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RESERVOIRS OPTIMIZATION WITH DYNAMIC PROGRAMMING

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Abstract. Differential Evolution (DE) and Dynamic Programming (DP) are important optimal methods in reservoir regulation. In the previous work [1], we presented the outline of DE, and applied it into Pleikrong reservoir, a big one in the Highland of Vietnam for dry season of 2010 year. Continuing from that, in this work, we present the outline of DP and then again, apply it to Pleikrong reservoir; and also apply it to Ialy, the biggest reservoir in Sesan cascade in the Highland of Vietnam; to reach optimal regulation for the maximum power production in the dry season of two years: 2010 and 2012. The results getting from DP are compared to the results by using DE. The results by these two methods have the same trend of releases which is storing the water at the beginning and significantly releasing at the end of the calculation time.

Keywords: Dynamic programming, reservoirs, cascade, Sesan, optimization.

1. INTRODUCTION

Many problems in economics, mathematics, engineering, agriculture, and so on, require optimal solution. Therefore, the methods to reach the optimal solution have been researched for a long time and been enhanced, expanded and combined, modified and developed over time with the development of computer science and the greater and greater need of practical calculation. Optimization methods are different analyses to get the best target under a set of constraints. Optimization problems have many categories: linear or nonlinear, deterministic or stochastic, static or dynamic, continuous or discrete, single or multi objective [2, 3]. This classification is based on the character of both objective functions and the constraints. Many methods are studied to solve these problems such as Linear Programming (LP), Dynamic Programming (DP), Nonlinear Programming (NLP), Genetic Algorithm (GA), etc [4–6]. These methods are also being improved and developed into many versions to adapt to specific problems with specific characteristics separately [7–10].

Dynamic Programming (DP) is a recent, large-scale-applied optimization method which is researched and used worldwide. In Vietnam, this method is used more frequently in many researches. In 2003, Ha Van Khoi shown that DP could be applied to plan and regulate a power plant cascade [11]. That research tested DP to 3 reservoirs of Da river and claimed that DP had overcome the local optimal than other linear methods, however, the amount of calculation is big and the test cases are made with many suggestions of the inflow. In 2012, Le Hung set the optimal reservoir problem and used DP to apply for some reservoirs to show that this method is a good tool for planning water resource [12].

In Vietnam, to operate and regulate reservoirs, many ministries get involved such as the Ministry of Industry and Trade, Ministry of Agriculture and Rural Development, Ministry of Natural Resources and Environment, etc. The reservoir operation must not only be reasonable, consistent with agricultural seasons, time of electricity using peak, but also need reasonably discharged to avoid flood in rainy season, and to store enough water in dry season. Therefore, reservoir optimization problem is a significant practice problem in the world and Vietnam in particular. Even in a specific river cascade, regulation of a reservoir to get maximum electricity production objective only is not easy because the optimal standards are various, only applicable to each separate project, and not general.

2. PROCESS OF OPTIMIZATION BY DYNAMIC PROGRAMMING

2.1. Outline of Dynamic Programming

The concept called Dynamic Programming was first introduced by Bellman in 1957 [13]. Since then, Dynamic Programming (DP) has become a major research sector in applied mathematics, basic researches and computer science. It has also become an optimization method that has been widely using in many fields such as technology, economy, planning and management, etc [14–17].

Bellman used the concept "Dynamic Programming" to describe the process of solving problems to find the best one in a group of decisions. "Programming" means using different methods to get the optimal solution, and "dynamic" mentions the aspect of time in the problem. The optimality principle of Bellman about the optimization problem in recursive form is the core of DP:

"An optimal policy has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decisions" [13].

The process of an optimization can be described as [18] (Fig. 1), where S_t: states variables, depended on controls and inputs, r_t: control variables, selected to maximize or minimize the benefit, I_t: input variables, $r_t\left(S_t\right)$: decision rule, is the function that gives u_t for any x_t, $S_{t+1} = g\left(S_t, r_t, I_t\right)$: state equation, S_0: initial condition.

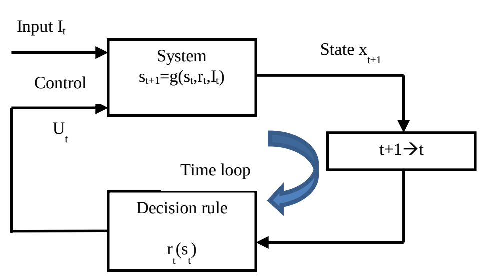

Fig. 1. Process of optimization

This method is usually applied to problems that are sequential. The simple idea of this method is: instead of solving a complicated problem, we solve each separated part of it then combine the result of each part into an overall result. Each small part of the problem only done once by DP and thus the number of calculations is reduced [12].

In other words, DP discomposes an N-decision problem into N of separate, inter-related sub-problems, each sub-problem is a stage. Each stage is characterized by chosen state variables. The decision analysis from this stage to the next one until reaching the last one could be done through space or time by control the decision variables.

The frame work of DP could be described as below [18] (Fig. 2).

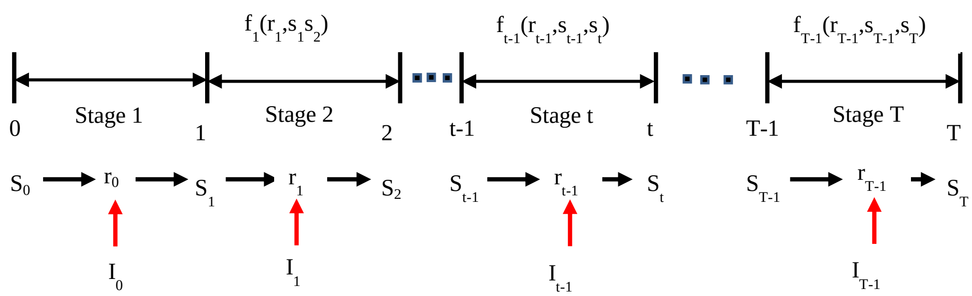

Fig. 2. Frame work of DP

The optimization problem is selecting $r_0, \ldots, r_{T-1}$ to maximize the benefit. The benefit of the problem could be described in the recursive equation as below [16]

$$F_j^*\left(S_j\right) = \max_{R_j} \left\{ E_j\left(S_j, r_j\right) + F_{j+1}^*\left(S_{j+1}\right) \right\}, \tag{1}$$

where j: current stage, F_{j+1}^*: accumulated sub-optimal benefits for future stages $j+1, j+2, \ldots, T$, could be the power production in reservoir optimal problem, T: total number of stages, S_j: system state at stage j, is the water level or storage in reservoir optimal problem, $S_{j+1} = g\left(S_j, r_j, t\right)$: state transformation equation, r_j: decision taken at stage j, is the *release* in reservoir optimal problem, $E_j\left(S_j, r_j\right)$: benefit or contribution of the decision r_j given state S_j at the initial stage, could be energy or power production at state j in reservoir optimal problem.

In the reservoir optimization problem, states variables could be water levels or the corresponding reservoir volumes; the control variables could be the releases, and the objective function could be the hydro power production. In this case, the chart and the flow diagram of the optimal trajectory as Figs. 3 and 4 below [16], which could be shortly explained as:

We separate the storage volume of the reservoir into n parts which is called n states of the reservoir.

We separate the calculating time, in this case is 130 days, into 14 parts, then call them 14 stages. Each stage is 10 days.

The volume of the reservoir at the starting time is V_0 at stage 0.

From volume V_0 at stage 0, there are many ways to go to stage 1 by many ways of release water. With each way of release, we can calculate the corresponding electricity. The release give the highest electricity from stage 0 to stage 1 is the optimal solution for the first step.

Repeat the same sequence with other steps to reach the optimal releases to get optimal electricity production.

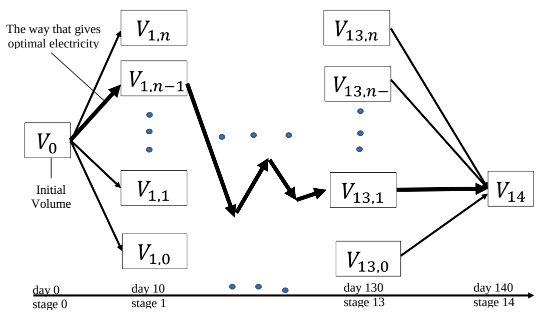

Fig. 3. Chart of the optimal trajectory in forward computational procedure

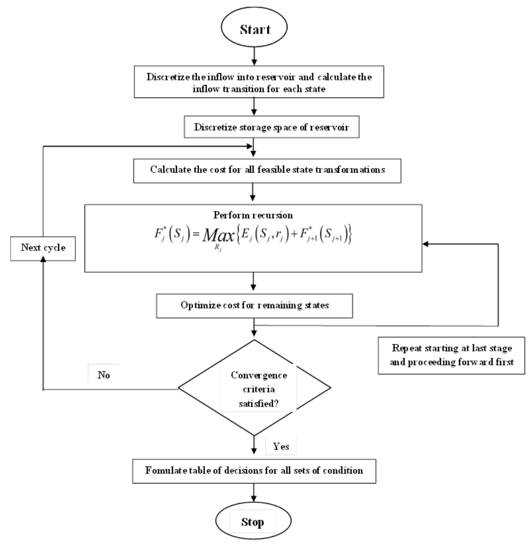

Fig. 4. Flow diagram for the DP model of a reservoir [16]

2.2. Application of dynamic programming

We now apply DP to maximize the total electricity production over a prespecified time period of two biggest reservoirs of Sesan cascade: Pleikrong and Ialy.

In this work, we apply DP to find the optimal set of releases to get the maximum power production of each reservoir in the years of 2010 and 2012.

The objective function in this case is:

$$E = \text{maximum} \sum E_j (j = 1, \ldots, T), \tag{2}$$

where

$$E_i = 9.8 * h_i * Q_i * k * 24 * 10/1000 \text{ (MWh)}, \tag{3}$$

h_i - water height at time period i,

Q_i - release at time period i,

k - overall generation efficiency.

The problem is created as in [1] which means calculation time is in dry season of Sesan cascade. It starts at the beginning of December and ends in the next June. The reservoir management is followed the operation rules in the decision No. 1182 QD-TTg of the Government of Vietnam signed in July 17, 2014 [19].

The data that is used in this paper was provided by the team of Institute of Mechanics of VAST in project of building the reservoir operation for Sesan cascade in dry season (under contract N 01/2011/QTVH - SESAN on June 02, 2011).

The results by DP of Pleikrong and Ialy reservoirs are shown as the charts and Tables below. These results are compared to the results by Differential Evolution which is a method in Genetic group that we presented in our previous work [1].

2.2.1. *Application to Pleikrong reservoir*

Pleikrong hydropower plant is located at the upstream of Sesan river in the Highland of Vietnam. It has a significant affect to the other plants of this cascade. The main objectives of Pleikrong reservoir are storing water for the whole Sesan cascade and producing electricity to regulate flooding in rainy season and enhance the capacity of the lower plants, see Fig. 5.

Fig. 5. Pleikrong damp

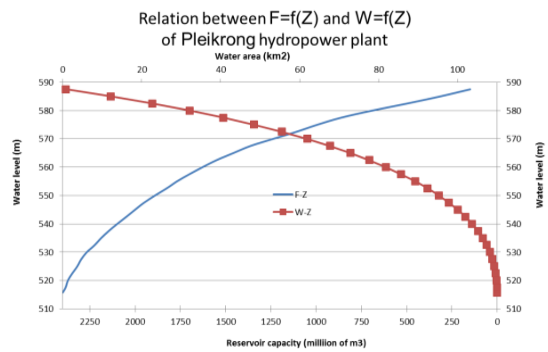

Fig. 6. Relationship between water levels Z and storage volumes W of the Pleikrong reservoir

Water height is non-linearly related to the volume of the reservoir. To calculate the objective function, we need to have the corresponding water heights of the volumes. The relationship between water levels and the volumes is given in the operation rules in the decision No. 1182 QD-TTg of the Government of Vietnam signed on July 17, 2014 [19] in Fig. 6.

There are some constraints that the calculating must be satisfied, such as the storage volume of the reservoir must be lower the given useful volume, the releases must be lower than the designed maximum release. We also need some other data such as the inflows to the reservoir, the initial water level is given by the initial volume. These inputs of the problem are shown in Tabs. 1 and 2.

Table 1. Other input of Pleikrong reservoir in the year 2010 and 2012

	Useful capacity W ($\times 10^3$ m^3)	Initial water level Z (m)	Maximum of average release Q_i (m^3/s)
2010	948.43	569.0	330
2012	948.43	569.06	330

Table 2. Inflow of Pleikrong reservoir in the year 2010 and 2012

No period	Time of period	Inflow of Pleikrong reservoir (m^3/s) in 2010	Inflow of Pleikrong reservoir (m^3/s) in 2012
1	11/Feb - 20/Feb	47.73	48.7
2	21/Feb - 02/Mar	53.35	52.2
3	03/Mar - 12/Mar	23.79	51.5
4	13/Mar-22/Mar	51.8	31.6
5	23/Mar - 01/Apr	31.98	24.5
6	02/Apr - 11/Apr	10.86	46.1
7	12/Apr - 21/Apr	44.56	37.8
8	22/Apr - 01/May	41.18	45.3
9	02/May - 11/May	14.54	37.3
10	12/May - 21/May	23.54	45.6
11	22/May - 31/May	0.09	48.9
12	01/Jun - 10/Jun	17.15	62.2
13	11/Jun - 20/Jun	44.76	131.6
14	21/Jun - 30/Jun	36.48	235.6

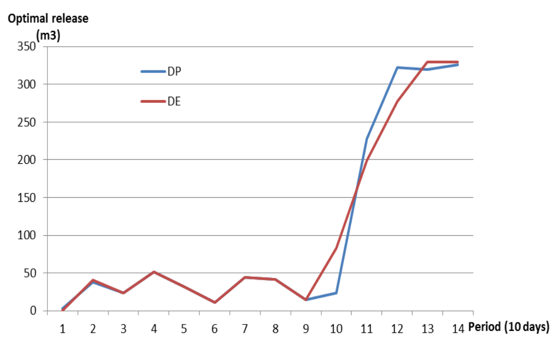

Fig. 7. Results for Pleikrong reservoir in 2010 by DP and DE

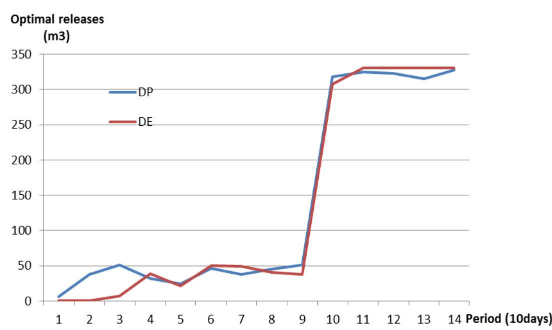

Fig. 8. Results for Pleikrong reservoir in 2012 by DP and DE

In Fig. 7, the releases from the first period to the 9^{th} period are equal to the inflows. This means that at the beginning of the dry season, water is stored to reach to the maximum water level. Then releases are higher and higher at the end of the season. This way of release could give the highest power production. The releases are also given in the table form in Tab. 3.

The results of Pleikrong reservoir in 2012 are shown in Fig. 8. In Fig. 8, water is also stored at the beginning periods and highly released at the ending periods. The detail optimal releases are also shown in table form as in Tab. 4.

Table 3. Optimal releases of Pleikrong reservoir by DP and DE in 2010

No period	Time of period	Optimal releases of Pleikrong reservoir by DP (m^3/s)	Optimal releases of Pleikrong reservoir by DE (m^3/s)
1	11/Feb - 20/Feb	2.540329861	0.0
2	21/Feb - 02/Mar	38.28677662	40.53416590
3	03/Mar - 12/Mar	23.79	23.78999995
4	13/Mar - 22/Mar	51.8	51.80000037
5	23/Mar - 01/Apr	31.98	31.98000024
6	02/Apr - 11/Apr	10.86	10.86000043
7	12/Apr - 21/Apr	44.56	44.56000130
8	22/Apr - 01/May	41.18	41.17999976
9	02/May - 11/May	14.54	14.53999904
10	12/May - 21/May	23.54	83.22103829
11	22/May - 31/May	228.1684399	199.33007812
12	01/Jun - 10/Jun	322.287357	277.37941453
13	11/Jun - 20/Jun	319.3836213	329.80134326
14	21/Jun - 30/Jun	326.3604892	329.97833269
	Electrical production	**141 060.59331938 MWh**	**141 079.0869 MWh**
	Real electrical production	**133 547 MWh**	

Table 4. Optimal releases of Pleikrong by DP and DE in 2012

N0 period	Time of period	Optimal releases of Pleikrong reservoir by DP (m3/s)	Optimal releases of Pleikrong reservoir by DE (m3/s)
1	11/Feb - 20/Feb	6.164844	0
2	21/Feb - 02/Mar	38.02161	0
3	03/Mar - 12/Mar	51.5	7.474769
4	13/Mar - 22/Mar	31.6	38.9
5	23/Mar - 01/Apr	24.5	21.69
6	02/Apr - 11/Apr	46.1	50.01
7	12/Apr - 21/Apr	37.8	49.51
8	22/Apr - 01/May	45.3	40.5
9	02/May - 11/May	51.47839	37.58
10	12/May - 21/May	317.7689	307.9099
11	22/May - 31/May	324.4605	330
12	01/Jun - 10/Jun	322.4516	330
13	11/Jun - 20/Jun	315.307	330
14	21/Jun - 30/Jun	327.4535	330
	Electrical production	**174 183.56436057 MWh**	**169 106.5166 MWh**
	Real electrical production	**134 058.35 MWh**	

2.2.2. *Application to Ialy reservoir*

Ialy hydropower plant is the biggest one in Sesan cascade and the second one in Vietnam, after Hoa Binh power plant, see Fig. 9. Pleikrong release and Dabla river are two inflows of Ialy reservoir. Ialy power plant supplies most of the energy for the Highland of Vietnam.

Fig. 9. Ialy damp

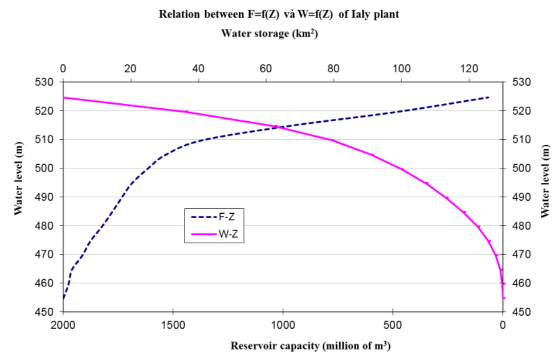

Fig. 10. Relationship between water levels Z and storage volumes W of the Ialy reservoir

The relationship between water levels and the volumes of Ialy reservoir is given [19], as Fig. 10. The constraints for Ialy reservoir and the initial data are given in Tab. 5 and Tab. 6.

Table 5. Other input of Ialy in the year 2010 and 2012

	Useful capacity W ($\times 10^3$ m^3)	Initial water level Z (m)	Maximum of average release Q_i (m^3/s)
2010	619,74	660,769548	420
2012	619,74	987,967877	420

The optimal releases of Ialy reservoir in 2010 are shown in Tab. 7 and Fig. 11. In this Fig. 11, we can see that water is firstly stored at the beginning periods to reach maximum water level. Then it starts to release because of the inflows are high. The releases from the 6^{th} period to 12^{th} period are up and down, depended on the inflows to keep the maximum water level. At the end of the season, water is released maximum for maximum power production.

The results calculating for Ialy reservoir in 2012 are shown as in Tab. 8 and Fig. 12. As shown in Tab. 8 and Fig. 12, water is released right at the beginning of the season. It is different to Tab. 7 and Fig. 10 for results in 2010. The reason is in 2012 water of Ialy reservoir is excess. Then water releases depend on the inflows to the reservoir. It needs to keep the maximum water level. Finally, the releases are increased at the end of the season to get maximum power production.

Table 6. Inflow of Ialy in the year 2010 and 2012

No period	Time of period	Inflow of Ialy reservoir in 2010 (m^3/s)	Inflow of Ialy reservoir in 2012 (m^3/s)
1	11/Feb - 20/Feb	87.73	125.43
2	21/Feb - 02/Mar	119.4	130.5
3	03/Mar - 12/Mar	117.63	150.78
4	13/Mar - 22/Mar	126.35	167.21
5	23/Mar - 01/Apr	189.48	207.33
6	02/Apr - 11/Apr	248.25	146.28
7	12/Apr - 21/Apr	215.98	190.29
8	22/Apr - 01/May	169.2	170.35
9	02/May - 11/May	223.25	190.68
10	12/May - 21/May	188.08	228.04
11	22/May - 31/May	124.88	235.81
12	01/Jun - 10/Jun	109.95	256.51
13	11/Jun - 20/Jun	100.65	267.05
14	21/Jun - 30/Jun	81.13	266.87

Table 7. Optimal releases of Ialy reservoir by DP and DE in 2010

No period	Time of period	Optimal releases of Ialy reservoir by DP (m^3/s)	Optimal releases of Ialy reservoir by DE (m^3/s)
1	11/Feb - 20/Feb	8.386377	0
2	21/Feb - 02/Mar	0.384566	0
3	03/Mar - 12/Mar	58.12228	72.479129904
4	13/Mar - 22/Mar	126.35	101.31
5	23/Mar - 01/Apr	189.48	188.70
6	02/Apr - 11/Apr	248.25	236.27
7	12/Apr - 21/Apr	215.98	223.31
8	22/Apr - 01/May	169.2	165.54
9	02/May - 11/May	223.25	238.79
10	12/May - 21/May	188.08	134.46
11	22/May - 31/May	124.88	118.99
12	01/Jun - 10/Jun	179.3757	163.57164806
13	11/Jun - 20/Jun	418.9286	420
14	21/Jun - 30/Jun	410.7174	420
	Electrical production	**1 107 901.5565 MWh**	**1 081 234.9943 MWh**
	Real electrical production	**1 063 948 MWh**	

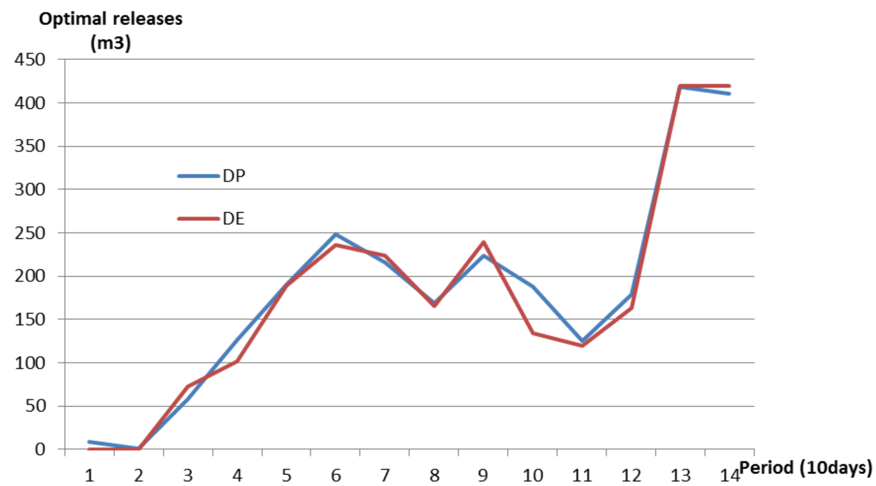

Fig. 11. Reults for Ialy reservoir by DP and DE in 2010

Table 8. Optimal releases of Ialy reservoir by DP and DE in 2012

No period	Time of period	Optimal releases (m^3/s) by DP	Optimal releases (m^3/s) by DE
1	11/Feb - 20/Feb	246.2649961	246.2609
2	21/Feb - 02/Mar	130.5	130.5
3	03/Mar - 12/Mar	150.78	150.78
4	13/Mar - 22/Mar	167.21	167.21
5	23/Mar - 01/Apr	207.33	207.33
6	02/Apr - 11/Apr	146.28	146.28
7	12/Apr - 21/Apr	190.29	190.29
8	22/Apr - 01/May	170.35	170.35
9	02/May - 11/May	190.68	190.68
10	12/May - 21/May	307.7390741	291.5716
11	22/May - 31/May	415.1329167	420
12	01/Jun - 10/Jun	415.9081481	420
13	11/Jun - 20/Jun	416.4857639	420
14	21/Jun - 30/Jun	416.3057639	420
	Electrical production	**1 486 726.6824 MWh**	**1 486 667.4 MWh**
	Real electrical production	**1 392 585 MWh**	

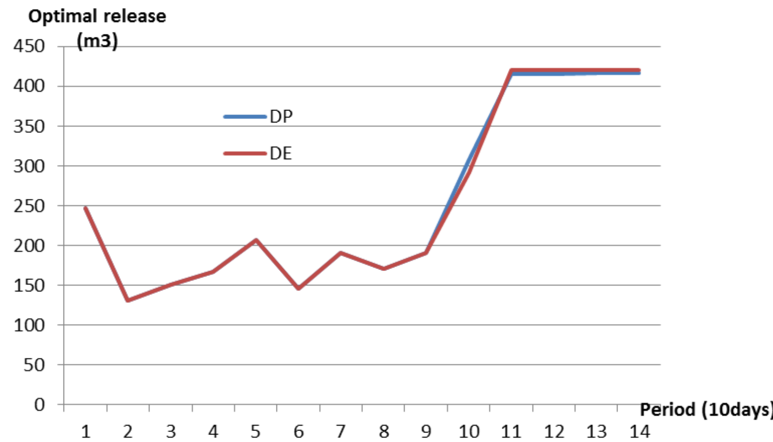

Fig. 12. Results for Ialy reservoir in 2012 by DP and DE

3. CONCLUSION

The electricity productions and the set of the releases getting by these two methods are close to each other, respectively. The slight differences between them depend on the input of the year, for example, in 2010, the water levels and the inflows are low, DE gives higher amount of electricity production; while in 2012, the water levels and the inflows are high, then DP gives a slightly higher production. However, electricity productions using both these two optimization methods are much higher than the real production. That could be concluded that DP and DE are reliable methods for reservoir optimization problems.

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