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CONTROL ALGORITHM FOR FEEDBACK ACTIVE CONTROLLED STRUCTURES

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Abstract. Linear quadratic regulator (LQR) is an effective method of feedback active control theory. However, the LQR control is not truly optimal because it is only a feedback algorithm, i.e. the external excitation term is ignored in the optimal equation. In a previous paper [1], the identification algorithm is presented for feedback active controlled systems to identify the excitation from the structural response measured. The aim of this paper is to propose a feedback-feedforward control algorithm using the identified excitation to improve the classical LQR control. A numerical simulation is applied to an eight story building subjected to base acceleration and controlled by active mass damper system.

1. INTRODUCTION

In recent years, much progress and new concepts have been achieved in reducing the structure response due to environmental and man-made loading. Among those innovative means, passive damping and active control systems represent fundamental approaches for response reduction in structures. Passive damping systems encompass a range of devices, which are characterized by a capability to enhance energy dissipation in the structure. This effect may be achieved by using devices which operate by conversion of kinetic energy to heat such as friction dampers, viscous dampers ... or by transferring of energy among vibrating modes such as tuned mass dampers or tuned liquid dampers. Passive devices, however, have inherent limitations which can be solved by active control systems. For example, the passive tuned mass damper is often tuned to the first natural frequency of the structure, means that it is designed to reduce only the first mode vibration. An active mass damper, on the other hand, can be effective over a much wider frequency range. Hence, the study of active control is a logical extension of passive control technology. Active control systems are force delivery devices integrated with real-time processing controllers and sensors within the structure. When only the responses can be measured, the method is called *feedback* or *closed loop* control. A *feedforward* or *open loop* control results when the control force are regulated only by the measured excitation. In the case where the both information of excitations and responses are utilized for controller design, the term *feedback-feedforward* or *closed-open loop* control is used. Many control strategies have been proposed, such as LQR/LQG control [2], H_2/H_∞ control [3, 4], sliding mode control [5], fuzzy control [6], neural control [7]... We also proposed some control algorithms applying to structures, in which some components of excitation can be known [8] or the control forces are bounded [9] or the number of sensor is limited [10]. As it has been observed in many papers the well-known linear quadratic regulator (LQR) control is one of very effective methods to attenuate undesired vibrations in structure. However, in optimal control theory, the LQR control is not truly optimal because the feedforward (excitation) term is ignored in the optimal equation. In fact, we presented a method called identification algorithm [1], which identifies the external excitation from the structural response measured. Thus, the aim of this paper is to improve the classical LQR control by the information of the excitation identified by the identification algorithm.

2. OPTIMAL CONTROL PROBLEM

The general control problem has form:

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = L_u u(t) + L_f f(t), \quad x(0) = x_0, \tag{2.1}$$

$$y(t) = Cx(t), \tag{2.2}$$

where $x(t)$ is the n-dimensional displacement vector , $f(t)$ is the r-dimensional external force vector, $u(t)$ is the m-dimensional control force vector, three $n \times n$ matrices M, D and K are mass, damping and stiffness matrices, respectively. The $n \times m$ matrix L_u and $n \times r$ matrix L_f are location matrices which define locations of the control force and the excitation, respectively. The p-dimensional measurement vector $y(t)$ is defined by the $p \times n$ measurement matrix C. If the rank of location matrix L_u is smaller than n, the number of control forces is limited. Similarly, the number of sensors is limited when the rank of measurement matrix C is smaller than n. In the previous paper [10], we discussed the case, in which the number of sensor is limited. In this paper, the case of limited number of control force is considered. The general case in which both control force and sensor are limited can be solved by using the Luenberger observer or Kalman-Bucy filter [2] but that is beyond the scope of this study. Because only the number of control force is limited, the measurement vector $y(t)$ is identical with the displacement vector $x(t)$. To facilitate the problem, one use the state-space representation to rewrite Eq. (2.1) as in the form

$$\begin{cases} \dot{z}(t) = Az(t) + Bu(t) + Hf(t), \\ z(0) = z_0, \end{cases} \tag{2.3}$$

where $z(t) = \begin{bmatrix} x(t) & \dot{x}(t) \end{bmatrix}^T$ is the $2n$-dimensional state vector, the superscript ^{T} indicates vector or matrix transpose, A is the $2n \times 2n$ system matrix, B and H are $2n \times$m and $2n \times r$ location matrices specifying the locations of controllers and external excitations in the state-space, respectively:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, B = \begin{bmatrix} 0 \\ -M^{-1}L_u \end{bmatrix}, H = \begin{bmatrix} 0 \\ -M^{-1}L_f \end{bmatrix}$$

where O and I denote, respectively, the null matrix and the identity matrix of appropriate dimensions. The control force $u(t)$ is to be chosen in such a way that a performance index J, defined as

$$J = \frac{1}{2}\int_0^{t_f} [z^T(t)Qz(t) + u^T(t)Ru(t)]dt, \tag{2.4}$$

is minimized. In the above, the time interval $[0,t_f]$ is defined to be longer than that of the external excitation, Q and R are weighting matrices. Q is a $2n \times 2n$ positive semi-definite matrix and R is an $m \times m$ positive definite matrix. Their magnitudes depend on the relative importance attached to the state variables and to the control forces in the minimization procedure. The optimal control problem with J defined by Eq. (2.4) subject to the constraint (2.3) is well documented in the literature. The truly control law is taken as [11]:

$$u(t) = -R^{-1}B^T\left[Pz(t) + p(t)\right], \tag{2.5}$$

where P is the Riccati matrix satisfying the Riccati equation

$$PA - PBR^{-1}B^TP + A^TP + Q = 0 \tag{2.6}$$

and $p(t)$ is determined from the differential equation

$$\begin{cases} \dot{p}(t) + \left(A^T - PBR^{-1}B^T\right)p(t) + PHf(t) = 0, \\ p(t_f) = 0. \end{cases} \tag{2.7}$$

The control law (2.5) contains 2 separate terms: the feedback term $Pz(t)$ depending on state vector and the feedforward term $p(t)$ depending on external excitation $f(t)$. The system of equations given by (2.5), (2.6) and (2.7) provides optimal solution. Unfortunately, the truly optimal control law is generally infeasible. This is because the feedforward term from Eq. (2.7) must be solved backwards from the terminal time t_f, requiring that the excitation $f(t)$ over the entire control interval be known a priori. This is not possible in most of structural control applications. The classical linear quadratic regulator (LQR) control is the well-known control law, in which only feedback term is used, i.e:

$$u(t) = -R^{-1}B^TPz(t). \tag{2.8}$$

From Eq. (2.7), it can be seen that the LQR control ignores the following term

$$E_{LQR} = PHf(t). \tag{2.9}$$

In the next section, we show that, in some case, the ignored term can be reduced by adding a feedforward term, which can be identified.

3. FEEDBACK-FEEDFORWARD ALGORITHM

Although the external excitation can not be known over the entire control interval, it still can be online identified by the identification algorithm [1], which is briefly presented in section 4. Therefore, we propose here a control law using the online excitation to improve the LQR control in case the external excitations have low frequency. Considering the following form of the feedforward term $p(t)$:

$$p(t) = SHf(t). \tag{3.1}$$

The feedback-feedforward (FB-FF) control law is:

$$u(t) = -R^{-1}B^T\left[Pz(t) + SHf(t)\right]. \tag{3.2}$$

Substituting (3.1) into Eqs (2.7), the optimal equation reduces to:

$$\left[\left(A^T - PBR^{-1}B^T\right)S + P\right]Hf(t) + SH\dot{f}(t) = 0. \tag{3.3}$$

We choose

$$S = -\left(A^T - PBR^{-1}B^T\right)^{-1}P. \tag{3.4}$$

Then the ignored term is:

$$E_{FB-FF} = SH\dot{f}(t) = -\left(A^T - PBR^{-1}B^T\right)^{-1}PH\dot{f}(t). \tag{3.5}$$

Comparing (2.9) and (3.5), we see that, if the excitation frequency is sufficiently low, the ignored term of the FB-FF control is smaller than that of the LQR control. We also expect that the control performance is improved when the ignored term is reduced. A question is addressed: which value of the excitation frequency is considered as "sufficiently low". We answer it by the well-known eigenfunction technique. Assuming that the matrix $A^T - PBR^{-1}B^T$ is transformed by the modal matrix as:

$$A^T - PBR^{-1}B^T = T\Lambda T^{-1}$$

where T is the modal matrix whos columns are the eigenvectors of $A^T - PBR^{-1}B^T$ and Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i (i=1, ... $2n$) of matrix $A^T - PBR^{-1}B^T$. The expression (3.5) is rewritten as:

$$E_{FB-FF} = -T\Lambda^{-1}T^{-1}P\dot{f}(t)\,.$$

We have the following estimation:

$$\|E_{LQR}\| = \|PHf(t)\|\,, \tag{3.6}$$

$$\|E_{FB-FF}\| = \left\|T\Lambda^{-1}T^{-1}PH\dot{f}(t)\right\| \le \frac{1}{\min\limits_i|\lambda_i|}\left\|PH\dot{f}(t)\right\|. \tag{3.7}$$

The excitation frequency is considered as "sufficiently low" when

$$\frac{1}{\min\limits_i|\lambda_i|}\left\|PH\dot{f}(t)\right\| < \|PHf(t)\|\,. \tag{3.8}$$

We remark that the absolute values of the eigenvalues are the natural frequency of the structure controlled by LQR algorithm [2]. In the special case, when the excitation is harmonic with frequency ω, the condition (3.8) reduces to:

$$\omega < \min_i|\lambda_i|$$

This means that the excitation frequency is lower than the smallest frequency of the structure. As seen from (3.1), the implementation of FB-FF control process requires the knowledge of the excitation vector $Hf(t)$. In fact, it is usually that one is unable to measure the external excitation while the structural response can often be measured. Therefore, the idea involved in the control law (3.1) is used in a modified way, in which the history of the external excitation can be identified with a time delay by a so called identification process [1]. The following section briefly presents the identification algorithm to complete the control law.

4. IDENTIFICATION CONTROL ALGORITHM

Let all the components of the displacement vector $x(t)$ can be measured and all components of its first and second order derivatives can be calculated in a short time. Then the state vector $z(t)$ and its derivative can be known. The control interval $[0, t_f]$ is divided into n small equal intervals of the length Δ where Δ is a small positive number whose value depends on computation speed and accuracy of computer. Thus one has:

$$t_f = q\Delta.$$

For any given function vector $m(t)$, the following notations are introduced:

$$m^{[k]}(t) = \begin{cases} m(t) & (k-1)\Delta \le t \le k\Delta \\ 0 & \text{otherwise} \end{cases} \qquad k = 1, 2, ..., q. \tag{4.1}$$

The identification control is implemented in the following inductive way.

* In the initial subinterval $T_1 = [0 \le t < \Delta]$, two tasks are carried out simultaneously:

+***Task 1***: The control force is set to zero: $u^{[1]}(t) = 0$.

+***Task 2***: The excitation is identified from the state equation (2.3).

$$Hf^{[1]}(t) = \dot{z}^{[1]}(t) - Az^{[1]}(t).$$

* In the next subintervals $T_k = [(k-1)\Delta \le t < k\Delta]$ with k¿1, the similar tasks are carried out simultaneously:

+***Task 1***: The control force is determined from the FB-FF control law (3.2) except that the excitation is replaced by the delayed excitation in the previous subinterval:

$$u^{[k]}(t) = -R^{-1}B^T\left[Pz^{[k]}(t) + SHf^{[k-1]}(t-\Delta)\right]. \tag{4.2}$$

+***Task 2***: The excitation is identified from the state equation (2.3)

$$Hf^{[k]}(t) = \dot{z}^{[k]}(t) - Az^{[k]}(t) - Bu^{[k]}(t). \tag{4.3}$$

Using two tasks above for each time interval T_k, the FB-FF control law is completely feasible.

5. NUMERICAL SIMULATION

The example given below is taken from [12]. An eight-storey structure in which every storey unit is identically constructed is considered. The characteristics of the building are the same for each story: floor mass m, elastic stiffness k and internal damping coefficient c. Assuming that the structure is subject to the earthquake ground acceleration, whose history is taken from the $N-S$ component recorded at Hachinohe City during the Tokachioki earthquake of May 16, 1968. The absolute peak acceleration of the earthquake record is 2.25 m/s^2. In the previous paper [10], we simulated this structure, which is controlled by a set of tendons placed between each of two floors. In this paper, another control mechanism is considered. The control is accomplished through an active mass damper system installed at the top of the structure as shown in Fig. 1. An active mass damper (AMD) is a system, in which an auxiliary mass m_d is connected to the main structure through a spring k_d, a damping device c_d and a hydraulic actuator producing an active force u. Without the active force, the mass damper is passive and is called tuned mass damper (TMD). Passive TMD system was widely used for motion control of tall buildings [13]. Therefore, AMD system is also the most popular mechanism in active structural control [14].

The optimum values of spring and damping device are available in literature. They are in general tuned to the first fundamental frequency of the structure. It is not difficult to derive the structural motion equation having the form of Eq. (2.1):

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = L_u u(t) + L_f \ddot{x}_g(t), \tag{5.1}$$

where the mass, damping and stiffness matrices have form

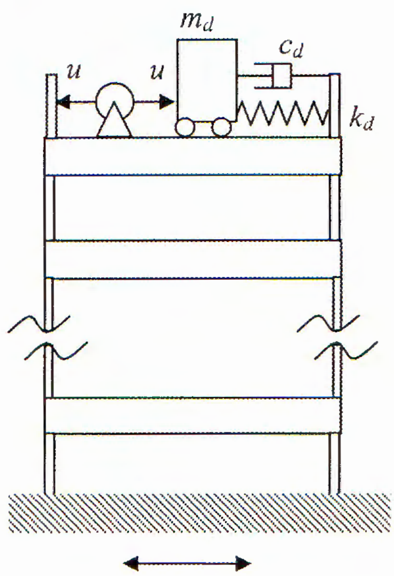

Fig. 1. Structure with an active mass damper

$$M = m \begin{bmatrix} 8+\mu & 7+\mu & 6+\mu & \dots & 1+\mu & \mu \\ 7+\mu & 7+\mu & 6+\mu & \dots & 1+\mu & \mu \\ 6+\mu & 6+\mu & 6+\mu & \dots & 1+\mu & \mu \\ \vdots & \vdots & \vdots & \ddots & 1+\mu & \mu \\ 1+\mu & 1+\mu & 1+\mu & 1+\mu & 1+\mu & \mu \\ \mu & \mu & \mu & \mu & \mu & \mu \end{bmatrix};$$

$$C = diag\left(c, c, ..., c, c_d\right); K = diag\left(k, k, ..., k, k_d\right)$$

In which μ=m_d/m is the mass ratio, $\ddot{x}_g$ denotes the base acceleration. The displacement vector x, the location matrices L_u and L_f have form:

$$x = \begin{bmatrix} x_1, & x_2 - x_1, & x_3 - x_2, & \dots & x_8 - x_7, & x_d - x_8 \end{bmatrix}^T,$$

$$L_u = \begin{bmatrix} 0 & 0 & 0 & \dots & 1 \end{bmatrix}^T,$$

$$L_f = -\begin{bmatrix} 8+\mu, & 7+\mu, & 6+\mu, & \dots & 1+\mu, & \mu \end{bmatrix}^T m,$$

where x_i (i=1,..., 8) and x_d, respectively, are the relative displacement of the i^{th} floor and the auxiliary mass with respect to the foundation. The motion Eq. (5.1) then is represented in the state-space form. Let the parameters take values as [12]: m= 345.6 metric tons, elastic stiffness k= 3.404×10^5 kN/m, internal damping coefficient c= 2937 metric tons/sec. Some first natural frequencies of this building are 0.92, 2.73, 4.45, 6.02, 7.38 Hz. For the active mass damper, m_d=29.3 tons, c_d=25.0 tons/sec and k_d=957.2 kN/m. Thus, the damper frequency is tuned to 98% of the first natural frequency of the structure and the damping ratio of the damper is 7.3%. For the purpose of comparison between control laws, the time scale of the base acceleration is varied to change the external excitation frequency. Fig. 2 shows the time histories and the power spectrums of the base acceleration with different time scales.

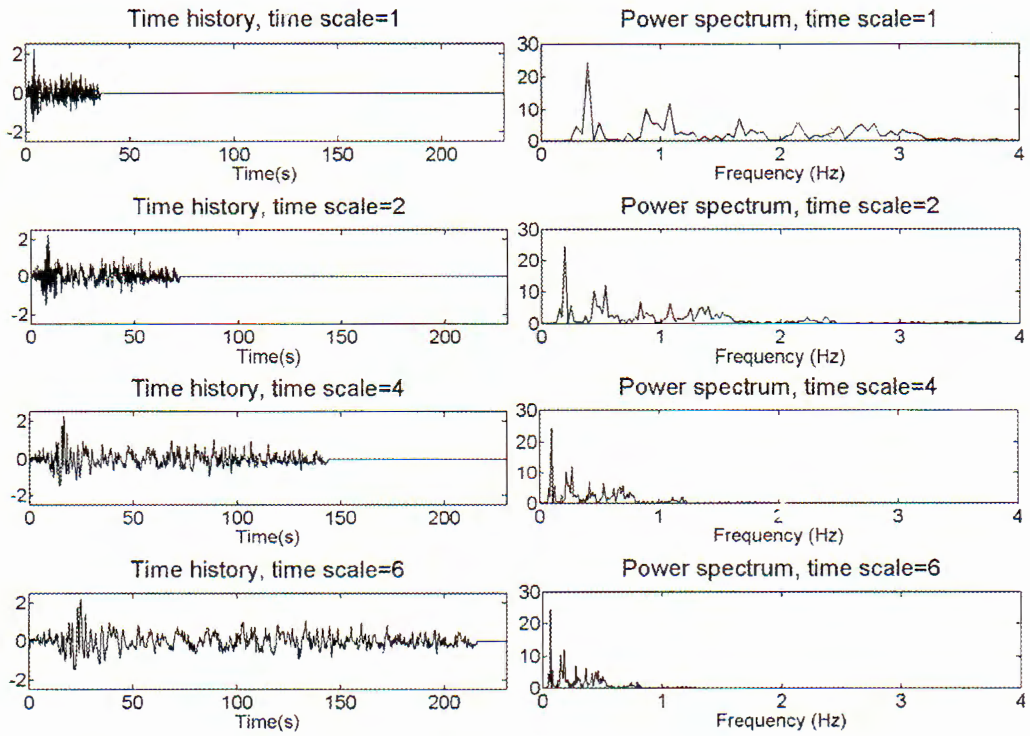

Fig. 2. The time histories and the power spectrums of the base acceleration
From top to bottom: time scale = 1 , 2, 4 and 6

It is clearly to see that the larger time scale makes the lower frequency and conversely. In the numerical simulation, we consider 4 cases of time scale as showed in Fig. 2. In the power spectrum of the base acceleration, the excitation frequency is lower than the first structure frequency (0.92 Hz) when time scale=4 and =6. Therefore, two last case of time scale is considered as "low frequency" excitation. The time delay Δ is taken to be 4% of the first natural period of the structure. The objective of control is to reduce the top floor displacement x_8. Therefore, in the expression (2.4) of performance index J, the 18×18 weighting matrix Q is chosen so that all its elements are zero except for the 8th element in the diagonal, i.e.

$$Q = diag\left(\underbrace{0, 0 ..., 0,}_{7\,\text{elements}} \quad 1, \quad \underbrace{0, 0, ... 0}_{10\text{ elements}} \right)$$

Because there is only one control force, the weighting matrix R in this case is a scalar and is assigned a value of 10^{-8}. The values of performance index J, the root mean square (RMS) values of the top floor displacement x_8, the mass damper relative displacement $x_d - x_8$ and control force u are tabulated in Table 2 for each case of time scale. Each column in this table is for different control type:

- Column (1) is for the structure without mass damper.
- Column (2) is for the passive TMD case (control force u=0).
- Column (3) is for the LQR control law (2.8)

- Column (4) is for the FB-FF identification control law (4.2), (4.3). We denote this control law as FB-FF-ID

-Column (5) is for the truly optimal solution (2.5), where $p(t)$ is obtained from (2.7). We note that this solution is infeasible in practice because the optimal equation must be solved backwards from the terminal time t_f.

Table 2. The performance indexes and the RMS responses in the numerical simulation

	Response	Without mass damper (1)	Passive TMD (2)	LQR control (3)	FB-FF-ID control (4)	The truly optimal solution (5)
Time scale=1	J	0.059	0.023	0.009	0.012	0.003
	x_8 (cm)	2.86	1.79	1.09	1.35	0.54
	$x_d - x_8$ (cm)	0.00	8.96	14.39	17.56	15.63
	u (kN)	0.00	0.00	47.17	84.14	42.79
Time scale=2	J	0.096	0.049	0.028	0.029	0.010
	x_8 (cm)	3.19	2.33	1.60	1.90	1.02
	$x_d - x_8$ (cm)	0.00	10.43	20.66	22.17	20.10
	u (kN)	0.00	0.00	82.53	106.43	59.30
Time scale=4	J	0.111	0.076	0.073	0.058	0.029
	x_8 (cm)	3.18	2.65	2.28	2.35	1.61
	$x_d - x_8$ (cm)	0.00	9.72	30.83	24.83	26.19
	u (kN)	0.00	0.00	127.73	92.30	96.79
Time scale=6	J	0.096	0.075	0.077	0.057	0.036
	x_8 (cm)	2.94	2.62	2.25	2.21	1.73
	$x_d - x_8$ (cm)	0.00	8.52	28.32	19.83	21.81
	u (kN)	0.00	0.00	139.64	68.29	73.33

Comparing the performance index between two columns (3) and (4), we see that, the FB-FF-ID control law is more efficient than the classical LQR control law in case of low frequency excitation (time scale=4 and 6). For example, when timescale=4, the RMS top floor responses (2.28 cm and 2.35 cm) are nearly the same but the RMS control force and the RMS mass damper response of the LQR control (127.73 kN and 30.83 cm) are significantly larger than that of the FB-FF-ID control (92.3 kN and 24.83 cm). In the opposite case, when the excitation frequency is high (time scale=1 and 2), the numerical simulation indicates that FB-FF-ID control is less efficient than LQR control. As we expect, the results in this example show that the performance of control is improved when the ignored term of the optimal equation is reduced. Besides, in comparison with the truly optimal case (column (5)), the performance indexes of LQR control and FB-FF-ID control still have a large distance. Therefore, the problem finding the better control laws is still interesting.

6. CONCLUSION

The aim of this paper is to propose a feedback-feedforward control law to improve the classical LQR control law for feedback active controlled structures. The proposed FB-FF control law is better than LQR control when the excitation frequency is low in comparison with the structure natural frequency. The improvement is achieved by adding the feedforward term to the LQR control. The feedforward term is identified by a so called identification algorithm. To illustrate the algorithm, a numerical simulation is applied to an eight story building subjected to earthquake ground acceleration and controlled by active mass damper system. The effect of the excitation frequency is considered in the simulation.

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VỀ MỘT THUẬT TOÁN ĐIỀU KHIỂN NHẬN DẠNG HỒI TIẾP-DẪN TIẾP ỨNG DỤNG CHO CÁC KẾT CẤU ĐƯỢC ĐIỀU KHIỂN TÍCH CỰC PHẢN HỒI

Điều chỉnh tuyến tính với chỉ tiêu dạng toàn phương (LQR) là một phương pháp hiệu quả trong lý thuyết điều khiển tích cực phản hồi. Tuy nhiên, điều khiển LQR không thật sự tối ưu vì đây mới chỉ là thuật toán phản hồi, nghĩa là thành phần kích động ngoài đã bị bỏ qua trong phương trình tối ưu. Trong 1 bài báo trước [1], thuật toán nhận dạng đã được trình bày cho các hệ được điều khiển tích cực phản hồi với mục đích nhận dạng kích động ngoài từ các đáp ứng đo được của kết cấu. Mục đích của bài báo là đề xuất 1 thuật toán điều khiển hồi tiếp-dẫn tiếp sử dụng các kích động đã được nhận dạng để cải thiện điều khiển LQR. Mô phỏng số được thực hiện cho mô hình nhà 8 tầng chịu tải gia tốc nền và được điều khiển bởi hệ thống điều khiển tích cực khối lượng.