Close loop step test used for tuning PID controller by genetic algorithms

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Summary

The identification of multiple points on the process frequency response from a single step feedback test is used. These identified points are there employed to design a PID controller using the multiple-point fifting controller design method. The PID controller is design by minimizing the error between the actual and desired close-loop response in a certain frequency region. The control problem is stated as a nonlinear least squares unconstrained minimization problem. The optimization problem is solved with a simple genetic algorithm.

Keywords: FFT, genetic algorithm, nonlinear least squares optimization, PID controller.

1. Introduction

Proportional-Integral-Derivative (PID) controllers are widely used in many control systems. In process control, more than ninety-five percent of the control loops are of PI or PID type [1, 2]. Since Ziegler and Nichols [3], proposed their empirical method to tune PID controllers, to date, many relevant methods to improve the tuning of PID controllers has been reported at the control literature, one of them is a tutorial given by Hang et al. [4].

As is well known, the dynamics of a process can be known from the transient response, so when it gets the step response is possible to determine both the process gain and the process dynamics. Due to this statement, in this work, the frequency response is obtained from the step response in a close loop system. The size of the step can be as small as it has desired, this is a great advantage because it can apply a small step near the operation point, without significantly affecting process safety.

Step response test have been widely used for model identification in the process industry [5], and has remained attractive owing to its simplicity. Several researchers have made important contributions on Control-oriented model identification methods [6, 7-8, 9-10]. A significant tutorial review on process identification from step or relay feedback was presented by Liu *et al*, [5], where the most important identification methods developed in the past three decades are surveyed. In the first proposals on auto-tuning methods, one estimated point over Nyquist curve is enough to tune a PID controller. In recently studies, it has been shown that the multiple identified points allow better PID tuning controller [4-5]. This work presents an application of the multiple-point identification method, in order to tune PID controllers. The control problem is posed as a nonlinear least squares unconstrained problem.

A genetic algorithm is proposed to solve the optimization problem. Nonlinear least squares methods involve an iterative improvement of parameter values in order to reduce the sum of the squares of the errors between the function and the measured data points. Problems of this type occur when fitting model functions to experimental data. The Levenberg-Marquardt algorithm [11-12], is the most common method for nonlinear least-squares minimization, nevertheless it can suffer from a slow convergence, and it is possible to finds only a local minimum [12].

The PID's designed with this method takes into account the effect of the sensitivity function values of the closed-loop system as a measure of robustness against possible variations in the parameters of the plant [1-2, 13-14]. The contents of the paper are described as follows: In **section 2** the basic definitions of a nonlinear least squares unconstrained minimization problem, and the use of close loop step transient test, are shown. **Section 3**, presents applications of the multiple point identification method to a PID controller tuning. In **Section 4** the conclusions are presented.

2. Basic concepts

2.1. Unconstrained minimization problem

In a large number of practical problems, the objective function f(x) is a sum of squares of nonlinear functions

$$f(x) = \frac{1}{2}\sum_{j=1}^{m}(r_j(x))^2 = \frac{1}{2}\|r(x)\|_2^2 \qquad (1)$$

that needs to be minimized. We consider the following problem

$$\min_x f(x) = \min_x \frac{1}{2}\sum_{j=1}^{m}(r_j(x))^2 \qquad (2)$$

This is an unconstrained nonlinear least squares minimization problem. It is called least squares because the sum of squares of these functions is the quantity to be minimized. Problems of this type occur when fitting model functions to data: if φ(x; t) represents the model function with *t* as an independent variable, then each $r_j(x) = \varphi(x; t_j) - y_j$, where φ (t_j, y_j) is the given set of data points [11-12].

2.2. Use of close loop step transient

It was shown by Wang *et al*. [15-16] who propose a method that can identify multiple points simultaneously under one relay test. For a close loop step transient system in Fig. 2, the process input *u(t)* and output *y(t)* are recorded from the initial time until, the system reaches a steady value, after the transient step response. *U(t)* and *y(t)* are not integrable since they do not die down in finite time (at T_{ss} time). They cannot be directly transformed to frequency response meaningfully using *FFT*. A decay exponential $e^{-\alpha t}$ is then introduced to form

$$\tilde{u}(t) = u(t)e^{-\propto t} \tag{3}$$

and

$$\tilde{y}(t) = y(t)e^{-\propto t} \tag{4}$$

such that *u(t)* and *y(t)* will decay to zero exponentially as *t* approaches infinity. Applying the Fourier transform to (3) and (4) yields

$$\tilde{U}(t) = \int_0^\infty \tilde{u}(t)e^{-jwt}\,dt = U(jw+\propto)$$

and

$$\tilde{Y}(t) = \int_0^\infty \tilde{y}(t)e^{-jw}\ \ dt = Y(jw+\propto)$$

For a process *G(s)=Y(s)/U(s), at s=jw+α*, one has

$$G(jw+\propto) = \frac{Y(jw+\propto)}{u(jw\ \ \propto)} = \frac{\tilde{Y}(jw)}{\tilde{U}(jw)} \tag{5}$$

$\tilde{Y}\,(jw)$ and $\tilde{U}(jw)$ can be computed at discrete frequencies with the standard *FFT* technique [15-16]. Therefore, the shifted process frequency response *G(jw+α)* can be obtained from (5). To find *G(jw)* from *G(jw+α)*, we firts take the inverse *FFT* of *G(jw+α)* as

$$\tilde{g}(kT) = FFT^{-1}\big(G(jw{+}\propto)\big) = g(kt)e^{-\propto kt}$$

It then follows that the process impulse response g(kT) is

$$g(kT) = \tilde{g}(kT)e^{\propto kT} \quad (6)$$

Applying the *FFT* again to *g(kT)* leads to the process frequency response:

$$G(jw) = FFT(g(kt))$$

Since the identification process is based on sampled values, it is convenient to think that the sequences under study are simply a period of infinite periodic succession. This fact justifies the application of the Discrete Fourier Transform (fft).

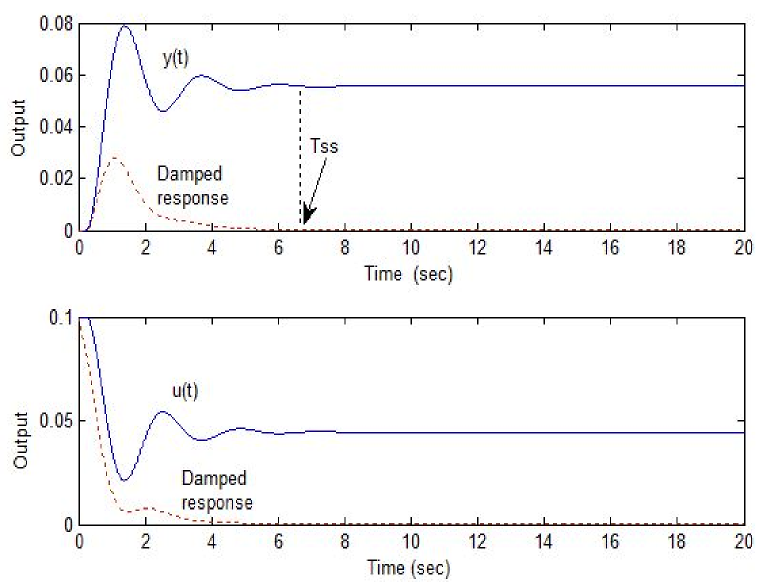

Fig. 1. Signals under step feedback.

In this identification problem is very important the adequate selection of α value, in [18] a rule to compute the α value in terms of the T_{ss} time (see Fig. 1) is proposed, where the system reaches a steady value, after the transient step response. The value of α, it can be computed by means of:

$$\propto < \frac{1}{Tss} ln \frac{\Delta y(Tss)}{\delta}$$

Where $\Delta y(T_{ss})=y(T_{ss})-y(0)$, denotes the dynamic output response in terms of the settling time (T_{ss}) to the step change, in which *y(0)* indicates initial steady output value before the step test. δ is a computational threshold which may be practically taken smaller than $\Delta y(T_{ss})x10^{-6}$

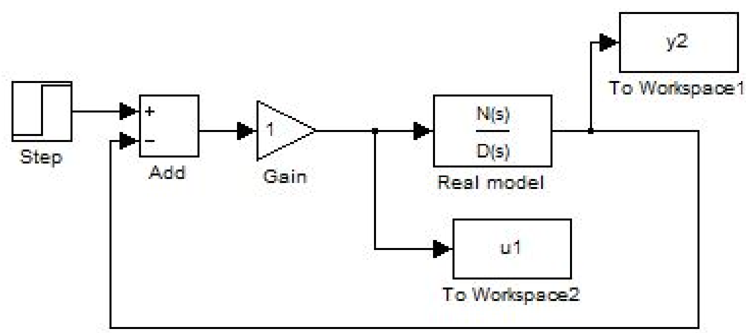

Fig. 2. Schematic of feedback system.

The method can accurately identify as many as desired frequency response points with one step experiment. They may be very useful for improving the performance of PID and other model-based controllers. In both applications: PID tuning and transfer function modeling, the shifted frequency response may be used without the needing to computer *G(jw)*. To illustrate the method, a model with oscillatory dynamics is considered in simulation.

$$G(s) = \frac{1.25}{0.25s^2+.7s+1} e^{-.234s} \quad (7)$$

Fig. 3 shows the identified frequency responses for these processes using this method, for *G(jw)*.

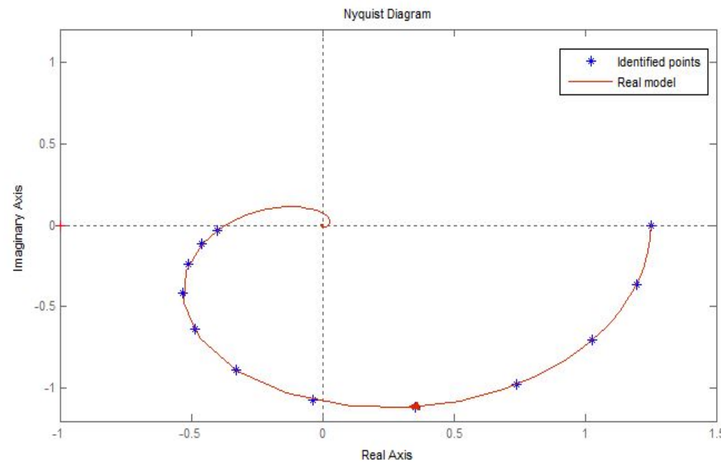

Fig. 3. Nyquist plot for *G(jw).*

And G*(jw+α)* plot, where *α=0.85,* is given by Fig. 4

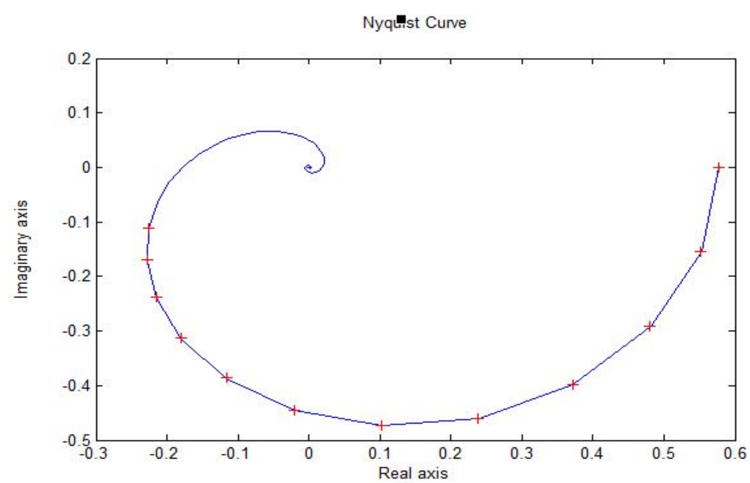

Fig. 4. Nyquist plot for *G(jw+α).*

2.3. Simple genetic algorithms

The genetic algorithm is a useful tool to solve both constrained and unconstrained optimization problems that takes principles of biological evolution [19, 20-22, 14,8]. At present work, each of the individuals in the population (chromosomes), contain the parameters included in the fitness function, as an example, in the process to tune the

PID controller, each chromosome contains the coded parameters of the controller [*Kp, Ki, Kd*].

3. PID tuning

Tuning via frequency response fitting is a simple but efficient solution to this kind of processes, that was developed [4, 15-16]. It shapes the loop frequency response to optimally match the desired dynamics over large range of frequencies. Thus the closed-loop performance is more firmly guaranteed than in the case of only one or two points PID or PI tuning laws.

Suppose that multiple process frequency response points $G(jw_i)$, $1=1,2,\ldots,m$, are available. The control specifications can be formulated as a desirable closed loop transfer function

$$H_d(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n+\omega_n^2}e^{-Ls} \qquad (8)$$

where L is the *apparent dead-time* of the process, w_n and ζ dominate the behavior of the desired closed-loop response, [9]. Specifications are given as the phase margin Φ_m, and gain margin A_m. The default settings for ζ and w_nL values are =0.707 and $w_nL=2$, which imply that the overshoot of the objective set-point step response is about 5%, the phase margin is 60° and the gain margin is 2.2 [4]. The open-loop transfer function corresponding to G_d is

$$G_d = \frac{H_d}{1-H_d} \qquad (9)$$

The controller *C(jw)* is designed such that the actual *GC(jw)* is fitted to the desired transfer function $G_d(jw)$, as well as possible. Thus the resultant system will have the desired performance. The PID controller desired can be obtained by minimizing the objective function given from the sum of squared differences between computed and recorded frequency response points

$$CG(jw_i) = \frac{Kpjw_i + Ki + Kd(jw_i)^2}{jw_i} G(jw_i) \quad (10)$$

$$CG'(jw_i) = \begin{bmatrix} Real\big(GC(jw_i)\big) \\ Imag\big(GC(jw_i)\big) \end{bmatrix}$$

$$G'_d\ (jw_i) = \begin{bmatrix} Real(G_d(jw_i)) \\ Imag(G_d(jw_i)) \end{bmatrix}$$

The objective function

$$y = \sum_1^m |CG'(jw_i) - G'_d\ (jw_i)|^2 \quad (11)$$

If the PID controller is designed from *G(jw+α)*, then

$$G_m(j\omega_i{+}\propto) = \frac{1}{a(j\omega_i{+}\propto)^2 + b(j\omega_i{+}\propto) + c}$$

$$CG(jw_i + \alpha) = C(jw_i + \alpha)G(jw_i + \alpha)$$

$$CG'(jw_i + \alpha) = \begin{bmatrix} Real\big(GC(jw_i + \alpha)\big) \\ Imag\big(GC(jw_i + \alpha)\big) \end{bmatrix}$$

$$G'_d\ (jw_i + \alpha) = \begin{bmatrix} Real(G_d(jw_i + \alpha)) \\ Imag(G_d(jw_i + \alpha)) \end{bmatrix}$$

The objective function

$$y = \sum_1^m |CG'(jw_i + \alpha) - G'_d\ (jw_i + \alpha)|^2 \quad (13)$$

The solution of the problem is obtained by minimizing *y*.

In this work the identified points were obtained from a schematic Simulink® system where the system feedback is simulated. To solve the optimization problem, the MATLAB® Genetic Algorithm Optimizations Using the Optimization Tool GUI is used.

Example 1. Consider a model with oscillatory dynamics

$$G(s) = \frac{1.25}{0.25s^2 + .7s + 1} e^{-.234s} \quad (14)$$

The identified points for this model are showed in Fig. (3)-(4). In this example the apparent dead-time *L=0.23,* is proposed.

The designed PID is solved by minimizing the equation 11 by means of a simple genetic algorithm. The PID parameters are coded and arranged into each individual (chromosome), of population in the genetic process. Multiple points are acquired from *G(jw)*

$$C(s) = (1.453 + \frac{2}{s} + 0.561s] \quad (15)$$

And from *G(jw+α)*, the tuned PID is

$$C(s) = (1.45 + \frac{2}{s} + 0.561s] \quad (16)$$

Eq, (15)-(16) show that both PID's controllers have very close values as might be expected. Performance of the PID designed is shown in the Fig. 5. The time response shows that the overshoot value is close of 5%, as it was proposed.

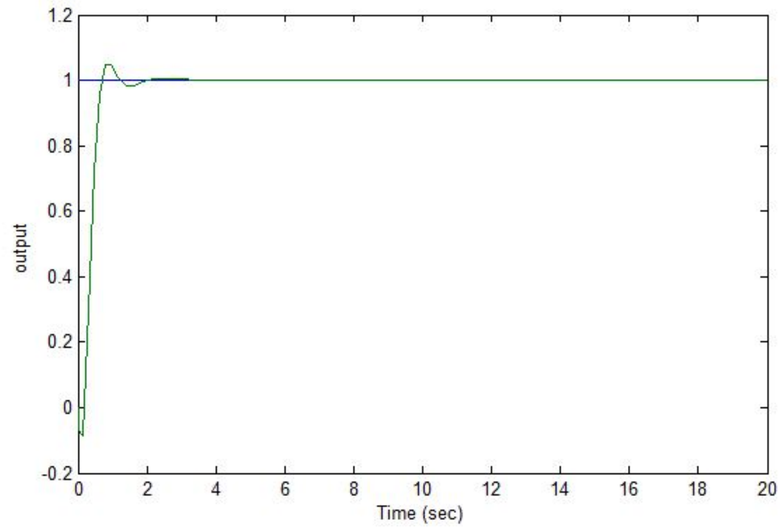

Fig. 5. Control performance for an oscillatory process.

Example 2. Considerer a high order model

$$G(s) = \frac{1}{(s+1)^{10}} \tag{17}$$

For this model the value of *apparent dead-time* of the process *L=4.5* was proposed. The modeling error for this example was 0.0097%

Estimated model From *G(jw)* the design PID is

$$C(s) = \left(0.808 + \frac{0.132}{s} + 1.99s\right) \tag{18}$$

Performance of the PID designed is shown in the Fig. 6

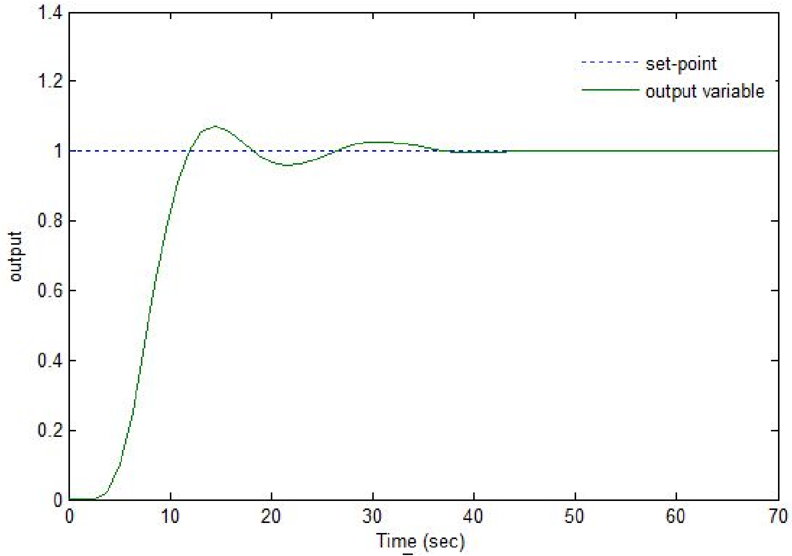

Fig. 6. Control performance for high order model process.

Example 3. Considerer a high order model

$$G(s) = \frac{1}{(s+1)(5s+1)^2} e^{2.5s} \qquad (19)$$

For this model the value of *apparent dead-time* of the process *L=5.48* was proposed.

Estimated model From *G(jw)* the design PID is

$$C(s) = (1.096 + \frac{0.108}{s} + 1.53s) \qquad (20)$$

Performance of the PID designed is shown in the Fig. 7

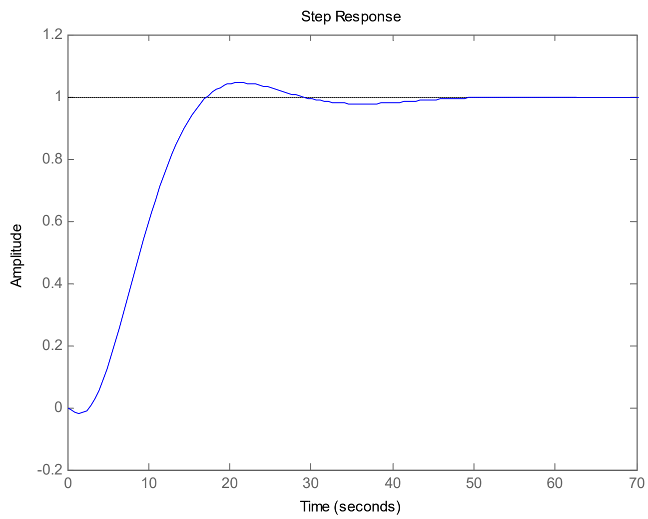

Fig. 7. Control performance for high order model process.

3.1. The sensitivity to modeling errors

Since the controller is tuned for a particular process, it is desirable that the closed loop system is not very sensitive to variations of the process dynamics. A convenient way to express the sensitivity of the closed loop system is through the sensitivity function *S(s), defined as:*

$$S(s) = \frac{1}{1+L(s)}$$

where *L(s)* denotes the loop transfer function [13, 22,14,16, 23]. *L (s)* is given by:

$$L(s) = C(s)G(s) = G(s)k\left\{1 + \frac{1}{T_i s} + T_d s\right\}$$

The maximum sensitivity (frequency response) is then given by $M_s = \max_{\omega} |S(i\omega)|$.

Therefore M_s is given by $M_s = \| S(s) \|_{\infty}$. On the other hand, it is known that the quantity M_s, is the inverse of the shortest distance from the Nyquist curve of loop transfer function to the critical point s=-1 [13]. Typical values of M_s are in the range from *1.2 to 2.0*.

Table 1 shows the values of M_s, A_m and Φ_m for the presented examples

Model	*Ms*	*Gain margin*	*Phase margin*
$\frac{1.25}{0.25s^2+.7s+1}e^{-.234s}$	1.685	3.13	59.4°
$\frac{1}{(s+1)^{10}}$	1.89	2.15	62.3°
$\frac{\mathbf{1}}{(\boldsymbol{s}+\mathbf{1})(\mathbf{5}\boldsymbol{s}+\mathbf{1})^2}\boldsymbol{e}^{\mathbf{2.5}\boldsymbol{s}}$	1.73	2.07	43°

Table 1. Values of *Ms,* A_m and Φ_m.

The operation of genetic algorithm was configured with the following parameter values:

- Population size: 100.
- Stochastic uniform Selection
- Crossover function: Scattered
- Mutation function: Gaussian
- Number of generation: 500
- Crossover probability: 0.8
- Mutation Probability: 0.09
- Elite count: 2

4. Conclusion

The genetic algorithm was an excellent tool to solve the optimization problem. The obtained results were more accurate from the identified points of $G(jw_i+\alpha)$ to $G(jw_i)$; It was due to the fact that using $G(jw_i+\alpha)$ is more direct than $G(jw_i)$. Nonlinear least squares method, was successfully applied in all cases to adjust the parameters values in order to reduce the sum of the squares of the errors between the function and the measured data points. It is remarkable to say that used method has a good performance to identify the proposed models: long dead time process, oscillatory process and high order model.

It is also important to mention that M_s value was always a referent in relation to a good performance of the designed PID's, especially at the relative stability; on the other hand, when the *Ms* Value is within the proposed range, this ensures that the controlled systems are insensitive to possible changes in plant models [1]. So it, the values of Gain Margin and Phase Margin were very close as expected.

On the other hand, with regard to the convergence of the genetic algorithm, it is known that in practice there is no way to know whether it has reached or not to the optimal solution (that applies any GA). A possible stopping criterion is the consecutive lack of new solutions that dominate the ones which are better up to the moment. If there is no progress after a certain number of iterations, it is reasonable to assume that the algorithm converged already, but obviously there is no guarantee of that. This is the handicap of heuristic strategies.

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