

NUMERICAL EVALUATION OF PERIODIC TRANSVERSE VIBRATION OF ELASTIC CONNECTING RODS IN A SIX-LINK MECHANISM

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Abstract. Application of the substructure method and d'Alembert's principle to deriving the differential equation of motion of a six-link mechanism with two elastic connecting rods is presented

In the case of stationary motion, the generalized Ritz's method has been applied to obtain system of linear differential equations with periodic coefficients. We have written computer programs to check conditions of dynamic stability and to find periodic solutions of the obtained equations. Numerical examples are given and from which the effect of elastic factors on articulation reactions is evaluated.

1. Introduction

Informative development has made the solution of certain mechanism vibration problems possible. However, it is usual that only natural frequencies or solution in the starting period are found. The determination of periodic solutions for stationary motion in mechanisms with both solid and elastic elements is still under investigation [3-8].

In [3] we examined the periodic transverse vibration problem of a connecting rod in a four-link mechanism. The objective in this paper is to extend the method into a mechanism having six links. The substructure method has been applied to deriving the dynamic equations of the mechanism. From which equations of transverse vibration of the connecting rods are obtained for the case of the driving link rotating uniformly. The dynamic stability conditions and periodic solutions of the mentioned equations are found by using numerical method [1, 2].

2. Derivation of dynamic equations

We consider a six-link mechanism (Fig.1) moving on the vertical plane, in which the crank O_1A (link 2), the rocker O_2BC (link 4), the rocker O_3D (link 6) are solid bodies of weighs P_2, P_4, P_6 , and centers of mass S_2, S_4, S_6 respectively. And $O_1S_2 = s_2, O_2S_4 = s_4, O_3S_6 = s_6, \angle AO_1S_2 = \alpha_2, \angle BO_2S_4 = \alpha_4, \angle BO_2C = \beta_4, \angle DO_3S_6 = \alpha_6$.

The connecting rods AB (link 3), CD (link 5) are elastic bodies. Supposing that the connecting rods are rectilinear and their axes without deformation coincides with the elastic one and that longitudinal vibration is negligible. The connecting rods AB, CD have area of the cross section F_j , mass density ρ_j , mass per unit length μ_j , elastic modules E_j , moment of inertia J_j , relative transverse vibration w_j , where $j = 3$ for the element AB, $j = 5$ for the element CD.

Fig.1 shows the mechanism with global axes $O_1\xi\eta$. The horizontal $O_1\xi$ forms with the segments $O_1O_2, O_2O_3, O_1A, AB, O_2B, O_2C, CD, O_3D$ the angles $\theta, \bar{\theta}, \varphi_2, \varphi_3, \varphi_4, \bar{\varphi}_4, \varphi_5, \varphi_6$ respectively. We have:

$$\bar{\varphi}_4 = \varphi_4 - \beta_4. \tag{2.1}$$

Given $O_1O_2 = l_1$, $O_2O_3 = \bar{l}_1$, $O_1A = l_2$, $AB = l_3$, $O_2B = l_4$, $O_2C = \bar{l}_4$, $CD = l_5$, $DO_3 = l_6$.

The kinematics relations between links are:

$$\begin{aligned} l_2 \cos \varphi_2 + l_3 \cos \varphi_3 - l_4 \cos \varphi_4 - l_1 \cos \theta &= 0, \\ l_2 \sin \varphi_2 + l_3 \sin \varphi_3 - l_4 \sin \varphi_4 - l_1 \sin \theta &= 0, \\ \bar{l}_4 \cos \bar{\varphi}_4 + l_5 \cos \varphi_5 - l_6 \cos \varphi_6 - \bar{l}_1 \cos \bar{\theta} &= 0, \\ \bar{l}_4 \sin \bar{\varphi}_4 + l_5 \sin \varphi_5 - l_6 \sin \varphi_6 - \bar{l}_1 \sin \bar{\theta} &= 0. \end{aligned} \quad (2.2)$$

From (2.2), we can find $\varphi_i = \varphi_i(\varphi_2)$ ($i = 3, 4, 5, 6$). In the stationary regime we have $\varphi_2 = \Omega t$ (Ω is constant), thus φ_i are periodic functions with period $2\pi/\Omega$.

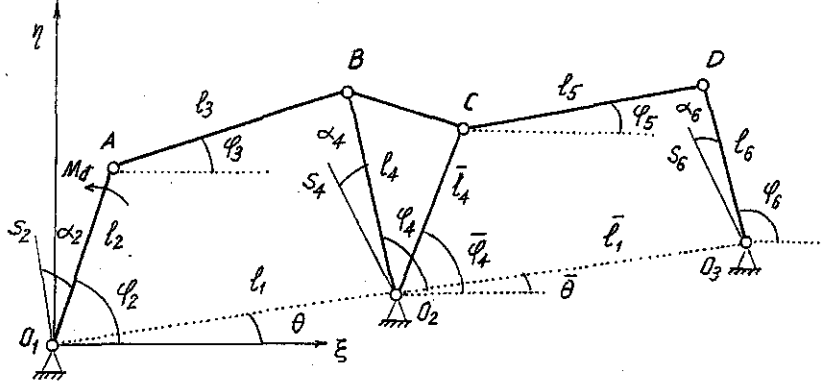


Fig. 1

Applying the substructure method we divide the mechanism into 5 sub-structures. Each of the links represents a substructure.

The symbols J_{O_1} , J_{O_2} , J_{O_3} indicate moments of inertia of O_1A , O_2BC and O_3D with respect O_1 , O_2 , O_3 respectively.

The application of d'Alembert's principle to each link yields:

For the crank O_1A (Fig. 2):

$$X_{O_1} = -X_A + P_2 \sin \varphi_3 - \frac{P_2}{g} s_2 \left[\dot{\varphi}_2^2 \cos(\varphi_2 - \varphi_3 + \alpha_2) + \ddot{\varphi}_2 \sin(\varphi_2 - \varphi_3 + \alpha_2) \right], \quad (2.3)$$

$$Y_{O_1} = -Y_A + P_2 \cos \varphi_3 - \frac{P_2}{g} s_2 \left[\dot{\varphi}_2^2 \sin(\varphi_2 - \varphi_3 + \alpha_2) - \ddot{\varphi}_2 \cos(\varphi_2 - \varphi_3 + \alpha_2) \right], \quad (2.4)$$

$$M_d = J_{O_1} \ddot{\varphi}_2 + l_2 \left[X_A \sin(\varphi_2 - \varphi_3) - Y_A \cos(\varphi_2 - \varphi_3) \right] + s_2 P_2 \cos(\varphi_2 + \alpha_2). \quad (2.5)$$

For the rocker O_2BC (Fig. 3):

$$\begin{aligned} X_{O_2} = & -X_C \cos(\varphi_5 - \varphi_3) + Y_C \sin(\varphi_5 - \varphi_3) + X_B + P_4 \sin \varphi_3 \\ & - \frac{P_4}{g} s_4 \left[\dot{\varphi}_4^2 \cos(\varphi_4 - \varphi_3 + \alpha_4) + \ddot{\varphi}_4 \sin(\varphi_4 - \varphi_3 + \alpha_4) \right], \end{aligned} \quad (2.6)$$

$$\begin{aligned} Y_{O_2} = & -X_C \sin(\varphi_5 - \varphi_3) - Y_C \cos(\varphi_5 - \varphi_3) + Y_B + P_4 \cos \varphi_3 \\ & - \frac{P_4}{g} s_4 \left[\dot{\varphi}_4^2 \sin(\varphi_4 - \varphi_3 + \alpha_4) + \ddot{\varphi}_4 \cos(\varphi_4 - \varphi_3 + \alpha_4) \right], \end{aligned} \quad (2.7)$$

$$X_B = l_4 \frac{X_C \sin(\bar{\varphi}_4 - \varphi_5) - Y_C \cos(\bar{\varphi}_4 - \varphi_5)}{l_4 \sin(\varphi_4 - \varphi_3)} + \frac{P_4 s_4 \cos(\varphi_4 + \alpha_4) + J_{O_2} \ddot{\varphi}_4}{l_4 \sin(\varphi_4 - \varphi_3)} + Y_B \cotg(\varphi_4 - \varphi_3). \quad (2.8)$$

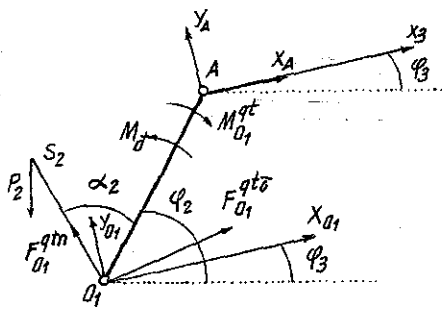


Fig. 2

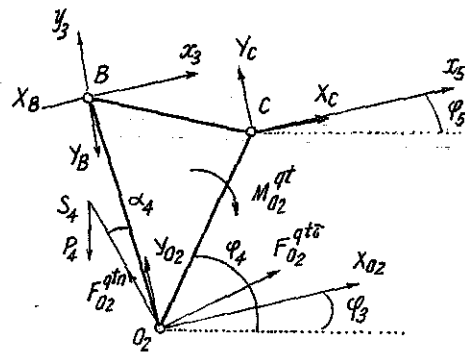


Fig. 3

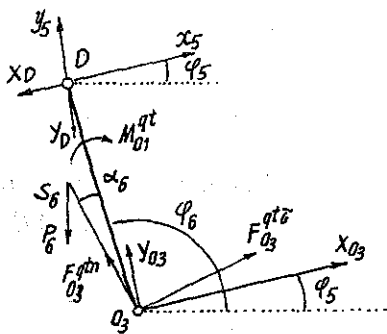


Fig. 4

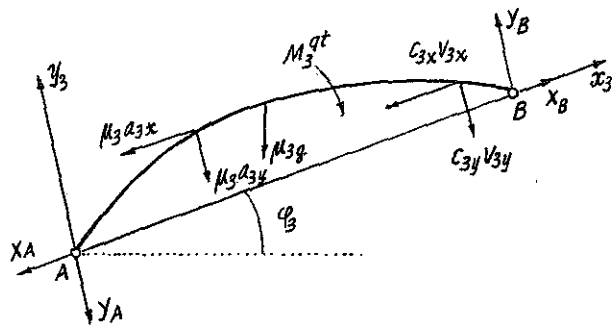


Fig. 5

For the rocker O_3D (Fig. 4):

$$X_{O_3} = X_D + P_6 \sin \varphi_5 - \frac{P_6}{g} s_6 \left[\dot{\varphi}_6^2 \cos(\varphi_6 - \varphi_5 + \alpha_6) + \ddot{\varphi}_6 \sin(\varphi_6 - \varphi_5 + \alpha_6) \right], \quad (2.9)$$

$$Y_{O_3} = Y_D + P_6 \cos \varphi_5 - \frac{P_6}{g} s_6 \left[\dot{\varphi}_6^2 \sin(\varphi_6 - \varphi_5 + \alpha_6) + \ddot{\varphi}_6 \cos(\varphi_6 - \varphi_5 + \alpha_6) \right], \quad (2.10)$$

$$X_D = \frac{P_6 s_6 \cos(\varphi_6 + \alpha_6) + J_{O_3} \ddot{\varphi}_6}{l_6 \sin(\varphi_6 - \varphi_5)} + Y_D \cotg(\varphi_6 - \varphi_5). \quad (2.11)$$

For the connecting rod AD (Fig. 5):

$$X_A = X_B - \int_0^{l_3} [\mu_3 (g \sin \varphi_3 + a_{3x}) + c_{3x} v_{3x}] dx_3, \quad (2.12)$$

$$Y_A = Y_B - \int_0^{l_3} [\mu_3 (g \cos \varphi_3 + a_{3y}) + c_{3y} v_{3y}] dx_3, \quad (2.13)$$

$$\begin{aligned}
Y_B = \frac{1}{\ell_3} \left\{ \int_0^{\ell_3} \left[\rho_3 J_3 \left(\ddot{\varphi}_3 + \frac{\partial^3 w_3}{\partial x_3 \partial t^2} \right) dx_3 + \int_0^{\ell_3} [\mu_3 (g \cos \varphi_3 + a_{3y}) \right. \right. \\
\left. \left. + c_{3y} v_{3y} \right] x_3 dx_3 - \int_0^{\ell_3} [\mu_3 (g \sin \varphi_3 + a_{3x}) + c_{3x} v_{3x}] w_3 dx_3 \right\}. \quad (2.14)
\end{aligned}$$

As for the connecting rod CD, it is seen that the model is completely similar to that of AB by changing the index 3 into 5 and the letters A, B into C, D respectively. So we have:

$$X_C = X_D - \int_0^{\ell_5} [\mu_5 (g \sin \varphi_5 + a_{5x}) + c_{5x} v_{5x}] dx_5, \quad (2.15)$$

$$Y_C = Y_D - \int_0^{\ell_5} [\mu_5 (g \cos \varphi_5 + a_{5y}) + c_{5y} v_{5y}] dx_5, \quad (2.16)$$

$$\begin{aligned}
Y_D = \frac{1}{\ell_5} \left\{ \int_0^{\ell_5} \left[\rho_5 J_5 \left(\ddot{\varphi}_5 + \frac{\partial^3 w_5}{\partial x_5 \partial t^2} \right) dx_5 + \int_0^{\ell_5} [\mu_5 (g \cos \varphi_5 + a_{5y}) + c_{5y} v_{5y}] x_5 dx_5 \right. \right. \\
\left. \left. - \int_0^{\ell_5} [\mu_5 (g \sin \varphi_5 + a_{5x}) + c_{5x} v_{5x}] w_5 dx_5 \right\}. \quad (2.17)
\end{aligned}$$

In which c_{jx} , c_{jy} are external damping coefficients per unit length and v_{jx} , v_{jy} , a_{jx} , a_{jy} ($j = 3, 5$) are velocity and acceleration components in the moving reference frame (x_j y_j), which are determined as follows:

$$\begin{aligned}
v_{jx} &= -\ell_{j-1}^* \dot{\varphi}_{j-1} \sin(\varphi_{j-1}^* - \varphi_j) - \dot{\varphi}_j w_j, \\
v_{jy} &= \ell_{j-1}^* \dot{\varphi}_{j-1} \cos(\varphi_{j-1}^* - \varphi_j) + \dot{\varphi}_j x_j + \frac{\partial w_j}{\partial t}, \\
a_{jx} &= -\ell_{j-1}^* \ddot{\varphi}_{j-1} \sin(\varphi_{j-1}^* - \varphi_j) - \ell_{j-1}^* \dot{\varphi}_{j-1}^2 \cos(\varphi_{j-1}^* - \varphi_j) - 2\dot{\varphi}_j \frac{\partial w_j}{\partial t} - \ddot{\varphi}_j w_j - \dot{\varphi}_j^2 x_j, \\
a_{jy} &= \ell_{j-1}^* \ddot{\varphi}_{j-1} \cos(\varphi_{j-1}^* - \varphi_j) - \ell_{j-1}^* \dot{\varphi}_{j-1}^2 \sin(\varphi_{j-1}^* - \varphi_j) - \dot{\varphi}_j^2 w_j + \ddot{\varphi}_j x_j + \frac{\partial^2 w_j}{\partial t^2}.
\end{aligned} \quad (2.18)$$

Note that

$$\ell_2^* = \ell_2, \quad \ell_4^* = \bar{\ell}_4, \quad \varphi_2^* = \varphi_2, \quad \varphi_4^* = \bar{\varphi}_4. \quad (2.19)$$

By analogy with the calculations in [3] the following equations of relative transverse vibration of links AB and CD can be written:

$$\begin{aligned}
\frac{\partial^2}{\partial x_j^2} \left[E_j J_j \left(\frac{\partial^2 w_j}{\partial x_j^2} + \alpha_{jc} \frac{\partial^3 w_j}{\partial x_j^2 \partial t} \right) \right] - \frac{\partial}{\partial x_j} \left[\rho_j J_j \left(\dot{\varphi}_j + \frac{\partial^3 w_j}{\partial x_j \partial t^2} \right) \right] \\
- \left\{ X_j - \int_{x_j}^{\ell_j} \mu_j(x) [g \sin \varphi_j + a_{jx}(x)] dx - \int_{x_j}^{\ell_j} c_{jx}(x) v_{jx}(x) dx \right\} \frac{\partial^2 w_j}{\partial x_j^2} \\
- [\mu_j (g \sin \varphi_j + a_{jx}) + c_{jx} v_{jx}] \frac{\partial w_j}{\partial x_j} + \mu_j (g \cos \varphi_j + a_{jy}) + c_{jy} v_{jy} = 0 \quad (j = 3, 5)
\end{aligned} \quad (2.20)$$

with the boundary conditions

$$w_j(0, t) = 0, \quad w_j(\ell_j, t) = 0, \quad \left(\frac{\partial^2 w_j}{\partial x_j^2} \right)_{x_j=0} = 0, \quad \left(\frac{\partial^2 w_j}{\partial x_j^2} \right)_{x_j=\ell_j} = 0. \quad (2.21)$$

In (2.20) $X_3 = X_B$, $X_5 = X_D$, α_{jc} is internal damping coefficient coming from the expression relating strain ε and stress σ in an elastic beam: $\sigma = E \left(\varepsilon + \alpha_c \frac{\partial \varepsilon}{\partial t} \right)$.

In total we have 17 equations: (2.3) \rightarrow (2.17) and (2.20) to determine 17 unknown quantities: w_3 , w_5 , reactions at joints O_1 , O_2 , O_3 , A, B, C, D and relation between the driving moment M_d and the angle φ_2 .

Note that after having found w_3 , w_5 it is easy to determine other unknowns. Therefore the determination of relative transverse vibrations w_3 , w_5 is essential.

3. Small transverse vibration of connecting rods during uniform rotation of the crank

We consider small vibration of rectilinear, homogeneous connecting rods having constant section and constant damping coefficient. Supposing that the crank OA rotates with a constant angular velocity Ω

$$E_j, F_j, \rho_j, J_j, c_{jx}, c_{jy}, \alpha_{jc}, \Omega = \text{const} \quad (j = 3, 5); \quad \varphi_2 = \Omega t. \quad (3.1)$$

$$\text{Putting } \psi_3 = \varphi_2 - \varphi_3 = \Omega t - \varphi_3, \quad \psi_5 = \bar{\varphi}_4 - \varphi_5 = \varphi_4 - \beta_4 - \varphi_5. \quad (3.2)$$

Eliminating X_B , X_D in (2.20), neglecting nonlinear terms we obtain after some calculation:

$$\begin{aligned} \frac{\partial^4 w_j}{\partial x_j^4} + \alpha_{jc} \frac{\partial^3 w_j}{\partial x_j^3 \partial t} - \frac{\rho_j}{E_j} \frac{\partial^4 w_j}{\partial x_j^2 \partial t^2} - \left[{}^j f_4(t) + {}^j f_3(t) x_j - {}^j f_2(t) \frac{x_j^2}{2} \right] \frac{\partial^2 w_j}{\partial x_j^2} - \left[{}^j f_3(t) - {}^j f_2(t) x_j \right] \frac{\partial w_j}{\partial x_j} \\ + \frac{\mu_j}{E_j J_j} \frac{\partial^2 w_j}{\partial t^2} + \frac{c_{jy}}{E_j J_j} \frac{\partial w_j}{\partial t} - {}^j f_2(t) w_j = -{}^j f_0(t) - {}^j f_1(t) x_j \quad (j = 3, 5) \end{aligned} \quad (3.3)$$

where

$$\begin{aligned} {}^j f_0(t) &= \frac{\mu_j}{E_j J_j} \left[g \cos \varphi_j + \ell_{j-1}^* \bar{\varphi}_{j-1} \cos \psi_j - \ell_{j-1}^* \dot{\varphi}_{j-1}^2 \sin \psi_j \right] + \frac{c_{jy}}{E_j J_j} \ell_{j-1}^* \dot{\varphi}_{j-1} \cos \psi_j, \\ {}^j f_1(t) &= \frac{\mu_j}{E_j J_j} \bar{\varphi}_j + \frac{c_{jy}}{E_j J_j} \dot{\varphi}_j, \quad {}^j f_2(t) = \frac{\mu_j}{E_j J_j} \dot{\varphi}_j^2, \\ {}^j f_3(t) &= \frac{\mu_j}{E_j J_j} \left[g \sin \varphi_j - \ell_{j-1}^* \bar{\varphi}_{j-1} \sin \psi_j - \ell_{j-1}^* \dot{\varphi}_{j-1}^2 \cos \psi_j \right] - \frac{c_{jx}}{E_j J_j} \ell_{j-1}^* \dot{\varphi}_{j-1} \sin \psi_j, \\ {}^j f_4(t) &= \frac{H_j}{E_j J_j} + \cotg(\varphi_{j+1} - \varphi_j) \left[\frac{\rho_j}{E_j} \bar{\varphi}_j + {}^j f_0(t) \frac{\ell_j}{2} + {}^j f_1(t) \frac{\ell_j^2}{3} \right] - {}^j f_3(t) \cdot \ell_j + {}^j f_2(t) \frac{\ell_j^2}{2}, \end{aligned} \quad (3.4)$$

in which: ℓ_2^* , ℓ_4^* are determined by (2.19) and note that: $\bar{\varphi}_2 = 0$, $\dot{\varphi}_2 = \Omega$, and $\bar{\varphi}_k$, $\dot{\varphi}_k$, φ_k ($k = 3, \dots, 6$) are evaluated from (2.2). Therefore ${}^j f_i$ are periodic functions of t with the period $2\pi/\Omega$

$$\text{Also: } H_5 = \frac{P_6 s_6 \cos(\varphi_6 + \alpha_6) + J_{O_3} \ddot{\varphi}_6}{\ell_6 \sin(\varphi_6 - \varphi_5)} \quad (3.5)$$

$$H_3 = \ell_4 \frac{X_C \sin(\bar{\varphi}_4 - \varphi_3) - Y_C \cos(\bar{\varphi}_4 - \varphi_3)}{\ell_4 \sin(\varphi_4 - \varphi_3)} + \frac{P_4 S_4 \cos(\varphi_4 + \alpha_4) + J_{O_2} \ddot{\varphi}_4}{\ell_4 \sin(\varphi_4 - \varphi_3)} \quad (3.6)$$

As an application of the generalized Ritz's method, the solution of the equations (3.3) with the boundary conditions (2.21) is found in the form:

$$w_j(x_j, t) = \sum_{i=1}^I {}^j q_i(t) \sin\left(i \frac{\pi}{\ell_j} x_j\right) \quad (j = 3, 5). \quad (3.7)$$

Substituting (3.7) into equations (3.3) we obtain after some calculation a system of linear differential equations with periodic coefficients of period $2\pi/\Omega$ as follows

$${}^j \underline{M} {}^j \ddot{\vec{q}} + {}^j \underline{B} {}^j \dot{\vec{q}} + {}^j \underline{C}(\Omega t) {}^j \vec{q} = {}^j \vec{h}(\Omega t) \quad (j = 3, 5), \quad (3.8)$$

where: ${}^j \vec{q} = ({}^j q_1, {}^j q_2, \dots, {}^j q_I)^T$; ${}^j \vec{h} = ({}^j h_1, {}^j h_2, \dots, {}^j h_I)^T$; ${}^j \underline{M}$, ${}^j \underline{B}$, ${}^j \underline{C}$ are square matrices of order I , the elements of which are calculated form:

$${}^j m_{ik} = \left[\frac{{}^j \rho}{{}^j E} \left(k \frac{\pi}{\ell_j}\right)^2 + \frac{\mu_j}{E_j J_j} \right] \delta_k^i, \quad (3.9)$$

$${}^j b_{jk} = \left[\alpha_{jc} \left(i \frac{\pi}{\ell_j}\right)^4 + \frac{c_{jy}}{E_j J_j} \right] \delta_k^i, \quad (3.10)$$

$${}^j c_{ik} = \begin{cases} \left(k \frac{\pi}{\ell_j}\right)^4 + \left(k \frac{\pi}{\ell_j}\right)^2 {}^j f_4(t) + k^2 \frac{\pi^2}{2\ell_j} {}^j f_3(t) - \left(\frac{5}{4} + k^2 \frac{\pi^2}{6}\right) {}^j f_2(t) & \text{if } i = k, \\ -\alpha_{ik} \left[\frac{2}{\ell_j} {}^j f_3(t) - {}^j f_2(t) \right] & \text{if } i + k - \text{odd}, \\ -\alpha_{ik} {}^j f_2(t) & \text{if } i \neq k; \quad i + k - \text{even}, \end{cases} \quad (3.11)$$

in which:

$${}^j h_i = \begin{cases} \frac{1}{i} \frac{2\ell_j}{\pi} {}^j f_1(t) & \text{if } i - \text{even}, \\ -\frac{1}{i} \left[\frac{4}{\pi} {}^j f_0(t) + \frac{2\ell_j}{\pi} {}^j f_1(t) \right] & \text{if } i - \text{odd} \end{cases} \quad (3.12)$$

and

$$\delta_j^i = \begin{cases} 0 & \text{if } j = i \\ 1 & \text{if } j \neq i \end{cases}; \quad \alpha_{ij} = ij \left[\frac{1}{(i-j)^2} + \frac{1}{(i+j)^2} \right] \quad i \neq j. \quad (3.13)$$

After having determined the values of w_j ($j = 3, 5$), we substitute them into the equations from (2.3) to (2.17) and easily get other unknowns, reaction components in the moving reference for example. A program based on the mentioned algorithm for evaluating condition of dynamic stability and periodic solutions of relative transverse vibration of connecting rods is introduced. It can also determine reaction components in the moving reference. It is written in FORTRAN 77, easily used for personal computer.

4. Numerical Simulation

The following data numerical calculations are given:

$c_{3x} = c_{3y} = c_{5x} = c_{5y} = 10^{-6} \text{ kg}/(\text{mm}\cdot\text{s})$; $\alpha_{3c} = \alpha_{5c} = 10^{-4} \text{ s}$; $J_{O_2} = 18300 \text{ kg}\cdot\text{mm}^2$; $J_{O_3} = 11500 \text{ mm}^2$; $\ell_1 = 270 \text{ mm}$; $\ell_2 = 55 \text{ mm}$; $\ell_3 = 259 \text{ mm}$; $\ell_4 = 200 \text{ mm}$; $\ell_5 = 258 \text{ mm}$; $\ell_6 = 220 \text{ mm}$; $\bar{\ell}_1 = 300 \text{ mm}$; $\bar{\ell}_4 = 200 \text{ mm}$; $s_4 = 118 \text{ mm}$; $s_6 = 113 \text{ mm}$; $\alpha_4 = -0.15$; $\alpha_6 = 0$; $\beta_4 = 0.3$; $\theta = \bar{\theta} = 0$. The links O_2BC and O_3D have mass 1.84 kg and 1.35 kg respectively.

The connecting rods AB and CD made of steel have mass 0.165 kg and 0.163 kg respectively; elastic modulus $E_3 = E_5 = 21 \cdot 10^7 \text{ kg}\cdot\text{mm}^{-1}\text{s}^{-2}$, area of the cross section $F_3 = 10 \text{ mm}^2$, $F_5 = 9 \text{ mm}^2$ and moment of inertia $J_3 = J_5 = 4500 \text{ mm}^4$.

Some of the results are shown in the table and figures 6 to 15.

PHASE TRAJECTORY ($I = 1$)

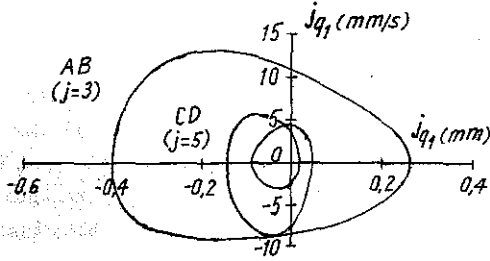


Fig. 6

REACTION

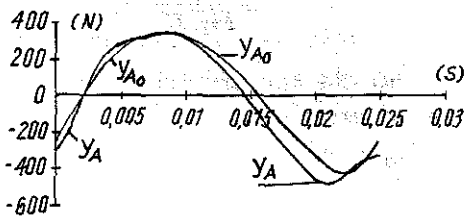


Fig. 8

REACTION

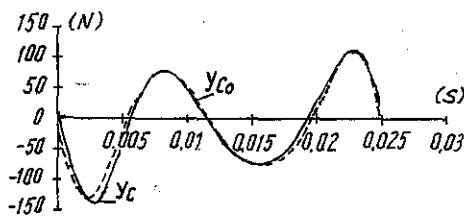


Fig. 10

PHASE TRAJECTORY ($I = 5$)

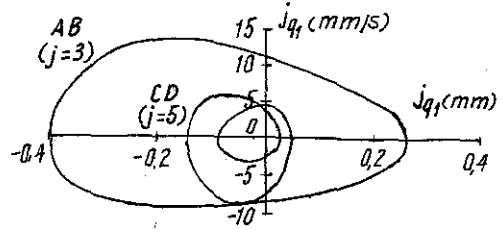


Fig. 7

REACTION

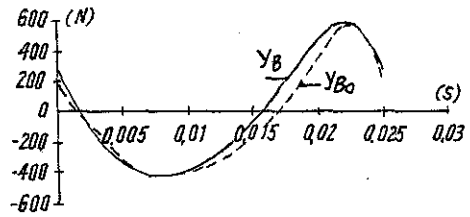


Fig. 9

REACTION

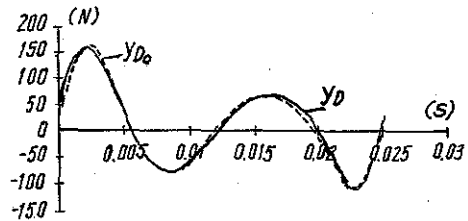


Fig. 11

REACTION

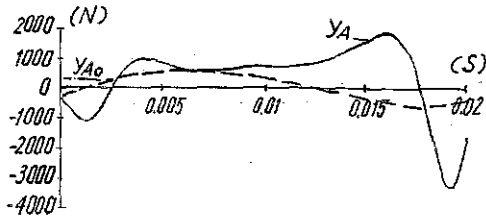


Fig. 12

REACTION

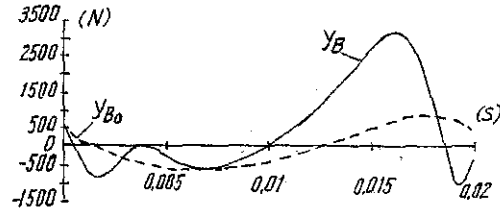


Fig. 13

REACTION

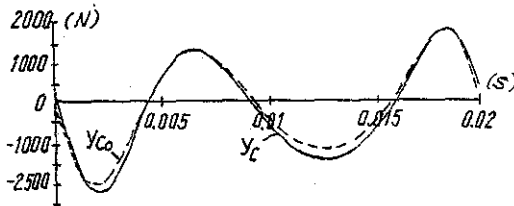


Fig. 14

REACTION

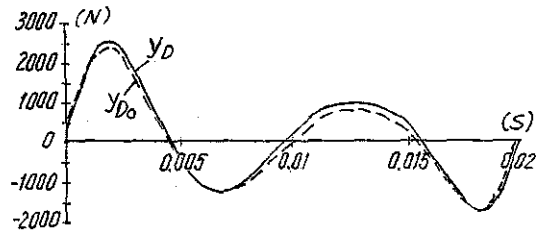


Fig. 15

After having determined condition of dynamic stability, the program evaluates the values of initial conditions and then determine the solution during a period.

Table 1 gives a comparison between the amplitudes of ${}^j q_i(t)$ ($j = 3, 5; i = 1, \dots, 5$) at the angular velocity $n = 300$ rotations per minute.

Table 1. Maximum values of $|{}^j q_i(t)|$ (unit: mm)

Connecting Rod	$I = 1$			$I = 5$		
	q_1	q_1	q_2	q_3	q_4	q_5
AB(${}^3 q_i$)	0.40151	0.39812	0.05008	0.00162	0.00016	0.00013
CD(${}^5 q_i$)	0.14537	0.14541	0.01042	0.00060	0.00003	0.00005

Figures 6 and 7 show phase trajectories of variables ${}^3 q_1$ and ${}^5 q_1$ for $I = 1$ and $I = 5$ at the angular velocity $n = 300$ rotations per minute. The horizontal represents ${}^j q_1$ and the vertical ${}^j \dot{q}_1$ ($j = 3, 5; i = 1, \dots, 5$). The phase trajectory of the variable corresponding to rod AB labeled with symbol AB(3) is outside the phase trajectory of the variable corresponding to rod CD labeled with CD(5).

The fact that the phase trajectory is closed proves that the obtained solutions are periodic.

From table 1 and figures 6 and 7 we can see that the difference in value between ${}^j q_1(t)$ for $I = 1$ and $I = 5$ ($j = 3, 5$) is negligible. As for $I = 5$ the values of ${}^j q_k(t)$ ($j = 3, 5; k = 2, 3, 4, 5$) obtained from table 1 are very small in comparison with ${}^j q_1(t)$, for example $\frac{\max(|{}^j q_2|)}{\max(|{}^j q_1|)} \approx 1\%$, $\frac{\max(|{}^j q_3|)}{\max(|{}^j q_1|)} \approx 0.4\%$, the values of ${}^j q_4$ and ${}^j q_5$ are even much smaller. Similar results can be obtained for different values of angular velocity n . Therefore it is suggested finding the solution in the form: $w_j(x_j, t) = {}^j q(t) \sin\left(\frac{\pi}{l_j} x_j\right)$ ($j = 3, 5$).

Figures 8, 9, 10, 11 give responses of the reaction components Y_A, Y_B, Y_C, Y_D along the direction y_i in the moving reference (x_i, y_i) at articulations A, B, C, D respectively in case of taking elasticity in account. On the other hand $Y_{A_0}, Y_{B_0}, Y_{C_0}, Y_{D_0}$ denote the same quantities in case of neglecting elastic factors. The figures plotted during a period at the angular velocity $n = 2400$ rotations per minute, show the effect of elasticity upon articulation reactions in the y_i direction. Figures 12, 13, 14, 15 are similar with the figures 8, 9, 10, 11 but at the angular velocity $n = 3000$ rotations per minutes. The difference in value between reaction components in the x_i direction is negligible.

When evaluating the effect of elasticity upon articulation reactions we find that elasticity has little effect for $n < 2000$ rot/min., the effect becomes significant for $n > 2000$ rot/min. and very significant for $n > 3000$ rot/min.. This result appears reasonable because the value of relative transverse vibration $w_j(x_j, t)$ also increases rapidly for $n > 3000$ rot/min..

5. Conclusions

In this paper the substructure method and d'Alembert's principle have been applied to the derivation of differential equations of motion of a six-link mechanism, in which two connecting rods are elastic and the other links are solid. In case of uniform rotation of the driving link the generalized Ritz's method is used to transform the partial differential equations into ordinary differential equations with periodic coefficients.

A program for relative transverse vibration analysis of the connecting rods and for evaluation of reactions at joints has been written. Numerical examples are given, some remark on the obtained solutions as well as the evaluation of the effect of elasticity on joint reaction are made.

The presented method can be applied to any type of six-link mechanisms of the order 2. It is also applicable to any mechanism as a combination of solid and elastic links.

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TÍNH TOÁN BẰNG SỐ DAO ĐỘNG UỐN TUẦN HOÀN CỦA CÁC THANH TRUYỀN ĐÀN HỒI TRONG CƠ CẤU SÁU KHÂU BẢN LỀ

Trong bài báo này áp dụng phương pháp tách cấu trúc và nguyên lý d'Alembert để thiết lập phương trình vi phân chuyển động cơ cấu của một dạng cơ cấu 6 khâu có hai thanh truyền là các thanh đàn hồi.

Trong trường hợp chuyển động bình ổn, áp dụng phương pháp Ritz suy rộng nhận được hệ phương trình vi phân tuyến tính hệ số tuần hoàn. Đã thiết lập được chương trình tìm điều kiện ổn định và nghiệm tuần hoàn của hệ phương trình nhận được. Đã đưa ra ví dụ minh họa và có so sánh ảnh hưởng của yếu tố đàn hồi đối với phản lực của các khớp.