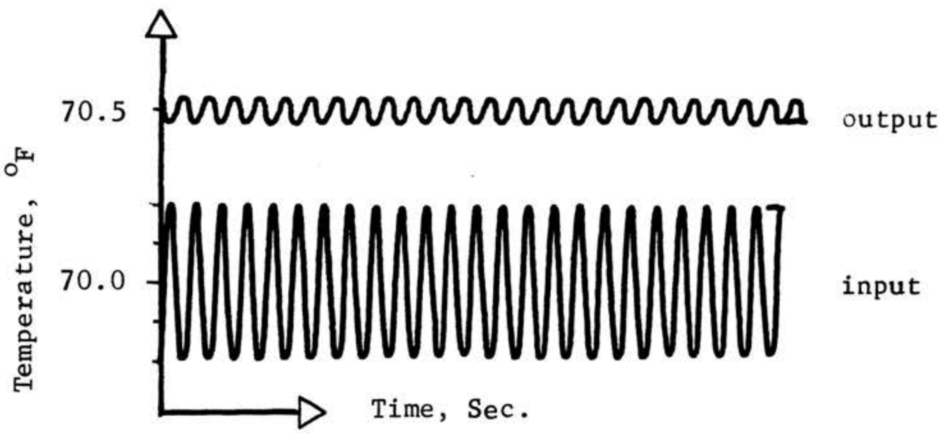


Figure 28. Proportional Plus Derivative Controller, K = 100, $\tau_1/\tau_2 = 10$

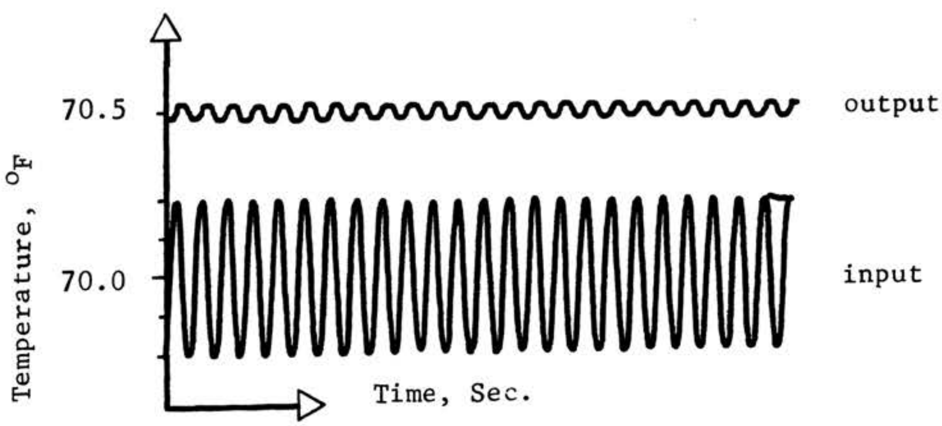


Figure 29. Proportional Plus Derivative Controller, K = 500, $\tau_1/\tau_2 = 10$

500 and with a $\tau_1/\tau_2$ ratio of 10, Figure 29 shows an output to input ratio fo 0.10, meaning that the temperature fluctuation leaving the system would be 0.04 $^oF$. This is considered as a reasonable value for the system and for the simulation chamber receiving the fluid.

The next test was to increase the size of the heating element to 2000 watts and test with both controllers. The gain for both tests was 100 and $\tau_1/\tau_2$ for the second test was 10. The results are shown in Figure 30 and 31. The ratio for the proportional controller is smaller, 0.32, and for the proportional plus derivative controller is the same as the previous test with a gain of 500.

The next series of tests were performed on the large tank capacitance system. The tests consist of running three tanks (Figure 5) of different sizes and observing the outflow temperature. The tanks were 2 ft x 2 ft x 2 ft, 2.5 ft x 2.5 ft x 2.5 ft and 3 ft x 3 ft x 3 ft. The results are shown in Figures 32, 33 and 34 and the output to input ratio was: 0.25, 0.20 and 0.13 respectively.

The final tests were to hook up the control system as shown in Figure 20 with heat input to the 2 ft x 2 ft x 2 ft tank. Two types of controllers were used, a proportional controller with a gain of 100 and a bang-bang controller (Figure 15). The results from the two test are shown in Figures 35 and 36. The output to input ratios were the same for each case with a value of 0.12.

Table I has been prepared for all the runs, defining all the

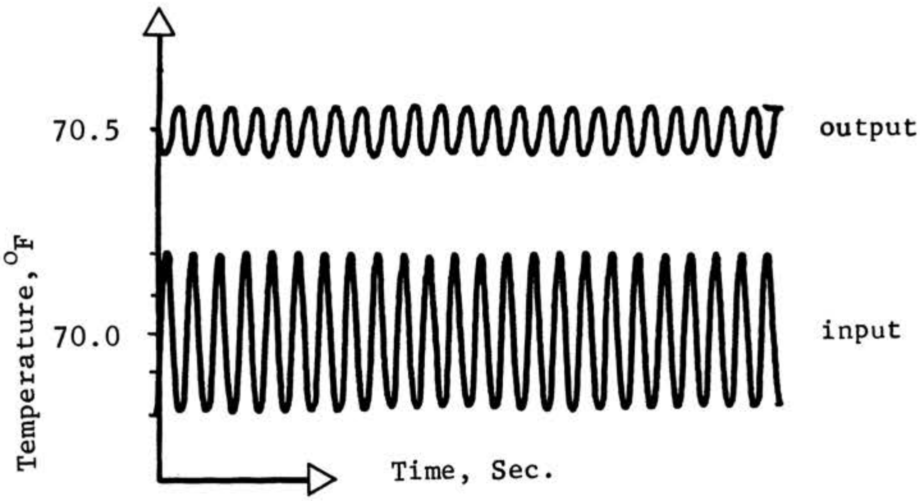


Figure 30, Proportional Controller, K = 100, 2000Watt

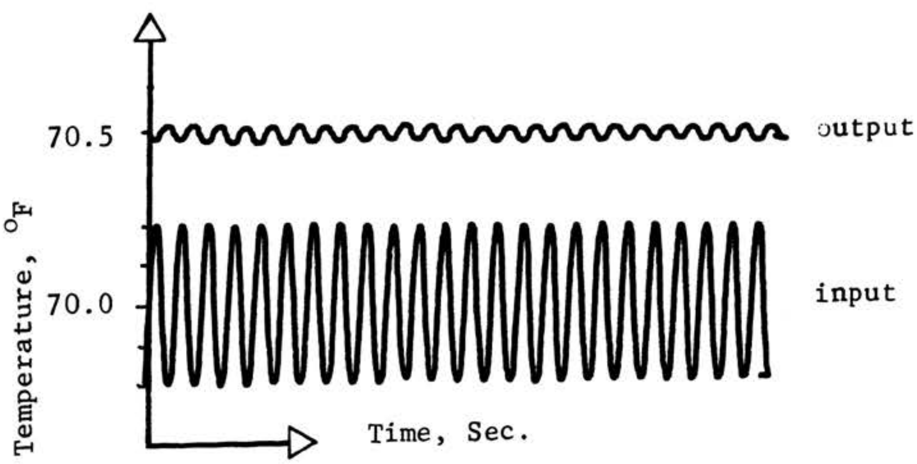


Figure 31. Proportional Plus Derivative Controller, K = 100
$\tau_1/\tau_2$ = 10, 2000 Watt.

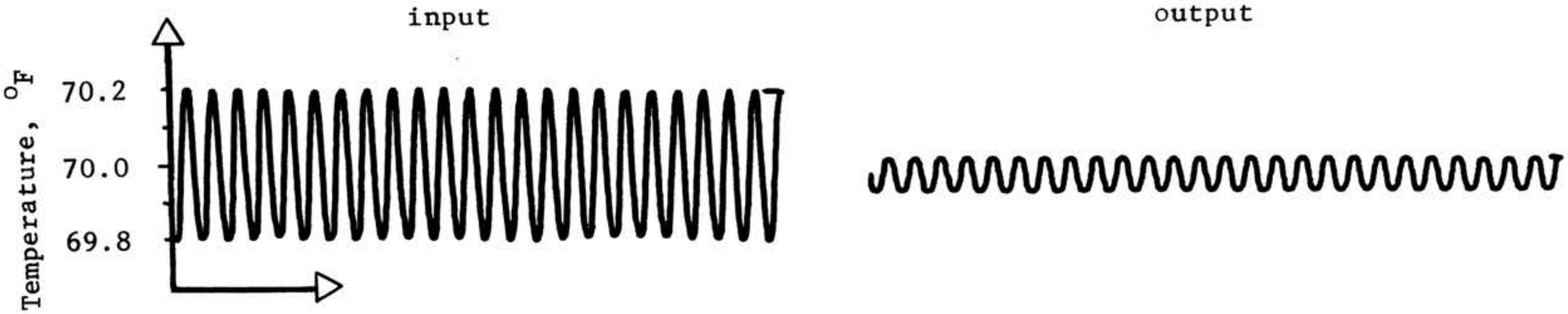


Figure 32. Single Lumped Tank, 500 lb.

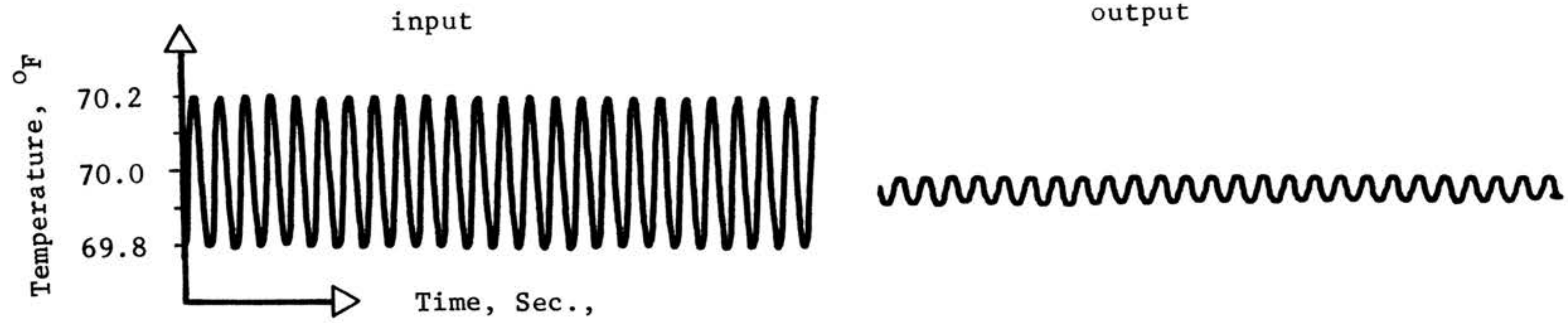


Figure 33. Single Lumped Tank, 1000 lb

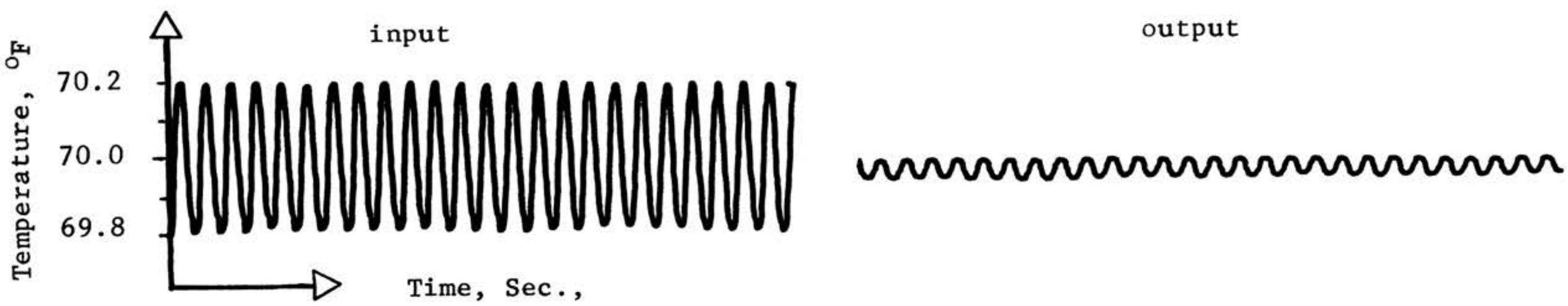


Figure 34. Single Lumped Tank, 1500 lb.

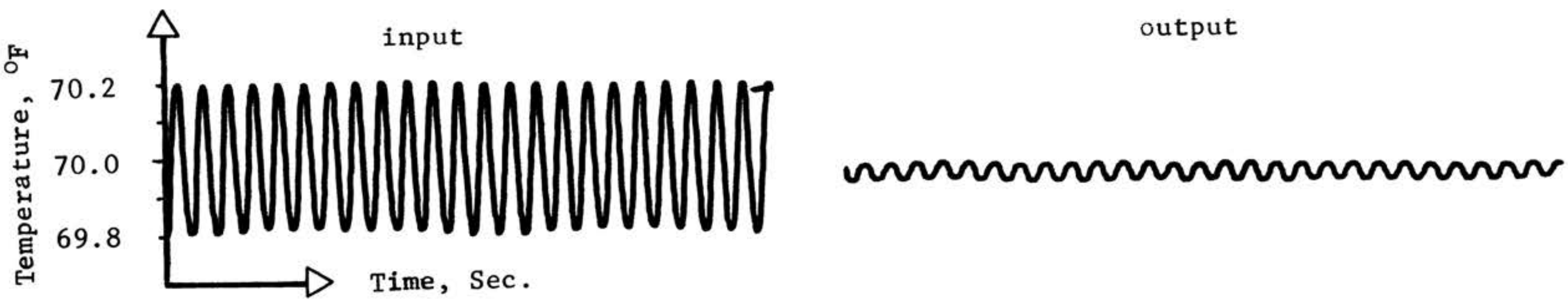


Figure 35. Single Lumped Tank, 500 lb; P. Controller, K = 100.

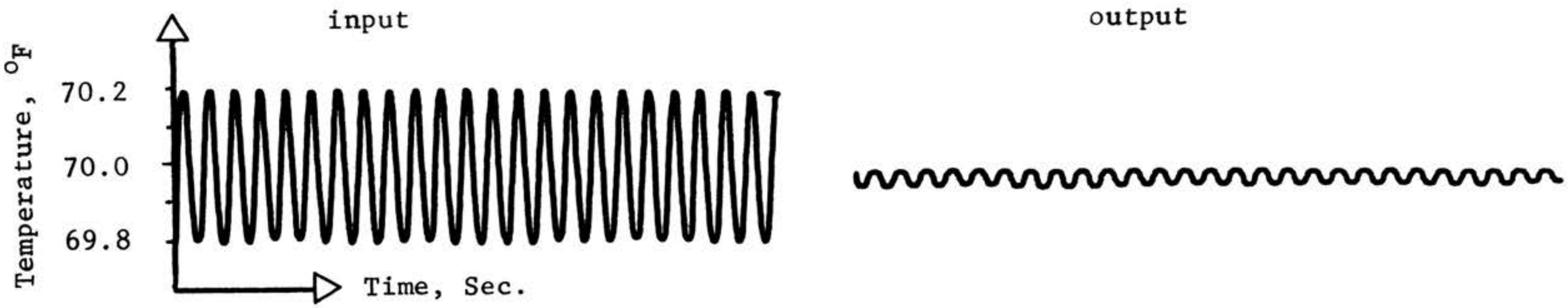


Figure 36. Single Lumped Tank, 500 lb; Bang-bang Controller.

controllers used with their appropriate gains and controller parameters with the output to input ratio for each of the cases.

Table I

Output to Input Amplitude Ratios for Different Configurations

| Run Number | Controller Type | Controller Parameters K | $\tau_1$ | $\tau_2$ | $\frac{\tau_1}{\tau_2}$ | Figure | Output-Input Amplitude Ratio |
|---|---|---|---|---|---|---|---|
| P1 | P | 10 | | | | F. 21 | .58 |
| P2 | P | 100 | | | | 22 | .40 |
| P3 | P | 500 | | | | 23 | .37 |
| PD1 | PD | 5 | .15 | .1 | 1.5 | 24 | .52 |
| PD2 | PD | 10 | .15 | .1 | 1.5 | 25 | .42 |
| PD3 | PD | 50 | 1. | .2 | 5. | 26 | .32 |
| PD4 | PD | 50 | 2. | .2 | 10. | 27 | .24 |
| PD5 | PD | 100 | 2. | .2 | 10. | 28 | .13 |
| PD6 | PD | 500 | 2. | .2 | 10. | 29 | .10 |
| P4* | P | 100 | | | | 30 | .32 |
| PD7* | PD | 100 | 2. | .2 | 10. | 31 | .10 |
| T1 | | | | | | 32 | .25 |
| T2 | | | | | | 33 | .20 |
| T3 | | | | | | 34 | .13 |
| TP | P | 100 | | | | 35 | .12 |
| TBB | BB | | | | | 36 | .12 |

* 2000 watt heating element

# VII. CONCLUSIONS

As indicated in a previous chapter a proportional controller did not produce results as good as was expected. The output to input ratio reached only a value of 0.37 with the 1000 watt heater of a high gain.

With a proportional plus derivative or a big tank, the output to input ratio reached values from 0.12 to 0.10. This would seen to be satisfactory and either system could be chosen.

A proportional plus derivative controller is more expensive and more complicated to operate but it does not take as much space and higher gains are still possible. Noise could be also become a problem in this system.

A tank is less expensive, very simple to operate but it takes a large amount of space, and a circulating pump is also required. The tank system will require the incorporation of a good stirring device and a large amount of cooling fluid will be required.

# REFERENCES


1. Cohen, W. C., and Johnson, E. F., "Dynamic Characteristics of Double Pipe Heat Exchangers", Industrial and Engineering Chemistry Vol. 48, 1956, pp. 1031-1034.

2. Paynter, H. M., "A New Method of Evaluating Dynamic Response of Counter Flow and Parallel Flow Heat Exchangers", ASME Trans., May 1956, pp. 749-758.

3. Lawson, D. I, "The Solution of Transient Heat Flow Problems by Analogous Electrical Networks", Proceeding of the Institution of Mechanical Engineers, A, 1953, 167, p. 275.

4. Malavard, L., "Electrical Analogies for Heat Transfer Problems", Engineers Digest, 1952, 13, p. 417.

5. Takahashi, Y., "Transfer Function Analysis of Heat Exchange Processes", Automatic and Manual Control, Butterworths, London, England, 1952, p. 235.

6. Ford, R. L., "Electrical Analogies for Heat Exchangers", Proceedings of IEE Journal, Jan. 1956, pp. 65-82.

7. Tivy, V. V. and Hainsworth, B. D., "Dynamic Analysis of Heat Exchanger Control". ISA Journal, June 1957, pp. 230-235.

8. Masubuchi, M., "Dynamic Response and Control of Multipass Heat Exchangers", ASME Journal of Basic Engineering, March 1960, pp. 51-65.

9. Schmidt, J. R., "Analog Simulation Techniques for Modeling Parallel Flow Heat Exchangers", Simulation Journal, Jan. 1969, pp. 15-21.

10. McAdams, W. H., "Heat Transmission", 3rd Edition, McGraw-Hill Book Company, Inc., New York, 1954, p. 219.

11. Brown and Marco, "Introduction to Heat Transfer", McGraw Hill Book Company, Inc., Third Edition, 1958, p. 302.

Vita

Le Thai Son, son of Mr. and Mrs. Le Van Ba was born in Saigon, Viet Nam, on July 18, 1948.

He attended Petrus Truong Vinh Ky High School in Saigon, Viet Nam and graduated in July 1966. In September, 1966 he enrolled in The National Mechanical Engineering School in PhuTho, Viet Nam and completed requirements for a Bachelor of Science degree in Mechanical Engineering in June 1970. He spent the next six months in industry and came to the States at the end of 1970. In January of 1971, he enrolled at the University of Missouri - Rolla as a graduate student in Mechanical Engineering.

He married Thoai Hoa and at present, they have one daughter, Ngoc Diep, two years old.

# APPENDIX

## TABLE II THROUGH XV

Table II

Operational Amplifier Assignments for Circuit in Figure 6

| Amplifier Number | Function | Output Variable |
|---|---|---|
| 05 | I | [$-T_1/5$] |
| 07 | SC | [$+T_1/5$] |
| 08 | SC | [$+ T_2$] |
| 09 | I | [$- T_2$] |
| 10 | I | [$- T_3$] |
| 11 | SC | [$+ T_{out}$] |
| 12 | SC | [$- T_{out}$] |
| 15 | SC | [$+ T_3$] |

I: integrator

SC: sign changer

Table III

Potentiometer Assignments for Circuit in Figure 6

| Pot. Number | Pot. Setting |
|---|---|
| 07 | 0.0265 |
| 08 | 0.19* |
| 09 | 0.958 |
| 10 | 0.09 |
| 12 | 0.0054 |
| 14 | 0.161 |
| 15 | 0.161 |
| 16 | 0.2* |
| 20 | 0.19* |
| 22 | 0.94 |

* gain of 10

Table IV

Amplifier Assignments for Circuit in Figure 8

| Amplifier Number | Function | Output Variable |
|---|---|---|
| 01 | I | [$- T_{11}/5$] |
| 02 | I | [$- T_{31}$] |
| 05 | I | [$-T_{21}$] |
| 06 | I | [$- T_{12}/5$] |
| 07 | SSC | [$T_{01}$] |
| 08 | SC | [$-T_{01}$] |
| 09 | I | [$- T_{22}$] |
| 10 | I | [$- T_{32}$] |
| 11 | SSC | [$T_{02}$] |
| 12 | SC | [$- T_{02}$] |
| 13 | I | [$- T_{23}$] |
| 14 | I | [$-T_{13}/5$] |
| 15 | SC | [$T_{11}/5$] |
| 16 | SC | [$T_{21}$] |
| 23 | SC | [$T_{22}$] |
| 24 | SC | [$T_{12}/5$] |
| 25 | SC | [$T_{32}$] |
| 26 | SC | [$T_{33}$] |

Table IV(continued)

| Amplifier Number | Function | Output variable |
|---|---|---|
| 27 | I | $[-\ T_{33}]$ |
| 29 | SSC | $[T_{out}]$ |
| 31 | SC | $[T_{23}]$ |
| 32 | SC | $[T_{13}/5]$ |
| 35 | SC | $[T_{31}]$ |
| 36 | SC | $[-\ T_{out}]$ |

Table V

Potentiometer Settings for Circuit in Figure 8

| Pot. Number | Pot. Setting |
|---|---|
| 01 | 0.0265 |
| 02 | 0.0054 |
| 03 | 0.0265 |
| 07 | 0.57* |
| 08 | 0.161 |
| 09 | 0.0054 |
| 10 | 0.958 |
| 14 | 0.57* |
| 16 | 0.958 |
| 17 | 0.57* |
| 19 | 0.57* |
| 20 | 0.958 |
| 21 | 0.161 |
| 22 | 0.09 |
| 23 | 0.161 |
| 24 | 0.161 |
| 25 | 0.94 |
| 26 | 0.94 |
| 32 | 0.09 |

Table V (continued)

| Pot. Number | Pot. Setting |
|---|---|
| 33 | 0.0265 |
| 34 | 0.2* |
| 35 | 0.09 |
| 36 | 0.94 |
| 37 | 0.2* |
| 38 | 0.161 |
| 39 | 0.161 |
| 40 | 0.2* |
| 41 | 0.0054 |
| 42 | 0.57* |
| 43 | 0.57* |

* gain of 10

Table VI

Amplifier Assignments for Circuit in Figure 10

| Amplifier Number | Function | Output variable |
| --- | --- | --- |
| 38 | SSC | [e(t)] |

Table VII

Potentiometer Settings for Circuit in Figure 10 for 3 Runs

| Run Number / Pot. Number | P1 | P2 | P3 |
|---|---|---|---|
| 27 | 1.0* | 1.0* * | 0.5*** |

Note: * gain of 10

* * gain of 100

*** gain of 1,000

Table VIII

Amplifier Assignments for Circuit in Figure 12

| Amplifier Number | Function | Output Variable |
|---|---|---|
| 18 | SC | - |
| 28 | I | - |
| 33 | SSC | e(t) |
| 37 | SSC | m(t) |
| 38 | SC | - |

Table IX

Potentiometer Settings for Circuit in Figure 12

| Pot \ Run | PD1 | PD2 | PD3 | PD4 | PD5 | PD6 |
|---|---|---|---|---|---|---|
| 27 | 1.* | 1.* | .5* | .5* | .5* | .5* |
| 29 | .15* | .15* | .5* | 1.* | 1.* | 1.* |
| 30 | 1.* | 1.* | .5* | .5* | .5* | .5* |
| 31 | .5* | 1.* | .5* | .5* | 1.* * | .5*** |

Note: * gain of 10

* * gain of 100

*** gain of 1,000

Table X

Amplifier Assignment for Circuit in Figure 14

| Amplifier Number | Function | Output Variable |
|---|---|---|
| 30 | SC | [x(t)] |
| 43 | I | [- x(t)] |
| 44 | I | [x(t)] |
| 45 | SSC | [$T_{in}$] |

Table XI

Potentiometer Settings for Circuit in Figure 14

| Pot. Number | Pot. Setting |
|---|---|
| 13 | 0.1 |
| 45 | 0.0436 |

Table XII

Amplifier Assignments for Circuit in Figure 16

| Amplifier Number | Function | Output Variable |
|---|---|---|
| 9 | I | $[-T_f]$ |
| 11 | SSC | $[T_{out}]$ |
| 12 | SC | $[-T_{out}]$ |
| 13 | I | $[-T_c]$ |
| 15 | SC | $[T_f]$ |
| 16 | SC | $[T_c]$ |

Table XIII

Potentiometer Setting for Circuit in Figure 16

| Run / Pot | T1 | T2 | T3 |
|---|---|---|---|
| 10 | .093 | .069 | .070 |
| 14 | .280* | .140* | .930 |
| 15 | .280* | .140* | .930 |
| 16 | .200* | .200* | .200* |
| 18 | .942 | .925 | .105* |
| 19 | .942 | .925 | .105* |
| 20 | .093 | .069 | .070 |

Note: * gain of 10

Table XIV

Amplifier Assignments for Circuit in Figure 20

| Amplifier Number | Function | Output Variable |
| --- | --- | --- |
| 05 | I | [- $T_1$/5] |
| 07 | SC | [$T_1$/5] |
| 08 | SC | [$T_f$] |
| 09 | I | [- $T_f$] |
| 10 | I | [- $T_c$] |
| 11 | SSC | [$T_{out}$] |
| 12 | SC | [- $T_{out}$] |
| 15 | SC | [$T_c$] |

Table XV

Potentiometer Settings for Circuit in Figure 20

| Pot. Number | Pot. Settings |
|---|---|
| 07 | .0265 |
| 08 | .280* |
| 09 | .093 |
| 10 | .0265 |
| 12 | .0054 |
| 14 | .942 |
| 15 | .942 |
| 16 | .200* |
| 20 | .280* |
| 22 | .093 |