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Analysis of a char-forming ablator

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ANALYSIS OF A CHAR-FORMING ABLATOR

BY

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THESIS

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ABSTRACT

The purpose of this thesis is to conduct an analytical analysis of a char-forming ablation material. This is one phase of ablation heat transfer.

Fourier's general equation for one dimensional heat conduction is transformed to moving coordinates and is then solved using the applicable boundary conditions. Solutions of this equation are valid for initial studies in ablation heat transfer.

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LIST OF SYMBOLS

Q..... applied heat flux, Btu/hr-ft^2

q_r.... heat radiated, Btu/hr-ft^2

q_g.... heat absorbed by gas, Btu/hr-ft^2

q_a.... heat absorbed by virgin material during ablation, Btu/hr-ft^2

q_m.... heat conducted into virgin material, Btu/hr-ft^2

x..... coordinate normal to ablative surface

x'.... $x-V_a\theta$, coordinate normal to ablative surface in moving reference frame

α.... thermal diffusivity, Ft^2/hr

V_a.... ablation velocity, Ft/hr

t..... temperature, °F

t_a.... ablative temperature, °F

t_b.... char burn-off temperature, °F

t_∞.... temperature at $x = \infty$, °F

σ.... Stefan-Boltzmann constant, Btu/hr-ft^2-°R^4

ϵ..... emissivity

m_g.... mass rate of gas formation, Lb/hr-ft^2

C_m.... specific heat of virgin material, Btu/lb-°F

C_g.... specific heat of gas, Btu/lb-°F

δ.... thickness of char layer, Ft

h..... heat of ablation of virgin material, Btu/lb

ρ_M.... density of virgin material, Lb/ft^3

K_m.... thermal conductivity of virgin material, Btu/hr-ft-°F

θ..... time, Hr

P..... percentage of ablated material given off as a gas

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INTRODUCTION

The coming of the Space Age has brought about temperature control problems that had never before been experienced. One of these problems was thermal protection against the intense aerodynamic heating experienced by a body during atomspheric re-entry. Whether the body be a manned space vehicle or a ballistic missile the problem of aerodynamic heating is present. The most successful solution to this heating problem has been provided by ablation heat shields.

The first work with ablation shields involved materials which ablated completely, i.e. surface ablators. As re-entry velocities became greater the thickness of surface ablating materials required became too large. Since the temperature of the bond line where the shield joins the vehicle is the critical factor a method of blocking the heat flow as well as dissapating the heat was sought. The charring ablator satisfied both of these requirements. This material, usually a phenol-nylon or phenol-fiberglas compound, does not ablate completely. As the ablation process takes place a porous residue or char, predominately carbon, is left in its wake. Heat is dissapated by ablation of the virgin material. The char and the gas flowing through it form a block to the incoming heat flux.

The purpose of this thesis is to obtain equations for the temperature distribution and ablation rate for a charring ablator. A semi-infinite section of material which is subjected to a high heat flux at one end will be considered. Fourier's general equation for one-dimensional heat transfer will be solved, with coordinate transformations, to yield these equations.

REVIEW OF LITERATURE

Analytical studies of ablation heat transfer have only been undertaken in recent years. Most of these studies are found in journals rather than textbooks. Since this subject is quite varied the usual analytical approach encompasses a very small and definite aspect of the problem.

An analytical approach to the problem of transient heat conduction in solids is presented in the text of H. S. Carslaw*(1). One method of solution for this type of problem is that of finite-differences, using a digital computer. Material for this numerical analysis approach is found in the work by Dusinberre (2). Schneider (3) and Kreith (4) show graphical methods of solution.

A paper by Swann (5) gives several possible configurations for thermal protection of manned re-entry vehicles. An analysis of different modes of ablation is given in a paper by Adams (6).

Blecher and Sutton (7) compare three approximate methods for calculating re-entry ablation. Miller (8) presents a transient one-dimensional analysis for an ablating surface. An analysis of a charring ablation is made by Barriault and Yos (9) using three reference frames. By the use of three frames of reference, rather than two as used in this thesis, the ablation rates of the char and virgin material need not be assumed equal. The thermal degradation of a char-forming plastic during hypersonic fight is investigated by Scola and Gilbert (10).

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*Typed numbers refer to Bibliography

The text by Rohsenow and Choi (11) has a short section on surface ablation. The parameters involved during surface ablation are determined in a thesis by Lin (12).

The phenomenon of transpiration cooling is treated in a paper by Weinbaum and Wheeler (13). They have demonstrated that the solid and coolant temperatures are very nearly the same throughout the porous structure provided the pores are sufficiently small. Their findings are applicable to the gas flow through the char layer in this thesis problem. A thesis by Posgay (14) presents a numerical solution of a transpiration problem.

STATEMENT OF THE PROBLEM

A semi-infinite section of ablation material will be investigated. This section has one end and extends to infinity in the other, or x, direction. The origin of the coordinate system will be taken at one end and at $x = \infty$ the temperature will be an assumed constant t_{∞}. The solid from which this section is taken is assumed to be large enough that the heat transfer is one-dimensional. A large heat flux, Q, is applied to the end of the section. The assumptions made for the analysis of this problem are summarized below:

(1) The size of the voids which constitute the porosity of the charred layer is sufficiently small, compared with the charred layer thickness, so that the counter-flowing gases are always at the same temperature as the solid skeleton at corresponding points where they are in contact.

(2) The gas flow is unimpeded by the porous layer; this means that the flow rate and gas pressure are not functions of the coordinate x.

(3) The thermal and physical properties of all phases are independent of temperature.

(4) Heat transfer by radiation across the voids and by back diffusion in the gas stream will be neglected.

(5) The ablation velocities or ablation rates of the char and virgin material are equal.

The ablative process and char formation begins when the face $x = 0$

reaches the ablative temperature t_a. Once ablation is initiated the face of solid material will recede in the x-direction at the velocity of ablation V_a leaving a porous char. Under the continued application of the heat flux the face of the char will reach the char burn-off temperature t_b and the face of the char will then recede at the velocity of ablation. Since there is a time differential between the material reaching t_a and the char face reaching t_b, when both faces are moving at V_a there will remain between them a layer of char of thickness δ . The gas formed when the material ablates must travel through this layer.

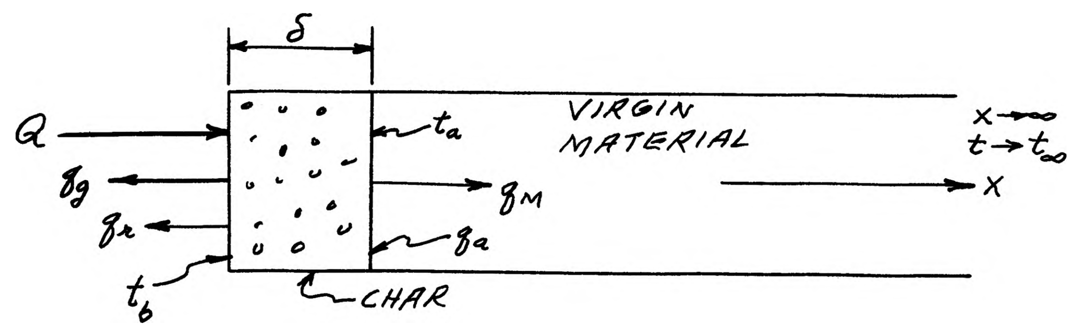

Figure 1. Ablative Section Under Quasi-steady Conditions.

A heat balance on the section yields,

$$Q = q_r + q_g + q_a + q_M \qquad (1)$$

where Q is the applied heat flux, q_r is the heat radiated, q_g is the heat absorbed and carried away by the gas, q_a is the heat absorbed by the material during ablation, and q_m is the heat conducted into the virgin material.

Once the material face and the char face have reached a steady burning rate V_a, this configuration may be considered as a moving heat source (15). The heat source is on the x'-axis of a coordinate system moving with a velocity, V_a, with respect to the stationary coordinate system and directed in the x direction. With this scheme a stationary observer on the x-axis would notice a change in temperature of his surroundings as the source moved along, while if the observer were stationed at a point on the moving x'-axis he would notice no such change in temperature. Expressed mathematically this condition is $\frac{\partial t}{\partial \theta} = 0$, (2) in the moving coordinate system. This "apparent" steady-state temperature condition is known as the quasi-steady state.

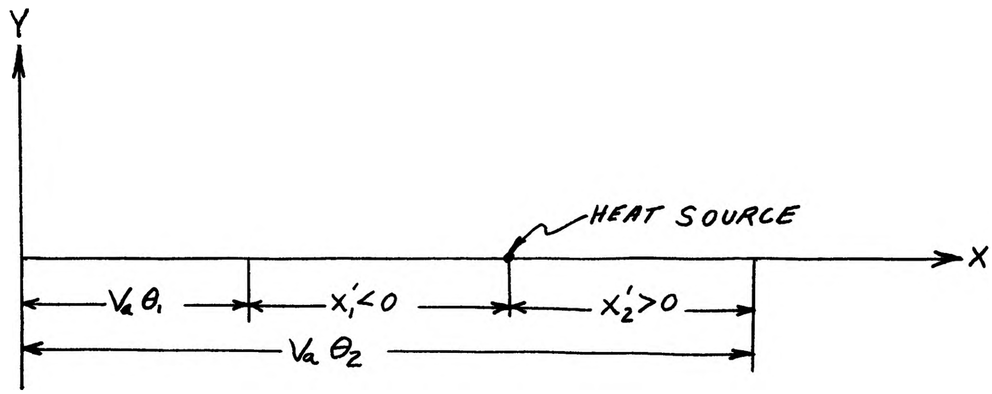

Figure 2. Dual Coordinate System For Moving Heat Sources

In the stationary system the temperature must satisfy the well known Fourier equation for one-dimensional heat transfer $\frac{\partial^2 t}{\partial x^2} = \frac{\partial t}{\partial \theta}\frac{1}{\alpha}$ (3). This equation is transformed from stationary to moving coordinates. Two new variables are defined as follows:

$$x' = x - V_a\theta, \quad \theta' = \theta \qquad (4)$$

Now differentiating equations (4)

$$\frac{\partial x'}{\partial x} = 1, \quad \frac{\partial x'}{\partial \theta} = -V_a, \quad \frac{\partial \theta'}{\partial x} = 0, \quad \frac{\partial \theta'}{\partial \theta} = 1 \qquad (5)$$

Therefore,

$$\frac{\partial t}{\partial x} = \frac{\partial t}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial t}{\partial \theta'}\frac{\partial \theta'}{\partial x} = \frac{\partial t}{\partial x'}, \quad \frac{\partial^2 t}{\partial x^2} = \frac{\partial^2 t}{\partial x'^2} \qquad (6)$$

and

$$\frac{\partial t}{\partial \theta} = \frac{\partial t}{\partial x'}\frac{\partial x'}{\partial \theta} + \frac{\partial t}{\partial \theta'}\frac{\partial \theta'}{\partial \theta} = -V_a\frac{\partial t}{\partial x'} + \frac{\partial t}{\partial \theta'} \qquad (7)$$

Substituting these partial derivatives in the Fourier equation (3)

$$\frac{\partial^2 t}{\partial x'^2} = \frac{1}{\alpha}\left(-V_a \frac{\partial t}{\partial x'} + \frac{\partial t}{\partial \theta'}\right) \qquad (8)$$

Since in the moving coordinate system $\frac{\partial t}{\partial \theta'} = 0$, then

$$\frac{\partial^2 t}{\partial x'^2} = -\frac{V_a}{\alpha}\frac{\partial t}{\partial x'}$$

or

$$\frac{d^2 t}{dx'^2} = -\frac{V_a}{\alpha}\frac{dt}{dx'} \qquad (9)$$

The general solution of equation (9) is,

$$t = C_1 e^{-\left(\frac{V_a}{\alpha}\right)x'} + C_2 \qquad (10)$$

which must satisfy the following boundary conditions stated in terms of the transformation variable x':

$$\frac{dt}{dx'} \rightarrow 0,\ t \rightarrow t_\infty,\ \text{AS } x' \rightarrow \infty \qquad (11)a$$

$$Q = q_r + q_g + q_a - K_M \frac{dt}{dx'},\ \text{AS } x' \rightarrow 0 \qquad (11)b$$

Then

$$\frac{dt}{dx'} = -C_1\left(\frac{V_a}{\alpha}\right) e^{-\left(\frac{V_a}{\alpha}\right)x'} \qquad (12)$$

To satisfy the first boundary condition (11)a

$$t_\infty = C_1(0) + C_2$$

hence

$$C_2 = t_\infty \qquad (13)$$

For the second boundary condition (11)b

$$Q = q_r + q_g + q_a - K_M\left[-C_1\left(\frac{V_a}{\alpha}\right)e^{-\left(\frac{V_a}{\alpha}\right)x'}\right]_{x'=0}$$

whereby

$$C_1 = \left(Q - q_r - q_g - q_a\right)\frac{\alpha}{K_M V_a} \qquad (14)$$

Substituting C_1 and C_2 into equation (10)

$$t = \left(Q - q_r - q_g - q_a\right)\frac{\alpha}{K_M V_a} e^{-\left(\frac{V_a}{\alpha}\right)x'} + t_\infty \qquad (15)$$

In equation (15) there are several terms that must be determined. These are q_r, q_g, q_a, and the ablation velocity V_a.

The heat radiated from the face of the char q_r will be considered first. The Stefan Boltzmann law of total radiation (16) states that the energy emitted from a body is proportional to the fourth power of its absolute temperature. The body under consideration may be thought of as nearly a black body, emissivity above .9. The equation for this radiant energy emission is:

$$q_r = \epsilon \sigma t_b^4 \qquad (16)$$

where ϵ is the emissivity of the char, σ is the Stefan-Boltzmann constant, and t_b is the char burn-off temperature.

As the gas formed by the ablating material passes through the layer of char it will absorb an amount of heat given by this equation:

$$q_g = m_g C_g \delta \frac{\partial t}{\partial x'} \qquad (17)$$

$$m_g = P V_a \rho_m \qquad (18)$$

where m_g is the mass rate of gas formation, C_g is the specific heat of the gas at constant pressure, δ is the thickness of the char, P is the percentage of ablated material given off as a gas, V_a is the ablation velocity, ρ_m is the density of the virgin material, and $\frac{\partial t}{\partial x'}$, is the temperature gradient in the char in the x' direction. The gas is formed at a constant rate at a temperature t_a. In the quasi-steady state V_a is constant, the char face will be at t_b and the layer thickness will be δ. Therefore the temperature gradient will be constant and the amount of heat absorbed by the gas will also be constant.

To continue ablation the material must absorb a definite amount of heat. This quantity of heat is given by:

$$q_a = h \rho_m V_a \qquad (19)$$

where h is the heat of ablation, ρ_m is the density of the virgin material, and V_a the velocity of ablation. Once the quasi-steady state is established the ablation velocity will be constant. Since h and ρ_m are also constant, the rate of heat absorption is constant.

To obtain a statement for V_a, a relationship between V_a and some known quantity must be found. The known quantity selected in this case was the heat conducted into the virgin material q_m. The relationship between V_a and q_m is:

$$q_M = -K_M \frac{\partial t}{\partial x'} = V_a \rho_M C_M (t_a - t_\infty) \qquad (20)$$

The right-hand expression of this statement gives the amount of heat transferred by conduction through the ablation area to the virgin material. The ablation velocity, V_a, is the governing parameter since all other terms in the expression are constant. Differentiating equation (15) with respect to x' gives:

$$\frac{dt}{dx'} = (Q - q_r - q_g - q_a)\frac{\alpha}{K_M V_a}\left(-\frac{V_a}{\alpha}\right)e^{-\left(\frac{V_a}{\alpha}\right)x'}$$

$$-K_M \frac{dt}{dx'}\Big|_{x'=0} = Q - q_r - q_g - q_a \qquad (21)$$

From equation (20)

$$V_a = \frac{-K_M \frac{dt}{dx'}}{\rho_M C_M (t_a - t_\infty)}$$

$$= \frac{Q - q_r - q_g - q_a}{\rho_M C_M (t_a - t_\infty)} \qquad (22)$$

After transposing, combining with equations (17) and (19), and rearranging,

$$V_a = \frac{Q - q_r}{\rho_M \left[C_M (t_a - t_\infty) + h + PC_g (t_b - t_a)\right]} \qquad (23)$$

By replacing x' with $x-V_a\theta$ in equation (15), the final form of the temperature distribution equation becomes:

$$t = (Q - q_r - q_g - q_a)\frac{\alpha}{K_M V_a} e^{-\left(\frac{V_a}{\alpha}\right)(x - V_a\theta)} + t_\infty \qquad (24)$$

where q_r, q_g, q_a, and V_a are defined by their respective equations numbered (16), (17), (19), and (23).

CONCLUSIONS

The temperature distribution and ablation velocity in a semi-infinite section of ablative material are given by equation (24) and equation (23). In these equations the applied heat flux and time are the controlling variables since all the other parameters are constant.

The equations derived in this thesis are valid for initial studies of charring ablation. The method used in the derivation of these equations can be used, with different assumptions and suitable boundary conditions, to produce equations that are more detailed, but are also quite complex. It must be pointed out that, at present, values for some of the parameters, particularly the properties of the gas, are not available. Extensive experimental work is needed to determine these properties. By assuming values for the properties one can readily obtain "order of magnitude" numerical results from the given equations.

The ablation velocity is a determining factor in the selection of the heat shield thickness required. If the ablation velocity can be decreased the thickness can be reduced. The heat blocking effect of the gas flowing through the char layer reduces the ablation velocity in a charring ablator. A material that ablates completely, surface ablation, does not produce this heat blocking. Another factor to be considered is the heat radiated, q_r. It is shown by equation (16) that the amount of heat radiated is proportional to the fourth power of the absolute temperature. Heat is radiated from the char face during charring ablation and from the ablating surface during surface ablation. Since the char burn-off temperature, t_b, is greater than the ablation temperature, t_a, more heat is radiated from a charring ablator.

To demonstrate the reduction of ablation velocity provided by a charring ablator a comparison of two ablation materials will be made. The materials selected are a phnolic-nylon compound, a charring ablator, and Teflon, a surface ablator. The variables for which values are assumed are: Q, t_∞, and the properties of the gas. A typical value for the heat flux, Q, encountered during atomspheric re-entry is 500 Btu/sec-ft^2. The limiting bond-line temperature, t_∞, is assumed to be 500°F. The gas formed during ablation is assumed to have the properties of carbon dioxide and it is further assumed that P, the percentage of ablated material that is given off as a gas, is 10%. The assumptions made for the gas properties apply to the charring ablator only.

The surface ablator will be treated first. Lin (12) states the equation for surface ablation velocity as:

$$V_a = \frac{Q - q_r}{\rho_M\left[c_M(t_a - t_\infty) + h\right]} \qquad (25)$$

The properties of Teflon are:

t_a = 1040 °F	h = 3120 Btu/lb
C_m = .40 Btu/lb-°F	ϵ = .9
ρ_m = 135 Lb/ft^3	

The given properties are substituted into equation (25), with equation (16) being substituted for q_r to give:

$$V_a = \frac{(500)(3600) - (.9)(.171\times10^{-8})(1500)^4}{135\left[(.4)(1040-500) + 3120\right]} = 3.98\ \frac{FT}{HR} \qquad (26)$$

The ablation velocity of the phnolic-nylon compound will be calculated, using the statement for ablation velocity derived in this thesis, equation (16).

The properties of the phnolic-nylon compound are:

t_a = 1832 °F $\qquad$ ρ_m = 102 Lb/ft^3

t_b = 2960 °F $\qquad$ h = 4180 Btu/lb

C_m = .275 $\qquad$ ϵ = .9

The specific heat, C_g, of the assumed gas, carbon dioxide, is .3 Btu/lb-°F.

Substituting these properties into equation (23) and again substituting equation (16) for q_r the ablation velocity is given as:

$$V_a = \frac{q - q_r}{\rho_M\left[C_M(t_a - t_\infty) + h + PC_g(t_b - t_a)\right]}$$

$$= \frac{(500)(3600) - (.9)(.171\times10^{-8})(3420)^4}{102\left[(.275)(1832-500) + 4180 + (.1)(.3)(2960-1832)\right]} = 3.43\ \frac{FT}{HR} \qquad (27)$$

By comparing the results of equations (26) and (27) it can be seen that a 13.8% reduction in ablation velocity is provided by the phnolic-nylon compound, a charring ablator. The magnitude of the percentage reduction in ablation velocity is, of course, dependent upon the assumptions made. A charring ablator that produces a large amount of gas, whose specific heat, C_g, is large, will provide a more efficient heat shield.

The high heat flux associated with atomspheric re-entry is produced by aerodynamic heating. If this heat is to flow into the re-entry body a temperature difference must exist between the boundary layer and the surface of the body. This is true whether the surface is a charring or surface ablator. The amount of heat that can flow into the body is directly proportional to the temperature difference. Again the char surface temperature, t_b, is greater than the ablation temperature, t_a, so the boundary layer-char surface temperature difference will be smaller. This smaller temperature difference will decrease the amount of heat flowing into the body. An analysis which includes a variable heat flux is beyond the scope of this thesis but it was felt that this advantage of a charring ablator should, at least, be mentioned.

The charring ablator is not the ultimate in heat protective materials. Today's rapidly advancing technology will, undoubtedly, produce a more satisfactory material or method for thermal protection. At present, however, the char-forming ablation materials provide an effective solution to the temperature control problem. This thesis has developed a simplified method of approach to the analysis of a char-forming ablation material.

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