1971

Reservoir design: simulation techniques

Larry W. Mays

160

RESERVOIR DESIGN: SIMULATION TECHNIQUES

by

LARRY WESLEY MAYS, 1948-

A THESIS

Presented to the Faculty of the Graduate School of the

UNIVERSITY OF MISSOURI-ROLLA

In Partial Fulfillment of the Requirements for the Degree

MASTER OF SCIENCE IN CIVIL ENGINEERING

1971

Approved by

J E Haubaugh (Advisor) G Stevens Jr

M. E. Aughenbaugh

PUBLICATION THESIS OPTION

This thesis has been prepared in the style utilized by the Journal of the Hydraulics Division, American Society of Civil Engineers. The Vita has been added for purposes normal to thesis writing. The simulation computer program is contained in the publication "Computer Aided Spillway Design - Hydrologic Routing," Hydrologic Series Bulletin, Civil Engineering Studies, University of Missouri-Rolla, Missouri, November 1971.

RESERVOIR DESIGN: SIMULATION TECHNIQUES

By Larry W. Mays[1], M. ASCE

KEY WORDS: computer, continuity equation; costs; numerical model; reservoirs; simulation; spillway design.

Abstract: A simulation model is developed to aid in the analysis of small dams to reduce the possibility of inadequate spillway designs. Basic assumptions concerning the geometric aspects of the natural reservoir are made to develop the model which is based upon describing the timing and magnitude of a design flood passing through a reservoir. Simulation equations are derived from the basic continuity equation and describe reservoir outflow and storage as functions of reservoir depth. Newton's Iteration Technique is utilized to solve the simulation equations for the reservoir depth. The resulting simulation model determines an optimum size auxiliary spillway having a minimum crest length for a range of spillway elevations. Estimated project cost equations are developed for an aid in the comparative analysis of alternative projects.

---

[1]Graduate Student, Civil Engineering Department, University of Missouri-Rolla, Rolla, Missouri 65401.

TABLE OF CONTENTS

Page

LIST OF FIGURES

LIST OF TABLES

INTRODUCTION

A rapid population growth in recent years has resulted in the construction of many new small lakes and reservoirs. For example, Missouri has more than 1500 lakes with surface areas greater than 5 acres, 410 of these have surface areas greater than 15 acres. During the last 10 years, 17 dam failures have occurred in Missouri, and of these, 10 were due to inadequate spillways which usually result from lack of engineering advice during the design of the dam (5).

An apparent need exists for a rapid method of analyzing small dams to aid in reducing the possibility of an inadequate spillway design. This method should also allow a comparative analysis of alternative spillway sizes. The purpose of this paper is to describe the development and application of a method of analysis for small reservoir spillway design. Basic hydrologic information such as design storm hydrographs, reservoir storage, hydraulic design coefficients, etc. are used with a simulation technique to develop the overall analysis of the spillway design. The result of the simulation procedure is an optimum spillway described as having a minimum crest length.

The spillway size is optimized within the simulation program by using either of two design criteria: a maximum allowable reservoir depth or a maximum allowable outflow. These criteria are satisfied by determining an emergency spillway having a minimum crest length for a range of emergency spillway elevations.

Cost is one of the important factors of reservoir design often difficult to estimate; therefore, a need exists to rapidly define and analyze the economic aspects. The analysis employed in this paper limits the variables to those having the most direct bearing on the variation of cost, such as reservoir capacity and physical size, and assuming that the optimum spillway size having a minimum crest length is the most economical design. Based upon regional data, cost estimates are developed to provide comparative values for the selected range of design alternatives.

BASIC ASSUMPTIONS

Reservoir Routing. - Routing is the technique used to determine the timing and magnitude of a flood wave in a stream or reservoir from known or assumed data. The location and capacity of reservoirs, and the size of outlet structures and spillways are determined by use of routing. Routing techniques are classified into two types: hydrologic routing and hydraulic routing. The hydraulic method requires the use of the equation of continuity and an equation of motion. Hydrologic routing on the other hand only requires the use of the equation of continuity and a relationship describing the storage and outflow. The hydrologic routing procedure is used for the simulation model in this investigation. The necessary relationship of stage and storage, and outflow are determined assuming a normal pool depth. The inflow instantaneously spreads evenly throughout the reservoir, thus maintaining a level surface.

Design Procedure. - Various design standards for small reservoir projects are used by state and federal agencies (11). For example the Missouri Department of Conservation uses a design policy based upon determining the freeboard for the auxiliary spillway and then routing a floodwave through the reservoir to determine the spillway size.

Two floods, a spillway and a surcharge flood, are employed when a principal (pipe) spillway is used in conjunction with an auxiliary (emergency) spillway. The surcharge flood, a 50 year frequency flood, is routed to determine the height of freeboard between the crest of the principal spillway and the crest of the auxiliary spillway. Assuming a normal pool elevation prior to the commencement of a flood wave, the spillway flood is then routed through the reservoir.

Modification of this approach will allow a wide range of alternatives to be explored. For example, assuming a range of possible crest elevations for the auxiliary spillway, the spillway geometry can be computed by routing a design flood through the reservoir. The spillway size can be optimized using either of two design criteria: by comparing the reservoir depth to a maximum allowable depth or by comparing the spillway outflow to a maximum allowable outflow. Several ranges of the auxiliary spillway crest elevation could be assumed to find the optimum spillway size for the chosen design criteria. The result would be an auxiliary spillway geometry having a minimum crest length for a range of spillway crest elevations. This procedure eliminates the need to calculate a surcharge flood since the maximum allowable depth in the reservoir can be controlled by the design criteria.

Continuity Equation. - The continuity equation is utilized to develop a relationship describing the reservoir outflow resulting from an upstream discharge hydrograph. The basic continuity equation is given as

$$I - 0 = \frac{ds}{dt} \qquad (1)$$

in which I = inflow to reservoir in cfs; 0 = outflow from reservoir in cfs; and $\frac{ds}{dt}$ = rate of change of storage within a reservoir. This equation can be expressed in finite difference form as

$$\frac{I_n+I_{n+1}}{2} - \frac{0_n+0_{n+1}}{2} = \frac{S_{n+1}-S_n}{DT} \qquad (2)$$

where $DT=t_{n+1}-t_n$. The n subscripted values are at a time t and the n+1 subscripted values are at a DT time increment later.

Generally to solve the above equation, it is assumed that I_n, I_{n+1}, 0_n, and S_n are known and 0_{n+1} and S_{n+1} must be determined. By substituting the known values into Eq. 2, the value of $S_{n+1} + \frac{1}{2}(0_{n+1})DT$ is calculated. Then the outflow 0_{n+1} can be obtained from a rating curve for the relationship between 0_{n+1} and $S_{n+1} + \frac{1}{2}(0_{n+1})DT$. The inflow is determined from a known or assumed upstream discharge hydrograph.

To solve the continuity equation using a simulation technique, the outflow and storage can be expressed as functions of water depth by describing reservoir geometry and assuming a normal pool elevation. A simulation equation can then be solved for the depth of water at each DT time increment of inflow.

NUMERICAL SIMULATION MODEL

Reservoir Geometry. - A typical reservoir can be divided into a number of prismatic sections, each being described by a length, bottom width, left-hand side slope and a right-hand side slope. The side slope of the reservoir cross-section is the slope noted as a rise of 1: vertical to ZHL or ZHR: horizontal. The cross-sectional geometry is assumed trapezoidal with the reservoir cross-section area given by

$$A = B(Y_n) + \frac{ZHL(Y_n)^2}{2} + \frac{ZHR(Y_n)^2}{2} \quad \text{.........................(3)}$$

in which B = bottom width, ft; Y_n = water depth, ft; ZHL = left side slope; ZHR = right side slope; and A = cross-sectional area, ft^2.

Storage Equation. - The relationships to express the storage in the continuity equation are functions of water depth Y_n for time t_n and Y_{n+1} for time t_{n+1} a DT time increment later. The unknown depth Y_{n+1} is used to express the storage S_{n+1}. Storage for a section at time t_n is represented by

$$S_n = L(B(Y_n-Z) + (\frac{ZHL+ZHR}{2})(Y_n-Z)^2) \quad \text{..............................(4)}$$

and the storage for a section at time t_{n+1} is

$$S_{n+1} = L(B(Y_{n+1}-Z) + (\frac{ZHL+ZHR}{2})(Y_{n+1}-Z)^2) \quad \text{........................(5)}$$

where Z accounts for the slope of the reservoir valley given by

$$Z = L(SO)/2 \quad \text{..(6)}$$

in which L = total length of reservoir in feet and SO = slope of the reservoir valley in feet per feet.

The reservoir used in this paper is described by five prismatic sections; see Figure 1. Thus the total reservoir storage at time t_n is given by

$$S_n = (Y_n-Z)(L_1B_1 + L_2B_2 + L_3B_3 + L_4B_4 + L_5B_5) + (Y_n-Z)^2$$

$$(L_1ZH_1 + L_2ZH_2 + L_3ZH_3 + L_4ZH_4 + L_5ZH_5) \quad \text{..................(7)}$$

and at time t_{n+1}

FIGURE 1. IDEALIZED RESERVOIR GEOMETRY FOR BLIND PONY LAKE

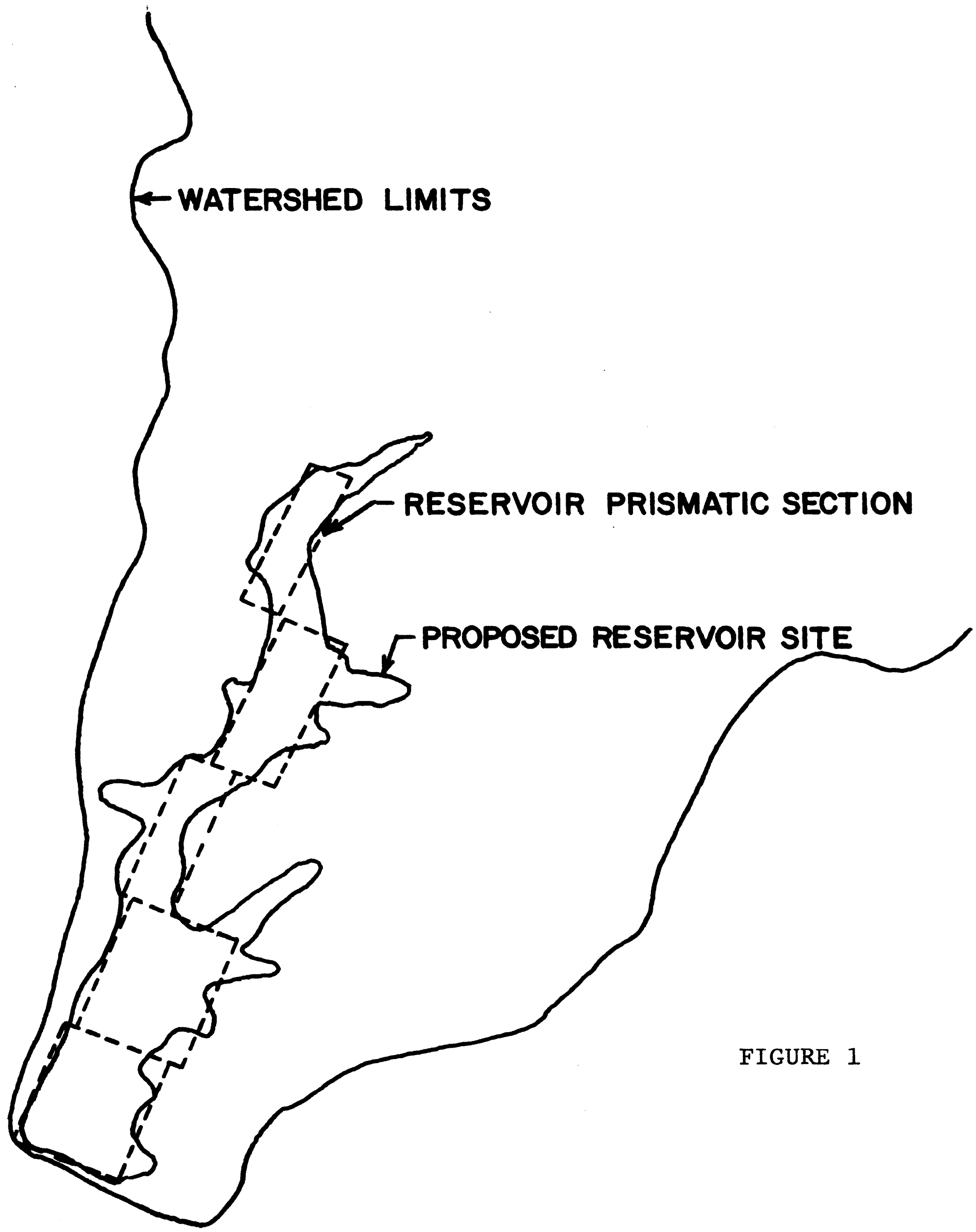

WATERSHED LIMITS
RESERVOIR PRISMATIC SECTION
PROPOSED RESERVOIR SITE

FIGURE 1

$$S_{n+1} = (Y_{n+1}-Z)(L_1B_1 + L_2B_2 + L_3B_3 + L_4B_4 + L_5B_5) + (Y_{n+1}-Z)^2$$

$$(L_1ZH_1 + L_2ZH_2 + L_3ZH_3 + L_4ZH_4 + L_5ZH_5) \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where average slope ZH_{k1}, k1=1,2,3,...denoting reservoir section, is

$$ZH_{k1} = \left(\frac{ZHL_{k1}+ZHR_{k1}}{2.}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Storage in the upper reaches of the reservoir is ignored since sediment deposits accumulate in this area and eventually reduces the storage to a negligible quantity (2). Only the prismatic portion of the reservoir is considered to contribute to the outflow for spillway design purposes.

Outflow Equations. - To express the outflow in the continuity equation, reference is given to the outflow equations presented by Orgosky and Mockus (10). Outflow spillways for small reservoir design generally include a principal (drop inlet or pipe spillway) and an auxiliary (overflow type) spillway. The outflow equation for the pipe spillway is given as

$$OF_{PRIN} = AP\left(\frac{2.(G)(Y_n-DEP)}{1.+CKE+CKB+CKP(LEN)}\right)^{0.5} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which OF_{PRIN} = discharge from the principal spillway, cfs; AP = cross-sectional area of pipe, ft^2; CKE = coefficient for entrance loss; CKB = coefficient for bend loss; CKP = coefficient for pipe friction loss; LEN = length of pipe for the principal spillway, ft; G = acceleration force due to gravity, ft/sec^2; and DEP = distance of invert of the pipe spillway above the reservoir floor, ft. The value of $(Y_n$-DEP) represents the total head in feet of water above the invert of the principal spillway.

The outflow from the auxiliary spillway is given by the overflow equation (8).

$$OF_{AUX} = C3(WSPL)(Y_n - HSPL)^{3/2} \quad (11)$$

in which OF_{AUX} = discharge from the auxiliary spillway, cfs; C3 = coefficient for discharge of the spillway crest; WSPL = width of auxiliary spillway, ft; and HSPL = elevation of the auxiliary spillway crest above the reservoir floor, ft.

Outflow Conditions. - Using the above relations for outflow and storage with an inflow hydrograph the continuity equation in finite difference form can be solve for the unknown depth at t_{n+1}. These equations must be developed for three different reservoir outflow conditions.

$$\text{I.} \quad Y_n \leq DEP \quad (12)$$

$$\text{II.} \quad DEP < Y_n \leq HSPL \quad (13)$$

$$\text{III.} \quad Y_n > HSPL \quad (14)$$

For condition I, the outflow is

$$OF = 0.0 \quad (15)$$

Since the water surface elevation is less than the elevation of the principal and auxiliary spillways.

For condition II, the outflow is

$$OF = AP\left(\frac{2.(G)}{1.+CKE+CKB+CKP(LEN)}\right)^{0.5}(Y_n - DEP)^{0.5} \quad (16)$$

where the elevation of the water surface is greater than the elevation of the principal spillway and less than the elevation of the auxiliary spillway.

For condition III, the outflow is

$$OF = AP\left(\frac{2.(G)}{1.+CKE+CKB+CKP(LEN)}\right)^{0.5}(Y_n-DEP)^{0.5}+C3(WSPL)$$

$$(Y_n-HSPL)^{1.5}\ldots\ldots(17)$$

where the elevation of the water surface is greater than the elevation of the auxiliary spillway crest.

Simulation Equations for Outflow Condition I. - The continuity equation for Outflow Condition I is given by

$$F1 - \frac{1}{DT}(Z4(Y_{n+1}-Z)+Z3(Y_{n+1}-Z)^2) = 0.0\ldots\ldots(18)$$

F1 represents the sum of all known variables at t_n, given by

$$F1 = \frac{I_n+I_{n+1}}{2.} + \frac{1}{DT}(Z4(Y_{n+1}-Z)+Z3(Y_n-Z)^2)\ldots\ldots(19)$$

where

$$Z4 = L_1B_1 + L_2B_2 + L_3B_3 + L_4B_4 + L_5B_5 \ldots\ldots(20)$$

$$Z3 = L_1(ZH_1)+L_2(ZH_2)+L_3(ZH_3)+L_4(ZH_4)+L_5(ZH_5)\ldots\ldots(21)$$

Equation 18 is nonlinear with respect to the unknown variable Y_{n+1}; consequently, Newton's Iteration Technique can be utilized to solve for the unknown depth. The general formula for Newton's Iteration Technique (3) is

$$Y_{k+1} = Y_k - \frac{F(Y_k)}{F'(Y_k)} \qquad (22)$$

where the subscript k denotes the number of iterations. This iterative equation applied successively with a known starting value Y_k will converge to a root of the nonlinear equation with the following error tolerance.

$$| Y_{k+1} - Y_k | \leq ER \qquad (23)$$

Rewriting Eq. 18 results in

$$Y2 = Y1 - (F1-1/DT(Z4(Y1-Z)+Z3(Y1-Z)^2)/(-1/DT(Z4+2.(Z3)(Y1-Z)) \qquad (24)$$

for Outflow Condition I. The initial approximation Y1 is taken as the normal pool depth before the flood commences flowing through the reservoir. Care must be taken to choose a small DT in relation to the total duration of the flood so that the change in depth between time intervals will be relatively small. The initial approximation of depth at each successive time increment is taken as the previously calculated value of Y1. This iterative procedure for Condition I is continued for DT increments of time until the reservoir surface exceeds the elevation of the principal spillway crest.

Simulation Equation for Outflow Condition II. - When the water surface elevation is greater than the elevation of the principal spillway and less than the elevation of the auxiliary spillway, the continuity equation that applies is

$$F2 - \frac{Z2}{2}(Y_{n+1}-DEP)^{0.5} - \frac{1}{DT}(Z4(Y_{n+1}-Z)+Z3(Y_{n+1}-Z)^2)=0.0\ldots\ldots(25)$$

F2 represents the sum of all known quantities at t_n, given by

$$F2 = \frac{I_n+I_{n+1}}{2} - \frac{Z2}{2}(Y_n-DEP)^{0.5} + \frac{1}{DT}(Z4(Y_n-Z)+Z3(Y_n-Z)^2)\ldots\ldots(26)$$

where

$$Z2 = AP\left(\frac{2.(G)}{1.+CKE+CKB+CKD(LEN)}\right)^{0.5}\ldots\ldots(27)$$

Equation 25 is nonlinear with respect to the unknown depth Y_{n+1}; therefore, Newton's Iteration Technique is again utilized. The iterative equation that results for Condition II is

$$Y2 = Y1 - ((F2 - \frac{Z2}{2}(Y1-DEP)^{0.5} - \frac{1}{DT}(Z4(Y1-Z)+Z3(Y1-Z)^2)/$$

$$(-\frac{Z2}{4}\left(\frac{1}{Y1-DEP}\right)^{0.5} - \frac{1}{DT}(Z4+2.(Z3)(Y1-Z)))\ldots\ldots(28)$$

This iterative procedure is continued until the reservoir surface exceeds the elevation of the assumed auxiliary spillway crest.

Simulation Equations for Outflow Condition III. - When the water surface elevation is greater than the elevation of the auxiliary spillway, the continuity equation that applies is expressed as

$$F3 - \frac{Z1}{2}(Y_{n+1}-DEP)^{0.5} - \frac{Z1}{2}(Y_{n+1}-HSPL)^{1.5} - \frac{1}{DT}(Z4(Y_{n+1}-Z) +$$

$$Z3(Y_{n+1}-Z)^2)=0.0\ldots\ldots(29)$$

F3 represents the sum of all known quantities at t_n given by

$$F3 = \frac{I_n + I_{n+1}}{2} - \frac{Z2}{2}(Y\text{-}DEP)^{0.5} - \frac{Z1}{2}(Y\text{-}HSPL)^{1.5} + \frac{1}{DT}(Z4(Y\text{-}Z)$$

$$+Z3(Y_n\text{-}Z)^2) \qquad (30)$$

where

$$Z1 = C3(WSPL) \qquad (31)$$

Equation 29 is nonlinear with respect to the unknown depth Y_{n+1}, and the general iterative equation results in

$$Y2 = Y1 - (-\frac{Z2}{2}(Y1\text{-}DEP)^{0.5} - \frac{Z1}{2}(Y1\text{-}HSPL)^{1.5} - \frac{1}{DT}(Z4(Y1\text{-}Z) +$$

$$Z3(Y1\text{-}Z)^2)+F1)/(-\frac{Z2}{4}(\frac{1}{Y1\text{-}DEP})^{0.5} - \frac{3.(Z1)}{4.}(Y1\text{-}HSPL)^{0.5} -$$

$$\frac{1}{DT}(Z4+2.(Z3)(Y1\text{-}Z))) \qquad (32)$$

COMPUTER PROGRAM DESCRIPTION

A computational scheme of solving for the reservoir depths using the iterative equations that apply for the outflow conditions can be utilized in a design procedure. Application of this procedure in the optimization of spillway sizes results in a lengthy hand computation; consequently, the numerical simulation model has been programmed on an IBM 360/50 computer (8). This simulation program optimizes the auxiliary spillway size of a proposed reservoir site by use of the iterative Eqs. 24, 28, and 32 to solve for the water depths of an imposed design flood.

The design flood is introduced at the upstream boundary of the reservoir assuming no lateral inflow and a normal pool elevation. As the floodwave proceeds through the reservoir the inflow is assumed to instantaneously spread evenly throughout the reservoir surface. The increased water depth causes an increased spillway head. This results in an increased outflow and depth over each interval of time DT until peak inflow occurs at which time the inflow decreases however outflow continues to increase.

For each interval of time, the reservoir depth or outflow is compared with a maximum depth or maximum outflow, respectively. Either or both design criteria can be utilized. If the depth or outflow exceeds the maximum allowable, the spillway dimensions are changed accordingly and the floodwave computations are again performed for the changed spillway dimensions. The spillway crest elevation is changed, decreased if maximum depth is the design criteria and increased if maximum outflow is the criteria, over a range of possible elevations. If this range of crest elevations fail to provide an adequate spillway, the crest length is changed, increased if maximum depth is the design criteria and decreased if maximum outflow is the design criteria. This procedure results in an auxiliary spillway geometry having a minimum crest length for a range of spillway crest elevations. Several other ranges of crest elevations and crest lengths can be assumed in the program to find the optimum design.

The upstream boundary condition of a reservoir considered for design is a discharge hydrograph determined as a function of time. Considering continuity, the upstream relationship is given by

$$I = A(t)v(t) \text{..} (33)$$

An accurate means of simulating the design inflow hydrograph is essential to properly compute the reservoir depth during each interval of time. This can be accomplished by describing the inflow as step functions for gaged areas or by using Gray's Synthetic Unit Hydrograph Technique for ungaged areas (4).

VERIFICATION OF NUMERICAL SIMULATION MODEL

Comparison with Blind Pony Lake Project. - In order to verify the numerical simulation model a comparative analysis of the model to an actual reservoir design situation was made. The reservoir is known as the Blind Pony Creek Project, Saline County, Missouri (6). This is a homogeneous earth fill dam which impounds approximately 210 acres of water over a reservoir length of approximately 8000 feet. The spillways consist of an asbestos-cement conduit with a concrete box-drop-inlet spillway plus a grass emergency spillway. The watershed lying above the proposed dam site is 5.03 square miles and the length of the longest watercourse within the drainage area is approximately 19,000 feet.

For a comparative analysis the design inflow hydrograph, Maximum Probable Flood, used by the Missouri Conservation Department is described as step-functions. The design inflow hydrograph and the resulting outflow hydrograph using the simulation model are shown in Fig. 2. Using the numerical simulation model, the optimum design for the criteria of a 31. ft. maximum depth results in an auxiliary spillway crest elevation of 26.0 feet above the reservoir floor at the dam site and a spillway crest length of 250.0 feet. The peak outflow is reduced to 7946 cfs and occurs 73 minutes after the time of peak inflow. Use of the simulation model for the spillway design of Blind Pony Lake has comparable results to the Conservation Department's design. Their spillway crest elevation was 27.0 feet above the reservoir floor as compared to 26.0 feet using the simulation model and both designs resulted in a spillway crest length of 250.0 feet.

Shape of Inflow Hydrograph. - Changing the shape of the inflow hydrograph, by shifting the time to peak TPINF to one-half TPINF results in the inflow and outflow hydrographs shown in Fig. 3. Using the same design criteria as in the previous routing, the optimum spillway design has an auxiliary spillway crest elevation of 26.5 feet above the reservoir floor at the dam and a spillway crest length of 250.0 feet. The peak outflow is reduced to 7056 cfs and occurs 119 minutes after the time of peak inflow.

The effect of shifting the time to peak inflow is further shown by increasing TPINF to one-half the original time to peak. The inflow and outflow hydrographs are shown in Fig. 4. Using the same design criteria, the optimum auxiliary spillway design does not change from that of the original inflow hydrograph; however, the peak outflow is 8601 cfs and occurs 37 minutes after the time of peak inflow. Shifting the time to peak does have a definite effect upon the outflow hydrograph, but has little effect upon the spillway design as shown in Table 1.

Design Inflow Hydrographs by Combining Gray's Synthetic Unit Hydrograph Method and the Critical Storm Method. - Design flood hydrographs used by the Missouri Department of Conservation are developed by constructing a theoretical unit hydrograph and then determining the runoff for successive intervals using the SCS runoff equations. The flood hydrograph is developed for the "Maximum Probable Flood, Assumption A". Harbaugh (12) has presented the Critical Storm Method which utilizes a synthetic unit hydrograph and a critical storm pattern developed from the most critical sequence of rainfall excess pattern.

For the purposes of this investigation, the synthetic unit hydrograph is developed by Gray's Method (4) which is based upon dimensionalizing an incomplete gamma distribution. This method for generating synthetic unit hydrographs for midwestern watersheds is convenient for ungaged areas. A 13.05 inch runoff over a 24 hour period is used by the Missouri Conservation Department for the Blind Pony Lake project. The synthetic unit hydrograph using Gray's Method has a 9.28 minute duration. Using this unit hydrograph, a six hour duration unit hydrograph is developed by use of the S-Curve Method. The six hour duration is chosen because 96 percent of the cumulative storm occurs in this time for a runoff period of 24 hours when a 10% probability level is assumed for a first-quartile storm. This 10% probability level is interpreted as the distribution typical of an intense, prefrontal squall line such as thunderstorms (12). The rainfall excess pattern of the 10% probability level for the 13.05 inches of runoff is applied to the six hour unit hydrograph to develop the storm hydrograph.

Using the above procedure results in a peak inflow of 10,400 cfs for the Blind Pony Lake Project. The 31.0 foot maximum depth-design criteria used in the simulation model produces a design crest elevation of 25.5 feet and a design crest length of 250.0 feet. This design

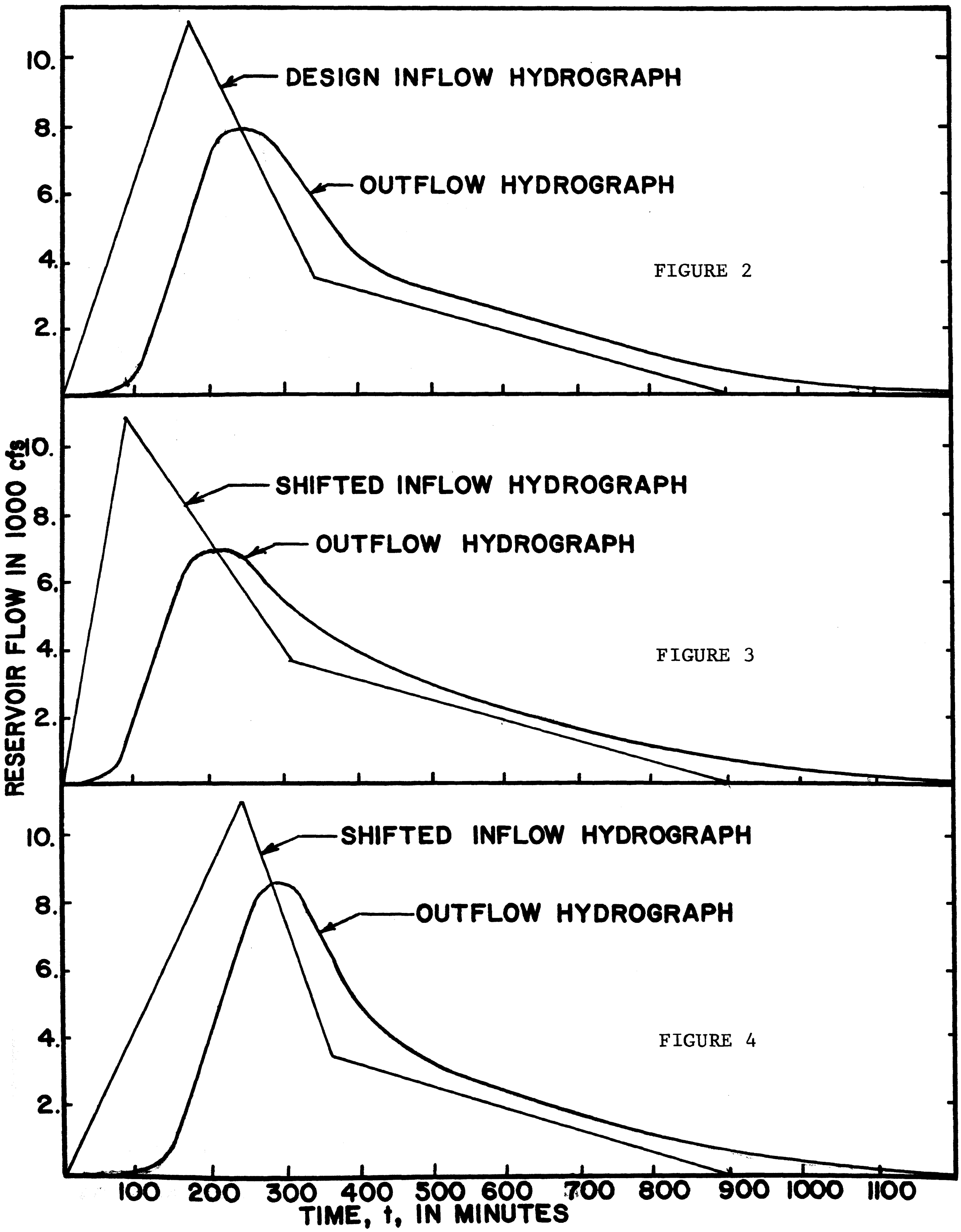
TABLE 1. EFFECT INFLOW HYDROGRAPHS HAVE ON SPILLWAY DESIGN

DESIGN INFLOW HYDROGRAPH METHOD	TIME OF PEAK INFLOW (min)	PEAK INFLOW (cfs)	TIME TO PEAK OUTFLOW (min)	PEAK OUTFLOW (cfs)	WSPL (ft)	HSPL (ft)	MAXIMUM DEPTH (ft)
Maximum Probable Flood (shifted peak inflow)	81	11000	200	7056	250	26.5	30.85
Maximum Probable Flood	162	11000	235	7946	250	26.0	30.71
Maximum Probable Flood (shifted peak inflow)	243	11000	280	8601	250	26.0	30.97
Critical Storm Method	90	10400	280	8893	250	25.5	30.58

FIGURE 2. DESIGN INFLOW HYDROGRAPH USED BY MISSOURI DEPARTMENT OF CONSERVATION AND RESULTING OUTFLOW HYDROGRAPH USING NUMERICAL SIMULATION MODEL (TPINF = 162. MINUTES)

FIGURE 3. SHIFTED INFLOW HYDROGRAPH AND RESULTING OUTFLOW HYDROGRAPH USING NUMERICAL SIMULATION MODEL (TPINF = 81. MINUTES)

FIGURE 4. SHIFTED INFLOW HYDROGRAPH AND RESULTING OUTFLOW HYDROGRAPH USING NUMERICAL SIMULATION MODEL (TPINF = 243.)


DESIGN INFLOW HYDROGRAPH
OUTFLOW HYDROGRAPH
FIGURE 2
SHIFTED INFLOW HYDROGRAPH
OUTFLOW HYDROGRAPH
FIGURE 3
SHIFTED INFLOW HYDROGRAPH
OUTFLOW HYDROGRAPH
FIGURE 4
RESERVOIR FLOW IN 1000 cfs
10.
8.
6.
4.
2.
100 200 300 400 500 600 700 800 900 1000 1100
TIME, t, IN MINUTES

differs by only 0.5 feet in the crest elevation when compared with the simulation design using the Maximum Probable Flood of the Missouri Department of Conservation. Therefore, combining Gray's Synthetic Unit Hydrograph and the Critical Storm Method has very good correlation with the method of assuming a Maximum Probable Flood, Assumption A.

COST ANALYSIS

To aid the designer in the comparative analysis of alternative projects, estimated costs are normally used as a parameter. For cost estimating purposes an analysis can be limited to those variables having the greatest effect on cost, i.e. reservoir capacity and size. The study made at the Illinois State Water Survey by Dawes (1) on cost of various elements of water-resource development has been adapted to the following analysis.

The relationship of lake surface area and storage capacity vary with physiographic conditions; consequently, this relationship must be modified for different geographic areas. A regression equation for lake surface area versus storage in Missouri can be shown by

$$L_a = 0.483(\mathrm{STORAF})^{0.87} \qquad (34)$$

in which L_a = lake surface area, acres and STORAF = reservoir storage, acre-ft. This equation was developed from cost data for the Blind Pony Lake Project.

The factors involved in the analysis of project cost are construction cost, land cost, engineering services, and contingencies. Using the above expression for lake surface area, the costs for Missouri projects are estimated as a sum of the above factors which results in

TABLE 2. ESTIMATED COSTS FOR THREE MISSOURI PROJECTS

	PROJECT	SURFACE WATER AREA (Acres)	AVERAGE OF BIDS ($)	LAND ACQUISITION ($)	TOTAL PROJECT COST ($)
I.	Perry Co. Lake	103	229,000.	123,000.	352,000.
II.	Binder Lake	150	236,000.	180,000.	416,000.
III.	Blind Pony Lake	210	204,000.*	244,000.	448,000.

* No bids are available from the Missouri Conservation Department for this project. This figure is the amount spent by the Department excluding land acquisition and adding 10% engineering contingency costs.

FIGURE 5. PROJECT COSTS VERSUS STORAGE CAPACITY IN MISSOURI FOR LAND VALUE OF $400 PER ACRE

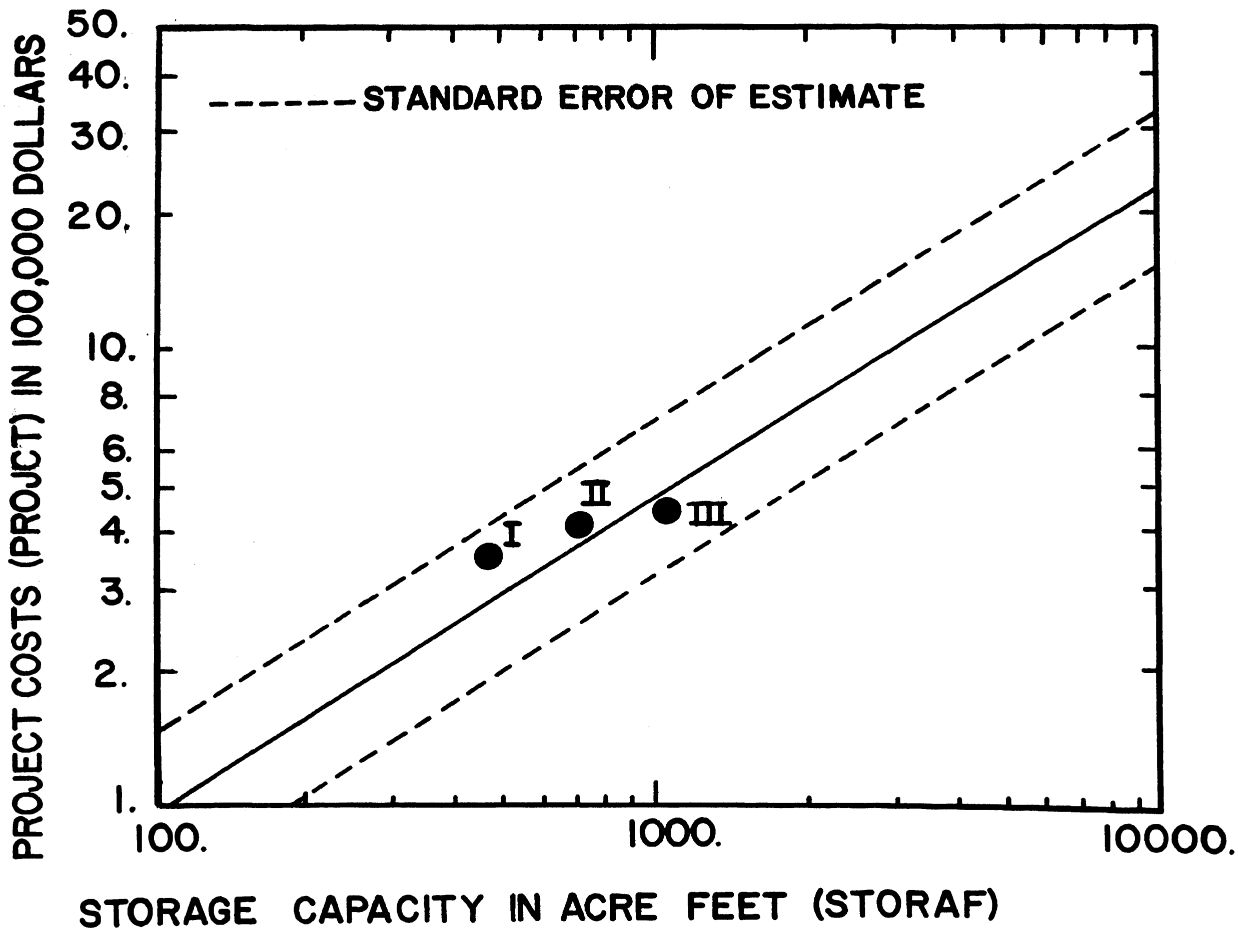

50.
40.
30.
20.
10.
8.
6.
5.
4.
3.
2.
1.
PROJECT COSTS (PROJECT) IN 100,000 DOLLARS
------STANDARD ERROR OF ESTIMATE
I
II
III
100.
1000.
10000.
STORAGE CAPACITY IN ACRE FEET (STORAF)

FIGURE 5

$$\text{PROJCT} = 5360.(\text{STORAF})^{0.54} + 1.45(\text{PRAC})(\text{STORAF})^{0.87} \quad \ldots\ldots\ldots(35)$$

in which PROJCT = estimated project cost in dollars and PRAC = project land value per acre in dollars. Incorporated in the above equation is the assumption used by the Missouri Department of Conservation that the required land to meet project objectives is three times the normal pool surface area. Equation 35 is plotted in Fig. 5, the standard error of estimate is assumed to be the same as that used for the Illinois data (1).

Three projects, listed in Table 2, are used to establish the cost equation for Missouri. The only available costs for the Perry County Lake and the Binder Lake were itemized bids which excluded land acquisition. Estimated costs for these projects are the averages of the bids plus land acquisition costs. Discrepancies in this analysis are apparent because the bid prices varied as much as 72 percent. Bid prices are not available for the Blind Pony Lake, but records indicate Conservation Department expenditures were equal to 448,000.00 dollars for the completed project. The cost relationships of these projects are plotted in Fig. 5.

SUMMARY AND CONCLUSION

A simulation model has been developed to aid in the design of small dams to alleviate the possibility of inadequate spillways. This simulation model is based upon describing the timing and magnitude of a floodwave passing through the reservoir to optimize the auxiliary spillway size. Simulation equations are presented which have been derived from the basic continuity equation by representing the three outflow conditions and the storage as functions of depth. Newton's Iteration Technique is employed to solve the simulation equations for the reservoir depth at each time interval. The spillway size is optimized using either of two design criteria to determine an auxiliary spillway having a minimum crest length for a range of spillway crest elevations.

Another numerical simulation model (9) which considers the transient nature of reservoir flow has been developed and computer programmed. Derivation of this model involves solving the Saint Venant unsteady flow equations using the Method of Characteristics. This method involves much more sophisticated mathematics and a greater knowledge of unsteady flow. The computer program for the Method of Characteristics requires approximately 300k of storage as compared to 105k for the hydrologic model discussed in this paper. Also, the computer time was 3 to 4 times greater for the Method of Characteristics.

A comparative analysis of the models accuracy to optimize the spillway size of an actual reservoir situation has indicated excellent results. Use of the simulation model on the Blind Pony Creek Project provided an auxiliary spillway differing by only one foot in crest elevation as compared to the Missouri Conservation Department's design. The effect of shifting the shape of the design inflow hydrograph by decreasing and then increasing the time to peak inflow showed a decreased and increased peak outflow, respectively. Differences between time to peak inflow and time to peak outflow decreased as TPINF was increased. The change in shape of the inflow hydrograph showed only a minor effect upon the optimized spillway size.

The equation given for estimated project costs in Missouri is to be used only as a parameter in the comparison of alternatives. This cost equation has been modified from the Illinois data to meet the requirements for land acquisition by the Missouri Department of Conservation and to conform to the data available for the Blind Pony Project. Costs for the three projects, listed in Table 1 resulted in close correlation to the derived cost equation for Missouri. The optimum spillway design having a minimum crest length for a range of spillway elevations is assumed to be the most economical spillway size.

A method of applying Gray's Synthetic Unit Hydrograph to the Critical Storm Method has been presented. The 10% probability level for a first-quartile storm has been used as the most critical sequence of rainfall excess pattern. Applying the total runoff and rainfall excess pattern to the unit hydrograph resulted in a storm hydrograph comparable to the Maximum Probable Flood, Assumption A.

ACKNOWLEDGEMENTS

The writer would like to express special appreciation to Dr. T.E. Harbaugh of the Civil Engineering faculty, University of Missouri-Rolla for his advice concerning the research reported herein and for his extensive review of the paper.

APPENDIX I - REFERENCES

1. Dawes, Julius, H., "Tools for Water-Resource Study," Journal of the Irrigation and Drainage Division, ASCE, Vol. 96, No. IR4, Proc. Paper 7720, December 1970, pp. 403-408.

2. Fread, D.L. and T.E. Harbaugh, "Simulation Program for the Transient Hydraulics Produced by Gradually Breached Earth Dams," Hydraulic Series Bulletin, Civil Engineering Department, University of Missouri-Rolla, Rolla, Missouri, May 1971.

3. Fread, D.L. and T.E. Harbaugh, "Gradually Varied Flow Profiles by Newton's Iteration Technique," Journal Hydrology, Vol. 13, 1971, pp. 70-80.

4. Gray, D.M., "Synthetic Unit Hydrographs for Small Watersheds," Journal of the Hydraulics Division, ASCE, Vol. 87, No. HY4, Proc. Paper, July 1961, pp. 33-54.

5. Harbaugh, T.E. and Thompson, J.E., "Spillway Design Floods for Small Dams in Rural Missouri," Hydrologic Series Bulletin, Civil Engineering Studies, University of Missouri-Rolla, Rolla, Missouri, June 1970.

6. Hooker, C.E. and Henson, D.L., "Proposed Lake Site, Pony Creek - Saline County," Engineering Feasibility Report, Job No. 65-12, Engineering Section, Missouri Department of Conservation, Jefferson City, Missouri.

7. Linsley, R.K., Kohler, M.A., and Paulhus, H., Hydrology for Engineers, McGraw-Hill Book Co., New York, 1958.

8. Mays, L.W. and T.E. Harbaugh, "Computer Aided Spillway Design - Hydrologic Routing," Hydrologic Series Bulletin, Civil Engineering Studies, University of Missouri-Rolla, Rolla, Missouri, November 1971.

9. Mays, L.W. and T.E. Harbaugh, "Computer Aid Spillway Design - Hydraulic Routing," Hydrologic Series Bulletin, Civil Engineering Studies, University of Missouri-Rolla, Rolla, Missouri, November 1971.

10. Ogrosky, Harold O. and Victor Mockus, "Hydrology of Agricultural Lands," pp. 21-63, Handbook of Applied Hydrology, Edited by Ven T. Chow, McGraw-Hill Book Co., New York, 1964.

11. U.S. Bureau of Reclamation: Design of Small Dams, 1960.

12. Viesman, W., Harbaugh, T.E. and Knapp, J.W., Introduction To Hydrology, Pending Publication, 1971.

APPENDIX II - NOTATION

The following symbols are used in this paper:

A = Area of reservoir cross-section

AP = Cross-sectional area of pipe spillway

$B, B_1, B_2, B_3, B_4, B_5$ = Bottom widths of cross-sections describing reservoir geometry

CKB = Coefficient for bend loss in pipe spillway

CKE = Coefficient for entrance loss in pipe spillway

CKP = Coefficient for pipe friction loss in pipe spillway

C3 = Coefficient for discharge from auxiliary spillway

DEP = Elevation of the invert of the pipe spillway above the reservoir floor at the dam

DT = Time increment

ER = Error tolerance for Newton's Iteration Technique

F1 = Sum of known variables in simulation equation

F2 = Sum of known variables in simulation equation

F3 = Sum of known variables in simulation equation

G = Acceleration constant due to force of gravity

HSPL = Elevation of the auxiliary crest above the reservoir floor

I = Inflow to reservoir

K1 = Subscript denoting a prismatic section

K = Subscript denoting number of iterations

L = Total length of reservoir

L_a = Lake surface area

LEN = Length of pipe spillway

L_1, L_2, L_3, L_4, L_5 = Lengths of prismatic sections of the reservoir

n,n+1 = Subscripts denoting time

O = Outflow from reservoir

OF_{AUX} = Discharge from auxiliary spillway

OF_{PRIN} = Discharge from principal spillway

PRAC = Project land value per acre

PROJCT = Total project cost

S = Storage within reservoir

SO = Slope of reservoir valley

STORAF = Reservoir storage in acre - feet

t = Time

TPINF = Time to peak of design inflow hydrograph

WSPL = Crest length of auxiliary spillway

Y = Depth of reservoir at dam

Y1,Y2 = Iterative depths used in simulation Eqs.

Z = Constant to account for effect of slope on depth given by Eq.

ZHL = Left-hand side slope of reservoir cross-section

ZHR = Right-hand side slope of reservoir cross-section

$ZH_1, ZH_2, ZH_3, ZH_4, ZH_5$ = Average of left side slope and right side slope of prismatic sections

Z1 = Constant given by Eq. 31

Z2 = Constant given by Eq. 27

Z3 = Constant given by Eq. 21

Z4 = Constant given by Eq. 20

VITA

Larry Wesley Mays was born on February 7, 1948, in Pittsfield, Illinois. He received his primary and secondary education in Pittsfield, completing his high school education in 1966. He attended the University of Missouri-Rolla, Rolla, Missouri where he received his Bachelor of Science in Civil Engineering in 1970.

In June 1970, he accepted a career-conditional appointment with the Water Resources Division of the U.S. Geological Survey as a hydraulics engineer. He remained in this position until August 1970, when he was inducted into the U.S. Army. Since his induction he has been stationed at Fort Leonard Wood, Missouri while attending part time graduate school at the University of Missouri-Rolla.

He is an associate member of the American Society of Civil Engineers and a member of Chi Epsilon.