A determination of the economic feasibility of producing marginal gas wells

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A DETERMINATION OF THE ECONOMIC FEASIBILITY OF PRODUCING MARGINAL GAS WELLS

BY

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A

THESIS

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INTRODUCTION

The purpose of this investigation is to show that it is economically feasible to produce marginal gas wells in such a manner that all the gas from the edge of the field will be captured by the first row of wells. In this problem the permeability of the sand is to be variable but known, and the effects of this variable permeability on the pressure distribution, and resulting gas production, will be discussed.

In many gas fields the point at which it is generally considered unprofitable to drill new wells is well inside the boundary of the reservoir This is due primarily to a reduction of permeability and porosity as the boundary of the reservoir is approached. The quantity of gas which is outside the economic limit is frequently quite large in relation to the amount within the drainage area of any interior well. It will subsequently be shown that all this large quantity of marginal gas will flow into the first, or marginal, row of wells as long as they are maintained at a bottom hole pressure below that which would exist at their point of location if they were not producing, i.e., capped.

Two problems have been solved. The first to show the effect of variable permeability on the pressure distribution about a single marginal well; and the second to show the pressure distribution, and consequently the production, of a line array of three wells leading into the center of the field over a period of four years as the field is depleted.

In both problems the relaxation method of R. V. Southwell [(1)] has been used to determine the pressure distribution at the interior points of the drainage area. The basic idea in solving steady state fluid flow

(1) Southwell, R. V., Proc. Royal Society, Series A, Vol. 168, pp. 317-350, 1938.

problems by the relaxation method is that the fluid mass in any closed system can neither be created nor destroyed. In other words, at steady flow conditions the total quantity of fluid, Q, at any interior point, at any instant of time, must be zero, e.g., as much fluid is flowing toward the point as is flowing away from it.

The above idea is the basis for the solution of fluid flow problems involving steady state conditions. The drainage area is to be replaced by an equivalent network of pipes through which all the gas is assumed to flow. Essentially the solution proceedure is to assume values of pressure at each interior intersection of the pipes and calculate Q for the intersection using D'Arcy's Law.

Since, for steady flow the net $Q = 0$, it is necessary to reassign values of pressure until Q approaches zero to the desired degree of accuracy.

It has been shown by A. A. Zwierzchowski [(2)] that the relaxation

(2) Zwierzchowski, A. A., The Application of the Relaxation Method to the Solution of Problems Involving the Flow of Fluids Through Porous Media, Thesis, Missouri School of Mines and Metallurgy, Rolla, Mo., 1949.

method is applicable to steady state fluid flow problems, so it will not be the purpose of this paper to prove the validity of the method. However, the proceedure has been expanded to include variable permeability.

In the first problem of this thesis it was found that after first relaxing the entire drainage area to an approximate value it was quicker to assume a number of new simpler networks involving a few of the interior points in the original problem and solve these to completion. After this was done the values found from the simpler networks were applied to the solution of the entire drainage area. The original network solution was then completed using the values obtained from the simplifications.

REVIEW OF LITERATURE

Numerical methods of solution of steady flow problems appear to have had their beginning in the work of Cross [3] who used an approximate

(3) Cross, H., Analysis of Flow in Networks, Univ. of Illinois Eng. Expt. Sta., Bull. 286, Nov. 1936.

method in the solution of municipal water distribution networks. Independently and with essential modifications, a similar method was developed by Southwell. [4] For the solution of mechanics problems Southwell called

(4) Southwell, R. V., op. cit., p. 1.

it the relaxation method but did not use it to solve fluid flow problems. The application to the field of heat transmission was first made by Emmonds, [5] in 1933-34. Since that time many texts, papers, and articles

(5) Emmonds, H. W., The Numerical Solution of Heat Conduction Problems, Trans. A.S.M.E., Vol. 65, No. 6, 1943.

have been written on the subject of the relaxation method as a method of obtaining approximate solutions to problems in the fields of heat transmission, electrical conduction, etc. Notable among these are the text by Dusinberre, [6] and a report by Shaw, [7] which was not available to

(6) Dusinberre, G. M., The Numerical Analysis of Heat Flow, 1st Ed., N. Y., McGraw-Hill, 215 p.

(7) Shaw, F. S., The Torsion of Solid and Hollow Prisms in the Elastic and Plastic Range by Relaxation Methods, Australian Council for Aeronautics Report, ACA 11, Nov. 1944.

the author, which extends the relaxation method well beyond the limits of heat transmission problems.

The only work known to the author in the field of the flow of homogeneous fluids through porous media which treats the problem from the relaxation method standpoint is by Zwierzchowski.[8] This work considers

(8) Zwierzchowski, A. A., op. cit., p. 2.

only flow into a single well from regions of constant permeability. Muskat's[9] text considers a great variety of fluid flow problems but

(9) Muskat, M., Flow of Homogeneous Fluids, J. W. Edwards, Inc., Ann Arbor, Mich., 749 p., 1946.

only from an analytical standpoint and again with constant permeability.

PROBLEM 1

SOLUTION OF A PROBLEM INVOLVING A SINGLE MARGINAL WELL DRAINING A RECTANGULAR DRAINAGE AREA OF VARIABLE PERMEABILITY

For this problem the drainage area is to be that shown in Fig. 1. The following information is given:

Well bore - 0 feet (point sink)

Well pressure - 0 psia

Drainage area

500 feet inward from the well

500 feet on either side of the well

5500 feet to the edge of the field

Drainage pressure - 1200 psi over the entire boundary.

The thickness of the sand is constant, i.e., A = 1.

The quantity is to be 100 units for a distance of 500 feet on all sides of the well. From this point outward to the edge of the field, the value of $\frac{k\gamma_0 A}{2\mu}$ is to decrease lineally with distance to a value of 10 units at the edge of the field. This is shown graphically in Fig. 2.

Steady flow conditions prevail.

To solve the problem the following assumptions will be made:

1. The flow is to be isothermal.
2. The pressure distribution need be found for only one half of the drainage area since there is an axis of symmetry running through the figure from a0 to a14. Consequently, only one half of the total drainage area is shown in Fig. 1.
3. The initial pressure assumptions at the unknown points are to be any convenient value.

The equation which describes the mass flow into a gas well is

$$Q = \frac{k\gamma_0 A}{2\mu L}\left(P_1^2 - P_2^2\right)$$

Fig. 1 (above) Showing one half of the drainage area of problem 1. The equivelent pipe network is shown and the values for $k\sigma_0 A/24L$ are indicated on each pipe (to scale)

Fig. 2 (below) Showing a graphical representation of the values for $k\sigma_0 A/24L$ for the drainage area of problem 1 (to scale) where:

Q is the mass flow

k is the permeability

γ_0 is the gas density at the well

A is the sand area

μ is the gas viscosity

L is the length of flow path

$p_1^2 - p_2^2$ is the difference of the squares of the source and sink pressures

Since for any area of flow the term $\frac{k\gamma_0 A}{2\mu L}$ can be taken as constant for the area, this equation can be given in the form

$$Q = \text{const.}\ (p_1^2 - p_2^2)$$

And since the relaxation method uses a single pipe to represent an area, it only remains to evaluate $\frac{k\gamma_0 A}{2\mu L}$ for each pipe to be able to apply the method to an area of variable permeability and variable length. Also, since the mass flow is proportional to the difference of the squares of the pressures, values of p^2 must be used to calculate the Q's. In the pipes where there is a change in the value of $\frac{k\gamma_0 A}{2\mu}$ from one end of the pipe to the other, and arithmetic average of the two end values is used. A sample calculation for pipe [c1 - c2] where the value of $\frac{k\gamma_0 A}{2\mu}$ is constant (100 units) and also for pipe [b11 - b12] where the value of $\frac{k\gamma_0 A}{2\mu}$ is the average of the values at b11 and b12 is shown.

The values for $\frac{k\gamma_0 A}{2\mu L}$ for each pipe are tabulated in Table 1. These values in Table 1 are also shown on the proper pipe in Fig. 1.

For pipe [c1 - c2]

$$\frac{k\gamma_0 A}{2\mu L} = \frac{100}{200} = 0.5000$$

TABLE I

Pipe —	Value of $\frac{k\gamma_0 A}{2\mu L}$	Pipe —	Value of $\frac{k\gamma_0 A}{2\mu L}$	Pipe —	Value of $\frac{k\gamma_0 A}{2\mu L}$	Pipe —	Value of $\frac{k\gamma_0 A}{2\mu L}$
a0 - a1	0.5000	b10 - b11	0.1550	a3 - b3	1.000	a11 - b11	0.7300
a1 - a2	0.5000	b11 - b12	0.0640	b3 - a3	0.5000	b11 - c11	0.3650
a2 - a3	1.000	b12 - b13	0.460	c3 - d3	0.5000	c11 - d11	0.3650
a3 - a4	1.000	b13 - b14	0.0157	a4 - b4	1.000	a12 - b12	0.5500
a4 - a5	0.5000	c0 - c1	0.5000	b4 - c4	0.5000	b12 - c12	0.27500
a5 - a6	0.5000	c1 - c2	0.5000	c4 - d4	0.5000	c12 - d12	0.2750
a6 - a7	0.9910	c2 - c3	1.000	a5 - b5	1.000	a13 - b13	0.3700
a7 - a8	0.4820	c3 - c4	1.000	b5 - c5	0.5000	b13 - c13	0.1850
a8 - a9	0.3063	c4 - c5	0.5000	c5 - d5	0.5000	c13 - d3	0.1850
a9 - a10	0.2140	c5 - c6	0.5000	a6 - b6	1.000		
a10 - a11	0.1550	c6 - c7	0.9910	b6 - c6	0.5000		
a11 - a12	0.0640	c7 - c8	0.4820	a6 - d6	0.5000		
a12 - a13	0.0460	c8 - c9	0.3063	a7 - b7	0.9820		
a13 - a14	0.0157	c9 - c10	0.440	b7 - c7	0.4910		
b0 - b1	0.5000	c10 - c11	0.1550	c7 - d7	0.4910		
b1 - b2	0.5000	c11 - c12	0.0640	a8 - b8	0.9460		
b2 - b3	1.000	c12 - c13	0.0460	b8 - c8	0.4730		
b3 - b4	1.000	c13 - c14	0.0157	c8 - d8	0.4730		
b4 - b5	0.5000	a1 - b1	1.000	a9 - b9	0.8920		
b5 - b6	0.5000	b2 - c1	0.5000	b9 - c9	0.4460		
b6 - b7	0.9910	c1 - d1	0.5000	c9 - d9	0.4460		
b7 - b8	0.4820	a2 - b2	1.000	a10 - b10	0.8200		
b8 - b9	0.3063	b2 - c2	0.5000	b10 - c10	0.4100		
b9 - b10	0.2140	c2 - d2	0.5000	c18 - d10	0.4100		

For pipe [b11 - b12]

Point b11 $\frac{k\gamma_0 A}{z\mu} = 55$ (see Fig. 2)

Point b12 $\frac{k\gamma_0 A}{z\mu} = 73$ (see Fig. 2)

Ave. $\frac{k\gamma_0 A}{z\mu L} = \frac{55 + 73}{2 \times 1000} = 0.0640$

The initial pressure assumptions, or more precisely, the initial values for p^2 are given for each point in Table II.

A sample calculation is now shown for the Q of an interior point using point c2

$$Q_{c2} = 0.5\ (p^2_{d2} - p^2_{c2}) + 1.0\ (p^2_{c3} - p^2_{c2}) + 0.5\ (p^2_{c1} - p^2_{c2}) + 0.5\ (p^2_{b2} - p^2_{c2})$$

$$= 0.5(1{,}000{,}000 - 810{,}000) + 1.0(810{,}000 - 810{,}000) + 0.5(1{,}440{,}000 - 810{,}000) + 0.5(360{,}000 - 810{,}000)$$

$$= +185{,}000 \text{ units}$$

Table III shows the results of the computations up to the point where the first simplified network was introduced.

At this point the first simplified network for the drainage area of Problem 1 was set up as shown by the heavy lines in Fig. 3. Since the length of the pipes is not the same as those in Fig. 1, new values for the constant $\frac{k\gamma_0 A}{z\mu L}$ must be calculated for each new pipe. The method of calculation is the same as before. These new values of $\frac{k\gamma_0 A}{z\mu L}$ are tabulated in Table IV. The initial assumptions for p^2 in this simplification are the last determined in the tabulation of Table III.

Table V shows the results of the computations for the values of p^2 in the first simplification.

TABLE II

Point	Value of p^2	Point	Value of p^2	Point	Value of p^2
a1	58,400	b1	640,000	c1	1,000,000
a2	57,600	b2	360,000	c2	810,000
a3	0 - well	b3	250,000	c3	810,000
a4	625	b4	250,000	c4	810,000
a5	10,000	b5	250,000	c5	810,000
a6	40,000	b6	360,000	c6	1,000,000
a7	90,000	b7	360,000	c7	1,000,000
a8	160,000	b8	490,000	c8	1,000,000
a9	250,000	b9	640,000	c9	1,000,000
a10	360,000	b10	640,000	c10	1,000,000
a11	490,000	b11	810,000	c11	1,210,000
a12	640,000	b12	1,000,000	c12	1,210,000
a13	1,000,000	b13	1,210,000	c13	1,322,500

TABLE IV

Pipe	Value of $\frac{k\gamma_0 A}{2\mu L}$	Pipe	Value of $\frac{k\gamma_0 A}{2\mu L}$	Pipe	Value of $\frac{k\gamma_0 A}{2\mu L}$
a0 - a2	0.2500	b0 - b2	0.2500	a2 - c2	0.3300
a2 - a3	1.0000	b2 - b3	1.0000	c2 - d2	0.5000
a3 - a6	0.2000	a3 - a6	0.2000	a3 - c3	0.3300
a6 - a10	0.0910	a6 - a10	0.0910	c3 - d3	0.5000
a10 - a14	0.0115	a10 - a14	0.0115	a6 - c6	0.3300
				c6 - d6	0.5000
				a10 - c10	0.2930
				c10 - d10	0.4100

TABLE III

Q_{a1}	P_{a1}	Q_{b1}	P_{b1}	Q_{c1}	P_{c1}	Q_{a2}	P_{a2}	Q_{b2}	P_{b2}
	518,000		640,000		1,000,000		57,600		360,000
+818,000		+318,000		+165,000		+1,013,000		−46,000	
	1,000,000	+800,000		+445,000		+438,000		+45,000	
+1,120,000		−100,000		−55,000			160,000	+135,000	
−100,000			1,000,000		1,200,000	+120,000		+330,000	
−160,000		0		+40,000		+500,000			500,000
	1,200,000	+70,000				0	400,000	−90,000	
−160,000		+240,000				+200,000		+60,000	
−880,000		+320,000				+400,000		+320,000	
			1,200,000				600,000	+20,000	
								−100,000	
								+120,000	

Q_{c2}	P_{c2}	Q_{a3}	P_{a3}	Q_{b3}	P_{b3}	Q_{c3}	P_{c3}	Q_{a4}	P_{a4}
	810,000	0 (well)	0		250,000		810,000		625
+185,000				+140,000		+35,000		+530,000	
+290,000				+280,000		+230,000		+750,000	
	1,000,000			+460,000			1,000,000		100,000
−95,000					420,000	+310,000		+250,000	
−250,000				−135,000		−320,000		+610,000	
+70,000				−40,000		−70,000			230,000
+120,000				−120,000				−40,000	
								0	

Q_{b4}	P_{b4}	Q_{c4}	P_{c4}	Q_{a5}	P_{a5}	Q_{b5}	P_{b5}	Q_{c5}	P_{c5}
	250,000		810,000		10,000		250,000		810,000
+306,000		−150,000		+500,000		+95,000		+130,000	
+130,000		+125,000		+760,000		+335,000		+230,000	
+230,000		−65,000			250,000	+380,000		+325,000	
	430,000	+505,000		−200,000			450,000	−55,000	
+310,000		−65,000		−100,000		−170,000			1,000,000
−140,000			1,000,000	+300,000		−30,000			
−45,000				+430,000		+450,000			
+85,000					300,000	+210,000			
				+230,000					
				+370,000					

Q_{a6}	P_{a6}	Q_{b6}	P_{b6}	Q_{c6}	P_{c6}	Q_{a7}	P_{a7}	Q_{b7}	P_{b7}
	40,000		360,000		1,000,000		90,000		360,000
+710,000		−550,000		−195,000		+365,000		+350,000	
+1,420,000		+205,000		−150,000		+825,000		+465,000	
	300,000	+190,000		−55,000			450,000		500,000
+128,000			450,000			−940,000		+50,000	
−228,000		+75,000				−426,000		+140,000	
+550,000		+150,000				−150,000		+204,000	
+600,000		+295,000				+46,000			
	400,000								
+100,000									

Q_{c7}	P_{c7}	Q_{a8}	P_{a8}	Q_{b8}	P_{b8}	Q_{c8}	P_{c8}	Q_{a9}	P_{a9}
	1,000,000		160,000		490,000		1,000,000		250,000
−98,000		+306,000		−6,000		−33,000		+688,000	
−29,000		+776,000		+235,000		+22,000			500,000
			500,000	+278,000				−10,000	
		−416,000		+620,000				+19,000	
		−72,000			620,000			+526,000	
		+24,000		+15,000					650,000
		+178,000						−88,000	

Q_{b9}	P_{b9}	Q_{c9}	P_{c9}	Q_{a10}	P_{a10}	Q_{b10}	P_{b10}	Q_{c10}	P_{c10}
	640,000		1,000,000		360,000		640,000		1,000,000
+235,000		+36,000		+550,000		−56,000		+67,000	
+10,000				+565,000		+222,000			
−123,000					700,000				
−100,000				−244,000					
				−50,000					
				−116,000					

Q_{a11}	P_{a11}	Q_{b11}	P_{b11}	Q_{c11}	P_{c11}	Q_{a12}	P_{a12}	Q_{b12}	P_{b12}
	490,000		810,000		1,210,000		640,000		1,000,000
+446,000		−105,000		−94,000		+408,000		−142,000	
+858,000		+198,000				+464,000		+111,000	
	900,000						1,100,000		
−228,000						−146,000			
−38,000						−122,000			

Q_{c12}	P_{c12}	Q_{a13}	P_{a13}	Q_{b13}	P_{b13}	Q_{c13}	P_{c13}
	1,121,000		1,000,000		1,210,000		1,324,000
+10,000		+136,000		−31,000		−2,000	
		+180,000		+50,000			
			1,250,000				
		−38,000					

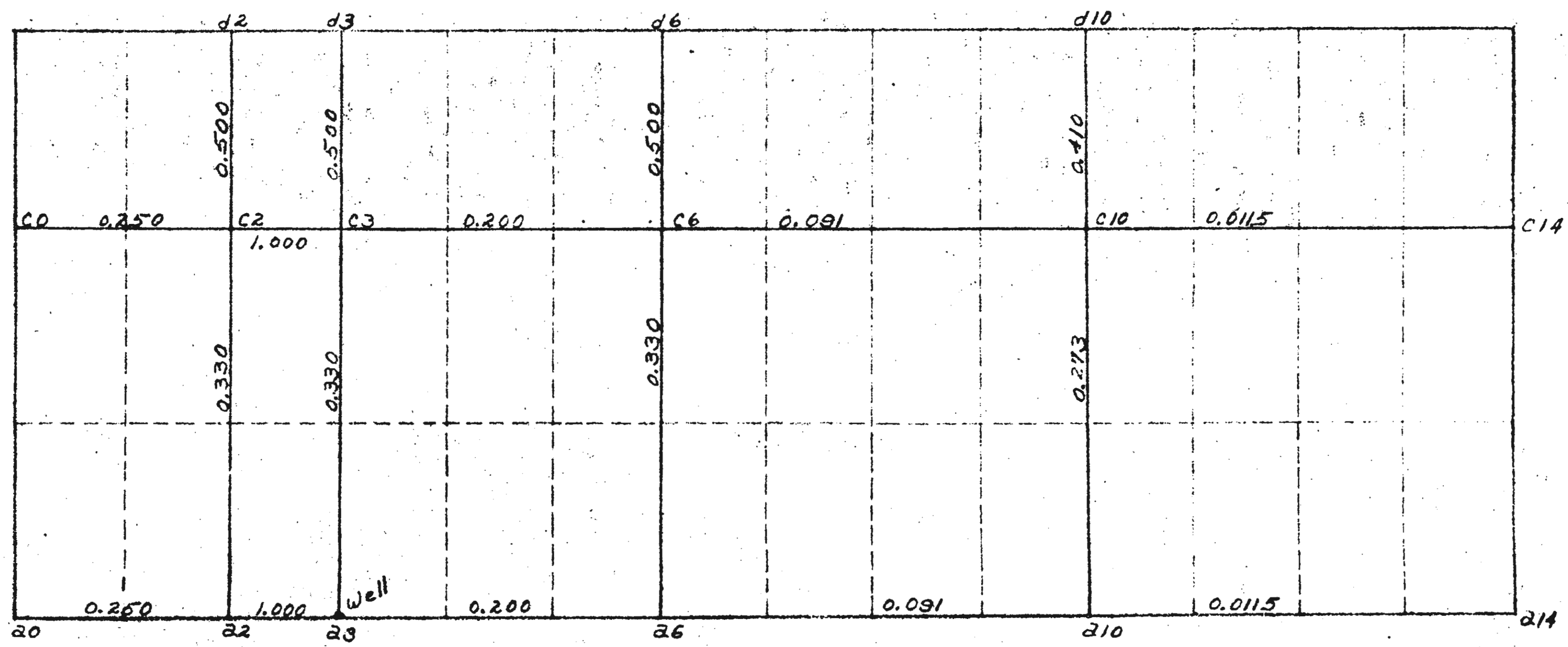

Fig. 3 Showing the network of the first simplification of problem 1 (solid lines - not to scale) The value of $k\gamma_0 A/2\mu L$ is shown for each pipe. e.g. pipe [a6-a10] = 0.091

TABLE V

Q_{a2}	P_{a2}	Q_{c2}	P_{c2}	Q_{c3}	P_{c3}	Q_{a6}	P_{a6}
	600,000		1,000,000		1,000,000		400,000
-580,000		+230,000		-110,000		+348,000	
	450,000	+147,000		+180,000			500,000
-160,000			1,200,000		1,100,000	+170,000	
+94,000		-285,000		-113,000		+270,000	
+60,000		-185,000		-107,000		+326,000	
+28,000			1,150,000	-137,000			800,000
+14,000		-160,000			1,050,000	-50,000	
+8,000		-116,000		-35,000		-32,000	
	458,000		1,100,000	-85,000		-22,000	
	453,000	-13,000			1,020,000	-18,000	
-1,000		-42,000		-24,000			780,000
	452,500		1,080,000	-46,000		+7,000	
+412		0			1,000,000		785,000
-250		-20,000		-3000		+1,300	
			1,070,000	-13,000		+4,000	
		+16,000			995,000		787,000
		+1000		-3000		+2,000	
		+2800		-2000		+2500	
		-3300		-500		+8,000	
			1,069,000	+1200		-9,200	
		-1250		-1166			795,000
						+800	
						+400	
						+980	
						+1,000	

Q_{c6}	P_{c6}	Q_{a10}	P_{a10}	Q_{c10}	P_{c10}
	1,000,000		700,000		1,000,000
+20,000		+126,000		-18,000	
+53,000		+144,000		+89,000	
+73,000		+254,000			1,200,000
	1,150,000		1,000,000	-49,000	
-111,000		+8,000		+33,000	
+22,000		+84,000		+63,000	
+12,000			1,100,000	+69,000	
+6,500		+8,000			1,300,000
+16,000		+62,000		-10,000	
+12,000			1,150,000	+40,000	
+11,000		+24,000		+13,000	
+5,000			1,180,000		1,305,000
+6,300		+2,000		+9,700	
	1,155,000	0			1,320,000
-6,100		-3,600		-1,800	
	1,163,000	+3000		-1,100	
-3,500		+2,600		+900	
-3,000			1,183,000	±2,800	
	1,160,000	+400			1,325,000
+400		+6,000		-1,200	
			1,197,000	-400	
		-2,000		-650	
		+3,600			
			1,200,000		
		-1,400			

A second simplification will now be solved for the drainage area. The network is shown in Fig. 4. The proceedure is the same as for the first simplification and the original assumptions for p^2 will again be the last determined in the tabulation in Table III. Table VI tabulates the value of $\frac{k\gamma_o A}{2\mu L}$ for the pipes in the second simplification.

TABLE VI

Pipe	Value of $\frac{k\gamma_o A}{2\mu L}$	Pipe	Value of $\frac{k\gamma_o A}{2\mu L}$	Pipe	Value of $\frac{k\gamma_o A}{2\mu L}$
a0 - a1	0.5000	b0 - b1	0.5000	a1 - b1	1.0000
a1 - a3	0.3333	b1 - b3	0.3333	b1 - d1	0.2500
a3 - a5	0.3333	b3 - b5	0.3333	a3 - b3	1.0000
a5 - a8	0.1968	b5 - b8	0.1968	b3 - d3	0.2500
a8 - a12	0.0238	b8 - b12	0.0238	a5 - b5	1.0000
a12 - a14	0.0110	b12 - b14	0.0110	b5 - d5	0.2500
				a8 - b8	0.9460
				b8 - d8	0.2365
				a12 - b12	0.5500
				b12 - d12	0.1375

Table VII shows the results of the computations for the values of p^2 in the second simplification.

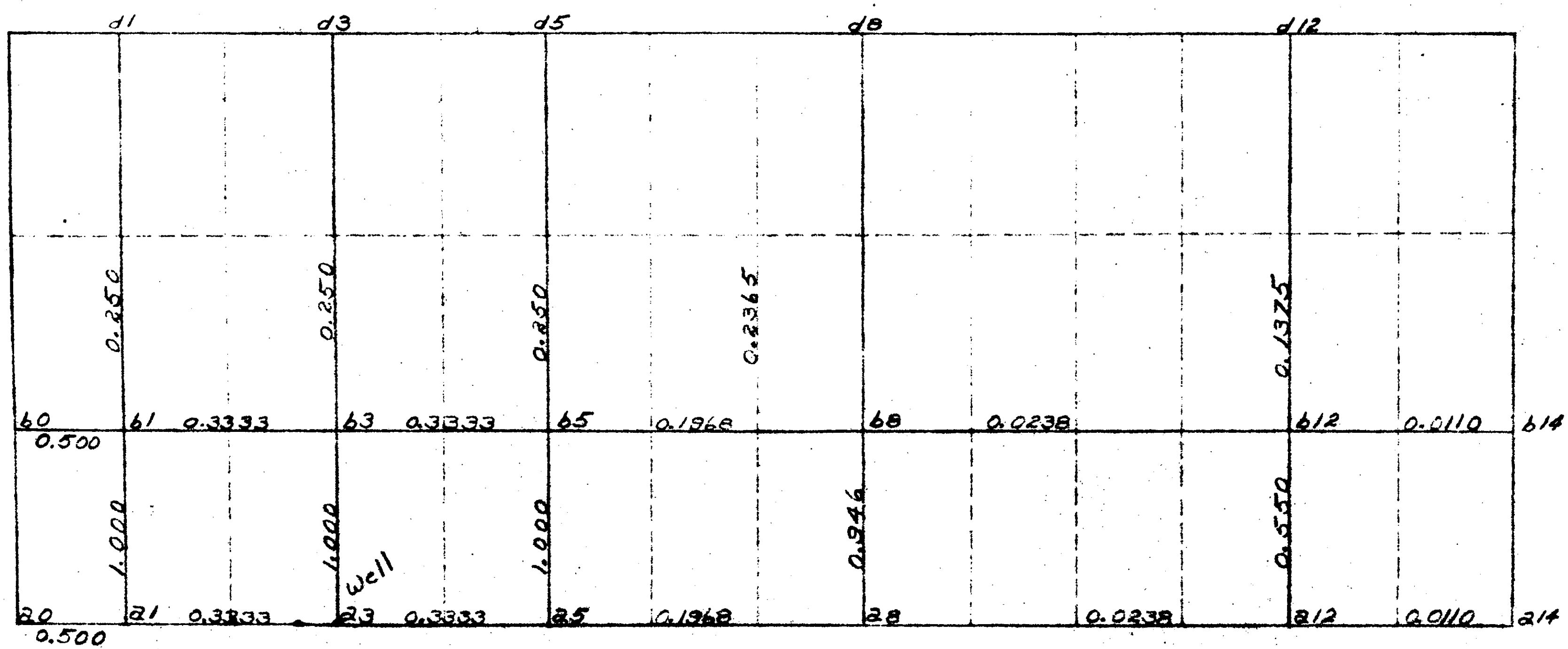

Fig. 4 Showing the network of the second simplification of problem 1 (solid lines.-not to scale) The value of $k\bar{\sigma}_0 A/2\mu L$ is shown for each pipe e.g. pipe [b8-d8] = 0.2365

TABLE VII

Q_{a1}	P_{a1}	Q_{b1}	P_{b1}	Q_{b3}	P_{b3}	Q_{a5}	P_{a5}
	1,100,000		1,250,000		400,000		600,000
−98,000		−291,000		+263,000		+58,000	
+444,000			1,075,000	+202,000		+97,000	
	950,000	+64,000			500,000		630,000
+106,000		−77,000		+9,000		+5,000	
	970,000	−43,000		+17,000		+45,000	
+34,000		−23,000			505,000	+26,000	
	980,000	−13,000		+4,000		−38,000	
−5,000		−11,000		+3,750			650,000
−24,000			1,065,000	+450		−22,000	
	975,000	+9,000				−36,000	
−5,000		+5,000				−44,000	
	973,000	+2,380				−56,000	
+2,340							620,000
						+36,000	
							630,000
						+6,000	
						+3,850	
						−20,000	
							620,000
						+10,000	
						+4,600	

Q_{b5}	P_{b5}	Q_{a8}	P_{a8}	Q_{b8}	P_{b8}	Q_{a12}	P_{a12}	Q_{12}	P_{b12}
	750,000		1,000,000		1,175,000		1,350,000		1,370,000
−9,000		+208,000		−182,000		+8,000		−5000	
+23,000			1,100,000	−67,000		+16,000		−5,500	
+20,000		−40,000			1,160,000	+10,000		−8,000	
+50,000		−70,000		−62,000		+11,000		−1,000	
	770,000	−60,000		−109,000		+10,000			1,360,000
+14,000			1,050,000		1,100,000	+9,000		−10,000	
+40,000		+58,000		−25,000		+8,000		−10,000	
+33,000		−58,000		+2,400			1,340,000	−5,000	
+18,000			1,080,000	−26,000		+600		−1,100	
−12,000		−13,500		−45,000		−600			
−5,000		−120,000			1,160,000	−800			
−7,000			1,050,000	−13,000					
−5,600		−48,000		−15,000					
−12,400			1,030,000		1,050,000				
	760,000	−3,000		−9,000					
+9,000		−78,000			1,045,000				
−1,000			1,000,000	−2,800					
−2,000		−8,000		−7,500					
−1,800		−14,000		−9,500					
	759,000	−8,000			1,040,000				
			990,000	−3,500					
		−4,000			1,030,000				
		−14,000		−1,700					
			985,000						
		−1,600							
		−6,000							
		−15,000							
			970,000						
		+1,400							

The network shown in Fig. 5 has now been isolated with known pressures at all surrounding points and will be worked as simplification number three. Since the pipes are the same as those in the original problem the same values for $\frac{k\gamma_0 A}{2\mu L}$ will be used. These values for p^2 will be assumed for:

c4 - 950,000

c5 - 1,100,000

b4 - 600,000

a4 - 250,000

Table VIII shows the results of the computations for the values of p^2 in the third simplification.

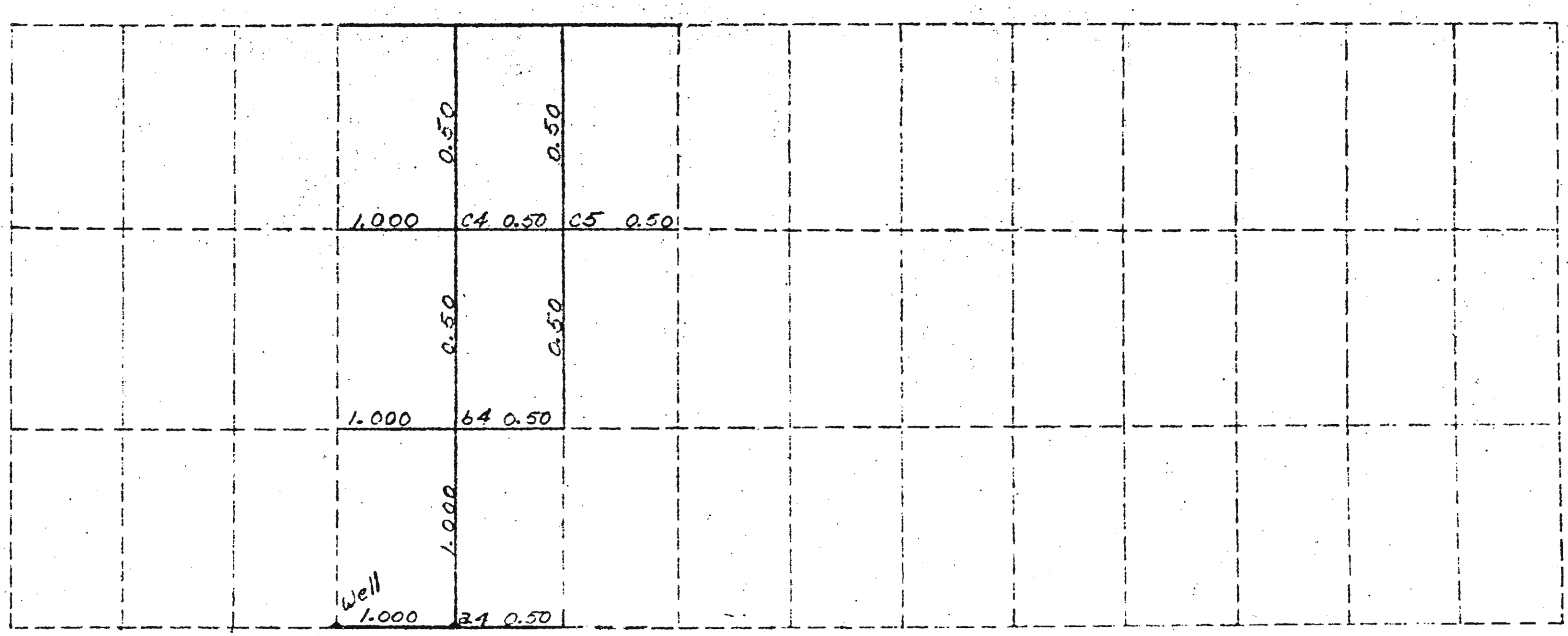

Fig 5 Showing the network of the third simplification of problem 1 (solid lines - not to scale) The value of $K\sigma_o A/24L$ is shown for each pipe. e.g. pipe [24-b4] = 1.000

TABLE VIII

Q_{a4}	P_{a4}	Q_{b4}	P_{b4}	Q_{c4}	P_{c4}	Q_{c5}	P_{c5}
	250,000		600,000		950,000		1,100,000
+ 570,000		−190,000		+190,000		−450,000	
	350,000	− 90,000			1,000,000	−20,000	
+ 75,000		− 65,000		+ 65,000		−7,500	
	370,000	− 45,000			1,025,000		1,095,000
− 30,000			595,000	+ 2,500		+2,000	
− 40,000		− 18,000		0		+500	
	360,000	− 28,000		− 2,500			
+ 10,000			590,000	− 5,000			
0		− 15,000		−7,500			
− 10,000			585,000		1,023,000		
+ 5,000		0		0			
+ 3,000		− 3,000		−500			
		− 4,500					
		− 1,500					

Now returning to the original problem, further corrections for the interior values of p^2 have been made as shown in Table IX. The final values for p^2 in Table IX are close enough to be considered exact and the problem is complete. It now remains to convert these values of p^2 back to values for pressure. These values of pressure for each interior point are shown on a diagram of the drainage area (Fig. 6).

A plot of pressure versus distance along a0 - a14, b0 - b14, c0 - c14, d0 - d14 is shown in Plate 1.

If the drainage area of Problem 1 was of constant permeability, it would be possible to determine the pressure distribution by the conventional analytical method. This has been done along the line a0 - a14, and is shown as a dotted line on Plate 1.

From Plate 1, the following conclusions can be drawn. First, that for constant boundary pressures all the gas from the edge of the field must flow into the first, or marginal, well. The boundary pressure is the highest value of the entire region and gas or any fluid cannot flow of its own accord from a region of low pressure to a region of high pressure. In other words, there is no way for the gas to flow away from the well toward the boundarys because to do this it would have to overcome a negative pressure gradient.

Second, for ratios of permeability change to length equal to or greater to that of this problem (ten to one permeability change in 5000 feet with 1000 feet well spacing) the effect of variable permeability is negligible. In Plate 1 the curve which would represent pressure distribution in a region of constant permeability is so close to the curve for pressure distribution with variable permeability that the error would have negligible effect on the direction of gas flow.

With these two things in mind, the second problem is now to be solved.

TABLE IX

Q_{a1}	P_{a1}	Q_{b1}	P_{b1}	Q_{c1}	P_{c1}	Q_{a2}	P_{a2}	Q_{b2}	P_{b2}
	973,000		1,065,000		1,235,000		452,000		675,000
+131,000		−15,000		+47,500		+62,000		0	
	1,025,000	+38,000				+120,000		+17,000	
−78,000							470,000		
−60,000						+25,000			

Q_{c2}	P_{c2}	Q_{a3}	P_{a3}	Q_{b3}	P_{b3}	Q_{c3}	P_{c3}	Q_{a4}	P_{a4}
	1,069,000		0		505,000		995,000		360,000
−3,000		0 (well)		−1,000		+40,000		−10,000	
								+40,000	

Q_{b4}	P_{b4}	Q_{c4}	P_{c4}	Q_{a5}	P_{a5}	Q_{b5}	P_{b5}	Q_{c5}	P_{c5}
	585,000		1,023,000		620,000		759,000		1,095,000
+1,000		−2,500		+193,000		+12,500		+73,000	
					670,000	+62,000			
				−7,000					

Q_{a6}	P_{a6}	Q_{b6}	P_{b6}	Q_{c6}	P_{c6}	Q_{a7}	P_{a7}	Q_{b7}	P_{b7}
	795,000		900,000		1,160,000		900,000		1,000,000
+68,000		+53,000		+37,000		+70,000		−75,000	
+118,000							950,000	−41,000	
+38,000						−11,000		+108,000	
									1,020,000
								−50,000	

Q_{c7}	P_{c7}	Q_{a8}	P_{a8}	Q_{b8}	P_{b8}	Q_{c8}	P_{c8}	Q_{a9}	P_{a9}
	1,220,000		970,000		1,030,000		1,260,000		1,100,000
+9,000		+180,000		+10,2000		−30,000		+212,000	
		+188,000			1,100,000	+1,000			1,150,000
		+288,000		−52,000				+72,000	
		+12,000		+22,000				+122,000	
			1,050,000	+13,000					
		+60,000							

Q_{b9}	P_{b9}	Q_{c9}	P_{c9}	Q_{a10}	P_{a10}	Q_{b10}	P_{b10}	Q_{c10}	P_{c10}
	1,240,000		1,300,000		1,200,000		1,290,000		1,325,000
−151,000		+30,000		+138,000		−63,000		+35,000	
−130,000				+156,000			1,260,000		
−85,000				+30,000		−20,000			
−86,000									
	1,210,000								
−35,000									

Q_{a11}	P_{a11}	Q_{b11}	P_{b11}	Q_{c11}	P_{c11}	Q_{a12}	P_{a12}	Q_{b12}	P_{b12}
	1,300,000		1,340,000		1,375,000		1,340,000		1,380,000
+32,000		−23,000		+5,000		+24,000		+5,000	
	1,225,000		1,300,000				1,275,000	−31,000	
+17,000		−26,000				+10,000			1,350,000
								−22,000	

Q_{c12}	P_{c12}	Q_{a13}	P_{a13}	Q_{b13}	P_{b13}	Q_{c13}	P_{c13}
	1,410,000		1,420,000		1,430,000		1,435,000
−6,000		+600		−5,000		−1,000	
			1,335,000	−36,000			
		+20,000			1,400,000		
				−19,500			

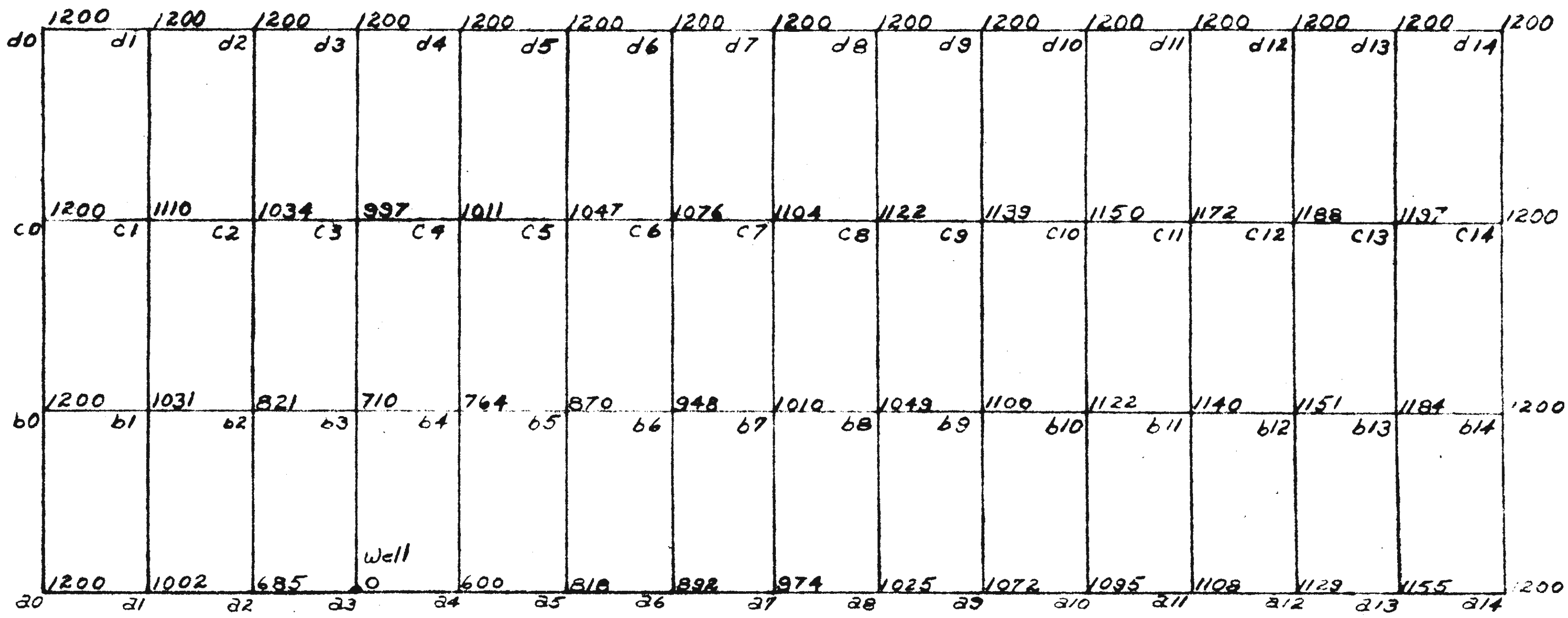

Fig. 6 Showing the final calculated pressures of the pipe network representing the drainage area of problem 1

Plate 1 Pressure distribution curves along a0-a14, b0-b14, c0-c14, d0-d14, for the drainage area of problem 1

PROBLEM 2

SOLUTION OF A PROBLEM INVOLVING FOUR WELLS IN LINEAR ARRAY LEADING INWARD TOWARD THE CENTER OF THE RESERVOIR

In this problem a new drainage area is to be considered. This drainage area is to be drained by three wells. These three wells can be considered as part of a much larger field which is laid out in a square array. Only one row of wells need be considered (as is done here) since for a square array and rectangular boundarys there will be symmetry between rows. These three wells represent the first, or marginal, well and the two adjacent wells leading inward toward the center of the field. It will be shown that for the given boundary conditions that all of the gas from beyond the economic limit of the reservoir will flow into the marginal well. This will be done by considering a field production history over a period of four years starting with the initial year of production and continuing for the following three years.

The solution of the problem involves assuming a number of shut-in pressures over the four year period, which will represent boundary conditions for each of the three wells. The boundary pressures for the three well array will decrease over the four years for the interior boundarys, while the pressure at the extent of the reservoir will remain constant.

As in the first problem, the value of $\frac{K\gamma_o A}{Z\mu}$ is to be variable, but known. The solution for the four years is shown on the following pages.

In this problem the given data for each of the four years is the same, with the exception of the boundary conditions which change from year to year. Consequently, the following information pertains to the entire problem except where noted.

1. Well No. 1, the marginal well, is located at a4.

   Well No. 2 is located at a-4.

   Well No. 3 is located at a-12.

   (See Figs. 7, 9, 10, and 11.)

2. Well pressure for all wells = 0 psi.

   Well bore = 0 feet (point sink).

3. Total drainage area represented by the boundarys a-16, b-16, b9, and a9.

4. The thickness of the sand is constant, i.e., A = 1.

5. The boundary conditions are shown for each year on the figures which represent the drainage areas as follows:

   Year one - Fig. 7

   Year two - Fig. 9

   Year three - Fig. 10

   Year four - Fig. 11

To solve the problem, the following assumptions must be made:

1. The flow is isothermal.

2. The pressure distribution need be found for only one half of the drainage area since there is an axis of symmetry running through the figure from a-16 to a9. Consequently, only one half of the total drainage area is shown in Fig. 7.

3. The initial pressure assumptions at the unknown pipe intersections are to be any convenient value.

4. Since, in the first problem of this thesis it was shown that the effect of variable permeability is negligible, an assumption as to the pressure distribution of the two interior wells will be made. The drainage area and boundary pressures on the interior side of the first well are the same as those of the two interior wells, so the values of pressure computed for the interior side of the first well will be used for the pressure distribution of the other wells. This assumption will not be exact, but will suffice since the exact pressure at any point is not necessary. As long as the direction of the pressure gradient can be determined, the direction of the gas flow can be determined.

As in the preceding problem, values of P^2 will be used since the pressure distribution is proportional to the difference of the squares of the pressure. In the pipes where there is a change in the value of $\frac{K\gamma_0 A}{Z\mu}$ from end of the pipe to the other, an arithmetic average of the two end values will be used. The value of $\frac{k\gamma_0 A}{Z\mu}$ for the entire drainage area is shown graphically in Fig. 8. The method of calculating these pipe constants $\left(\frac{k\gamma_0 A}{Z\mu L}\right)$is the same as that illustrated in Problem 1.

Tables X through XIII show the results of the computations for pressure distribution for each of the four years, Table X for year one, Table XI for year two, etc.

It now remains to plot the pressure distribution for each of the four years. This is done along lines a-16 to a9, and b-16 to b9, on Figures 7, 9, 10, and 11 for the respective years.

TABLE X

Q_{a1}	P_{a1}	Q_{a2}	P_{a2}	Q_{a3}	P_{a3}	Q_{a4}	P_{a4}	Q_{a5}	P_{a5}
	125,000		37,500		12,500	0 (well)	0		1,200
+11,500		+20,000		+21,000				+24,000	
	170,000	−42,000			50,000				25,000
+9,900			160,000	−4,500				+9,000	
	210,000	−3,500		−9,000					50,000
−4,800			120,000		60,000			−4,500	
	200,000	−14,000		+20,000					37,000
−1,600		+4,000			40,000			+14,000	
	180,000		90,000	+5,000				+5,000	
+650		+8,000			50,000				50,000
	185,000	+1,000		−9,000				−4,500	
+1,000					35,000				45,000
	180,000			+1,300				+1,900	
+800									47,000
								−400	

Q_{a6}	P_{a6}	Q_{a7}	P_{a7}	Q_{a8}	P_{a8}
	3,500		23,000		114,000
+22,500		+19,000		+7,500	
+8,150		+13,900			120,000
	50,000		75,000	+7,000	
−6,650		+6,300			200,000
	100,000		150,000	+2,440	
−10,900		+1,800			210,000
	80,000		180,000	+2,000	
+1,080		+3,500			220,000
−4,000		−750		+1,600	
+6,000			200,000		225,000
	100,000	−700		+1,200	
−5,500			195,000		230,000
		−350		+850	

TABLE XI

Q_{a1}	P_{a1}	Q_{a2}	P_{a2}	Q_{a3}	P_{a3}	Q_{a4}	P_{a4}	Q_{a5}	P_{a5}
	125,000		50,000		25,000	0 (well)	0		25,000
+3,700		+12,600		+4,800				+4,400	
+5,750			100,000		75,000				45,000
	140,000	+31,500		−45,100				−2,000	
+1,100		−3,800			42,000				40,000
	150,000		90,000	−11,000				+2,500	
−1,400		−8,200		−8,000					42,000
	160,000		95,000		37,000			−50	
+220		−500		−18,500					35,000
0			87,500	−55,000				−1000	
	155,000	−1,000			55,000				
+310			90,000	−4,500					
	152,500	−3,000			43,000				
−400			85,000	−3,300					
		−800			40,000				
				−1,000					

Q_{a6}	P_{a6}	Q_{a7}	P_{a7}	Q_{a8}	P_{a8}
	50,000		100,000		150,000
+5,000		+6,200		+3,700	
	100,000		180,000		200,000
−11,800		−2,300		+1,000	
−10,500			183,000		210,000
	95,000	−3,800		+400	
−7,500			185,000	+440	
	70,000	−3,500			200,000
+3,550			160,000	+875	
+600		−900			

TABLE XII

Q_{a1}	P_{a1}	Q_{a2}	P_{a2}	Q_{a3}	P_{a3}	Q_{a4}	P_{a4}	Q_{a5}	P_{a5}
	80,000		45,000		20,000	0 (well)	0		20,000
− 1,000		−1,600		+ 500				+ 300	
	75,000		42,000	− 850					17,000
+ 140		−270						− 900	
								0	

Q_{a6}	P_{a6}	Q_{a7}	P_{a7}	Q_{a8}	P_{a8}
	45,000		90,000		140,000
−4,000		−1,000		+ 1,000	
	42,000		95,000		145,000
− 2,400		−1,870		+630	
	35,000		85,000		155,000
− 2,000		−700		− 60	
+ 330					

TABLE XIII

Q_{a1}	P_{a1}	Q_{a2}	P_{a2}	Q_{a3}	P_{a3}	Q_{a4}	P_{a4}	Q_{a5}	P_{a5}
−120	38,000	−130	21,000	−240	9,900	0 (well)	0	−30	8,950

Q_{a6}	P_{a6}	Q_{a7}	P_{a7}	Q_{a8}	P_{a8}
+680	18,500	−150	56,000	−235	128,000

Year One

Fig 7

Above - Showing pipe network for the three well drainage areas. Values of P^2 representing known boundary cond. are shown, e.g. at b8 P^2 = 250000. The value for μL is shown for each pipe where necessary

Below. - Showing the pressure distribution curves along a-16 to a9 and b-16 to b9 for the entire drainage area. To scale.

Fig. 8 - Graphic representation of the value of $k\gamma_0 A/24$ for the drainage area of problem 2 (to scale)

Year Two

Fig 9

Above - Showing the pipe network for the three well drainage area of problem 2. The values of P^2 representing known boundary conditions are shown. eg. at a0 $P^2 = 200{,}000$. The value of $\frac{K\delta h}{24L}$ is shown for each pipe where necessary

Below - Showing the pressure distribution curves along a-16 to a9 and b-16 to b9 for the entire drainage area

Year Three

Fig 10

Above - Showing the pipe network for the three well drainage area of problem 2. The values of P^2 representing known boundary conditions are shown. eg. at a-8 $P^2 = 100,000$. The value of $\frac{k\,r_0\,\Delta}{2\,\mu\,L}$ is shown for each pipe where necessary.

Below - Showing the pressure distribution curves along a-16 to a9 and b-16 to b9 for the entire drainage area

Year Four

Fig 11

Above - Showing the pipe network for the three well drainage area of problem 2. The values of P^2 representing known boundary conditions are shown. eg at b6 $P^2 = 53000$

Below - Showing the pressure distribution curves along a-16 to a9 and b-16 to b9 for the entire drainage area

CONCLUSIONS

From the solutions of the two problems, the following conclusions can be drawn. Considering first the initial problem of the thesis, two things are apparent from its solution. One, for normal variance of permeability with distance, the pressure distribution curves are only slightly different from the curve of a similar region of constant permeability. That is, permeability changes have little effect on pressure distribution where distances are on the order of those in problem one, or greater. Two, for boundary conditions such as those given in problem one, all the gas from the given drainage area will flow into the single well. The proof of this lies in the shape of the curves in Plate One and a paraphrase of one of the laws of thermodynamics. Namely, gas cannot, in a natural process, flow from a region of low pressure to a region of high pressure. From the shape of the curves in Plate One, it can be seen that for any point in the drainage area the pressure gradient is toward the well.

In the second problem three wells in line array leading toward the center of the field were considered over a period of four years. From the pressure distribution curves for each year the following conclusions can be drawn. First, that for the given boundary conditions all the gas from beyond the economic limit of the reservoir will flow into the first, or marginal well, at all times during the four year production period. Also, the marginal well will drain half of the area between it and the next well. Since the volume of gas in the drainage area of the marginal well may at times be quite large in relation to the volume of gas in the drainage area of any interior well, it may be quite advantageous to drill and produce marginal wells. Any bottom hole pressure below that which would exist if the well was not producing would cause the pressure gradient of the marginal drainage area to be toward the marginal well, for the given problem in this thesis. Naturally, the bottom hole pressure of the marginal well will influence its production rate, zero pressure giving the maximum.

Some comment on the use of the relaxation method in solving steady state fluid flow problems is in order here for those who may wish to continue work in this field. Technically, the relaxation method is valid for problems of this type and will lead to satisfactory solutions, but it has some drawbacks. For problems involving a large number of unknown points, say thirty or more, the work required for solution of the problem may be out of reason. It might be, however, that an electric circuit could be set up to duplicate the pipe network and thereby eliminate the tedium and time required for thousands of calculations.

Also, the relaxation method may not give satisfactory solutions where multiple sinks are involved if there is no symmetry of drainage areas. It is necessary to locate all the highs and valleys of pressure distribution before laying out the equivalent pipe network, and have pipes in close proximity to the regions of maximum and minimum to obtain valid solutions. In effect, the pressure distribution must be known to some extent, or else the equivalent pipe network must involve enough points to insure that all unknown maximum and minimum pressure changes be located.

Finally, all boundary conditions must be known or obtained from some other data prior to solution, as the relaxation method requires known boundary conditions.

Two problems have been solved by the relaxation method in this thesis. The first involves a single marginal well draining a rectangular drainage area of variable permeability. With the given boundary conditions two things are shown. First, that variable permeability has negligible effect on pressure distribution and second, that all the gas from the entire drainage area will flow into the well.

The second problem involves three wells in line array draining a rectangular area of variable permeability. By calculating a four year pressure distribution history it has been shown that for the given boundary conditions all the gas from beyond the economic limit of the reservoir will flow into the first, or marginal well.

Some comments on the relaxation method, as a method of solution of fluid flow problems involving multiple sinks, are made in the conclusions.

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