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DERIVATION OF A METHOD TO DETERMINE REACTIONS AND DEFLECTION OF A CONTINUOUS BEAM ON AN ELASTIC FOUNDATION

BY

LOUIS RICHARD FUKA

A

THESIS

submitted to the faculty of the

SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI

in partial fulfillment of the requirements for the

Degree of

MASTER OF SCIENCE, CIVIL ENGINEERING

Rolla, Missouri

1963

Approved by

John L. Best (advisor) Bill L. Atchley

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ABSTRACT

The purpose of this thesis is to develop a method for determining the reactions and deflection curve of continuous beams partially supported by an elastic foundation. The method involves representation of an arbitrary loading and the intermediate redundant reactions as Fourier Series. Redundant intermediate reactions are determined by the solution of an equal number of linear equations. The remaining reactions are determined by the application of statics.

This same method can be used for simply supported beams on an elastic foundation and for continuous beams without an elastic foundation.

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LIST OF SYMBOLS

1. $a_1, a_2, a_3, \ldots, a_m$: span ratios
2. $A_n, A_{n1}, A_{n2}, \ldots A_{nm}$: Fourier coefficients of loading and intermediate reactions.
3. $B_1, B_2, B_3, \ldots, B_n$: amplitudes of the sine curves of which they are coefficients.
4. $D_n = A_n - A_{n1} - A_{n2} - A_{n3} - \ldots - A_{nm}$
5. E : modulus of elasticity of the beam
6. F_{uv} : Coefficients of Intermediate Reactions
7. I : constant moment of inertia of the beam
8. K. : constant coefficient of elasticity of the elastic foundation
9. L : length of the beam
10. m : number of intermediate reactions
11. P : concentrated load
12. p(x) : loading on beam composed of q(x) and intermediate reactions
13. q(x) : external loading on system
14. r : infinitesimal width
15. $R_1, R_2, R_3, \ldots, R_m$: intermediate reactions
16. R_L and R_R: left and right reactions, respectively
17. $S_n = \frac{EI\pi^4}{L^4}\left[n^4 + \frac{KL^4}{EI\pi^4} \right]$
18. U_a : Work done by applied forces
19. U_b: strain energy of the beam in bending
20. U_d : strain energy of deformation of the elastic foundation

21. U_t : total strain energy in the system composed of both elastic foundation and beam

22. x : horizontal distance from left end of beam

23. y : vertical deflection of the beam

24. y" : second derivative of the deflection of the beam

25. $\delta_1, \delta_2, \delta_3, \ldots, \delta_m$: deflections at intermediate supports

I. INTRODUCTION

The object of this investigation is to determine a reasonably rapid and accurate means of obtaining the reactions and deflection curve of a continuous beam on an elastic foundation. Additional desirable qualities of the method should include relative ease of application, need of minimum computing equipment (preferably just a slide rule), and generalization of the method to make it applicable to other types of problems. The method described in this thesis has these qualities to a greater or lesser extent depending upon the particular problem.

A few possible applications might include:

1. Footings on piling
2. Pipe on piling
3. Continuous highway slab on piling
4. Braced laminate construction

The first three of these problems are prevalent in areas which have soil that is poor in bearing. The structures are usually built neglecting the elastic supporting effect of the soils and designing the piling to take full load and any possible uplift effect the soil may have. The fourth possible application concerns laminates with bracing ribs which are relatively stiff compared to the filler material.

The author selected this subject because of an interest in applying mathematics to the development of new methods in civil engineering and engineering mechanics. This particular problem seemed worth solving because, while it has several feasible applications, there is to his knowledge no other method of solution.

The author owes many thanks to his beloved wife Mary and to the teachers who helped and encouraged him during his studies at Missouri School of Mines and Metallurgy, especially Professor John Best, his advisor, and Professor S. J. Pagano.

II. REVIEW OF LITERATURE

This method is actually a generalization and extension of the methods developed by Seng-Lip Lee[1] and M. Hetenyi.[2] Lee developed a method to determine reactions and the deflection curve of continuous beams by arbitrary load function and the intermediate redundant reactions in infinite trigonometric series. Hetenyi developed a method to determine the deflection curve of a simply supported beam on an elastic foundation by representing the deflection curve of a simply supported beam by a Fourier sine series and evaluating the Fourier coefficient by application of the strain energy equations. Neither method could be applied to solve both the continuous beam problem and the simply supported beam on an elastic foundation. The method developed in this thesis is a synthesis of the two methods outlined above. It can be applied to solve either of these problems in addition to the more complex continuous beam on an elastic foundation problem. As far as the author knows, there is no other practical method to solve this latter problem.

III. DERIVATION OF METHOD

The following is a succinct worded outline of the procedure used to derive the equations necessary for solution of continuous beam on elastic foundation problems.

A. Determination of the total strain energy in the system composed of beam and elastic foundation in terms of trigonometric series.

B. Representation of beam loading and intermediate reactions as Fourier sine series.

C. Evaluation of Fourier coefficient in strain energy equation in terms of the intermediate supports and beam loading.

D. Check of derived deflection with results obtained by Lee and Hetenyi.

E. Insertion of boundary conditions (deflections at intermediate supports) and obtaining m equations in m unknown intermediate reactions. If the system satisfies Hooke's law (stress is proportional to strain) the same m equations can be obtained by applying the Theorem of Castigliano to the total energy in the system.

F. Isolation of unknown intermediate supports.

A. DETERMINATION OF TOTAL STRAIN ENERGY IN THE SYSTEM

Consider the beam shown in Figure 1, page 6. The strain energy U_b of a beam in bending is:[2]

$$U_b = \frac{EI}{2} \int_0^L (y'')^2 \, dx$$

The strain energy of deformation U_d, in the elastic foundation is:[2]

$$U_d = \frac{K}{2} \int_0^L y^2 \, dx$$

where:

U_b = strain energy of the beam in bending

E = modulus of elasticity of the beam

I = constant moment of inertia of the beam

L = length of the beam

y = vertical deflection of the beam

y'' = second derivative of the deflection of the beam

K = constant coefficient of elasticity of the elastic foundation

U_d = strain energy of deformation of the elastic foundation

x = horizontal distance from left end of the beam

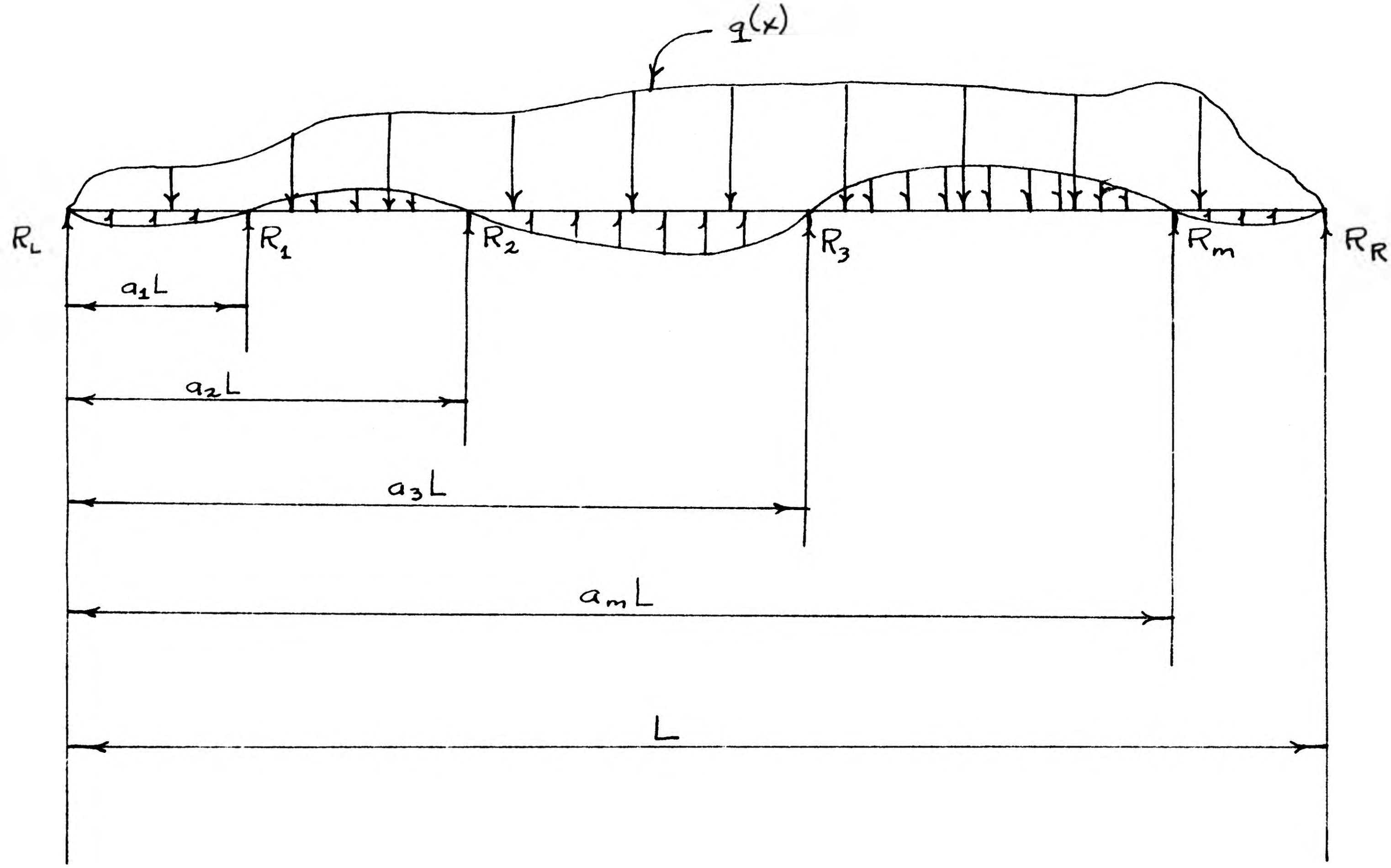

Figure 1

Then the total strain energy U_t in the system composed of both elastic foundation and beam is equal to:

$$U_t = U_b + U_d$$

$$(1) \quad U_t = \frac{EI}{2}\int_0^L (y'')^2\,dx + \frac{K}{2}\int_0^L y^2\,dx$$

The deflection curve of beams can be represented in the form:

$$(2) \quad y = B_1 \sin\frac{\pi x}{L} + B_2 \sin\frac{2\pi x}{L} + \ldots + B_n \sin\frac{n\pi x}{L} + \ldots$$

This form of representing the deflection is especially amenable to simply supported beams since y and y" are zero at $x = 0$ and $x = L$. $B_1, B_2, B_3, \ldots B_n$ are the amplitudes of the sine curves of which they are coefficients. These coefficients will be determined in Part C.

Equation (2) can be represented in the more compact form:

$$(2a) \quad y = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L}$$

$$(2b) \quad y'' = -\sum_{n=1}^{\infty} \frac{n^2\pi^2}{L^2} B_n \sin\frac{n\pi x}{L}$$

Substituting equations (2a) and (2b) in equation (1) yields:

$$(3)\quad U_t = \frac{EI}{2}\int_0^L \left[\sum_{n=1}^{\infty} -\frac{n^2\pi^2}{L^2} B_n \sin\frac{n\pi x}{L}\right]^2 dx$$

$$+\frac{K}{2}\int_0^L \left[\sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L}\right]^2 dx$$

Expansion of the integrands yields two distinct types of integrals:

$$\int_0^L \sin\frac{u\pi x}{L}\sin\frac{v\pi x}{L}\,dx$$

where u and v are any positive integers.

(a) $u \neq v$

(b) $u = v$

These two general sets of integrals fulfill the conditions of orthogonality. (The first general set of integrals equals zero; the second general set equals $\frac{L}{2}$.)

Keeping this in mind and evaluating Equation (3) we obtain:

$$(3a)\quad U_t = \sum_{n=1}^{\infty} B_n^2 \left[\frac{EIn^4\pi^4}{4L^3} + \frac{KL}{4}\right]$$

B. REPRESENTATION OF BEAM LOADING AND INTERMEDIATE REACTIONS BY FOURIER SERIES

The applied load $q(x)$ and the intermediate reactions R_1, R_2, R_3, ..., R_m can be represented by Fourier series in the following manner:

$$(4a)\quad q(x) = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L}$$

$$(4b)\quad R_1 = \sum_{n=1}^{\infty} A_{n1} \sin\frac{n\pi x}{L}$$

$$(4c)\quad R_2 = \sum_{n=1}^{\infty} A_{n2} \sin\frac{n\pi x}{L}$$

$$(4d)\quad R_3 = \sum_{n=1}^{\infty} A_{n3} \sin\frac{n\pi x}{L}$$

$$\vdots$$

$$(4e)\quad R_m = \sum_{n=1}^{\infty} A_{nm} \sin\frac{n\pi x}{L}$$

where A_n, A_{n1}, A_{n2}, A_{n3}, ..., A_{nm} are Fourier coefficients determined as follows:

$$(5a)\quad A_n = \frac{2}{L}\int_0^L q(x) \sin\frac{n\pi x}{L}\, dx$$

$$(5b)\quad A_{n1} = \frac{2}{L} \lim_{r \to 0} \int_{a_1L-r}^{a_1L+r} \frac{R_1}{2r} \sin\frac{n\pi x}{L}\, dx$$

$$= \lim_{r \to 0} -\frac{2}{L}\frac{R_1}{2r}\left[\frac{L}{n\pi} \cos\frac{n\pi x}{L} \Big|_{a_1L-r}^{a_1L+r} \right]$$

$$= \lim_{r \to 0} -\frac{R_1}{rn\pi}\left[\cos\frac{n\pi}{L}(a_1L + r) - \cos\frac{n\pi}{L}(a_1L - r) \right]$$

Expanding the cosines of the sum and difference of two arguments yields:

$$A_{n1} = \lim_{r \to 0} \frac{-2R_1}{rn\pi}\left[-\sin(n\pi a_1)\, \sin\frac{n\pi r}{L} \right]$$

Application of L'Hospital's Rule yields:

$$(5b)\quad A_{n1} = \frac{2}{L} R_1 \sin(n\pi a)$$

In a similar manner:

$$(5c)\quad A_{n2} = \frac{2}{L} \lim_{r \to 0} \int_{a_2L-r}^{a_2L+r} \frac{R_2}{2r} \sin\frac{n\pi x}{L}\, dx = \frac{2R_2}{L} \sin(n\pi a_2)$$

$$(5d)\quad A_{n3} = \frac{2}{L} \lim_{r \to 0} \int_{a_3L-r}^{a_3L+r} \frac{R_3}{2r} \sin\frac{n\pi x}{L}\, dx = \frac{2R_3}{L} \sin(n\pi a_3)$$

.
.
.

(5e) $$A_{nm} = \frac{2}{L} \lim_{r \to 0} \int_{a_m L - r}^{a_m L + r} \frac{R_m}{2r} \sin\frac{n\pi x}{L}\, dx = \frac{2R_m}{L} \sin(n\pi a_m)$$

Consider the continuous beam on an elastic foundation shown in Figure 1 as a simply supported beam of length L with a loading p(x).

(6) Then $p(x) = q(x) - R_1 - R_2 - R_3 - R_4 - \dots - R_m$

$$p(x) = (A_n - A_{n1} - A_{n2} - A_{n3} - \dots - A_{nm}) \sin\frac{n\pi x}{L}$$

(6a) Let $D_n = (A_n - A_{n1} - A_{n2} - A_{n3} - \dots - A_{nm})$

Then $$p(x) = \sum_{n=1}^{\infty} D_n \sin\frac{n\pi x}{L}$$

C. EVALUATION OF COEFFICIENT B_n

Consider the beam shown in Figure 1 to be a simply supported beam of length L with external loading p(x). From equation (3a), if a small variation is produced in a B_n term, the resulting change in the total strain energy U_t is:

$$\frac{\partial U_{tn}}{\partial B_n} = \frac{\partial}{\partial B_n}\left\{ B_n^{\,2}\left[\frac{EIn^4\pi^4}{4L^3} + \frac{KL}{4}\right]\right\}$$

$$(7)\quad d\,U_{tn} = B_n\left[\frac{EI\,n^4\pi^4}{2L^3} + \frac{KL}{2}\right] dB_n$$

At the same time the loading p(x) will do work of the amount:

$$\int_0^L p(x)\,\frac{\partial y}{\partial B_n}\,dB_n\,dx = dU_a = \int_0^L p(x)\,\sin\frac{n\pi x}{L}\,dB_n\,dx$$

$$(8)\quad dU_a = dB_n \int_0^L p(x)\,\sin\frac{n\pi x}{L}\,dx$$

where U_a = work done by applied load.

Work done by applied load equals energy added to the system.

Equating (7) and (8):

$$dB_n \int_0^L p(x) \sin \frac{n\pi x}{L} \, dx = B_n \left[\frac{EIn^4\pi^4}{2L^3} + \frac{KL}{2} \right] dB_n$$

$$(9) \quad B_n = \frac{\int_0^L p(x) \sin \frac{n\pi x}{L} \, dx}{\frac{EIn^4\pi^4}{2L^3} + \frac{KL}{2}} = \frac{\int_0^L D_n \sin^2 \frac{n\pi x}{L} \, dx}{\frac{EIn^4\pi^4}{2L^3} + \frac{KL}{2}}$$

Let $S_n = \frac{EI\pi^4}{L^4} \left[n^4 + \frac{KL^4}{EI\pi^4} \right]$

Then $B_n = \dfrac{\int_0^L D_n \sin^2 \frac{n\pi x}{L} \, dx}{\frac{L}{2} S_n}$

Since D_n is not a function of x:

$$B_n = \frac{D_n \int_0^L \sin^2 \frac{n\pi x}{L} \, dx}{\frac{L}{2} S_n} = \frac{D_n \frac{L}{2}}{\frac{L}{2} S_n}$$

Then:

$$(9a)\quad B_n = \frac{D_n}{S_n}$$

Inserting Eqmation (9a) in Equation (2a):

$$(10)\quad y = \sum_{n=1}^{\infty} \frac{D_n}{S_n} \sin\frac{n\pi x}{L}$$

D. CHECK

Consider a continuous beam without an elastic foundation. In this case K = 0. Then from Equation (10):

$$y = \sum_{n=1}^{\infty} \frac{D_n}{S_n} \sin\frac{n\pi x}{L} = \sum_{n=1}^{\infty} \frac{D_n}{\frac{EIn^4\pi^4}{L^4} + K} \sin\frac{n\pi x}{L}$$

For K = 0:

$$y = \frac{L^4}{EI\pi^4} \sum_{n=1}^{\infty} \frac{D_n}{n^4} \sin\frac{n\pi x}{L}$$

which concurs with the deflection derived by Seng-Lip Lee.[1]

To compare with Hetenyi, consider a simply supported beam on an elastic foundation with a load P at x = c.

$$D_n = (A_n - A_{n1} - A_{n2} - A_{n3} - \dots - A_{nm})$$

All A's except A_n are equal to zero since there are no intermediate reactions.

$$A_n = \lim_{r \to 0} \frac{2}{L} \int_{c-r}^{c+r} \frac{P}{2r} \sin\frac{n\pi x}{L} \, dx = \frac{2P}{L} \sin\frac{n\pi c}{L} = D_n$$

From Equation (2a):

$$y = \sum_{n=1}^{\infty} \frac{\frac{2P}{L} \sin\frac{n\pi c}{L} \sin\frac{n\pi x}{L}}{\frac{EIn^4\pi^4}{L^4} + K}$$

$$y = \frac{2PL^3}{EI\pi^4} \sum_{n=1}^{\infty} \frac{\sin\frac{n\pi c}{L} \sin\frac{n\pi x}{L}}{n^4 + \frac{KL^4}{EI\pi^4}}$$

This agrees with Hetenyi.[2]

E. DETERMINATION OF INTERMEDIATE REACTIONS

a. When the deflection at intermediate supports is zero:

$$y = 0 \quad \text{at } x = a_1L,\ a_2L,\ a_3L,\ \ldots,\ a_mL$$

Inserting these boundary conditions in Equations (6a) and(10):

$$y = 0 = \sum_{n=1}^{\infty} \frac{A_n - \frac{2}{L}\left[R_1\sin(n\pi a_1) + R_2\sin(n\pi a_2) + \ldots + R_m\sin(n\pi a_m)\right]}{S_n} \sin(n\pi a_1)$$

$$y = 0 = \sum_{n=1}^{\infty} \frac{A_n - \frac{2}{L}\left[R_1\sin(n\pi a_1) + R_2\sin(n\pi a_2) + \ldots + R_m\sin(n\pi a_m)\right]}{S_n} \sin(n\pi a_2)$$

$$y = 0 = \sum_{n=1}^{\infty} \frac{A_n - \frac{2}{L}\left[R_1\sin(n\pi a_1) + R_2\sin(n\pi a_2) + \ldots + R_m\sin(n\pi a_m)\right]}{S_n} \sin(n\pi a_3)$$

$$\vdots$$

$$y = 0 = \sum_{n=1}^{\infty} \frac{A_n - \frac{2}{L}\left[R_1\sin(n\pi a_1) + R_2\sin(n\pi a_2) + \ldots + R_m\sin(n\pi a_m)\right]}{S_n} \sin(n\pi a_m)$$

Thus there are m linear equations in m unknown intermediate supports. Transposing the intermediate reactions and their coefficients to the left side of the equation:

$$(11)\quad F_{11}R_1 + F_{12}R_2 + F_{13}R_3 + \ldots + F_{1m}R_m = \frac{L}{2} \sum_{n=1}^{\infty} \frac{A_n}{S_n} \sin(n\pi a_1)$$

(11) continued

$$F_{21}R_1 + F_{22}R_2 + F_{23}R_3 + \dots + F_{2m}R_m = \frac{L}{2}\sum_{n=1}^{\infty} \frac{A_n}{S_n} \sin(n\pi a_2)$$

$$F_{31}R_1 + F_{32}R_2 + F_{33}R_3 + \dots + F_{3m}R_m = \frac{L}{2}\sum_{n=1}^{\infty} \frac{A_n}{S_n} \sin(n\pi a_3)$$

$$\vdots$$

$$F_{m1}R_1 + F_{m2}R_2 + F_{m3}R_3 + \dots + F_{mm}R_m = \frac{L}{2}\sum_{n=1}^{\infty} \frac{A_n}{S_n} \sin(n\pi a_m)$$

where:

$$F_{11} = \sum_{n=1}^{\infty} \frac{\sin^2(n\pi a_1)}{S_n}$$

$$F_{12} = \sum_{n=1}^{\infty} \frac{\sin(n\pi a_2)\sin(n\pi a_1)}{S_n}$$

$$\vdots$$

$$F_{1m} = \sum_{n=1}^{\infty} \frac{\sin(n\pi a_m)\sin(n\pi a_1)}{}$$

and: so (12) $$F_{uv} = F_{vu} = \sum_{n=1}^{\infty} \frac{\sin(n\pi a_u)\sin(n\pi a_v)}{S_n}$$

where u and v are any positive integers.

To solve for intermediate reactions R_1, R_2, R_3, ..., Rm, use Equation (12) to evaluate their coefficients in Equation (11). Evaluate the right sides of Equations (11) and solve the m simultaneous equations.

b. When the deflections at the intermediate supports are not zero:

$$y = \delta_1 \text{ at } x = a_1 L, \quad y = \delta_2 \text{ at } x = a_2 L, \quad \ldots, \quad y = \delta_m \text{ at } x = a_m L$$

where δ_1, δ_2, ..., δ_m are the deflections at the intermediate supports.

These boundary conditions are substituted into Equations (6a) and (10) and the intermediate reactions are determined as outlined in part a above.

IV. APPLICATION OF METHOD TO AN EXAMPLE PROBLEM

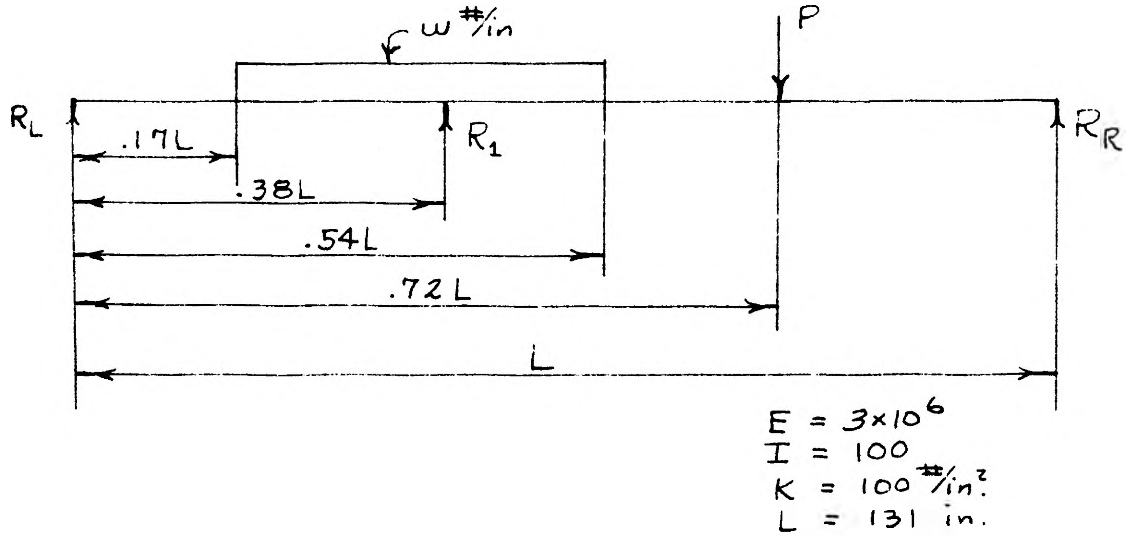

Figure 2

Since $R_2, R_3, \ldots, R_m = 0$ and $\delta_1 = 0$, Equation (11) reduces to:

$$F_{11}R_1 = \frac{L}{2}\sum_{n=1}^{\infty}\frac{A_n}{S_n}\sin(n\pi a_1)$$

$$R_1 = \frac{\frac{L}{2}\sum_{n=1}^{\infty}\frac{A_n}{S_n}\sin(.38)n\pi}{F_{11}}$$

From Equation (5a):

$$A_n = \frac{2}{L}\int_0^L q(x)\sin\frac{n\pi x}{L}\,dx$$

$$A_n = \frac{2}{L}\int_{.17L}^{.54L} w\sin\frac{n\pi x}{L}\,dx + \lim_{r\to 0}\frac{2}{L}\int_{.72L-r}^{.72L+r}\frac{P}{2r}\sin\frac{n\pi x}{L}\,dx$$

$$A_n = \frac{-2w}{L} \frac{L}{n\pi} \left[\cos(.54)n\pi - \cos(.17)n\pi \right] + \frac{2P}{L} \sin(.72)n\pi$$

From Equation (12):

$$F_{11} = \sum_{n=1}^{\infty} \frac{\sin^2 n\pi a_1}{S_n} = \sum_{n=1}^{\infty} \frac{\sin^2(.38)n\pi}{S_n}$$

From the definition:

$$S_n = \frac{EI\pi^4}{L^4} \left[n^4 + \frac{KL^4}{EI\pi^4} \right]$$

n	$\cos(.54)n\pi$	$\cos(.17)n\pi$	Δ
1	− .12533	+ .86074	− .98607
2	− .96858	+ .48175	−1.45033
3	+ .36812	− .03141	+ .39953
4	+ .87631	− .53583	+1.41214

$\Delta = \cos(.54)n\pi - \cos(.17)n\pi$

n	$\sin(.72)n\pi$	$\sin(.38)n\pi$	$\sin^2(.38)n\pi$	$\frac{L^4}{EI\pi^4} S_n$
1	+ .77051	+ .92978	+ .8649	2
2	− .98229	+ .68455	+ .4692	17
3	+ .48175	− .42578	+ .1815	82
4	+ .36812	− .99803	+.9960	257

$$R_1 = \frac{\frac{L}{2}\sum_{n=1}^{\infty}\frac{A_n}{S_n}\sin(.38)\,n\pi}{F_{11}}$$

$$R_1 = \frac{L}{2}\,\frac{\frac{L^4}{EI\,\pi^4}}{\frac{L^4}{EI\,\pi^4}}\;\frac{\dfrac{\left[\frac{-2w}{\pi}(-.9861) + \frac{2}{L}P(.7705)\right].9298}{2} + \dfrac{\left[\frac{-2w}{2\pi}(1.4503) + \frac{2}{L}P(-.9823)\right].6846}{17} + \dfrac{\left[\frac{-2w}{3\pi}(.3995) + \frac{2}{L}P(.4818)\right](-.4258)}{82}}{\frac{.8649}{2} + \frac{.4692}{17} + \frac{.1815}{82}}$$

$$R_1 = .929\frac{wL}{\pi} + .675\,P$$

From Equation (10):

$$y = \sum_{n=1}^{\infty}\left[A_n - \frac{2R_1}{L}\sin(.38)n\pi\right]\frac{n\pi x}{L}$$

V. CONCLUSION

The method developed gives a practical means of solving continuous beam on elastic foundation problems. In addition, it can be used to solve problems of a less difficult nature, i.e., simply supported beam on an elastic foundation and continuous beam problems. This generalization is an important asset since increased application promotes thorough familiarity with the method which in turn promotes greater efficiency.

Although the derivation of the method is long and arduous, the method of application is quite simple and requires only a working knowledge of integral calculus and high school trigonometry. Setting up and evaluating the loading integral for most conceivable types of loads requires only a few minutes of an engineer's time and the remainder of the computation can be relegated to an engineering technician with some knowledge of trigonometry. Another advantage is that the method can be applied using only a slide rule, although use of mathematical tables facilitates solution.

The series approximation for the intermediate supports converges rapidly for small values of $\frac{KL^4}{EI\pi^4}$. The length of the solution depends on the size of this factor and the number of intermediate supports.

The method can easily be extended to include the use of influence lines, determination of bending moment, and slope of the beam under consideration.

The application of the outlined method is also an asset financially. The engineer does a minimum of work with the more routine work done at a lower cost by the engineering technician Application of this method also allows for consideration of the advantageous properties of the elastic material, saving cost in the design. In the past only the deleterious effects of the elastic foundation were considered in the design.

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VITA

Louis Richard Fuka was born December 19, 1937 in New York City. He received his elementary education in public school in New York and at Holy Family School in St. Louis, Missouri, after his family moved to St. Louis in 1947. He received a scholarship to Christian Brothers College High School in St. Louis. On June 6, 1959, he received the degree of Bachelor of Science in Civil Engineering from Saint Louis University.

From June, 1959, until September, 1961, Mr. Fuka was employed with the U. S. Army Corps of Engineers. He left his position as engineer in charge of contracted flood control sewer projects to enroll at Missouri School of Mines and Metallurgy for graduate study in the field of Civil Engineering.

Mr. Fuka was married to Mary Alice Swenson in June, 1959. They now have one daughter, Mary, who is two years old.