Fall 2015

# Multi-objective optimal dispatch of distributed energy resources

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MULTI-OBJECTIVE OPTIMAL DISPATCH OF DISTRIBUTED

ENERGY RESOURCES

by

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A THESIS

Presented to the Faculty of the Graduate School of the

MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

In Partial Fulfillment of the Requirements for the Degree

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

2015

Approved by

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# PUBLICATION THESIS OPTION

This thesis consists of the following two articles that have been submitted for publication as follows that have been formatted according to the journals from which they came:

Pages 4-26 are intended for submission to the IEEE TRANSACTIONS ON SMART GRIDS.

Pages 27-53 are intended for submission to the IEEE TRANSACTIONS ON SMART GRIDS.

# ABSTRACT

This thesis is composed of two papers which investigate the optimal dispatch for distributed energy resources. In the first paper, an economic dispatch problem for a community microgrid is studied. In this microgrid, each agent pursues an economic dispatch for its personal resources. In addition, each agent is capable of trading electricity with other agents through a local energy market. In this paper, a simple market structure is introduced as a framework for energy trades in a small community microgrid such as the Solar Village. It was found that both sellers and buyers benefited by participating in this market. In the second paper, Semidefinite Programming (SDP) for convex relaxation of power flow equations is used for optimal active and reactive dispatch for Distributed Energy Resources (DER). Various objective functions including voltage regulation, reduced transmission line power losses, and minimized reactive power charges for a microgrid are introduced. Combinations of these goals are attained by solving a multi-objective optimization for the proposed ORPD problem. Also, both centralized and distributed versions of this optimal dispatch are investigated. It was found that SDP made the optimal dispatch faster and distributed solution allowed for scalability.

## ACKNOWLEDGMENTS

I would like to thank my advisor, Dr. Pourya Shamsi, for guiding me throughout the arduous process of writing my thesis. Thank you for being a great mentor who has been instrumental to my success at MS&T. I also thank the members of my committee; Dr. Joo and Dr. Ferdowsi, for being always readily available and for making this project a success.

I would like to thank my parents, Mr. and Mrs. Abiodun and Olufunke Longe, and my siblings, Temitayo, Eniola, Olumide and Adedayo Longe, for supporting me and encouraging me in the pursuit of my dreams. I would not be here without your love and prayers. I would also like to thank Uncle Taofik for always being concerned about the progress of my thesis and my progress in life in general; you have been more like a dad to me.

I would like to thank my special friend, Ibukun Adegbulu, for his constant encouragement and patience throughout the writing process. Your wisdom, advice and kindness kept me going during the tough times.

Finally, I would like to thank my eternal Lord and savior, Jesus Christ, for keeping me alive to witness this day. I found assurance and encouraged myself in Him because it is written, “I can do all things through Christ who strengthens me”, Philippians 4:13. All glory and honor be to His name forever and ever, Amen.

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# 1. INTRODUCTION

This thesis introduces two papers including economic dispatch for an agent based community microgrid and multi-objective optimal dispatch of Distributed Energy Resources (DER). Both papers address microgrids which incorporate distributed energy resources. Migration to microgrids provides new opportunities in energy planning and load flow management in Electric Power Systems (EPS). With respect to available resources, a microgrid can dynamically optimize its energy resources for various criteria such as maximum reliability, minimum cost of operation, and minimum CO2 emissions. In traditional microgrids, a central entity is responsible for monitoring, energy planning, and control of the microgrid. For large numbers of energy resources within a microgrid, a centralized approach is not feasible due to the excessive requirements for memory and processing power.

In markets, agents can participate to sell their excess resources by providing bids without completely disclosing their planning information. This approach reduces the computational burden of a centralized economic dispatch and provides more flexibility to individual agents in operating their resources. In the first paper, a simple solution involving dynamic economic dispatch is provided that can be incorporated by residential agents in the energy planning and bidding mechanism. The interest is on a closed electricity market which is available to members of a community microgrid. In this market, bids are only submitted by the suppliers (active) and not by the demand (demanding agents act passive). The focus is on the members of a local community who share their resources to minimize the total cost of acquiring their demand or getting profit from their excess resources. The main challenge in this market is the presence of the utility grid with a pre-determined rate for electricity. For this reason, if the clearing price of this market exceeds the regional price of electricity, then the grid will dominate the market. Hence, unlike a traditional market, in a distribution level community market, lower and upper bounds limit the spot price of the market. This microgrid does not have a single owner nor a central control system. Within this microgrid, each node has full control over its local energy resources and can participate in microgrid energy planning based on its own personal benefits and without any obligations (hence, the set of providers can vary with time). The incentive for the proposed definition is the structure of the community microgrid installed at Missouri University of Science and Technology (S&T) where the users can trade power without any interference from the utility grid. In the experimental community microgrid, a dynamic economic dispatch method for each agent is reviewed which will be used to derive the bids.

The second paper characterizes proper power control within a distribution network that provides a better voltage profile regulation, increased voltage stability, and reduced active power losses on the distribution lines. DER is performed by solving a multi-objective semi definite programming optimization problem for distribution level networks. Conventionally, in many power systems with DER such as wind or solar resources, these DERs are not allowed to participate in grid voltage regulation procedures. This is mainly enforced to prevent voltage instabilities and oscillations in the power system. However, power grids with a high number of DERs have a potential for injecting sufficient active and reactive power to regulate the voltage of the network and control the power flow. In this paper, the goal is to provide a centralized and distributed framework for an optimal DER dispatch to provide combinatorial goals including voltage regulation, transmission loss minimization, and voltage stability maximization. The dispatch is performed at a set of time steps and considering the expected load and generation within the network for the upcoming time period. Both Optimal Power Flow (OPF) and Optimal Reactive Power Dispatch (ORPD) are considered to determine both active and reactive power flow injections. Optimal Power Flow is concerned with finding the optimal minimal cost for generating active power. Optimal Reactive Power Dispatch problem as a sub-problem of the OPF is a very important optimization problem in power systems as proper management of reactive power injection into the system can minimize real power loss and voltage profile deviations and improve voltage stability.

ORPD is particularly useful in smart micro grids where the renewables also known as distributed energy resources DER are connected in a distribution network. It is widely known that the ORPD problem is nonconvex in nature. ORPD is concerned with commanding the renewables to generate fixed reactive power to control voltage. One objective of ORPD is to find the optimal reactive power to be injected by the renewables in order to keep the power system bus voltages as close to one per unit (p.u) as possible. Another objective that can be achieved is the minimization of the active power losses in the power system. The non-convexity of the ORPD problem is as a result of the nonlinearity of the voltages and active and reactive powers injected at each bus of the power system. The nonlinear power flow equations pose a technical challenge in solving the optimization problem under a low computational burden. This burden is reduced by converting the multi-objective optimal dispatch problem into a Semi-Definite Programming (SDP) Problem. This enables the nonconvex rank constraint to be eliminated.

# I. Economic Dispatch for an Agent-Based Community Microgrid

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## Abstract

In this paper an economic dispatch problem for a community microgrid is studied. In this microgrid, each agent pursues an economic dispatch for its personal resources. In addition, each agent is capable of trading electricity with other agents through a local energy market. In this paper, an energy market operating in the presence of the grid is introduced. The proposed market is mainly developed for an experimental community microgrid at Missouri University of Science and Technology (S&T) and can be applied to other distribution level microgrids. To develop the algorithm, first, the microgrid is modeled and a dynamic economic dispatch algorithm for each agent is developed. Afterwards, an algorithm for handling the market is introduced. Lastly, simulation results are provided to demonstrate the proposed community market and show the effectiveness of the market in reducing the operation costs of passive and active agents.

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## NOMENCLATURE

| | |
|---|---|
| $b_i$ | $i$-th agent / busbar $i$ |
| $R_i$ | $i$-th intermittent resource |
| $D_i$ | $i$-th dispatchable resource |
| $S_i$ | $i$-th storage system |
| $t$ | Time |
| $\mathcal{D}_i$ | Set of Dispatchable Resources (DR) of node $i$ |
| $\bar{\mathcal{D}}_i$ | Set of Intermittent Resources (IR) of node $i$ |
| $P_{g_i}(t)$ | Power injected at time $t$: grid to node $i$ |
| $P_{l_i}(t)$ | Power consumed by the agent $b_i$ at time $t$ |
| $P_{d_{i_j}}(t)$ | Power injected at time $t$: DR $i_j$ to node $i$ |
| $P_{r_{i_j}}(t)$ | Power injected at time $t$: IR $i_j$ to node $i$ |
| $P_{b_{i_j}}(t)$ | Power injected at time $t$: battery $i_j$ to node $i$ |
| $P_k^{max}$ | Max (or min) bounds of the $k$-th resource |
| $E_{i_j}(t)$ | Energy level at time $t$: battery $j$ at node $i$ |
| $E_k^{max}$ | Max (or min) levels of the $k$-th battery |
| $\Delta t$ | Time period of each dispatch cycle |
| $T_i$ | Dynamic dispatch horizon of agent $i$ |
| $C_k$ | Cost/kWh associated with the resource $k$ |
| $E^{stp}$ | Discretized energy-level step-size for DP |
| $\hat{C}_g^i(t)$ | Market spot price estimation at iteration $i$ |
| $\hat{P}_g^i(t)$ | Trade opportunity estimation at iteration $i$ |

## I. INTRODUCTION

Migration to microgrids provides new opportunities in energy planning and load flow management in Electric Power Systems (EPS). With respect to available resources, a microgrid can dynamically optimize its energy resources for various criteria such as maximum reliability, minimum cost of operation, and minimum CO2 emissions.

Economic dispatch (ED) problem in a power system is a well-known process and has been studied since the formation of power grids. In general, ED can be divided into three categories: static economic dispatch [1], dynamic economic dispatch [2]–[4], and dynamic economic dispatch with unit commitment [2], [5]. Various algorithms for ED are available in the literature. In traditional microgrids, a central entity is responsible for monitoring, energy planning, and control of the microgrid [5]–[7]. For large numbers of energy resources within a microgrid, a centralized approach is not feasible due to the excessive requirements for memory and processing power. Furthermore, in a practical system, independent owners of distributed generation are participating in the electricity market for a personal benefit and may not wish to completely share their pricing and planning policies. In markets, agents can participate to sell their excess resources by providing bids and without completely disclosing their planning information. This approach reduces the computational burden of a centralized economic dispatch and provides more flexibility to individual agents in operating their resources. Various research has studied aspects of distributed planning and markets in power systems [8]–[10]. Distributed agents can participate in energy planning in different ways. Various market structures, game theoretic methods, and bidding policies have been applied to power systems [11]–[14]. Majority of electricity markets are competitive [15], [16]. In such markets, each participant provides a bid and the spot price is determined based on the ascending list of bids and the total demand. In many markets, auctions are closed and no information on submitted offers/bids are available to other agents. Even if offers/bids are openly announced, various techniques are required to gather information on inner states of competitors to generate a successful bid [17]. In this paper, the interest is not to investigate such methods. However, a simple solution is provided that can be incorporated by residential agents in the energy planning and bidding mechanism.

Some common electricity markets are studied in [18]. Markets can be formed by independent agents and a utility or as a group of agents trading their resources [19]. In a simple auction market, operator clears the market by finding the intersection of the ascending supply and the demand [20], [21]. In this paper, the interest is on a close electricity market which is available to members of a community microgrid. In this market, bids are only submitted by the suppliers and not by the demand (demanding agents act passive). The focus is on the members of a local community who share their resources to minimize the total cost of acquiring their demand or to get profit from their excess resources. This process is also compatible with a demand responsive framework [22], [23] where the demand varies with the price. The main challenge in this market is the presence of the utility grid with a pre-determined rate for electricity. For this reason, if the clearing price of this market exceeds the regional price of electricity, then the grid will dominate the market. Hence, unlike a traditional market, in a distribution level community market, lower and upper bounds limit the spot price of the market.

In this paper, a community microgrid is defined as a microgrid that supports a community of residents. This microgrid does not have a single owner nor a central control system (it might have a central monitoring system). Within this microgrid, each node has full control over its local energy resources and can participate in microgrid energy planning based on its own personal benefits and without any obligations (hence, the set of providers can vary with time). The incentive for the proposed definition is the structure of the community microgrid installed at Missouri University of Science and Technology (S&T) where the users can trade power without any interference from the utility grid. Although the algorithm does not depend on the size of the system, expansion of this algorithm to other communities has a fundamental requirement: there should be no utility meter inside the boundaries of the microgrid. The utility meter should be placed at the Point of Common Coupling (PCC). This is to prohibit the local electric cooperative from monitoring the flow of power within the microgrid. Otherwise, the price of selling and purchasing energy will be set by the electric cooperative.

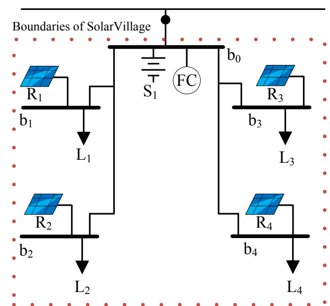
The structure of this paper is as follows: after introduction of this experimental community microgrid, a dynamic economic dispatch method for each agent is reviewed which will be used to derive the bids. Then the market is introduced and the overall algorithm is provided. Simulation results are provided to demonstrate the behavior of this system and cost reduction due to internal trades.

## II. A COMMUNITY MICROGRID: GREEN COMMUNITY

The selected microgrid is based on *Solar Village* microgrid at Missouri S&T. This microgrid consists of four houses with their individual access to solar energy resources and storage systems. Also, a central 60kWh battery storage system with a 50kW bidirectional inverter and a 5kW Fuel Cell (FC) Distributed Energy Resource (DER) are shared among these houses and are managed by a central microgrid controller. The physical microgrid is shown in Fig. 1a. The schematic of this system is illustrated in Fig. 1b.

In this figure, bus $b_0$ is the central bus for shared resources. This bus is the Point of Common Coupling with the utility grid. $b_1$ through $b_4$ are the available solar houses and $R_1$ through $R_4$ represent their individual solar resources, respectively. Similarly, each house $i$ has its local load $L_i$.

(a)




(b)

Fig. 1. (a) Solar Village at Missouri S&T, (b) schematic of *Solar Village* phase I.

The overall microgrid is a property of S&T and the local utility provider, Rolla Municipal Utility, has no information on the power flow within this microgrid (which is part of S&T's agreement). Currently, the university is paying for the electricity usage of all tenants through the installed smart meter shown with a *black* circle in Fig. 1b.

In the second phase of this project which is called the *Green Community*, several houses and local businesses will form a microgrid. This microgrid is shown in Fig. 2. Currently, this system is under construction and a market structure for energy trades within this community microgrid is developed. In Fig. 2, $R_i$, $S_i$, and $D_i$ represent renewable energy resources, storage systems, and dispatchable generation systems, respectively. In this system, each house or business will pay for their individual electricity usage. However, this payment will be in the form of a cost share on the single electricity bill for the overall microgrid which is recorded by the utility meter at the PCC. Electricity usage of each house is recorded by the microgrid controller using multiple smart meters (shown in *green*). In this expansion, instead of having a common bus (i.e. bus $b_0$) as the shared DER and storage system, each resident will acquire their own energy systems. Furthermore, flow of power within the microgrid will remain under control of this community and outside of the utility power grid. Interconnection with the utility grid is through the utility meter at the point of common coupling located after bus 0. From this point, microgrid can disconnect from the main grid and operate autonomously.

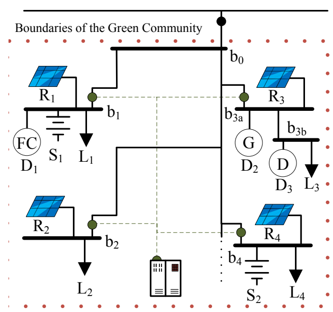



Fig. 2. Schematic of Solar Village phase II which is called the *Green Community*.

In this market, the goal is to find the spot price of electricity based on available bids on offered energy resources. At each time step (usually an hour), each agent will announce whether it demands energy or sells excess energy. Hence, the list of bidders will change at each time step (in this sense, the market is dynamic). If an agent is a buyer, it announces the amount to be purchased (buyers are passive). If an agent is a seller, it announces available power levels with their corresponding price. A seller can have multiple bids for its energy resources. A simple market clearing process is performed based on the intersection of the supply and the demand. In this paper, the iterative bidding where an agent can modify its bid is not considered. However, the same algorithm will work for that case with a constraint on the maximum number of iterations. After clearing the market, the price for the upcoming time step is set.

Sorting of the bids is based on the ascending price rates of electricity. Hence, the market operator will aggregate the received bids as shown in Fig. 3. Then solve for the spot price by intersecting the demand and the ascending plot of the bids. A difference between this market and an ordinary market is the presence of the utility grid. With respect to the power levels of the microgrid, utility grid has no limit in offering power at its set price. Therefore, if any offer is higher than the price of electricity from the grid, the offer is naturally neglected and the required demand is purchased from the grid. For this reason, we do not consider any bids above the price of the grid. Also, there can be a case where large incentives are in place for utilization of distributed resources. Hence, grid

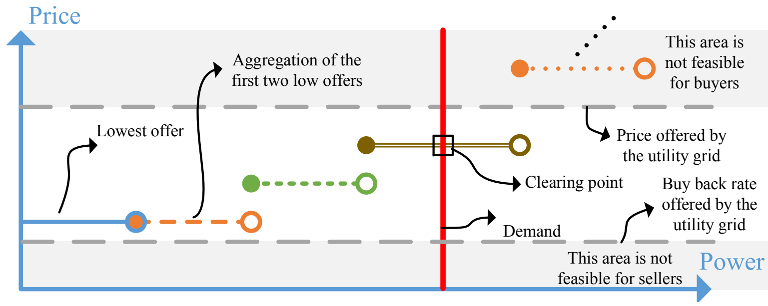



Fig. 3. Aggregation of the bids.

can buy electricity at a higher price than what it sells. This scenario is not suitable for a microgrid with multiple nodes and one PCC as the sum of the power will pass through the PCC. So even if the sellers want to sell their energy to the utility grid, they first need to supply the local demand. Therefore, first, they need to sell their electricity at a lower clearance price of the market, and then sell the excess energy to the grid at the higher rate of the incentives. In this scenario, users with large distributed resources will not benefit from being a member of the microgrid and they might seek their own connection to the grid. Fortunately, this is not the case for the microgrid located at Missouri S&T. In this region, the buy back rate is at most \$0.04/kWh which is about 2 times lower than the cost of purchasing electricity. Hence, sellers will profit if they sell power locally at a higher price than selling it back to the grid. There are two possible outcomes for this market.

1. There is more total demand than the total offer: In this case, to meet the demand, power has to be purchased from the grid. Hence, the intersection of the demand and the offer occurs on the price level of the grid. Therefore, in this case, the spot price will be equal to the price of the electricity from the utility grid and the bidders will receive this rate.

2. The total offer is more than the demand: In this case, first, the market is cleared by meeting the local demand using the ascending price curve. Afterwards, the flow of power can be outwards at the point of PCC and the sellers can sell their power back to the utility grid. Usually this process occurs at a lower rate as it was mentioned that the average rate for our geographical location is \$0.04/kWh.

## III. ECONOMIC DISPATCH FOR A SINGLE ENTITY

The problem of Economic Dispatch (ED) is to minimize the total cost of energy within a window of optimization. $\mathcal{D}_i = \{g_i, d_{i_1}, d_{i_2},\cdots, b_{i_1}, b_{i_2},\cdots\}$ is the set of dispatchable resources at node $i$ (each agent can posses multiple resources of a same kind), $\bar{\mathcal{D}}_i = \{r_{i_1}, r_{i_2},\cdots\}$ is the set of intermittent resource, and $P_{l_i}$ is the load. $E_{i_j}(t)$ is the energy stored in the $j$-th battery resource at node $i$. The economic dispatch problem for agent $i \in \{1, 2, ..., N\}$ is formulated as

$$\min_{P_k|k\in\mathcal{D}_i} \; C = \sum_{t=1}^{T_i} \sum_{k\in\mathcal{D}_i} C_k.P_k(t) \tag{1a}$$

$$s.t. \; \sum_{k\in\mathcal{D}_i} p_k(t) + \sum_{k\in\bar{\mathcal{D}}_i} p_k(t) = p_{l_i}(t) \tag{1b}$$

$$p_k^{min} \leq p_k(t) \leq p_k^{max}, \; k \in \mathcal{D}_i \tag{1c}$$

$$E_{i_j}^{min} \leq E_{i_j}(t) \leq E_{i_j}^{max}, \; j \in \{1,\cdots, n\} \tag{1d}$$

$$E_{i_j}(t) = E_{i_j}(t-1) + P_{b_{i_j}}(t).\Delta t \tag{1e}$$

where $T_i$ is the length of the dispatch window (optimization horizon). This value is usually selected to be 24-hours to support a day of dispatch. Larger values of this dispatch window results in a better sub-optimal solution at a higher computational costs. $\Delta t$ is the time period between two consequent dispatch steps. $n$ is the number of batteries at node $i$. Power balance equation is calculated in (1b). Each energy resource has power limitations which are considered in (1c).

Problem (1) can be also solved using Dynamic Programming (DP). In this way, the problem can be reduced to subproblems which are solved independently. If a node $i$ owns $d$ dispatchable resources including $b < d$ battery storage systems, by using DP, a $T_i \times d/\Delta t$ dimensional problem will be reduced to $N_{stp_1} \times\cdots\times N_{stp_b} \times T_i/\Delta t$ problems of $(d-b)$ dimensions where $N_{stp_j}$ is the number of steps selected for the dispatch of the $j$-th battery system.

To do so, the possible levels of energy in each battery system is discretized to a set of levels with a step size of $E_{stp}$. The optimization is performed every $\Delta t$ (usually an hour). Therefore, the dispatch level of each battery is no longer an independent variable and is calculated as $P_{b_{i_k}}(t) = (E_{i_k}(t+1) - E_{i_k}(t))/\Delta t$. Feasible dispatch levels for the battery should comply with (1d) and (1e), otherwise, the cost of transition from $E_{i_k}(t)$ to $E_{i_k}(t+1)$ is infinity.

After solving each sub-problem, a graph of all possible transitions is formed. In this graph, nodes are possible energy levels in battery resources at a time $t_k$. Hence, set of graph columns are defined as $\mathcal{N} = \{\mathcal{N}_1, ..., \mathcal{N}_{T_i}\}$, $\mathcal{N}_k = \{E_{ij}^{min}, E_{ij}^{min} + E_{stp}^k, ..., E_{ij}^{max}\}$ while the set of directed transitions (arcs) are defined as $\mathcal{W} = \{w_1, ..., w_{T_i}\}$ and $w_k \in \mathcal{N}_k \times \mathcal{N}_{k+1}$ $(k \in \{1, \cdots, T_i - 1\})$. Fig. 4 illustrates the transition graph for this DP by illustrating the directed graph $(\mathcal{N}, \mathcal{W})$.

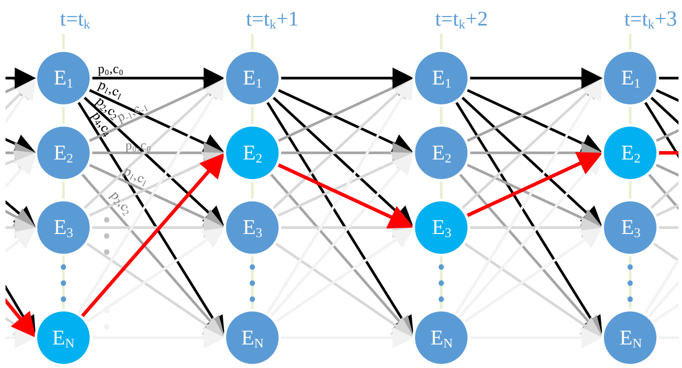



Fig. 4. Dynamic programing graph of the ED problem.

Based on the arc weights, the shortest path (lowest sum of weights) from the last column (i.e. $E_{end}^{T_i}$) to the starting energy level (i.e. $E_{start}^1$) is calculated using dynamic programming. This calculation will not only define the shortest path, but also will define the final energy level. It is assumed that the starting energy level is known which is the level at $t = t_0$.

The energy level in the battery systems will create a dependency between the optimal solution of the dispatch at each time step. Hence, the economic dispatch problem is in fact an infinite horizon optimization problem where the starting point is known and the optimal path can be calculated using the extended bellman method. However, there is no significant point in solving the solution for an infinite horizon case as the true stochastic variations of loads, intermittent resources, and policies selected by other agents are not known. Therefore, for long optimization windows, the covariance of stochastic process will become large and the optimization cannot provide any practical benefit compared to a smaller time window. For this reason, in many applications, the time window for a dynamic economic dispatch is selected as an integer multiple of days such as $T_i = 24$h or $48$h. Also, since the problem is now a sub-optimal solution of the original infinite horizon problem, it is sufficient to find the best solution without any concerns for upcoming windows. Therefore, by knowing the starting energy level for the battery system, one can find the shortest path to $t = t_0 + T_i$ without enforcing any constraints on the final state of the battery. In the simulations provided, each agent will have a different optimization window.

The importance of a DP approach will be described in the next section. Using this method, each agent can reduce the dispatch problem to a much smaller set of problems by re-utilizing the DP graph from the previous steps and only updating the required elements. This can significantly reduce the computational burden of the market procedures on each individual agent.

## IV. ELECTRICITY MARKET IN A COMMUNITY MICROGRID

### A. Announcing the Bids

The proposed method is mainly developed for linear cost functions. Recently a non-linear non-convex auction based method has been introduced which considers transmission losses [10]. In this work, a set of bids for various power levels is generated by each agent and transmitted to the neighboring agents. This algorithm is based on consensus between agents. Due to non-convexity of the problem, each agent requires to provide a set of feasible operation points. Instead, in this method, the cost functions are linear and agents need to provide a list of bids including the rating of their resources and the price of each resource. If a resource has a non-linear but a convex cost function (such as a second order function), then the agent can break its operation region into a set of linearized cost functions. Afterwards, the agent provide bids regarding the capacity of each linearized section and the corresponding price.

In order to solve the ED problem, an agent needs the cost function of the grid $C_g$ in (1). For positive acquires from the grid, this value is at most the price of electricity offered by the electric cooperative. However, this value can be lower as the clearance price of the market depends on available offers. Hence, an agent can have a price estimate of the grid for this time period as $\hat{C}_g(t)$. First, this agent can assume $\hat{C}_g(t)$ is equal to the price offered by the local electric cooperative. Eventually, this agent can train a price model based on the observations of the price at each market cycle. For instance, a simple learning mechanism as $\hat{C}_g^{(i+1)}(t) = \hat{C}_g^i(t) + \gamma(C_g(t) - \hat{C}_g^i(t))$ where $\hat{C}_g^i(t)$ is the estimate of the spot price at time $t$ during the $i$-th cycle of the market procedure. $C_g(t)$ is the clearing price of the market at the time $t$ during the $i$-th cycle of operation of the market. $\gamma$ is the learning (filtering) rate.

At a time $t$, using the vector of price estimates for each optimization step and for a $T_i$ window of time in the future, each agent solves the optimization problem and derives the optimal dispatch. Based on the dispatch, if $P_{g_i}(t) \geq 0$, then this agent is a buyer and acts passively in the market. This agent will only announce the required amount of power. If $P_{g_i} < 0$, then it is optimal for the agent to sell power back to the grid. Agent will generate the ascending cost plot of its resources. Lower cost resources will be used to supply the internal demand (i.e. $P_{l_i}(t)$). The remainder of the plot is announced to the market as a set of available capacity and the price of each capacity.

### B. Clearing the Market

At this point, the market has received all the demands and offers for the time step $t$. If the demand is higher than the available capacity, then the remainder of the power has to come from the utility grid. Since the grid is an infinite capacity market (with respect to the nominal rating of the microgrid), the price of the grid will become the dominant price as the intersection of the demand and the bids occur on the price of the grid. Therefore, for the case where $\sum_k P_{g_k} > 0$, the clearing price of the market is price of the grid and every seller will receive this rate. If $\sum_k P_{g_k} < 0$, then there are more offers than the demand and the market can clear without considering the grid. As it was shown in Fig. 3, the clearance price of the market should remain between the price at which the utility grid sells power and the rate at which it buys back power.

If the spot price is higher than the grid's price, buyers would complain and will demand their individual connection to the utility grid. If the spot price is lower than the grid's buyback rate, then the sellers would seek direct connection to the grid. Hence, for feasible operation of the microgrid, the spot price is bounded within these two margins.

Therefore, the market will intersect the ascending offers and the demand to find the spot price. The spot price is limited to the rate at which the grid sells power at, and the rate at which the grid buys back power at. The results are announced to the agents. At this time, the demand is fulfilled. However, some of the bids are not used. Based on the preferences of the remaining bidders, their capacity can be sold to the grid. However, this rate is the low buyback price rate of the grid and agents should verify if it is still in their benefit to do so.

After clearing the market, the price is announced to agents and the system will redo this process for the next time step. Here, it was assumed that the market is static and modifications of bids are not applicable. However, one can simply allow for modification of the bids and agents can compete further by modifying their bids. In this case, a maximum limit on the number of iterations is necessary to ensure a final settlement before the dispatch period begins.

### C. Post Market Procedures

At this time, the dispatch levels and the spot price of the electricity for the time step $t$ are derived. Agents will use the information regarding the amount of power that was traded as well as the spot price to form an estimation for the similar time period in upcoming days. In a simple approach, each agent can track the spot price using a learning mechanism such as $\hat{C}_g^{(i+1)}(t) = \hat{C}_g^i(t) + \gamma_1(C_g(t) - \hat{C}_g^i(t))$ and track the demand level as $\hat{P}_g^{(i+1)}(t) = \hat{P}_g^i + \gamma_2(P_g^i(t) - \hat{P}_g^i(t))$. Tracking the demand is important for the sellers as the local demand is cleared at a higher rate than what the grid pays for electricity. So an agent needs to know how much power can be sold at a rate of the market and the remainder will be sold at the rate of the grid.

The importance of the dynamic programing appears in the post market step. If an agent updates the price and demand estimates only for a similar time period, there is no change in the DP graph of upcoming hours. Therefore, the agent can simply update the DP graph by calculating the affected sub-problems without re-calculating the whole graph. Whether agents use DP or not, they need to recalculate their optimal dispatch for the upcoming hours and announce their bids for the next cycle of the market.

**Algorithm 1** A community energy market

Initialize a 24h vector of $\hat{C}_g(t)$ and $\hat{P}_g(t)$ with a prior assumption based on the location.
Set $t = 0$. 1 i=1:#(Agents) {Announcing bids}
Solve (1) for $t = \{t+1, t+2, \cdots, t+T_i\}$. $P_{g_i}(t+1) > 0$
Announce the total demand $P_{g_i}(t+1)$.
Announce the total offer $|P_{g_i}(t+1)|$.
Breakdown resources used to form $P_{g_i}(t+1)$:
Per resource, announce the capacity/price rate.
Do not announce any resource with a price rate
higher than grid's rate at $t+1$.
Place all bids in $O = [o]_j = [p_j, c_j]$ where $p_j$ is the
capacity of the $j$-th bid and $c_j$ is the corresponding rate. Demand $>$ total bids {Clearing the market}
Set $C_g(t+1) =$ grid's rate at time $t+1$.
Use all bidders, buy the remaining demand from
the utility grid.
Sort $O$ and find the intersection of the cumulative
bids and the demand. Set this bid as $C_g(t+1)$.
Limit: grid's buyback rate $\leq C_g(t+1)$.
Pay the selected bidders at the rate $C_g(t+1)$.
Remainder of the bids can be sold back to the utility
grid at a rate $C'_g(t+1) =$ grid's buyback rate.
Enforce the dispatch. $t = t+1$.
Update the estimations: {Post-market process}
Each agent can track the settled $C_g(t)$ and update $\hat{C}_g(t)$.
Each agent can track the demand.
Each agent can track its estimated share of the market.

The overall process for the proposed algorithm is shown in Fig. 5. It should be noted that the decision making policies and the estimation methods are not discussed in details in this paper. The main objective of this paper is to provide a market procedure

TABLE I

SIMULATED RESOURCES/COSTS BASED ON FIG. 2

| Node | Resource | Power [kW] | | Energy [kWh] | | Price |
|---|---|---|---|---|---|---|
| | (id.) | min. | max. | min. | max. | \$/kWh |
| $b_1$ | Solar PV ($R_1$) | 0 | 1 | - | - | 0.00 |
| | Battery ($S_1$) | -1 | 1 | 0.5 | 2 | 0.00 |
| | Fuel Cell ($D_1$) | 0 | 2 | - | - | 0.07 |
| $b_2$ | Solar PV ($R_2$) | 0 | 0.8 | - | - | 0.00 |
| $b_3$ | Solar PV ($R_3$) | 0 | 0.6 | - | - | 0.00 |
| | Diesel gen. ($D_2$) | 0 | 2 | - | - | 0.08 |
| | Gas gen. ($D_3$) | 0 | 1 | - | - | 0.06 |
| $b_4$ | Solar PV ($R_1$) | 0 | 0.5 | - | - | 0.00 |
| | Battery ($S_2$) | -2 | 2 | 1 | 4 | 0.00 |

for a microgrid in presence of the utility grid. However, the simple recursive estimation methods provided can effectively handle the ED for residential agents as it is shown later in the simulation results. The summary of the proposed method can be described as Algorithm 1.

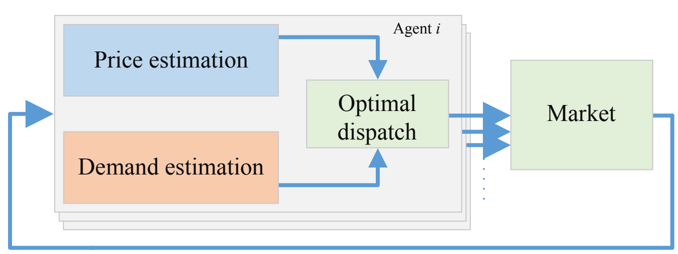



Fig. 5. The proposed community economic dispatch scheme.

## V. CASE STUDY

In this section, several case studies are provided for the microgrid shown in Fig. 2. With respect to this figure, parameters of each load and resource are presented in

Table I. In this scenario, it is assumed that the price of buying energy from, and selling energy back to the grid are given by Fig. 6. To solve the problem for each agent, (1) was used. For each node of the DP graph, linear programing was used to find the cost of the transition based on the dispatch level of the battery. Lastly, the overall optimal path was found by finding the shortest path on th DP graph. Using a standard 4-core Intel 4-th generation i-7 laptop and MATLAB, the 48 hour optimal dispatch for agent $b_1$ was solved in 50ms.

In a real-world implementation, each agent will solve the ED and will update his/her estimates of the market behavior locally. However, all the stages of Algorithm 1 were solved using this MATLAB model. The overall processing time for 1 cycle of the market including ED of the 4 agents as well as the clearing process and post-market updates is 0.1s. It should be noted that agents $b_2$ and $b_3$ have an optimization window of 1 hour due to the lack of storage systems and agent $b_1$ has an optimization window $T_4 = 36$ hours to maintain generality.

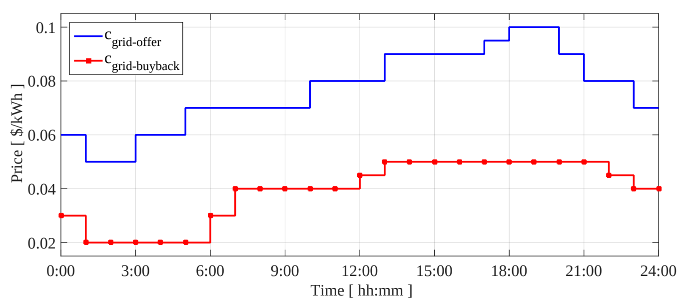



Fig. 6. The 24h price of energy to and from the grid.

Load located at each bus are presented in Fig. 7. Solar production profile is depicted in Fig. 8. It should be noted that due to the close proximity of houses, their solar profile is similar and only varies in amplitude.

Buses $b_1$ and $b_4$ have energy storage systems. Therefore, to perform an ED, these buses need to consider an optimization over a window of time. As it was mentioned before, the selection of the window itself is a trade-off between optimality and computational complexity. In many low power applications, a 24-hour window is selected for ED. Without a loss of generality, $b_1$ selects its optimization window to be 48h and $b_4$ selects 36h. Also, the algorithm is started with an assumption that the price of electricity is

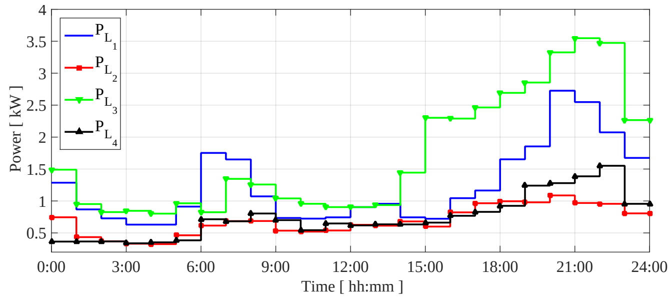



Fig. 7. The 24h load profile of each bus.

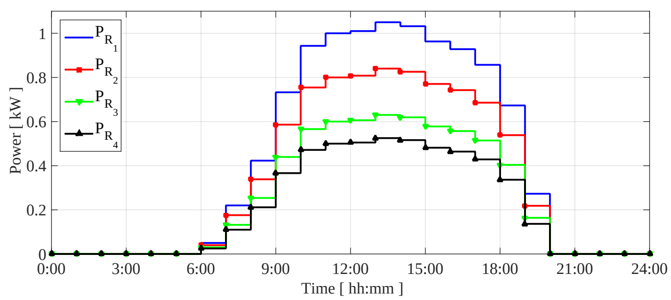



Fig. 8. The 24h solar production profile of each bus.

\$0.07/kWh at each time step. Each bus assumes that it is possible to sell 1kWh to the market (at a higher rate than what the utility buys back at). These are starting assumptions and based on each agent's learning mechanism, the agent will soon find a better estimate of each of these parameters as is shown later. With the above assumptions, the system is simulated for 48 hours or equivalently, 48 market cycles for dispatch windows of 1 hour each. Currently, the market is not settled for the 48-th hour. Therefore, up to the hour 47, the price of the market is known and energy has been traded. The time instance of hour 48 when each agent has calculated its optimal bid [demand] to [from] the market is looked at. At this time step, agent $b_1$ has calculated an ED with an optimization window of 48h in the future. Fig. 9 illustrates the dispatch performed by $b_1$ through time.

Figure 9 illustrates the evolution of the optimal dispatch through time. It can be observed that as the number of market cycles increased, the ED solution is changing. During the first day, for 24h, there is no correct estimation of the price of the grid/market. Therefore, this agent is assuming \$0.07/kWh as the price of the electricity from the grid (which is a fair assumption throughout the U.S.). Also, this agent assumes that there is

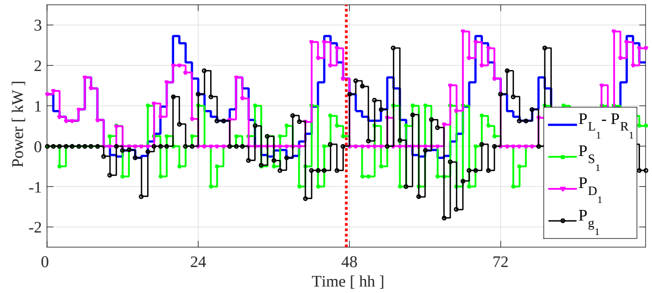



Fig. 9. ED performed by $b_1$, hours 48-96 are the optimal dispatch in the future.

a chance of selling 1kW to the neighboring agents at any time. In the second day when there are prior knowledge of the trades which took place in the first day, this agent has a better understanding of the possible spot prices of the market at each market cycle. It can be observed that the battery storage system is optimally charged at times with lower cost of electricity and is sold to other agents during the peak usage times. The period between 48h to 96h shows the ED for a 48h window in the future. However, this agent will update this dispatch after every market cycle to maintain its optimality based on the settling price of the market and based on the microgrid demand.

Similar to this agent, other agents dynamically solve the ED problem and participate in the market. The agent at node $b_3$ has no energy storage system. Hence, for this agent, there is no need to solve a dynamic ED in time and derivation of the ED for only one cycle in the future will suffice. Fig. 10 illustrates the dispatch for this agent. Based on the price of each resource provided in Table I and the starting assumption for the price of electricity to be sold to the grid, during the first day, this agent tends to use its gas generator to sell power to the microgrid. This resource is only \$0.06/kWh and can easily compete in the market. As more information is collected in the first day, on the second day, ED involves a significant dispatch for this gas resource. However, it can be observed that it takes one additional day for this agent to get a sufficiently accurate estimate for the settling price of the market to start using its diesel resource at a rate of \$0.08/kWh.

To observe the evolution of the market and growth of the benefits for each agent, the value function of the ED of agent $b_1$ is considered. At $t = 1$, this agent has no realistic estimate of the market. Hence, it is calculating the ED based on the prior assumptions and

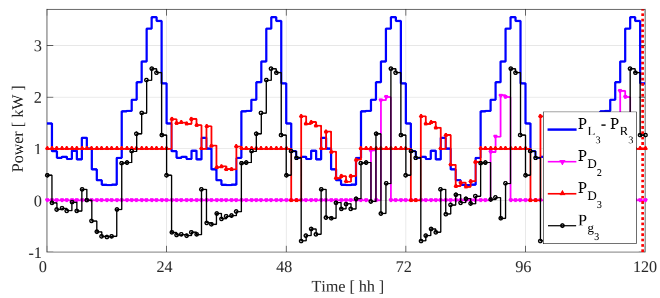



Fig. 10. ED performed by $b_2$ during the first 5 days.

without high expectations of profits. Fig. 11 illustrates the plot of the cumulative value functions for the dynamic ED of agent $b_1$ with its 48h dispatch window. Each green curve in this figure represents the growth of a the value function of the optimization period $t = t_o$ to $t = t_o + T_1$ where $T_1 = 48$h is the dynamic ED window for agent $b_1$. Each value function starts from zero and grows based on the expected cost of energy during the upcoming 48h. At $t = t_o + T_1$, the final value of the total expected cost of energy is achieved. These final points are connected using a blue line with small squares. It is observed that the expected final cost of a 48-hour operation is decreasing as the agent gains more knowledge about the operation of the market and can integrate more accurate pricing in its ED. In addition to profits for agents with storage systems, other agents can benefit from this market. For instance, Fig. 12 illustrates the daily cost of energy paid by agent $b_3$. It can be observed that the total is higher for this agent during the first two days of operation. However, as this agent acquires an estimate of the price/demand of the market, it can utilize its gas and diesel resources to reduce its costs of operation.

Lastly, the evolution of the spot price of the market is observed. As it was mentioned before, this price is limited to the price of electricity from the grid and buy back price of the grid. On the first day, agents start with an assumption of \$0.07/kWh. However, as the market operates, new prices are settled and agents update their estimation. To reduce the number of days required, large learning factor for both cost estimation and demand estimation are used ($\gamma_1 = \gamma_2 = 0.3$). In Fig. 13, red line-dot illustrates the price settlement of the market for each cycle. It can be observed that during the daytime, the market price is settled to a lower value than the offer from the grid. This shows that

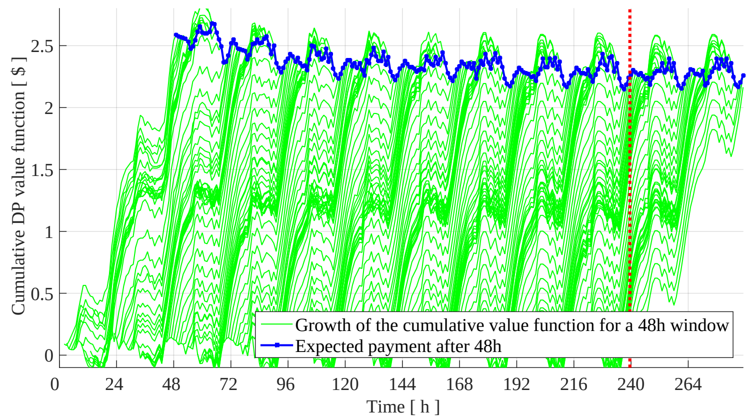



Fig. 11. Reduction is the cost of operation for a 48h optimization window of agent $b_1$.

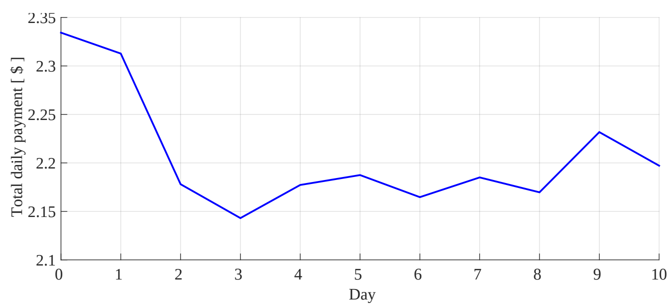



Fig. 12. Daily cost of energy for $b_3$ for the first ten days of operation.

the microgrid has enough capacity to support its demand and agents with distributed resources are able to compete in a local community market to sell their excess energy (due to the large number of solar resources). Also, since the spot price is lower than the regular price of electricity from the grid, agents who buy energy are benefiting as well.

The blue line-dash curve illustrates the estimation of the price used by agent $b_1$. It is shown that agent $b_1$ is improving its price estimation. For the 48 hours of dispatch after the current time $t = 360$h, this agent is utilizing the shown curve as the cost model for the market which is much more accurate than the stating constant assumption of $0.07/kWh.

## VI. CONCLUSION

In this paper, a market for economic dispatch in a community microgrid was introduced. This market was based on a standard auction market with passive buyers where sellers provide bids by announcing their available capacity and its linear cost model.

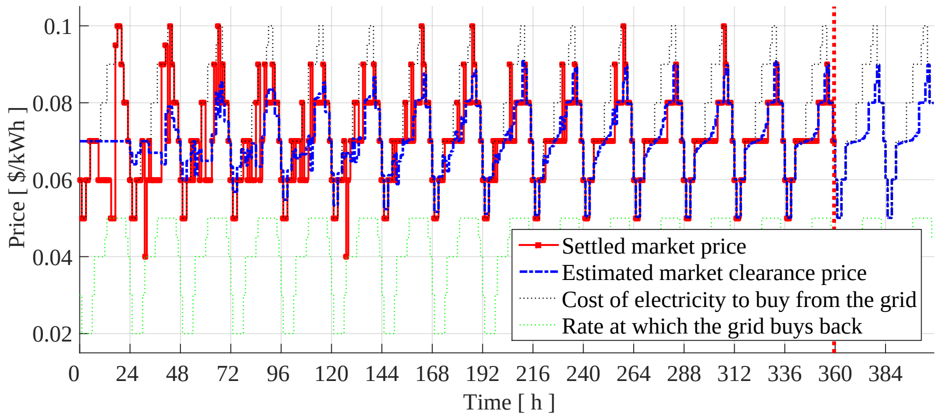



Fig. 13. Evolution of the market spot price.

Market was cleared by intersecting the demand and the ascending list of offers. It was shown that in such community markets, agents can estimate the operation of the market and effectively dispatch their resources. Since the spot price of the market is always lower or equal to that of the grid and higher or equal to the buyback price of the grid, both sellers and buyers will always benefit from participating in this market.

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## II. Multi-objective Semidefinite Optimal Dispatch of Distributed Energy Resources

**Ayomide Longe†, Pourya Shamsi††**

### Abstract

Proper power flow control within a distribution network provides a better voltage profile regulation, lower operation costs, and reduced active power losses. In such networks, due to the large impedance of lines, dispatch problems for active and reactive power have to be solved simultaneously to achieve optimal results. In this paper, optimal dispatch of Distributed Energy Resources (DER) is performed by solving a convex multi-objective optimization problem to calculate the optimal dispatch levels of active and reactive power for a distribution network. Convexification is performed by deriving the power flow equations in the form of semidefinite constraints and neglecting the rank constraint. After formulating the problem, various objective functions including voltage regulation, minimum network power losses, minimum cost of operation, and minimum curtailment of renewable energy resources are introduced. Lastly, linear and nonlinear combination of these objective functions are incorporated to form the multi-objective dispatch problem. In addition, a distributed solution for this multi-objective optimization is introduced. Afterwards, simulation results are provided to analyze the behavior of the developed framework.

### Index Terms

OPF, ORPD, SDP, distributed power flow, inverter dispatch, multi-objective convex optimization, voltage regulation, reactive power dispatch.

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## I. INTRODUCTION

Various power systems do not allow Distributed Energy Resources (DER) such as wind or solar resources to participate in reactive power control and grid voltage regulation procedures. This is mainly enforced to prevent voltage instabilities and oscillations. In the U.S., limitations induced by IEEE 1547 standard enforces small DERs (in particular, Renewable Energy Resources (RER)) to operate at the unity power factor. However, power grids with a large number of DERs have a good potential for reactive power injections as well as optimal curtailment of RERs to regulate the voltage of the network and control the power flow. Additionally, optimal reactive power injections will reduce the total cost of operation by minimizing losses over the network.

Traditionally, Optimal Power Flow (OPF) and Optimal Reactive Power Dispatch (ORPD) problems were solved separately. There are a variety of method for solving OPF problem. Recently, OPF has been investigated for dc distribution networks [1] as well as hybrid ac-dc distribution networks in [2]. For dynamic dispatch problems, dynamic programing is often incorporated to reduce the computation burden of the problem by quantizing the dispatch levels of energy storage systems [3], [4]. ORPD is similar to OPF but for reactive power dispatch and it has significant impacts on the safe and economic operation of power systems. The nature of the ORPD problem is to allocate reactive power generation to minimize the line losses and improve the voltage regulation [5]. Unlike OPF which is solved mainly for the minimum cost (under constraints for transmission limits), various objective functions can be considered for an ORPD problem. One objective is the voltage profile improvement in the distribution micro grid [6], [7], another objective is the maximization of the voltage stability index [8]. These objectives are often combined to form a multi-objective optimization problem [9]. Also, ORPD and OPF have been combined as a multi-objective optimization problem [10]–[12].

In the distribution networks, the assumption of a completely reactive power line impedance is no longer valid. Hence, separation of ORPD and OPF problems is not feasible and to attain the optimal solution, both problems have to be solved simultaneously. Various research has investigated dispatch of DERs within a distribution network [13]. In recent years, massive integration of RER has introduced new challenges as well as new opportunities for the ORPD problem and voltage regulation. High production levels from RERs can lead to over voltages within distribution networks. To cope with this problem, curtailment of RER has been used during high production periods. Curtailment of RER/flexible loads has been integrated with the OPF problem and is investigated in [14]. On the other hand, the power electronic inverters of RERs can be used as a distributed reactive power generation fleet.

In general, solving the power flow problem is a technical challenge. For OPF, various methods have been considered including linear programming [15], successive quadratic programming [16], mixed-integer linear programming, [17], and many more. In many applications with binary selection of generators, distribution network losses are neglected to relax the OPF problem to a mixed integer linear programing. However, if the nonlinear power flow equations are included, the problem becomes a non-convex NP-hard problem which cannot be dealt with using many conventional methods. In these conditions, heuristic optimization methods such as evolutionary algorithms [11], [15], [18] and swarm optimizations [5], [19] are utilized. Unfortunately, these methods suffer from a high computational burden and slow convergence.

Convexification of the problem has been used to reduce the computational burden [20]. A suboptimal approach of sequential convex programing was studied in [21]. A promising convexification approach is the Semi-Definite Programing (SDP) relaxation [1], [13], [22], [23]. In this approach, the problem is converted to a SDP and the non-convex rank constraint is eliminated. The challenge in derivation of the SDP relaxation is meeting the optimality under the rank one condition [22]. If the rank is higher than one, the solver has reached a sub-optimal point or the problem is not feasible. Fortunately, for many practical distribution networks, the SDP relaxation finds the optimal point. In particular, if the network is radial or is resistive, this method is very effective [24]. Details on accuracy and feasibility of SDP is studied in [25]. Recently, applications of SDP has been investigated for mesh networks in addition to the radial distribution networks [22].

To decompose this central problem into a distributed problem, [12] has utilized the alternating direction method of multipliers (ADMM) which has been used to ensure a robust decomposition of convex programs [26]. Also, decomposing using Lagrangian dual problem has been investigated in [27]. Comparison of such methods has been studied in [28]. Similarly, this problem can be solved using other methods such as a distributed consensus formulation [29]. Also, distributed heuristic methods have been incorporated to solve this problem [30].

In this paper, a semidefinite programing approach is used for optimal dispatch of energy resources within a distribution network. After introduction of the optimization problem, various objective functions are introduced to be used in a multi-objective framework for dispatch of these energy resources. Contributions of this paper include introduction of realistic boundaries for power electronic inverters, investigation of multiple objectives including voltage profile regulation and power loss minimization, introduction of various growth functions to be used with the multi-objective semi-definite programing, and investigation of regularization methods for sparsification of the inverter dispatch. The proposed framework is simulated to analyze the behavior of each objective and growth function.

## II. PROBLEM FORMULATION

The nature of the optimal dispatch problem is to allocate optimal active and reactive power generation/curtailment to achieve the desired objectives within the electrical region under study. However, the nonlinear power flow equations pose a technical challenge in solving the optimization problem under a low computational burden. In this paper, the goal is to reduce this burden by converting the multi-objective optimal dispatch problem into a Semi-Definite Programming (SDP).

### A. Notations

Before introducing the remainder of the method, a brief review of useful notations and matrices is provided. In this paper, bold-lower-case letters denote a vector and bold-upper-case letters denote a matrix. $\Im(\cdot)$ and $\Re(\cdot)$ extract the imaginary and real parts of the input argument, respectively. $[\mathbf{A}]_{ij}$ denotes the $ij$-th element of $\mathbf{A}$. $\mathbf{a}^T$ denoted the transpose, $\mathbf{a}^*$ denotes the complex-conjugate, and $\mathbf{a}^{T*}$ denoted the complex-conjugate transpose of $\mathbf{a}$. $\mathbf{0}$ and $\mathbf{1}$ denote all zero and all one matrices of appropriate dimension, respectively. $\mathbf{I}$ is the unity matrix.

$\mathrm{Diag}(\mathbf{a})$ returns a matrix $\mathbf{A}$ where $[\mathbf{A}]_{ii} = [\mathbf{a}]_i$. If this operator acts on a matrix, $\mathrm{diag}(\mathbf{A})$ returns a vector $\mathbf{a}$ where $[\mathbf{a}]_i = [\mathbf{A}]_{ii}$. $\mathrm{Tr}(\mathbf{A})$ is the trace, $\lambda_{max}(\mathbf{A})$ is the largest singular value, and $\mathrm{Rk}(\mathbf{A})$ is the rank of the matrix $\mathbf{A}$. Lastly, $\mathbf{a} \circ \mathbf{b} = \mathrm{diag}(\mathbf{a})\mathbf{b}$ is the element-wise (Schur) product of the two matrices.

$|\cdot|$ is the absolute value, $\parallel \mathbf{a} \parallel_1 = \sum_j |[\mathbf{a}]_j|$ is the linear norm, and $\parallel \mathbf{a} \parallel_2^2 = \mathbf{a}^{T*}\mathbf{a}$ is the Euclidean norm of $\mathbf{a}$. Additionally, $\mathbf{e}_q$, $q \in \{1, \cdots, \#\mathcal{N}\}$ is the basis of $\mathbb{R}^{\#\mathcal{N}}$ and $\mathbf{E}_{q,w} = \mathbf{e}_q \mathbf{e}_w^T$. Also, $\mathbf{EE}_{q,q} = [\mathbf{E}_{q,q}, \mathbf{0}; \mathbf{0}, \mathbf{E}_{q,q}]$ and $\mathbf{EE}_{q,w} = [(\mathbf{e}_q - \mathbf{e}_w)(\mathbf{e}_q - \mathbf{e}_w)^T, \mathbf{0}; \mathbf{0}, (\mathbf{e}_q - \mathbf{e}_w)(\mathbf{e}_q - \mathbf{e}_w)^T]$.

Additionally, Schur complement of the block $\mathbf{A}$ of the matrix $\mathbf{M} = [\mathbf{A}, \mathbf{B}; \mathbf{B}^T, \mathbf{C}]$ is defined as $\mathbf{S} = \mathbf{C} - \mathbf{B}^T \mathbf{A}^{-1} \mathbf{B}$. If $\mathbf{A}$ and $\mathbf{M}$ are both positive semidefinite, then $\mathbf{S}$ is also positive semidefinite. This property will be used to reformulate some of the objective functions.

## B. SDP Relaxation of the Optimal Dispatch Problem

Since the proposed method is mainly used for dispatch and voltage regulation of the distribution networks, for simplicity, the electrical region under study is refereed to as a Local Distribution System (LDS). In general, an optimal dispatch problem is in the form of

$$\min_{\{\mathbf{p}^g, \mathbf{q}^g, \mathbf{v}\}} \mathcal{V}(\mathbf{v}, \mathbf{i}, \mathbf{p}_g, \mathbf{q}_g) \tag{1a}$$

$$s.t. \quad \mathbf{p}^g + \jmath \mathbf{q}^g - \mathbf{p}^d - \jmath \mathbf{q}^d = \mathbf{v} \circ \mathbf{i}^* \tag{1b}$$

$$\mathbf{i} = \mathbf{Y}\mathbf{v} \tag{1c}$$

$$\mathbf{p}^{min} \le \mathbf{p}^g \le \mathbf{p}^{max} \tag{1d}$$

$$\mathbf{q}^{min} \le \mathbf{q}^g \le \mathbf{q}^{max} \tag{1e}$$

$$P^t_{q,w} \le P^{t-max}_{q,w}, \ \ \forall\, q, w \in \mathcal{N} \tag{1f}$$

$$S^t_{q,w} \le S^{t-max}_{q,w}, \ \ \forall\, q, w \in \mathcal{N} \tag{1g}$$

$$|v_q - v_w| \le v^{drop-max}_{q,w} \tag{1h}$$

where $n \in \mathcal{N}$ is the set of nodes within the LDS with a cardinality of $\#\mathcal{N}$. $p^g_n$ and $q^g_n$ are the generated power and reactive power at the node $n$, respectively. Similarly, $p^d_n$

and $q_n^d$ are the consumed active and reactive power at this node, respectively. $\mathbf{p}^g$, $\mathbf{p}^d$, $\mathbf{q}^g$, and $\mathbf{q}^d$ represent the vectors of the generated and consumed active and reactive power throughout the LDS, respectively. If a bus does not have each of these entities, then a value of zero is considered for the corresponding vector elements. $v_q$ and $i_q$ are the voltage and current of bus $q \in \mathcal{N}$, respectively. $\mathbf{v}$ and $\mathbf{i}$ are the vectors of of voltages and currents within the LDS, respectively. $\mathbf{p}^{min}$, $\mathbf{q}^{min}$, $\mathbf{p}^{max}$, and $\mathbf{q}^{max}$ are the vectors of the minimum and maximum limitations on the active and reactive power dispatch levels of each node, respectively. $P_{q,w}^t$ and $S_{q,w}^t$ are the active and complex power flowing between buses $q$, and $w$, respectively. These powers have nominal limits of $P_{q,w}^{t-max}$ and $S_{q,w}^{t_max}$. Additionally, the voltage drop on this line is limited to $v_{q,w}^{dop-max}$.

$\mathcal{V}(\cdot)$ is the objective function and is formed as a combination of the objectives introduced later.

The power flow problem relies on the vector "$\mathrm{diag}(\mathbf{v})\mathbf{Y}^*\mathbf{v}^*$". In particular, the entry related to node $q$ can be extracted as $\mathbf{v}^T\mathbf{E}_{q,q}\mathbf{Y}^*\mathbf{v}^*$. In order to convert this problem to a SDP, the rotational property of the trace function is used (i.e. $\mathbf{v}^T\mathbf{E}_{q,q}\mathbf{Y}^*\mathbf{v}^* = \mathrm{Tr}(\mathbf{v}^T\mathbf{E}_{q,q}\mathbf{Y}^*\mathbf{v}^*) = \mathrm{Tr}(\mathbf{E}_{q,q}\mathbf{Y}^*\mathbf{v}^*\mathbf{v}^T)$). This approach suggest using a new variable $\mathbf{V} = \bar{\mathbf{v}}\bar{\mathbf{v}}^T$ as the main voltage information where $\bar{\mathbf{v}}$ is the decomposed voltage vector as $\bar{\mathbf{v}} = [\Re(\mathbf{v})\ ;\ \Im(\mathbf{v})]$. Also, $\forall\, q, w \in \mathcal{N}$, the admittance matrix can be decomposed as

$$\bar{\mathbf{Y}}_{q,w} = \frac{1}{2}\begin{bmatrix} (\mathbf{Y}_{q,w} + \mathbf{Y}_{q,w}^T)^* \jmath(\mathbf{Y}_{q,w}^T - \mathbf{Y}_{q,w})^* \\ \jmath(\mathbf{Y}_{q,w} - \mathbf{Y}_{q,w}^T)^* (\mathbf{Y}_{q,w} + \mathbf{Y}_{q,w}^T)^* \end{bmatrix} \tag{2}$$

where $\mathbf{Y}_{q,q} = \mathbf{E}_{q,q}\mathbf{Y}$ and $\mathbf{Y}_{q,w} = (y_{q,w}^C + y_{q,w})\mathbf{E}_{q,q} - y_{q,w}\mathbf{E}_{q,w}$ where $y_{q,w} = [\mathbf{Y}]_{q,w}$ and $y_{q,w}^C$ is the admittance associated with the shunt element of the $\pi$-model of the line between buses $q$ and $w$. Now, one can define the symmetric matrices

$$\mathbf{Y}_{q,w}^{\Re} = \Re(\bar{\mathbf{Y}}_{q,w}) \tag{3}$$

$$\mathbf{Y}_{q,w}^{\Im} = \Im(\bar{\mathbf{Y}}_{q,w}) \tag{4}$$

to generate the following relations

$$p_q^g - p_q^d = \mathrm{Tr}(\mathbf{Y}_{q,q}^{\Re}\mathbf{V}),\ q_q^g - q_q^d = \mathrm{Tr}(\mathbf{Y}_{q,q}^{\Im}\mathbf{V}) \tag{5}$$

$$p_{q,w} = \mathrm{Tr}(\mathbf{Y}_{q,w}^{\Re}\mathbf{V}),\ q_{q,w} = \mathrm{Tr}(\mathbf{Y}_{q,w}^{\Im}\mathbf{V}) \tag{6}$$

$$p_{q,w}^{line} = p_{q,w} + p_{w,q},\ q_{q,w}^{line} = q_{q,w} + q_{w,q} \tag{7}$$

$$|v_q|^2 = \mathrm{Tr}(\mathbf{E}\mathbf{E}_{q,q}\mathbf{V}),\ |v_q - v_w|^2 = \mathrm{Tr}(\mathbf{E}\mathbf{E}_{q,w}\mathbf{V}) \tag{8}$$

which are useful to derive various objective functions for the optimization. These objective functions will then be combined to form the multi-objective optimization.

Lastly, (1) can be relaxed into a convex SDP as

$$\min_{\{p_q^g, q_q^g, \mathbf{V}\}} \mathcal{V}(\mathbf{V}, \mathbf{p}^g, \mathbf{q}^g) \tag{9a}$$

$$s.t. \quad \forall q, w \in \mathcal{N}$$

$$p_q^g - p_q^d = \mathrm{Tr}(\mathbf{Y}_{q,q}^{\Re}\mathbf{V}),\ q_q^g - q_q^d = \mathrm{Tr}(\mathbf{Y}_{q,q}^{\Im}\mathbf{V}) \tag{9b}$$

$$p_q^{min} \leq p_q^g \leq p_q^{max} \tag{9c}$$

$$q_q^{min} \leq q_q^g \leq q_q^{max} \tag{9d}$$

$$\{v_q^{min}\}^2 \leq \mathrm{Tr}(\mathbf{E}\mathbf{E}_{q,q}\mathbf{V}) \leq \{v_q^{max}\}^2 \tag{9e}$$

$$\begin{bmatrix} (S_{q,w}^{t-max})^2 \mathrm{Tr}(\mathbf{Y}_{q,w}^{\Re}\mathbf{V}) \mathrm{Tr}(\mathbf{Y}_{q,w}^{\Im}\mathbf{V}) \\ \mathrm{Tr}(\mathbf{Y}_{q,w}^{\Re}\mathbf{V}) 10 \\ \mathrm{Tr}(\mathbf{Y}_{q,w}^{\Im}\mathbf{V}) 01 \end{bmatrix} \succeq 0 \tag{9f}$$

$$-p_{q,w}^{t-max} \leq \mathrm{Tr}(\mathbf{Y}_{q,w}^{\Re}\mathbf{V}) \leq p_{q,w}^{t-max} \tag{9g}$$

$$\mathrm{Tr}(\mathbf{E}\mathbf{E}_{q,w}\mathbf{V}) \leq \{v_{q,w}^{dop-max}\}^2 \tag{9h}$$

$$\mathbf{V} \succeq 0 \tag{9i}$$

$$\mathrm{Tr}(\mathbf{E}_{ref}^{\mathcal{R}}\mathbf{V}) = 1,\ \mathrm{Tr}(\mathbf{E}_{ref}^{\mathcal{I}}\mathbf{V}) = 0 \tag{9j}$$

by eliminating a non-convex constraint of $\mathrm{Rk}(\mathbf{V}) = 1$.

The power flow constraints are enforced by (9b). (9c) and (9d) set the limits on the generated active and reactive power for bus $q$. (9e) limits the square Euclidean norm of the voltage of bus $q$ based on the grid requirement (such as ANSI C84.1-2011 standard). (9f) controls the square Euclidean norm of the complex power passing through the line between buses $q$ and $w$. Similarly, (9g) and (9h) control the square Euclidean norm of the total active power passing through this line and the total voltage drop on this line, respectively. (9i) is the semidefinite constraint on $\mathbf{V}$.

In (9j), $\mathbf{E}_{ref}^{\mathcal{R}} = [\mathbf{E}_{1,1}, \mathbf{0}; \mathbf{0}, \mathbf{0}]$ and $\mathbf{E}_{ref}^{\mathcal{I}} = [\mathbf{0}, \mathbf{0}; \mathbf{0}, \mathbf{E}_{1,1}]$ extract the real and imaginary parts of the reference bus voltage, respectively. Therefore, this constraint sets the voltage of the slack or reference bus of the LDS. If this constraint is eliminated, the optimization will converge to a wrong feasible point even with the rank constraint satisfied.

### C. Extraction of the Optimized Variables

If the above optimization finds a feasible point, vectors $\mathbf{p}_g$ and $\mathbf{q}_g$ are readily available. However, $\mathbf{v}$ needs to be extracted from the solution. In a fast and simple approach, one can select the first column of $\mathbf{V}$ to represent $\bar{\mathbf{v}}$ (i.e. $\bar{\mathbf{v}} = [\mathbf{V}_{:,1}]_{:}$). However, this approach is not accurate.

If the optimization reaches the optimal solution, $\mathbf{V}$ is rank 1. To calculate the rank, one should use a discriminatory methods to eliminate small singular values generated as a result of numerical errors. To do so, singular value decomposition can be used to generate $V = \mathbf{U}\Sigma\mathbf{V}^{T*}$ where $\mathbf{U}$ and $\mathbf{V}$ give the orthogonal basis associated with singular values. $\Sigma = [\sigma_i]_{ii}$ is a diagonal matrix containing singular values and $\sigma_i$ is the i-th singular value. It is common to sort $\Sigma$ from the largest singular value to the smallest. In a dispatch problem such as (9), if $\sigma_2 < 0.01\%\sigma_1$, one can assume the rank 1 condition is satisfied and calculate $\bar{\mathbf{v}} = \sqrt{\sigma_1}\mathbf{u}_1$ where $\mathbf{u}_1$ is the vector associated with the $\sigma_1$. Lastly, $\mathbf{v} = [v]_i = [\bar{v}]_i + \jmath[\bar{v}]_{(\mathcal{N}+i)}$ is the complex voltage vector for the underlying system.

## III. MULTI-OBJECTIVE DISPATCH

In this section, various objective functions are introduced and convexified. These objectives are combined later to form a multi-objective optimization problem. To add flexibility in combining various objectives, each objective function $i$ will be defined as an auxiliary variable $O_i$ where $\mathbf{o} = [O]_i$ is the vector of auxiliary variables defined to form various objective functions. Using this approach, objectives are enforced using additional constraints as described in the following sections.

### A. Active Power Cost Minimization

The most common objective function is the cost of active power generation. This function is often in the form of $c_{0_q} + c_{1_q}p^g_q + c^2_{2_q}(p^g_q)^2$. To make this objective function suitable for SDP, Schur complement is used as

$$\min_{\{p^g_q, q^g_q, \mathbf{V}, \mathbf{o}\}} O_1 \tag{10a}$$

$$s.t. \quad (9b) - (9j),\ \forall q \in \mathcal{N}_D$$

$$\begin{bmatrix} C_q - c_{1_q} p_q^g - c_{0_q} + \kappa c_{2_q} p_q^g \\ c_{2_q} p_q^g 1 \end{bmatrix} \succeq 0,\ O_1 = \sum_q C_q \tag{10b}$$

where $\mathcal{N}_D$ is the set of nodes with fuel consuming resources. $\kappa$ is a large constant to ensure $C_q - c_{1_q} p_q^g - c_{0_q} >$, otherwise, the correctness conditions for the Schur complement will not be satisfied.

## B. Voltage Regulation

In many industrial applications, voltage regulation is an important objective function as the performance and life span of equipment is a function of the voltage quality. This objective function can be achieve using two different approaches. In the first approach, the voltage amplitude of each bus is regulated to a predefined set point of $v_q^{ref}$. To do so, $O_2$ can be minimized as

$$\min_{\{p_q^g, q_q^g, \mathbf{V}, \mathbf{o}\}} O_2 \tag{11a}$$

$$s.t. \quad (9b) - (9j),\ \forall q \in \mathcal{N} \setminus \{b_1\}$$

$$(v_q^{ref})^2 - O_2 \leq \mathrm{Tr}(\mathbf{EE}_{q,q}\mathbf{V}) \leq (v_q^{ref})^2 + O_2 \tag{11b}$$

where $b_1$ is the reference or slack bus. In the second approach, the voltage profile is smoothened by adding costs to deviations from the average. Therefore, the cost associated to a voltage deviation is proportional to $(v_q - \sum_i v_i / \#\mathcal{N})^2$. Therefore, in (11b), $v_q^{ref}$ can be updated as $v_q^{ref} = \mathrm{Tr}(\mathbf{V})/\#\mathcal{N}$ is the average of the squares of the bus voltage amplitudes.

## C. Line Power Loss Minimization

In some applications, an objective is to minimize the total losses over the distribution lines. These losses can be simply calculated using (7). Therefore, this objective can be obtained using

$$\min_{\{p_q^g, q_q^g, \mathbf{V}, \mathbf{o}\}} O_3 \tag{12a}$$

$$s.t. \quad (9b) - (9j),\ \forall q, w \in \mathcal{N}$$

$$= \mathrm{Tr}(\mathbf{Y}_{q,w}^{\Re}\mathbf{V}) + \mathrm{Tr}(\mathbf{Y}_{w,q}^{\Re}\mathbf{V}) \leq O_3 \tag{12b}$$

### D. Reactive Power Cost Minimization

In various countries, industrial consumers have to pay for the reactive power purchased from the grid. For this reason, an objective function can be formed to reduce this acquisition. To do so, the reactive power passing through the Point of Common Coupling (PCC) has to be minimized. One can notice that this reactive power is infact the total reactive power generated on the reference bus. Hence,

$$\min_{\{p_q^g, q_q^g, \mathbf{V}, \mathbf{o}\}} O_4 \tag{13a}$$

$$s.t. \quad (9b) - (9j)$$

$$-O_4 \leq q_1^g \leq O_4 \tag{13b}$$

### E. Minimum Renewable Resource Curtailment

To reduce the total curtailment of renewable energy resources, a cost function can be associated with the total curtailed power $\hat{p}_q^g - p_q^g$ where $\hat{p}_q^g$ is the expected production level and $p_q^g$ is the dispatched production level. Hence, a second order curtailment penalty function can be considered as

$$\min_{\{p_q^g, q_q^g, \mathbf{V}, \mathbf{o}\}} O_5 \tag{14a}$$

$$s.t. \quad (9b) - (9j), \ \forall q \in \mathcal{N}_R$$

$$\begin{bmatrix} C_q - c_{3_q}(\hat{p}_q^g - p_q^g) + \kappa c_{4_q}(\hat{p}_q^g - p_q^g) & \\ c_{4_q}(\hat{p}_q^g - p_q^g) & 1 \end{bmatrix} \succeq 0 \tag{14b}$$

$$O_5 = \sum_q C_q \tag{14c}$$

where $\mathcal{N}_R$ is the set of nodes with renewable energy resources.

### F. Inverter Dispatch

This paper has considered the dispatch problem for DER within a LDS. Hence, the results are readily applicable to power electronic converters within a LDS. In this section, proper upper and lower generation limits for an inverter are introduced which can be integrated with the generation limits of (9).

The power generated by an inverter is limited to the capacity of its energy resource. In practice, inverters are over-designed for their applications. Hence, the peak power that an inverter can generate is higher than the limits induced by its resource such as photovoltaic panels. In this section, the maximum rated power of an inverter is considered at 1 p.u. without any further assumption on its resource.

Grid ties inverters can generate reactive power. However, their reactive power generation is controlled to ensure a unity power factor at the inverter terminals. The unity power factor regulation is enforced by IEEE 1547 standards within the U.S. This is mainly to maintain the voltage stability of the grid and to prevent voltage oscillations as a result of excessive number of voltage controllers. In the future, IEEE will relax this regulation to allow for a controlled dispatch of reactive power by low power DER inverters similar to some European regulations.

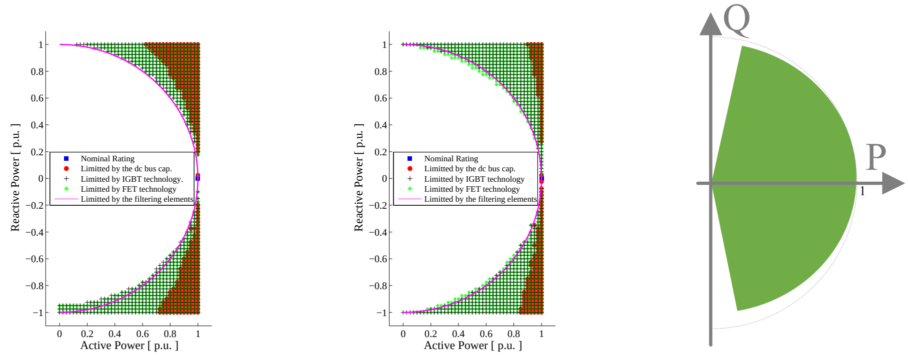
A three-phase inverter, even during a unity power factor operation, needs to generate sufficient reactive power to cancel the reactive power consumption of its filtering inductors. If the filtering inductor is considered ideal (i.e. $R_L = 0$), the power generated by the inverter is $P = |V_i||V_t|sin(\delta)/X$ where $V_i$ is the internal voltage of the inverter generated by the modulator and $V_t$ is the terminal voltage. $X$ is the impedance of the inductor and $\delta$ is the power factor of the inverter. Meanwhile, $Q_L = (|V_i|^2 + |V_t|^2 - 2|V_i||V_t|cos(\delta))/X$ is the reactive power consumed in the filtering inductor. This suggests that an inverter is readily utilizing some of its reactive power generation capacity. This amount is a function of the generated power (i.e. $\delta$). Later, it is shown in a numerical study of an inverter boundaries that the graph is shifted towards the reactive power generation. This is mainly due to the reason discussed above.

In addition, thermal limitations are enforced mainly by the switches and not the filtering elements. The current passing through the switch has a heavily nonlinear equation which is influenced by the modulation strategy, voltages, currents, and the switch technology. Hence, derivation of a closed-form equation is not possible as the equation depends on an exhaustive number of parameters. It is known that as the power factor of the inverter varies, the share of the current passed through the switches and the free wheeling diodes will change [31]. For this reason, at low power factors (high ratios of generated $Q/P$), diodes tend to have higher currents.

This limiting factor will reduce the total reactive power generation capability of an inverter to lower than the nominal active power. Therefore, assumption of a circular boundary such as those introduced in synchronous machines is not accurate (i.e. $P^2 + Q^2 = 1$). This effect is worsened as the active power generation becomes nearly zero. Meanwhile, the reactive power generation is also influenced by the dc link capacitor current limits [32]. In three phase inverters, this effect is not comparable to switch limitations, however, in single phase inverters, this effect is highly influential as the dc bus capacitor is required to support the second harmonic ripples.

Many manufacturers limit the output of the inverter to a boundary defined by the inverter power factor. However, this boundary is excessive and a wider region can keep the inverter in a safe operation area. To get an estimate of this boundary, numerical simulations of the inverter can be incorporated. In an example, a three-phase 100 kW inverter with an output voltage of 480 V and a dc bus voltage of 800 V is considered. The output inductor is design to have 5 $\%$ voltage drop at the rated power. This inverter has a switching frequency of 20 kHz and a space vector modulation (inverse park) pattern. The modulation is set to maximum efficiency by minimizing the number of switching per ac cycle (any modification in the switching pattern can influence the generated boundary [31]). Also, field effect transistors are considered to formulate the switch loss equation. The results will be different for bipolar transistors such as IGBTs. Under these conditions, the inverter is simulated and the switch losses are calculated. The boundaries generated by this inverter is shown in Fig. 1c. The circular boundary is enforced by the inductor current limits while the remainder of the boundary is set by the switch losses and the dc link current rating. Figure 1c is not symmetrical as the inverter needs to compensate for the reactive power consumption of the filtering elements at all times as mentioned earlier.

To model such behavior as a constraint in the optimization problem, the figure can be approximated by an ellipse as is shown in Fig. 1a. The parameters of the ellipse can be calculated numerically for each inverter by the manufacturer. $c_p$ and $c_q$ represent the centers for $P$ and $Q$, respectively.




(a) (b) (c)

Fig. 1. Boundaries for (a) a 1-phase, (b) a 3-phase converter, (c) approximated boundaries.

Similarly, $r_p$ and $r_q$ represent the radius on the $p$ and $q$ axis, respectively. This new constraint is added to (9) as

$$\begin{bmatrix} 1(p_q^g - c_{p_q})(q_q^q - c_{q_q}) \\ (p_q^g - c_{p_q})r_p^2 0 \\ (p_q^g - c_{p_q})0 r_q^2 \end{bmatrix} \succeq 0 \tag{15}$$

$$q_q^g \leq a_q p_q^g, \; q_q^g \geq -b_q p_q^g \tag{16}$$

for all $q \in \mathcal{N}_I$ where $\mathcal{N}_I$ is the set of nodes with inverters.

### G. Regularization

To this point, the framework was addressing the non-convexity induced by the power flow equation. However, sparsification of the system is another technical challenge in development of an optimization framework. In many applications, OPF or ORPD has to be performed on a limited number of generators. To do so, a binary selection variable of $s_i \in \{0, 1\}$ is assigned to the $i$-th generator. If $s_i = 0$ the generator is exempt from the dispatch and if $s_i = 1$ the generator is controlled. In general, this variable can be defined for both active and reactive power as

$$p_q^{min} s_q^p \leq p_q^g \leq p_q^{max} s_q^p \tag{17a}$$

$$q_q^{min} s_q^q \leq q_q^g \leq q_q^{max} s_q^q \tag{17b}$$

$$s_q^p,\ s_q^q \in \{0,1\},\ \sum_j s_j^p \leq S_p,\ \sum_j s_j^q \leq S_q \tag{17c}$$

where $S_p$ and $S_q$ set the maximum number of generators participating in the active and reactive power control, respectively. In general, this problem is NP-hard. Therefore, a treatment to this sparsification problem is required. Instead of the hard sparse norm of $\|\cdot\|_0$, one can use convex norms to relax the optimization problem. In is important to note that $p$-norms for $p < 1$ which are the sparse norms are not feasible for implementation with the SDP. Instead, regularization norms of $\ell_1$ and $\ell_2$ can be incorporated. For $\ell_1$, if all the generation levels are positive numbers, one can generate a cost function as $O_6^p = \sum p_j^g$ with minimization over $O_6$ (similarly for the reactive power). In general, one can use

$$\min_{\{p_q^g, q_q^g, \mathbf{V}, \mathbf{o}\}} O_5 \tag{18a}$$

$$s.t. \quad (9b) - (9j),\ \forall q \in \mathcal{N} \setminus \{b_1\}$$

$$-C_{q_p} \leq p_q^g \leq C_{q_p},\ -C_{q_q} \leq q_q^g \leq C_{q_q} \tag{18b}$$

$$O_6 = \sum_k (\gamma_p C_{p_k} + \gamma_q C_{q_k}) \tag{18c}$$

where $\gamma_p$ and $\gamma_q$ are the regularization gains for active and reactive powers. Quadratic regularization is not recommended as a sparsification method. However, if one decides to use this function, Schur's complement can be incorporated. Some solvers directly support minimizing over the $\ell_1$ norm which is similar to the above optimization.

### H. Combining Objectives

In order to form the multi-objective optimization problem, the objective functions introduced in the previous sections are combined. To do so, various approaches can be taken. In a variety of applications, linear combination of the objectives is used to form the multi-objective cost function. In this approach, the optimization problem is formed as

$$\min_{\{\mathbf{p}^g, \mathbf{q}^g, \mathbf{V}, \mathbf{o}\}} \sum_i \gamma_i O_i \tag{19}$$

$$s.t. \quad (9b) - (18c)$$

where $\gamma_i$ is a linear gain to set the influence of the $i$-th objective function. Although this approach is simple, it cannot enforce non-linear preferences between multiple objectives.

To clarify this problem, consider an LDS with multiple voltage sensitive nodes such as a hospital complex with a variety of safety equipment which require a high quality input voltage. An objective is to minimize the operation cost of this LDS while maintaining a good voltage regulation by performing a multi-objective optimization over the energy assets located within this LDS. As the voltage regulation is enforced further, the strength of the cost minimization objective is relatively reduced. In the linear approach, linear gains do not allow for flexibility in the selection of objective functions. However, one desire to relax the voltage regulation objective by allowing for $\pm 3\%$ regulation to achieve a better cost optimization over the assets. Outside this boundary, one can enforce a non-linear growth on the voltage regulation objective to attain a safe operation zone for the equipment while achieving the best relative dispatch for the energy assets. Therefore, in this section, various functions are introduced to form the multi-objective optimization problem.

Assuming that $\forall i \in \{1,\cdots,\#\mathbf{o}\}; 0 \leq O_i$, the first form of growth function is the linear growth which is extensively used in many applications. This function is a linear gain of the objective function as $\gamma_i O_i$ (we define $\bar{O}_i = O_i$). Another useful growth function is the quadratic form of the cost function. In this approach, Schur complement can be incorporated to form a growth function as $\bar{O}_i = (O_i^2 + c_{1_i} O_i + c_{2_i})$ which is often simplified as $\bar{O}_i = O_i^2$ (some solvers, directly support $f(x) = x^p$ , $1 \leq p$). Higher orders of the objective functions can be similarly generated using Schur complement. Another interesting function is a hard limit which is achieved by introducing a new constraint as $O_i \leq k$ where $k$ is the desired limit.

Lastly, the multi-objective optimization is formed as

$$\min_{\{\mathbf{p}^g,\mathbf{q}^g,\mathbf{V},\mathbf{o}\}} \sum_i \gamma_i \bar{O}_i \tag{20}$$
$$s.t. \quad (9b) - (18c),\ f(O_i, \bar{O}_i),\ \forall i \in \{1,\cdots,\#\mathbf{o}\}$$

where $f(\cdot)$ denotes one of the explored growth functions which are mostly formed using Schur complements.

## IV. DISTRIBUTED SOLUTION

The previous sections introduced the centralized multi-objective optimization of the dispatch problem. In this section, a distributed solution is studied to improve the scalability of the dispatch problem. The graph of the network $\mathcal{G}$ consists of $n \in \mathcal{N}$ nodes which are connected with arcs $a_{q,w} \in \mathcal{N}\times\mathcal{N}$. This graph can have $\mathfrak{r}$ dispatch regions. In this case, each node is a member of one region $n \in \mathcal{R}_i$ where $\mathcal{R}_i \in \{\mathcal{R}_1,\cdots,\mathcal{R}_\mathfrak{r}\}$. In this paper, it is assumed that each node can be only a member of one region. Also, it is assumed that the underlying network is a tree and hence, only one path exists between two different nodes. Under these conditions, the convergence of the distributed solution is guaranteed [33]. The set of neighboring nodes to region $\mathcal{R}_i$ is defined as $\partial\mathcal{R}_i = \{j|a_{i,j} = 1,\ i \in \mathcal{R}_i,\ j \notin \mathcal{R}_i\}$. Also, the extended region is defined as $\bar{\mathcal{R}}_i = \mathcal{R}_i \cup \partial\mathcal{R}_i$.

To perform th distributed optimization, each region $i$ needs to solve the power flow constraints over its extended regional graph $\bar{\mathcal{R}}_i$. Hence, the optimal dispatch for region $i$ is derived as

$$\min_{\{\mathbf{p}_i^g,\mathbf{q}_i^g,\mathbf{V}_i\}} f(\mathbf{p}_i^g,\mathbf{q}_i^g,\mathbf{V}_i) \tag{21a}$$

$$s.t. \quad (9b)-(18c)$$

$$[V_i]_q,w = [V_j]_q,w|q,w \in \bar{\mathcal{R}}_i,\ q,w \in \bar{\mathcal{R}}_j,\ i \neq j \tag{21b}$$

where the constraint (22b) ensures that in each regional optimization, the edge nodes have the same voltage solution. This constraint leads to 16 constraints on $\mathbf{V}_i$ at locations $\mathcal{H}_i = \{(q,q),\ (q,w),\ (w,q),\ (w,w),\ (q{+}\#\bar{\mathcal{R}}_i,q),\cdots,\ (w,w{+}\#\bar{\mathcal{R}}_i),\ (q{+}\#\bar{\mathcal{R}}_i,q{+}\#\bar{\mathcal{R}}_i),\cdots$. In theory, less than half of these constraints are required and the remaining will be satisfied by the symmetric nature of $\mathbf{V}$. However, to ensure numerical convergence, it is best to include all 16. To practically implement the distributed optimization, one should introduce auxiliary variables to satisfy $[\mathbf{V}_i]_{(q,w)\in\mathcal{H}_i} = A$ in the first optimization and $A = [\mathbf{V}_j]_{(q,w)\in\mathcal{H}_j}$ in the second optimization. Using this approach, the distributed optimization can be solved iteratively using the Alternating Direction Method of Multipliers (ADMM) [33]. At each iteration $k$, this method depends on the first and second order norms of $\varepsilon = ([\mathbf{V}_i]^k_{(q,w)} - A^{k-1}_{(q,w)})$ where $[\mathbf{V}_i]^k_{(q,w)}$ is the optimization variable at iteration $k$ and $A^{k-1}_{(q,w)}$ is the average of all optimizations solved by regions containing this element. For instance, if these nodes are shared by two regions, $A_{q,w}^{k-1} = ([\mathbf{V}_i]_{(q,w)}^{k-1} + [\mathbf{V}_j]_{(q,w)}^{k-1})/2$. Unfortunately, many optimizers such as CVX do not allow for addressing an individual matrix entry. In this case, the matrix $\bar{\mathbf{V}}_i^{k-1}$ can be constructed by averaging the 16 desired entries outside of the optimizer and a selector matrix $\mathbf{S}_{q,w} \in \mathbb{R}^{\#\bar{\mathcal{R}}_i \times \#\bar{\mathcal{R}}_i}$ can be introduced which contains zeros except for the entry $q,w$ which is one. Then $\varepsilon = \mathrm{Tr}(\mathbf{S}_{(q,w)}(\mathbf{V}_i^k - \bar{\mathbf{V}}_i^{k-1})^T)$. The first order norm of $\varepsilon$ is readily available (the constant $\bar{\mathbf{V}}_i^{k-1}$ can be dropped from the first order norm). For the second order norm, Schur complement is used to calculate the Frobenius norm of the above difference. By introducing Lagrangian multiplier $\Lambda_i$, one can generate the primal optimization as

$$\min_{\{\mathbf{p}_i^g, \mathbf{q}_i^g, \mathbf{V}_i\}} f(\mathbf{p}_i^g, \mathbf{q}_i^g, \mathbf{V}_i) + \gamma_{(\hat{q},\hat{w})} + \mathrm{Tr}(\Lambda_i^{k-1}\mathbf{V}_i^T) + \sum_{(q,w)\in\mathcal{H}_i} \beta_{q,w} \tag{22a}$$

$$s.t. \quad (9b) - (18c),\ \forall (q,w) \in \mathcal{H}_i$$

$$\begin{bmatrix} \beta_{(q,w)} & \mathrm{Tr}(\mathbf{S}_{(q,w)}(\mathbf{V}_i - \bar{\mathbf{V}}_i^{k-1})^T) \\ \mathrm{Tr}(\mathbf{S}_{(q,w)}(\mathbf{V}_i - \bar{\mathbf{V}}_i^{k-1})^T)2/\rho & \end{bmatrix} \succeq 0 \tag{22b}$$

$$\gamma_{(\hat{q},\hat{w})} >= g(\mathrm{Tr}(\mathbf{Y}_{(\hat{q},\hat{w})}^{\mathfrak{R}}\mathbf{V})),\ \hat{q} \in \mathcal{R}_i,\ \hat{q} \notin \mathcal{R}_j,\ \hat{w} \in \mathcal{R}_j,\ \hat{w} \notin \mathcal{R}_i \tag{22c}$$

Where do to the linearity of $\sum\sum \mathrm{Tr}(\Lambda_{i_{(q,w)}}^{k-1}(\mathbf{S}_{(q,w)}\mathbf{V}_i)^T)$, it can be simply replaced with $\mathrm{Tr}(\Lambda_i^{k-1}\mathbf{V}_i^T)$. $\rho$ is an arbitrary positive number which is required by the ADMM [33]. It should be noted that (22b) represents 16 constraints per each connection between two different regions. To make the problem more practical, $\gamma_{q,w}$ assigns a cost function $g(\cdot)$ to the power flowing between the two regions.

Using this step, a new value for $\mathbf{V}_i$ is calculated. At this point, neighboring regions announce their relevant entries from $\mathbf{V}_i^k$ so that each region can construct the new average $\bar{\mathbf{V}}_i^k$. Now, it is time to update the dual variable $\Lambda_i$.

$$\Lambda_i^k = \Lambda_i^{k-1} + 2\rho(\mathbf{V}_i^k - \bar{\mathbf{V}}_i^k) \tag{23}$$

where $2\rho(\mathbf{V}_i^k - \bar{\mathbf{V}}_i^k)$ is equal to $\rho(\mathbf{V}_i^k - \mathbf{V}_j^k)$ which is the standard error in the formulation of ADMM dual update. This iterative process will continue until the convergence criterion is met.

## V. CASE STUDY

In this section, several case studies will be performed on the system shown in Fig. 2. This system consists of three regions A, B, and C. First, the centralized optimal dispatch of this system is studied and later, the distributed optimization over these three regions will be investigated. Parameters of this system are introduced in Table I. These parameters are selected to drop the voltage of the region A and increase the voltage of the region B and C to improve the quality of the case studies. The series impedance of each transmission line is $0.1 + \jmath 0.1$.

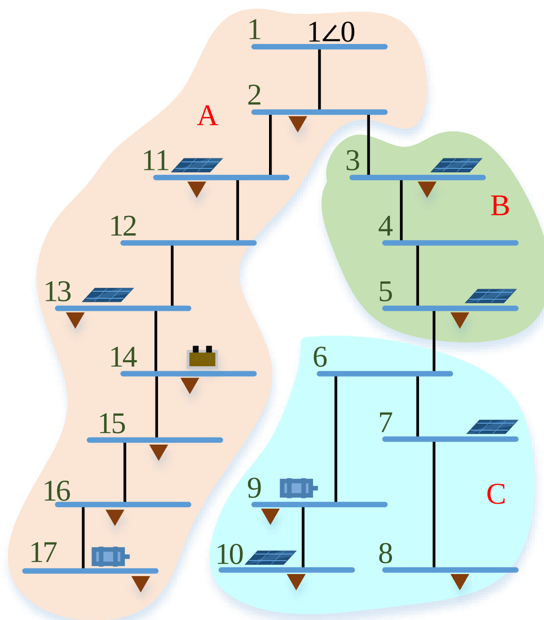



Fig. 2. The distribution network under study which consists of three dispatch regions.

In the first scenario, the centralized optimal dispatch is solved using only active power cost functions. To reduce the effects of numerical errors, one can include a very small cost for voltage regulations to emphasis the impacts of individual bus bar voltages. Also, one can include a very small second order cost function for reactive power dispatch to create a minimum over the reactive dispatch of zero. Although the cost of the reactive power dispatch is zero, addition of this second order cost function prevents multiple minimums for the overall problem. With these assumptions, the optimized dispatch of the resources considering the active generation cost functions is shown in Fig. 3.

TABLE I

PARAMETERS OF THE UNDERLYING SYSTEM

| Node | Demand | $P_g^{max.}$ | $P_g^{min.}$ | Cost model |
|---|---|---|---|---|
| 1 | - | - | - | $c_1 P_g$ |
| 2 | $0.02 + \jmath 0.01$ | - | - | - |
| 3 | $0.005 + \jmath 0.005$ | 0 | 0.01 | $c_0 - c_1 P_g$ |
| 4 | - | - | - | - |
| 5 | $0.005 + \jmath 0.001$ | 0 | 0.2 | $c_0 - c_1 P_g$ |
| 6 | - | - | - | - |
| 7 | - | 0 | 0.2 | $c_0 - c_1 P_g$ |
| 8 | $0.01 + \jmath 0.001$ | - | - | - |
| 9 | $0.005 - \jmath 0.005$ | 0 | 0.1 | $c_1 P_g + c_2 P_g^2$ |
| 10 | $0.02 - \jmath 0.001$ | 0 | 0.2 | $c_0 - c_1 P_g$ |
| 11 | $0.02 + \jmath 0.01$ | 0 | 0.005 | $c_0 - c_1 P_g$ |
| 12 | - | - | - | - |
| 13 | $0.02 + \jmath 0.02$ | 0 | 0.01 | $c_0 - c_1 P_g$ |
| 14 | $0.03 + \jmath 0.02$ | -0.05 | 0.05 | $c_2 P_g^2$ |
| 15 | $0.02 + \jmath 0.005$ | - | - | - |
| 16 | $0.02 + \jmath 0.02$ | - | - | - |
| 17 | $0.02 + \jmath 0.02$ | 0 | 0.2 | $c_1 P_g + c_2 P_g^2$ |

From Fig. 3, it can be observed that the voltage of the wing 2:10 increases due to the large amount of solar generation. Also, the wing 11:17 observes voltage drop due to the large demand. To improve the voltage regulation, an objective is formed with linear combination of the active power generation costs as well as the voltage regulation cost function. The results of this optimization is shown in Fig. 4. It can be observed that the voltage regulation is much better compared to the previous scenario. Also, no significant change in the active power dispatch is detected. The optimizer has utilized optimal dispatch of reactive power to regulate the voltage of buses.

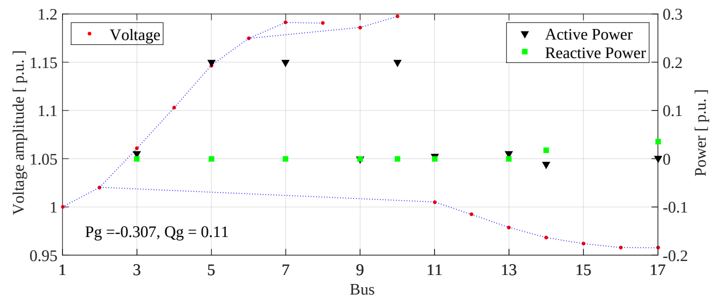



Fig. 3. Active power dispatch.

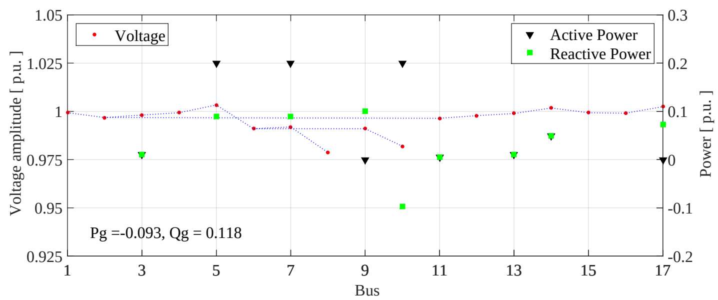



Fig. 4. Linear combination of the active power dispatch and voltage regulation.

In practical applications, the cost of active power has priority to the voltage regulation. Hence, linear combination of these objectives might not be feasible. In a second approach, the voltage regulation can be combined as a quadratic order cost function. Hence, if the voltage of the bus exceeds from the reference voltage, the cost is increased nonlinearly. Therefore, this approach promotes a more relaxed voltage regulation constraint for lower voltage variations and a larger penalty if the variations are large. Results for the scenario with a quadratic voltage regulation cost function is shown in Fig. 5.

As a result of the quadratic cost function, it can be observed that the larger variations of the voltage are suppressed. In the next scenario, the cost function for reducing the transmission losses is linearly added to the previous objective function. Transmission loss minimization acts similar to a voltage profile regulator. If the adjacent buses have similar voltages, the current passing through the transmission lines will be reduced and smaller transmission losses will be attained. Results for this scenario are shown in Fig. 6.

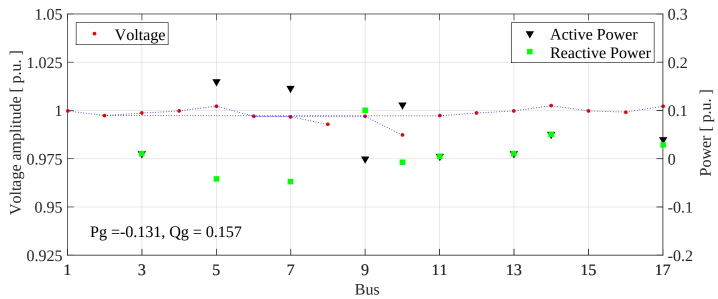



Fig. 5. Linear combination of the active power dispatch and quadratic voltage regulation.

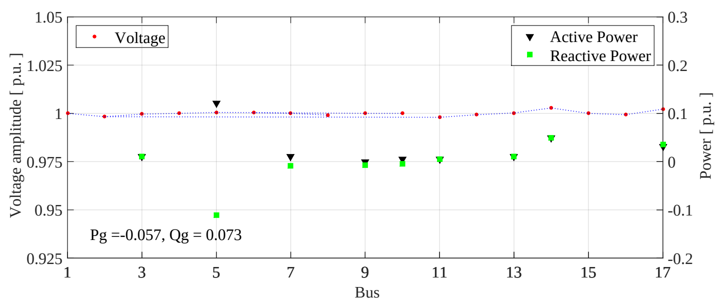



Fig. 6. Linear combination with the transmission loss minimization.

In the next scenario, to demonstrate the close relation between the voltage profile regulation and transmission loss minimization cost functions, voltage regulation cost is eliminated and the objective function is a linear combination of the generation costs and transmission losses. Results for this scenario are illustrated in Fig. 7.

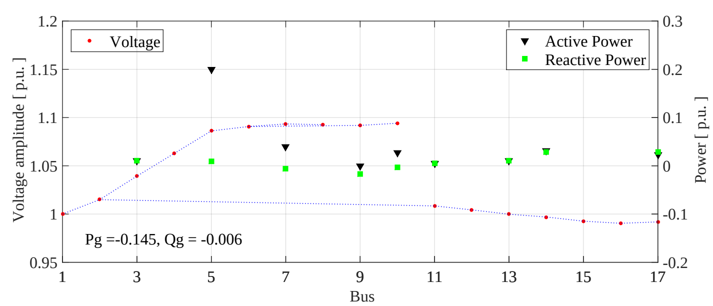



Fig. 7. Linear combination of active power dispatch and transmission loss minimization.

It can be observed that the transmission loss minimization objective function tends to keep the voltage of adjacent buses similar. However, this does not guarantee a good voltage regulation as it can be observed from Fig. 7.

Lastly, it is observed from the Fig. 6 that some reactive power is being purchased from the grid. In some industrial applications, purchasing reactive power from the utility grid is costly. Hence, one can linearly add an objective function to reduce this reactive power purchase. The results for this scenario are shown in Fig. 8.

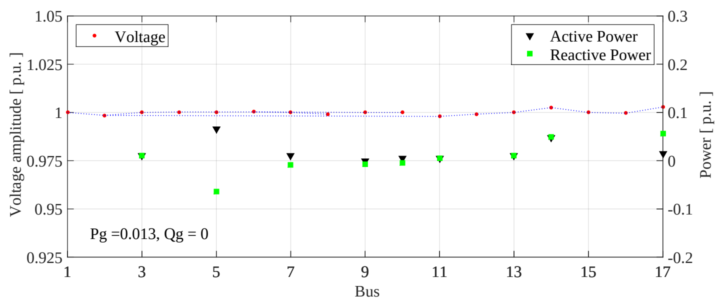



Fig. 8. Adding quadratic voltage regulation cost function.

Scenarios with distributed optimization over the three regions of A, B, and C are now focused on. In the first scenario, the only cost function used is the active power generation cost. Results for this scenario are shown in Fig. 9. It can be observed that the results are different compared to the centralized solution of Fig. 3. This is due to the nature of distributed optimization where each region seeks its regional minimum cost. Results show inferior voltage regulation. To this end, in the next scenario, voltage regulation cost functions are added to improve the voltage profile. Results for this scenario are shown in Fig. 10.

## VI. CONCLUSION

In this paper, centralized and distributed formalisms for multi-objective dispatch of distributed energy resources were introduced. First, this paper investigated semidefinite relaxation of the power flow equations. Later, various cost functions suitable for distribution network applications were introduced in the semidefinite framework. Afterwards, combination of these objective functions were investigated. The centralized solution was

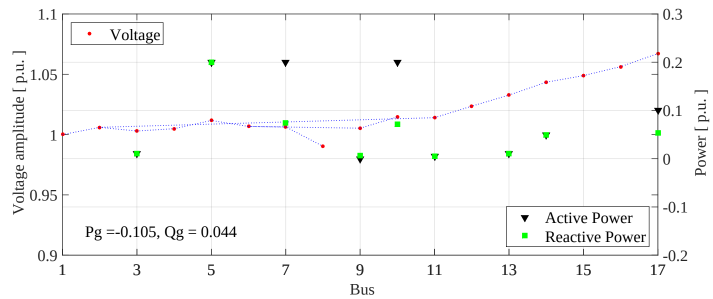



Fig. 9. Distributed active power generation cost optimization.

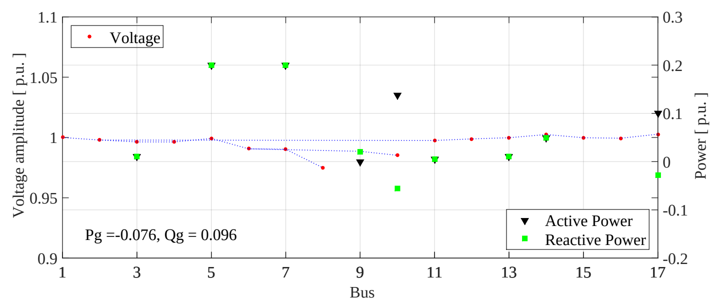



Fig. 10. Distributed optimization of the active power generation and voltage regulation.

extended to support distributed optimization using the alternating direction method of multipliers. In the end, various case studies were provided to demonstrate the behavior of objective functions.

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## SECTION

## 2. CONCLUSIONS

This thesis proposed two papers in which the optimal dispatch for distributed energy resources was investigated. In the first paper, an economic dispatch problem for a community microgrid was studied. In this microgrid, each agent pursued an economic dispatch for its personal resources. In addition, each agent was capable of trading electricity with other agents through a local energy market. A simple market structure was introduced as a framework for energy trades in a small community microgrid such as the Solar Village. It was found that agents were able to estimate the operation of the operation of the market and effectively dispatch their resources. Both buyers and sellers benefited from participating in the community market. In the second paper, Semidefinite Programming (SDP) for convex relaxation of power flow equations was used for optimal active and reactive dispatch for Distributed Energy Resources (DER). Because SDP drops the rank constraint, it made the optimal dispatch process faster. Various objective functions including voltage regulation, reduced transmission line power losses, and minimized reactive power charges for a microgrid were introduced. Combinations of these goals were attained by solving a multi-objective optimization for the proposed ORPD problem. The different combinations allowed objectives to be prioritized. Also, both centralized and distributed versions of this optimal dispatch were investigated and were utilized depending on privacy concerns.

# VITA

Ayomide Longe was born on April 12, 1991 in Lagos, Nigeria, West Africa. She was inducted into the Eta Kappa Nu (HKN) Electrical Engineering and Tau Beta Pi (TBP) engineering honor societies in 2011. She earned her Bachelor's degree with Honors in Electrical Engineering and Computer Science from the University of Texas, Arlington in December 2013. She earned her Engineer In Training (EIT) certification that would enable her to qualify as a Professional Engineer (PE) in the future in December 2013. She earned her Master's degree in Electrical Engineering with a focus on Power Systems from Missouri University of Science and Technology in August 2015.

Ayomide completed an internship with ALCOA in the summer of 2013, where she conducted an Arc Flash Hazard Analysis on the substations in the Alumina processing plant in Point Comfort, Texas. She worked as a Graduate Research Assistant under Dr. Pourya Shamsi in the Department of Electrical Engineering from January 2014 to July 2015, from which this thesis came to fruition.

Ayomide is passionate about delivering reliable power to the public. She began full-time employment as an Electrical Engineer at Aiken Electric Cooperative, an electric utility company serving Aiken, South Carolina and extended areas, in September 2015. She will explore options of earning a Ph.D. in Electrical Engineering and Computer Science, tackling problems in big data analytics in power systems. She is a member of IEEE Power and Energy Society (PES), and will serve as General Secretary for the IEEE PES Power Africa conference in 2016.