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AN EFFECTIVE ALGORITHM FOR RELIABILITY-BASED OPTIMIZATION OF STIFFENED MINDLIN PLATE

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Abstract. Nowadays, stiffened plates have been widely used in many branches of structural engineering such as aircraft, ships, bridges, buildings etc... In comparision with common bending plate structures, stiffened plates not only have larger bending stiffness but also use less amount of material. Hence, it usually has higher economic efficiency. However, to obtain high effectiveness in solving the design problems of the stiffened plate, the reliability-based optimization problems need to be established together with the ordinary numerical computing methods. Therefore, the paper presents an approach to establish and solve the reliability-based optimization problem for the stiffened Mindlin plate. To analyze the behavior of Mindlin plate, we use the recently proposed CS-DSG3 element. The random variables are chosen to be elastic modulus, density of mass and external force. The design variables are the thickness, the width and the height of the stiffened plate. The objective function can be the strain energy or the mass of the structure and subjected to the constraints of displacement or vibration frequency. The reliability-based optimization algorithm used in this paper is a three-step closed loop: 1) Estimating the random variables by the Reliability Index (RI) method; 2) Solving the optimization problem using Sequential Quadratic Programming (SQP) method; 3) Checking and estimating the reliability by the first-order reliability method (FORM) in which the limit state function is the limit of displacement or vibration frequency of the structure.

Keywords: Stiffened plate, reliability-based optimization, sequential quadratic programming - SQP, cell-based smoothed discrete shear gap method (CS-DSG3), reliability index method, first order reliability method.

1. INTRODUCTION

Together with many effective applications in many branches of structural engineering such as aircraft, ships, bridges, buildings etc..., the researches on numerical simulation of stiffened plate has been developed quickly, in which the analyses of static respond, free vibration and compressive stabilization play a major role. Some researches can be mentioned as: Bhimaraddi et al. [1] presented a method using FEM to analyze the static respond and free vibration of the ring-sectional plate stiffened with orthogonal beams; Mukhopadhyay [2] applied the Finite Difference Method to analyze the vibration and the stabilization of stiffened plate; Manoranjan Barik [3] used FEM for the static, dynamic and stable analysis of randomly stiffened plate; Bui Xuan Thang et al. [4] proposed a method using CS-DSG3 elements to solve the problem of static, dynamic and stable analysis of stiffened plate, etc...

Recently, together with the successive development of computer science and many effective optimization methods, the optimization problems of stiffened plates has been carried out and become one of the interesting research tendencies. Some noticeable works can be listed as: Ravi Bellur Ramaswamy [5] solved the design optimization problem of stiffened plate in which the objective function is the minimum mass of the structure under the respond constraints of frequency and ultimate load. Karoly Jarmai [6] presented a method for optimizing stiffened plates by Massonnet and Gience technique. In this work, the objective function is the minimum cost of the construction work, etc... However so far, the reliability has not been considered in optimization problems of stiffened plate. Therefore, in this paper, we attempt to contribute an approach to establish and solve the reliability-based optimization problem for stiffened Mindlin plate. To analyze the respond of Mindlin plate, we used the CS-DSG3 element recently proposed by Nguyen Thoi Trung et al. [7]. The reliability-based optimization algorithm used in the paper is a three-step closed loop, in which the random variables chosen are elastic modulus, density of mass and external force. The design variables are the thickness, the width and the height of the stiffened plate. The objective function can be the strain energy or the mass of the structure and subjected to the constraints of displacement or vibration frequency.

2. THEORETICAL FUNDAMENTAL

The reliability-based optimization problem can be defined by the following mathematical model:

$$\min_{\mathbf{x}} f(\mathbf{x}) \quad \text{s.t.} \quad \begin{cases} h_i(\mathbf{x}) = 0 & i = 1, ..., l. \\ g_j(\mathbf{x}) \leq 0 & j = 1, ..., m. \\ \beta_t \geq \beta & \end{cases} \tag{1}$$

where $\mathbf{x}$ is the vector of design variables such as material type, geometrical form, or size of the section, etc... ; $h_i(\mathbf{x}) = 0$ and $g_j(\mathbf{x}) \leq 0$ are inequality and equality constraints; l, m are the number of inequality and equality constraints, respectively; $\beta \geq \beta_t$ is the constraint on reliability; $f(\mathbf{x})$ is the objective function which can be the function of mass, cost or other characteristics of the structure.

The aim of this problem is to find the values of design variables in design space such that the objective function is minimum. In this paper, the Sequential Quadratic Programming method will be applied to solve the optimization problem to find out the suitable value of design variables.

2.1. Brief on the sequential quadratic programming method

Sequential Quadratic Programming Method proposed by Wilson [8] is one of many effective algorithms applied for solving nonlinear optimization problems. This algorithm gives out the solution with high convergence, reliable results and has been built in many optimization software such as: NPSOL, NLPQL, OPSYC, OPTIMA, MATLAB, etc... The main idea of SQP is presented as follows:

Consider the structural optimization problem with the objective function and constraints as follows

$$\min_{\mathbf{x}} f(\mathbf{x}) \quad \text{s.t.} \quad \begin{cases} h_i(\mathbf{x}) = 0 & i = 1, ..., l. \\ g_j(\mathbf{x}) \le 0 & j = 1, ..., m. \end{cases} \tag{2}$$

where $\mathbf{x} \in \mathbf{R}^n$, $f : \mathbf{R}^n \rightarrow \mathbf{R}$, $h : \mathbf{R}^n \rightarrow \mathbf{R}^l$, and $g : \mathbf{R}^n \rightarrow \mathbf{R}^m$ are second-order differentiable functions. At k-th iterate, the SQP algorithm will generate a search direction d_x by solving the Quadratic Programming subproblem as follows

$$\min_{d_x} \ \nabla f(\mathbf{x}_k)\mathbf{d}_x + \frac{1}{2}\mathbf{d}_x^T \mathbf{B}_k \mathbf{d}_x \quad \text{s.t} \quad \begin{cases} h_i(\mathbf{x}_k) + \nabla h_i(\mathbf{x}_k)\mathbf{d}_x = 0 & i = 1, ..., l. \\ g_j(\mathbf{x}_k) + \nabla g_j(\mathbf{x}_k)\mathbf{d}_x \le 0 & j = 1, ..., m. \end{cases} \tag{3}$$

where $\mathbf{B_k}$ is the positive definite Hessian matrix of the Lagrangian

$$L(\mathbf{x}, \lambda, \mu) = f(\mathbf{x}) + \sum_{i=1}^{l} \lambda_i^T h_i(\mathbf{x}) + \sum_{j=1}^{m} \mu_j^T g_j(\mathbf{x}) \tag{4}$$

Hessian matrix $\mathbf{B}_k$ will be updated throughout the optimization process by Broyden-Fletcher-Goldfarb-Shanno algorithm.

$$\mathbf{B}_{k+1} = \mathbf{B}_k + \frac{\hat{\mathbf{y}}_k \hat{\mathbf{y}}_k^T}{\hat{\mathbf{y}}_k^T \mathbf{d}_k} - \frac{\mathbf{B}_k \mathbf{d}_k (\mathbf{B}_k \mathbf{d}_k)^T}{\mathbf{d}_k^T \mathbf{B}_k \mathbf{d}_k} \tag{5}$$

where $\hat{\mathbf{y}}_k = t\mathbf{y}_k + (1-t)\mathbf{B}_k\mathbf{d}_k$ with $\mathbf{y}_k = \nabla \mathbf{L}(\mathbf{x}_{k+1}, \lambda_{k+1}, \mu_{k+1}) - \nabla \mathbf{L}(\mathbf{x}_k, \lambda_{k+1}, \mu_{k+1})$,

$$t \text{ is reduced factor, } t = \begin{cases} 1, \text{ if } \mathbf{d}_k^T \mathbf{y}_k > 0.2 \mathbf{d}_k^T \mathbf{B}_k \mathbf{d}_k \\ \dfrac{0.8 \mathbf{d}_k^T \mathbf{B}_k \mathbf{d}_k}{\mathbf{d}_k^T \mathbf{B}_k \mathbf{d}_k - \mathbf{d}_k^T \mathbf{y}_k}, \text{ otherwise} \end{cases}$$

so that $\mathbf{B}_k$ is always positive definite. $\mathbf{B}_k$, $\mathbf{d}_k$ and $\mathbf{y}_k$ must satisfy Newton equation:

$$\mathbf{B}_k \mathbf{d}_k^{\mathrm{T}} = \mathbf{y}_k \tag{6}$$

Optimization process will be executed at the next iterate with $\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{d}_k$, in which α_k is chosen such that merit function always decreases $\varphi(\mathbf{x}_k + \alpha_k \mathbf{d}_k) < \varphi(\mathbf{x}_k)$, where the merit function is expressed by

$$\varphi(\mathbf{x}) = f(\mathbf{x}) + r\left[\sum_{i=1}^{l} |h_i(\mathbf{x})| + \sum_{j=1}^{m} \max(0, g_j(\mathbf{x}))\right], \quad r > \max(\lambda_i, \boldsymbol{\mu}_j) \tag{7}$$

2.2. Reliability index method

The solution of the optimization problem without reliability always lies on the limit state surface between the safe and failure domains. As a consequence, the stability of the structure may not be ensured if the input data such as load or material parameters, etc... oscillates with certain distribution rules. To ensure the random variables oscillating in the safe domain, the Reliability Index (RI) method [9] was proposed with the aim of determining suitable values of the random variables before solving the optimization problem.

Specifically, the values of these random variables are determined by solving the problem of finding the minimizer of distance function as follows

$$\beta = \min \sqrt{\mathbf{u}^T\mathbf{u}} \quad \text{subject to} \quad \beta \geq \beta_t \tag{8}$$

where β_t is the objective reliability index and relates to the failure probability of the structure by the following formula

$$P_f \approx \varphi(-\beta_t) \quad \Rightarrow \quad \beta_t \approx \varphi^{-1} P_f, \tag{9}$$

where $\varphi(.)$ is the standard Guassian cumulated function. Depending on the requirement of the problem and the importance of the structure, the value of β_t will be determined specifically; $\mathbf{u}$ is the vector of normalized variables transformed from the vector of physical variables $\mathbf{y}$ as follows

$$u_i = (y_i - \mu_i)/\sigma_i; \tag{10}$$

where μ_i, σ_i are respectively the mean value and standard-deviation of random variables y_i.

2.3. First-order reliability method – FORM

Reliability analysis is a tool for computing the failure probability of the structure, in which all random variables related to design process are taken into account. In normalized space, the failure probability of the structure is calculated by

$$P_f = P\{g(\mathbf{U}) < 0\} = \int_{g(\mathbf{U})<0} \phi_{\mathbf{U}}(\mathbf{u})\mathrm{d}u \tag{11}$$

The direct evaluation of the integral (11) is not easy because the number of the random variables is large and the limit state function $g(\mathbf{U})$ and the density function of the random variables are usually high order nonlinear functions. Hence, many methods were proposed to the compute the reliability index instead of evaluating the integral (11). Among these methods, the First-Order Reliability Method (FORM) [10] is the most popular. The failure probability P_f then can be calculated easily from the RI by the relation (9). In the FORM, the limit state function $g(\mathbf{U})$ is approximated by a linear function based on first order Taylor series expansion as follows

$$g(\mathbf{U}) = g(\mathbf{u}^*) + \nabla g(\mathbf{u}^*)(\mathbf{U} - \mathbf{u}^*)^T; \tag{12}$$

where $\mathbf{u}^* = (u_1^*, u_2^*, ..., u_n^*)$ is the vector of the present values of the normalized variables u_i; $\nabla g(\mathbf{u}^*)$ is the gradient of $g(\mathbf{U})$ at $\mathbf{u}^*$, and determined by $\nabla g(\mathbf{u}^*) = \left.\left[\dfrac{\partial g}{\partial U_1}, \dfrac{\partial g}{\partial U_2}, ..., \dfrac{\partial g}{\partial U_i}\right]\right|_{\mathbf{u}^*}$.

In the FORM, one needs to search for the Most Probable Point (MPP) which has the minimum distance β from the origin of coordinate O to the limit state function $g(\mathbf{U})$ in normalized space. This problem can be illustrated in Fig. 1 and presented in the form of

$$\beta = \min \mathrm{d}(\mathbf{u}) = \sqrt{u_1^2 + u_2^2 + ... + u_n^2} \quad \text{subjected to} \quad g(\mathbf{u}) = 0; \tag{13}$$

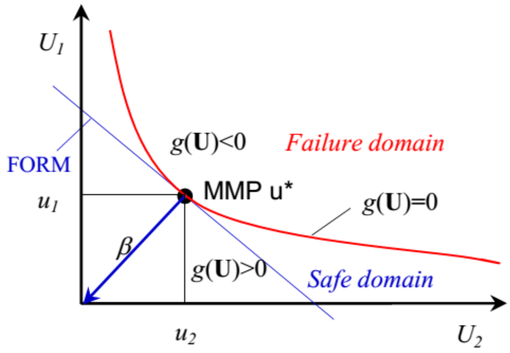

Fig. 1. Most Probable Point $\mathbf{u}^*$ in normalized space $\mathbf{U}$

where β is the reliability index used to compute failure probability of the structure by Eq. (9). The algorithms used to solve (13) can be found in the work of Hasofer and Lind [10]. After obtaining the failure probability, the reliability of the structure will be determined by the following formula

$$R = 1 - P_f = 1 - \Phi(-\beta) = \Phi(\beta) \tag{14}$$

2.4. A simple and effective reliability-based optimization algorithm

Combining the RI method with the SQP method and the FORM, we propose a three-step algorithm for the reliability-based design optimization problem as illustrated in Fig. 2. The content of each step is presented as follows:

+ Step 1: Give a reliability index and the values of the random variables for the optimization problem are determined by the RI method without using the limit state function.

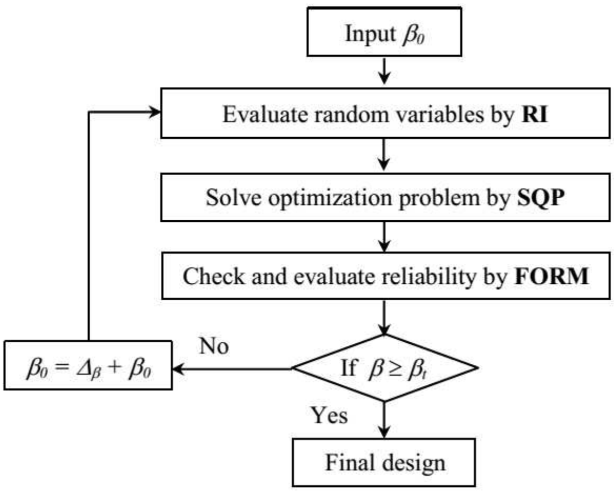

Fig. 2. A reliability-based optimization algorithm

+ Step 2: Using the set of the random variables obtained in step 1, we use the SQP method to solve the optimization problem to find the optimum solution of the design variables.

+ Step 3: From the optimum value of the design variables in step 2, we carry out checking and estimating the reliability level of the limit state functions by the FORM. These limit state functions are the constraints in step 2. If the reliability value satisfies the condition $\beta \geq \beta_t$, the loop stops. Otherwise, go back to step 1 and increase the given reliability value.

2.5. Brief on the behavior equation of Mindlin plate stiffened with Timoshenko beam

Stiffened plate can be seen as the combination between Mindlin elements and the stiffening Timoshenko beam elements, as illustrated in Fig. 3. The stiffening beam is set parallelly with the axes in the surface of plate and the centroid of beam has a distance e from the middle plane of plate. The plate-beam system is discretized by a set of node. The degree of freedom (DOF) of each node of the plate is $\mathbf{d} = [u, v, w, \beta_x, \beta_y]^T$, in which u, v, w are the displacements at the middle of the plate and β_x, β_y are the rotations around the y-axis and x-axis. The DOF of each node of the beam is $\mathbf{d}_s = [u_{sc}, v_{sc}, w_{sc}, \beta_{sx}, \beta_{sy}]^T$, in which u_{sc}, v_{sc}, w_{sc} are respectively the centroidal displacements of beam and are expressed by the middle surface displacements of plate as

$$u_{sc} = u + e\beta_x,\ v_{sc} = v + e\beta_y,\ w_{sc} = w, \tag{15}$$

and β_{sx}, β_{sy} are the rotations of beam around y-axis and x-axis.

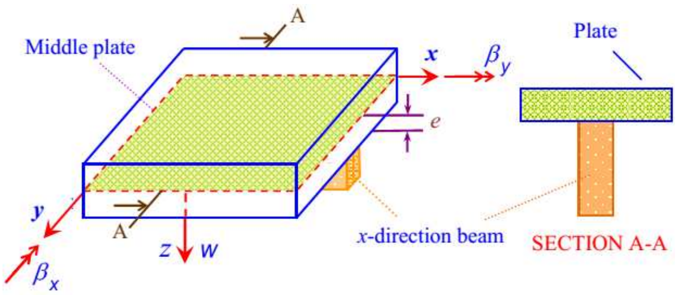

Fig. 3. A plate stiffened by an x-direction stiffener

To connect these DOFs of plate and beams, we used the compatible conditions on displacement proposed by Peng et al. [11] as follows

$$\mathbf{d}_s = \mathbf{T}\mathbf{d} \tag{16}$$

where $\mathbf{T} = 5N_{sn} \times 5N_{pn}$ is the transforming matrix of beam nodes and plate nodes; N_{pn}, N_{sn} are the total nodes of plate and beam, respectively.

2.5.1. *Energy equation of plates, beams and stiffened plates*

The strain energy of Mindlin plate is given as

$$U_P = \frac{1}{2}\iint\limits_A \boldsymbol{\varepsilon}_0^T \mathbf{D}^m \boldsymbol{\varepsilon}_0 dA + \frac{1}{2}\iint\limits_A \boldsymbol{\kappa}_b^T \mathbf{D}^b \boldsymbol{\kappa}_b dA + \frac{1}{2}\iint\limits_A \boldsymbol{\gamma}^T \mathbf{D}^s \boldsymbol{\gamma} dA, \tag{17}$$

where $\boldsymbol{\varepsilon}_0$, $\boldsymbol{\kappa}_b$, $\boldsymbol{\gamma}$ are respectively the membrane, bending and shear strains of plate, and are expressed as follows

$$\boldsymbol{\varepsilon}_0 = [u_{,x}, v_{,y}, u_{,y} + v_{,x}]^T; \boldsymbol{\kappa}_b = [\beta_{x,x}, \beta_{y,y}, \beta_{x,y} + \beta_{y,x}]^T; \boldsymbol{\gamma} = [w_{,x} + \beta_x, w_{,y} + \beta_y]^T. \tag{18}$$

$\mathbf{D}^m$, $\mathbf{D}^b$ and $\mathbf{D}^s$ are the material matrices involving with the strains of membrane, bending, and shearing components of the plate. The kinetic energy of Mindlin plate is given by the following formula

$$T_P = \frac{1}{2}\iint\limits_A \dot{\mathbf{u}}^T \mathbf{m}_P \dot{\mathbf{u}}\, dA, \tag{19}$$

where $\dot{\mathbf{u}}$ is the derivative with respect to time of the vector of displacement field and the matrix $\mathbf{m}_p$ has the form of

$$\mathbf{m}_p = \rho\, \mathrm{diag}(t, t, t, t^3/12, t^3/12), \tag{20}$$

where ρ is the density of plate. The strain and kinetic energy of beam are expressed by the centroidal displacements as

$$U_s = \frac{1}{2}\int_l \varepsilon_{st}^T \mathbf{D}^{st} \varepsilon_{st}^T dx; T_s = \frac{1}{2}\int_l \dot{\mathbf{u}}^T \mathbf{m}_{st} \dot{\mathbf{u}} dx, \tag{21}$$

where $\boldsymbol{\varepsilon}_{st} = \begin{bmatrix} u_{,x} + e\beta_{x,x} & \beta_{x,x} & w_{,x} + \beta_x & \beta_{y,x} \end{bmatrix}^T; \mathbf{D}^{st} = \mathrm{diag}(E_s A_s, E_s I_{sy}, kG_s A_s, G_s J_s)$ and

$$\mathbf{m}_{st} = \rho_s A_s \begin{bmatrix} 1 & 0 & 0 & e & 0 \\ 0 & 1 & 0 & 0 & e \\ 0 & 0 & 1 & 0 & 0 \\ e & 0 & 0 & e^2 + I_{sy}/A_s & 0 \\ 0 & e & 0 & 0 & e^2 + I_{sx}/A_s \end{bmatrix}; \tag{22}$$

where A_s is the sectional area of beam; J_s is the torsion moment; I_{sx}, I_{sy} are the second moments of the cross-sectional area of the stiffness about the axis parallel to the x-axis and y-axis.

Using the superposition principle, the total strain and kinetic energy of the stiffened plate are obtained by

$$U = U_P + \sum_{i=1}^{N_{stx}} U_{sx}^i + \sum_{i=1}^{N_{sty}} U_{sy}^i; \quad T = T_P + \sum_{i=1}^{N_{stx}} T_{sx}^i + \sum_{i=1}^{N_{sty}} T_{sy}^i \tag{23}$$

where N_{stx}, N_{sty} are the number of stiffness in x and y directions, respectively.

2.5.2. *Finite element method for stiffened plate problem*

FEM is applied independently for each element of plate and beam in stiffened plate. First, the displacement of plate is approximated by a three-node triangular element as

$$\mathbf{d} = \sum_{i=1}^{N_{Pn}} \mathbf{N}_i \mathbf{d}_i = \sum_{i=1}^{N_{Pn}} N_i(\mathbf{x}) \mathbf{I}_5 \mathbf{d}_i, \tag{24}$$

where $\mathbf{d}_i = [u_i, v_i, w_i, \beta_{xi}, \beta_{yi}]^T$ is the vector of displacement field of plate at i^{th} node. $N_i(\mathbf{x})$ is the linear shape function of triangular element. The displacement field of beam is approximated by a two-node bar element as

$$\mathbf{d} = \sum_{i=1}^{N_{sn}} \varphi_i \mathbf{d}_{si} = \sum_{i=1}^{N_{sn}} \varphi_i(x) \mathbf{I}_5 \mathbf{d}_{si}, \tag{25}$$

where $\mathbf{d}_{si} = [u_{si}, v_{si}, w_{si}, \beta_{xsi}, \beta_{ysi}]^T$ is the vector of displacement field of beam at node i-th; $\varphi_i(x)$ is the linear shape function of bar element. Substituting Eqs. (25) and (24) into Eq. (23), we obtain the total strain and kinetic energy of the stiffened plate as

$$U = \frac{1}{2}\mathbf{d}^T \mathbf{K} \mathbf{d}; \quad T = \frac{1}{2}\dot{\mathbf{d}}^T \mathbf{M} \dot{\mathbf{d}}, \tag{26}$$

where $\mathbf{K}$, $\mathbf{M}$ are respectively the stiffness matrix and mass matrix of the stiffened plate and computed by

$$\mathbf{K} = \mathbf{K}_P + \mathbf{T}_x^T \mathbf{K}_{sx} \mathbf{T}_x^+ \mathbf{T}_y^T \mathbf{K}_{sy} \mathbf{T}_y; \quad \mathbf{M} = \mathbf{M}_P + \mathbf{T}_x^T \mathbf{M}_{sx} \mathbf{T}_x^+ \mathbf{T}_y^T \mathbf{M}_{sy} \mathbf{T}_y, \tag{27}$$

where $\mathbf{T}_x, \mathbf{T}_y$ are transforming matrices and $\mathbf{K}_p, \mathbf{K}_{sx}, \mathbf{K}_{sy}, \mathbf{M}_p, \mathbf{M}_{sx}, \mathbf{M}_{sy}$ are respectively the global stiffness and mass matrices of plate and beam with respect to x and y directions and computed by

$$\mathbf{K}_P = \frac{1}{2}\iint_A \left(\mathbf{B}_m^T \mathbf{D}^m \mathbf{B}_m + \mathbf{B}_b^T \mathbf{D}^b \mathbf{B}_b + \mathbf{B}_s^T \mathbf{D}^s \mathbf{B}_s\right) dA; \mathbf{M}_P = \frac{1}{2}\iint_A \mathbf{N}^T \mathbf{m}_P \mathbf{N}\, dA \tag{28}$$

$$\mathbf{K}_{sti} = \frac{1}{2}\int_{l_i} \mathbf{B}_{st}^T \mathbf{D}^{st} \mathbf{B}_{st}\, dA; \quad \mathbf{M}_{sti} = \frac{1}{2}\int_{l_i} \varphi^T \mathbf{m}_{sti} \varphi dl, \quad i = x, y \tag{29}$$

For the problem of static analysis, we solve the following discretized equations

$$\mathbf{K}\mathbf{d} = \mathbf{f} \tag{30}$$

where $\mathbf{f}$ is the load vector, $\mathbf{d}$ is the vector of displacement.

For the problem of free vibration analysis, we solve the following system of equations

$$(\mathbf{M} - \omega^2 \mathbf{K})\mathbf{d} = 0, \tag{31}$$

where ω is the eigen-frequencies that need to be determined.

In this paper, we use CS-DSG3 element recently proposed by Nguyen Thoi Trung et al. [7] to analyze the behavior of Mindlin plate. According to this method, each three-node triangular elements will be divided into 3 sub-triangular elements and these sub-elements are connected to each other through the centroid of the triangle as illustrated in Fig. 4.

In each sub-element, the Discrete Shear Gap (DSG) method is applied to compute bending, shearing and membrane components. Then, the Cell-based smoothed technique

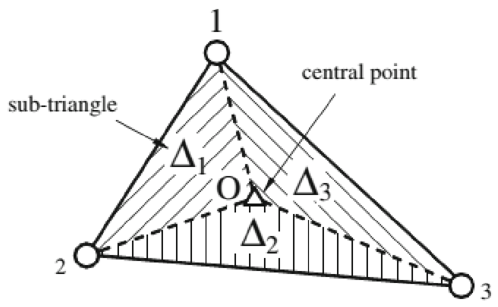

Fig. 4. Three sub-triangles $\Delta_1, \Delta_2, \Delta_3$ created from triangle 123 in the CS-DSG3

[12] will be used to smooth the bending, shearing and membrane strain fields of sub-triangles. The smoothed strain components on the element are then used to compute the bending, shearing and membrane stiffness matrices, respectively. And the remaining computing process is carried out as in the standard FEM.

3. NUMERICAL RESULTS

3.1. Static problem

Consider a Mindlin clamped plate stiffened with Timoshenko beam in two directions x, y and subjected to a uniformly distributed load of $q = 0.4$ (N/mm^2) as shown in Fig. 5a. The parameters for the problem are given as follows: The sizes of plate $L = 700$ (mm), $H = 1500$ (mm); Young's modulus $E = 2.06845 \times 10^5$ (N/mm^2); Possion's ratio $\nu = 0.3$.

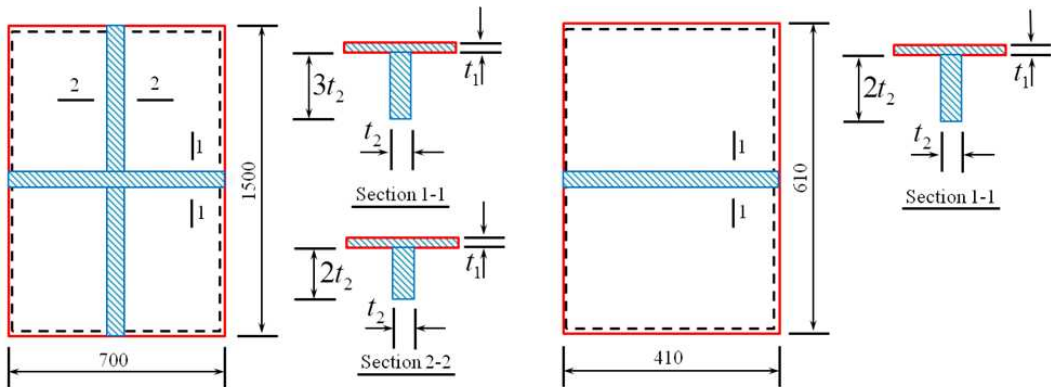

Fig. 5. a) Plate stiffened in two directions x, y; b) Plate stiffened in x-direction

The optimization problem is then established as

$$\min_{t_i} U(t_i) \text{ s.t.} \begin{cases} u(\mathbf{x}) \leq 6\,(\text{mm}); \beta(\mathbf{u}) \geq \beta_t; \beta_t = 2.3263 \\ t_i \in \mathbf{X} = \{t_i \in \mathbf{R}^n, 9 \leq t_1 \leq 15; 15 \leq t_2 \leq 25\,(\text{mm})\}, \; i = 1 \div 4 \\ t_3 = 3t_2; t_4 = 2t_2 \end{cases} \tag{32}$$

where U is the strain energy function; t_1, t_2 are respectively the design variables of the thickness of plate and the width of beam. t_3, t_4 are the height of beam along the directions of x and y; $u(\mathbf{x})$ is the displacement function of the structure; β_t is the objective reliability index corresponding to the given reliability index of 99%.

3.2. Dynamic problem

Consider a Reissner-Mindlin clamped plate stiffened with Timoshenko beam along x-direction, as shown in Fig. 5b. The parameters for the free vibration problem are given as: the sizes of plate $L = 0.41$, H= 0.61 (m); Young's modulus $E = 211 \times 109$ (kg/m^2); Possion's ratio $\nu = 0.3$, the density of plate $\rho = 7830$ (kg/m^3). The optimization problem is presented as

$$\min_{t_i} \; M(t_i) \quad \text{s.t.} \quad \begin{cases} \omega(\mathbf{x}) \leq 220; \; \beta(\mathbf{u}) \geq \beta_t; \; \beta_t = 2.3263 \\ t_i \in \mathbf{X} = \{t_i \in \mathbf{R}^n, 5 \leq t_1 \leq 10; \; 8 \leq t_2 \leq 20 \, (\text{mm})\}, i = 1 \div 3 \\ t_3 = 2t_2 \end{cases} \tag{33}$$

where M is the mass function of the structure; t_1, t_2, t_3 are design variables of thickness, width and height of beam along x-direction; $\omega(\mathbf{x})$ is the eigen-frequency function of the structure; β_t is the objective reliability index corresponding to the given reliability index of 99%.

The stiffened plates used in two above problems are simply supported plates, and are discretized by a mesh of $8 \times 8 \times 2$ three-node triangular elements.

3.3. Numerical results

The results of two problems obtained by the SQP and compared with Genetic Algorithm (GA - a global optimization method based on the laws of natural evolution) are presented in Tab. 1 for two cases: a) without considering the reliability index; and b) with reliability index

Table 1. Results for the two problems by the SQP and GA

Result	Problem	Method optimization	Strain Energy W(N.m)	Mass of structure M(kg)	Thickness of plate t_1(mm)	Width of beam t_2(mm)	Computational cost (seconds)
Without reliability	Static	SQP	457748	—	9	15	7
		GA	457750	—	9	15	8250
	Dynamic	SQP	—	10.43	0.5	1	9
		GA	—	10.47	0.51	1	6432
With the reliability of 98%	Static	SQP	579568	—	10.4	15	45
		GA	583230	—	10.37	15.1	15655
	Dynamic	SQP	—	12.93	0.5	1	30
		GA	—	12.93	0.5	1	7545

According to the results in Tab. 1, the solutions by the SQP agree very well with those by the GA. In addition, in comparison about the objective function value and computational cost, it is seen that the objective function values of problems by the SQP are smaller than that by GA, and the computational cost by the SQP is much less than that by the GA. These results hence illustrate the efficiency of the SQP compared with those of the GA.

Note that, for both problems, the values of the objective function of the problem with reliability are larger than those of the problem without the reliability. This reflects correctly the logic of the problems because the problem with reliability tends toward the safety of the structure. These results hence illustrate the efficiency of the proposed three-step algorithm for the reliability-based design optimization.

Also note that the SQP is a gradient-based optimization method and hence the process of finding the global minimum solution will depend strongly on the initial trial value of the solution, especially for the complicated optimization problems with many local extreme points and many design variables. For such complicated optimization problems, it is hence advised to combine the SQP with a global optimization method for finding the suitable initial trial value. This hence will increase significantly the computational cost of the SQP.

4. CONCLUSIONS AND REMARKS

The paper presents an approach to establish and solve two reliability-based design optimization problems for Mindlin plate stiffened by Timoshenko beams. In the first problem, static analysis is considered. The objective function is the minimum of the strain energy with constraints on displacement. In the second problem, free vibration analysis is considered. The objective function is the minimum of the mass of the structure under the constraints of eigen-frequencies. The design variables in two problems are the thickness of plate and the width, the height of stiffened beam. The random variables are chosen to be elastic modulus, density and external load.

A three-step reliability-based optimization algorithm was proposed including: 1) Estimating the random variables by the Reliability Index (RI) method; 2) Solving the optimization problem using Sequential Quadratic Programming (SQP) method; 3) Checking and estimating the reliability by the first-order reliability method (FORM).

Numerical results of two reliability-based design optimization problems for Mindlin plate stiffened by Timoshenko beams illustrated the efficiency of the SQP compared with those of the GA and the efficiency of the proposed three-step algorithm for the reliability-based design optimization.

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